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A New Reading of *Method* Proposition 14 : Preliminary Evidence from the Archimedes Palimpsest (Part 1)

Reviel Netz
Stanford University

Ken Saito
Osaka Prefecture University

Natalie Tchernetska
Trinity College, Cambridge

To the memory of Wilbur Knorr (1945–1997)

I Introduction[1]

The Archimedes Palimpsest, referred to in Heiberg [1910–1915] as ‘Codex C’, had a complicated history that is understood only in part (see e.g. Netz [2000]). Originally a tenth century manuscript containing several works by Archimedes, it was palimpsested as a Greek prayer book in the twelfth or thirteenth century, and then remained unknown to the scholarly world until it was rediscovered in Istanbul at the end of the nineteenth century. The manuscript was then briefly studied by Heiberg, mostly during a visit made in 1906.[2] This led to Heiberg’s second edition of Archimedes’ works. The manuscript then disappeared from Istanbul, perhaps in the aftermath of the First World War. Following a long period of obscurity it finally re-appeared in 1998 in a Christies’ sale in New York. The current owner has deposited the manuscript at the Walters Art Museum, Baltimore, for the purposes of conservation, imaging and research, which will lead to a complete facsimile and edition of the manuscript.

As is well known, the manuscript is unique in several ways: it provides our only Greek text for *On Floating Bodies* and for the *Stomachion*, and, most important,

[1]The study leading to this work was enabled by many people. Our deepest words of thanks and appreciation go to all: to William Noel, curator of manuscripts at the Walters Art Museum, and director of the Archimedes Palimpsest Project; to Abigail Quandt, senior conservator of manuscripts, who is conserving the manuscript; to the imagers of the palimpsest — Roger Easton of the Rochester Institute of Technology, William Christens-Barry of Johns Hopkins University, and Keith Knox of the Xerox Corporation; to Michael Toth of R. B. Toth Associates; and to the owner of the Archimedes palimpsest, for allowing the study of the palimpsest, and for the many ways in which he assisted in this difficult task.

[2]Heiberg’s study was mostly based on an incomplete set of photos taken during that 1906 visit. This set of photos is in the Royal Danish Library, Ms. Phot. 38, currently on loan to the Walters Art Museum, and provides important insight to Heiberg’s approach as an editor.

it provides our only text for the *Method*. It is also in very bad physical shape, so that the usual difficulties of reading an inferior writing in a palimpsest are compounded by the difficulties of reading faint, abraded or heavily molded text. Much of the deterioration has occurred during the twentieth century (as can be clearly learned from Heiberg's photos). Still, Heiberg's success in recovering so much of the manuscript is a tribute to his genius. Where he had guidance for his readings (in works attested by other manuscripts), his readings were often very thorough, and even for the previously unknown *Stomachion* and the *Method* he was capable of reading over 80 percent of the extant text. (It is important to note also that, through the process of palimpsesting the codex, as well as through later losses, a significant proportion of the original Archimedes text was lost, apparently irretrievably: the extant percentage, once again, seems to be somewhat above 80 percent). It is only through the help of modern technology — ultra-violet light, as well as through digital image capture and enhancement — that substantial progress can now be made over Heiberg. Even so, Heiberg's own readings greatly help the decipherment: when we are lost in the jungle of faint traces yielded by digital imagery, it is Heiberg's voice that leads us through the thicket.

The portion that Heiberg was able to recover from the *Method* formed a major discovery. Archimedes, we learned, had advanced the application of mechanics to geometry (by "mechanics" we here mean such results as are developed in *Planes in Equilibrium*, or results analogous to them), much beyond the level of the *Quadrature of the Parabola*. Furthermore, unlike anywhere else in the previously known works, Archimedes had boldly applied the use of "indivisibles"[3], composing $n + 1$ dimensional figures from infinitely many n-dimensional ones to derive the properties of the $n + 1$ dimensional figures from those of the n-dimensional ones. Finally, in his introduction, Archimedes offered several tantalizing meta-mathematical observations. In particular, Achimedes stated that the *Method* was in some sense non-rigorous, falling short of strict geometrical proof, referring to it as "mechanical", in distinction from "geometrical".[4]

It is not surprising, then, that during the twentieth century study of Archimedes was dominated by the fascinating questions opened up by the *Method*. The best study remains Dijksterhuis [1987], with an extensive bibliographical essay by Knorr covering the literature up to that time. In recent years one can mention especially, among studies of the *Method* relevant to this article, the articles Sato [1986], and Knorr [1996].

It is a feature of the *Method* that, with its 'methodological' interest, it either

[3] We use the term "indivisibles" as a convenient label for the slicing technique used in the *Method* by Archimedes, and we do not make any special claim concerning its possible similarity to the notion and technique used by Cavalieri (from whom the term is borrowed).

[4] Heiberg [1913] 428.18–430.18.

proves results already proved elsewhere through new techniques or, when it proves new results, it proves them through more than a single technique, allowing us to see side by side the nature of those techniques. The proof of theorems already proved elsewhere (propositions 1–11) always combines the two novel features: application to geometry of mechanics, and the use of indivisibles. The final sequence of theorems in the extant text (props. 12–15)[5], has a more complex structure. Archimedes first proves a certain result through the same combination of mechanics and indivisibles (props. 12–13); he then proves the same result again, through indivisibles alone (prop. 14); and finally proves the same result in a 'classical' manner, using neither mechanics nor indivisibles (prop. 15). Here the extant text breaks off. Based on Archimedes' introduction, it appears that the conclusion of the work went through a similar set of several proofs, for another new result.

Among the three options covered by this final sequence, the middle one has a special position. Proofs combining mechanics and indivisibles are well attested from elsewhere in the *Method*, while 'classical' proofs are to be found everywhere in the Archimedean corpus. Here, however, is the only extant proof by Archimedes relying on the use of indivisibles alone. Perhaps for this very reason, Heiberg had rather more difficulties with this proposition — which he called '14', and which we shall call 'the Indivisibles Proof' — than with most others in the treatise. Admittedly, the physical state of the manuscript provided Heiberg with severe problems. The text occupies the whole of the bifolium 105–110 of *Euchologion*, the prayer book written over the Archimedean text, going on into a small part of the bifolium 158–159[6]. The bifolium 158–159 is fairly readable in its small part occupied by Proposition14, but the bifolium 105–110, already in Heiberg's time, was much abraded in places, especially in its lower half, leaf 105. Heiberg did not even bother to take photos of this leaf, and many of his readings here were either conjectural or lacunose. In particu-

[5] A caveat ought to be mentioned here. As can be clearly seen in Heiberg's apparatus itself, there is no textual authority for the proposition numberings he had introduced. To a certain extent, those numbers distort the structure of the work: for instance, the meta-mathematical observation between what Heiberg calls 'propositions 1 and 2' (Heiberg [1913] 438.16–21) belongs to neither proposition: by inserting the numberings, Heiberg assigned it arbitrarily to proposition 2. Thus such numbers are to be used with caution; they are of course indispensable for reference purposes.

[6] It is probable, though not certain, that the proof starts at the very start of bifolium 105–110. The bifolium immediately preceding this one was lost already when the book was palimpsested, in the twelfth or thirteenth century, and so there is a major lacuna which cannot be filled, immediately prior to the beginning of our text. Because of the way the palimpsest was made, each original leaf of the Archimedes manuscript was folded and made into two leaves of the prayer book; the order of the original leaves was greatly disturbed — first 110r–105v, then its verso side 110v–105r, continuing into 158r–159v where the diagram appears.

lar, he left the column 105r. col. 1 nearly totally unread.[7] We have now been able to complete Heiberg's reading through most of the text, confirming and sometimes refining his conjectures, and filling in the lacuna. We believe the new reading has sufficient interest to merit a separate publication, anticipating the planned publication of the entire manuscript. Heiberg noted, concerning 105r. col. 1, this (Heiberg [1913] 499 n. 1): 'Quid in tanta lacuna fuerit dictum, non exputo' — 'I do not guess what was written in such a long lacuna'. This lacuna, it turns out, changes the nature of the proof and thus, potentially, of our understanding of Archimedes' use of indivisibles — and, consequently, of much else.

II *Method* Proposition 14: Translation

The following translation of *Method* proposition 14 is based on a study, *in situ*, of the bifolia 105–110 and 158–159 with the aid of ultra-violet light. Further, and more important, the 'verso' side of the bifolium 105–110 (110v–105r) has been digitally imaged and processed by the imaging team[8]. We have studied those images with the aid of Adobe Photoshop and other prgrams, and the edition in the Appendix is based on this study. Though we give the traslation of the whole proposition 14 in the following, only the part from 110v–105r is based on this new edition. This is the part where the crucial mathematical issues are located.

As for the text before and after 110v–105r, we offer here scattered improvements on Heiberg's text in our translation, and we plan to publish its new edition as part 2 of the present article.

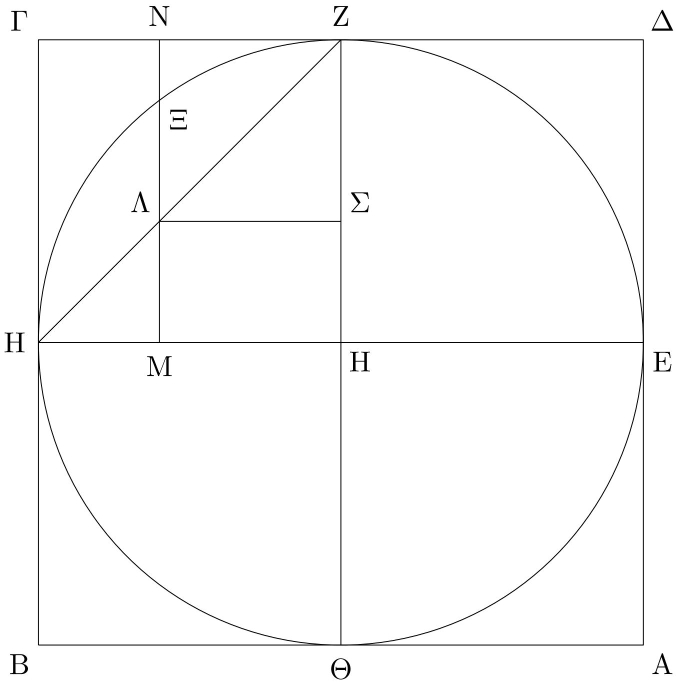
Being as it is in the bifolium 158–159, the diagram, as well, is not edited from the manuscript. We do however attempt to keep close to the "spirit" of the manuscript diagram. We believe that, in general, manuscript diagrams offer valid evidence for the form of diagrams in antiquity. In this case, we note the interesting convention adopted at some stage of the transmission (possibly, by Archimedes himself), to represent the parabolic segment by a triangle. This convention is helpful for allowing a clear resolution of the section from the semi-circle within which it is inscribed. The convention is thus comparable to another one observed in the manuscripts of *Sphere and Cylinder*, where the polygon inscribed (or circumscribed) in (or around) a circle, is often represented by a series of arcs drawn in the opposite direction to the circle. In the proposition before us, a curved line is represented by a straight line; in the *Sphere and Cylinder*, a straight line is represented by a curved line. The structural

[7] The Archimedean text is written in two columns; notice that columns cross from one leaf to another in the same bifolium, so that 110r. col. 1, for instance, is followed not by 110r. col. 2, but by 105v. col. 1.

[8] See above n. 1. It is hoped that an image of this leaf shall soon be made available to the public in some electronic form.

principle is the same, and confirms that such conventions are not the residues of textual corruption, but represent genuine visual habits of the ancient audience. We believe that, just as reading Greek mathematics demands that we adjust to certain linguistic habits, it also demands that we adjust to certain visual habits. We thus keep the representation of the segment by triangle. With little practice you, too, will see a parabola.

For ease of reference, we insert Latin letters to enumerate steps of construction ((a), (b), (c) etc.), and Arabic numerals to enumerate steps of argument ((1), (2), (3) etc.). References in the footnotes to the *Elements* or to the *Conics* are not intended to be complete, nor do we suggest that Archimedes or his audience referred to those works; they merely serve to signal the Greek mathematical tool-box relevant for the claim made.



(a) Let there be a right prism having square bases, (b) and let one of its bases be the square ΑΒΓΔ, (c) and let a cylinder be inscribed inside the prism, and let the base of the cylinder be the circle ΕΖΗΘ, touching the sides of the <square> ΑΒΓΔ at the <points> Ε, Ζ, Η, Θ, (d) and let a plane be drawn through its <= the circle's> centre, and <through> the side, above ΓΔ, of the square in the plane opposite to ΑΒΓΔ; (1) so it shall cut, of the whole prism, another <prism>, which shall be a fourth part of the whole prism[9]. (2) This, <other> prism shall be contained by three parallelograms, and two triangles opposite to each other. (e) So, let a section of a right-angled cone[10] be drawn in the semi-circle ΕΖΗ, (f) and let

[9] *Elements* I.41, XI.32.

[10] What we call a 'parabola'.

its diameter be ZK, (g) and let the same <line> ZK also be that, <applied> on which, the <lines> drawn in the section are equal in square[11], (h) and let some <line>, <namely> MN, be drawn in the parallelogram ΔH, being parallel to KZ. (3) So it shall cut the circumference of the semi-circle at Ξ, and that of the section of the cone at Λ. (4) And the <rectangle> contained by the <lines> MNΛ is equal to the <square> on NZ. (5) For this is clear. [12] (6) So, through this, it shall be: as MN to NΛ, the <square> on KH to the <square> on ΛΣ [13]. (i) And let a plane be set up on MN, right to the <line> EH. (7) So the plane shall make a right angled triangle in the prism cut off from the whole prism, of which <= triangle> one of the <sides> around the right angle shall be MN, while the other <shall be> drawn up from N in the plane on ΓΔ, right to the <line> ΓΔ, equal to the axis of the cylinder, and the hypoteneuse <shall> be in the cutting plane itself; (8) and it shall also make a cut, a right-angled triangle in the segment cut off from the cylinder by the plane that was drawn through EH and <through> the side of the square opposite to ΓΔ, of which <= triangle> one of the <sides> around the right angle shall be MΞ, and the other <shall be> in the surface of the cylinder drawn up from Ξ, right to the plane KN, and the hypoteneuse <shall be> in the cutting plane. (9) Now, similarly, since the <rectangle> contained by MN, MΛ is equal to the <square> on MΞ ((10) for this is obvious),[14] (11) it shall be: as MN to MΛ, so the <square> on MN to the

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[11] A formulaic expression, well-known from Apollonius but also attested in Archimedes (*Conoids and Spheroids* 3, 272.16–17), for what we call the '*latus rectum*' of a conic section. In the case of a parabola and in terms of the diagram at hand, this is the line L satisfying the property that, for every line on the segment such as ΛΣ, r(ΣZ, L) = q(ΛΣ) (r(A, B) and q(AB) represent *rectangle contained by lines A and B*, and *square on AB,* respectively). Archimedes' definition provides an elegant way of making the parabola cut the circle at H, E (r(HK, ZK) = q(ZK)). For constructing a parabola from a diameter, implied vertex, and *latus rectum*, see Apollonius' *Conics* I.52. Heiberg was unable to read the text of Step g, which is the most important improvement on his text we offer outside the edited text below. The Greek seems to read: ἔστω δὲ καὶ παρ' ἣν δύνανται αἱ καταγόμεναι ἐν τῇ τομὴ αὕτη ἡ ZK.

[12] Directly from the property of the parabola: r(ZK, ZΣ) = q(ΛΣ), but ZK = MN, ZΣ = NΛ, ΛΣ = NZ (*Elem.* I.41).

[13] MN : NΛ :: q(MN) : r(MN, NΛ) (*Elem.* VI, 1); substituting r(MN, NΛ) with q(NZ) (Step 4), we have MN : NΛ :: q(MN) : q(NZ). But since MN = KH and NZ = ΛΣ, we get MN : NΛ :: q(KH) : q(ΛΣ). Note that this Step 6 has considerable texual and logical difficulties. The three points K, H, and Λ are dotted in Heiberg's text (i.e., they were illegible). Though Heiberg's edition yields a mathematically correct relation as we have just explained, this relation is never used again in the rest of the proposition. This text might perhaps be altered in the second part of this article.

[14] Step 9 provides the main geometrical relation of the proposition — the beautiful observation that, as it were, a circle is the mean between a line and a parabola. The reasoning can be provided in this manner: MN = KZ = KΞ, therefore q(MN) = q(KΞ). But q(MN) = r(MN, NΛ) + r(MN, MΛ)

square on MΞ. (12) But as the square on MN to the square on MΞ, so the triangle on MN, coming about in the prism, to the triangle on MΞ, taken away in the segment by the surface of the cylinder; (13) Therefore as MN to MΛ, so the triangle to the triangle. (14) And similarly we shall also prove that if any other <line> is drawn in the parallelogram circumscribed around the section, parallel to KZ. And a plane is set up on the drawn <parallel line>, right to the <line> EH, it shall be: as the triangle made in the prism to the triangle in the segment cut off from the cylinder, so the <line> drawn in the parallelogram ΔH, being parallel to KZ, to the <line> taken by the section of the right-angled cone HZ and <by> the diameter EH. (15) Now, the parallelogram ΓEΔH being filled by the <lines> drawn parallel to KZ, (16) and the segment contained by both: the section of the right-angled cone, and <by> the diameter EH, <being filled> by the <lines> in the segment, (17) and also the prism being filled by the triangles that come to be in it, (18) as well as the segment cut off from the cylinder, (19) there are certain magnitudes equal to each other — the triangles in the prism; (20) and there are other magnitudes, which are lines in the parallelogram ΔH, being parallel to ZKΘ, which are both equal to each other (21) and equal in multitude to the triangles in the prism; (22) and there are other triangles, in the segment cut off, equal in multitude to the triangles that come about in the prism. (23) And other lines taken away from the lines drawn parallel to KZ between the section of the right-angled cone and EH, shall be equal in multitude to the <lines> drawn parallel to KZ in the parallelogram ΔH, (24) it shall be, as well: as all the triangles in the prism to all the triangles taken away in the segment cut off from the cylinder, so all the lines in the parallelogram ΔH to all the lines between the section of the right-angled cone and the line EH. (25) And, from the triangles in the prism, is composed the prism; (26) while, from the <triangles> in the segment cut off from the cylinder, <is composed> the segment; (27) and, from the <lines> in the parallelogram ΔH, parallel to KZ, <is composed> the parallelogram ΔH; (28) and, from the lines between the section of the right-angled cone and EH, <is composed> the segment [of the parabola][15]; (29) therefore as the prism to the segment <cut off> from the cylinder, so the parallelogram ΔH to the segment EZH contained by the section of the right-angled cone and <by> the line EH. (30) But the parallelogram ΔH is half as much again as the segment so contained by the section of the right-angled cone and <by> the line EH ((31) for this has been proved in the <treatises> sent out previously[16]); (32) therefore the

(*Elem.* II–2), and $q(K\Xi) = q(MK) + q(M\Xi) = q(NZ) + q(M\Xi)$. Hence $r(MN, N\Lambda) + r(MN, M\Lambda) = q(NZ) + q(M\Xi)$, of which $r(MN, N\Lambda) = q(NZ)$ (Step 4). Therefore, $r(MN, M\Lambda) = q(M\Xi)$.

[15] The square brackets follow Heiberg's view — no doubt correct — that the word 'parabola' is a late gloss introduced inito the text (the term of course dates from Apollonius).

[16] An equivalent result (one also requires *Elements* I.41) is proved in the first proposition of the *Method* itself, but Archimedes clearly refers to the more rigorous proof in the treatise *Quadrature*

prism, too, is half as large again as the segment taken away from the cylinder; (33) therefore, of such <parts> that the segment of the cylinder is <made> of two, the prism is <made> of three, (34) but, of such <parts> that the prism is <made> of three, the whole prism containing the cylinder is made <of> 12, (35) through the one being 4 <times> the other. (36) Therefore, of such <parts> that the segment of the cylinder is <made> of two, the whole prism is made <of> 12; (37) so that the segment cut off from the whole cylinder is a sixth part of the prism.

III Discussion[17]

III.1 The Indivisibles Proof: Structure of the Argument

The proof of the proposition falls into three parts. The first is Steps 1–14, reaching the conclusion that, for any chance line drawn in the parallelogram (that is, rectangle) ΗΓΔΕ, parallel to ΚΖ, and triangles being set up on it inside the prism and the cylindrical segment, we have:

(triangle on ΜΝ):(triangle on ΜΞ) :: (ΜΝ):(ΜΛ)

That is:

(triangle in prism):(triangle in cylinderical segment)
::(line in parallelogram-rectangle):(line in parabolic segment)

or, to put this succinctly:

$$\boldsymbol{\triangle_{pr}:\triangle_{cyl} :: l_{rect}:l_{segm}}$$

This is the claim of Step 14.

The second part, Steps 14–29, transforms Step 14 into the proportion of Step 29:

(prism):(cylindrical segment)::(parallelogram-rectangle):(parabolic segment)

pr:cyl :: rect:segm

In other words, this part transforms the proportion of Step 14, holding between two plane areas and two lines, into the proportion of Step 29, holding between two solids and two plane areas, each object transformed into the corresponding object in the higher dimensionality.

The third part, Steps 29–37, simplifies the proportion of Step 29: the ratio of the parallelogram-rectangle to the parabolic segment is known from elsewhere, and

of the Parabola.

[17]For this section, we thank Henry Mendell and Pier Daniele Napolitani for important suggestions.

then simple arithmetical operations yield the conclusion of the proposition: the cylindrical segment is one-sixth the entire prism (since it is two-thirds the triangular prism directly enclosing it).

The first part is an application of proportion theory (using conic sections) to the construction. This part is ingenious and beautiful, but also straightforward, posing no conceptual difficulties. The third part is even simpler, combining a previously known result for parabolic segments, together with mere arithmetical manipulations.

The conceptual questions concerning this proposition are confined to the second part: the transformation of Step 14 to Step 29. This is also the part affected by the new reading. Heiberg's Greek text broke off right at the beginning of Step 17, and resumed in the middle of Step 23. He could easily guess the content of Steps 17–18, but was at a loss for the content of Step 23. In logical terms, then, he had a lacuna for Steps 19–23. To see the significance of these steps, we shall now summarize the second part of the proof, Steps 14–29, first skipping Steps 19–23 (that is, setting out the argument as it was known during the twentieth century), and then bringing those steps in.

Step 14: The Fundamental Proportion $\triangle_{\mathbf{pr}}\mathbf{:}\triangle_{\mathbf{cyl}} \mathbin{\mathbf{::}} \mathbf{l_{rect}:l_{segm}}$

Steps 15–18: Four reconceptions, an object being, effectively, reconceived as its constituents:

Step 15	**rect**	as made up of	**the** $\mathbf{l_{rect}}$ **in it**
Step 16	**segm**	as made up of	**the** $\mathbf{l_{segm}}$ **in it**
Step 17	**pr**	as made up of	**the** $\triangle_{\mathbf{pr}}$ **in it**
Step 18	**cyl**	as made up of	**the** $\triangle_{\mathbf{cyl}}$ **in it**

Step 24: A restatement of the Fundamental Proportion, not in "any" terms, now, but in "all" terms: **all** $\triangle_{\mathbf{pr}}$**:all** $\triangle_{\mathbf{cyl}}$ **:: all** $\mathbf{l_{rect}}$ **: all** $\mathbf{l_{segm}}$

Steps 25–28: Four assertions of identity (expressed in terms of "composition")

Step 25	**pr**	is composed of	**the** $\triangle_{\mathbf{pr}}$ **in it**
Step 26	**cyl**	is composed of	**the** $\triangle_{\mathbf{cyl}}$ **in it**
Step 27	**rect**	is composed of	**the** $\mathbf{l_{rect}}$ **in it**
Step 28	**segm**	is composed of	**the** $\mathbf{l_{segm}}$ **in it**[18]

Step 29: the Fundamental Proportion transformed in dimensions

pr:cyl :: rect:segm

The argument, without Steps 19–23, seems to be merely moving, gingerly, from the Fundamental Proportion — itself rigorously proved — to the conclusion of Step 29. This conclusion appears finally like a sheer leap of faith, based on nothing but

[18]It is typical of the Greek mathematical style that the sequence of objects in Steps 15–18 differs from that of Steps 25–28.

a trust in indivisibles. Perhaps to accommodate this leap, then, the Fundamental Proportion is gradually restated. In Steps 15–19 its four terms are correlated with the higher-dimensional objects; in Step 24 the proportion is repeated, "all" substituted for "any"; in Steps 25–28 the four terms are once again correlated with the higher-dimension objects, though this time more explicitly identified with them; finally Step 29 makes the bold claim of the higher-dimensional proportion itself. This long stretch of argument does not appear to do anything except appeal to a principle of indivisibles. Indeed it can be argued that Steps 15–24, under this reading, are redundant.

Why did no one raise the alarm? How could scholarly literature be content with a verbose, even a sloppy Archimedes? There were two major schools of thought. The dominant one, whose best known proponent is Dijksterhuis, applies here its basic understanding of the *Method*, as a heuristic text where requirements of rigour are put aside. A certain appeal to a principle of indivisibles is made throughout the *Method* (props. 1–13), though elsewhere accompanied by an appeal to mechanics. Here, mechanics being dropped, indivisibles remain, still as a mere heuristic tool. The argument, then, has no pretensions for validity. Another, more recent and less well known school of thought, is found in Sato (1986) and Knorr (1996). Both, through different avenues, reached the same conclusion: Greek mathematicians accepted indivisibles arguments of the kind above as valid. Thus the argument here might be a bit ponderous, but this is just the Greek way of following through the formulae of a well-established sequence of proof.

Briefly, we now know that both schools are at least partly wrong, and while either might be partly right, we still cannot tell which, if any. The argument is clearly not a simple application of indivisibles, so it is not merely "propositions 1–13 minus mechanics" as Dijksterhuis took it. Nor can we say on the basis of this passage that the Greeks considered the argument from indivisibles to be valid as such, as Sato and Knorr thought. On the other hand, we still cannot say whether Archimedes considered this as a merely heuristic argument, or as a rigorous one.

Here is how the structure of the argument appears with Steps 19–23 reinstated:

Step 14: The Fundamental Proportion $\boldsymbol{\triangle_{pr}:\triangle_{cyl} :: l_{rect}:l_{segm}}$

Steps 15–18: Reconceptions: **rect** as made up of **the $\mathbf{l_{rect}}$ in it**, **segm** as made up of **the $\mathbf{l_{segm}}$ in it**, **pr** as made up of **the $\boldsymbol{\triangle_{pr}}$ in it**, **cyl** as made up of **the $\boldsymbol{\triangle_{cyl}}$ in it**.

Step 19: **The $\boldsymbol{\triangle_{pr}}$ are "magnitudes" equal to each other**

Step 20: **The $\mathbf{l_{rect}}$ are "other magnitudes" equal to each other**, and

Step 21: **The $\mathbf{l_{rect}}$ are "equal in multitude" to the $\boldsymbol{\triangle_{pr}}$**

Step 22: **The $\boldsymbol{\triangle_{cyl}}$ are "equal in multitude" to the $\boldsymbol{\triangle_{pr}}$**

Step 23: **The $\mathbf{l_{segm}}$ are "equal in multitude" to the $\mathbf{l_{rect}}$**

Step 24: The Fundamental Proportion in "all" terms:

all $\triangle_{\mathbf{pr}}$:all $\triangle_{\mathbf{cyl}}$ :: all $\mathbf{l}_{\mathbf{rect}}$:all $\mathbf{l}_{\mathbf{segm}}$

Steps 25–28: Four assertions of identity: **pr** is composed of **the $\triangle_{\mathbf{pr}}$ in it**, **cyl** is composed of **the $\triangle_{\mathbf{cyl}}$ in it**, **rect** is composed of **the $\mathbf{l}_{\mathbf{rect}}$ in it**, **segm** is composed of **the $\mathbf{l}_{\mathbf{segm}}$ in it**

Step 29: The Fundamental Proportion transformed in dimensions: **pr:cyl :: rect:segm**.

The logic of the argument is still not self-obvious. The sense of each individual statement is clear, even the difficult expression "equal in multitude" which must mean "being in the same number of objects".[19] But how to make sense of the contribution of Steps 19–23 to the argument as a whole?

Steps 22–23 could have been there — had they been on their own — merely to guard against the possibility that the proportions "overlap" in such ways that, although each four-term set is proportional, the four sums of sets are no longer proportional. This could have been the case had, say, all infinitely many "triangles in the prism" been in the said ratio to only a *single* "triangle in the cylinder", so that the summation of the "triangles in the prism" would go a dimension up and yield a prism while the summation of "triangles in the cylinder" remained a mere triangle. Assume then that the lines in the rectangle and in the segment behave as they should, summing into plane areas: clearly then a prism to a triangle is not the same as an area to an area.

However, how does one make sense of Steps 19–21? These make three claims, of no obvious inherent relevance to the argument: **$\triangle_{\mathbf{pr}}$ are "magnitudes" equal to each other, $\mathbf{l}_{\mathbf{rect}}$ are "other magnitudes" equal to each other, $\mathbf{l}_{\mathbf{rect}}$ are "equal in multitude" to the $\triangle_{\mathbf{pr}}$**. In other words: the two sets, $\triangle_{\mathbf{pr}}$, $\mathbf{l}_{\mathbf{rect}}$, are each made of magnitudes all equal to each other; the number of objects in both sets is the same. What is the contribution of these claims to the argument as a whole?

The tell-tale signs are the formulaic, redundant words in these Steps: "there are certain magnitudes . . . there are other magnitudes . . . ". In the economy of the Greek mathematical lexicon, such redundancy points to an implicit reference to a previous result. In fact, we do not need to go far in search of this result. Archimedes has explicitly quoted it in the introduction to the treatise. Here, then, is what Heiberg called "Lemma 11":

'If however many magnitudes have the same ratio (equal <to them> by multitude), two by two, with other magnitudes as those similarly ordered, and the first magnitudes — whether all or some of them — are to other magnitudes in however many ratios, and the latter magnitudes are in the same ratios to other magnitudes, respectively, <then> all the first magnitudes to all the <magnitudes> they stand in ratio to, have a ratio that all the latter magnitudes have to all the <magnitudes>

[19] This is difficult, because the sets in question are all infinite; more on this below.

they stand in ratio to.'

This lemma sets out certain conditions holding for a collection of four sets, which we may call, for the sake of this anachronistic exposition, A, B, C and D with the members (a_1, a_2, a_3, ..., a_n), (b_1, b_2, b_3, ..., b_n), (c_1, c_2, c_3, ..., c_n), (d_1, d_2, d_3, ..., d_n):

(i) A and B are isomorphic under ratio. For each k, m we have $a_k:a_m :: b_k:b_m$.
(ii) A and B have the same number of terms.
(iii) C and D are produced, term by term, from corresponding terms in A, B, always through proportions (which need not be all the same). For every applicable k, we have $a_k:c_k :: b_k:d_k$.
(iv) C and D need not correspond to all the members of A and B but, instead, may correspond merely to subsets of them. However (this condition is not fully explicit, but is implied by the way in which the previous condition is set out), the subsets to which they correspond must have the same number of terms.

When all these conditions (i)–(iv) are met, then the sums of the sets fulfil:

$$\Sigma a:\Sigma c :: \Sigma b:\Sigma d$$

As we shall mention again below, this result was proved by Archimedes in *Conoids and Spheroids* 1; the explicit reference to *Conoids and Spheroids*, in the text of the *Method*, is probably, as suggested by Heiberg, a (correct) interpolation.[20]

The structure of the entire argument of Steps 14–29 can now finally be clarified.

In Step 14, the Fundamental Proportion $\boldsymbol{\triangle_{pr}:\triangle_{cyl} :: l_{rect}:l_{segm}}$ is stated.

In Steps 15–18, the higher-dimensional objects are logically reconceived as sets of objects of lower dimensionality — the four sets of Lemma 11. A may stand for the $\boldsymbol{\triangle_{pr}}$, B for the $\boldsymbol{l_{rect}}$, C for the $\boldsymbol{\triangle_{cyl}}$ and D for the $\boldsymbol{l_{segm}}$.

For Lemma 11 to apply to these four sets, certain conditions need to be met, and these are the conditions of Steps 14, 19–23:

(iii) Step 14: C and D are produced, term by term, from A and B, through proportions.
(i) Steps 19–20, taken together, show that the sets A and B are isomorphic under ratio: since all members of A are equal to each other, and all members of B are equal to each other, then it follows trivially that for each pair of A terms and B terms, $a_k:a_m :: b_k:b_m$.
(ii) Step 21: A and B have the same number of terms.

[20]Heiberg [1913] 434 n. 1.

(iv) Steps 22–23 (together with Step 21): C and D correspond to sets with the same number of terms.

Step 24: The application of Lemma 11 to these four sets is now seen to hold, and it yields:

$$\boldsymbol{\Sigma\triangle_{pr}:\Sigma\triangle_{cyl} :: \Sigma l_{rect}:\Sigma l_{segm}}$$

This can now be seen as a logical conclusion of the entire sequence of Steps 14–23, taken as a whole.

Steps 25–28: Now the logical sets — “all the triangles”, “all the lines” — are transformed into the geometrical objects they constitute.

Step 29: The claim of Step 24 is now reiterated for the sets conceived as the geometrical objects they constitute.

Notice how deeply the structure of the argument depends upon Steps 19–23. Without them, Steps 14 and 15–18 appear to be merely a proportion, and its mapping into a higher dimensionality; in fact these steps set out conditions for the applicability of a specific condition, that of Lemma 11. Step 24 appears like the conclusion of an indivisibles argument; in fact it is the conclusion of Lemma 11. Steps 25–28, 29, appear to restate respectively, with minor variation, Steps 15–18, 24; in fact those steps all differ considerably in nature, and it is only from Steps 25 onwards that higher-dimensional objects are directly contemplated. There is nothing redundant: the proof is, in fact, a logical masterpiece, based not on a simple assumption of indivisibles but on a very sophisticated argument in proportion theory.

All of which opens a can of worms. We move on to note some of the difficulties and questions arising.

III.2 The Indivisibles Proof: Some Difficulties and Questions

We have seen how deeply the Indivisibles Proof relies upon Lemma 11. This immediately gives rise to a serious difficulty. To the extent that Lemma 11 is indeed meant to be supported by the proof of *Conoids and Spheroids* 1, Archimedes is in danger of committing a fallacy. This is because the proof of *Conoids and Spheroids* 1 is clearly conceived to apply for a finite number of magnitudes. For instance, the proof makes use of the ratio of the sum of a set to one of its members. In the case of an infinite set of the kind we have here, this results in the ratio of a plane, say, to a line, which normally we would consider meaningless.[21] The proof of *Conoids and Spheroids* 1 does not lend itself to restatement without such operations.

There are many ways one can react to this difficulty. First, the proof may indeed

[21] In this context of bold departures from standard practice, can one envisage, perhaps, Archimedes admitting such a ratio as valid? We believe this is a possible, but certainly a very extreme suggestion.

have been meant as "heuristic", the lack of rigour wholly due to the extension of Lemma 11 into the infinite case. Or the other extreme: the proof was considered to be perfectly valid: Archimedes believed that Lemma 11 in fact held for the infinite case, its method of proof notwithstanding. (After all, Greek proofs are often considered to hold for a case more general than that for which they are proved.) Or Archimedes may have thought Lemma 11 was true but as yet unproved for the infinite case, the Indivisibles Proof being, for the time being, *hypothetically* valid. Many such possibilities may be further suggested and, in this article, we do not try to decide between them.

Related to the question of how Lemma 11 is taken is the question of the relationship between the Indivisibles Proof and the mechanical proofs of the *Method* (Propositions 1–13). The situation can now be seen as follows:

- Both the mechanical proofs, and the Indivisibles Proof, employ a reconception of an object as composed of its lower dimensionality constituents.
- The mechanical proofs supplement this reconception with specific mechanical assumptions, for instance that, if infinitely many objects all have their centre of weight at a certain point X, then, when they are summed up and considered as a geometrical object of a higher dimension, they shall still have the same centre of weight X.
- The Indivisibles Proof supplements the reconception of the object, with Lemma 11.

Thus a certain parallelism may be perceived between the mechanical propositions, and the Indivisibles Proof. None uses indivisibles alone. Both essentially rely upon deriving proportions for infinitely many objects. (Of course, statements about centres of weight are equivalent to geometrical proportion statements. These are proportions holding between weights, i.e. geometrical measures such as length, area, etc.; and distances). Both transform those infinitely many proportions into a single proportion. Both do so through specific extra-assumptions, different in each case. Now the question arises: which of those extra-assumptions is considered fully to validate its conclusion? The answer might be all, none, or either. Perhaps both kinds of arguments were considered valid by Archimedes (the "mechanical" propositions being un-geometrical simply because of their mention of the principle of the balance, un-geometrical in spite of being valid). Or perhaps both the balance assumptions, as well as Lemma 11, are no more than stopgap measures, designed to make somewhat more probable an argument that is seen as deeply problematical. Or perhaps one is considered valid, and not the other (but then which?). Once again, we cannot offer an answer. It is important to stress, however, that it has now become impossible to understand the mechanical propositions independently of the Indivisibles Proof. The interpretation of the mechanical propositions depends, we find out, on judging their special assumptions concerning infinite summations, by a comparison to Lemma 11.

The place of the Indivisibles Proof inside the sequence of propositions 12–15 is another question opened up by the new reading. To recall: the sequence is made of a mechanical proof (props. 12–13), an Indivisibles Proof, which we have read, and a geometrical proof, especially fragmentary in the Palimpsest (prop. 15). It was always assumed that Archimedes cited Lemma 11 so as to use it in Proposition 15, the geometrical one. This proposition goes through a construction similar to that of the Indivisibles Proof, dividing the prism not into triangles, but into prismatic slices, as thin as required for the application of the method of exhaustion.[22] (This is exactly analogous to the standard practice elsewhere in Archimedes, in particular in *Conoids and Spheroids* itself, from which Lemma 11 is taken.) The application of Lemma 11 would then obtain for finite sets of prisms and rectangles, rather than for infinite sets of triangles and lines. We now see that the Indivisibles Proof and the geometrical proof are very similar indeed. On the other hand, one notes the deep gap between these two, on the one hand, and the mechanical proof, on the other. That ingenious and very complicated mechanical proof works through an altogether different route. It first balances the cylindrical segment with another, geometrically distinct cylindrical sector, and then balances and measures that other sector.[23] In other words, the geometrical proof is nothing but an upgrade of the Indivisibles Proof using the method of exhaustion, while both differ sharply from the considerably more complicated mechanical proof. It seems that Archimedes could have chosen no worse set of examples, then, for the alleged "heuristic" value of the mechanical method. The sequence of propositions 12–15 suggests, if anything, that Archimedes first thought of the Indivisibles Proof, then transformed it, straightforwardly, into a geometrical proof, finally, and independently, came up with a mechanical proof. Propositions 12–13 lend credence to the intuition Archimedes expresses in the introduction to the *Method*: it is through knowing the result that one can find a proof. In this case, however, it seems impossible to find the *mechanical* proof without first knowing what it should prove! This, then, requires a rethinking of the nature of "heuristics" in the *Method*.

It must be stressed at this point that our answers to such questions as raised above still face the fragmentary state of the *Method*. In particular, the Indivisibles Proof starts immediately following a missing leaf, discarded already in the twelfth or thirteenth century when the manuscript was palimpsested. We have only the beginning of the mechanical proof. It is possible that, in that missing leaf, following the conclusion of the mechanical proof, Archimedes offered some general observations. Obviously, whether such observations existed or not, and then what they

[22] For the method of exhaustion in general, see Dijksterhuis [1987] 130–133.

[23] Knorr [1996] provides a clear exposition of this mechanical proof.

might possibly have been, are all questions of great bearing on the interpretation of the *Method*. Realizing how much we have been in ignorance merely by failing to read Steps 19–23 of the Indivisibles Proof, we feel humble before the lacuna of a full bifolium.

We are similarly humbled before the final lacuna of the *Method*. As mentioned above, the Palimpsest contains (some of) the discussion for the cylindrical segment, but none of the discussion for another, equally remarkable object: the figure resulting from the intersection of two cylinders within a cube. Several authors have tried in the past to reconstruct Archimedes' arguments for this intersection.[24] We shall not attempt such a reconstruction ourselves, and merely note that Archimedes could have offered any combination of mechanical, indivisibles, geometrical or indeed other kinds of proofs. The puzzle of the *Method*, then, was many-dimensional: the puzzle of the mechnical proofs on their own; the puzzle of the Indivisbles Proof(s?) on its own; the puzzle of the internal relationship inside the sequence of Propositions 12–15; the puzzle of the internal relationship inside the final (lost) sequence; finally, the puzzle of the global relationship between all parts. The more we read of the *Method*, the more we come to glimpse the intricacy of this many-dimensional structure. One thing the *Method* does not possess is, so to speak, mechanical repetition. Variation, rather than repetition, seems to be the dominant theme.

The difficulties of interpreting such a structure are evident, and are obviously compounded by its fragmentary state. Possibly, these difficulties may have been intentional. Whatever else the *Method* was, it was also a letter sent to Eratosthenes. It is in keeping with Archimedes' character, and with the character of his times, to have sent a puzzling, tantalizing text, meant as a challenge. "Find the logical structure of the *Method*" is the implicit task it sets — deliberately, we suggest — as an intricate puzzle: to Eratosthenes and, through him, to us.

We leave aside the puzzle of the *Method* and point to two general questions, one mathematical, the other philosophical, that arise from the new reading.

First, it appears that in the one recorded text where Archimedes uses the method of indivisibles without an appeal to mechanics, he does not use it directly, but attempts to justify the move to indivisibles through a special, complicated condition — that of Lemma 11. Of course, it is easy to see how Lemma 11 is brought into the picture: Archimedes is extending, in an intuitive way, an application of Lemma 11 which unproblematically applies to the finite case of Proposition 15. All the Indivisibles Proof does, then, is to move — intuitively — from the indefinitely many of Proposition 15 to the infinitely many of the Indivisibles Proof. Still, it is clear also that Archimedes does not think indivisibles, as it were, do the job on their own. At the very least, he must have thought that proportion theory statements, when applied in infinite cases, ought to be grounded in specific proportion theory

[24]See in particular Heiberg-Zeuthen [1907], Reinach [1907], Rufini [1926], Sato [1986].

conditions: one can not simply move one dimension up and trust that the proportions still hold. In a way, then, we see Archimedes in a different light from that in which the twentieth century tended to see him. He was not a mere heuristic precursor of the calculus. Rather — comparable to the more perceptive authors of the pre-calculus — he was interested simultaneously in obtaining results, and, at least to some extent, in obtaining them rigorously. It is an open question, now, how well he had chosen his conditions for rigour. What is already clear is that this new reading ought to affect our overall interpretation of the pre-history of the calculus.

The specific way in which Archimedes seeks this rigour is especially remarkable, and gives rise to the second general question, more philosophical in character. We refer to Steps 21-23. In these steps, Archimedes takes three pairs of magnitudes infinite in number and asserts that they are, respectively, "equal in multitude":

Step 21: **The $\mathbf{l_{rect}}$ are "equal in multitude" to the $\triangle_{\mathbf{pr}}$**
Step 22: **The $\triangle_{\mathbf{cyl}}$ are "equal in multitude" to the $\triangle_{\mathbf{pr}}$**
Step 23: **The $\mathbf{l_{segm}}$ are "equal in multitude" to the $\mathbf{l_{rect}}$**

We suspect there may be no other known places in Greek mathematics — or, indeed, in ancient Greek writing — where objects infinite in number are said to be "equal in multitude." Greek authors are usually taken to be extra-cautious about infinity, using the potential infinity of the indefinitely many to deal with such problems that, in other intellectual traditions, are studied through actual infinities. Here actual infinities are manipulated in specific ways, suggesting a degree of comfort which is quite surprising.

Two features in particular are striking:

- The very suggestion that certain objects, infinite in number, are "equal in multitude" to others implies that not all such objects, infinite in number, are so equal.
- All three pairs are transparently related through a geometrical pairing: the lines in the rectangle are each a base for a separate triangle in the prism; the triangles in the cylindrical segment are each contained by a separate triangle in the prism; the lines in the parabolic segment are, each, contained by a separate line in the rectangle. One is led to believe that, for Archimedes, "equality in multitude" for magnitudes infinite in number could be related to some sort of one-to-one correspondence arguments.

We have here infinitely many objects - having definite, and different multitudes (i.e., they nearly have number); such multitudes are manipulated in a concrete way, apparently by something rather like a one-to-one correspondence. Now, our understanding of the Greek treatment of infinity is largely shaped by the influence of

Aristotle, and we are often impressed by his great care to avoid actual infinities. As ever, we find that one should not generalize across all ancient authors, and that, at least in this case, Archimedes discusses actual infinities almost as if they possessed numbers in the usual sense — which of course calls for further study of the infinite in Archimedes, and in Greek thought.

To sum up, then, the new reading from Archimedes' "Indivisibles" Proof does not so much solve the puzzle of the *Method*, as add to its complexity. We believe that the new reading totally alters the logical structure of the Indivisibles Proof itself and that, as a consequence, the overall structure of the *Method* has to be reconsidered. Finally, this should call for some reconsideration of the position of Archimedes in some key areas of the history of mathematics, especially the two related conceptual fields of the calculus, and of infinity.

Appendix : New text of *Method* Proposition 14, 110v–105r

The following text, that of the verso side of the Archimedes leaf 105–110, starts at the middle of Step 14, and breaks at the very beginning of Step 31 (see the translation above, especially note 6). It thus encompasses almost all of the main part of the argument, Steps 14–29.

Standard conventions are used in the transcription, roughly following a 'papyrological' model. Note that, to avoid clutter, the critical apparatus does not note as variants the places where the new edition conflicts with Heiberg's.

We add breathings, accents and punctuation (all rare in the manuscript). We print Iota Adscriptum following the manuscript, rather than modern conventions, since our readings sometimes depend upon such characters.

H498,2 ΕΗ, (ὅτι) ἔσται ὡς τὸ τρίγωνον τὸ γε 110v. col. 1
νόμενον ἐν τῶ πρίσματι (π̣ρ̣ὸ̣ς) τὸ
⟨τ⟩ρ⟨ίγωνον ἐν τῶ ἀ⟩π̣⟨ο⟩τμήμα(τι)
τ̣ὸ̣ ἐ̣ν̣ τ̣ῶ̣ ἀ̣π̣ὸ̣ τ̣ο̣ῦ̣ κ̣υ̣λ̣ί̣ν̣δρου (οὕτως)
⟨ἡ ἀ⟩χ̣θ̣εῖσα ⟨ἐν⟩ τ̣ῶ̣ι̣ ⟨Δ⟩Η̣ παραλλη
⟨λογ⟩ρ̣ά̣μ̣μ̣ω̣ π̣α̣ράλληλο̣⟨ς⟩ οὖσα τ̣ῆ̣
Κ̣Ζ (π̣ρ̣ὸ̣ς) ⟨τὴν ἀπολ⟩ηφθεῖσαν ἀπὸ
τῆς Η̣Ζ τοῦ ὀρθογωνίου κώνου
τομῆς καὶ τῆς ΕΗ (δια)μέτρου. συμ
πληρωθέντος οὖν τοῦ ΔΗ πα
ραλληλογράμμου ὑπὸ τῶν ἀ̣γομέ
H498,10 νω̣⟨ν⟩ παρὰ τὴν ΚΖ, καὶ τοῦ τμή
ματος τοῦ περιεχομένου ὑ⟨πό⟩
τ̣ε̣ τ̣ῆ̣ς τοῦ ὀρθογωνίου κώνου τ⟨ομῆς⟩

καὶ τῆς ΕΗ (δια)μέτρου ⟨ὑ⟩πὸ τ̣⟨ῶν ἀ⟩
πολαμβανομένων ⟨ἐν τῶ τμή⟩
ματι· συμπληρωθέντ⟨ος δὲ καὶ⟩
τοῦ πρίσματος ὑπὸ τῶν τ̣⟨ριγώ⟩

7 ἀπὸ] ὑπὸ 10 τοῦ ΓΕ] τούτου ΓΕ?

νων τῶν γενομένων ⟨ἐν αὐτῶι⟩, 105r. col. 1
(καὶ) τοῦ τμήματος τοῦ ἀποτμη
θέντος ἀπὸ τοῦ κ̣⟨υλίνδρου· (καὶ)⟩ ἔ̣σ̣τ̣⟨ι⟩
τινα μεγέθ̣⟨η⟩ ἴ̣σ̣α ἀλλ⟨ήλοις, τὰ τρί⟩
γων̣⟨α⟩ τὰ ἐν τῶι πρ⟨ίσμ⟩α⟨τι⟩, (κ̣α̣ὶ̣) ⟨ἔστι
ἑτέ⟩ρ̣α̣ μ̣ε̣γ̣έ̣θ̣⟨η, αἵ⟩ ε̣ἰ̣⟨σιν εὐθεῖαι ἐν⟩
τῶ ΔΗ ⟨παραλληλογράμμω πα⟩
ράλληλ⟨οι οὖσαι τῆ Ζ⟩ΚΘ, ⟨ἃ καὶ ἀλ⟩
λ̣ή⟨λοις ἴσα⟩ ἐ̣σ̣τ̣ὶ̣, ⟨καὶ πλ⟩ήθ̣ε̣ι̣ ἴ̣σ̣⟨α
τοῖ⟩ς ἐ̣ν̣ ⟨τ⟩ῶ̣ πρίσματι τριγών⟨οις⟩·
ἔ̣σ̣τ̣α̣ι̣ δ̣ὲ̣ ⟨καὶ ἑτέρα τρίγω⟩να ⟨τ⟩ὰ̣
⟨ἐν τῶι τμήματι τῶι απο⟩τμηθέν
⟨τι ἴ⟩σ̣⟨α⟩ τ̣ῶ̣⟨ι πλ⟩ήθ̣⟨ει τοῖς γεν⟩ο̣⟨μέν
οις⟩ ἐ̣ν̣ ⟨τῶ⟩ι̣ ⟨π⟩ρ̣⟨ίσματι τρι⟩
γ⟨ών⟩ο̣⟨ις⟩· κ̣α̣ὶ̣ ⟨αἱ ἑτέραι εὐθεῖαι
ἀ⟩π̣ο̣λ̣α̣μ̣⟨βα⟩ν̣⟨όμεναι ἀπο τῶν⟩

8 ΖΚΘ] ΖΚΒ?

ἀγομένων π̣(α̣ρ̣ὰ̣) τὴν Κ⟨Ζ μετα⟩ξ⟨ὺ⟩ 110v. col. 2
τῆς ⟨τοῦ ὀρθο⟩γων⟨ίου κώ⟩νου
τομῆς καὶ τῆς Ε̣⟨Η εἰσὶ ἴσ⟩α̣⟨ι⟩
H498,20 πλῆθει ταῖς ἐν τῶι ΔΗ ⟨παραλ⟩
ληλογράμμωι ἠγμέναις ⟨(παρὰ)⟩
τὴν ΚΖ· καὶ ἔσται ὡς πάντα τὰ
τρίγωνα τὰ ἐν τῶι πρίσματι
πρὸς πάντα τὰ τρίγωνα τὰ
ἐν τῶ ἀποτμηθέντι τῶι ἀπὸ
τοῦ κυλίνδρου ἀφηρημένα
οὕτως πᾶσαι αἱ εὐθεῖαι αἱ ἐν
τῶι ΔΗ παραλληλογράμμωι (π̣ρ̣ὸ̣ς)
πάσας τὰς εὐθείας τὰς μετα
ξὺ τῆς τοῦ ὀρθογωνίου κώνου
τομῆς καὶ τῆς ⟨ΕΗ⟩ εὐθείας. (καὶ)

ἐκ μὲν τῶν ἐν τῶι πρίσματι τρι
γώνων σύγκειται τὸ πρίσμα, ἐκ
δὲ τῶν ἐν τῶι ἀποτμήματι τῶι
⟨ἀποτμηθέντι ἀπὸ τοῦ κυλίνδρου τὸ ἀπότμη⟩

6 ὡς] om.

μα, ἐκ δὲ τῶν ⟨εὐθείων⟩ τ̣ῶ̣ν̣ ἐ̣ν̣ 105r. col. 2
τῶι ΔΗ παραλληλογράμμωι π
H498,30 αρὰ τὴν ΚΖ τὸ ΔΗ παραλλη
λόγραμμον, ἐκ δὲ τῶν ⟨εὐθ⟩ε̣ί̣ω̣ν̣
μεταξὺ τῆς τοῦ ὀρθογωνίου ⟨κώ⟩
H499,1 νου τομῆς καὶ τῆς Ε⟨Η⟩ τὸ τμῆ
⟨μα⟩ τ̣ῆ⟨ς πα⟩ρ̣α̣β̣ολ̣⟨ῆς⟩· ὡς (ἄρα) τ̣⟨ὸ⟩ πρ⟨(ίσ)⟩
μ̣⟨α⟩ (π̣ρ̣ὸ̣ς) τὸ ἀπὸτμῆμα τὸ ἀπὸ (τοῦ)
⟨κυλ⟩ίνδρου οὕτως τὸ Δ̣⟨Η παρα⟩λ̣
ληλόγραμμον (π̣ρ̣ὸ̣ς) τὸ ⟨Ε⟩Ζ̣Η̣ τμῆμα
⟨τὸ περι⟩ε̣χ̣ό̣μ̣⟨εν⟩ο̣ν̣ ⟨ὑ⟩π̣⟨ὸ⟩ τῆς τοῦ
ὀρθογωνίου κώνου τομῆς καὶ
τῆς Ε̣⟨Η⟩ εὐθείας· ⟨ἡ⟩μ̣ι̣ό̣λ̣⟨ιο⟩ν̣ ⟨δὲ⟩
τὸ ΔΗ π̣⟨αραλληλό⟩γ̣ρ̣α̣μ̣⟨μον⟩ τ̣ο̣ῦ̣
τμήμα⟨τος τ⟩ο̣⟨ῦ περι⟩ε̣χ̣⟨όμενου⟩ (οὕτως)
ὑ̣⟨(πὸ⟩ τ̣ῆ⟨ς τοῦ⟩ ὀ̣ρ̣⟨θογω⟩ν̣⟨ίου κώνου
τομῆς καὶ τ⟩ῆ⟨ς Ε⟩Η̣ ⟨εὐθείας⟩· δε–

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