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The *Caturacintāmaṇi* of Giridharabhaṭṭa: A Sixteenth-Century Sanskrit Mathematical Treatise

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I Introduction

I.1 Author

Very little is known about Giridharabhaṭṭa, the author of the *Caturacintāmaṇi* (Clever Wish-Fulfilling Gem), but we can probably identify him with the Giridharabhaṭṭa who wrote a *Jaganmaṇi*, since both authors call their fathers Vīrabhaṭṭa (Bīrābhaṭṭa in the present manuscript) and most of the extant manuscripts of both works have come from the north-western India (see CESS A2, 126b, and A5, 87a). He mentions the date Śaka 1509 = A.D. 1587 in the latter work (CESS A2, 126b). He is also, probably, the author of a *Tājakaśabdaugha*, who calls his father Śrī Vīrabhaṭṭa in the colophon and Vanibhaṭṭa in the first verse (CESS A5, 87a).

In two verses of the *Caturacintāmaṇi*, a king (*nṛpati/śāha*) called Śrīdāni ("wealth-giver") is the main character in mathematical examples. Śrīdāni is a generous king who offers horses and money to meritorious men (in Verse 21) and towns to his devotees (in Verse 85). Giridhara seems to have had some connection with the king Śrīdāni, if he was a real king.

I.2 Contents and sectioning of the *Caturacintāmaṇi*

The *Caturacintāmaṇi* is a book on pāṭī (algorithms), and consists of 89 (and a few additional) verses of rules and examples on arithmetical problems including mensuration (or geometry). Most of the examples are given answers without a working process. There is no new topic in this work, but some topics are treated from a new point of view, and others are given new formulas or solutions (see the next section).

As is usual with a small mathematical treatise in Sanskrit, the *Caturacintāmaṇi* does not have a clear division of its contents into chapters, but seems to more or less follow the sectioning of the *Līlāvatī* (A.D. 1150) of Bhāskara II. This is but natural since the *Līlāvatī* was the best known, standard book of pāṭī in medieval India.

The contents of the *Caturacintāmaṇi* are as follows. The sectioning and the section numbers are tentative. Notation: ⟨A⟩ indicates that A does not actually exist in the manuscript and has been supplied by me. v. = verse. a, b, c, and d after a verse number designate the four quarters of the verse.

1. Introduction (v. 1).
2. Weights and measures (*paribhāṣā*, v. 2).
3. Eight elementary operations of integers and fractions (*karmāṣṭaka*, v. 3).
4. Miscellaneous (⟨*prakīrṇaka*⟩) operations (vv. 4–36).
   Reversed operation.
   ⟨⟨Vv. 4cd–15 and 17ab are missing; Rule of concurrence was probably included here (see under vv. 54–57).⟩⟩
   Divisibility.
   Property of a traveling merchant.
   Various equations of the first degree.
   Rules of three, of five, of seven, and of nine.
   Inverse rule of three.
   Barter.
5. Practical mathematics of mixture (⟨*miśrakavyavahāra*⟩, vv. 37–40).
   Proportional distribution.
   Equation of properties.
   Interest.
6. Practical mathematics of series (*śreḍhīvyavahāra*, vv. 41–51).
   Natural ser., square ser., cubic ser., and geometric progression.
   Arithmetic progression.
   Equations of journeys of two travelers.
7. Practical mathematics of plane figures (*kṣetravyavahāra*, vv. 52–76).
   Sides and area of a right-angled triangle.
   Areas of regular polygons.
   ⟨⟨Vv. 58b–64 are missing.⟩⟩
   Circle and sphere.
   Segment of a circle.
   Irregular figures (fish-like, moon-digit-like, and drum-like figures).
8. Rules for shadows (*dīpacchāyāvidhi*, vv. 77–80).
   Shadow of a gnomon illuminated by a lamp.
   Height of a bamboo stalk.
9. Rules for magic squares (*sarvatobhadravidhi*, vv. 81–86).
   Quasi-magic squares.
   Magic squares of odd and even orders.
   Magic squares having any optional constant sum.
10. Concluding remarks (vv. 87–89).

The contents and the sectioning of the *Līlāvatī* are as follows.

1. Weights and measures (*paribhāṣā*, including introductory remarks).
2. Determination of the names of decimal places (*saṃkhyāsthānanirṇaya*).
3. Eight elementary operations (*parikarmāṣṭaka*).
   Integers.
   Fractions.
   Zero.
4. Miscellaneous (*prakīrṇaka*) operations.
   Reversed operation.
   Operation with an optional number.
   Operation with inequality (normal forms of equations).
   Operation with squares (a certain type of quadratic equations).
   Operation with a multiplier (a certain type of quadratic equations).
   Rule of three and inverse rule of three.
   Rules of five, of seven, etc.
   Barter.
5. Practical mathematics of mixture (*miśrakavyavahāra*).
   Interest.
   Investment and proportional distribution of gain.
   Filling a pond with water through several pipes.
   Buying and selling.
   Equation of properties after the exchange of jewels.
   Purity of gold.
   Combination.
6. Practical mathematics of series (*śreḍhīvyavahāra*).
   Natural ser., square ser., etc.
   Arithmetic progression.
   Geometric progression.
   Number of meters.
7. Practical mathematics of plane figures (*kṣetravyavahāra*).
   Right-angled triangles.
   Triangles and quadrilaterals.
   Circle and sphere.
8. Practical mathematics of ditches (*khātavyavahāra*).
9. Practical mathematics of brick-piling (*citivyavahāra*).
10. Practical mathematics of timber-sawing (*krākacikavyavahāra*).
11. Practical mathematics of heaped-up grain (*rāśivyavahāra*).
12. Practical mathematics of shadows (*chāyāvyavahāra*).
13. Pulverizer (*kuṭṭaka*, indeterminate equations of the first degree).
14. Net of digits (*aṅkapāśa*, combinatorics).

I.3 Characteristic features of the *Caturacintāmaṇi*

A comparative study of the *Caturacintāmaṇi* with other Sanskrit mathematical works including the *Līlāvatī* (abbr. *L*) shows the following features of the *Caturacintāmaṇi*.

1. "Weights and measures" (v. 2) and "Eight elementary operations" (v. 3) have been greatly abridged.
2. The section on various equations of the first degree (vv. 22–29) under the "Miscellaneous operations" has been elaborated, only a few of them having been treated in the *L*. The topic is common in books on algebra like Bhāskara II's *Bījagaṇita*, but Giridhara's treatment of systems of linear equations (vv. 24–27) is unique.
3. The application of the *iṣṭakarman* ("computation by optional number" or *regula falsi*) to a system of linear equations (v. 30) is rare.
4. The vertical arrangement of the three terms of the rule of three (vv. 32–35ab) is very rare.
5. Out of the eight kinds of "practical mathematics", which have been fully dealt with in the *L* (Sections 5–12), only four, namely, mixture, series, plane figures, and shadows are taken up; and only two sections, namely, those for series and plane figures, retain the name, "practical mathematics".
6. Giridhara's approximate formula for the area of an equilateral trilateral (v. 58a), given in terms of its side, is very rare in India, though several other attempts for the same purpose are known to have existed.
7. The value of π, 1889/600 ($= 3;8,54$ when expressed sexagesimally), used in the *Caturacintāmaṇi* (vv. 64–65) has not been attested in any other Indian works.
8. The "correction by one-twentieth" is made in the calculations of the surface area of a sphere (v. 65) and of the area of a segment of a circle (v. 70^a). The resulting formula in each case implicitly corresponds to: $\pi = 63/20$ ($= 3;9$), although it is not certain whether this value was recognized as π by Indian mathematicians.
9. The formula for an accurate calculation of the area of a segment of a circle (v. 70) is peculiar to Giridhara.
10. Like Śrīdhara's *Triśatikā* (rule 44), Āryabhaṭa II's *Mahāsiddhānta* (15.101) and Nārāyaṇa's *Gaṇitakaumudī* (part 2, pp. 10–13), the *Caturacintāmaṇi* (vv. 73–76) treats irregular plane figures such as a fish-like figure. Giridhara's treatment is, however, different from others': Giridhara reduces them into segments of a circle (curvilinear figures) while others mostly reduce them into triangles or quadrilaterals (rectilinear figures). The *L* contains none of them.
11. The two topics, the pulverizer and the net of digits, which have been included in the *L* and in Nārāyaṇa's *Gaṇitakaumudī*, have been omitted by Giridhara.
12. As in Nārāyaṇa's *Gaṇitakaumudī*, the last section of the *Caturacintāmaṇi*

is devoted to magic squares, which are not treated in the *L*. Nārāyaṇa and Giridhara use similar terms for "a magic square": the former calls it a *bhadra* ("good one" or "lucky one") while the latter a *sarvatobhadra* ("one which is good for all directions or purposes"). Giridhara also uses *cakra* ("a disk" or "a diagram") for a magic square, which is unique to him. Ṭhakkura Pherū uses *jaṃta* (= Skt. *yantra*, "apparatus" or "diagram") in his *Gaṇitasāra*. There is no indication that Giridhara has been influenced by Nārāyaṇa or by Ṭhakkura Pherū. Instead, Giridhara's methods for constructing magic squares of even and oddly-even orders contain Islamic elements, while his method for odd orders seems to be unique. The word, *śāha* ("a king"), of Persian origin, occurs once in an example for magic squares (v. 85).

I.4 Manuscripts

Ms.: Jaipur City Palace Library, Puṇḍarīka Collection, Jyotiṣa Section, No. 57. Folios 1–14. Incomplete: folios 2, 3, 9 and 10 are missing. Devanāgarī. Written finely. 7 to 9 lines to a page, and about 30 letters (*akṣaras*) to a line.

Use of "b" for "v" is a characteristic feature in the phonology of the present manuscript (bilomabidhi for vilomavidhi, sarba for sarva, bada for vada, etc.).

The numbering of verses in the manuscript is made according to the order of the second hemistitches of the verses. Therefore, when a verse is divided into two halves, each of which prescribes a rule, and another verse for an example is inserted in between the two halves, the first half of the first verse is not numbered in the manuscript and the second verse is numbered before the second half of the first verse. See, for example, Verse 22ab + Verse 21 + Verse 22cd, Verse 35ab + Verse 34 + Verse 35cd, etc. I have supplied the number for the first half of the first verse in such cases.

A *Caturacārucintāmaṇi* of Giridhara Bhaṭṭa is listed in the *Catalogue of Sanskrit Manuscripts in the Punjab University Library*, Lahore 1932/41 (see CESS A2, 126b). This seems to be another manuscript of the same work (see the second quarter of Verse 1 for the addition of the word, *cāru*, in the title), but it has not so far been available to me.

Acknowledgment

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II Text

|[1] śrīgaṇeśāya namaḥ // śrīgurubhyo namaḥ // śrīsūryāya namaḥ // 1b

praṇamya gurum acyutaṃ gaṇapatiṃ girīśaṃ giraṃ
suvṛttam amalaṃ sphuṭaṃ caturacārucintāmaṇiṃ /
camatkṛtikaraṃ[2] paraṃ gaṇakamaṇḍalīmaṇḍanaṃ
pravacmi gaṇitaṃ kṛtī giridharaḥ sudhīsaṃmude // 1 //[3]

deśakālajanitāḥ paribhāṣāḥ lokataḥ prathamatas tv avagamya
kalpanāṃ[4] tadapavarttanakaṃ vā saṃvidhāya gaṇitaṃ parisādhyaṃ // 2 //[5]

yogo viyogo guṇanaṃ vibhāgo
vargo ghanaś cātra tayoś ca mūle /
karmmāṣṭakaṃ bhinnam abhinnakaṃ vā
jñeyaṃ tu tat sadgurusaṃpradāyāt // 3 //[6]

atha vilomavidhau[7] sūtraṃ /

yutau viyogo viyutau ca yogo guṇe hṛtir hāravidhau[8] guṇa⟨ṃ ca / 4ab /[9]

...[10]

... ⟩|kair nniśi / 4a
hṛtvā cauryeṇa tacchiṣṭaṃ prātar nītaṃ vibhajya ca /
svasvāṃśaṃ tat kiyad dravyaṃ dakṣo ⟨'⟩si gaṇite vada / 16 /[11]

dhanaṃ 1024 ⟨//⟩
sūtraṃ /

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[1]The symbol, |, indicates the beginning of a page, obverse (designated a) or reverse (b), of a folio with the folio number in margin.

[2]*-kṛtiṃ karaṃ.*

[3]Pṛthvī meter. Hereafter, I provide the name of the meter at the end of each stanza.

[4]*tkalpanāṃ.*

[5]Svāgatā.

[6]Indravajrā.

[7]*bilomabidhau.*

[8]*hārabidhau.*

[9]Upendravajrā (half stanza).

[10]Folios 2 and 3, which contain Verses 4cd–15 and 17ab, are missing.

[11]Anuṣṭubh. */ 16 /* at the end of the previous line (after *ca*). See Verse 34 for a similar confusion in numbering.

āgatadvijavarasya ca saṃkhyāsaṃhatiś ca gaṇitaṃ draviṇaṃ syāt / 17⟨cd⟩ /[12]

udā° ⟨/⟩

navabhir brāhmaṇair nītaṃ dhanaṃ niśśeṣatāṃ gataṃ /
viśvaiś ca manubhis tattvais[13] taddhanaṃ syāt kiyad vada / 18 //[14]

dhanaṃ 40950 ⟨//⟩
sūtraṃ /

iṣṭaṃ rūpaṃ kalpa cādha⟨ḥ⟩sthitaṃ tad[15]
ūrdhvaṃ[16] dattaṃ sthāpayet tad yathoktaṃ /
hatvā da⟨t⟩tvā tad guṇeṣṭāṃtareṇa
bhaktaṃ mānaṃ jāyate 'jñātarāśeḥ / 19 //[17]

udā /

yadi gatā dhanino vyayato daśa
dviguṇam urvaritaṃ ca tathā daśa /
samabhavan nagaratritaye tv idaṃ
dviguṇam eti dhanaṃ vada tat kiyat[18] / 20 //[19]

dhanaṃ 35 ⟨//⟩
sūtraṃ /

dravyāṃtaraṃ vājiviyogabhaktaṃ
tad vājimūlyaṃ dhanamūlyatulye / ⟨22ab //⟩[20]

udā° /

ekasmai guṇine ⟨'⟩rpitaṃ nṛpatinā śrīdāninā vājināṃ | 4b
ṣaṭkaṃ rūpaśataṃ tathānyaguṇine rūpāṣṭakaṃ vājinaḥ /
aṣṭau tulyadhanāv ubhāv[21] api yadā jātau tu tau vājinaḥ[22] /

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[12]Svāgatā (half stanza).

[13]*viśvai13ścamanubhi14stvatvai25s.*

[14]Anuṣṭubh.

[15]*-sthitaṃcatad.*

[16]*rūpaṃ.*

[17]Śālinī.

[18]*titkiṃyat.*

[19]Drutavilambita.

[20]Half Indravajrā.

[21]*-dhanābubhāv.*

[22]vājine.

mūlyaṃ cāpi kiyad dhanaṃ vada tadā dakṣo ⟨'⟩si ced dakṣiṇa / 21 /[23]

aśvamūlyaṃ 46 samadhanaṃ 376 ⟨//⟩
sūtraṃ /

gajādikāś[24] cet pṛthag eva hatvā-
bhīṣṭena yogaṃ ca vidhāya sādhyaṃ // 22 ⟨cd⟩ /[25]

udā° /

da⟨t⟩tvā pūrvoditaṃ caikaṃ dve ratne dvau trayo gajāḥ /
dattā⟨ḥ⟩ krameṇa ratnebhavājimūlyāni[26] me vada / 23 ⟨//⟩[27]

ratna 1	gaja 2	a[28]6	rū 100
ratna 2	gaja 3	a 8	rū 8

gajā viṃśatyā guṇā[29] aśvā[30] daśaguṇā rūpeṣu kṣepyāḥ ⟨/⟩ pūrvaval labdhaṃ ratnamūlyaṃ 52 / gajamūlyaṃ 20 aśvamūlyaṃ 10 samadhanaṃ 252 / paraṃ tu tatheṣṭena[31] guṇanīyaṃ yathā rūpāṃtarād vastvaṃtaram anyapakṣagaṃ syāt //
sūtraṃ /

ced dhanino bahavas tatra kramatas ta⟨d⟩dvayor dvayoḥ /
yogaṃ kṛtvāsakṛd[32] yāva⟨d⟩ dvayaṃ pūrvoktavat[33] tataḥ / 24 /[34]

udā° /

dvi|catuḥpaṃcasaptāśvāś caturṇāṃ dhaninām ⟨tathā /⟩ 5a
arkaghnāḥ[35] śivasaptākṣabhuvaś cen mūlyam ādiśa / 25 //[36]

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[23] Śārdūlavikrīḍita.

[24] *ṣṭe*, between *kā* and *śce*, crossed out.

[25] Upajāti (half stanza).

[26] *ratnoścabhavāji-*. *śca*, between *no* and *bha*, crossed out.

[27] Anuṣṭubh.

[28] *ā*.

[29] *guṇāḥ*.

[30] *aśvāḥ 1*.

[31] *ṣṭe*, between *ta* and *the*, crossed out.

[32] *kṛtvā'sakṛd*.

[33] *-bat*.

[34] Anuṣṭubh (1st pāda is hypermetric).

[35] *arka12ghnā*.

[36] Anuṣṭubh.

nyāsaḥ

a 2	rū 132
a 4	rū 84
a 5	rū 60
a 7	rū 12

aśvamūlyaṃ 24 samadhanaṃ 180 //

sūtraṃ /

gajādikāś ced dhaninām tadānīṃ yatheṣṭam ekasya vidhāya mūlyaṃ /
rūpeṣu nikṣipya puroktavat tanmūlyaṃ hi sādhyaṃ samamūlyavitte / 26 /[37]

udā° /

yadi gajā[38] dvicatuḥśarasaṃmitā
nagadiśākṣimitāś[39] ca turaṃgamāḥ /
gajayamā viṣayāś[40] ca guṇā dhanaṃ
daśaguṇā vada mūlyadhanaṃ kiyat / 27 /[41]

ga 2	a 7	rū 280
ga 4	a 10	rū 50
ga 5	a 2	rū 30

gajāḥ śataguṇāḥ aśvamūlyaṃ 10 ⟨/ atha⟩ vā daśaguṇitā aśvāḥ jātaṃ gajamūlyaṃ 100 // samadhanaṃ 550 ⟨//⟩

sūtraṃ /

yatheccham ekasya vidhāya vittaṃ tad anyarūponitam aśvabhaktaṃ /
mūlyaṃ tu tau sto yadi tulyavittau mithohateḥ syād yadi nāsti rūpaṃ / 28 /[42]

udā° // ¦ 5b

yadāśvās ta evārpitā rūpakāś ced
vilomena tau tulyavittau bhavetāṃ /
tadā vājimūlyaṃ tayor vā navāśvā
nakhāśvā na rūpaṃ pṛthag me pracakṣva / 29 /[43]

a 6	rū 8
a 8	rū 100

ādyasyāśvamūlyaṃ kalpitaṃ 50 ⟨/⟩ ādyasya dhanaṃ 308 ⟨//⟩ dvitīyasyāśvamūlyaṃ 26 ⟨/⟩ samadhanaṃ 308 // atha dvitīyodāharaṇe

a 9
a 20

ādyasyāśvamūlyaṃ 20 / dvitīyasyāśvamūlyaṃ 9 ⟨/⟩ samadhanaṃ 180 //

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[37]Upajāti.

[38]*gajāḥ*.

[39]*ka*, between *kṣi* and *mi*, crossed out; a vertical stroke written left of *tā* crossed out.

[40]*biṣayāś*.

[41]Drutavilambita. *127* / corrected.

[42]Upendravajrā.

[43]Bhujaṅgaprayāta.

sūtraṃ /

iṣṭo 'śvādikamityāptaḥ phalaikyena vibhājitaṃ[44] /
aśvādikaikamūlyaikyam iṣṭaghnaṃ sarvamūlyakaṃ[45] / 30 /[46]

udā° //

arkkadvyabdhimitā⟨ḥ⟩ krītā aśvebhoṣṭrāḥ samena cet /
aśvādikaikamūlyaikyaṃ triśatī taddhanaṃ vada / 31 /[47]

jātaṃ dhanaṃ 360 aśvamūlyaṃ 30 gajamūlyaṃ 180 uṣṭramūlyaṃ 90 //
atha trairāśike sūtraṃ /

samajātī pramāṇecche kārye cecchāhataṃ phalaṃ / | 6a
icchāphalaṃ pramāṇāttaṃ vyaste vyastavidhir bhavet / 32 /[48]
eṣa trairāśikavidhiḥ paṃcasaptanavādike /
svalparāśivadhenaiva bahurāśivadhaṃ bhajet / 33 /[49]
icchāpakṣaṃ phale nīte tadvadho bahurāśijaḥ / ⟨35ab /⟩[50]

udā /

paṃcakena yadi ṣaṭ kim aṣṭabhir māsi ced daśasu vā tadā kati /
labhyate yadi phalaṃ janatrayāt paṃcakāt kati tadā pṛthag vada // 34 //[51]

5	mā 1	10	ja 3	5	trai · pha 48 paṃcarāśiphalaṃ 96 saptarā ·
6	5	8	mā 1	10	5
8	6		5	8	
			6		
trairā	paṃcarā[52]		saptarā		

pha 161 //
sūtraṃ // //

mūlyam anyonyapakṣasthaṃ bhāṇḍakapratibhāṇḍake / 35 ⟨cd /⟩[53]

[44] *bibhājiaṃ*.

[45] *sarba-*.

[46] Anuṣṭubh.

[47] Anuṣṭubh.

[48] Anuṣṭubh.

[49] Anuṣṭubh.

[50] Anuṣṭubh (half stanza).

[51] Rathoddhatā. *// 34 //* at the end of the prose comment on this verse (after *pha 161 //*).

[52] *paṃca //*.

[53] Anuṣṭubh (half stanza).

udā°[54] /

śrīphalāni yadi saptakena ṣaṭ pūgaṣaṣṭir iha rūpakais tribhiḥ
śrīphalatritayakena[55] pūgakān ānayasva vigaṇayya vetsi cet / 36 //[56]

jātaṃ 70 ⟨//⟩
sūtraṃ /

miśrahataṃ svasvadhanaṃ
dhanasaṃyogair bhajed dhanaphalaṃ | syāt / ⟨38ab /⟩[57] 6b

udā° /

paṃca sapta nava mānavais tribhiś cārpitāḥ samakalāṃtareṇa cet /
kasyacit tu tata eva saptatiś cāgatā vada kiyad dhanaṃ pṛthak / 37 /[58]

16	23	30
2	1	
3	3	

sūtraṃ /

narahatadānavihīnai ratnair iṣṭe hṛte mūlyaṃ / 38⟨cd⟩ /[59]

udā° /

māṇikyanīlarathavājigajān krameṇa
dignāgabhānudinasaptamitān vilabdhā⟨ḥ⟩ /[60]
smṛtvā puroktaśapathaṃ nijavastu caikaṃ
dat⟨t⟩vā mitho dvijavarāś ca tadā samā⟨ḥ⟩ syuḥ ⟨/⟩ 39 /[61]

mā 10 nī 8 ra 12 vā 15 ga 7 ⟨/⟩ jātāni krameṇa mūlyāni mā 42 nī[62] 70 ra 30 vā 21 ga 105 samadhanaṃ 478 ⟨//⟩
sūtraṃ ⟨/⟩

kālo bhaven māsaphalāṃtarāptaṃ

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[54] *śrīudā°*.

[55] *-tṛtayakena.*

[56] Rathoddhatā.

[57] Āryā (first half).

[58] Rathoddhatā.

[59] Āryā (second half).

[60] An unknown letter, like *mṛ* in Śāradā, after the daṇḍa.

[61] Vasantatilakā.

[62] *mī.*

dhanāṃtarālāt sati saṃbhave tu / ⟨39^a /⟩[63]

udā° /

dviṣaṭkalāṃtaravṛddhau triśataṃ dviśataṃ samarpitaṃ yena /
yadi sakalāṃtaradhanayoḥ samatā syāt kena kālena / 40 /[64]

labdhaḥ[65] kālaḥ 25 // //

| atha śreḍhīvyavahāre sūtraṃ // 7a

saikapadena hataṃ padakhaṃḍaṃ saṃkalitaṃ ca tad ekacayena /
dvighnapadena hataṃ kuyutena[66] rāmahṛtaṃ kṛtisaṃyutir atra / 41 /[67]
saṃkalitasya kṛtir ghanayoge[68] yugmapade dalite sati vargaḥ ⟨/⟩
ojapade vividhau guṇakaḥ syād eṣa vidhis tu padasya layāṃtaṃ / 42 /[69]
vyutkramato guṇajaṃ kṛtijaṃ yat tatphalam ekavihīnitam āptaṃ /
vyekaguṇena[70] phalaṃ mukhanighnaṃ tad bhavatīha guṇe guṇitaikyaṃ / 43 /[71]

udā° /

māsi saṃkalitaṃ vargayutiṃ ghanayutiṃ tathā /
dvimukhaṃ paṃcaguṇitaṃ tadaikyaṃ ca pṛthag vada / 44 /[72]

padaṃ 30 / saṃ 465 vargaikyaṃ 9455[73] ghanaikyaṃ 216225 guṇitaikyaṃ ⟨4656612873077392578 12 //⟩
sūtraṃ /

vyekapadaghnacayārddhayug ādir gacchahataś ca ya|thecchacaye syāt / 7b
vyekapadaghnacayārddhavihīnaṃ saṃkalitaṃ padabhājitam ādiḥ ⟨/⟩ 45 /[74]
padahṛtaṃ gaṇitaṃ mukhahīnitaṃ vikupadārddhahṛtaṃ[75] pracayo bhavet /

---

[63] Upajāti (half stanza). Its counterpart is not found.

[64] Āryā. The jagaṇa as the 1st Caturmātra is irregular.

[65] *labdhaṃ*.

[66] *kuyutaṃ tad*.

[67] Dodhaka.

[68] ghanayoge /.

[69] Dodhaka.

[70] *aikaguṇena*.

[71] Dodhaka.

[72] Anuṣṭubh.

[73] *9320*.

[74] Dodhaka.

[75] *vikulpadā-*.

cayaguṇair dviguṇair[76] gaṇitair yutā cayadalādiviyogakṛtiḥ padaṃ / 46 /[77]
vimukham uttarakhaṃḍayutaṃ ca tad
bhavati gaccha ihottarabhājitaṃ / ⟨48ab /⟩[78]

udā° /

ādye dine daśa tataś caturuttareṇa
kaścij janaḥ pracalito ⟨'⟩nudinaṃ ca māsi /
svasthānam eti vada[79] me kati yojanāni
tebhyo mukhaṃ pracayam atra padaṃ vicārya / 47 /[80]

yo 2040 /
sūtraṃ /

mṛdugatir guṇit⟨ā⟩ divasāṃtarair gativiyogahṛtāptadinair yutiḥ / 48⟨cd⟩ /[81]

udā° /

yo yojanatrikagatiś calitaś ca paścān
māsatraye ⟨'⟩pi ca gate calito jano 'nyaḥ /
yo|gas tayor[82] nava gati⟨ḥ⟩ prativāsaraṃ ca 8a
dakṣo ⟨'⟩si ced vada dinaiḥ katibhir drutaṃ me / 49 /[83]

di 45 //
sūtraṃ //

niyatagatir yā dviguṇā dviguṇamukhonā cayānvitā pracayalabdhā / ⟨50ab /⟩[84]

udā° /

eko janaḥ pratidinaṃ śatam eti cānyaḥ
paṃcottareṇa ca kadā yutir etayoḥ syāt // 51[85]⟨ab⟩ /[86]

sūtraṃ /

---

[76] *ddhiguṇair.*

[77] Drutavilambita.

[78] Drutavilambita (half stanza).

[79] *bada.*

[80] Vasantatilakā.

[81] Drutavilambita (half stanza).

[82] *tuyor.*

[83] Vasantatilakā.

[84] Meter? Without *pracaya* it would be the first half of an Āryā stanza.

[85] *50.*

[86] Vasantatilakā (half stanza).

dviguṇā gatir virūpāthaikacayenobhayor yogaḥ / ⟨50cd /⟩[87]

udā° /

sūtāśvinī pathi śataṃ yadi yāti bālo
hy ekottareṇa ca kadā yutir etayoḥ syāt // 51⟨cd⟩ /[88]

di 199 //

atha kṣetravyavahāre sūtraṃ /

karṇo doḥkoṭivargaikyān mūlaṃ doḥkarṇayos tathā ⟨/⟩
vargāṃtarāt padaṃ koṭiḥ śrutikoṭyos tathā bhujaḥ / 52 /[89]

udā° /

yatra koṭir nṛpaś cārkko bhujas tatra śrutiṃ vada /
koṭiṃ ca[90] doḥśru|tibhyāṃ me bhujaṃ ca śrutikoṭitaḥ / 53 //[91] 8b

atha kayościd yogāṃtare jñāte bhujakoṭikarṇānām ekatame ca jñāte pṛthakkaraṇārthaṃ sūtraṃ /

bhujakarṇaikyabhaktaś cet koṭivargas tadaṃtaraṃ /
tathā tadaṃtareṇāptaḥ koṭivargas tu tadyutiḥ / 54 ⟨/⟩[92]
koṭikarṇaikyabhaktaś ced bhujavargas tadaṃtaraṃ /
karṇakoṭyaṃtareṇāpto bhujavargas tu tadyutiḥ / 55 ⟨/⟩[93]
bhujakoṭyaikyavargonāt karṇavargā⟨d⟩ dvisaṃguṇāt /
mūlaṃ tadaṃtaraṃ jñeyaṃ aṃtareṇa tathā yutiḥ // 56 /[94]
tataḥ saṃkramasūtreṇa kuryān mānadvayaṃ tataḥ /
bhujakoṭivadhasyārddhaṃ tatra kṣetraphalaṃ bhavet // 57 /[95]

bhuja 12 ko 16 kṣetraphalaṃ 96 /
atha samatryasrādīnāṃ[96] kṣetraphalārthaṃ sūtraṃ /

[87] Āryā (second half).

[88] Vasantatilakā (half stanza).

[89] Anuṣṭubh.

[90] *va.*

[91] Anuṣṭubh.

[92] Anuṣṭubh.

[93] Anuṣṭubh.

[94] Anuṣṭubh.

[95] Anuṣṭubh.

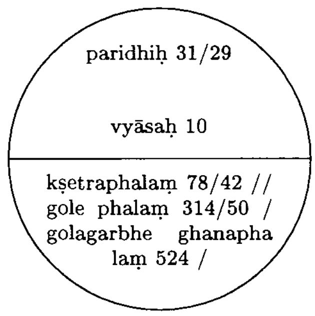
[96] *samaḥ tryasrā-.*

bhujakṛtir amarābdhiguṇā[97] ⟨sahasrabhaktā ... / 58ab /

...⟩[98] | paridhiḥ 31/29 paridher vyāsaḥ 10 kṣetraphalaṃ 78/42 gole phalaṃ 11a
314/50 ⟨/⟩
sūtraṃ ⟨/⟩

trighnī vyāsakṛti⟨s⟩ sthūlaṃ phalaṃ gole nakhāṃśayuk /
vyāsaghnaṃ tac ca ṣaḍbhaktaṃ golagarbhe ghanātmakaṃ / 65 /[99]

golaphalaṃ[100] sthūlaṃ ⟨315⟩ sū 314/50 ghanaphalaṃ sū 524/47 //



101

atha vṛtte jyāvyāsaśarānayanaṃ /

vyāsajyāyutivivarāhateḥ padaṃ yat tadūnitaḥ karṇaḥ /
dalito vāṇo viśaravyāsahateṣoḥ padaṃ dvighnaṃ / 66 /[102]
jīvā tadarddhavargā⟨c⟩
charabhaktāptāḍhyasāyako vyāsaḥ /
śrutyabdhighātayugjyā-
labdhaṃ[103] jīvāṃghripaṃcasaṃguṇaṃ vargam[104] / 67 /[105]
paridher vṛtikṛtipādas tadūnito 'smāt padaṃ tena /
hīnaṃ paridhidalaṃ syāc cāpaṃ vakṣ⟨y⟩e phalaṃ cātra / 68 /[106]

---

[97] *amarābdhi433guṇā.* End of folio 8.

[98] Folios 9 and 10, which contain Verses 58b–64, are missing.

[99] Anuṣṭubh.

[100] *sthūgolaphalaṃ.*

[101] Fig. 1: Circle and sphere. The figure contains the statement: *vyāsaḥ 10 paridhiḥ 31/29 kṣetraphalaṃ 78/42// gole phalaṃ 314/50/ golagarbhe ghanaphalaṃ 524/.* See Fig. 1a in Appendix C.

[102] Āryā.

[103] *śrutyavdhiḥ 4 ghāta-.*

[104] *-saṃguṇād vargāt.*

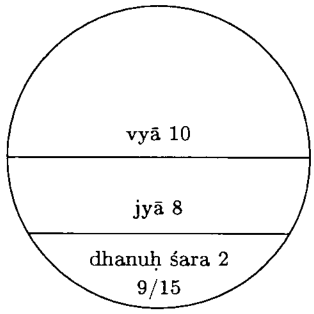
[105] Gīti with an extra guru at the end.

[106] Upagīti.

udā° /

vṛttakṣetre daśa|vyāse jyāṣṭau tatra śaraṃ vada[107] / 11b
śarāj jīvāṃ śarajyābhyāṃ vyāsaṃ cāpamitiṃ pṛthak ⟨/⟩ 69 /[108]

vyāsa 10 vāṇa 2 jyā 8 cāpa 9/15 /



[109]

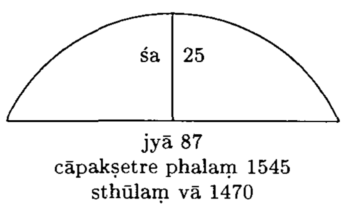
cāpaphale sūtraṃ /

cāpajyāyutikhaṃḍaṃ dvidheṣuṇāḍhyaṃ vihīnitaṃ[110] ca tayoḥ ⟨/⟩
ghātāc charakṛtiguṇitān mūlaṃ dvighnaṃ trihṛt phalaṃ cāpe / 70 ⟨/⟩[111]
vāṇo sthūlaṃ vā śarajīvāyogārddhaghno nakhāṃśasaṃyuktaḥ ⟨/ 70[a] /⟩[112]

udā° //

vāṇas tat⟨t⟩vamito yatra cāpaṃ paṃcottaraṃ śataṃ //
jyā saptāṣṭamitā mitra tatra kṣetraphalaṃ vada / 71 /[113]

cāpaphalaṃ 1545 /



[114]

---

[107] *bada.*

[108] Anuṣṭubh.

[109] Fig. 2: Arc and chord in a circle. The figure contains the statement: *vyā 10 jyā 8 dhanuḥ 915/* (uncorrected) *śara 2.* See Fig. 2a in Appendix C.

[110] *vihahīnitaṃ.*

[111] Gīti.

[112] Meter? Without *vāṇo* or *sthūlaṃ* it would be the first half of an Āryā stanza, but there seems no counterpart to this hemistitch.

[113] Anuṣṭubh.

[114] Fig. 3: Segment of a circle. The figure contains the statement: *śā 25 jyā 87 cāpakṣetre phalaṃ 1545 sthūlaṃ vā 1470.* See Fig. 3a in Appendix C.

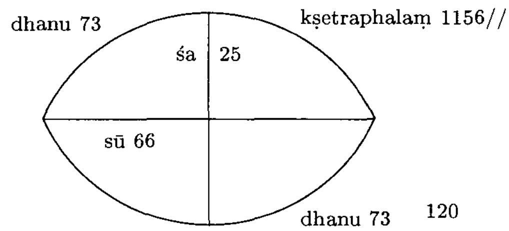
atha matsyakṣetraphale sūtraṃ /

matsyakṣetre matsyapucchāsyasūtraṃ
tat syāj jīvā cāpavattatphalaikyaṃ // ⟨73ab /⟩[115]

udā° //

ṣaṭṣaṣṭipramitaṃ sūtraṃ[116] yatra matsyodare dhanuḥ /
trisa|ptatimitaṃ[117] tat⟨t⟩vamito bāṇa⟨ḥ⟩[118] phalaṃ kiyat / 72 /[119] 12a

phalaṃ 1156 /



120

atha caṃdrakalākṣetraphale sūtraṃ ⟨/⟩

caṃdrakṣetre 'dhodhanur jyāṃ prakalpya
tat syāt spaṣṭaṃ cāpavat sādhayitvā / 73⟨cd⟩ /[121]

udā° //

navanakhapramitaṃ śaradigmitaṃ[122]
yadi dhanuḥ khaśarapramitaḥ[123] śaraḥ ⟨/⟩
śaśikalāsu phalaṃ vada me tadā
yadi tavātitarā gaṇite gatiḥ // 74 //[124]

pha 4960 /

---

[115]Śālinī (half stanza).

[116]*sūtraṃ 66.*

[117]*-mitaṃ 73.*

[118]*bāṇa25.*

[119]Anuṣṭubh.

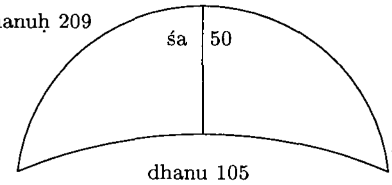
[120]Fig. 4: Fish-like figure. The figure, which has been written in between the first and the second quarters of the udāharaṇa, contains the statement: *dhanu 73 śa 25 sū 66 dhanu 73 kṣetraphalaṃ 1156 //*. See Fig. 4a in Appendix C.

[121]Śālinī (half stanza).

[122]*navanakhapramitaṃ 209 śaradigmitaṃ 105.*

[123]*khaśara50pramitaḥ.*

[124]Drutavilambita.



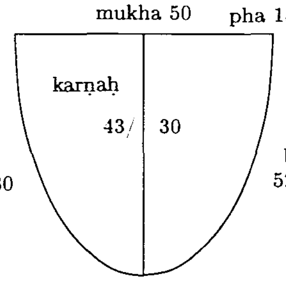
atha duṃdubhikṣetre sūtraṃ /

pārśvabhujaikyaṃ cāpaḥ śaro mukhārddhaṃ[126] jyakā śrutir dvighnaḥ /
sādhyaṃ tato dhanurvat phalaṃ sphuṭaṃ duṃdubhikṣetre / 75 /[127]

udā° //

duṃdubhau śatadalaṃ ca tanmukhaṃ
pārśvabāhuyutir akṣadigmitā[128] ⟨/⟩
sārddha|vahniyugasaṃmitaḥ[129] śrutis 12b
tatra me vada phalaṃ kiyad bhavet / 76 /[130]

pha 1545 ⟨/⟩



atha dīpacchāyāvidhau sūtraṃ //

dīpaśaṃkuvivaraghnaśaṃkuto bhā bhaved vinaradīpabhājitāt /

[125]Fig. 5: Moon-digit-like figure. The figure, which has been written in between the first and the second quarters of the udāharaṇa, contains the statement: *dhanuḥ 209 śa 50 dhanu 105 kṣetraphalaṃ 4960*. The *dhanuḥ 209* has been written vertically in margin. See Fig. 5a in Appendix C.

[126]*muṣārddhaṃ.*

[127]Āryā.

[128]*akṣadigmitā 105.*

[129]*-mitaḥ 43/30.*

[130]Rathoddhatā.

[131]Fig. 6: Drum-like figure. The figure contains the statement: *mukha 50 karṇaḥ 43/30 pārśvabhuja 52/30 bhujaḥ 52/30 pha 1545*. See Fig. 6a in Appendix C.

bhāhṛtān narayutaṃ phalaṃ tato dīpakaucyam athavā tu tad bhavet / 77 /[132]
bhāguṇo vinaradīpako hṛtaḥ śaṃkunā bhavati dīpaśaṃkubhūḥ / ⟨79ab /⟩[133]

udā° //

śaṃkudīpatalabhū radonmitā dīpakocchritir ihābhraṣaṇmitā /
dvādaśāṃgulanarasya bhām ito dīpakaucyam iha cāṃtaraṃ vada[134] / 78 /[135]

dīpaśaṃkvaṃtarabhū⟨ḥ⟩ 32 dīpocchritiḥ 6⟨0⟩ chāyā ⟨8 /⟩[136]

sūtraṃ //

vaṃśabhāguṇitaśaṃkur āhṛtaḥ śaṃkubhābhir iha vaṃśamānakaṃ / 79⟨cd⟩ /[137]

udā° //

samabhuvi yadi veṇor bhā bhavet khāṣṭi|tulyā[138] 13a
daśaparimitaśaṃkor bhā yadā vedatulyā /
gaṇaka vada[139] mamāgre vaṃśamānaṃ kiyat syād
yadi gaṇitavidhāne tvaṃ pradhāno ⟨'⟩si vidvan / 80 /[140]

vaṃśamānaṃ 400 ⟨//⟩

atha sarvatobhadravidhau sūtraṃ //

āditaḥ kramataś cāṃkā lekhyāḥ paṃktyaṃtakoṣṭhakāt[141] /
adho vilikhya[142] tatpṛṣṭam āpūrya[143] ca punaḥ punaḥ / 81 /[144]
viṣame vā same ⟨'ṃ⟩kānām evaṃ samā[145] yutir bhavet /
tadādir madhyapaṃktyāṃ ca sthāpanīyas tataḥ kramāt / 82 /[146]

---

[132] Rathoddhatā.

[133] Rathoddhatā (half stanza).

[134] *badaḥ.*

[135] Rathoddhatā.

[136] A space for two akṣaras is left blank after *chāyā*.

[137] Rathoddhatā (half stanza).

[138] *khāṣṭi180tulyā.*

[139] *bada.*

[140] Mālinī.

[141] *-koṣṭakāt.*

[142] *viliṣya.*

[143] *āpūrvya.*

[144] Anuṣṭubh.

[145] *samaṃ.*

[146] Anuṣṭubh.

ekakoṣṭhāpacitayā[147] vāmaṃ sādhyās tu paṃktayaḥ ⟨/⟩
viṣame sarvatobhadre vidhir eṣa prakīrttitaḥ / 83 /[148]
same ca prathamāt koṣṭhād[149] ekatas tad⟨d⟩vayāṃtare /
dvike dvike likhed[150] aṃkān tato vāmaṃ tathaikataḥ / 84 /[151]

atha tryādinavāṃtānāṃ koṣṭhānāṃ[152] nyāsaḥ[153]

1	2	3
5	6	4
9	7	8

[a]

1	2	3	4
6	7	8	5
11	12	9	10
16[c]	13	14	15

[b]

1	2	3	4	5
7	8	9	10	6
13	14	15	11	12
19	20	16	17	18
25	21	22	23	24

[d]

[147] *ekakoṣṭā-*. *ekako* is filled in by a different hand.

[148] Anuṣṭubh.

[149] *koṣṭād.*

[150] *liṣed.*

[151] Anuṣṭubh.

[152] *koṣṭānāṃ.*

[153] *nāṃ nyāsaḥ* is written by a different hand in margin.

[a] Fig. 7: The quasi-magic square of order three.

[b] Fig. 8: The quasi-magic square of order four.

[c] The last line of cells, 16-13-14-15, is missing.

[d] Fig. 9: The quasi-magic square of order five.

| 13b

1	2	3	4	5	6
8	9	10	11	12	7
15	16	17	18	13	14
22	23	24	19	20	21
29	30	25	26	27	28
36[f]	31	32	33	34	35

[e]

1	2	3	4	5	6	7
9	10	11	12	13	14	8
17[h]	18	19	20	21	15	16
25	26	27	28	22	23	24
33	34	35	29	30	31	32
41	42	36	37	38	39[i]	40
49	43	44	45	46	47	48

[g]

1	2	3	4	5	6	7	8
10	11	12	13	14	15	16	9
19	20	21	22	23	24	17	18
28	29[k]	30	31	32	25	26	27
37	38	39	40	33	34	35	36
46	47	48	41	42	43	44	45
55	56	49	50	51	52	53	54
64	57	58	59	60	61	62	63

[j]

[e]Fig. 10: The quasi-magic square of order six.

[f]*39.*

[g]Fig. 11: The quasi-magic square of order seven.

[h]*11.*

[i]*37.*

[j]Fig. 12: The quasi-magic square of order eight.

[k]*28.*

1	2	3	4	5	6	7	8	9
11	12	13	14	15	16	17	18	10
21	22	23	24[m]	25	26	27	19	20
31	32	33	34	35	36	28	29	30
41	42	43	44	45	37	38	39	40
51	52	53	54	46	47	48	49	50
61	62	63	55	56	57	58	59	60
71	72	64	65	66	67	68	69	70
81	73	74	75	76	77	78	79	80

[l]

6	1	8
7	5	3
2	9	4

[n]

1	15	14	4
12	6	7	9
8	10	11	5
13	3	2	16

[o]

15	8	1	24	17
16	14	7	5	23[q]
22	20	13	6	4
3	21	19	12	10
9[r]	2	25	18	11

[p]

[l] Fig. 13: The quasi-magic square of order nine. This square has been written in between the magic squares of orders 7 and 8 below.

[m] *?2.* The digit for the second decimal place is illegible.

[n] Fig. 14: A magic square of order three.

[o] Fig. 15: A magic square of order four.

[p] Fig. 16: A magic square of order five.

[q] *27.*

[r] The last line, 9-2-25-18-11, has been shifted to the right by one cell due to the magic square of order four written below the bottom left corner.

6	28[s]	34	2	36	5
30	18	21	24	11	7
29[u]	23	12	17	22	8
10	13	26	19	16	27
4	20	15	14	25	33
32	9	3	35	1	31

[t]

28	19	10	1	48	39	30
29	27	18	9	7	47	38
37	35	26	17[w]	8	6	46
45	36	34	25	16	14	5
4	44	42	33	24	15	13
12	3	43	41	32[w]	23	21
20	11	2	49	40[w]	31[w]	22

[v]

1	63	62	4	5	59	58	8
56	10	11	53	52	14	15	49
48	18	19	45	44	22	23	41
25	39	38	28	29	35	34	32
33	31	30	36	37	27	26	40
24	42	43	21	20	46	47	17
16	50	51	13	12	54	55	9
57	7	6	60	61	3	2	64

[x]

[s] *27*.

[t] Fig. 17: A magic square of order six. See Fig. 17a in Appendix C.

[u] *22*.

[v] Fig. 18: A magic square of order seven.

[w] Illegible due to a correction made on the manuscript.

[x] Fig. 19: A magic square of order eight.

|	45	34	23	12	1	80	69	58	47 [y]	14a
	46	44	33	22	11	9	79	68[z]	57	
	56	54	43	32	21	10	8	78	67	
	66	55	53	42	31	20	18	7	77	
	76	65	63	52	41	30	19	17	6	
	5	75	64	62	51	40	29	27	16	
	15	4	74	72	61	50	39	28	26	
	25	14	3	73	71	60	49	38	36	
	35	24	13	2[a]	81[b]	70[c]	59	48	37	

// udā° /

ekāśītiṃ purāṇāṃ nṛpativaraśiroratnanīrājitāṃghriḥ[145]
prādāc chrīdāniśāhaḥ samanagaradhanaṃ sevakebhyo navabhyaḥ /
vṛddhyā caikottarāṇāṃ vada vimalamate yat pṛthaksthaṃ dhanaṃ tat[146]
tvaṃ ce⟨d⟩ dakṣo ⟨'⟩si bīje vinimayasamaye[147] ⟨'⟩py evam anyonyato vā / 85 /[148]

sūtraṃ //

paṃktihṛtena śreḍhīphalena rāśir vibhājitaḥ[149] pracayaḥ
paṃktyā hṛtam avaśiṣṭaṃ[150] mukhaṃ tataḥ pūrvavat[151] pūrttiḥ / 86 //[152]

---

[y]Fig. 20: A magic square of order nine.

[z]*19.*

[a]*13*
2

[b]*8*
1

[c]Illegible due to a correction made on the manuscript.

[145]*-nīrājināṃghriḥ.*

[146]*vimalamatapūḥpṛthaktāṃdhanaṃte.*

[147]*binimaya-.*

[148]Sragdharā.

[149]*bi-.*

[150]*avasiṣṭaṃ.*

[151]*pūrbavat.*

[152]Āryā.

tripaṃkticakre triśatīsamayogārthaṃ nyāsaḥ /

120[a]	20	160
140	100	60
40	180	80

[b]

ekapaṃcāśadyogārthaṃ vā /

20	5	26
23	17	11
8	29[d]	14

[c]

catuḥpaṃkticakre triśatīyogārthaṃ nyāsaḥ //

15	127	119	39
103	55	63	79[f]
71	87	95	47
111	31	23	135

[e]

paṃcāśadyogārthaṃ ca //

5	19	18	8
16[h]	10	11	13
12	14	15	9
17	7	6	20

[g]

vināvyaktayuktyā kilāvyaktam uktaṃ
vinā yaṃtrarītyā ca vaṃśādikaucyaṃ ⟨/⟩
vinānalpakaṣṭaṃ na kaṣṭaikagamyaṃ
mayedaṃ mude tadvidāṃ kiṃcid uktaṃ / 87 /[150]
yan mayā nigaditaṃ samāsatas tat sudhībhir avalokya sādaraṃ /
śodhanīyam apahāya matsaraṃ prārthaneti ca mamāsti tān prati / 88 /[151]
mitākṣarārthagaṃbhīrā dhīrāṃtarmodadāyinī /

---

[a] *12*

[b] Fig. 21: A magic square of order three with the constant sum, 300. These four squares have been written together after the third introduction, *catuḥ ... nyāsaḥ //*.

[c] Fig. 22: A magic square of order three with the constant sum, 51.

[d] *27*

[e] Fig. 23: A magic square of order four with the constant sum, 300.

[f] *69*

[g] Fig. 24: A magic square of order four with the constant sum, 50. The introduction for this square, *paṃcāśad ... ca //*, has been written in smaller letters in margin.

[h] *11*

[150] Bhujaṅgaprayāta.

[151] Rathoddhatā.

kṛtir giridharasyaiṣā[152] bhātu yāvad dineśvaraḥ // 89 //[153]

// iti śrībīrābhaṭṭātmajena giridharabhaṭṭena viracitaś caturacintāmaṇiḥ samāptaḥ // śubham astu[154] //

[152] *giridhasyeṣā.*

[153] Anuṣṭubh.

[154] *astuḥ.*

III Translation

III.1 Introduction

Salutation to the Honorable Gaṇeśa. Salutation to the Honorable Gurus (preceptors). Salutation to the Honorable Sun.

1. Having saluted the preceptor, the imperishable (Acyuta), the lord of Gaṇas (Gaṇeśa), the lord of mountains (Śiva), and speech (Sarasvatī), I, a learned one, Giridhara, speak, for the sake of the pleasure of intelligent people, a faultless, clear science of calculation (*gaṇita*) composed of beautiful verses, ⟨entitled⟩ *Caturacārucintāmaṇi* (clever, lovely wish-fulfilling gem), which produces admiration, which is the greatest, and which decorates the circle of calculators.

III.2 Weights and measures

2. Having understood first, according to common use, the terminology ⟨for weights and measures⟩ produced in a specific area at a specific time, and having made either composition or contraction of those ⟨ratios⟩, one should perform calculation.

III.3 Eight elementary operations

3. Addition, subtraction, multiplication, division, squaring, cubing, and the root-extraction corresponding to these two (i.e., squaring and cubing); this eight-fold ⟨mathematical⟩ operation, either of fractions or of integers, should be known from the traditional instruction of good teachers.

III.4 Miscellaneous operations

Now, a versified rule on the reversed operation:

4ab. In the case of an addition, a subtraction ⟨is made⟩; in the case of a subtraction, an addition; in the case of a multiplier, a division; and in the case of an operation of division, a multiplier ⟨is made⟩.

4cd – ⋮ – *15* – *17ab*: Missing.

⟨An example:⟩

16. ... When ... have taken away ... by robbery at night, and when they have divided its remainder, which has been brought away, ⟨equally among themselves⟩ in the next morning, how much property is the share of each? Say, if you are skillful in calculation.

The property ⟨of each⟩ is 1024.

A versified rule:

17cd. The product of the numbers of the best of twice-born men (i.e., brāhmaṇas) who have come shall be the calculated property.

An example:

18. A ⟨certain amount of⟩ property did not leave a remainder when it was ⟨divided and separately⟩ brought away by nine brāhmaṇas, nor did it ⟨when it was brought away⟩ by thirteen, fourteen, and twenty-five ⟨brāhmaṇas⟩. Say, how much will that property be?

The property is 40950.

A versified rule:

19. Suppose the optional ⟨quantity⟩ is unity. It is placed below. One should place the given ⟨quantity⟩ above. When one has multiplied it (i.e., both terms) ⟨by the multiplier⟩ and given (i.e., added) ⟨the "given" quantity to the upper term repeatedly⟩ as has been told ⟨in the problem⟩, that ⟨result⟩, divided by the difference of the multiplier and the optional ⟨quantity finally obtained⟩, becomes the value of the unknown quantity.

An example:

20. ⟨In each town⟩, ten ⟨units of money⟩ of a rich man are spent for expenditure, the remainder is doubled, and likewise ten ⟨are spent⟩: if this took place in three towns and ⟨his⟩ property becomes twice ⟨the original property⟩, then say how much is that ⟨original property⟩?

The property is 35.

A versified rule:

22ab. The difference of the money ⟨of two persons⟩ is divided by the difference of ⟨their⟩ horses. That ⟨quotient⟩ is the price of a horse when the prices (i.e., values) of ⟨their⟩ properties are the same.

An example:

21. Six horses and a hundred *rūpas* (monetary units) were given to a meritorious man by the king (*nṛpati*) Śrīdāni,[155] and likewise eight *rūpas* and eight horses to another meritorious man. When both of them came to have equal properties, how much is the price of a horse and ⟨how much is⟩ the ⟨equal⟩ property? Say, clever one, if you are skillful ⟨in calculation⟩.

The price of a horse is 46 ⟨*rūpas*⟩, and the equal property is 376 ⟨*rūpas*⟩.

A versified rule:

22cd. If there exist elephants, etc. ⟨in addition to money and horses in a problem⟩, then, having multiplied each ⟨except one⟩ by an optional ⟨number⟩ and made the sum ⟨of the products with the money⟩, one should obtain ⟨the price of the remaining one as before⟩.

An example:

23. When he had given the above said ⟨articles to the two persons⟩, one and two jewels and two and three elephants, in order, were given ⟨again to the same two persons⟩. Tell me the prices of a jewel, of an elephant and of a horse ⟨when their properties are the same⟩.

j 1	e 2	h 6	*rū* 100
j 2	e 3	h 8	*rū* 8

[156] The ⟨number of⟩ elephants multiplied by twenty and the ⟨number of⟩ horses multiplied by ten are to be added to the *rūpas*. The price of a jewel obtained as before is 52. The price of an elephant is 20, the price of a horse 10, and the equal property 252. However, the multiplication ⟨by an optional number⟩ should be made in such a way that, ⟨in the equation⟩, the difference of the things will go to the side (*pakṣa*) opposite to the difference of the *rūpas*.

A versified rule:

[155] The word śrīdāni, which means "a wealth-giver", may or may not be a proper noun. See Verse 85 for the same word.

[156] j = jewels, e = elephants, h = horses, *rū* = *rūpas*.

24. If there are many (i.e., more than two) rich men, when one has combined them two by two in order until two ⟨men only remain⟩, then ⟨one should treat them⟩ just as stated before.

An example:

25. If four rich men possess ⟨in order⟩ two, four, five, and seven horses and eleven, seven, five, and one ⟨*rūpas* each⟩ multiplied by twelve, then point out the price ⟨of a horse⟩.

Setting-down

h 2	*rū* 132
h 4	*rū* 84
h 5	*rū* 60
h 7	*rū* 12

[157] The price of a horse is 24. The equal property is 180.

A versified rule:

26. If rich men possess elephants, etc. at that time ⟨in addition to horses⟩, then when one has laid down the price of one ⟨kind⟩ according to one's will and added ⟨the known prices⟩ to the *rūpas* ⟨of each person⟩, its price should be obtained as stated before, when ⟨their⟩ properties have the same value.

An example:

27. If the elephants ⟨possessed by three persons⟩ are measured ⟨in order⟩ by two, four, and five ⟨in number⟩, and the horses by seven, ten, and two, and the money ⟨of each⟩ is eight=two (28), five, and three, ⟨each⟩ multiplied by ten, say what is the ⟨equally⟩ priced property?

e 2	h 7	*rū* 280
e 4	h 10	*rū* 50
e 5	h 2	*rū* 30

[158] The ⟨numbers of⟩ elephants are multiplied by ⟨an assumed number,⟩ one hundred; ⟨then⟩ the price of a horse is 10. Or else, the ⟨numbers of⟩ horses are multiplied by ⟨an assumed number,⟩ ten; ⟨then⟩ the price of an elephant is produced, 100. The equal property is 550 ⟨in either case⟩.

[157] h = horses, *rū* = *rūpas*.

[158] e = elephants, h = horses, *rū* = *rūpas*.

A versified rule:

28. When one has fixed the property of one ⟨of two persons⟩ according to one's will, that ⟨property⟩, decreased by the *rūpas* of the other and divided by the ⟨number of⟩ horses, is the price ⟨of a horse⟩ if the two ⟨persons⟩ have equal properties. From mutual multiplication ⟨of the numbers of horses possessed by the two persons⟩ there will be ⟨equality of their properties⟩ if there is no *rūpa* ⟨for them⟩.

An example:

29. When the same ⟨numbers of⟩ horses and ⟨of⟩ *rūpas* ⟨as in Example 21⟩[159] are given ⟨to two persons, the latter being given⟩ in inverse order, or else, when they have nine and twenty horses but no *rūpa*, if the two ⟨persons⟩ have equal properties, then tell me the price of a horse separately.

h 6	*rū* 8
h 8	*rū* 100

[160] The price of a horse of the first ⟨person⟩ is assumed to be 50. ⟨Then⟩ the property of the first is 308, and the price of a horse of the second ⟨person⟩ is 26. The equal property is 308. Now, for the second example,

h 9
h 20

The price of a horse of the first ⟨person⟩ is 20, and the price of a horse of the second is 9. The equal property is 180.

A versified rule:

30. Any optional ⟨number⟩ is divided by the number of horses, etc. ⟨separately⟩. The sum of the unit prices of the horses, etc., divided by the sum of the quotients and multiplied by the optional number, is the ⟨equal⟩ price of each ⟨kind of commodity⟩.

An example:

31. If horses, elephants and camels, measured respectively by twelve, two and four, are bought for the same ⟨amount of money⟩, and the sum of the unit prices of the horses, etc. is three hundred, then say that ⟨equal

[159]Note, however, that in this example the price of a horse of one person is assumed to be different from that of the other, while in Example 21 the price of every horse is the same.

[160]h = horses, *rū* = *rūpas*.

amount of⟩ money.

⟨The result⟩ produced is: the ⟨equal⟩ money 360, the price of a horse 30, the price of an elephant 180, and the price of a camel 90.

Now, a versified rule for the rule of three:

> *32.* The standard and the requirement should be made to have the same kind ⟨of measure⟩. The result, multiplied by the requirement and divided by the standard, is the result of the requirement. In the inverse ⟨rule of three⟩, there will be the inverse operation.
> *33.* This is the operation for the rule of three. In the case of ⟨the rules of⟩ five, seven, nine, etc., one should divide the product of the more numerous quantities by the product of the fewer quantities.
> *35ab.* When the result is brought to the side of requirement, the product of ⟨the quantities belonging to⟩ that ⟨side⟩ is what is produced from the more numerous quantities.

An example:

> *34.* If six is ⟨obtained⟩ by means of five, what is ⟨obtained⟩ by means of eight? Or else, if ⟨that is obtained⟩ in one month, then what is ⟨obtained⟩ in ten ⟨months⟩? If the result ⟨is obtained⟩ from three people, then what is ⟨obtained⟩ from five? Say separately.

5
6
8

r. of three

month 1	10
5	8
6	

r. of five

people 3	5
month 1	10
5	8
6	

r. of seven[161]

The result of the rule of three is $\frac{48}{5}$. The result of the rule of five is 96. The result of the rule of seven is 161.

A versified rule:

> *35cd.* The prices are placed in the mutually opposite sides in the case of barter (lit. in the case of commodity and counter-commodity).

An example:

---

[161] r. = rule.

36. When six bilva fruits are ⟨obtained⟩ for seven ⟨*rūpas*⟩ and sixty betel-nuts for three *rūpas* here, you, calculating if you know, bring betel-nuts for three bilva fruits.

⟨The result⟩ produced is 70.

III.5 Practical mathematics of mixture

A versified rule:

38ab. One should divide the property of each one multiplied by the mixture by the sum of the properties. There will be a result for ⟨each⟩ property.

An example:

37. If, ⟨when⟩ five, seven, and nine were offered to a certain person for an equal interest by three men, seventy were obtained ⟨in total⟩ from that ⟨investment⟩, say separately how much was ⟨each⟩ property?

⟨The result:⟩

16	23	30
2	1	
3	3	

A versified rule:

38cd. When an optional ⟨number⟩ is divided by ⟨the number of⟩ the jewels ⟨of each person⟩ decreased by the offer multiplied by ⟨the number of⟩ persons, the ⟨unit⟩ price ⟨of the jewels will be obtained⟩.

An example:

39. When the best of twice-born men, ⟨five in number⟩, who have been presented ⟨respectively⟩ rubies, sapphires, chariots, horses, and elephants measured in order by ten, eight, twelve, fifteen, and seven, have remembered the oath sworn ⟨by themselves⟩ before and have each mutually given one of his own thing, then they will become equal ⟨in property. How much is the unit price of each thing?⟩.

r 10, s 8, c 12, h 15, e 7.[162] The ⟨unit⟩ prices produced in order are: r 42, s 70, c 30, h 21, e 105. The equal property is 478.

[162] r = rubies, s = sapphires, c = chariots, h = horses, e = elephants.

A versified rule:

39[a]. If possible, the quotient from ⟨the division of⟩ the difference of the capitals by the difference of the monthly results will be the time.

An example:

40. Three hundred and two hundred are lent ⟨respectively⟩ on the interest of two and six ⟨per month⟩ by a certain man. If there be equality of the two capitals increased by ⟨each⟩ interest, in what time will it be?

The time obtained is 25 ⟨months⟩.

III.6 Practical mathematics of series

Now, a versified rule in the practical mathematics of series:

41. Half the ⟨number of⟩ terms is multiplied by the ⟨number of⟩ terms increased by unity. That is the sum (*saṅkalita*) ⟨of a natural series⟩ with unity as the increase (i.e., the common difference). ⟨The sum (*saṅkalita*)⟩, multiplied by twice the ⟨number of⟩ terms increased by unity and divided by three, is here the sum of a square ⟨series⟩.
42. The square of the sum (*saṅkalita*) is ⟨calculated⟩ for the sum of a cubic ⟨series⟩. When an even ⟨number of⟩ terms is halved, ⟨the word⟩ "square" should be ⟨put down⟩, and when an odd ⟨number of⟩ terms is decreased by unity, ⟨the word⟩ "multiplier". This operation is ⟨repeated⟩ until the ⟨number of⟩ terms disappears.
43. The result which is produced in the inverse order by multiplication and squaring ⟨as indicated⟩ is decreased by unity and divided by the multiplier decreased by unity and multiplied by the mouth (i.e., the first term). That is here the sum of what has been multiplied in this multiplication ⟨series⟩ (i.e., a geometric progression).

An example:

44. Say separately, in a month, the sum (*saṅkalita*), the sum of the square ⟨series⟩, the sum of the cubic ⟨series⟩, and the sum when the first term, two, is multiplied by five ⟨repeatedly⟩.

The ⟨number of⟩ terms is 30. Saṃ[163] is 465. The sum of the square ⟨series⟩ is 9455. The sum of the cubic ⟨series⟩ is 216225. The sum of the multiplied is 4656612873077392578l2.[164]

A versified rule:

45. The first term, increased by half the increase multiplied by the ⟨number of⟩ terms minus one and multiplied by the number of terms, will be ⟨the sum⟩ for any optional increase as desired. The sum (*saṅkalita*), divided by the ⟨number of⟩ terms and decreased by half the increase multiplied by the ⟨number of⟩ terms minus one, is the first term.
46. The sum (*gaṇita*), divided by the ⟨number of⟩ terms, decreased by the mouth (the first term), and divided by half the ⟨number of⟩ terms minus one, will be the increase. The square of the difference between half the increase and the first term is increased by the sum (*gaṇita*) multiplied by the increase and by two. The square root ⟨of the result⟩ is
48ab. decreased by the mouth (the first term) and increased by half the increase. That ⟨result⟩ divided by the increase will be the number of terms in this case.

An example:

47. A certain man, who has proceeded ten ⟨*yojanas*⟩ on the first day and then with the increase of four ⟨*yojanas*⟩ every succeeding day, comes back to his own place in one month. Tell me. How many *yojanas* ⟨did he travel⟩? ⟨Tell me also⟩ the mouth (the first term), the increase, and the ⟨number of⟩ terms in this case from them (i.e., from the known elements), after having considered well.

2040 *yojanas*.

A versified rule:

48cd. The slower speed is multiplied by the difference of the days ⟨of the departures⟩ and divided by the difference of the speeds. In the days obtained, ⟨there will be⟩ the meeting ⟨of the two travelers⟩.

An example:

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[163] Saṃ is an abbreviation of *saṅkalita*.

[164] The digits for the last answer are missing in the ms.

49. A man whose speed is three *yojanas* ⟨a day⟩ starts ⟨first⟩ and later, when as long as three months have elapsed, another man starts. The speed ⟨of the latter⟩ is nine ⟨*yojanas*⟩ a day. In how many days does the meeting of the two ⟨men take place⟩? Tell me quickly if you are versed well ⟨in this computation⟩.

45 days.

A versified rule:

50ab. The constant speed is multiplied by two, decreased by twice the mouth (the first term), increased by the increase, and divided by the increase. ⟨The result is the time required for the meeting of the two travelers⟩.

An example:

51ab. A man goes a hundred ⟨*yojanas*⟩ a day, and another ⟨goes one *yojana* on the first day and⟩ increases ⟨it⟩ by five ⟨*yojanas* every succeeding day⟩. When will the meeting of the two ⟨persons⟩ be?

A versified rule:

50cd. The ⟨constant⟩ speed is multiplied by two and decreased by unity. ⟨In the days obtained there will be⟩ the meeting of the two ⟨persons⟩ when the increase ⟨and the first term are⟩ unity ⟨in the previous rule⟩.

An example:

51cd. If Aśvinī, who has given birth to a child, goes a hundred ⟨*yojanas*⟩ on the road ⟨every day⟩ while the boy ⟨goes one *yojana* on the first day and⟩ increases ⟨it⟩ by one ⟨*yojana* every succeeding day⟩, when will the meeting of the two ⟨persons⟩ be?

199 days.

III.7 Practical mathematics of plane figures

A versified rule for the practical mathematics of plane figures:

52. The ear (hypotenuse) ⟨of a right-angled triangle⟩ is the square root of the sum of the squares of the arm (one of the two orthogonal sides) and the upright. Likewise, the square root of the difference between the squares of the arm and the ear is the upright, and ⟨that⟩ of the ear and the upright is the arm.

An example:

53. Tell me the ear when the upright is sixteen and the arm is twelve; the upright from the arm and the ear; and the arm from the ear and the upright.

Now, when the sum or the difference of any two ⟨sides⟩ and ⟨the remaining⟩ one of the arm, the upright and the ear are known, a versified rule for separating ⟨the two whose sum or the difference is given⟩:

54. The square of the upright, when divided by the sum of the arm and the ear, is their difference. Likewise, the square of the upright, when divided by their difference, is their sum.
55. The square of the arm, when divided by the sum of the upright and the ear, is their difference, but the square of the arm divided by the difference of the ear and the upright is their sum.
56. The square root from twice the square of the ear decreased by the square of the sum of the arm and the upright should be known as their difference. Likewise, by means of the difference ⟨of the arm and the upright⟩, the sum ⟨of them is calculated⟩.
57. Then, one should calculate (lit. make) the two ⟨unknown⟩ quantities by means of the rule of concurrence. Then, half of the product of the arm and the upright will be the area (lit. the field-fruit) in that case.

The arm is 12, the upright 16, and the area 96.

Now, a versified rule for the areas of ⟨regular polygons such as⟩ an equilateral trilateral, etc.:

58ab. The square of a side (lit. arm), multiplied by thirty-three=four (433) ⟨and divided by a thousand is the area of an equilateral trilateral. ...⟩

58cd ⋮ *64* } Missing.

... . The circumference is 31;29. The diameter for ⟨this⟩ circumference is 10. The area is 78;42. The area (lit. the fruit) on the sphere is 314;50.

A versified rule:

> *65.* The square of the diameter multiplied by three and increased by one twentieth ⟨of itself⟩ is a gross area on a sphere. That multiplied by the diameter and divided by six is the solid content inside the sphere.

The gross sphere-fruit (i.e., surface) is ⟨315, and⟩ the accurate one is 314;50. The accurate solid-fruit (i.e., volume) is 524;47 (sic).

(Here is Figure 1.)[165]

Now, computation of chords, the diameter and arrows in a circle:

> *66.* The ear[166] decreased by the square root of the product of the sum and the difference of the diameter and the chord, halved, is the arrow. The square root of the arrow multiplied by the diameter decreased by the arrow, multiplied by two,
> *67.* is the chord. The arrow increased by the quotient of the division of the square of half of it (i.e., the chord) by the arrow is the diameter. The square of the circumference[167] multiplied by a quarter of the chord and by five is divided by the chord increased by the product of the ear and four.
> *68.* A quarter of the square of the circumference is decreased by it (i.e., the quotient of the division). The square root of this ⟨is taken⟩. Half of the circumference decreased by it will be the bow (i.e., the arc). I will tell the area in this (i.e., of this figure) ⟨in the next rule⟩.

An example:

> *69.* In a circular figure whose diameter is ten, there is a chord of eight. Tell the arrow in that case, and ⟨also tell⟩ separately the chord from the arrow, the diameter from the arrow and the chord, and the size of the bow.

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[165] See Figure 1 in Section II. Statements in the figure: Circumference 31;29. Diameter 10. Field-fruit (i.e., area) 78;42. Fruit on the sphere (i.e., surface) 314;50. Solid fruit inside the sphere (i.e., volume) 524 (sic).

[166] The "ear" in this context means the diameter as the hypotenuse of a right triangle inscribed in a circle.

[167] The word *paridher* for "of the circumference" is placed at the beginning of the next verse.

The diameter is 10, the arrow 2, the chord 8, and the bow 9;15.

(Here is Figure 2.)[168]

A versified rule for the fruit of a bow (i.e., the area of a segment of a circle):

> *70.* Half of the sum of the bow (arc) and the chord, ⟨put down⟩ twice, is increased ⟨in one place⟩ and decreased ⟨in the other⟩ by the arrow. The square root of their product multiplied by the square of the arrow, multiplied by two and divided by three, is the fruit in the bow (i.e., the area of a segment of a circle).
> *70*^{a}. Or, a gross ⟨area of a segment of a circle⟩ is the arrow multiplied by half the sum of the arrow and the chord, increased by one twentieth ⟨of itself⟩.

An example:

> *71.* ⟨In a bow-like figure⟩ where the arrow is measured by twenty-five, the bow (arc) by a hundred and five, and the chord by seven=eight (87), O friend, tell ⟨me⟩ the field-fruit (the area) ⟨of the segment⟩.

The bow-fruit is 1545.

(Here is Figure 3.)[169]

Now, a versified rule for the fruit in a fish-field (i.e., the area of a fish-like plane figure):

> *73ab.* In a fish-field, there is a line through the tail and the mouth of the fish. That shall be the chord. The sum of the areas of those ⟨two halves obtained⟩ as in the case of a bow⟨-like figure is the fruit of the fish-like figure⟩.

An example:

> *72.* ⟨In a fish-like figure⟩ where the line ⟨through the tail and the mouth⟩ is measured by sixty-six, the bow (arc) at the belly of the fish by seventy-three, and the arrow by twenty-five, how much is the fruit?

[168] See Figure 2 in Section II. Statements in the figure: Diameter 10. Chord 8. Bow 915 (uncorrected). Arrow 2.

[169] See Figure 3 in Section II. Statements in the figure: Arrow 25. Chord 87. Fruit in the bow-like field 1545. Or, the gross ⟨area⟩ 1470.

The area is 1156.

(Here is Figure 4.)[170]

Now, a versified rule for the fruit in a moon-digit-field (i.e., the area of a crescent-shaped plane figure):

> *73cd.* In a moon-field, when one has regarded the lower bow (arc) as the chord and obtained ⟨its area⟩ as in the case of a bow (i.e., a segment of a circle), that will be an accurate ⟨area of the crescent-shaped figure⟩.

An example:

> *74.* When the ⟨upper⟩ bow is measured by nine=twenty (209) and ⟨the lower one⟩ by five=ten (105), and the arrow by zero=five (50) in a crescent-shaped figure (lit. in digits of the moon), then tell me the fruit if you are versed well in mathematics.[171]

The area is 4960.

(Here is Figure 5.)[172]

Now, a versified rule for a drum-field (i.e., a drum-shaped plane figure):

> *75.* The sum of the ⟨two⟩ side arms ⟨of a drum-shaped figure⟩ is ⟨regarded as⟩ the bow (i.e., the arc), half the face (the top line) as the arrow, and twice the ear[173] as the chord. Then, the accurate fruit in the drum field should be obtained as in the case of a bow (i.e., a segment of a circle).

An example:

> *76.* In a drum⟨-shaped figure⟩, its face is half of a hundred, the sum of the side arms is measured by five=ten (105), and the ear is measured by three=four (43) with a half. In that case, tell me, how much shall be the fruit?

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[170]See Figure 4 in Section II. Statements in the figure: Bow 73. Arrow 25. Line 66. Bow 73. Field-fruit 1156.

[171]Or, "if your speed in calculation is extra⟨fast⟩."

[172]See Figure 5 in Section II. Statements in the figure: Bow 209. Arrow 50. Bow 105. Field-fruit 4960.

[173]The "ear" in this context means the depth (or height) of the drum.

The fruit is 1545.

(Here is Figure 6.)[174]

III.8 Rules for shadows

Now, a versified rule on the lamp-shadow operation:

> *77.* From the gnomon multiplied by the distance between the lamp and the gnomon and divided by the lamp decreased by the gnomon, the shadow will be ⟨obtained⟩. Or else, the result (i.e., the quotient) from that (i.e., the gnomon multiplied by the distance between the lamp and the gnomon) divided by the shadow is increased by the gnomon. It (the result) will be the height of the lamp.
> *79ab.* The lamp decreased by the gnomon, when multiplied by the shadow and divided by the gnomon, becomes the ground between the lamp and the gnomon.

An example:

> *78.* The ground between the bases of the gnomon and the lamp is measured by thirty-two. and the height of the lamp here by zero=six (60). Tell the shadow of a gnomon of twelve digits (*aṅgulas*), and ⟨conversely⟩ the height of the lamp from this and ⟨also⟩ the distance ⟨between the lamp and the gnomon⟩ here.

The ground between the lamp and the gnomon is 32, the height of the lamp 60, and the shadow 8.

A versified rule:

> *79cd.* The gnomon multiplied by the shadow of a bamboo and divided by the shadow of the gnomon is the measure of the bamboo here.

An example:

> *80.* If, on level ground, the shadow of a bamboo is equal to zero=sixteen (160), and when the shadow of a gnomon measured by ten is equal to four, then, calculator, tell, in front of me, how much shall be the measure of the bamboo, if, O wise man, you are the best in mathematical operations.

[174] See Figure 6 in Section II. Statements in the figure: Face 50. Ear 43;30. Side arm 52;30. Arm 52;30. Area 1545.

The measure of the bamboo is 400.

III.9 Rules for magic squares

Now, a versified rule on the operation for magic squares[175] :

> *81.* The digits in order beginning with the first should be written down ⟨on the first horizontal row⟩. Having written down ⟨the next digit in the cell immediately⟩ below the last cell of the row[176] and having filled in ⟨the cells on⟩ its back side, ⟨one should apply the same procedure to the succeeding rows⟩ again and again.
> *82.* In this way, the sum of the digits ⟨in each vertical line⟩ will be the same whether in odd (*viṣama*) or in even (*sama*)[177] ⟨squares⟩. Its first ⟨column⟩ should be placed at the middle ⟨vertical⟩ line (column) ⟨of a new square⟩, and thence, in order,
> *83.* the ⟨vertical⟩ lines (columns) to the left should be accomplished with the ⟨succeeding columns of the original square⟩ from which one ⟨succesive⟩ cell ⟨in each⟩ has fallen away (that is, one top cell of the second column, two top cells of the third column, etc. have been shifted to the bottom of the same column). This rule has been said in the case of odd magic squares.
> *84.* In the case of even[178] ⟨magic squares⟩, one should write down the digits in pairs ⟨into cells⟩ beginning with the first cell in one direction at intervals of two of them (cells), and thence ⟨proceed⟩ in the other direction to the left in the same manner.

Now, the setting-down of ⟨the quasi-magic and magic squares with⟩ three to nine cells ⟨on their sides⟩.

(Here are Figures 7–20.)[179]

An example.

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[175] *sarvatobhadra*, lit. "good for all directions or purposes".

[176] *paṅkti*, which means a line or sequence in general. The same word is used also for vertical lines by Giridhara. See Verses 82 and 83 below.

[177] Here, "even" includes both "evenly-even" and "oddly-even".

[178] Here, "even" means "evenly-even" only.

[179] See Figures 7–20 in Section II.

85. The revered king (*śāha*) Śrīdāni,[180] whose feet are illuminated by jewels on the heads of the best kings ⟨who bow down to him⟩, gave away eighty-one towns to nine devotees, ⟨in other words⟩, a property of an equal ⟨number of⟩ towns ⟨to each⟩ with an increase of ⟨successive⟩ differences of one ⟨in their values⟩. Tell me, man of stainless intelligence, the property of each, if you are well versed in the seed ⟨mathematics⟩ (i.e., algebra) or in the rule of exchange (Verses 81–83) or else from ⟨their⟩ mutual ⟨interaction⟩.

A versified rule.

86. The quantity (i.e., the constant sum), divided by the fruit (i.e., the sum) of the ⟨natural⟩ series ⟨up to the square of the length of the line minus one⟩ divided by the ⟨length of the⟩ line, is the increase (i.e., the common difference) ⟨of the arithmetical progression to be used in a magic square⟩. The remainder ⟨of the division⟩, divided by the ⟨length of the⟩ line, is the mouth (i.e., the first term). Then, the filling ⟨of the cells of a square with the terms of this arithmetical progression is done⟩ as before.

The setting-down for the sum equal to three hundred in a diagram with three rows:

(Here is Figure 21.)[181]

Or for the sum, fifty-one:

(Here is Figure 22.)[182]

The setting-down for the sum, three hundred, in a diagram with four rows:

(Here is Figure 23.)[183]

And for the sum, fifty:

(Here is Figure 24.)[184]

[180]See Verse 21 for the same word.

[181]See Figure 21 in Section II.

[182]See Figure 22 in Section II.

[183]See Figure 23 in Section II.

[184]See Figure 24 in Section II.

III.10 Concluding remarks

87. The ⟨mathematics of⟩ unknown ⟨quantities⟩ has been told ⟨here⟩ indeed without the use of unknown ⟨symbols⟩; the height of a bamboo, etc., without instrumental means. This something which is not to be apprehended with ⟨even⟩ one difficulty has been told by me without a lot of difficulty for the pleasure of those who know this ⟨subject⟩.
88. What has been told by me in summary ⟨in this work⟩ should be revised without hostility by the wise men who have investigated ⟨it⟩ carefully. This is my request to them.
89. Let this work of Giridhara, which consists of a limited number of letters, which is deep in its meaning, and which gives pleasure to the hearts of wise people, shine as long as the Lord of the day (i.e., the Sun) ⟨shines⟩.

Thus ends the *Caturacintāmaṇi* composed by Giridharabhaṭṭa, the son of Śrī Bīrābhaṭṭa (= Vīrabhaṭṭa). Let there be prosperity.

IV Mathematical Commentary

Giridharabhaṭṭa, it should be noted, never uses algebraic expressions in this work even when he treats algebraic problems, although Indian mathematicians had developed and skillfully employed algebraic symbolism in algebra (bīja-gaṇita or avyakta-gaṇita) since, at least, the seventh century A.D. This is one of the main features of this work as he himself emphasizes in one of the concluding verses (Verse 87). In this Commentary, however, I shall use modern algebraic notation in order to make it easier for the reader to understand the problems treated and the solutions given. Also, multiplication of a by b is indicated by $a \cdot b$ or $a \times b$ or ab, and division of a by b by a/b or $a \div b$ or $\frac{a}{b}$.

IV.1 Introduction (v. 1)

Verse 1. Introductory salutation.

IV.2 Weights and measures (v. 2)

Verse 2. Weights and measures.

Giridhara simply says that one should perform calculations after having made either composition (*kalpanā*) or contraction (*apavartanaka*) of ratios circulated in a specific area (*deśa*) at a specific time (*kāla*).

IV.3 Eight elementary operations (v. 3)

Verse 3. Eight elementary operations for integers and fractions.

Giridhara does not give any specific rules but simply says that the rules for these calculations should be known "from the traditional instruction of good teachers" (*sadgurusampradāyāt*).

IV.4 Miscellaneous operations (vv. 4–36)

Verse 4ab. Rule of inversion.

Type of problem. To know the original quantity when the result of an operation (or of a series of operations) performed on the original quantity is known.

Solution.

$$x + a = b \to x = b - a, \quad x - a = b \to x = b + a,$$

$$x \times a = b \to x = b \div a, \quad x \div a = b \to x = b \times a.$$

Note. Rules for the remaining operations may have been contained in the second line of the same verse, which is lost.

Verses 4cd–15 and *17ab*. Missing.

Verse 16. Example. Answer: 1024. Not understood.

Verse 17cd. Rule for divisibility (an indeterminate problem).

Type of problem. A total sum of money, x, is divided equally among a_1 brāhmaṇas without residue on one occasion, and the same amount is again divided equally among a_2 brāhmaṇas, a_3 brāhmaṇas, etc. on the succeeding occasions. What is that amount?

$$x = a_1 q_1 = a_2 q_2 = \cdots = a_n q_n.$$

Solution.

$$x = a_1 a_2 \cdots a_n.$$

Verse 18. Example. Given: $a_1 = 9$, $a_2 = 13$, $a_3 = 14$, and $a_4 = 25$. Solution: $x = 9 \times 13 \times 14 \times 25 = 40950$.

Verse 19. Rule for the property of a traveling merchant.

Type of problem. A man visits n towns. At each town, he spends a certain amount of money (a), multiplies the remainder by m, and again spends the same amount of money (a). What remains in his hand at the end of his journey is m' times his original property (x). What is the original property?

$$x_0 = x,$$

$$x_i = (x_{i-1} - a) \times m - a \quad (i = 1, 2, \ldots, n),$$

$$x_n = m'x.$$

Solution. The quantity, a, is written above 1: $\begin{bmatrix} a \\ 1 \end{bmatrix}$. This arrangement seems to have been motivated by the division to be made at the last step of this solution. The upper quantity is called 'the given' (*datta*) since that amount of money is 'given away' (i.e., spent) twice in each town, while the lower is called 'the optional' (*iṣṭa*), presumably because it has originally been chosen as a tentative solution for x (see Note below). To this pair of quantities, the following operation is applied:

$$\begin{bmatrix} a \\ 1 \end{bmatrix} \to \begin{bmatrix} b_1 \\ m \end{bmatrix} \to \begin{bmatrix} b_2 \\ m^2 \end{bmatrix} \to \cdots \to \begin{bmatrix} b_n \\ m^n \end{bmatrix},$$

where $b_1 = a \times m + a$, $b_2 = (b_1 + a) \times m + a$, $\cdots$, $b_n = (b_{n-1} + a) \times m + a$. Then the lower number is decreased by 'the multiplier' (*guṇa*), m', and the upper number is divided by the lower; the result is the solution:

$$x = \frac{b_n}{m^n - m'}.$$

Note. The word *iṣṭa* ("optional") suggests that the above solution is based on the so-called iṣṭa-karman or "computation by an optional number" (cf. Verse 30 below). Tentatively let q be the solution of x. Then, on the one hand, we have, $x_1 = qm - b_1$, $x_2 = qm^2 - b_2, \cdots, x_n = qm^n - b_n$, where b_n is independent of the q; and on the other, $x_n = qm'$. Hence $qm^n - b_n = qm'$, or $qm^n - qm' = b_n$, if the q happens to be the true solution of x. If it is not, the ratio of the true solution of x to the q is equal to that of b_n to $qm^n - qm'$, that is,

$$x = \frac{b_n}{qm^n - qm'} \times q.$$

Taking $q = 1$, we have the above solution.

In his *Gaṇitamañjarī* (148–150, Ms: Eggeling 2771, fol. 26), Gaṇeśa (ca. 1575) treats the same problem and prescribes a similar solution. He calls the m 'the multiplier', the m^n 'the first multiplier', and the m' 'another multiplier'.

> *sthāpayen nagarajaṃ guṇakāraṃ sthānakeṣu nagarapramiteṣu /*
> *taddhatiḥ prathamasaṃjñaguṇaḥ syād yadguṇaṃ dhanam asau guṇako 'nyaḥ // 148 //*
> *ālāpavad budhaiḥ kāryaṃ dattavittasya melane /*
> *ādyānyaguṇaviśleṣavihṛtaṃ prāktanaṃ dhanaṃ // 149 //*
>
> One should place the multiplier caused by the towns in as many places as the towns. Their product shall be the multiplier called the first. That by which the ⟨original⟩ property is multiplied is another multiplier. ⟨Computation⟩ should be performed by the wise according to the statement ⟨in the problem⟩, while addition, ⟨instead of subtraction⟩, of the money given away ⟨is being made⟩. ⟨The result⟩ divided by the difference of the first and another multipliers is the original property.

A similar problem occurs in the *Bakhshālī Manuscript* (Example 1 for Sūtra C1), where it is solved by means of the rule of inversion (cf. Verse 4ab above), and in Bhāskara II's *Bījagaṇita* (101), where it is solved by means of a linear equation with the algebraic expression, *yā* (<*yāvattāvat*), for the unknown number (see Hayashi 1995a, 415).

Verse 20. Example. Property of a traveling merchant. Given: $n = 3$, $a = 10$,

$m = m' = 2$. That is to say,

$$[\{(\underbrace{x - 10) \times 2 - 10}_{\text{1st town}} \underbrace{- 10\} \times 2 - 10}_{\text{2nd town}} \underbrace{- 10] \times 2 - 10}_{\text{3rd town}} = 2x.$$

Solution: $x = 35$. Note: The working process, which is not given in the ms., must have been as follows. $\begin{bmatrix} 10 \\ 1 \end{bmatrix} \to \begin{bmatrix} 30 \\ 2 \end{bmatrix} \to \begin{bmatrix} 90 \\ 4 \end{bmatrix} \to \begin{bmatrix} 210 \\ 8 \end{bmatrix}$. Therefore, $x = 210/(8-2) = 35$. In Gaṇeśa's example (*Gaṇitamañjarī* 150), $n = 4$, $a = 10$, $m = 2$, $m' = 7$, and the answer, $x = 50$.

Verse 22ab. Rule for a linear equation of properties.

Type of problem. One equation with one unknown number. Each of two persons has money and one kind of article, whose unit price is unknown, and their properties are equal to each other.

$$ax + b = cx + d \ \langle = y \rangle.$$

Solution.

$$x = (d - b) \div (a - c).$$

Note. Verses 22, 24, 26, and 28 treat equations of properties, some of which are indeterminate. Giridhara seems to have had the following scheme in mind.

Verses	Persons	Articles	Equations	Unknowns
22ab	$m = 2$	$n = 1$	$m - 1 = 1$	$n = 1$
22cd	$m = 2$	$n > 1$ (3)	$m - 1 = 1$	$n > 1$ (3)
24	$m > 2$ (4)	$n = 1$	$m - 1 > 1$ (3)	$n = 1$
26	$m > 2$ (3)	$n > 1$ (2)	$m - 1 > 1$ (2)	$n > 1$ (2)
28	$m = 2$	$n = 2$	$m - 1 = 1$	$n = 2$

Here, the equations are classified according to the number of persons (m) or of equations ($m - 1$) and to that of articles (n) or of unknown numbers (n) involved. The numbers in parentheses are their values in the example accompanying each rule.

Verse 21. Example. Two persons with money and horses. Given: $6x + 100 = 8x + 8$ ($= y$). Solution: $x = 46$, $y = 376$. Note: The working process must have been as follows (h = horses, *rū* = *rūpas*). $\begin{bmatrix} \text{h} & 6 & \textit{rū} & 100 \\ \text{h} & 8 & \textit{rū} & 8 \end{bmatrix} \to \begin{bmatrix} \text{h} & 0 & \textit{rū} & 92 \\ \text{h} & 2 & \textit{rū} & 0 \end{bmatrix}$. Therefore, the price of a horse $= 92/2 = 46$ *rūpas*, and the equal property $= 6 \times 46 + 100 = 376$ *rūpas*.

Verse 22cd. Rule for a linear equation of properties (indeterminate).

Type of problem. One equation with more than one unknown number. Each of two persons has money and more than one kind of article, whose unit prices are unknown, and their properties are equal to each other. That is to say, for $n > 1$,

$$a_1x_1 + a_2x_2 + \cdots + a_nx_n + b = c_1x_1 + c_2x_2 + \cdots + c_nx_n + d \ \langle = y \rangle.$$

Solution. Let $x_i = e_i$ for $i > 1$. Then, the equation is reduced to the first type (Verse 22ab).

$$a_1x_1 + (a_2e_2 + \cdots + a_ne_n + b) = c_1x_1 + (c_2e_2 + \cdots + c_ne_n + d) \ \langle = y \rangle.$$

Note. Under the next verse, Giridhara gives the condition for e_i in order to make x_1 positive. That is, e_i should be determined in such a way that $(a_2e_2+\cdots+a_ne_n+b)$ is smaller or greater than $(c_2e_2 + \cdots + c_ne_n + d)$ according to whether a_1 is greater or smaller than c_1,

Verse 23. Example. Two persons with money, jewels, elephants and horses. Given: $x_1+2x_2+6x_3+100 = 2x_1+3x_2+8x_3+8$ $(= y)$. Solution: Let $x_2 = 20$ and $x_3 = 10$, then the equation is reduced to: $x_1 + 200 = 2x_1 + 148$. Therefore, $x_1 = 52$ and $y = 252$. Note: The working process must have been as follows (j = jewels, e = elephants, h = horses, *rū* = *rūpas*). The "setting-down" is: $\begin{bmatrix} \text{j} & 1 & \text{e} & 2 & \text{h} & 6 & \textit{rū} & 100 \\ \text{j} & 2 & \text{e} & 3 & \text{h} & 8 & \textit{rū} & 8 \end{bmatrix}$. Let the price of an elephant = 20 *rūpas* and that of a horse = 10 *rūpas* (v. 22cd). Then, $\begin{bmatrix} \text{j} & 1 & \textit{rū} & 200 \\ \text{j} & 2 & \textit{rū} & 148 \end{bmatrix} \xrightarrow{\text{v.22ab}} \begin{bmatrix} \text{j} & 0 & \textit{rū} & 52 \\ \text{j} & 1 & \textit{rū} & 0 \end{bmatrix}$. Therefore, the price of a jewel = 52 *rūpas* and the equal property = 52 + 200 = 252 *rūpas*.

Verse 24. Rule for a system of linear equations of properties.

Type of problem. More than one equation with one unknown number. Each of more than two persons has money and one and the same kind of article, whose unit price is unknown, and their properties are equal to each other. That is to say, for $m > 2$,

$$a_1x + b_1 \ \langle = y \rangle,$$

$$a_2x + b_2 \ \langle = y \rangle,$$

$$\cdots$$

$$a_mx + b_m \ \langle = y \rangle.$$

Solution. Combine the "persons" two by two. That is to say, if m is even,

$$(a_1 + a_2)x + (b_1 + b_2) \langle = 2y \rangle,$$

$$(a_3 + a_4)x + (b_3 + b_4) \langle = 2y \rangle,$$

$$\dots$$

$$(a_{m-1} + a_m)x + (b_{m-1} + b_m) \langle = 2y \rangle.$$

If m is odd, on the other hand, Giridhara perhaps added one of the m "sides" twice (cf. Note for Verse 27 below), although this case is not mentioned in the rule. By repeating the same procedure, one finally arrives at two "sides",

$$ax + b = \langle ky = \rangle\ cx + d,$$

to which the rule of Verse 22ab is applied.

Verse 25. Example. Four persons with money and horses. Given: $2x + 132 = 4x + 84 = 5x + 60 = 7x + 12$ $(= y)$. Solution: $x = 24$ and $y = 180$. Note: The working process must have been like this (h = horses, *rū* = *rūpas*).

$$\begin{bmatrix} \text{h} & 2 & \textit{rū} & 132 \\ \text{h} & 4 & \textit{rū} & 84 \\ \text{h} & 5 & \textit{rū} & 60 \\ \text{h} & 7 & \textit{rū} & 12 \end{bmatrix} \xrightarrow{\text{v.24}} \begin{bmatrix} \text{h} & 6 & \textit{rū} & 216 \\ \text{h} & 12 & \textit{rū} & 72 \end{bmatrix} \xrightarrow{\text{v.22ab}} \begin{bmatrix} \text{h} & 0 & \textit{rū} & 144 \\ \text{h} & 6 & \textit{rū} & 0 \end{bmatrix}.$$

Therefore, the price of a horse $= 144/6 = 24$ *rūpas* and the equal property $= 24 \times 2 + 132 = 180$ *rūpas.*

Verse 26. Rule for a system of linear equations of properties (indeterminate).

Type of problem. More than one equation with more than one unknown number. Each of more than two persons has money and more than one kind of article, whose unit prices are unknown, and their properties are equal to each other. That is to say, for $m > 2$ and $n > 1$,

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + b_1 \langle = y \rangle,$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + b_2 \langle = y \rangle,$$

...

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + b_m \ \langle = y \rangle.$$

Solution. Let $x_i = c_i$ for $i > 1$, then the problem is reduced to the previous case (Verse 24).

Note. Giridhara's rule itself is designed for the case, $n = 2$, but he seems to have had the general case in mind.

Verse 27. Example. Three persons with money, elephants, and horses. Given: $2x_1 + 7x_2 + 280 = 4x_1 + 10x_2 + 50 = 5x_1 + 2x_2 + 30$ $(= y)$. Solution: Let $x_1 = 100$, then $x_2 = 10$. Let $x_2 = 10$, then $x_1 = 100$. In either case, $y = 550$. Note: It is not mentioned that x_1 and x_2 can not be otherwise assumed. The working process must have been like this (e = elephants, h = horses, *rū* = *rūpas*). The "setting-down" is:

$$\begin{bmatrix} \text{e} & 2 & \text{h} & 7 & \textit{rū} & 280 \\ \text{e} & 4 & \text{h} & 10 & \textit{rū} & 50 \\ \text{e} & 5 & \text{h} & 2 & \textit{rū} & 30 \end{bmatrix}.$$

Let the price of an elephant = 100 *rūpas* (v. 26). Then,

$$\begin{bmatrix} \text{h} & 7 & \textit{rū} & 480 \\ \text{h} & 10 & \textit{rū} & 450 \\ \text{h} & 2 & \textit{rū} & 530 \end{bmatrix} \xrightarrow{\text{v.24}} \begin{bmatrix} \text{h} & 17 & \textit{rū} & 930 \\ \text{h} & 12 & \textit{rū} & 980 \end{bmatrix} \xrightarrow{\text{v.22ab}} \begin{bmatrix} \text{h} & 5 & \textit{rū} & 0 \\ \text{h} & 0 & \textit{rū} & 50 \end{bmatrix}.$$

Therefore, the price of a horse $= 50/5 = 10$ *rūpas* and the equal property $= 10 \times 7 + 480 = 550$ *rūpas*.

Verse 28. Rule for a linear equation of properties (indeterminate).

Type of problem. One equation with two unknown numbers. Each of two persons has money and one kind of article, whose unit price is unknown, and their properties are equal to each other.

$$ax_1 + b = cx_2 + d \ \langle = y \rangle.$$

Solution. Let $x_1 = k$, then $x_2 = \{(ak + b) - d\} \div c$. If $b = d = 0$, then $x_1 = c$ and $x_2 = a$.

Verse 29. Examples. Two persons with money and horses. 1st example — Given: $6x_1 + 8 = 8x_2 + 100$ $(= y)$. Solution: Let $x_1 = 50$, then $x_2 = 26$ and $y = 308$. 2nd example — Given: $9x_1 = 20x_2$ $(= y)$. Solution: $x_1 = 20$, $x_2 = 9$, $y = 180$. Note: The working process for the first example must have been as follow (h_1 = horses of the 1st person, h_2 = horses of the 2nd person, although this distinction was not made in Giridhara's notation). The "setting-down" is: $\begin{bmatrix} \text{h}_1 & 6 & r\bar{u} & 8 \\ \text{h}_2 & 8 & r\bar{u} & 100 \end{bmatrix}$. Let the price of a horse of the 1st person be 50 *rūpas*. Then, $\begin{bmatrix} \text{h}_1 & 0 & r\bar{u} & 308 \\ \text{h}_2 & 8 & r\bar{u} & 100 \end{bmatrix} \rightarrow$ $\begin{bmatrix} \text{h}_1 & 0 & r\bar{u} & 208 \\ \text{h}_2 & 8 & r\bar{u} & 0 \end{bmatrix}$. Therefore, the price of a horse of the 2nd person is $208/8 =$ 26 *rūpas* and the equal property 308 *rūpas*.

Verse 30. Rule for a system of linear equations.

Type of problem. The unit prices, x_i, of n kinds of commodities are unknown, but their sum, p, is known. When a_i of the i-th commodity ($i = 1, 2, \ldots, n$) are bought (or sold) for the same amount of money (y) for every i, what is that amount?

$$x_1 + x_2 + \cdots + x_n = p, \quad a_1x_1 = a_2x_2 = \cdots = a_nx_n = y.$$

Solution. Let q be any optional number, and calculate:

$$p' = \frac{q}{a_1} + \frac{q}{a_2} + \cdots + \frac{q}{a_n}.$$

Then,

$$y = p \div p' \times q.$$

Note. This is the "computation by an optional number" (iṣṭa-karman), though this term does not occur in the extant portion of the present work. Each price, x_i, can be obtained by $x_i = y/a_i$. This is not mentioned in Verse 30, but the solution for x_i is given in the example in Verse 31. For a similar use of the iṣṭakarman, see Hayashi 1995a, 396–399. Cf. also Verse 19 above.

Verse 31. Example. Prices of horses, elephants and camels. Given: $x_1 + x_2 + x_3 =$ 300, $12x_1 = 2x_2 = 4x_3 = y$. Answer: $y = 360$, $x_1 = 30$, $x_2 = 180$, $x_3 = 90$.

Verses 32–33, and *35ab*. Rule of three and its variations.

Rule of three:

Type of problem. When b is obtained from a, what (x) is obtained from c (if b increases in proportion to a when a increases) ?

$$a : b = c : x.$$

Solution. $x = b \times c \div a$.

Note. The three given quantities, a, b, and c, are called in order the "standard" (*pramāṇa*), the "result of standard" (*pramāṇa-phala*) or simply the "result", and the "requirement" (*icchā*). The three quantities are usually arranged horizontally as:

a	b	c

But in the present manuscript they are arranged vertically as:

a
b
c

.

The vertical arrangement of the three terms for the rule of three is very rare but does occur twice under Sūtra N19 in the *Bakhshālī Manuscript*. See Hayashi 1995a, 413.

Inverse rule of three:

Type of problem. When b is obtained from a, what (x) is obtained from c (if b decreases in inverse proportion to a when a increases) ?

$$a :: b = c :: x.$$

Solution. $x = b \times a \div c$.

Note. The three given quantities seem to have been arranged vertically as in the case of the rule of three, although no example for this case occurs in the present manuscript.

Rule of five:

Type of problem. When b is obtained from a in time period d, what (x) is obtained from c in time period e ?

Solution. The five terms are arranged in two vertical columns as:

d	e
a	c
b	

, where the left column is called the "standard-side" (*pramāṇa-pakṣa*) and the right column the "requirement-side" (*icchā-pakṣa*). Then the "result" (b) is moved to the opposite side, and then the product of the elements of the longer (right) side is divided by that of the shorter (left) side. That is, $x = (b \times c \times e) \div (a \times d)$.

Rule of seven:

Type of problem. When b is obtained by (or from) f persons from a in time period d, what (x) is obtained by (or from) g persons from c in time period e ?

Solution. Just as in the case of the rule of five, the seven terms are arranged in two vertical columns as:

f	g
d	e
a	c
b	

, and the same procedure is followed. $x = (b \times c \times e \times g) \div (a \times d \times f)$.

Verse 34. Examples. Rules of three, five, and seven. 1st example (rule of three) — Given: $a = 5$, $b = 6$, $c = 8$. Answer: $x = 48/5$. 2nd example (rule of five) — Given: $a = 5$, $b = 6$, $c = 8$, $d = 1$ month, $e = 10$ months. Answer: $x = 96$. 3rd example (rule of seven) — Given: $a = 5$, $b = 6$, $c = 8$, $d = 1$ month, $e = 10$ months, $f = 3$ persons, $g = 5$ persons. Answer: $x = 161$.

Verse 35cd. Rule for barter.

Type of problem. A certain quantity, a, of commodity I cost b, and quantity c of commodity II cost d. What amount, x, of commodity II is bartered for the amount, e, of commodity I ?

Solution. The five given terms are arranged in two vertical columns:

a	c
b	d
e	

where each column represents each commodity. To this is applied the procedure prescribed for the rules of five, etc., with an additional step. The additional step, which only is mentioned in Verse 35cd, is the exchange of the prices (b and d), that is, they are moved to the mutually opposite sides before the computation. Thus,

a	c
d	b
	e

, and $x = (b \times c \times e) \div (a \times d)$.

Verse 36. Example. Barter of bilva fruits and betel-nuts. Given: $a = 6$, $b = 7$ *rūpakas*, $c = 60$, $d = 3$ *rūpakas*, $e = 3$. Answer: $x = 70$.

IV.5 Practical mathematics of mixture (vv. 37–40)

Verse 38ab. Rule for proportional ditribution.

Type of problem. Each of n persons invests a_i for a joint enterprise, which produces p. They divide the p in proportion to the amount of each investment. What (x_i) is for each person ?

Solution.

$$x_i = (p \times a_i) \div \sum_{j=1}^{n} a_j.$$

Verse 37. Example. Three money-lenders. Given: $a_1 = 5$, $a_2 = 7$, $a_3 = 9$, $p = 70$. Answer: $x_1 = 16\frac{2}{3}$, $x_2 = 23\frac{1}{3}$, $x_3 = 30$.

Verse 38cd. Rule for equation of properties after the mutual exchange of part of them.

Type of problem. The i-th of n persons possesses a_i of one kind of article, whose unit price, x_i, is unknown. When they give k out of a_i to each other among themselves, their properties become equal to each other. What is the unit price of each kind of article ?

$$\{a_i - (n-1)k\}x_i + k(x_1 + \cdots + x_{i-1} + x_{i+1} + \cdots + x_n) = y \quad \text{for every } i.$$

Solution.

$$x_i = p \div (a_i - nk),$$

where p is any optional number.

Note. A rationale of this solution may be given as follows. The above equation can be rewritten as:

$$(a_i - nk)x_i + k\sum_{j=1}^{n} x_j = y, \quad \text{or} \quad (a_i - nk)x_i = y - k\sum_{j=1}^{n} x_j.$$

The right-hand side of the last equation is independent of i. Let it be any optional number (p):

$$y - k\sum_{j=1}^{n} x_j = p.$$

Then the above solution is obtained. If p is a multiple of all $(a_i - nk)$, every x_i is an integer.

Verse 39. Example. Five persons with rubies, sapphires, chariots, horses, and elephants. Given: $n = 5$, $k = 1$, $a_1 = 10$, $a_2 = 8$, $a_3 = 12$, $a_4 = 15$, $a_5 = 7$. Answer: $x_1 = 42$, $x_2 = 70$, $x_3 = 30$, $x_4 = 21$, $x_5 = 105$, $y = 478$. Note: Here, p is taken to be the least common multiple, 210, of $(a_i - nk)$. The value of y seems to have been calculated by $y = p + k\sum_{j=1}^{n} x_j$.

Verse 39^a. Rule for equation of the sums of the principal and interest.

Type of problem. Two principals, say a_1 and a_2, are lent separately at different rates, say r_1 and r_2 per month respectively. In how many (x) months does the sum of the principal and interest of the first case become equal to that of the second ?

$$a_1 + r_1x = a_2 + r_2x.$$

Solution.

$$x = (a_1 - a_2) \div (r_2 - r_1).$$

Verse 40. Example. Given: $a_1 = 300$, $r_1 = 2$ per month, $a_2 = 200$, $r_2 = 6$ per month. Answer: $x = 25$ months.

IV.6 Practical mathematics of series (vv. 41–51)

Verses 41-43. Rules for natural series, etc.

Type of problem. To calculate the sums of a natural series, of a square series, of a cubic series, and of a geometric progression.

$$S(n) = 1 + 2 + \cdots + n, \quad S^2(n) = 1^2 + 2^2 + \cdots + n^2,$$

30		931322574615478515625 (= 5^{30})
$\downarrow$	s	$\uparrow$
15		30517578125
$\downarrow$	m	$\uparrow$
14		6103515625
$\downarrow$	s	$\uparrow$
7		78125
$\downarrow$	m	$\uparrow$
6		15625
$\downarrow$	s	$\uparrow$
3		125
$\downarrow$	m	$\uparrow$
2		25
$\downarrow$	s/m	$\uparrow$
1		5
$\downarrow$	m	$\uparrow$
0		1

Figure 25: Computation of 5^{30} (s = square, m = multiplication).

$$S^3(n) = 1^3 + 2^3 + \cdots + n^3, \quad G = G(a,r,n) = a + ar + ar^2 + \cdots + ar^{n-1}.$$

Solution.

$$S(n) = \frac{n}{2} \cdot (n+1), \quad S^2(n) = S(n) \cdot (2n+1) \div 3,$$

$$S^3(n) = \{S(n)\}^2, \quad G = \frac{r^n - 1}{r-1} \cdot a,$$

where r^n is calculated by means of the popular algorithm (see Fig. 25), which is based on the two identities:

$$r^k = r^{k-1} \cdot r, \quad r^k = \left(r^{\frac{k}{2}}\right)^2.$$

Verse 44. Examples. Given: $n = 30$ for all series, and $a = 2$ and $r = 5$ for a geometric progression. Answer: $S(30) = 465$, $S^2(30) = 9455$, $S^3(30) = 216225$, $G(2,5,30) = 465661287307739257812$. Note: The last answer (in 21 decimal places) is missing in the manuscript. The computation of 5^{30} must have been made as shown in Fig. 25.

Verses 45–46 and *48ab.* Rules for an arithmetical progression.

Type of problem. To calculate, in order, the sum, the first term, the common difference, and the number of terms of an arithmetical progression, from the rest.

$$A = A(a,d,n) = a + (a+d) + (a+2d) + \cdots + \{a+(n-1)d\}.$$

Solution.

$$A = \left\{a + (n-1)\cdot\frac{n}{2}\right\}\cdot n, \quad a = \frac{A}{n} - (n-1)\cdot\frac{n}{2},$$

$$d = \frac{A/n - a}{(n-1)/2}, \quad n = \frac{\sqrt{(d/2-a)^2 + 2dA} - a + d/2}{d}.$$

Verse 47. Example. Journey of a person. Given: $a = 10$ *yojanas*, $d = 4$ *yojanas*/day, $n = 1$ month ($= 30$ days). Answer: $A(10,4,30) = 2040$ *yojanas*. Note: Verse 47 requires also to calculate the first term, the common difference, and the number of terms in order from the rest, but the answer given in the manuscript is the A only.

Verse 48cd. Rule for equation of journeys of two travelers.

Type of problem. One person goes at the constant speed, v_1. When the time period, t, has elapsed from his departure, another person starts traveling at the constant speed, $v_2\,(> v_1)$. How long (x) does it take for the second person to catch up with the first ?

$$v_1t + v_1x = v_2x.$$

Solution.

$$x = v_1t \div (v_2 - v_1).$$

Verse 49. Example. Given: $v_1 = 3$ *yojanas*/day, $v_2 = 9$ *yojanas*/day, $t = 3$ months. Solution: $t = 3 \times 30 = 90$ days, and $x = (3 \times 90) \div (9-3) = 45$ days.

Verse 50ab. Rule for equation of journeys of two travelers.

Type of problem. Two persons start at the same time and go on the same route. One of them travels at a constant speed, v, and the other goes a on the first day and increases his speed by d every day. When do they meet ?

$$\sum_{n=1}^{x} v_1(n) = \sum_{n=1}^{x} v_2(n), \text{ where } v_1(n) = v \text{ (constant), and } v_2(n) = a + (n-1)d.$$

Solution.

$$x = (2v - 2a + d) \div d.$$

Verse 51ab. Example. Given: $v = 100$ per day, $d = 5$; a not given. The linear measure is probably *yojana*. Answer: Not given. Note: The a intended by the author may have been unity as in the case of the example for the next rule (Verse 51cd). In that case, $x = 40\frac{3}{5}$ days.

Verse 50cd. Rule for equation of journeys of two travelers.

Type of problem. The initial speed (a) of the second person and the daily increase (d) of it in the previous rule (Verse 50ab) are both taken to be unity here.

Solution.

$$x = 2v - 1.$$

Verse 51cd. Example. Given: $v = 100$. The linear measure is probably *yojana*. Answer: $x = 199$ days.

IV.7 Practical mathematics of plane figures (vv. 52–76)

Verse 52. Rules for the sides of a right-angled triangle.

Type of problem. To calculate one of the three sides of a right triangle from the remaining two sides.

Solution. Let a, b and c be respectively the "arm" (*bhuja/dos/etc.*), the "upright" (*koṭi*) and the "ear" (*karṇa/śruti/etc.*) of a right triangle. Then,

$$c = \sqrt{a^2 + b^2}, \quad b = \sqrt{c^2 - a^2}, \quad a = \sqrt{c^2 - b^2}.$$

Verse 53. Example. A right triangle with three sides (12, 16, 20). Given: $a = 12$, $b = 16$. Answer: Not given. Note: Verse 53 requires to obtain also b, and then a, from the remaining two, but no answer is given.

Verses 54–57. Rules for the sides of a right-angled triangle and for its area.

Type of problem. To calculate two sides of a right triangle, when either the sum or the difference of the two sides and the remaining side are known, and to calculate the area, A, of the triangle.

Solution. When either $(c + a)$ or $(c - a)$ and b are given,

$$c \mp a = b^2/(c \pm a).$$

When either $(c + b)$ or $(c - b)$ and a are given,

$$c \mp b = a^2/(c \pm b).$$

When either $(a+b)$ or $(a-b)$ and c are given,

$$|a \mp b| = \sqrt{2c^2 - (a \pm b)^2}.$$

In each case, the two unknown numbers are obtained by means of the "rule of concurrence", that is,

$$x = \frac{(x+y)+(x-y)}{2}, \quad y = \frac{(x+y)-(x-y)}{2}.$$

$$A = \frac{ab}{2}.$$

Note. The "rule of concurrence" (*saṅkrama-sūtra*) is explicitly mentioned in Verse 57, but is not found in the extant portion of the manuscript. It may have been on folios 2–3, which are lost. For the Indian rules for algebraic normal forms including the rule of concurrence, see Hayashi and Kusuba 1998.

Verse 58ab. Rule for the areas of regular polygons.

Type of problem. It is required to calculate approximate areas of an equi-lateral triangle, etc. from their sides.

Solution. Only a formula for an equilateral triangle is extant, the rest being on the lost folio(s). Let a be the side of it, then the area is,

$$A \approx \frac{433a^2}{1000}.$$

Note. This approximate formula is based on the exact one, $A = (\sqrt{3}a^2)/4$, and on the approximation, $\sqrt{3}/4 \approx (1732/1000)/4 = 433/1000$. The formulas of Mahāvīra and of Bhaṭṭotpala are different from this. See Gupta 1990, 1992, and 1994.

Verses 58cd–64. Missing.

Verse 64. Examples of a circle and of a sphere. Note: The verse itself is missing but, judging from the extant portion of the answer, it seems to have dealt with an example of a circle and a sphere with the diameter, $d = 10$. The following values survive:

$$c = 31;29, \quad A = 78;42, \quad S_s = 314;50,$$

where c and A are respectively the circumference and the area of the circle and S_s the exact (*sūkṣma/sphuṭa*) surface area of the sphere. The fractional parts of these numerical values are given in the sexagesimal notation (31/29, etc. in the manuscript). The ratio used here of the circumference to the diameter is:

$$\frac{c}{d} = \frac{31;29}{10} = \frac{3;8,54}{1} = \frac{1889}{600} = \frac{3.14833\ldots}{1},$$

whose origin is not known. The formulas used for the area of a circle and for the surface of a sphere seem to be respectively,

$$A = \frac{c}{2} \cdot \frac{d}{2} \;\left(\text{or } A = \frac{cd}{4}\right) \quad \text{and} \quad S_s = cd \;(\text{or } S_s = 4A),$$

according to which we have $A = 78;42,30$ and $S_s = 314;50$. These rules must have been prescribed in the lost verses.

Verse 65. Rules for a sphere.

Type of problem. To calculate the surface (S) and the volume (V) of a sphere, whose diameter is d.

Solution.

$$S_v = 3d^2 + \frac{3d^2}{20} \;\text{(roughly)}, \quad V = Sd \div 6,$$

where S may be either S_s or S_v.

Note. It seems that this formula for S_v, which is meant for rough or practical (*sthūla/vyāvahārika*) calculation, is based on the correct formula, $S = \pi d^2$, with the rough value, 3, for π, and that the last term, $3d^2/20$, was added as a correction since 3 was too small for π. This formula implicitly corresponds to $\pi = 63/20 = 3.15 = 3;9$. This last expression in the sexagesimal notation suggests another possibility, namely, that it is an approximation to $\pi = 3;8,54$ employed in Verse 64 above. See also Verse 70^a below. Verse 65 is followed by a prose sentence, which states: $S_v = \langle 315 \rangle, S_s = 314;50, V_s = 524;47$, and by a figure, in which $d = 10$ and all the results obtained from it are shown, although the value of S_v is missing in the manuscript. The digits for V_s seem to be corrupt since we would have: $V_s = S_s d \div 6 = 524;43,20$.

Verses 66–68. Rules for a segment of a circle.

Type of problem. To calculate, in order, the "arrow" or height (h), chord (a) and "bow" or arc (b) in a "bow-like figure" (*cāpa-kṣetra*) or a segment of a circle, whose diameter and circumference are respectively d and c, from the rest.

Solution.

$$h = \frac{d - \sqrt{(d+a)(d-a)}}{2}, \quad a = 2\sqrt{h(d-h)}, \quad d = h + \left(\frac{a}{2}\right)^2 \div h,$$

$$b = \frac{c}{2} - \sqrt{\frac{c^2}{4} - \frac{(a/4) \times 5 \times c^2}{a + 4d}}.$$

Note. For the last formula, see Gupta 1967 and Hayashi 1991.

Verse 69. Example. The verse itself requires to calculate h from $d = 10$ and $a = 8$; a from d and h; d from a and h; and b from them. But the manuscript simply lists the four values once each: $d = 10$, $h = 2$, $a = 8$, $b = 9;15$. Note: Actually $b \approx 9;18,25$, since $c = 31;29$ for $d = 10$ (see under Verse 64).

Verse 70. Rule for a segment of a circle.

Type of problem. To calculate the area (B) of a bow-like figure or a segment of a circle.

Solution. An accurate area is:

$$B_s = \sqrt{\left(\frac{a+b}{2} + h\right)\left(\frac{a+b}{2} - h\right)} \times h^2 \times 2 \div 3.$$

Note. This is treated as an accurate formula by Giridhara, but actually it is even worse than his "gross" area, B_v, which is given in the next verse. It is not known how this formula, B_s, was obtained, but it seems to be an approximation by a rectangle whose orthogonal sides are e and $\frac{2}{3}h$, where e is a side of a right-angled triangle whose hypotenuse and remaining side are respectively $\frac{a+b}{2}$ and h:

$$B_s = e \times \frac{2}{3}h, \qquad \text{where} \quad e^2 + h^2 = \left(\frac{a+b}{2}\right)^2.$$

Verse 70^a. Rule for a segment of a circle.

Type of problem. To calculate the area (B) of a bow-like figure.

Solution. A gross (*sthūla*) area is:

$$B_v = h \cdot \frac{a+h}{2} + h \cdot \frac{a+h}{2} \cdot \frac{1}{20}.$$

Note. This formula has been attributed to Keśava of Nandigrāma by his son Gaṇeśa in his commentary, *Buddhivilāsinī*, on the *Līlāvatī*, and the verse for B_v cited by Gaṇeśa has been repeated verbatim in the *Gaṇitapañcaviṃśī* ascribed to Śrīdhara. See Hayashi 1995b, 242–244. This formula, too, implicitly corresponds to $\pi = 63/20 = 3.15 = 3;9$. Cf. Verse 65 above.

Verse 71. Example. Given: $h = 25$, $a = 87$, $b = 105$. Solution: $B_s = 1545$, $B_v = 1470$. Note: Actually, $B_s = 1544;47,38,... \approx 1545$. The given values of a and b seem to have been obtained from $d = 100$ and $h = 25$. In fact, we have $a = 86;36,\ldots$ and $b = 105;10,\ldots$, since $c = 314;50$ for $d = 100$. See under Verse 72 for the same circle.

Verse 73ab. Rule for the area of a fish-like figure (*matsya-kṣetra*).

Type of problem. To calculate the area (C) of a "fish-like figure", which is a plane figure made of two equal "bow-like figures" or segments of a circle.

Solution.

$$C = 2B.$$

In the calculation of B, the central line from the tail to the mouth of the fish is taken to be the chord (a) common to both segments.

Verse 72. Example. Given: the central line = 66 (= a), the "bow at the belly" = 73 (= b), the "arrow" = 25 (which is the greatest height of the fish, = $2h$). Solution: $C = 1156$ (*sic*). Note: This answer must be corrupted. Actually, we have: $2B_s = 1139; 26, 39, ...$, and $2B_v = 1030; 18, 45$. The given values of a and b seem to have been obtained from $d = 100$ and $h = 12; 30$. In fact, we have $a = 66; 8, \ldots$ and $b = 72; 22, \ldots$, since $c = 314; 50$ for $d = 100$. See under Verse 71 for the same circle.

Verse 73cd. Rule for the area of a moon-digit-like figure (*candra-kalā-kṣetra*).

Type of problem. To calculate the area (D) of a "moon-digit-like" or crescent-shaped figure, which is a plane figure delimited by two "bows" or arcs of two different circles.

Solution. The area in question is regarded as approximately equal to that of a "bow-like figure", whose chord and arc are respectively the lower (shorter) "bow" and the upper (longer) "bow", that is,

$$D = B.$$

Verse 74. Example. Given: $b_1 = 209$, $b_2 = 105$, $h = 50$. Solution: $D = 4960$. Note: If we assume $a = b_2 = 105$ and $b = b_1 = 209$ in the formulas given in Verses 70 and 70^a, we have $B_s = 4960; 50, 48, \ldots$ and $B_v = 4068; 45$.

Verse 75. Rule for the area of a drum-shaped figure (*dundubhi-kṣetra*).

Type of problem. To calculate the area (E) of a "drum-shaped figure", which is a plane figure like a half ellipse obtained by cutting an ellipse along its shorter axis.

Solution. The area in question is regarded as approximately equal to that of a "bow-like figure", which is formed when one cuts it into two equal parts along the longer axis and then combines them along the shorter axis. That is to say, when one takes twice the "side" to be the "bow" (b), half the "mouth" (*mukha*) or the top, horizontal line to be the "arrow" (h), and twice the "ear" (*śruti/karṇa*) or the depth to be the "chord" (a), then

$$E = B_s.$$

Verse 76. Example. Given: "mouth" $(2h) = 50$, "side" $(b/2) = 52;30$, "ear" $(a/2) = 43;30$. Solution: $E = B_s = 1545$. Note: The "bow-like figure" calculated here is exactly the same as the one in Verse 71.

IV.8 Rules for shadows (vv. 77–80)

Verses 77 and *79ab.* Rules for the shadow of a gnomon illuminated by a lamp.

Type of problem. There is a lamp on a post (height d) standing on a flat ground, and a gnomon (height s) at a certain distance (b) from the post. A shadow (c) of the gnomon is made by the lamp. One of the quantities is to be calculated when the rest are given.

$$(d - s) : b = s : c.$$

Solution.

$$c = \frac{bs}{d-s}, \quad d = \frac{bs}{c} + s, \quad b = \frac{c(d-s)}{s}.$$

Verse 78. Example. Given: $b = 32$, $d = 60$, $s = 12$ *aṅgulas*. Solution: $c = 8$ *aṅgulas*.

Verse 79cd. Rule for the height of a bamboo stalk.

Type of problem. To calculate the height (v) of a bamboo stalk when its own shadow (length b) together with a gnomon (height s) and its shadow (length c) is given.

$$c : s = b : v.$$

Solution.

$$v = \frac{bs}{c}.$$

Verse 80. Example. Given: $b = 160$, $s = 10$, $c = 4$. Solution: $v = 400$.

IV.9 Rules for magic squares (vv. 81–86)

Verses 81–84. Rules for magic squares.

Type of problem. To make magic squares, that is, to arrange the natural numbers, 1 to n^2, in a square in such a way that the sums of the numbers in each column, in each row, and in each diagonal, are equal to each other. The constant sum (s) can be expressed as $n(n^2 + 1)/2$.

Solution (with notes). Giridhara gives one rule each (1) for quasi-magic squares made of the numbers, 1 to n^2, where the sum of the numbers in every column is equal to the constant sum; (2) for magic squares of odd order ($n = 2k + 1$); and (3) for magic squares of even (that is, evenly-even) orders ($n = 4k$). (4) For magic

1	2	$\cdots$	j	$\cdots$	$n-2$	$n-1$	n
$n+2$	$n+3$	$\cdots$	$n+j+1$	$\cdots$	$2n-1$	$2n$	$n+1$
$2n+3$	$2n+4$	$\cdots$	$2n+j+2$	$\cdots$	$3n$	$2n+1$	$2n+2$
$\vdots$			$\vdots$				$\vdots$
$\cdots$	$\cdots$	$\cdots$	$a_{i,j}$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$\vdots$			$\vdots$				$\vdots$
n^2	$(n-1)n+1$	$\cdots$	$(n-1)n+j-1$	$\cdots$	n^2-3	n^2-2	n^2-1

Figure 26: Construction of the quasi-magic square of order n.

squares of oddly-even orders ($n = 4k + 2$), he only gives an example.

(1) Rule for quasi-magic squares (Verses 81–82cd). Starting from the top-left corner, proceed to the right, filling each cell of the rows with the numbers, 1, 2, 3, ...; when a row is completed, step down to the cell immediately below the cell just filled; when the last (n-th) cell of a row is filled and if the row is not yet completed, return to the first cell of the same row (Fig. 26). In a square arranged in this way, the sum of the numbers in each column is equal to the constant sum, although it is not yet a magic square. Let $a_{i,j}$ be the j-th element in the i-th row of the square. Then,

$$a_{i,j} = f(i,j) = (i-1)n + i + j - 1 \quad \text{for} \quad i + j \leq n + 1,$$

$$a_{i,j} = g(i,j) = (i-2)n + i + j - 1 \quad \text{for} \quad i + j \geq n + 2.$$

Therefore,

$$\sum_{i=1}^{n} a_{i,j} = \sum_{i=1}^{n-j+1} f(i,j) + \sum_{i=n-j+2}^{n} g(i,j) = \frac{n(n^2+1)}{2}.$$

This rule holds true for a square of any order as has been stated by Giridhara (Verse 82), and the manuscript actually gives quasi-magic squares of orders 3 to 9 (Figs. 7–13 in Section II).

(2) Rule for magic squares of odd orders ($n = 2k + 1$) (Verses 82cd–83). Put the numbers of the first column of the quasi-magic square in the cells of the central, $(k+1)$-th, column of a new square. Fill the j-th column to the left of the central column (inclusive) with the numbers of the j-th column of the quasi-magic square after having shifted the numbers of the upper $(j-1)$ cells to the bottom of the column (Fig. 27). When the left border of the square is reached, go to the right border and proceed again towards left. The result is a magic square (Figs. 14, 16, 18, 20 in Section II).

This method is unique to Giridhara, although the same magic squares can be obtained by means of the Oblique-Move Method taught by Nārāyaṇa in his

$$
\begin{array}{|ccc}
(1) & & (j) \\
\hline
a_{1,1} & \cdots & a_{1,j} \\
a_{2,1} & \cdots & a_{2,j} \\
\vdots & & \vdots \\
\vdots & & \vdots \\
\vdots & & \vdots \\
\vdots & & \vdots \\
a_{n,1} & \cdots & a_{n,j} \\
\hline
\end{array}
\quad \Rightarrow \quad
\begin{array}{|ccc}
(k-j+2) & & (k+1) \\
\hline
a_{j,j} & \cdots & a_{1,1} \\
a_{j+1,j} & \cdots & a_{2,1} \\
\vdots & & \vdots \\
a_{n,j} & \cdots & \cdot \\
a_{1,j} & \cdots & \cdot \\
\vdots & & \vdots \\
a_{j-1,j} & \cdots & a_{n,1} \\
\hline
\end{array}
$$

Figure 27: Transformation of a quasi-magic square into a magic square ($n = 2k + 1$).

Gaṇitakaumudī (A.D. 1356) (see Hayashi 1986, xxvi–xxviii; 1988, 649–650). The same magic squares can also be obtained by rotating Ṭhakkura Pherū's (ca. A.D. 1315) magic squares of odd orders, which he made by a method unique to him (see Hayashi 1986, vi–vii; 1988, 652–653).

(3) Rule for magic squares of evenly-even orders ($n = 4k$) (Verse 84). Giridhara states the rule for magic squares of evenly-even orders very briefly in one verse (Verse 84), only referring to two essential points of the procedure, namely, (i) that one starts from "the first cell" and proceeds "in one direction" and then "in the direction to the left", and (ii) that one writes numbers "in pairs". The details of the procedure are not clear, but what Giridhara intends here is probably the Diagonal Method taught by the anonymous author of a 12-th century Arabic manuscript (Fatih 3439, see Sesiano 1980, 191–192) and by Manuel Moschopoulos (ca. 1265–1315) (see Tannery 1920, 42–49; Sesiano 1998, 385–386). It is as follows (the left-right relation is reversed in the Arabic manuscript).

Divide the square of n^2 cells into k^2 small squares of 16 cells each and mark the diagonal cells of each small square (Fig. 28). Then, starting from the cell on the top-left corner, proceed to the right on each row, filling only the marked cells with the corresponding numbers beginning with 1. When the cell on the bottom-right corner is filled, proceed in the reversed way, filling the empty cells with the remaining numbers in ascending order. The square obtained is a magic square (Figs. 15 and 19 in Section II).

(4) A magic square of order 6. Giridhara does not give a general rule for magic squares of oddly-even orders ($n = 4k + 2$), but simply shows a magic square of order 6 without explanation (Fig. 17 in Section II). I have not so far been able to find this magic square elsewhere in Indian literature, but an analysis shows that it is a bordered magic square made by means of the Frame Method of Islam. The method employed by Giridhara was presumably as follows.

The first ten numbers, 1–10, are arranged on the border or frame consisting of 20 cells in such a way that only one of the two ends of each row, of each column,

•			•	•			•
	•	•			•	•	
	•	•			•	•	
•			•	•			•
•			•	•			•
	•	•			•	•	
	•	•			•	•	
•			•	•			•

Figure 28: Diagonal Method for evenly-even magic squares.

8	11	14	1
13	2	7	12
3	16	9	6
10	5	4	15

Figure 29: Islamic square used by Giridhara as the core of his square of order 6.

and of each diagonal is filled with a number. The next sixteen numbers, 11–26, are arranged in the inner square of 4×4 cells according to the pattern of a magic square of order 4 already known. The inner square itself, therefore, is a magic square of order 4 with the constant sum, 74. The last ten numbers, 27–36, are again arranged on the frame in such a way that the sum of the two numbers at both ends of each row, of each column, and of each diagonal, is equal to $n^2 + 1$.

Exactly the same magic square of order 6 occurs in a small compendium on bordered magic squares compiled by a Muḥammad ibn Yūnis and contained in a 12-th century Arabic manuscript (Hüsrev Pasa 257, see Sesiano 1991, 18, Fig. 8). He describes, in terms of the moves of chessmen, how to construct the magic square of order 4 (Fig. 29) employed here as the pattern for the inner, core magic square, and then states four methods for constructing the borders (or frames) of oddly-even magic squares, the first of which produces the border of Giridhara's square of order 6 (see Sesiano 1991, 16–17).

The same magic square of order 4 (Fig. 29) was frequently used by al-Būnī (d. 1225) and al-Zinjānī (ca. 1250) as a basic pattern for talismans (Figs. 30 and 31) (see Ahrens 1922, 162, etc.; Sesiano 1981, 260–264). Al-Būnī ascribes it to Plato (see

16	19	22	9
21	10	15	20
11	24	17	14
18	13	12	23

Figure 30: Al-Būnī's magic square with $s = 66$ for Allāh (1,30,30,5).

200	80	70	3
69	4	199	81
5	72	78	198
79	197	6	71

Figure 31: Al-Zinjānī's magic square for Jaʿfar (3,70,80,200) with $s = 353$.

15	21	27	1
25	3	13	23
5	31	17	11
19	9	7	29

Figure 32: A magic square of order 4 with $s = 64$ in the Ayasofya manuscript.

Ahrens 1922, 164; Bergsträsser 1923, 228), while al-Zinjānī, like ibn Yūnis, describes its construction in terms of the moves of chessmen (see Sesiano 1981, 258).

In a small Arabic treatise on magic squares, assignable to the beginning of the 11th century, which is contained in a 13th-century manuscript (Ayasofya 4801), the same magic square (Fig. 29) is constructed by means of a kind of Diagonal-Horse (or Knight)-Move Method, which the anonymous author prescribes for evenly-even magic squares in general (Sesiano 1996, 118–121 with Figs. 14 and 15). The same pattern is employed in that treatise in order to construct a magic square of order 4 (Fig. 32) and the core of a magic square of order 6 (Fig. 33), both with the numbers in an arithmetical progression; the pattern of the latter's border, however, is different from that of Giridhara's.

It is also used as the core of the Islamic square of order 6 (Fig. 34) incised with the Indo-Arabic numerals on an iron plate, which has been discovered at the ruins of the palace of the Chinese prince of Anxi (fl. 1278) (see Martzloff 1997, 365). The reversed form (mirror image) of the same magic square was used also by Moschopoulos as the basic pattern for the squares of order 4 constituting magic squares of evenly-even orders (Fig. 35) (see Tannery 1920, 48–51; Sesiano 1998, 388–390).

A magic square of order 6 having a border very close to that of Giridhara's is

19	55	13	65	63	1
61	35	41	47	21	11
57	45	23	33	43	15
3	25	51	37	31	69
5	39	29	27	49	67
71	17	59	7	9	53

Figure 33: A magic square of order 6 with $s = 216$ in the Ayasofya manuscript.

28	4	3	31	35	10
36	18	21	24	11	1
7	23	12	17	22	30
8	13	26	19	16	29
5	20	15	14	25	32
27	33	34	6	2	9

Figure 34: Islamic magic square of order 6 found in China.

1	62	59	8	9	54	51	16
60	7	2	61	52	15	10	53
6	57	64	3	14	49	56	11
63	4	5	58	55	12	13	50
17	46	43	24	25	38	35	32
44	23	18	45	36	31	26	37
22	41	48	19	30	33	40	27
47	20	21	42	39	28	29	34

Figure 35: Moschopoulos' magic square of order 8.

found in al-Kharaqī's (fl. 1100) work (Fig. 36) (see Sesiano 1995, 199), although its core square follows the pattern, not of this Islamic square (Fig. 29), but of another Islamic square of the Fatih manuscript mentioned above, which has the reversed form (mirror image) of Giridhara's square of order 4 (Fig. 15 in Section II).

I have pointed out elsewhere that that Islamic square (Fig. 29), with a rotation of 90°, coincides exactly with one of the four possible forms of the original square reconstructed from Varāhamihira's (ca. 550) irregular magic square of order 4 (Fig. 37) (see Hayashi 1987). But neither the Islamic square of order 4 itself nor the Frame Method have so far been found in any other Sanskrit works including those of Nārāyaṇa and Ṭhakkura Pherū.

Verses 81–84 are followed by illustrations of "the quasi-magic squares" of orders 3 to 9, and of the magic squares of orders 3 to 9 (see Figs. 7–20 in Section II).

Verse 85. Example. A king's equal donations of 81 towns to his 9 devotees. Given: The properties of the 81 towns are expressed by the natural series, 1,2,3, ..., 81.

6	28	34	2	36	5
4	14	24	25	11	33
30	19	17	16	22	7
29	15	21	20	18	8
10	26	12	13	23	27
32	9	3	35	1	31

Figure 36: Al-Kharaqī's magic square of order 6.

2	3	5	8
5	8	2	3
4	1	7	6
7	6	4	1

Figure 37: Varāhamihira's irregular magic square of order 4 with $s = 18$.

Solution: Although no explanation is given, the magic square of order 9 already given is perhaps meant to be the answer to this problem. Each row (or column) seems to be assigned to each devotee.

Verse 86. Rule for a magic square having any optional constant sum.

Type of problem. To construct a magic square of any order (n) having any constant sum (s) with the numbers constituting an arithmetical progression.

Solution. Divide the given "quantity" (*rāśi*) or the constant sum (s) by the sum of the natural series up to $(n^2 - 1)$ divided by the "line" (*paṅkti*), n; the quotient is the common difference of the arithmetical progression to be used. Next, divide the remainder of the division by the "line"; the quotient is the first term of the progression. Then, fill the square with the numbers of this progression according to the pattern of the magic square of order n already known.

Note. Let a and d be respectively the first term and the common difference of the arithmetical progression to be used for the magic square. Since the sum of the numbers arranged in the square is equal to the sum of the arithmetical progression,

$$ns = \frac{n^2}{2} \cdot \{2a + (n^2 - 1)d\}, \quad \text{or} \quad s = na + \frac{S(n^2 - 1)}{n} \cdot d,$$

where $S(n)$ is the sum of the first n terms of the natural series (see Verses 41–43 above). Hence follows the above rule. Nārāyaṇa (*Gaṇitakaumudī*, bhadragaṇita, v.9), on the other hand, rewrites the above equation as:

$$a = \frac{-S(n^2 - 1)d + ns}{n^2},$$

and obtains integer solutions (a, d) by means of the rule of pulverizer (*kuṭṭaka*) (see Hayashi 1986, xi; 1988, 677). In reality, however, Giridhara's rule is equivalent to Nārāyaṇa's, since Giridhara's "division" of s by $S(n^2 - 1)/n$ includes, so to speak, an extended division, where the remainder is greater than the divisor. See the following examples.

Four examples for Verse 86 (see Figs. 21–24 in Section II).

(1) Given $n = 3$ and $s = 300$. Solution: $a = 20$ and $d = 20$. Note: $S(n^2 - 1)/n = 36/3 = 12$. Hence $s = 300 = 12 \times 25 + 0$, but Giridhara takes: $300 = 12 \times 20 + 60$, and $60 = 3 \times 20$. The numbers are arranged according to the pattern of Fig. 14.

(2) Given $n = 3$ and $s = 51$. Solution: $a = 5$ and $d = 3$. Note: Here also $S(n^2 - 1)/n = 12$. Giridhara takes: $51 = 12 \times 3 + 15$, and $15 = 3 \times 5$. According to the pattern of Fig. 14.

(3) Given $n = 4$ and $s = 300$. Solution: $a = 15$ and $d = 8$. Note: $S(n^2 - 1)/n = 120/4 = 30$. Giridhara takes: $300 = 30 \times 8 + 60$, and $60 = 4 \times 15$. According to the pattern of Fig. 15.

(4) Given $n = 4$ and $s = 50$. Solution: $a = 5$ and $d = 1$. Note: Here also $S(n^2 - 1)/n = 30$. Hence $50 = 30 \times 1 + 20$, and $20 = 4 \times 5$. According to the pattern of Fig. 15.

IV.10 Concluding remarks (vv. 87–89)

In the first of the three concluding verses, Giridhara enumerates three characteristic features of this work. (1) This book treats algebraic topics without employing algebraic symbolism. Most books of pāṭī (algorithm) have more or less the same characteristic, but it is elaborated in this work, especially in Section 4 on "Miscellaneous operations", which includes various linear equations. (2) This book contains formulas that can be used in everyday life. With the formula given in Verse 79cd, for example, we are able to know the height of a bamboo without using mechanical instruments (*yantra*) other than a simple gnomon. (3) This book is concise but is easy to understand.

The remaining two verses contain ordinary colophonic statements. That is, Giridhara hopes that this work, with corrections if necessary, will continue to be used by people forever.

Appendix A: List of word numerals used by Giridharabhaṭṭa

The references in the following list are to the verse numbers.

0 = abhra, 78; kha, 74, 80
1 = ku, 41, 46; bhū, 25; rūpa, 50cd
2 = akṣi, 27; yama, 27
3 = guṇa, 27; rāma, 41; vahni, 76
4 = abdhi, 31, 58ab, 67; yuga, 76; veda, 80
5 = akṣa, 25, 76; viṣaya, 27; śara, 27, 74
7 = naga, 27
8 = gaja, 27; nāga, 39
10 = diś, 39, 74, 76; diśā, 27
11 = śiva, 25
12 = arka, 25, 31, 53; bhānu, 39
13 = viśva, 18
14 = manu, 18
15 = dina, 39
16 = aṣṭi, 80; nṛpa, 53
20 = nakha, 29, 65, 70[a], 74
25 = tatva/tattva, 18, 71, 72
32 = rada, 78
33 = amara, 58ab

Appendix B: List of abbreviations used in the manuscript

The word, udāharaṇa ("example"), which introduces verses for examples, is consistently abbreviated udā. The other abbreviations are sporadically used in the answers to the examples or in the figures attached to them. The references in the following list are to the verse numbers.

a = aśva, 23, 25, 27, 29
udā = udāharaṇa, 18, et passim.
ko = koṭi, 57
ga = gaja, 27, 39
ja = jana, 34
trai = trairāśika, 34
trairā = trairāśika, 34
di = dina, 49, 51cd
nī = nīla, 39
paṃcarā = pañcarāśika, 34
pha = phala, 34, 74, 76, Fig. 6

mā = māṇikya, 39; = māsa, 34
yo = yojana, 47
ra = ratha, 39
rū = rūpa, 23, 25, 27, 29
vā = vājin, 39
vyā = vyāsa, Fig. 2
śa = śara, Fig. 3, Fig. 4, Fig. 5
saṃ = saṅkalita, 44
saptarā = saptarāśika, 34
sū = sūkṣma, 65; = sūtra, Fig. 4

Appendix C: Figures in the manuscript

I reproduce here seven figures from the manuscript. Six of them (Figs. 1a–6a) are from the section on plane figures and one (Fig. 17a) from the section on magic squares. They correspond to Figs. 1–6 and 17 in Section II.

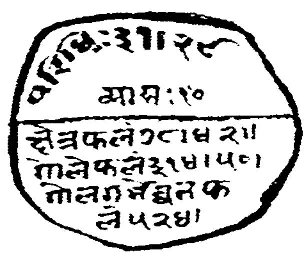

Figure 1a: Circle and sphere.

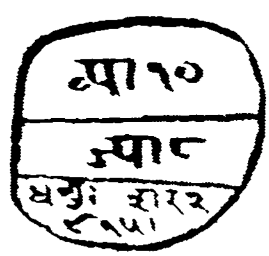

Figure 2a: Arc and chord in a circle.

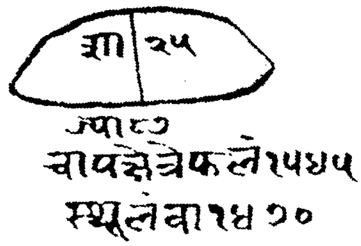

Figure 3a: Segment of a circle.

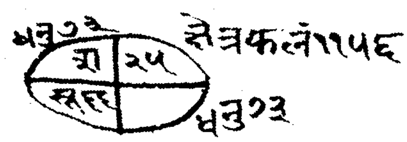

Figure 4a: Fish-like figure.

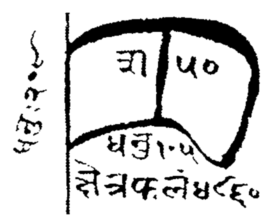

Figure 5a: Moon-digit-like (crescent-shaped) figure.

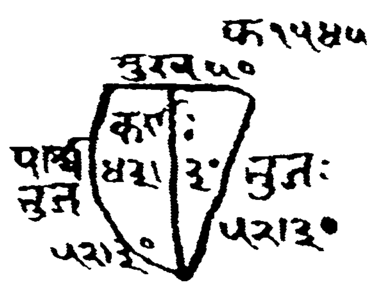

Figure 6a: Drum-shaped figure.

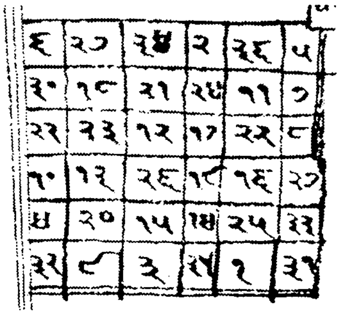

Figure 17a: Magic square of order 6.

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