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Mathematical cuneiform tablets in Philadelphia
Part 1: problems and calculations

Eleanor Robson
Wolfson College, Oxford
eleanor.robson@wolfson.ox.ac.uk

Introduction

The Babylonian Section of the University of Pennsylvania Museum of Archaeology and Anthropology, Philadelphia houses some thirty thousand ancient cuneiform tablets, around two hundred of which are mathematical in content. A few hundred tablets in the collection were bought from dealers or collectors, while others come from excavations at the southern Iraqi sites of Shuruppag, Ur, Tall-i Malyan, and Tell Billa, but the large majority were acquired through excavations at the ancient city of Nippur in the south of Iraq (Gerardi 1984: ix).[1] The Nippur expeditions were conducted in two phases—a pre-scientific series of digs before the First World War, and three controlled stratigraphic excavations conducted jointly with the Oriental Institute, University of Chicago, between 1948 and 1952.[2]

The finds from the early expeditions to Nippur were numerous, but sadly it was not considered necessary in those days to record the exact circumstances of the tablets found—texts were thought to be able to 'speak for themselves' (cf. Westenholz 1992: 292–3). So for most of the mathematical tablets in the museum's collections we know little more than they were found at Nippur, in the area dubbed 'Tablet Hill' due to the dense concentration of written artefacts found there (e.g. Peters 1897: II 418).[3] Epigraphic evidence dates the majority of them to the early second millennium BC, or Old Babylonian (OB) period. Under Turkish law, all the finds belonged to the Ottoman empire, and the American excavators were obliged to 'purchase such part of the objects found as may seem superfluous to the Imperial

[1] Zettler (1992) argues that the museum's very existence depends to a great extent on the University of Pennsylvania Expeditions to Babylonia's spectacular finds of tablets at Nippur in the 1890s.

[2] The University of Pennsylvania withdrew from the excavations after the third season, but the Chicago team continued work in conjunction with the Iraqi State Department of Antiquities at regular intervals until the outbreak of the Gulf War in 1990 (Bregstein and Schneider 1992: 365). For an overview of excavations at Nippur, see Zettler (1997).

[3] An account of the 1890s expeditions to Nippur is given by Kuklick (1996); see also Myer (1992); Westenholz (1992). There are first hand accounts by Peters (1897); Hilprecht (1903); Geere (1904); and Fisher (1907). See too the Nippur bibliography by Bregstein and Schneider (1992).

Museum if, after the price has been agreed upon, the supreme court of Bab el-Aali consent to the sale thereof' (Peters 1897: I 302). So most of the Nippur tablets were sent to Istanbul, and remain almost completely inaccessible to outsiders. A further fraction found its way to the University of Jena; and yet others have ended up in Israel. There are also Nippur tablets in Boston (Harvard) and New Haven (Yale).

Nevertheless, several thousand tablets from these early seasons have remained in Philadelphia, where they are catalogued in three series, with the following sigla:

- CBS 'Collection of the Babylonian Section', including purchased tablets not from Nippur, and some tablets excavated at Ur;
- N 'Nippur Collection';
- UM 'University Museum', where the first two digits give the year of cataloguing.

The division reflects museum cataloguing habits rather than circumstances of discovery, however, and it is common to find joins between tablet fragments across all three catalogue series and even across museum collections to:

- HS 'Hilprecht Sammlung', University of Jena;
- Ist Ni 'Istanbul Nippur' collection.

Where possible, I have included published Nippur tablets from Jena and Istanbul in the catalogue.

The post-World War II archaeological expeditions yielded fewer epigraphic finds, but were careful to make detailed records of their findspots. However, those records were never fully published, and it is necessary to refer to the handwritten field notebook held in Philadelphia to find exact information. Most of the mathematical tablets from these excavations come from the second and third seasons, especially from soundings TA and TB, which were residential areas of the Old Babylonian period (see McCown *et al.* 1967; Stone 1987). Frustratingly, the epigraphic finds were split between Chicago, Baghdad, and Philadelphia with little regard to their content. Where possible, I have endeavoured to record the published non-Philadelphia mathematical tablets from those seasons too.[4] The excavation numbers are of the form:

- 2N-T 'second season, Nippur tablets';
- 3N-T 'third season, Nippur tablets'.

Many, but not all, N-T tablets have museum numbers too:

- A Oriental Institute, Chicago;
- IM Iraq Museum, Baghdad;
- UM University Museum, Philadelphia.

[4] I am currently engaged in a fuller study of all the school tablets recovered from sites TA and TB in the second and third seasons of post-war excavations.

The Babylonian Section's mathematical holdings were catalogued by Abraham Sachs in the late 1940s and early 1950s. His card index can still be consulted in the Tablet Room, but all the information on it has since been incorporated into the Section's computerised catalogue. Neugebauer and Sachs published some of the Philadelpia mathematical tablets (*MCT*; Sachs 1947; Neugebauer and Sachs 1984[5]) and Anne Kilmer edited a very important tablet a decade later (Kilmer 1960). No work was done on the mathematics in the Babylonian Section for nearly 40 years thereafter.

I made my first brief trip to the Tablet Room in February 1997 to collate Kilmer's tablet while revising my D.Phil. thesis for publication. I also had time to start exploring the Section's other mathematical holdings, and some pieces were published in Robson (1999: 274–7). The tablets presented here were studied and copied in March and September 1998 and July 1999, on research trips to the Museum funded by Wolfson College, Oxford, and the British Academy. It is a particular pleasure to thank Linda Bregstein, Erle Leichty, Åke Sjöberg, Steve Tinney, Niek Veldhuis, and the other current and sometime denizens of the Tablet Room for their warm and generous hospitality, unfailing help, and good-natured bemusement at my interest in 'mathematical trash'.[6]

The University Museum's mathematical holdings are too large to be dealt with all at once, so this study is in three parts. This first part deals with the twenty-six tablets containing calculations, diagrams, questions and solutions, as well as the coefficient list; namely all the 'interesting', 'advanced' and 'active' mathematical documents. The remaining two parts will cover mathematics copied by trainee scribes as part of the rote-learning process (cf. Veldhuis 1997: 54–9): standard arithmetical tables in Part II (multiplications, reciprocals, squares and square roots, cubes and cube roots, etc.); and standard metrological lists and tables in Part III (lengths, areas, capacities, weights, with or without sexagesimal equivalents).

The following catalogue is in two parts. Table 1 lists the tablets in Philadelphia, while Table 2 shows the published tablets from Nippur which are housed in other museum collections. The tablets are catalogued by order of museum number or excavation number, with the fullest possible details of physical appearance, archaeological context, textual content, and previous publication. There follow drawings (copies) and editions of all sixteen previously unpublished texts, grouped by content; copies are also given of three tablets edited by Neugebauer and Sachs in transliteration only, as well as the reverse of a tablet whose obverse alone was published by Hilprecht. Comparisons are drawn with previously published material from Nippur and elsewhere, as appropriate.

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[5] This last article was submitted for publication in 1951.

[6] I would also like to thank the anonymous referee of this article, who made many useful suggestions for improving it. All errors and infelicities remain my own responsibility, of course.

Catalogue

Table 1: Mathematical tablets in Philadelphia

Museum No.	*Measurements*[7]	*Description and Publication*	*Date*	*Provenance*[8]	*No.*
CBS 43	5.0 × 7.0 × 2.0	Rectangular tablet, portrait orientation. Obverse 8 lines ruled into 4 problems; reverse 4 lines ruled into 2 problems. Six statements of problems about squares; related to CBS 154+921. Previously unpublished.	OB	Unprovenanced. Purchased from J. Shemtob in 1888.	17
CBS 154+921	5.5 × 7.0 × 2.0	Rectangular tablet, portrait orientation; top right corner and lower edge missing. Obverse 8 lines ruled into 4 problems; reverse 2 lines. Statements of problems about squares; related to CBS 43. Previously unpublished.	OB	Unprovenanced. Purchased from J. Shemtob in 1888.	18
CBS 165	5.0* × 6.0* × 1.5*	Surface flake; 9 ruled lines. List of at least 9 problems about rectangles. Previously unpublished.	OB	Unprovenanced. Purchased from J. Shemtob in 1888.	19
CBS 1215	7.0 × 10.5 × 2.0*	Rectangular tablet, portrait orientation. Bottom right corner missing with much surface damage. 3 columns of up to 40 lines on each side, with rulings between calculations. Originally contained 21 calculations of regular reciprocals. Edition: Sachs (1947: 230, 237–40); cf. *MCT*: 36 ('numbers written in a disorderly manner within square fields'). Copy previously unpublished.	OB	Unprovenanced. Khabaza collection 1895.	3

[7] Measurements are given to the nearest 5 mm in the order width—length—thickness.

[8] Quoted from the UM catalogue. The designation of the mounds at Nippur is often contradictory, due to Hilprecht's *post facto* desire to attribute all the finds to the 'Temple Library' on 'Tablet Hill': his 'mound IV', but Peters' 'mound V'. For a numbered map of the mounds and Peters' accusations against Hilprecht, see Peters (1905), with the map on page 146. Hilprecht's riposte is Hilprecht (1908). See Kuklick (1996: 123–40).

Museum No.	*Measurements*	*Description and Publication*	*Date*	*Provenance*	*No.*
CBS 3551	8.5 × 7.5 × 3.5	Rectangular tablet, landscape orientation. Sharp edges and corners; bottom right corner missing. Obverse 3 lines; reverse blank. Calculation of a square. Edition: *MCT*: 36. Copy previously unpublished.	OB	Nippur mound II, 'W side of Shatt en-Nil'.	4
CBS 7265	7.5 × 10.0 × 2.5	Incomplete rectangular tablet, portrait orientation. Obverse 3 lines; reverse blank. Calculation of a square. Previously unpublished.	OB	Nippur, 'Babylonian expedition'.	5
CBS 10201	8.0 × 6.0 × 2.0	Complete rectangular tablet, portrait orientation. Obverse 7 double-ruled lines; reverse 9. Eight calculations of regular reciprocals. Copy and photo: Hilprecht (1906: no. 25, pl. 15, pl. IX); editions: Sachs (1947: 225); Friberg (1983: 82).	OB	Nippur mound III, 'Temple library'.	—
CBS 10996	15.0 × 11.5* × 2.0	Three-column tablet, landscape orientation? Obverse 3 badly damaged columns; reverse 1 column preserved. Coefficient list with around 90 entries preserved in cols. II, III, and VI. Col. I contains musical instructions. Photo and edition: Kilmer (1960: text B); edition: Robson (1999: list B).	post-OB[9]	Nippur 'west side of Shatt en-Nil', 2nd Expedition (1895).	—
CBS 11318	6.5 × 6.5 × 2.5	Damaged square tablet; obverse 3 + 4 lines; reverse blank. Problem and calculation of a square. Copy and edition: Neugebauer and Sachs (1984: 246–8, 251).	OB	Nippur mound I.	—
CBS 11601	7.5 × 9.0 × 3.0	Rectangular tablet, portrait orientation. Some chips and surface damage. Remains of $5^?$ lines, ruled into $3^?$ columns, on both obverse and reverse. Badly preserved calculations. Previously unpublished.	OB	Nippur mound II, 'tablet hill'.	9

[9] 'Kassite' (Kilmer 1960: 273; Duchesne-Guillemin 1984: 6); 'MB' [= Middle Babylonian] (University Museum catalogue), but see Kilmer (1980–3: 575; 1992: 101): 'NB' [= Neo-Babylonian].

Museum No.	*Measurements*	*Description and Publication*	*Date*	*Provenance*	*No.*
CBS 11681	6.5 × 5.0* × 2.0	Rectangular tablet, landscape orientation; upper edge missing. Obverse 10 lines, reverse 7 lines, left edge 2 lines. Two problems on volume and sides of a cube. Previously unpublished.	OB	Nippur 2nd Expedition.	15
CBS 12648	7.0* × 11.0* × 2.5	Bottom left corner of two-column tablet, portrait orientation. Obverse 2 columns of 20 and 16 lines; reverse 15 lines. Remains of problems on volumes. Copy of obverse: Hilprecht (1906: no. 25a); editions: *MKT*: I 234-5; Muroi (1988). Reverse previously unpublished.	OB	Nippur 'East side of Shatt en-Nil', 2nd Expedition (1895).	14
CBS 19761	8.0* × 7.5* × 2.5	Top right corner of two-column tablet, portrait orientation. Obverse 2 columns of 11 and 12 lines; reverse 1 column of 6 lines, final column blank. Incompletely preserved problem(s) about squares. Previously unpublished.	OB	Nippur 4th Expedition, 'Temple library, mound IV' (1899).	16
IM 58446 + 58447 = 3N-T 362 + 366 (UM cast)	*ca.* 7.0 × 12.0	Casts of upper and lower halves of rectangular tablet; portrait orientation. Obverse 19 lines; reverse 3 + 7 lines. Extract from Sumerian literary composition Eduba C; calculation of regular reciprocal. Previously unpublished.	OB	Nippur. Locus TA 205 level X.2/XI.1 (3N-T 362) and X.3/XI.2 (3N-T 366) (1951).	2
N 837	6.0 × 7.5 × 2.5	Complete rectangular tablet with square edges and corners; portrait orientation. Obverse 3 lines over erasures; reverse blank. Calculation. Previously unpublished.	OB	Nippur	8
N 2873	3.0* × 5.5* × 1.5*	Surface flake. 2 columns of 12 lines, the second with 3 sub-columns. Incompletely preserved problems and calculations. Previously unpublished.	post-OB	Nippur	20

Museum No.	*Measurements*	*Description and Publication*	*Date*	*Provenance*	*No.*
N 3891	5.5* × 6.0 × 2.5	Incomplete square tablet. Obverse 1 line followed by double ruling; reverse 3 lines. Calculation of a regular reciprocal. Copy and edition: Sachs (1947: 234).	OB	Nippur	—
N 3914	7.0 × 7.0 × 2.0	Square tablet. Bottom left corner missing; much surface damage. Obverse 4 columns of 11 lines; reverse blank. Tabular calculations. Previously unpublished.	OB	Nippur	10
N 3958	6.5 × 9.5* × 3.0	Incomplete rectangular tablet; portrait orientation. Obverse 12 ruled lines; reverse 15. Originally contained at least 34 doublings of 2 05. Copy and edition: Sachs (1947: 229 (sic: 228)).	OB	Nippur	—
N 3971	5.5 × 5.5 × 3.0	Complete square tablet. Obverse 3 lines; reverse blank. Calculation of a square. Copy and edition: Robson (1999: 275).	OB	Nippur	—
N 4281	4.0* × 4.0* × 2.5*	Bottom right corner of multi-column tablet. Obverse 2 columns of 11 and 9 lines; reverse 5 lines. Multiplication table and calculations. Copy and edition: Robson (1999: 276).	OB	Nippur	—
N 4942	6.0 × 7.5 × 2.5	Incomplete rectangular tablet, portait orientation; left edge and bottom right corner missing. Obverse numbered diagram of a semicircle; reverse blank. Previously unpublished.	OB	Nippur	11
UM 29-13-021	8.0 × 4.5* × 2.0*	Lower third of multi-column tablet. Obverse 2 columns of 8 and 9 lines; reverse 3 columns of 10, 7, and 10 lines. Calculations of regular reciprocals. Edition and photo: *MCT*: 13–15, pl. 24; editions: Sachs (1947: 227); Friberg (1983: 83).	OB	Nippur	—

Museum No.	*Measurements*	*Description and Publication*	*Date*	*Provenance*	*No.*
UM 29-13-173	6.0* × 6.0* × 2.0	Roughly square tablet; right and bottom edges missing. Obverse 9 lines, reverse 7 lines. Calculations of quadrilateral areas? Previously unpublished.	OB	Nippur	13
UM 29-15-192	6.0 × 6.0 × 2.0	Damaged square tablet. Obverse 3 +4 lines; reverse blank. Problem and calculation of a square. Copy and edition: Neugebauer and Sachs (1984: 246, 248, 251).	OB	Nippur	—
UM 29-15-709	6.0 × 8.0 × 3.0	Rectangular tablet; landscape orientation. Sharp edges and corners. Numbered diagram of triangle on obverse and reverse. Previously unpublished.	OB	Nippur	12
UM 29-16-401	8.0 × 6.5 × 2.5	Rectangular tablet; landscape orientation. Bottom right corner missing. Obverse 3 lines; upper edge 1 line; reverse blank. Calculation of a square. Previously unpublished.	OB	Nippur	6
UM 29-16-504	6.5 × 7.5 × 2.5	Damaged square tablet. Obverse 3 lines; reverse 3 lines. Calculations. Copy and edition: Robson (1999: 276–7).	OB	Nippur	—
UM 55-21-076 = 2N-T 472	5.5 × 4.0* × 2.5	Lower half of square tablet. Obverse 5 ruled lines; reverse blank. Problem and calculation of a square area. Edition: Neugebauer and Sachs (1984: 246–7). Copy previously unpublished.	OB	Nippur. Locus TB 51 level E.2 (1949).	7
UM 55-21-357 = 3N-T 605	6.5 × 7.5 × 2.5	Complete rectangular tablet; portrait orientation. Obverse 2 ruled lines; reverse blank. Calculation of regular reciprocal; reverse blank. Previously unpublished.	OB	Nippur. Locus TA 205 level X.3/XI.2 (1951).	1

Table 2: Published mathematical tablets from Nippur in other collections

Museum No.	*Description and Publication*	*Date*	*Provenance*
A 29985 = 2 N-T 500	Broken rectangular tablet; portrait orientation. Obverse Sumerian proverb SP 2.52; reverse calculation of regular reciprocal. Photo: Gordon (1959: no. XXX); no copy published.	OB	Nippur. Locus TB 10, level II.2 (1949).
HS 231	Complete rectangular tablet; landscape orientation. Calculation of a square; reverse blank? Copy and edition: Friberg (1983; 83).	OB	Nippur
HS 232	Damaged square tablet. Calculations; reverse blank? Copy and edition: Friberg (1983; 82).	OB	Nippur
HS 245	Formerly HS 229. Recent editions: Rochberg-Halton (1983); Horowitz (1993); no copy published.	post-OB	Nippur
IM 57828 = 2 N-T 30	Damaged square tablet. Problem and calculation of a square. Photo: Steele (1951: pl. 7); edition: Neugebauer and Sachs (1984: 246–8); no copy published.	OB	Nippur. Locus TB 12, level II.1 (1949).
IM 57845 = 2 N-T 115	Fragment of tablet. Obverse: calculation of regular reciprocal and Sumerian proverb?; reverse: multiplication table. Edition: Neugebauer and Sachs (1984: 244–5); no copy published.	OB	Nippur. Locus TB 45, level II.1 (1949).
IM 57846 = 2 N-T 116	Problem and calculation of a square. Edition: Neugebauer and Sachs (1984: 246–7); no copy published.	OB	Nippur. Locus TB 45, level II.1 (1949).
IM 57863 = 2 N-T 174	Damaged square tablet. Calculation; reverse blank? Copy and edition: Neugebauer and Sachs (1984: 245, 250).	OB	Nippur locus TB 20 (50?), level E (1949).
IM 58966 = 2 N-T 496	Complete rectangular tablet; portrait orientation. Obverse Sumerian proverb, cf. SP 2.42; reverse calculation of regular reciprocal. Copy: Al-Fouadi (1979: no. 134).	OB	Nippur locus TB 10, level II.2 (1949).
Ist Ni 2265	Complete circular tablet. Calculation; reverse blank. Copy by H. V. Hilprecht, to be published by N. Veldhuis.	OB	Nippur 'Tablet Hill' 4th expedition (1889).

Copies and Editions

Tablet 1: UM 55-21-357 = 3N-T 605

This is a student's failed attempt at finding a reciprocal: the scribe has simply halved the starting number. The double ruling under the second line indicates that the scribe considered the exercise finished (Figure 1).

4 26 40	4 26 40:
IGI.BI 2 13 20	its reciprocal is 2 13 20

The correct method, exemplified on the similarly shaped 2N-T 500 (see Table 2), is to use the procedure which Sachs (1947: 222) dubbed 'the Technique'. The reverse of 2N-T 500 reads:

17 46 40	17 46 40:
⌜IGI⌝.BI 3 22 30	its reciprocal is 3 22 30

and underneath a double horizontal rule are the numbers:

17 46 40 [9]
2 40 22 30
3 22 30

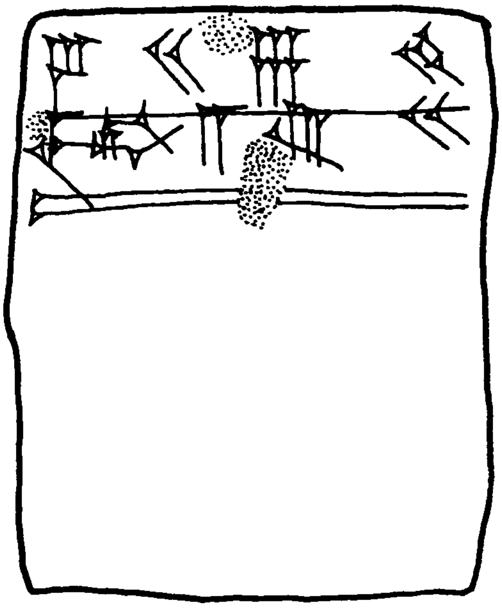


Figure 1: UM 55-21-357 = 3N-T 605

On the obverse, beneath a two-line proverb in Sumerian written first by the teacher then copied by the student (SP 2.52, cf. Alster 1997: 55), the calculation is partially repeated:[10]

dub-sar ⌜lú] [gu-ra-aḫ]	A [chattering] scribe,
nam-tag-ga-ni ab-⌜gu⌝-[ul]	his guilt is very great.

17 46 40 9
1 30

The procedure begins by breaking the given number into two strings of digits so that the second string is a two-place integer found in the standard table of reciprocals; its inverse is written to the right of the complete number. The scribe of 2N-T 500 chose to split 17;46 40 into 17;40 and 0;06 40. Following normal practice, he wrote the complete number and 9—the reciprocal of 0;06 40—to the right (now missing). The next step is to multiply that reciprocal with the first string, and add 1: here 17;40 × 9 + 1 = 2 39 + 1 = 2 40. This result is given on the left of the second line. This reciprocal is found (or a second iteration begins): here, although 2 40 is not in the standard table its half 1 20 is, so that the reciprocal 0;00 22 30 (on the right of

[10] Many of the calculations from Ur are also written on the back of tablets with single proverbs on (cf. Robson 1999: 245–77).

the second line) can be found immediately by halving 0;00 45—the reciprocal of 1 20. Multiplication by 9, the reciprocal of the small part, yields 0;03 22 30 on the bottom line—which is the reciprocal of the original number, 17 46 40. The entire procedure amounts to:

$$\frac{1}{a+b}=\frac{1}{a}\times\frac{1}{(1+b/a)}$$

in symbolic algebra.

Explicit instructions for using 'the Technique' are given on the unprovenanced OB tablet VAT 6505 (*MKT*: I 270–3; Sachs 1947: 226–7),[11] and many OB examples of its use are now known (see Table 3).

Tablet	*Provenance*	*Publication*
3N-T 362 + 366	Nippur	Tablet 2 below
BM 80150	Sippar?	*MKT*: I 49–50, II 61; Sachs (1947: 226); Nissen *et al.* (1993: 143–5, fig. 124).
CBS 1215	Unknown	Tablet 3 below
CBS 10201	Nippur	See Table 1
MLC 651	Unknown	Sachs (1947: 233)
N 3891	Nippur	See Table 1
UET 6/2 295	Ur	Robson (1999: 250)
VAT 5457	Unknown	*MCT*: 16; Sachs (1947: 234); Friberg (1983: 83)
YBC 1839	Unknown	Sachs (1947: 232)
YBC 10802	Unknown	*MCT*: 35; Sachs (1947: 233)

Table 3: OB calculations of regular reciprocals

The scribe of 2N-T 496 (see Table 2), from the same findspot as 2N-T 500, was able to solve his reciprocal without recourse to 'the Technique'; like his counterpart, he also had to copy a proverb about scribes on the obverse of his tablet underneath his teacher's version of it (cf. SP 2.42, Alster 1997: 304):

dub-sar ḫu-ru	A foolish scribe,
a-ga-aš-gi$_4$ gi$_4$-me-aš-e-ne	The most backward among his colleagues.

[11] The method of Str 366 (ii) from Uruk (*MKT*: I 257–9; Sachs 1947: 235), on the other hand, consists of dividing the (regular) number given by a (regular) factor so that a number in the standard reciprocal table is reached. Its reciprocal is then multiplied by those same factors to produce the reciprocal of the original number.

[16 40]	[16;40:]
IGI.BI 3 36	its reciprocal is 0;03 36.

3N-T 605 (Tablet 1) is from one of the largest documented finds of Old Babylonian school tablets. Level XI of House F (comprising loci 184, 189, 191, 192, 203 and 205) from Area TA in Nippur yielded some 1400 tablets and fragments, many if not all of which had been re-used as building material during minor alterations to that house. Fittings and furniture found there make it almost certain that the building was used as a scribal school (cf. Stone 1987: 56–59).

Tablet 2: IM 58446+58447 = 3N-T 362+366 (UM cast)

Figure 2: IM 58446+58447 = 3N-T 362+366 reverse (obverse not copied)

This rectangular tablet broke in two in antiquity; on excavation casts were made of the two separate fragments (Figure 2). The obverse and first three lines of the reverse contain lines 1–22 of the Sumerian school composition 'Advice of a Supervisor to a Young Scribe' (Eduba C) (Vanstiphout 1997: 590–2). Under the double ruling at the end of the literary extract, the 'Technique' is applied, first to 17 46 40 and then to its reciprocal 3 22 30:

17 46 40	9
2 40	«2» 22 30
3 22 30	[2]
6 4[5]	
9	⌜6 40⌝
8 53 20	
17 46 40	

The two fragments 3N-T 362+366 were, like 3N-T 605, found in House F at Nippur.

Tablet 3: CBS 1215

This tablet was edited by Sachs (1947), but a copy is presented here for the first time, with all of the unplaced fragments (cf. Sachs 1947: 225) joined by the author in 1998 (Figure 3). Reciprocals of successive doublings of 2 05 are found, using 'the Technique' (cf. 3N-T 605 and 3N-T 362+366, Tablets 1 and 2 above). All the reciprocal pairs discussed above are also from this same sequence.

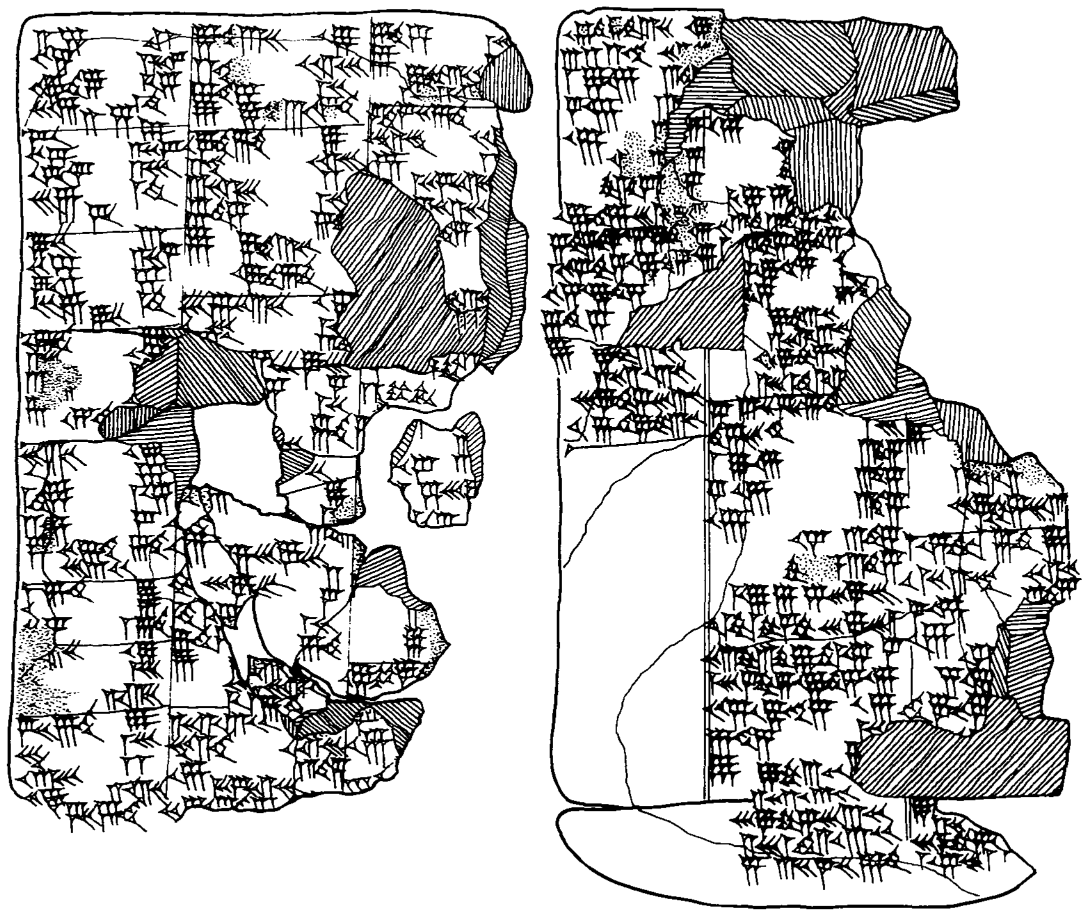


Figure 3: CBS 1215 obverse, reverse and lower edge (96% actual size)

Tablet 4: CBS 3551

This tablet displays a squaring calculation, laid out in the same manner as the Nippur tablets CBS 7265 and UM 29-16-491 (Tablets 5 and 6 below), HS 232 (see Table 2) and N 3971 (see Table 1), and the Ur tablets *UET* 6/2 211 and 311 (Robson 1999: 251). In the second sexagesimal place of the answer, the scribe has inadvertently repeated the 7 from the first place, instead of correctly writing a 5 (Figure 4).

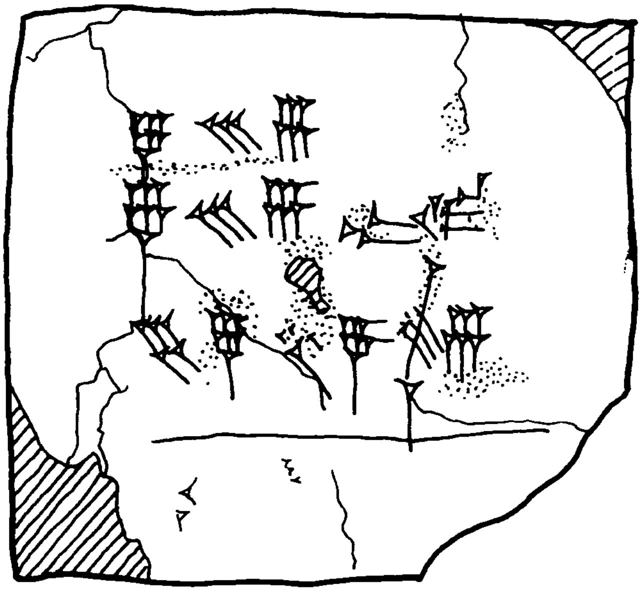


Figure 4: CBS 3551 obverse (reverse blank)

7 36	7;36
7 36 ⌜ÍB.SI$_8$⌝	× 7;36 square
57 ⌜41⌝ 36	= 57;45! 36

The tablet is unusually solid and square-edged, like N 837 and UM 29-15-709 (Tablets 8 and 12 below), UM 29-16-504 (see Table 1), and possibly HS 232 (see Table 2).

Tablet 5: CBS 7265

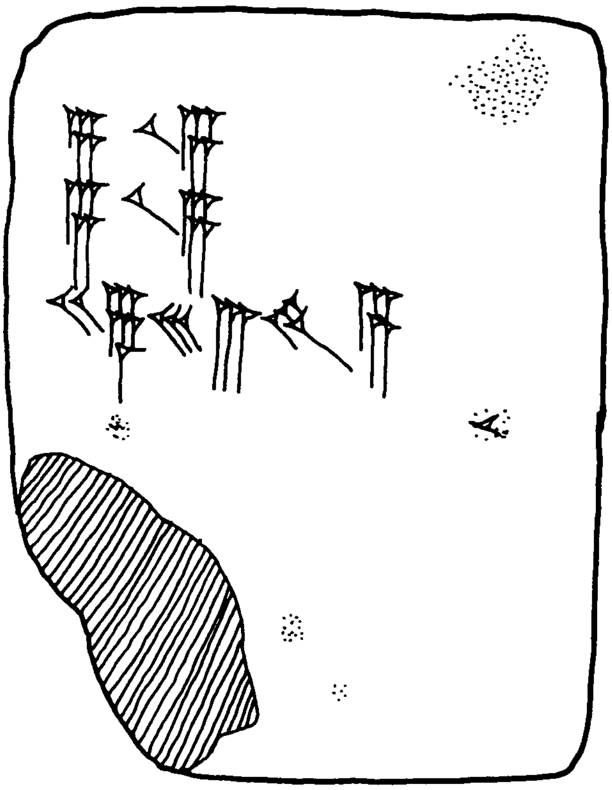


Figure 5: CBS 7265 obverse (reverse blank)

Correct squaring of a two-place number in the standard Nippur/Ur layout (Figure 5).

5 15	5;15
5 15	× 5;15
27 33 45	= 27;33 45

Tablet 6: UM 29-16-401

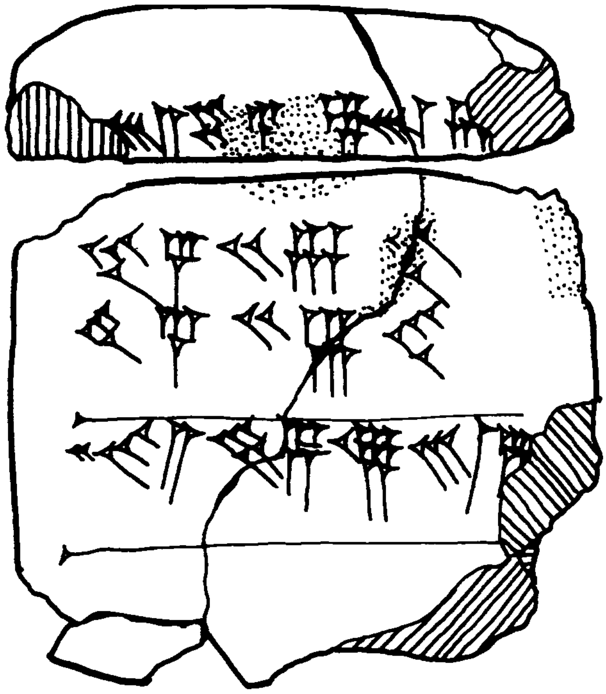


Figure 6: UM 29-16-401
upper edge and obverse (reverse blank)

Correct squaring of a two-place number in the standard Nippur/Ur layout, but with two additional features: horizonal rulings above and below the result on the obverse; and that result written out again in slightly neater writing on the upper edge of the tablet (Figure 6).

32 ⸢55 18⸣ 31 06 [40]

44 26 40
44 26 40
32 55 18 31 06 [40]

32 55;18 31 06 [40]
44;26 40
× 44;26 40
= 33 55;18 31 06 [40]

Tablet 7: UM 55-21-076 = 2N-T 472

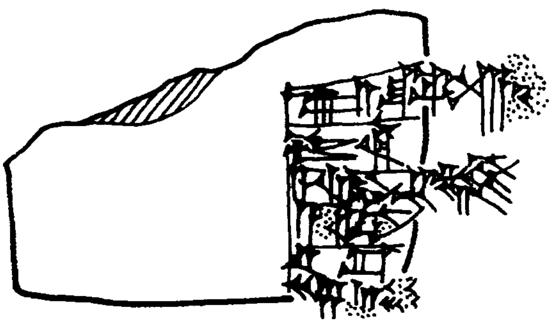


Figure 7: UM 55-21-076 = 2N-T 472
obverse (reverse blank)

This tablet was edited without illustrations by Neugebauer and Sachs (1984: 246–7); a copy is given here for completeness (Figure 7). The missing upper portion of the tablet presumably contained the squaring:

5 20	0;05 20
5 20	× 0;05 20
28 26 40	= 0;00 28 26 40

which is the sexagesimal calculation corresponding to the statement and answer on the extant portion of the tablet:

1 KÙŠ 2 ŠU.SI.TA.ÀM	1 cubit 2 fingers each
ÍB.SI$_8$	are the square-sides.
A.ŠÀ.BI EN.NAM	What is its area?

A.ŠÀ.BI	Its area is
$\frac{1}{3}$ GÍN 25$\frac{2}{3}$ ŠE	$^{1}/_{3}$ shekel, 25 $^{1}/_{3}$! grains.

2N-T 472 was one of ten tablets of various sorts found in House A (loci 50, 51, 60, and 61) of Area TB levels I–II. Most of the 2N-T mathematical tablets, though, come from House B (loci 10, 12, 17, 31, 45, 52, and 59), where some fifty tablets were recovered from levels II.1–2 (cf. Stone 1987: 84–5).

Tablet 8: N 837

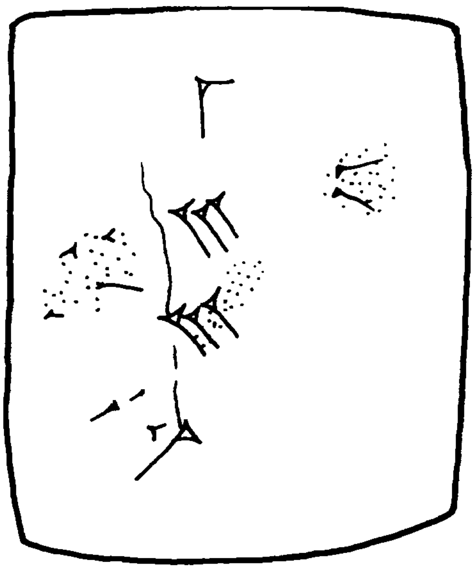


Figure 8: N 837 obverse (reverse blank)

This may be a simple multiplication, or just jottings (Figure 8). (For a multiplication, we might expect the multiplicands to be written to the left of a vertical ruling, in vertical alignment, with their product to the right; cf. the twenty multiplications from OB Ur (Robson 1999: 246).)

1	1
30	× 30
30	= 30

Tablet 9: CBS 11601

Very little comprehensible survives on either side of this tablet (Figure 9), but there is enough to suggest that these are perhaps the remains of two copies of a single tabular calculation or series of multiplications. One-place reciprocal pairs (20, 3) and (10, 6) are identifiable to the right of each side. Compare the calculations on HS 231 and Ist Ni 2265 (see Table 2).

obv.	15	1	⸢10?⸣[12]	rev.			⸢3 45⸣
	1⸢5?⸣	10?	5 ⸢3? 45?⸣		15	traces	20 3
	1	traces	20 3		1		
	⸢10?⸣	traces			1		10 6
	⸢30⸣		10 6		2	30	

[12] The apparent 1 here is in fact the top of the vertical column ruling.

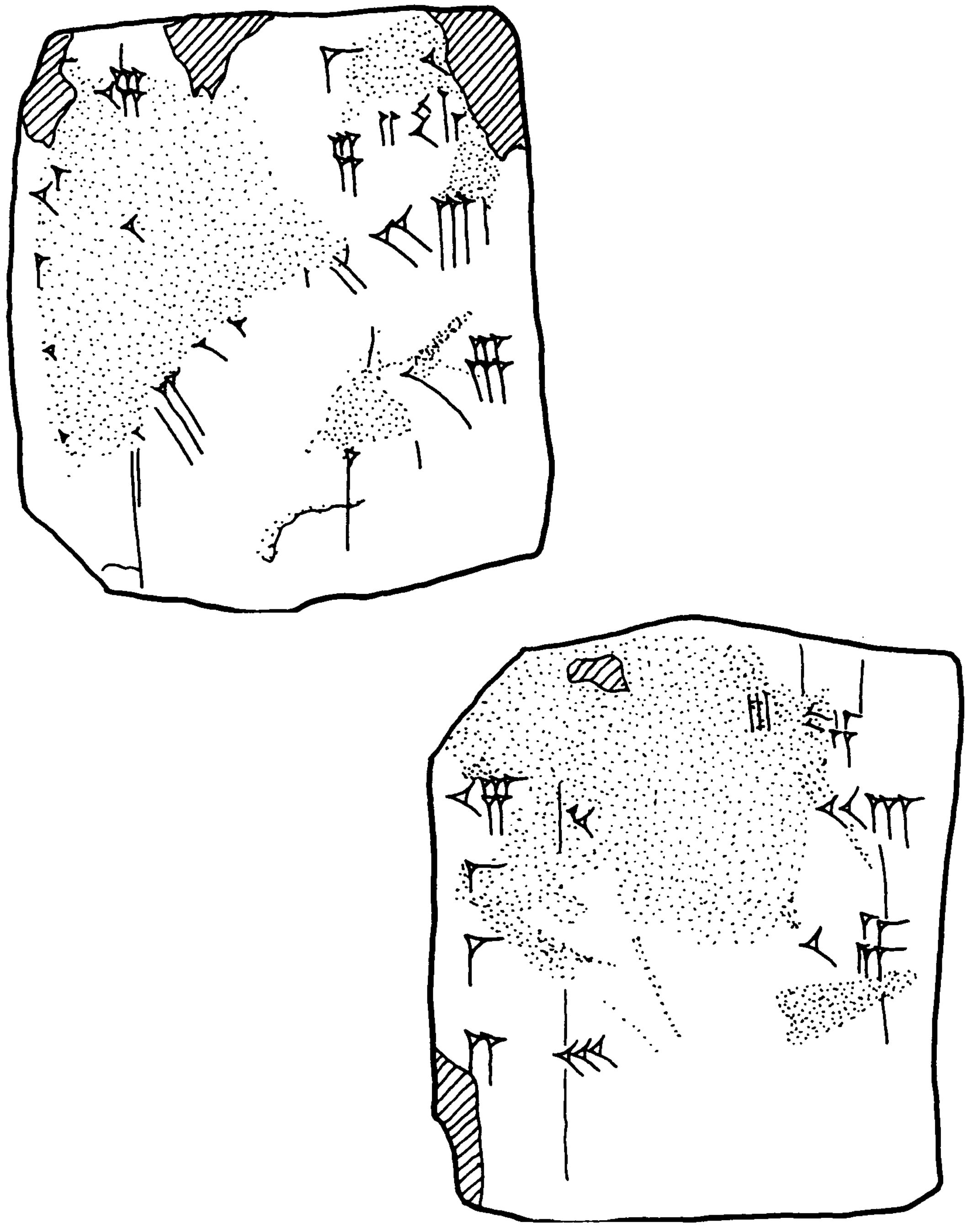


Figure 9: CBS 11601 obverse and reverse

Tablet 10: N 3914

This tablet contains caclulations laid out in a tabular format (Figure 10).The first column of the main table is a simple count 1–10. In the second column, the number 7 is divided by each of the Column I numbers in turn, and this result is multiplied by

10 in the third. The fourth and final column, which is blank from line 6 onwards, contains the products of Column I and Column III. The number at the top of Column III is the sum of the numbers in that column.

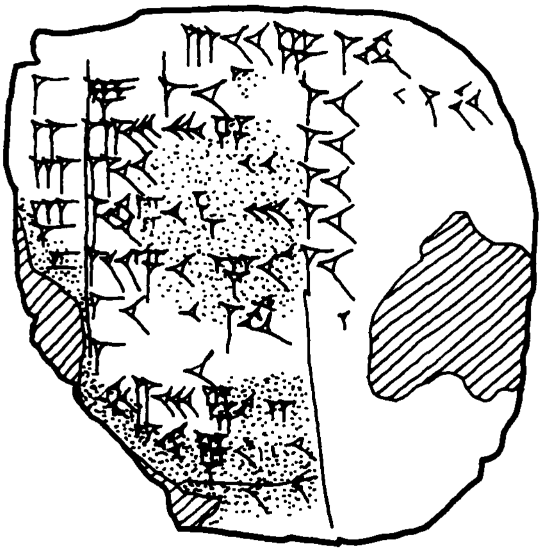


Figure 10: N 3914 obverse (reverse blank)

		3 25 01 40		
1	7	1 10	1 10	1 40
2	3 30	35	1 10	
3	2 20	[23] 20	1 10	
4	1 45	⌜17⌝ 30	1 10	
⌜5⌝	1 24	14 «40»	1 10	
[6]	1 10	11 40	<1 10>	
[7]	1	10	<1 10>	
[8]	52 30	8 ⌜45⌝	<1 10>	
[9	4]6 40	7 ⌜46 40⌝	<1 10>	
[10	42	7] ...	<1 10>	

The purpose of the 1 40, written to the right of the first row, is unclear.

Compare the unprovenanced OB tablets YBC 7234, YBC 7235, YBC 7353, YBC 7354, YBC 7355, YBC 7358, YBC 11125, and YBC 11127 (all *MCT*: 17), which are all a similar size and shape and contain analogously constructed tables, reminiscent of those at the end of three problems on AO 8862 (*MKT*: I 108–113).

Tablet 11: N 4942

This tablet shows a numbered diagram of a semicircle (Figure 11, Figure 12).

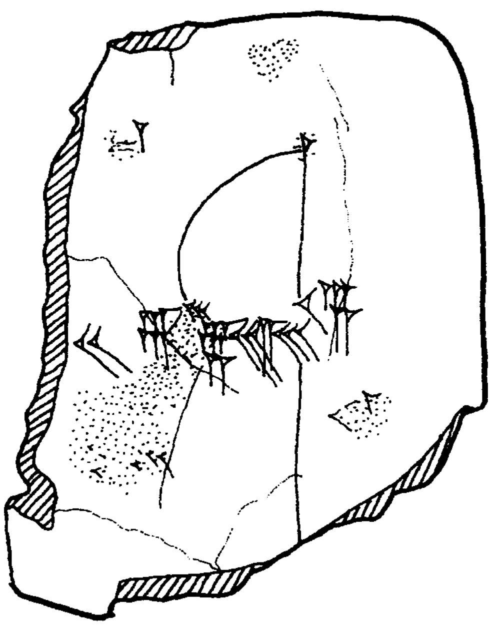


Figure 11: N 4942 obverse (reverse blank)

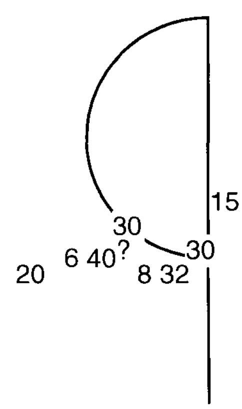


Figure 12: Transcription of the diagram on N 4942

At first sight this looks like the solution to a problem about a semicircle, but none of the relationships:

Area = 0;15 × semicircumference × diameter

Area = 0;22 30 × diameter^2

Area = 0;10 × semicircumference^2

are evident in the numbers (cf. Robson 1999: 39). Instead we may be dealing with powers of 2:

0;30 × 0;30 = 0;15 (i.e. $^1/_2 \times {}^1/_2 = {}^1/_4$)

8 32 (= 2^9) and 0;06 40 (= $^1/_9$).

Tablet 12: UM 29-15-709

The diagrams of triangles on this tablet are almost identical to each other, excepting the addition of two numbers to the left of the reverse (Figure 13, Figure 14). As usual in OB mathematics, they are not to scale, as can be seen by comparing the scaled drawings .

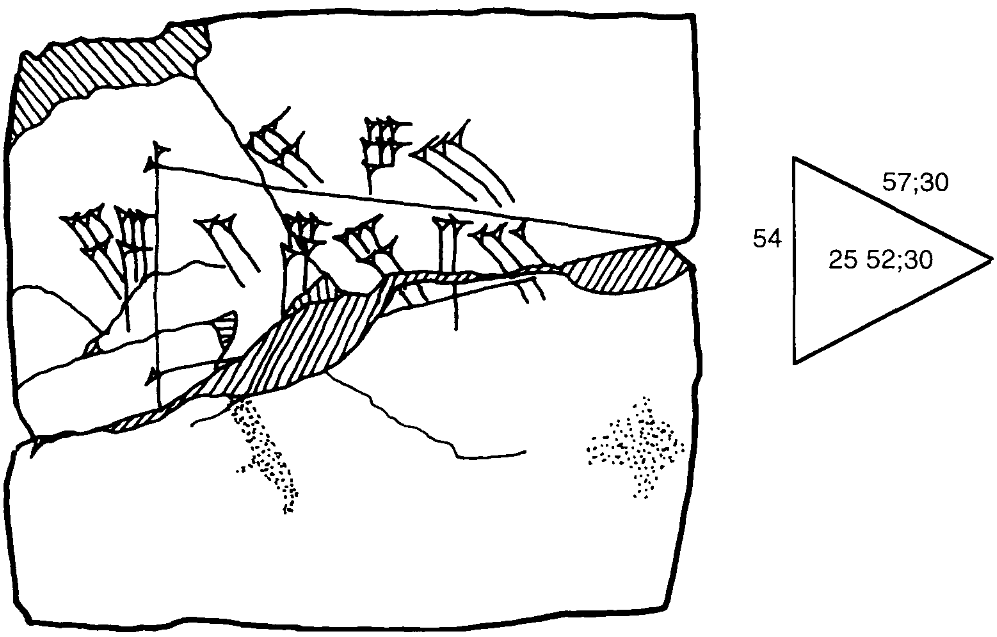


Figure 13: UM 29-15-709 obverse, with transcription of diagram

The sides of the triangle are given, and its area calculated; this is made explicit on the reverse by the halving of one of the sides: 54/2 × 57;30 = 27 × 57;30 = 25 52;30. (An arbitrary choice has been made in placing the 'sexagesimal points' in this transliteration.)

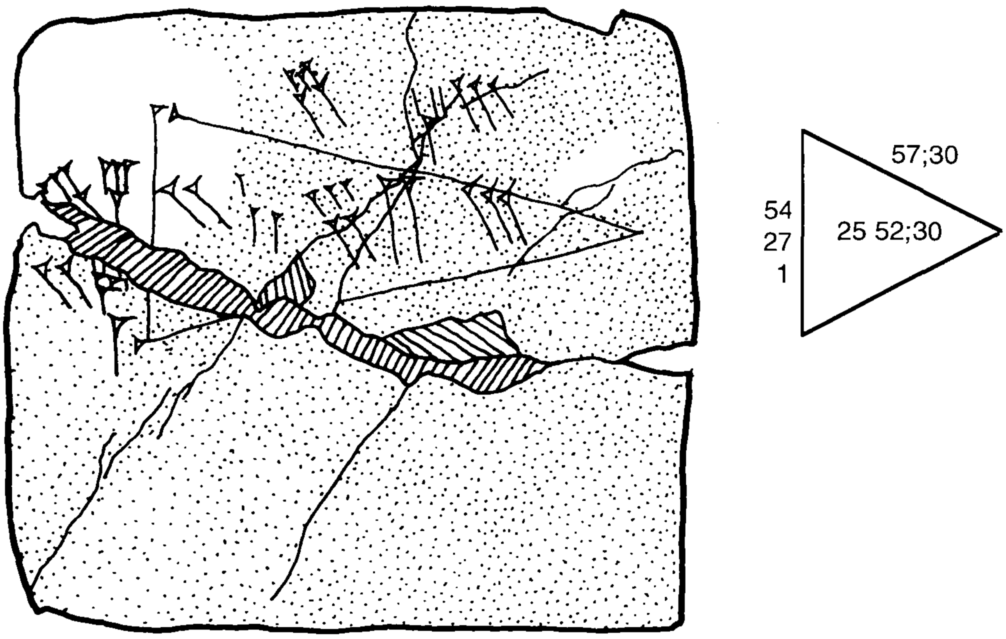


Figure 14: UM 29-15-709 reverse, with transcription of diagram

Tablet 13: UM 29-13-173

The obverse of this tablet appears to list the sides and areas of two irregular equilaterals (Figure 15). The sides are described simply as 'first' or 'second' except in lines 1–2, where the signs are unclear due to surface damage.

obv.	43 40 UŠ UR AN ⌜DA EL⌝	43;40, the ... length	rev.	⌜24⌝ [...]
	42 50 UŠ *e-bu-ra-*[...]	42;50, the ... long length;		1 32$^{?}$ [...]
	36$^{?}$30 SAG.1.⌜KAM⌝ [(...)]	*36*;30, the first width;		1 31 55 [...]
	35 SAG.2.KAM-⌜*ma*⌝	35, the second width.		23 [...]
5.	AŠA$_5$.BI $25\frac{1}{3}$ SAR [(...)]	Its area is $25\frac{2}{3}$! SAR [...]	5.	34 53 [...]
	1 11 UŠ 1.[KAM (...)]	1 11, the first length;		1 30 [...]
	1 08 30 UŠ.2.⌜KAM⌝ [(...)]	1 08;30, the second length;		
	⌜4$^{?}$9⌝ [(...)] ⌜SAG.1⌝.[KAM]	*49*;[..., the first width;]		
	⌜4$^{?}$2⌝ [(...) SAG.2.KAM]	*42*;[..., the second width.]		
	[AŠA$_5$.BI ... SAR]	[Its area is ... SAR ....]		

The method used for finding the area would have been the so-called 'agrimensor formula' of the product of the semi-sum of lengths and widths:[13]

[13] Cf. Ash 1922.168 (Robson 1999: 273–4), YBC 7290, and YBC 1126 (both *MCT*: 44), all unprovenanced but possibly from Larsa.

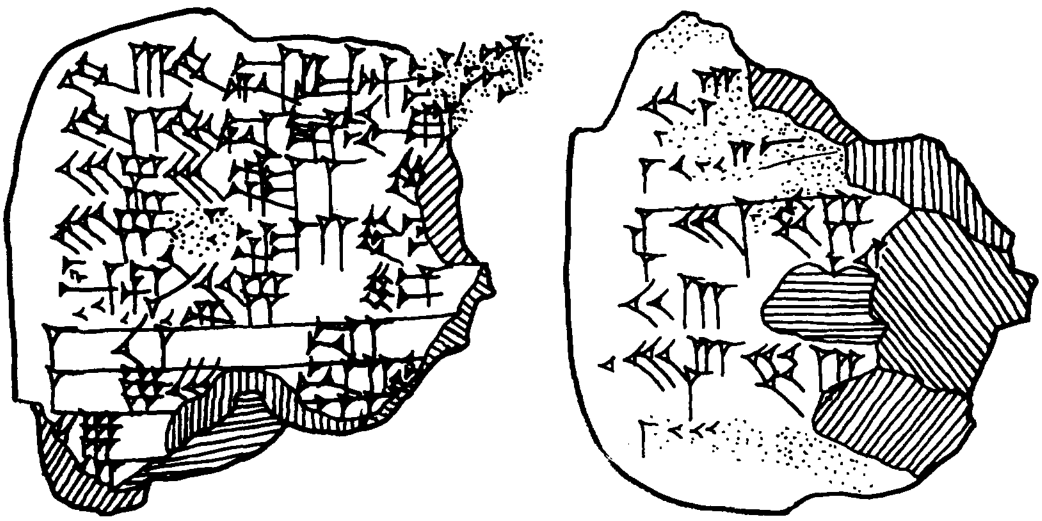


Figure 15: UM 29-13-173 obverse and reverse

$$A = \frac{u_1 + u_2}{2} \times \frac{s_1 + s_2}{2}.$$

For the numbers visible in the first four lines of the tablet, this yields average lengths and area:

(43;40 + 42;50)/2 = 43;15

(36;30 + 35)/2 = 35;45

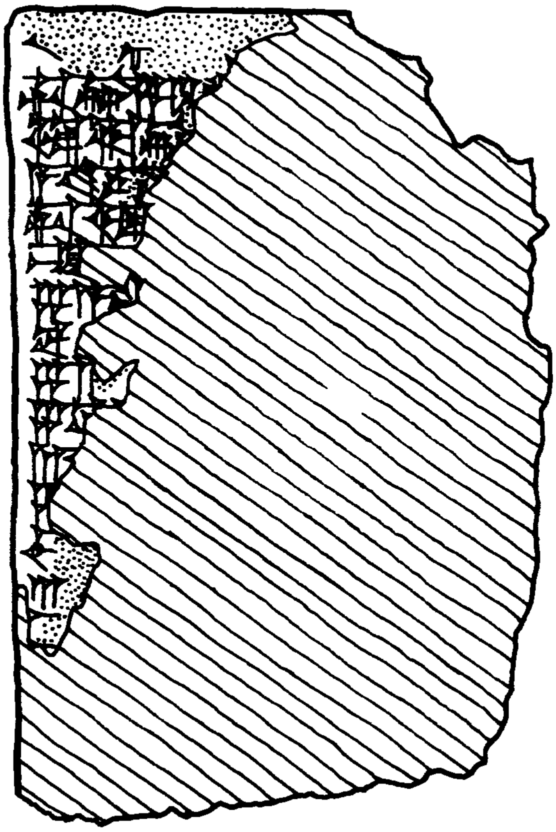
43;15 × 35;45 = 25 46;11 15

Now, this result differs by one sexagesimal place from that given on the tablet (which is given absolute value by the area unit SAR). Either we suppose that the tablet is concerned with real, strip-shaped fields whose lengths were over sixty times longer than their widths (cf. Liverani 1990: 160), or we infer that the tablet contains the school exercises of a student who had not yet fully mastered the concept of place value. This second interpretation is corroborated by the shape of the tablet, the rough jottings on the reverse, and the simple numbers—presumably carefully chosen by a scribal teacher. Too little survives of the lower half of the tablet to comment on the correctness of the second exercise.

The numbers on the reverse do not appear to comprise a connected calculation.

Tablet 14: CBS 12648

The obverse of this tablet was published by Hilprecht, who noted the existence of 'a few cuneiform characters' on the reverse (Hilprecht 1906: 62). A copy of that reverse is given here; the beginnings of fifteen lines are preserved (Figure 16).



rev.	⌜bur-bi⌝ [...]	Its depth [...]
	uš-bi 5 nindan ⌜X⌝ [...]	Its length is 5 rods [...]
	ù sag̃-bi [...]	and its width [...]
	a-šà-bi [...]	Its area [...]
	uš ù [...]	Length and [...]
	en-[nam ...]	*How much*? [...]
	5 a-[šà?/na?⌝ [...]	5, ... [...]
	ba-[...]	... [...]
	5 ⌜a⌝-[šà?/na? ...]	5, ... [...]
	5 ⌜sag̃⌝ [...]	5, the width [...]
	3 [...]	3 [...]
	⌜5⌝ [...]	5 [...]
	igi [...]	*reciprocal* [...]
	3 [...]	3 [...]
	1 [...]	1 [...]

Figure 16: CBS 12648 reverse (obverse published)

Tablet 15: CBS 11681

This small tablet contains the remains of two problems on finding the volume and the sides of a cube (Figure 17).

obv. [...] ⌜X⌝ [...]
[... $\frac{1}{2}$ NINDAN].⌜ÀM⌝ *ú*-⌜X⌝-[...]
[$\frac{1}{2}$] ⌜NINDAN *ú-ša⌝-ap-pi-il*$_5$ ⌜*e*⌝*-pe-ru-ú* [*ki ma-ṣi*]
i-na e-pe-ši-ka $\frac{1}{2}$ NINDAN *a-na* $\frac{1}{2}$ NINDAN *tu*-[*uš-ta-ka-al-ma*][14]
15 *i-il-li-a-am*
15 *a-na* 5 IGI.GUB.BA *tu-ub-ba-al-ma*
1 15 *i-il*$_5$*-li-a-am*
1 GÍN IGI.4.GÁL^{iku}
1 15 *a-na* $\frac{1}{2}$ NINDAN *šu-up-li-im*
tu-ub-ba-al-ma 7 30 *i-il*$_5$-⌜*li*⌝*-a-am*
7$\frac{1}{2}$ GÍN ⌜SAḪAR⌝

[14] This word, if written fully, would have extended onto the reverse of the tablet (cf. obv. 10).

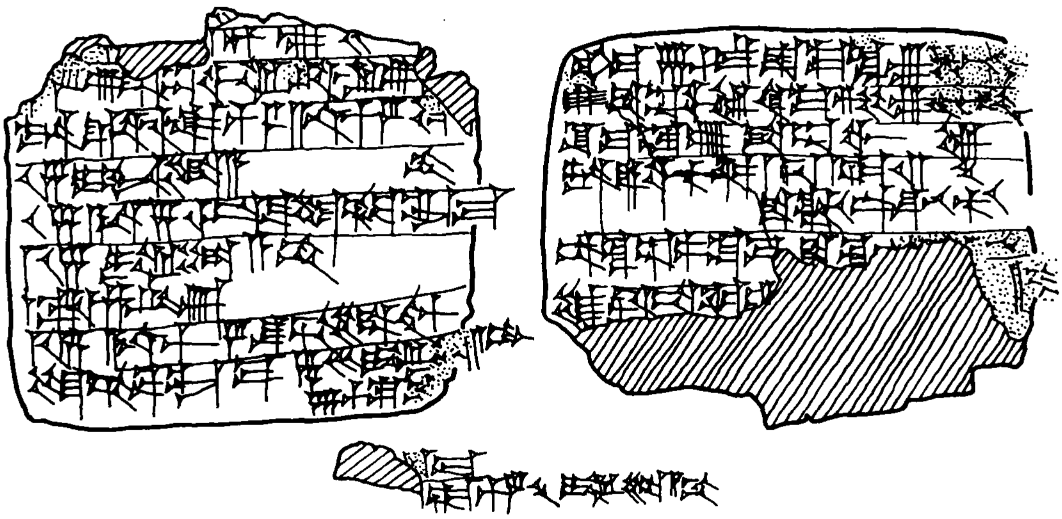


Figure 17: CBS 11681 obverse, reverse, and left edge

... [... I made (it) $^1/_2$ rod square] each way. I made (it) [$^1/_2$] rod deep. [What is] the earth? In your working: you [multiply (lit. 'make each other hold')] $^1/_2$ rod by $^1/_2$ rod, [and] 0;15 comes up. You multiply (lit. 'carry') 0;15 by 0;05, the coefficient, and 0;01 15 comes up: 1 shekel and a quarter, area measure. You multiply (lit. 'carry') 0;01 15 by $^1/_2$ rod, the depth, and 0;07 30 comes up: $7^1/_2$ shekels of volume.

rev. *šum-ma* $7\frac{1}{2}$ GÍN SAḪAR *ma-la ú-...*
ú-ša-ap-pi-il_5 *ki ma-ṣí ú-...*
ki ma-ṣi ú-ša-ap-pi-il_5
i-na e-pe-ši-ka 1 UŠ 1 SAG 12 GAM
ša la ti-du-šu-nu-ti
ta-la-ap-pa-at-ma ša la ⌜*ti*⌝*-*[*du-šu-nu*]*-*⌜*ti*⌝
tu-uš-ta-ka-⌜*al*⌝*-*[*ma* 12 *i-*il_5*-li*]*-*⌜*a-am*⌝
...

l.e. [............]*-ma*
[$\frac{1}{2}$ NINDAN UŠ $\frac{1}{2}$ NINDAN] SAG $\frac{1}{2}$ NINDAN GAM *i-*il_5*-li-a-am*

If it is $7^1/_2$ shekels in volume (and) *I made (it)* as *square* as I made deep, how *square did I make (it)*, (and) how deep? In your working, you write down 1, the length; 1, the width; 12, the depth—that which you do not know—and you multiply (lit. 'make each other hold') that which [you] do not [know, and 12 comes up]. ... ... and [$^1/_2$ rod, the length; $^1/_2$ rod], the width, $^1/_2$ rod, the depth comes up.

Philological notes

CBS 11681 and CBS 19761 (Tablet 16 below) are the first known examples of OB mathematics in Akkadian from Nippur. (CBS 12648, Tablet 14, is in Sumerian—or is at least highly logographic—as of course are the many arithmetical and metrological tables.) CBS 11681 exhibits many of the orthographic characteristics of southern OB spelling: full writing of doubled consonants (e.g. *ú-ša-ap-pi-il*$_5$ for *ušappil* 'I made deep', obv. 3);[15] alternation of *il* and *il*$_5$ (e.g. *i-il-li-a-am,* obv. 5 and *i-il-li*$_5$*-a-am*, obv. 7 for *illiam* 'it comes up'), and *ṣi* and *ṣí* (e.g. *ki ma-ṣí*, rev. 2 and *ki ma-ṣi*, rev. 3 for *kī maṣi* 'how much?').

Choice of vocabulary and grammatical forms are also characteristic (cf. Høyrup 1999). The solution is announced with the phrase *ina epēšika* 'in your working';[16] there is no closing formula. Instructions are given in the second person singular present (e.g. *ta-la-ap-pa-at* for *talappat* 'you put down', rev. 6),[17] and results are announced with the verb *elûm* in the ventive, *illiam* 'it comes up' (obv. 5, 7, 10; rev. 7; left edge 2).[18] The verb *šutākulum* (or *šutakūlum*: cf. Høyrup 1990: 42, 48–9) 'to make each other hold' is used for geometrical multiplication (i.e. of lengths, e.g. *tu-uš-ta-ka-al* for *tuštakkal* 'you make (the lengths) hold each other', rev. 7), while *wabālum* 'to carry' is used for non-geometrical multiplications (i.e. involving numerical constants, e.g. *tu-ub-ba-al* for *tubbal* 'you carry', obv. 6).[19] One verb remains undeciphered, despite the fact that it occurs three times on the tablet (obv. 2, rev. 1, 2). To judge from the context, and from comparison with IM 54478 below, it must mean 'I made square',[20] or possibly 'I measured out', 'I made long', 'I dug', or similar.[21]

On the three occasions that capacity units are given (obv. 8, 11, rev. 1), the associated numbers are in non-mathematical notation: compare the form of the two 7s in obv. 10–11.

---

[15] Cf. the more usual *ú-ša-pí-il*, obv. 2 of IM 54478 below.

[16] As in problems (i)-(ii) of YBC 6504, probably from Larsa (Høyrup 1999: 41). The more usual introductory phrase is *atta ina epēšika* 'you, in your working' (cf. IM 54478 obv. 6), or just *atta* 'you'.

[17] A typically southern usage (Robson 1999: 173 n. 19); cf. *lu-pu-ut* for *luput* 'put down' (imperative) IM 54478 obv. 6.

[18] Cf. *ta-mar* for *tammar* 'you see', IM 54478 rev. 3–4. The verb *amārum* 'to see' to announce results is not attested in southern texts (Høyrup 1999: 22).

[19] As in YBC 7997, probably from Larsa (Høyrup 1999: 41) and CBS 19761 rev. 2 (Tablet 16 below). The verb *našûm* 'to raise' is more usual; cf. *i-ši* for *iši* 'raise', IM 54478 obv. 9; rev. 3–4.

[20] Cf. *uš-ta-am-ḫi-ir* for *uštamḫir* 'I made square', IM 54478 obv. 4. This would not fit the traces here.

[21] It is almost certainly a verb, and in the first person singular preterite, as it is in a parallel construction with another of that person and tense: *mala* (unknown) *ušappil* 'as much as (unknown) I made deep' (rev. 1–2).

Mathematical commentary

There are two closely related problems on this tablet. The first concerns finding a cubic volume of side $^1/_2$ rod (≈ 3m); the second involves finding the sides of the cube given the volume. The first problem, with the same size of cube, also appears on IM 54478 (Baqir 1951: no. 1) from the small OB site of Shaduppûm in modern-day Baghdad, some 150 km north of Nippur:

obv. *šum-ma ki-a-am i-⸢ša⸣-al-ka um-ma šu-ú-ma*
ma-la uš-ta-am-ḫi-ru ú-ša-pí-il-ma
mu-ša-ar ù zu-uz$_4$ *mu-ša-ri*
e-pé-ri a-su-uḫ ki-ia uš-tam-ḫi-ir
ki ma-ṣí ú-ša-pí-il
at-ta i-na e-pé-ši-ka
[1 30] ⸢*ù* 12⸣ *lu-pu-ut-ma i-gi* 12 *pu-ṭú-ur-ma*
⸢5⸣ [*ta-mar* 5 *a-na* 1] ⸢30⸣ *e-pé-ri-ka*
rev. *i-ši-ma* 7 30 *ta-mar* 7 30
mi-nam ÍB.SI$_8$ 30 ÍB.SI$_8$ 30 *a-na* 1
i-ši-ma 30 *ta-mar* 30 *a-na* 1 *ša-ni-im*
i-ši-ma 30 *ta-mar* 30 *a-na* 12
i-ši-ma 6 *ta-mar* 30 *mi-it-ḫa-ar-ta-ka*
6 *šu-pu-ul-ka*

If someone asks you, saying this: "As much as I made square I made deep, and I removed a plot and half a plot of earth. What did I make square; how much did I make deep?" You, in your working, put down [1;30] and 12, and detach the reciprocal of 12, and [you see] 0;05. Multiply (lit. 'raise') [0;05 by 1];30, your earth, and you see 0;07 30. What is the cube root of 0;07 30? The cube root is 0;30. Multiply (lit. 'raise') 0;30 by 1, and you see 0;30. Multiply (lit. 'raise') 0;30 by the other 1, and you see 0;30. Multiply (lit. 'raise') 0;30 by 12, and you see 6. Your square-side is 0;30 (rods). Your depth is 6 (cubits).

While the Shaduppûm solution is mathematically correct, the Nippur version is not. The problem is made non-trivial by the nature of the Old Babylonian unit of volume. The unit of horizontal length is the NINDAN 'rod',[22] approximately 6 m, but the unit of vertical length is the KÙŠ/*ammatum* 'cubit', *ca.* 0.5 m. A (horizontal) area SAR/*mūšarum* 'garden plot' therefore measures 1 rod × 1 rod (≈ 36 m^2), while the corresponding volume SAR/*mūšarum* is not (1 rod)^3 but 1 rod × 1 rod × 1 cubit (≈ 18 m^3). Because there are 12 cubits in a rod, the conversion coefficient 12 (or its

[22] The Akkadian reading of this logogram remains unknown, as it was never written syllabically (cf. Powell 1987–90: 463).

reciprocal 0;05) must be used to convert vertical measures from rods to cubits for a volume to be calculated correctly (cf. Robson 1999: 111-3).

The reverse process happens in lines 7–9 of IM 54478, where the volume in SAR is divided by 12 to get a measurement in cubic rods. It is then easy to find the sides of the cube by inspection (this is, after all, a school exercise with trivial arithmetic), in rev. 2. The horizontal 'square-sides' can remain in rods (rev. 2–4), while the vertical 'depth' must be remultiplied by the height conversion coefficient to produce an answer in the correct unit—cubits (rev. 4–5).

But the scribe of CBS 11681 does not have a good grasp of this process. When finding the volume of the cube from the length measurements given, his mistake is to divide the (horizontal) area 0;15 SAR by the vertical conversion coefficient (obv. 6–7) instead of leaving it unconverted. He then silently (and correctly) transforms the depth from $^1/_2$ rod to 6 cubits in multiplying it by the area (obv. 9–10), but the net effect of this division and then multiplication is of course to leave the units unconverted—thereby producing a volume measured incorrectly (and unwittingly) in cubic rods (obv. 11). We do not know what procedure the scribe used in the second problem, but enough remains to show that he managed to get from the 'incorrect' volume to the 'correct' length measures, written on the edge of the tablet. He appears to have used some sort of 'false position' method, i.e. starting from the assumption that the sides were each 1 rod long (rev. 4–5).

Tablet 16: CBS 19761

The obverse of this tablet, as preserved, contains one problem on squares; the text on the reverse probably belongs to a separate problem (Figure 18).

obv. I	[A.ŠÀ 2 *mi-it*]-*ḫa-ra-tim*	[The areas of 2] squares
	⌈*ak-mu*⌉-*ur-ma* 13	I added: 13
	[*ù mi-it*]-⌈*ḫa*⌉-*ra-ti-ma*	[and] my square-sides
	[*ak*]-⌈*mu-ur*⌉-*ma* 5	[I] added: 5
	[............]-⌈X⌉-*tum*	[......] ...
	[... *mi-it-ḫa*]-*ar-tum*	[...] square
	[*i*?-*te*?]-*er*	*exceeds*
	[... *mi-it-ḫa*]-⌈*ra*⌉-*ti-ia*	[...] my squares
	[..................]-*in*	[......] ...
	[... *mi-it-ḫa*]-*ra-tim*	[...] squares
	[............] A.ŠÀ-*li*	[......] areas
	...	
II 1.	3 A.RÁ 3 9	3 times 3: 9
	9 A.ŠÀ *iš-te*$_{10}$-⌈*en-ma*⌉	9 is the first area.

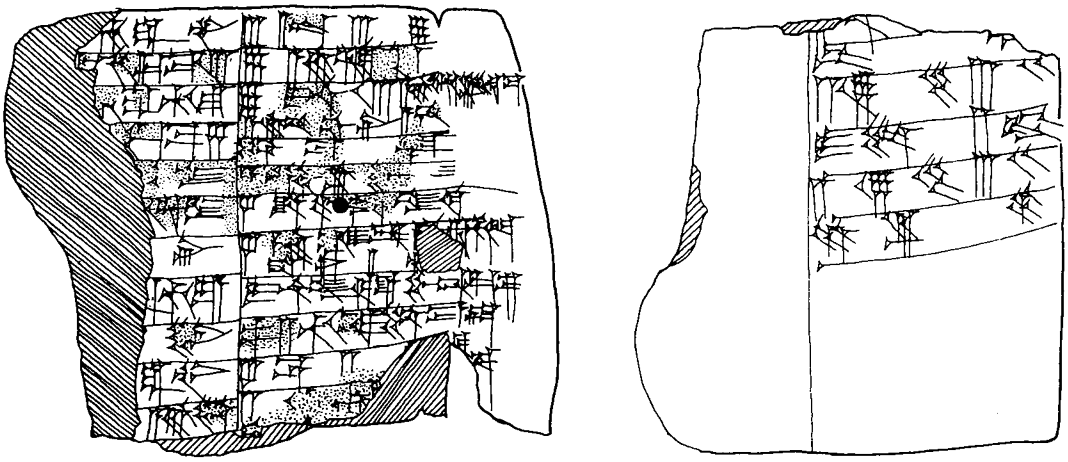


Figure 18: CBS 19761 obverse and reverse

	9 *i-na* 13 A.ŠÀ$^{-im}$ *hu-ru-uṣ-ma*	Tear out 9 from 13, the area, and
	A.ŠÀ$^{-am}$ *ša-ni-a-am*	the second area
	li-ma-ad	learn:
	⌈*a-na* X⌉ *ru-*⌈*bu-ú*?⌉	to ...
	4 *ša-pi-*⌈*il*⌉*-‹ti›* ÍB.SI$_8$	4 the *remainder* of the square(s)
	4 ⌈A.ŠÀ$^{-el}$⌉ *mi-it-*⌈*ḫa-ar*⌉*-ti-ka*	4 the area of your second square.
	ša-ni-a-⌈*am*⌉	
	i-na ÍB.SI$_8$.E *pu-ṭù-ur-ma*!	Detach the square-side, and
	pa-a-ti-ka li-ma-ad	learn your sides.
	4.E 2 [ÍB].⌈SI$_8$⌉	4 has the square-side 2
	2 ⌈X X⌉ [......]*-ka*	2 is your [......]
	⌈ÍB?⌉.[SI$_8$ ......]	*square*
	...	
rev. I	...	
	⌈*a-na*⌉ [.........]	*to* [......]
	bi-⌈*il*⌉*-*[*ma*?]	multiply (lit. 'carry'), *and*
	2 18 53 20	2 18 53 20
	i-li-a-kum	comes up for you.
	2 18 53 20	2 18 53 20
	te-ru-ub	you *enter*.

Philological notes

The surface of the tablet is badly damaged and the writing small, making some of the readings and translations given here quite tentative.

There is little to say about the first column. The restorations are based both on the remaining traces and on the mathematical evidence in Column II, as well as the parallel text in BM 13901 (viii), possibly from Larsa (Høyrup fc.: ch. 3). The remaining sign of I 7 may be the last syllable of *[i-te]-er* for *ittêr* 'it exceeds'. Almost all writings of A.ŠÀ for *eqlum* 'area' are followed by case-indicating phonetic complements (I 11, II 3, 4, 8?).

Much of the spelling and language in Column II is unusual or difficult. The use of the DI sign for *te* in line 2 is most unorthodox, while in lines 5 and 11 we are unusually told to 'learn' or 'find out' a result. (All instructions are apparently given in the imperative in this text.) Line 6 is almost completely illegible; we might expect a verb in the infinitive here ('in order to ...'). A sign has seemingly been omitted from line 7, but even so it appears to read *šapilti mitḫartim* 'the remainder of the square' instead of the expected *mitḫartum šapiltum* 'remaining square'; perhaps we should understand rather *šapilti mitḫarātim* 'the remainder of the squares'.

The gist of lines 10–11 is that the side of the square should be found by taking the square root of the area, but the language used is rather strange. The verb *paṭārum* in line 10 is otherwise attested in mathematical contexts solely for finding ('detaching') reciprocals or, rarely, for subtractions (cf. Høyrup 1990: 54 n. 71). Further, *pâtum*, the plural form of *pūtum* 'short side' of a figure, occurs in just one other context mathematically.[23]

On the sole epigraphic column of the reverse, there is a non-geometrical multiplication (imp. *bil* 'carry', from *wabālum*; cf. CBS 11681 obv. 6, 10: Tablet 15 above) whose result 'comes up for you' *illiakkum*. The scribe has not explicitly doubled the appropriate consonants, unlike the author of CBS 11681 (e.g. obv. 5). The text apparently ends with the problematic statement that *terrub* 'you enter' the result; but in the G stem *erēbum* is only used of oneself entering places or states. While the D stem *urrubum* can mean in Old Babylonian 'to enter something on a tablet', albeit rarely, the well-preserved signs fit neither of the expected forms *turrab* (2 ps. pres.) or *urrub* (imp.). This line may relate to the equally impenetrable II 6.

Mathematical commentary

Too little remains to make many comments about the mathematics of this text. The obverse is apparently concerned with two squares, the sum of whose areas is 13 (I 1–2) and the sum of whoses sides is 5 (I 3–4).[24] At the top of column II we discover

---

[23] See also *pa-a-at er-[be]-⌈et⌉-tam* 'four short sides' in BM 13901 (xxiii): rev. II 12 (*MKT*: III 5).

[24] It would also be possible, from the traces alone, to restore obv. I 4 as [... *aḫ-ru*]-⌈*uṣ*⌉-*ma* 5 '[... I] tore out: 5' and to understand lines 3–4 as describing the difference of the two square areas to be 5. But this interpretation is to excluded on several grounds. First, in obv. I 3 we are certainly dealing with 'squares' *mitḫarātum*, not a single 'square' *mitḫartum*; they cannot both be 'torn out' (subtracted).

that one has side 3 and area 9 (II 1–2), and it comes as no surprise to find that the second has area 4 (II 7–9) and side 2 (II 12). There are few clues as to nature of the solution originally given on the tablet but it would have almost certainly been along the lines of that restored by Høyrup (fc.: ch. 3) for BM 13901 (viii), whose statement is couched in similar terms, giving both the sum of the areas and the sum of the sides of the two squares to be found.

The remains on the reverse seem to be the conclusion of a different problem, as the numerical result, 2 18 53 20 (perhaps to be understood as 500,000), bears no relation to the numbers on the obverse. The text finishes without a double ruling, which suggests that the scribe had intended to write more.

Tablets 17 and 18: CBS 43 and CBS 154+921

These tablets are a pair, or two members of a larger group. They set problems about squares.

CBS 43 (Figure 19)

obv.	*a-na* A.ŠÀ LAGAB^{-ia} 1 DAḪ 1 ⸢41⸣	Add 1 to the area of my square: 1 41.
	LAGAB^{-ti} *ki-ia im-ta-ḫar*	How square is my square?
(ii)	*i-na* ŠÀ A.⸢ŠÀ⸣ LAGAB^{-ia} 1 BA.ZI 1 30	Take 1 from the area of my square: 1 20!
	LAGAB^{-ti} *ki-ia* ⸢*im*⸣*-ta-ḫar*	How square is my square?
	[2?]	
5. (iii)	*a-na* A.ŠÀ LAGAB^{-ia} 20 UŠ$^{-ia}$ DAḪ ⸢4⸣1	Add 0;20 of my length to the area of my square: 41
	LAGAB^{-ti} *ki-ia im-ta-ḫar*	How square is my square?
	⸢3⸣	
(iv)	*i-na* ŠÀ A.ŠÀ LAGAB^{-ia} 20 UŠ BA.ZI! ‹X›	Take 0;20 of the length from the area of my square …
	LAGAB^{-ti} *ki-ia im-ta-ḫar*	How square is my square?
	4	
rev. (v)	*a-na* A.ŠÀ LAGAB^{-ia} 40 UŠ$^{-ia}$ DAḪ 2 24	Add 0;40 of my length to the area of my square: 2 24
	LAGAB^{-ti} *ki-ia im-ta-ḫar*	How square is my square?
	5	
(vi)	*a-na* A.ŠÀ LAGAB^{-ia} 40 UŠ$^{-ia}$ *ku-mur* 1	Add 0;40 of my length to the area of my square: 1
	LAGAB^{-ti} *ki-ia im-ta-ḫar*	How square is my square?
	6	

Second, the verb *ḫarāṣum* 'to tear out' is, to my knowledge, used only in prescribing procedures of geometrical subtraction, not in describing relative size. The correct expression would be that the area of one square 'exceeds' the other, S_1 *eli* S_2 *ittêr* (cf. CBS 165, Tablet 19 below). Finally, I know of no other problem which is stated in terms of both sum and difference of two squares.

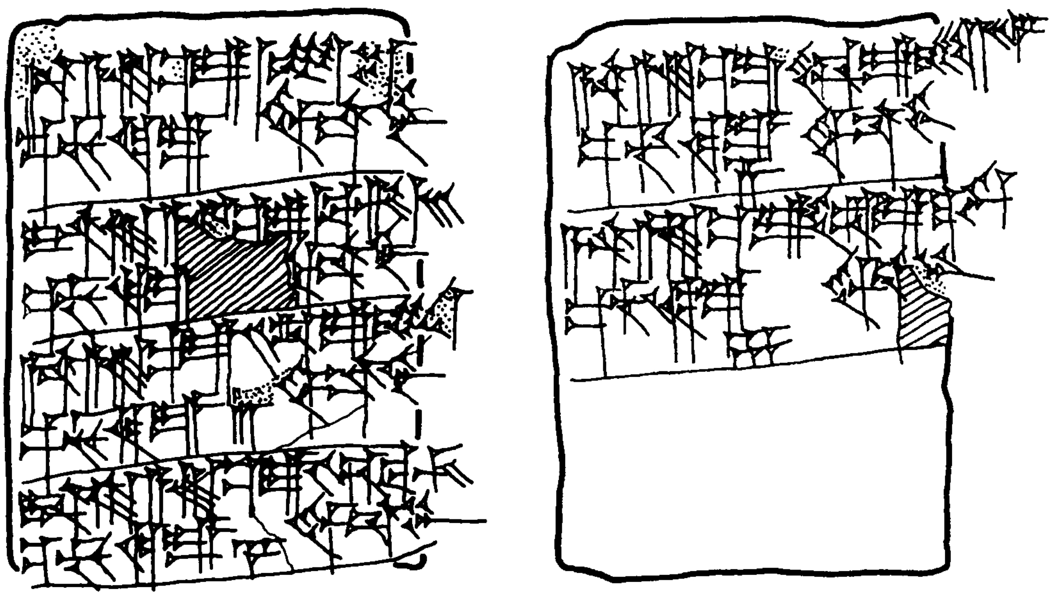


Figure 19: CBS 43 obverse and reverse

CBS 154+921 (Figure 20)

obv.	*i-na* ŠÀ A.ŠÀ LAGAB^{-ia} X [……]	[Take …] from the area of my square: […]
	LAGAB^{-ti} *ki-ia* ⸢*im*⸣-[*ta-ḫar*]	How [square] is my square?
(ii)	[*a-na* A].⸢ŠÀ⸣ LAGAB^{-ia} 5 UŠ$^{-ia}$ DAH 2 24	Add 5 of my lengths [to the] area of my square: 2 24
	LAGAB$^{-⸢ti⸣}$ *ki-ia im-ta-ḫar*	How square is my square?
5. (iii)	*a-na* A.ŠÀ LAGAB^{-ia} $\frac{2}{3}$ ŠU.SI^{-na} UŠ$^{-ia}$ DAḪ 2 06 40	Add $^{2}/_{3}$ (fingers) of my length to the area of my square: 2 06 40.
	⸢LAGAB^{-ti}⸣ *ki-ia im-ta-ḫar*	How square is my square?
(iv)	[*a-na* A.ŠÀ] ⸢LAGAB⸣$^{-ia}$ $\frac{1}{2}$ ⸢UŠ⸣$^{-ia}$ DAḪ 1 45 «30»	Add $^{1}/_{2}$ my length [to the area of] my square: 1 45 «30».
	[LAGAB^{-ti} *ki*]-⸢*ia*⸣ [*im*]-*ta-ḫar*	[How] square [is my square?]
	…	
rev.	…	
1′. (v′)	[*a-na* A.ŠÀ LAGAB^{-ia} …] ⸢UŠ$^{-ia}$ DAH X⸣	Add […] of my length [to the area of my square: …]
	[………………………] ⸢8 22⸣	[…………] 8 22

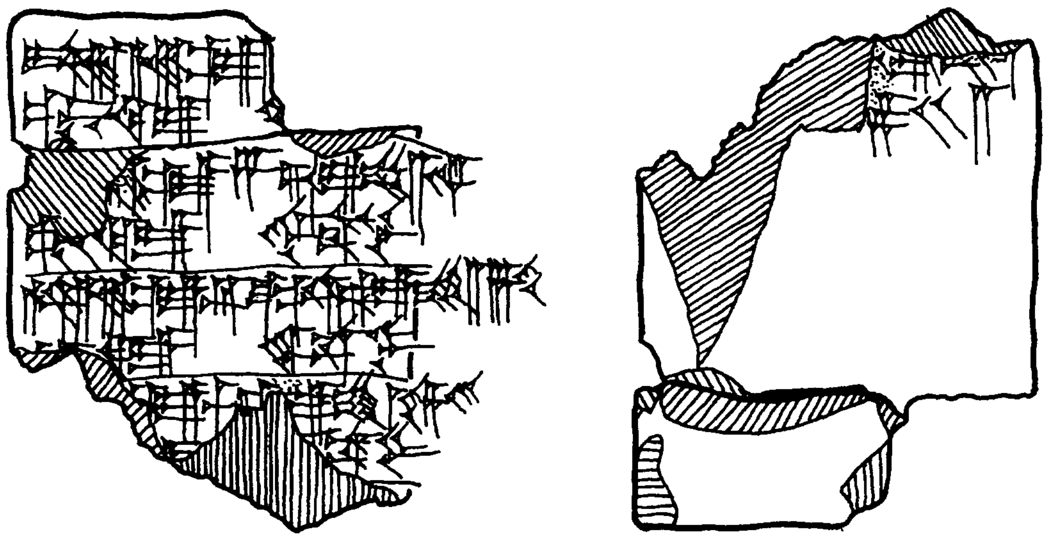


Figure 20: CBS 154+921 obverse and reverse

Philological notes

The tablets are by and large clearly written, though the crucial numbers at the end of the first line of each problem are often squashed and difficult to read. The script is quite densely logographic, with phonetic complements used to mark possession: by LAGAB^{-ia} and LAGAB^{-ti} (*passim*) we should understand *mitḫartia* and *mitḫartī* 'my square' in the genitive and nominitive respectively; similarly UŠ$^{-ia}$ for *šiddia* 'my length', *passim*. The logographically written verbs DAḪ 'to add' and BA.ZI 'to tear out' appear to stand for imperatives, *kumur* 'add' and *uṣuḫ* 'tear out', to judge by the sole syllabic writing *ku-mur* for *kumur* in CBS 43 rev. 3.

There is one difficult passage in obv. 5 of CBS 154+921. After a clearly written $^2/_3$ the signs ŠU, SI, and NA appear. The reading ŠU.SI^{-na} for *ubānā* 'fingers' suggests itself, perhaps implying that we should understand the $^2/_3$ not as 0;40 but as 0;00 40, of the same order of magnitude as the small length measure *ubānum* 'finger'.

On CBS 43, the scribe has counted up the number of problems on the tablet, marking his tally at the bottom of the gap between each *kīa* and *imtaḫḫar* which he had left in order to left- and right-justify the text.

Mathematical commentary

The problems on these two tablets are immediately recognisable as common types of cut-and-paste geometry exercises. CBS 43 (i) and (ii) ask for the side of a square given the area plus 1 and the area minus 1 respectively. In the first case the answer is $\sqrt{(1\ 41\ -\ 1)} = \sqrt{1\ 40} = 10$; in the second 1 30 appears to have been written erroneously for 1 20, giving $\sqrt{(1\ 20 + 1)} = \sqrt{1\ 21} = 9$.

The remaining problems are each one of two types. In the first, n square-sides are added to the square, whose square-side s must be found (CBS 43 (iii), (v), (vi), CBS 154 +921 (ii), (iii), (iv), (v′)). The standard OB geometrical procedure is to 'complete the square' (Høyrup 1990: 266–71), as shown in Figure 21—but of course no method of solution is given on these particular tablets. The intended numerical details are in many cases problematic, though, because the sums $s^2 + ns$ are each written on the very edge of the tablet—perhaps copied inaccurately by the ancient scribe, but in any case difficult to read with certainty.

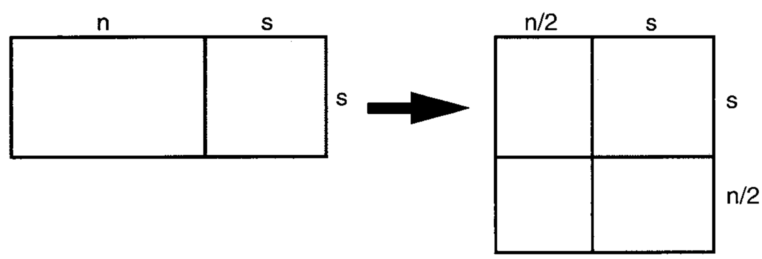


Figure 21: Completing the square for additive problems

The following problems seem to have set, many of which cannot yet be made to yield the expected integer or two-place values of s, however one chooses the absolute value of the numbers given: see Table 4.

Tablet	*Problem no.*	*Possible problem set*
CBS 43	(iii)	$s^2 + 20s = 41$
	(v)	$s^2 + 40s = 2\ 24$
	(vi)	$s^2 + 40s = 1$
CBS 154+921	(ii)	$s^2 + 5s = 2\ 24$
	(iii)	$s^2 + {}^2/_3 s = 2\ 06\ 40$
	(iv)	$s^2 + {}^1/_2 s = 1\ 45\ 30$

Table 4: Additive problems on CBS 43 and CBS 154+921

The remaining two problems extant, namely CBS 43 (iv) and CBS 154+921 (i), both concern the removal of n lengths from the square of side s (as in Figure 22) but in neither case can reconstructions be attempted as the numbers are not preserved.

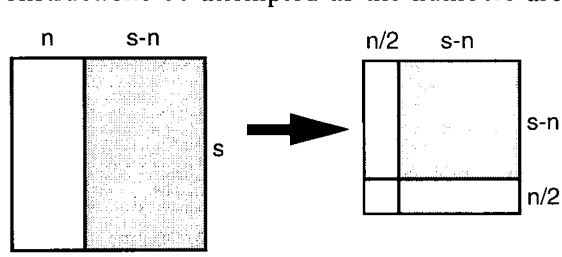


Figure 22: Completing the square for subtractive problems

Tablet 19: CBS 165

This series of nine highly abbreviated one-line statements of problems concerns rectangular areas whose long sides UŠ/*šiddum* and short sides SAG/*pūtum* have to be found, given the area and various relationships between the sides (Figure 23).

1′. [......UŠ] ⌈UGU 3 SAG⌉ [... DIRIG UŠ SAG EN.NAM]
[......UŠ *ù* 2] SAG UL.GAR-*ma* 2 [... UŠ SAG EN.NAM]
[...... UŠ UGU] 2 SAG 3 15 DIRIG ⌈UŠ⌉ [SAG EN.NAM]
[...... UŠ] ⌈*ù* $\frac{1}{2}$⌉ SAG UL.GAR-*ma* 2 UŠ ⌈SAG⌉ [EN.NAM]
5′. [......] ⌈UŠ⌉ UGU $\frac{1}{2}$ ⌈SAG⌉ 1 30 DIRIG UŠ ⌈SAG⌉ [EN.NAM]
[... *ši-ni*]-⌈*pí*⌉-*a-at* UŠ *ù* SAG UL.GAR-*ma* 53 [... UŠ SAG EN.NAM]
[... *ši-ni*]-⌈*pí*⌉-*a-at* UŠ UGU SAG 52 DIRIG [UŠ SAG EN.NAM]
[... *ši-ni-pí*]-*a-at* ⌈UŠ⌉ *ù* $\frac{1}{2}$ SAG UL.GAR-[*ma* ... UŠ SAG EN.NAM]
[...*ši-ni-pí-a-at*] ⌈UŠ UGU $\frac{1}{2}$⌉ [SAG DIRIG ... UŠ SAG EN.NAM]

1′. [...... length exceeds] 3 widths by [.... What are the length and width?]
[......length and 2] widths add: 2 [.... What are the length and width?]
[...... length] exceeds 2 widths by 3 15. [What are] the length [and width?]
[...... length] and $^1/_2$ the width add: 2. [What are] the length and width?
5′. [......] length exceeds $^1/_2$ the width by 1 30. [What are] the length and width?
[...] Add two thirds of the length and the width: 53 [.... What are the length and width?]
[...] Two thirds of the length exceeds the width by 52 [.... What are the length and width?]
[...] Add two thirds of the length and $^1/_2$ of the width: [.... What are the length and width?]
[... Two thirds of] the length [exceeds] $^1/_2$ the width by [.... What are the length and width?]

Although the areas are nowhere preserved (they would presumably have been to the left of the extant flake) we have:

[$u + 3s = \ldots$]
$u - 3s = \ldots$
$u + 2s = 2 \ldots$
$u - 2s = 3;15$
$u + {}^1/_2 s = 2$
$u - {}^1/_2 s = 1;30$
${}^2/_3 u + s = 53 \ldots$
${}^2/_3 u - s = 52$
${}^2/_3 u + {}^1/_2 s = \ldots$
[${}^2/_3 u - {}^1/_2 s = \ldots$]

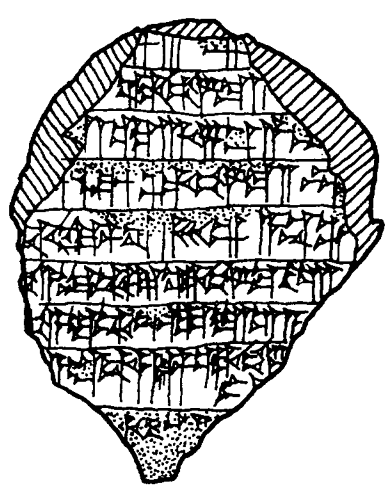


Figure 23: CBS 165 obverse (reverse missing)

We can imagine that the list continued with successive additions and subtractions of natural fractions of the length and width ($^2/_3$, $^1/_2$ $^1/_3$, $^1/_4$, $^1/_5$, ... $^1/_9$?), and had begun with integer values, perhaps of $3u + 3s$, or of $5u + 5s$.[25] The restoration [*ši-ni*]-*pí-a-at* for *šinepâtum*, the feminine form of *šinepûm* 'two-thirds', is almost certain, despite the extensive damage.

Tablet 20: N 2873

This is a surface fragment of a Late Babylonian (LB) mathematical tablet (Figure 24). The transliteration and translation presented here are highly tentative: although the signs are for the most part cleanly executed and well preserved, LB ductus is difficult and the mathematical technical terminology obscure. The traces on the far left belong to a separate column of text and are not in themselves meaningful as preserved.

[..................] 1 40 [......]		
[...............]	6 40 [......]	
[...........]	1 20 ⌜15?⌝ [...]	
[......]	5 37 30 1 07 30 ⌜25?⌝ [...]	
[...] ⌜X⌝	*ša* NIM^{TU} *ana* 20 ÍL [22 30 IGI 22 30]	
[...] ⌜X⌝	DU$_8$-*ma* 2 40 2 ŠU 25 [......]	
[...] ⌜X⌝	1 20 *lu-pu-ut ša* EN.⌜NAM?⌝ [......]	
[...].MEŠ	U$_4$ 7 ⌜X X⌝ *šum-šú*? [......]	
[...] ⌜SI⌝	40 ŠU 20 ÉŠ.GÀR *šu-kun* [......]	
[......]	4 ŠE.GUR 10 *šu-kun* [......]	
[...............]	5 [......]	
[..................] 1 ⌜40?⌝ [......]		

Multiply [0;01 07 30] that *resulted* by 0;20: [0;00 22;30. The reciprocal of 0;00 22 30]
solve: 2 40. 2 sixties 25? [......].
Put down 1 20 that ... [......]
7 days ...... *its name* [......]
Put 40 sixties 20, the workload [......]
Put 4 gur 10 [......]

The relationships between the numbers in the upper rows are as follows:

[25] In a similar way, IM 52916 and its partial duplicate IM 52304+652685 from Shaduppûm (Goetze 1951; Robson 1999: 196–8) lists around fifty problems of the form $s^2 + ns$ (cf. CBS 11681, Tablet 15 above), where n ranges from $^1/_{10}$ to 10 and consists of a natural fraction, and integer, or both.

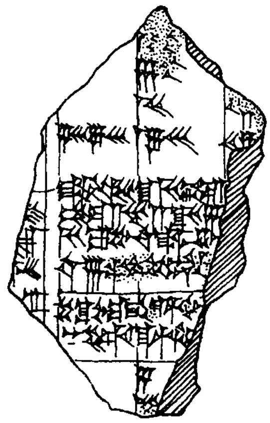


Figure 24: N 2873 obverse (reverse missing)

1 40 (× 4 =)
6 40 (× 12 or ÷ 5 =)
1 20
5 37 30 (× 12 or ÷ 5 =) 1 07 30.

The final 1 07 30 appears to be multiplied by 20 in line 5′; although the exact reading of NIM^{-ni} is unclear to me, it is presumably a derivative of *elû*/NIM 'to be(come) high', in mathematical contexts 'to result'. The reciprocal of the product is taken in line 6′ to give 2 40 (or perhaps it is 2 42, the reciprocal of 22 13 20). The ŠU signs here and in line 9′ are almost certainly the common LB abbreviation of ***šuššu*** 'sixty', used to indicate the absolute value of the preceding numerals. In line 7′ we are told to *luput* 'put down'—namely record on a rough-work tablet—1 20 which has reappeared from line 3′ and is described in a damaged passage.

The first sign of line 8′ may either be U_4 for *ūmū* 'days' or ÉRIN for *ṣābu* 'troops, workmen'; the rest of the line escapes interpretation for the moment. In the following two lines we are told to *šukun* 'put' 40 20 (i.e. 2420) as the ÉŠ.GÀR/*iškaru*. In OB mathematics this word can mean either 'workload'—a daily labouring rate measured variously in volume of earth to be dug, bricks to be carried over a standard distance, etc., or 'output'—; or the length of canal excavated thereby; or the bricks carried over a non-standard distance (Robson 1999: 172). However, *iškaru(m)* has many other related meanings too, and its exact significance is not clear in this context. Nevertheless, it is clearly twice the number 'put' in line 10′. One GUR of grain (ŠE) is equal to 300 SÌLA, itself roughly equivalent to a modern litre, so 4 ŠE.GUR 10 (SÌLA) = 1210 SÌLA. Exactly how these numbers relate to those on the preceding lines is not yet clear.

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