

The *Tithicintāmaṇi* of Gaṇeśa, A Medieval Indian Treatise on Astronomical Tables

Setsuro Ikeyama

Brown University.

E-mail: setsuro.ikeyama@nifty.ne.jp

Kim Plofker

Brown University.

E-mail: Kim_Plofker@Brown.edu

The *Tithicintāmaṇi* (TC) is one of the astronomical tables written for the convenience of Indian calendar-makers by Gaṇeśa, a famous astronomer who flourished in the early 16th century.

In this paper, after giving a minimal introduction to Indian astronomy and calendars, we give an overview of this treatise, explain the individual verses which describe rules to calculate the arguments for using the tables, and attempt to reconstruct the tables and demonstrate the meanings of each of their columns.

I Basic Description of the Indian Calendar

I.1 Elements of the Calendar

The traditional Indian calendar is called *pañcāṅga*¹ which literally means “having five subdivisions.” These subdivisions are *vāra*, *tithi*, *nakṣatra*, *yoga*, and *karaṇa*.

Vāra A *vāra* is a weekday, counted from Sunday to Saturday. Weekdays generally begin from sunrise. Therefore, each week begins from the sunrise of Sunday. Some schools of Indian astronomy, however, employ midnight epoch where weekdays begin at midnight. Hence it is sometimes necessary to convert the midnight epoch to the sunrise epoch, and vice versa. This kind of problem in the TC is discussed in section III.2.

Tithi A *tithi* is a “lunar day,” or 1/30 of a synodic month, during which the longitudinal elongation between the sun and moon increases by 12 degrees. When *tithis* are measured or calculated using mean longitudes of the sun and moon, the *tithis* are also mean. If the true longitudes are used, those are true. The first *tithi* of a synodic month begins with the conjunction of the sun and moon at new moon (hereafter NM), or opposition of those two at full moon (hereafter FM). *Tithis* are used to number each day of a month; when the sunrise of a day is included in the

¹For more information about *pañcāṅga* see, for example, IAS pp.100–109; and YANO, pp.46–61, 93–105, and 148–178.

n th tithi of a month, the day is the n th day of the month.

Nakṣatra A nakṣatra is a time unit during which the moon goes 13;20 degrees along the ecliptic. A sidereal month contains 27 nakṣatras. “Nakṣatra” originally refers to the 27 or 28 stars or groups of stars in the moon’s path, and the moon is said to conjoin with one of them each night.² It is very important astrologically to know in which nakṣatra the moon is located. The Indian months are named after the nakṣatras with which the (ideal) full moon of each month coincides.

Yoga The yoga, which literally means “sum”, is an artificial unit; during 1 yoga the sum of the longitudes of the sun and the moon increases by 13;20 degrees.

Karaṇa A karaṇa is half a tithi.

I.2 Days, Months, and Years

There are four standards for measuring years, months, and days: solar, lunar, civil, and sidereal. We explain some of them which are related to the calendar.

Day Civil days and lunar days (tithis) are commonly used in India. Civil days ordinarily begin from sunrise as explained in section I.1. Tithis have also been described above.

Month A synodic month is the interval between two successive occurrences of NM or FM.

A sidereal month is the rotation of the moon through 360° in its orbit.

A calendar month starts with the beginning of the first civil day (i.e., sunrise or midnight) immediately after NM or FM.

Year A solar year begins from the instant the sun enters into the first degree of Aries. This instant is called Meṣasaṅkrānti, “entry into Aries”. Because Indian astronomy does not employ precession for measuring a year, an Indian solar year is not tropical but sidereal.

A calendar year, on the other hand, starts with the first civil day of the month Caitra which begins immediately after NM or FM just before Meṣasaṅkrānti.

A standard calendar year includes 12 synodic months. Because 12 synodic months are shorter than 1 solar year, intercalary months are inserted periodically.

I.3 Miscellaneous

Astronomical Schools or Pakṣas Pakṣa (literally, “wing”) means a school or tradition within a discipline. Five pakṣas in Indian astronomy are traditionally

²See DSB, Supplement I, pp. 535 and 537.

distinguished by the parameters they use. Parameters of two of those,³ the Saura-pakṣa and the Gaṇeśapakṣa, are used in the TC.

The “Saura school” employs astronomical parameters based on the *Sūrya-siddhānta*, one of the most popular Sanskrit astronomical texts, which was compiled about the 9th century. The “Gaṇeśa school”, on the other hand, uses parameters devised by Gaṇeśa himself.

Era There are numerous different eras in India; the Śaka era is the most popular in astronomy. Its zero-year begins in A.D. 78. The epoch of the TC, Śaka 1447, corresponds to A.D. 1525.

Prime Meridian The Indian prime meridian or parallel of zero terrestrial longitude is generally supposed to pass through Laṅkā (Sri Lanka), Ujjayinī (modern Ujjain), and Sumeru.

Yojana The yojana is a unit of length on the order of 10 kilometers, though it does not have a standardized precise value. The circumference of the earth is traditionally assumed to be 5000 yojanas.

Sexagesimal Numbers Indian astronomy mainly uses sexagesimal numbers to express position and time. Numbers in the following sections expressed using semicolons and commas are sexagesimal. The semicolons separate the integer and fractional parts, and sexagesimal places are separated by commas. For example, 4;45,27 means $4 \times 60^0 + 45 \times 60^{-1} + 27 \times 60^{-2}$ (= 4.7575 in decimal).

Sub-Units of Time One sixtieth part of a “day” of whatever variety mentioned in section I.2 is called a ghaṭī. When the base unit is the civil day, 1 ghaṭī = 1/60 day. Ghaṭikā, nāḍī, and nāḍikā are synonyms for ghaṭī. 1/60 of a ghaṭī is called a pala or vighaṭī, vighaṭikā, etc. 4;45,27 days are 4 days, 45 ghaṭīs, and 27 palas.

Mean and True Longitudes Astronomical parameters are given in the form of integer numbers of rotations in a certain long period, either a yuga or a kalpa.

One kalpa contains 4320000000 solar years. Usually, a kalpa includes 14 subdivisions called manvantaras interspersed with 15 “twilight” (sandhi) periods. Each manvantara is divided into 71 mahāyugas or, simply, yugas. The yuga consists of 4320000 years and is divided into four sub-yugas: kṛta-yuga, tretā-yuga, dvāpara-yuga, and kali-yuga (See JYOTIḤ pp. 12–15). The intervening sandhis are equal to a kṛta-yuga, or 1728000 years.

When the number of complete rotations in such a period is given, the integer and fractional rotations in a shorter time interval can be found by proportion. In the Saura school, for example, the number of lunar rotations or sidereal months is 57753336 in 4320000 solar years. The longitude of the moon expressed in degrees at

³Though Gaṇeśa mentions in verse 18 of the TC that the parameter of the lunar node is from the Ārya pakṣa, the lunar node is never used in the TC.

the end of the y th year from the beginning of the yuga is easily calculated as:

$$\frac{57753336}{4320000} \cdot y \cdot 360 \pmod{360}.$$

This is the *mean* longitude of the moon at the end of the y th year. Any other values derived from the original parameters in this way are *mean*. Positions derived from other *mean* values are also called *mean*.

Mean longitudes of the sun etc. rectified by equations computed from their orbital anomalies are the *true* positions, and any other values calculated using these corrected longitudes are also *true*; e.g., the difference between the longitude of the *true* sun and that of the *true* moon divided by 12 is the number of *true* tithis from the *true* beginning of the synodic month.

II Overview of the *Tithicintāmaṇi*

II.1 Text of the *Tithicintāmaṇi*

We mainly used a published text edited with the *Udāharaṇa* of Viśvanātha (early 17th century) by Dattātreyā Āpaṭe, Ānandāśrama Sanskrit Series (ASS) 120, Part 1, Poona 1942. We also referred to part of a manuscript (Smith Indic 5, ff. 1–7 copied by D. Pingree. See CESS A5, 751b) of a commentary composed by Veṅkateśa in A.D. 1808. General information about the *Tithicintāmaṇi*, as well as about vast numbers of its manuscripts, is given in SATIUS pp. 47b–50b, SATE pp. 100–101, and CESS A2, 100b–103a; A3, 28a; A4, 74a–75a; and A5, 73a. For astronomical tables in general see JYOTIḤ pp. 41–46. For the author Gaṇeśa see DSB vol. V, 274–276.

II.2 Contents of the *Tithicintāmaṇi*

The published version of *Tithicintāmaṇi* we used consists of the following: an opening verse (verse 1); rules for calculating the arguments for tables and using the tables for calculating the weekday and time of day of the beginning of a given true tithi, nakṣatra or yoga in any given year (verses 2–13); general instructions to calendar-makers on using the data from the tables (verses 14–18); and three sets of tables.⁴

The Sanskrit texts of the verses with translations and explanations are given in section III (verses 1–13) and section IV (verses 14–18) of this paper. Some data from the tables is provided as an appendix.

II.3 Purpose of the *Tithicintāmaṇi*

Standard pañcāṅgas include not only the names of the tithi, nakṣatra, yoga, and karaṇa in which the beginning of each weekday is included but also the *true* time of

⁴These are classified in SATIUS as tables 1–3.

the day when a tithi, nakṣatra, yoga, or karaṇa changes to the next one. Determining these moments is the purpose of astronomical tables such as those in the TC.

The calendar-maker for whom the work is written is presumed to know the distance in yojanas from his locality to the prime meridian and also the current Śaka year. With those two items the calendar-maker can produce the required tabular arguments corresponding to the desired tithi, nakṣatra, and yoga according to the procedure described in verses 2–11. Once he obtains the arguments, he can get the required values for calendars using the tables in accordance with the instructions from verses 12 and 13.

II.4 How Do the Tables Work?

The tables seem to work as described in the following sections. We use the abbreviations below to stand for some characteristic events.

CA: mean NM or FM preceding Meṣasaṅkrānti. CA is the beginning of the synodic, not calender, month of Caitra.

MS: the entrance of the mean sun into the first degree of Aries. i.e., mean Meṣasaṅkrānti.

Tithi 0: the tithi in which MS falls.

VA: the end of the week preceding MS.

AA: the end of the anomalistic month preceding MS.

And we also use the subscripts t , n , and y to denote that given amounts with these subscripts are for the calculation of tithi, nakṣatra, and yoga respectively; and the superscripts i and g for integer and ghaṭikā portions of a time interval.

II.4.1 Tables for Tithis

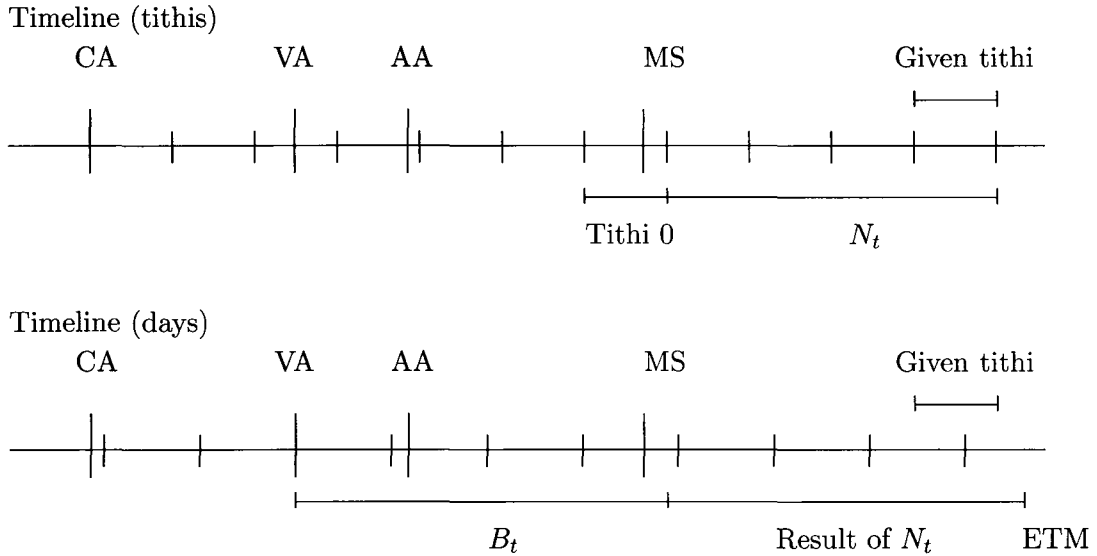
The set of tables for tithis includes three tables: vārādi table, parākhyā table, and hāra table.

Tithi-Vārādi Table The table of weekdays with fractions. The argument for this table is the number N_t of complete mean tithis from the end of mean tithi 0 to the end of the given tithi. The result is the difference in days and fractions of a day (modulo 7) between the end of mean tithi 0 and the end of the “true-mean” tithi corresponding to the given tithi.⁵ “True-mean” means that this result is calculated from the true sun and mean moon at the end of the given tithi. To this result is added the so-called tithibhoga B_t , or the actual day and time of day at the end of mean tithi 0, counted from VA. Then their sum (modulo 7) is the required weekday

⁵The value for $N_t = 0$ is, therefore, the difference in days between the end of “true-mean” tithi 0 and that of mean tithi 0.

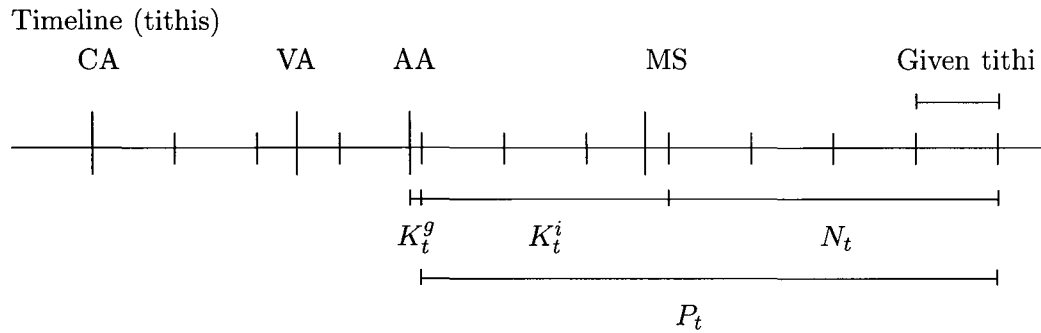
and time of day at the end of the given tithi. As mentioned above, this result seems to take into account only true solar and mean lunar motion, and must therefore be corrected with respect to the moon by results from the two subsequent tables.

Figure 1. Tabular Arguments and Results for Tithis (1)



Tithi-Parākhya Table The table of additional corrections. The argument is the sum P_t of the previous argument N_t and the integer part K_t^i of the tithikendra K_t , that is, the number of tithis from the end of the mean tithi including AA to the end of mean tithi 0.⁶ The result is an increment (or decrement) in ghaṭīs and smaller units which is applied to the day and time produced by the tithivārādi procedures. This result, depending as it does on the time elapsed from a point near AA (when the lunar anomaly is 0), is evidently a correction to account for the effect of lunar anomaly upon the interval up to the given tithi.

Figure 2. Tabular Arguments and Results for Tithis (2)



⁶For kendras see sections III.6–III.7.

Tithi-Hāra Table The table of divisors for another additional correction. The argument for this table is again the argument P_t . The fractional part K_t^g of the tithikendra K_t described above is divided by the divisor (hāra) in the table, and the result is an increment or decrement in ghaṭīs applied to the corrected result from the tithiparākhyā procedure. This is a calculation of the effect of the lunar anomaly upon the fractional part of the kendra, K_t^g , that was omitted in the previous step, and thus intended to compensate for that omission. With the parākhyā and hāra corrections, we finally get the weekdays with fractions from VA till the end of the true tithi.

Example We give a concrete example using the sample of the table of TC in the appendix.

We require the number of complete mean tithis since MS, or N_t , and also the number of tithis from AA to the end of mean tithi 0, which is K_t . The length in days of the fraction of a week from VA to the end of mean tithi 0 is B_t . Their supposed values are as follows:

$$\begin{aligned} N_t &= 10 \\ K_t &= 5; 20, 15 \quad (K_t^i = 5, \quad K_t^g = 0; 20, 15) \\ B_t &= 3; 30, 25 \\ P_t &= N_t + K_t^i = 10 + 5 = 15. \end{aligned}$$

Enter the column of tithi-varādi with $N_t = 10$ and get 3;0.57. Add $B_t = 3; 30, 25$ to it, and get

$$3; 0, 57 + 3; 30, 25 = 6; 31, 22.$$

Then enter the column of tithi-parākhyā and tithihāra with $P_t = 15$, and get $-6; 5$ and -13 respectively. Add $-6; 5 \times 60^{-1}$ (because they are expressed in ghaṭī) to 6; 31, 22:

$$6; 31, 22 + (-0; 6, 5) = 6; 25, 17.$$

Divide $K_t^g = 0; 20, 15$ by -13 :

$$\frac{0; 20, 15}{-13} \approx -0; 1, 33.$$

Add it multiplied by 60^{-1} to 6; 25, 17:

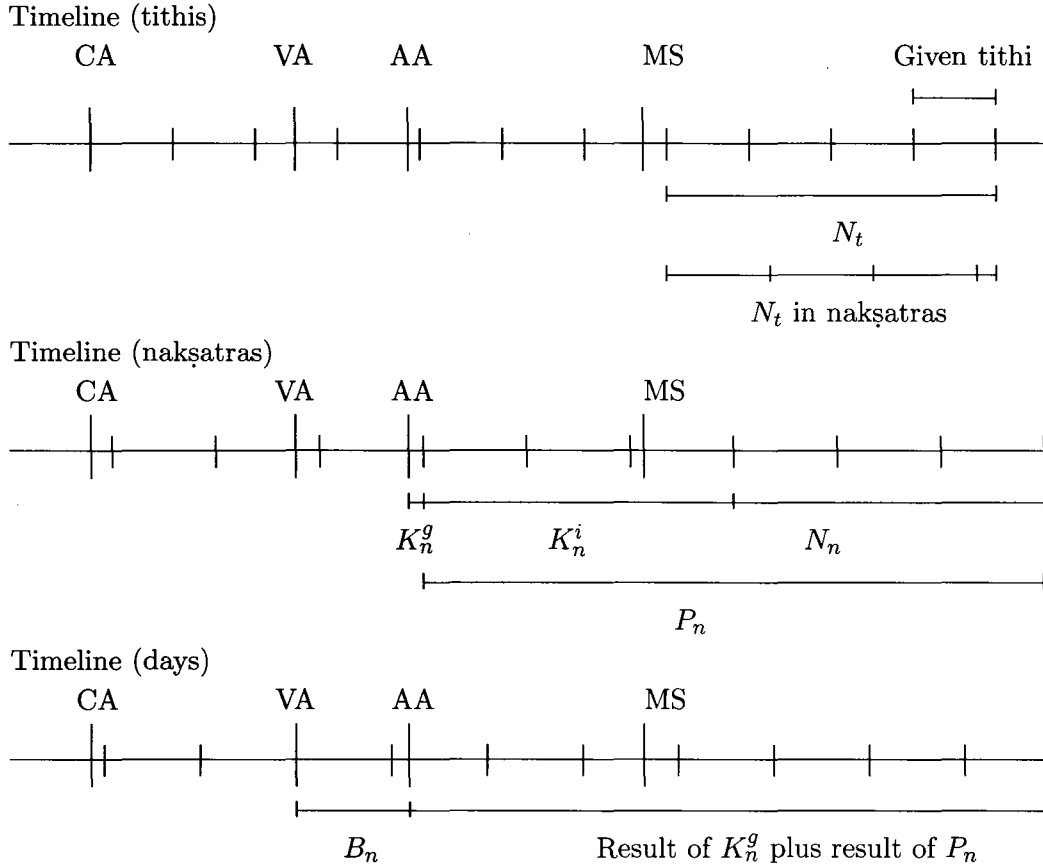
$$6; 25, 17 + (-0; 0, 1, 33) = 6; 25, 15, 27.$$

The final result, 6; 25, 15, 27, is the time of the weekday at the end of the given true tithi; that is, around sunset of a Saturday.

II.4.2 Tables for Nakṣatras

The set for nakṣatras includes just two tables. See section III.13.

Figure 3. Tabular Arguments and Results for Nakṣatras



Nakṣatra-Vārādi Table The tithivārādi argument N_t , the number of tithis, is converted to nakṣatras and truncated to the nearest integer, N_n . Thus N_n is an integer number of nakṣatras which may be considered as a time interval extending backwards from the end of the nakṣatra closest to the end of the given tithi (i.e., the given nakṣatra) to the end of the nakṣatra closest to the end of tithi 0 (namely, nakṣatra 0). To this is added the integer part K_n^i of the nakṣatrakendra K_n , the number of nakṣatras from the end of the nakṣatra including AA to the end of nakṣatra 0. Their sum is the argument P_n , the interval in nakṣatras between the end of that anomalistic month and the end of the given nakṣatra (minus the fractional part of the nakṣatrakendra, K_n^g). The result is the difference in days etc. (modulo 7) corresponding to that interval, and apparently takes into account true lunar motion.

Nakṣatra-Hāra Table The argument P_n is used again to find the resulting hāra by which the fractional part K_n^g of the nakṣatrakendra K_n must be divided. Their quotient, as in the tithihāra table, is applied to the previous result to compensate for the previous omission of K_n^g . To this is added the nakṣatrabhoga B_n , the actual

weekday and time of the end of the preceding anomalistic month. This sum (modulo 7) is the required weekday and time of the end of the given nakṣatra.

II.4.3 Tables for Yogas

This set contains three tables.

Yoga-Vārādi Table This table is intended to be essentially similar to the tithivārādi table. The argument N_y is the tithivārādi argument N_t converted to yogas and truncated to the nearest integer. Thus N_y is an integer number of yogas which may be considered as a time interval extending backwards from the end of the yoga closest to the end of the given tithi (i.e., the given yoga) to the end of the yoga closest to the end of tithi 0 (namely, yoga 0). The result is the difference in days and fractions of a day (modulo 7) between the end of yoga 0 and the end of the given yoga. To this is added the yogabhoga B_y , the day and time of the end of yoga 0 counted from the end of the preceding week. Their sum (modulo 7) is the weekday and time of day at the end of the given yoga, assuming true solar and mean lunar motion.

Yoga-Parākhya and Yoga-Hāra Tables These tables operate in precisely the same way as the corresponding tithi tables described above (using N_y , K_y etc. instead of the corresponding tithi parameters) to produce corrections (due to lunar anomaly) to the value of the weekday and time at the end of the given yoga.

III Rules for Computing the Tabular Arguments

We give in this section the Sanskrit texts of verses 1–13, their translations, and explanations of their meaning.

III.1 Verse 1: Salutation

*yaś cintāmaṇir aṅkalekhyabahulo 'tyalpakriyo matkṛtas
tithyādyāvagamagrado 'sya sukhino ye lekhane bhīravah |
tatprītyai laghum alpakṛtyam amalaṃ tithyādicintāmaṇiṃ
vighneśārkamukhān praṇamya kurute śrīmad gaṇeśaḥ kṛtī || 1*

A jewel of thought (*cintāmaṇi*) which I have made is rich in numbers and figures and gives knowledge about the tithi and so on, and is very concise. But some people are timid about this pleasant composition. Having saluted Gaṇeśa and (the planets) beginning with the sun, the skillful Gaṇeśa makes a jewel of thought about the tithi etc., which is easy, concise, and without errors for the sake of such people.

Gaṇeśa composed another set of tables named *Bṛhattithicintāmaṇi*.⁷ But its epoch is Śaka 1474 = A.D. 1555, thirty years after that of TC. So the *Tithicintāmaṇi* mentioned in the former half of this verse is presumably not this *Bṛhattithicintāmaṇi*. There is also an expanded version of the *Tithicintāmaṇi* in 73 verses.⁸ The relation between the two versions has so far not been clearly established.

III.2 Verse 2: Lord of the Year

*vyagayugamanuśākaḥ syāt samaugho hato 'yaṃ
svarakhakhakubhir āpto nāgaśatyā 'bdapaḥ syāt |
dyumukha iha paleṣu tryabdhīḥḍvarṣayuktaḥ
śrutibhir iṣusamudrair bhair yutaḥ saptataṣṭaḥ || 2*

(The number of) Śaka years diminished by 1447 is the number of years (from the epoch of the TC). Multiply this by 1007 and divide by 800. Add the year divided by 43 to its palas (1/3600 of a day). Increase (the sum) by 4;45,27 and divide by 7. The remainder is the lord of the year (L) beginning with days.

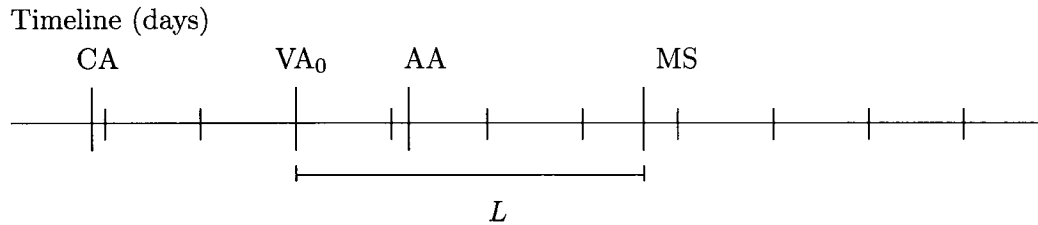
Let Y be the current Śaka year:

$$y = Y - 1447$$

$$L = \frac{1007}{800}y + \frac{y}{43} \cdot \frac{1}{3600} + 4;45,27 \pmod{7}.$$

The formula gives the excess in days over an integer number of weeks accumulated in the course of y years from the epoch of the TC (Śaka 1447): that is, the time in days between MS of the current year and the end of the preceding week at the prime meridian (VA_0). This is called the lord of the year, L , as it specifies which weekday begins or “rules” the current year. For example, when $0 \leq L < 1$, the day including MS is Sunday.

Figure 4. Lord of the Year at the Prime Meridian



⁷Published with the *Subodhinī* of Viṣṇu (without tables), ASS 120, Part 2.

⁸We referred to a copy in manuscript Oxford (Vyāsa) 11. See CESS A5, 73a.

The first y -term implies a year-length of $364 + 1007/800 = 6; 5; 15, 31, 30$ days, the year-length of the Ārdharātrikapakṣa.⁹ The second y -term increases this parameter to $6; 5; 15, 31, 31, 23, 43 \dots$ days; compare the Saurapakṣa parameter $6; 5; 15, 31, 31, 23, 59 \dots$ days. The accumulated excess weekdays in the y years since epoch are added to the epoch constant (kṣepaka), $4; 45, 27$ days, and the result (modulo 7) is the required fraction of a week, in days.

The given epoch constant is the lord of year at epoch, Śaka 1447 = A.D. 1525. Using the Saurapakṣa parameters of 4320000 years and 1577917828 days¹⁰ in the yuga, and 1955884626 years elapsed of the kalpa at epoch,¹¹ there results an integer number of weeks elapsed plus $4; 0.27$ days, which is less than the epoch constant, $4; 45, 27$, by $0; 45$.¹²

The $0; 45$ -day difference does not seem to be the result of a different choice of parameters. For if we recalculate the epoch constant with the *Tithicintāmaṇi* year-length, it produces a value of $3; 53, 28$ for the lord of the year. If the Ārdharātrikapakṣa year-length is used, the lord of the year becomes $3; 58, 39$. The $0; 45$ -day difference, then, is an extra quantity added to the result of the calculation with the Saurapakṣa parameters, apparently in order to shift the beginning of a day from the midnight epoch of the Saurapakṣa¹³ to a mean sunrise epoch. One may wonder why Gaṇeśa adds $0; 45$ and does not subtract $0; 15$ for getting the epoch constant, $3; 45, 27$. It seems likely that Gaṇeśa deliberately avoided changing the weekday of the first day of Śaka 1447; both $4; 0.26.56$ and $4; 45, 27$ imply the first day of Śaka 1447 is Thursday, while $3; 45.27$ indicates Wednesday.

III.3 Verse 3: Longitudinal Difference

rekhāyāḥ svapurasthayojanāni
svāṅghryūnāni palaiś ca tatpramāṇaiḥ |
hānādhyam vidadhīta varṣanātham
rekhāyāḥ parapūrvage svadeśe || 3

⁹See, for example, PS I 14 and XI 1.

¹⁰SS 1, 37.

¹¹The quantity of 1955884626 years corresponds to a combination of 7 twilights of 1728000 years each, 6 manvantaras of 306720000 years each, 27 mahāyugas of 4320000 years each in the current manvantara, 3888000 years of the kṛta-, treta- and dvāpara-yuga in the current mahāyuga, and 4626 years of the present kaliyuga up to Śaka 1447 = A.D. 1525, less the canonical 17064000 years of quiescence stipulated by the Saurapakṣa. (If these 17064000 years were included in the calculation, the epoch constant would be $1; 36, 26, 56$ days.)

¹² $1577917828 \times 1955884626 / 4320000 \pmod{7} = \text{an integer number of weeks} + 4; 0, 26, 56 \dots$

¹³The *Sūryasiddhānta* states (SS 1, 50) the epoch time to be midnight, and thus the period of quiescence or creation in the present kalpa ended at midnight Saturday/Sunday.

Diminish the (distance in) yojanas of one's own town from the prime meridian (δ) by their fourth part. Decrease or increase the lord of the year (L) by that amount in palas when one's own town is west or east of the prime meridian.

The rule given in this verse is:

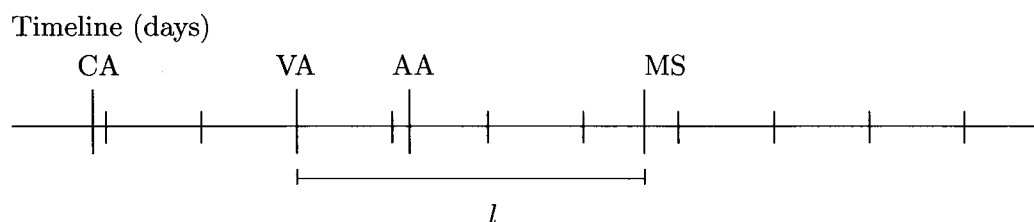
$$d = \frac{3}{4} \cdot \delta,$$

where δ is the distance in yojanas from the prime meridian to the observer's longitude and d is the time-difference in palas (1/3600 of a day) corresponding to that longitudinal distance. Then d is added to or subtracted from L :

$$l = L \pm d.$$

The rule replaces the initial value L for the current lord of the year with the value l corrected for the local longitude. Thus l gives the time in days between the current MS and the end (at mean sunrise) of the previous week (VA) at the local longitude.

Figure 5. Lord of the Year Corrected by the Longitudinal Difference



The derivation of the formula as reconstructed by Venkateśa is quite crude: one simply assumes that the circumference of the observer's parallel of latitude is 4800 yojanas. Since this corresponds to 1 solar circuit of the earth or 1 nychthemeron = 60 ghaṭikās, 1 ghaṭikā (= 60 palas) is equivalent to 80 yojanas, so the number of palas in d is 3/4 times the number of yojanas in δ .¹⁴

III.4 Verse 4: Accumulated Epact (Śuddhi)

bhavanighnasamāḥ svaṣaṭsahasrā-
ṁśakahīnāḥ śaradāṁ dināṁśayuktāḥ |
tithipūrvakaśuddhir anvitārthaiḥ
kṛtabāṇaiś ca jīnaiḥ kharāmataṣṭāḥ || 4

¹⁴Bhāskara's *Karaṇakutūhala* (KK 1, 14–15) uses a similar rule.

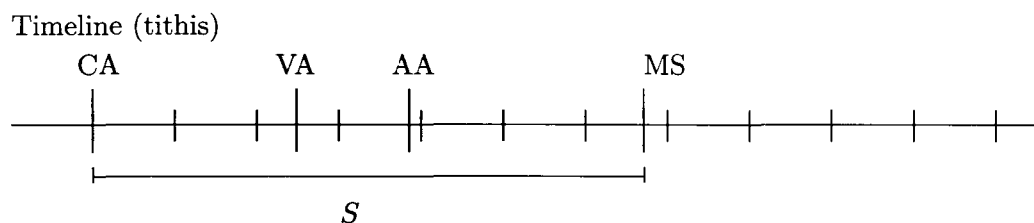
Multiply (the number of) years (y) by 11, diminish (that) by its 6000th part, add the 15th part of (the number of) years, add 5;54,24, and divide by 30. The remainder is the accumulated epact (śuddhi) beginning with tithis.

The formula given in this verse for calculating śuddhi or the accumulated epact S is:

$$S = 11y + \frac{y}{15} - \frac{11y}{6000} + 5;54,24 \pmod{30}.$$

The accumulated epact is the time in mean tithis between MS of the current year and the preceding CA.

Figure 6. Accumulated Epact (śuddhi)



The y -terms give a value for the epact of $11 \frac{389}{6000}$ or $11;3,53,24$ tithis, in accordance with the Saurapakṣa. This is derived from the ratio of tithis to years in a yuga: 1603000080 tithis¹⁵ in 4320000 years gives $6,11;3,53,24$ tithis per year, or $11;3,53,24$ tithis in excess of twelve synodic months. Equivalently, 57753336 lunar rotations in 4320000 years gives $1,20,12;46,40,48$ degrees of lunar motion per solar year, or $2,12;46,40,48$ of excess lunar motion over integer rotations. This equals the annual accumulation of lunisolar elongation (since the sun is at 0° at the end of the year), which when divided by 12° of elongation per mean tithi gives, as before, $11;3,53,24$ tithis. The tithis thus accumulated in the y years since epoch are added to the epoch constant, $5;54,24$ tithis, and the sum modulo 30 (i.e., after removal of integer intercalary months consisting of 30 tithis each) gives the accumulated epact for the current year.

The commentator Veṅkateśa explains the epoch constant, or epoch śuddhi, as the excess over integer rotations, in degrees, of the product of the epact in degrees and the number of years of the kalpa elapsed at epoch.¹⁶ This product gives an excess of $1,11;1,40,48 \approx 1,11;1,41$. Gaṇeśa diminishes this by $0;9$ to conform to observation.¹⁷ The result, $1,10;52,41$, is divided by 12 to produce the given epoch

¹⁵ $30 \times (57753336 - 4320000)$, that is, 30 times the excess of lunar over solar rotations, i.e. total synodic months. See the table of yuga parameters in section V.1.

¹⁶ $2,12;46,40,48 \times 1955884626 \pmod{360}$.

¹⁷See *Grahalāghava* 1, 16 and TC 18 (section IV.5 of this paper).

constant, 5;54,24 mean tithis.¹⁸

III.5 Verse 5: Śuddhi Complement (Dhruva)

śuddhiḥ khaṣaṭśodhitanāḍikā syāt
tithidhruvo 'tha svadaśāṃśahīnā |
śuddhis tu mūrdhny ekayutā khaṣaḍbhyah
saṃśuddhanāḍī bhayujor¹⁹ dhruvaḥ syāt || 5

The accumulated epact whose nāḍikās are subtracted from²⁰ 60 (produces) the dhruva of the tithi. Then, the accumulated epact is diminished by its 10th part. 1 is added to (its) first place (i.e., integer part) and (its) nāḍīs are subtracted from 60. (The sum of the results) is the dhruva of the nakṣatra and the yoga.

Using the subscripts t , n and y to denote amounts in tithis, nakṣatras and yogas, and superscripts i and g for integer and ghaṭikā portions, we have:

$$\begin{aligned} D_t^i &= S^i, \\ D_t^g &= 60 - S^g, \\ S_n &= S_y = \frac{9}{10}S, \\ D_n^i &= D_y^i = S_n^i + 1, \\ D_n^g &= D_y^g = 60 - S_n^g. \end{aligned}$$

The fractional tithidhruva D_t^g is the tithi-complement of the śuddhi, S ; that is, because there are 60 tithi-ghaṭikās in a tithi, one subtracts the fractional part of the śuddhi, S^g , from 60 to give the fractional tithidhruva, or time from MS to the end of the tithi in which it falls (tithi 0). The integer part D_t^i of the tithidhruva is considered to be the same as that of the śuddhi, S^i .

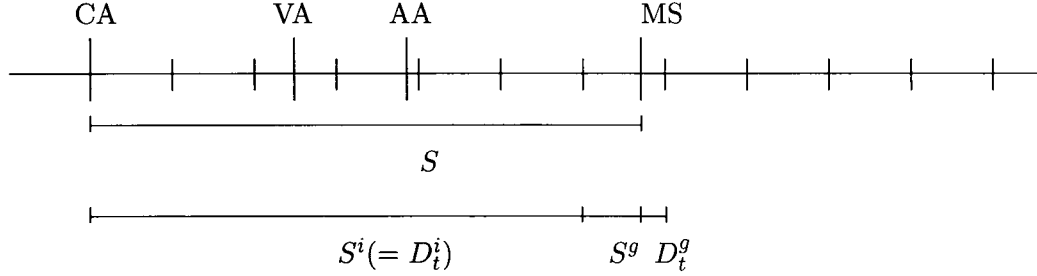
¹⁸If one computes this without Gaṇeśa's lunar correction, the epoch constant becomes 5;55,8,24 mean tithis.

¹⁹Published text: *bhujayor*.

²⁰The original Sanskrit of this part, *khaṣaṭśodhitanāḍikā*, literally means "whose nāḍikās are diminished by 60". This, however, makes no sense so we translate this part according to Viśvanātha's commentary.

Figure 7. Dhruva

Timeline (tithis)



The corresponding dhruvas of the nakṣatra and the yoga appear to be motivated by some such reasoning as the following. Since at MS the mean longitude of the sun $\bar{\lambda}_S$ is 0, using only the mean longitude of the moon $\bar{\lambda}_M$, we can calculate the mean tithi, nakṣatra and yoga corresponding to that moment relatively simply from

$$\begin{aligned} \text{tithi} &= \frac{\bar{\lambda}_M - 0}{12} = S, \\ \text{nakṣatra} &= \frac{\bar{\lambda}_M}{13;20} = \frac{12}{13;20}S = \frac{9}{10}S, \\ \text{yoga} &= \frac{\bar{\lambda}_M + 0}{13;20} = \frac{12}{13;20}S = \frac{9}{10}S. \end{aligned}$$

The integer part of the quantity $(9/10)S$ should then be $D_n^i = D_y^i$, and its fractional part ($S_n^g = S_y^g$) subtracted from 60 gives $D_n^g = D_y^g$, the nakṣatra- and yoga-complement of the śuddhi—i.e., the time from MS to the end of the nakṣatra or yoga in which it falls. This is true *only* if the beginnings of a nakṣatra and a yoga also coincide with CA, as does that of a tithi.

It is not clear why D_n^i and D_y^i are increased by 1. It may imply that S_n^g or S_y^g is subtracted from this 1 unit = 60 ghaṭikās, though both Viśvanātha and Veṅkateśa do not interpret the procedure in this way but simply add 1 to D_n^i and D_y^i without further discussion.

III.6 Verse 6: Mean Kendra for Tithi

saptāhato jaladhīśeṣitavarṣasaṃgho
'bdāṅgāṃśayuk śaradilādaśanāmśahīnaḥ |
kendraṃ tither bhavati madhyam idaṃ saṃudraiḥ
sāṅghryabdhivahnibhir upetaṃ ibhāśvitaṣṭham || 6

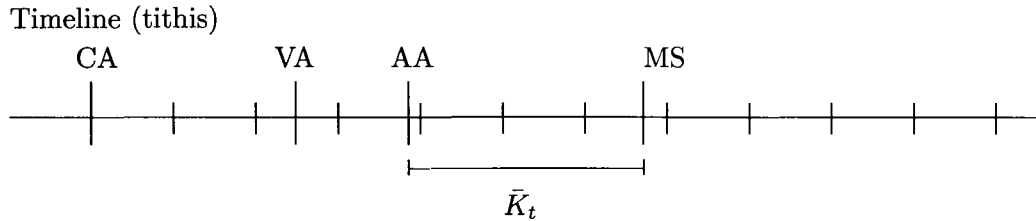
Multiply by 7 (the number of) years which remains after dividing (y) by 4, add a sixth part of the years, diminish by $1/321$ of the years, add 4;34,15, and divide by 28. The remainder gives the mean kendra of tithi (\bar{K}_t).

The formula given is:

$$\bar{K}_t = y \bmod 4 \cdot 7 + \frac{y}{6} - \frac{y}{321} + 4; 34, 15 \pmod{28}.$$

The mean tithikendra \bar{K}_t is the time in tithis between MS and the end of the preceding anomalistic month (AA).

Figure 8. Mean Kendra for Tithi



The portion of the kendra accumulated since epoch is calculated from the ratio of the number of anomalistic months in a yuga (57753336 lunar rotations minus 488203 rotations of the lunar apogee = 57265133 anomalistic months) to the corresponding number of years, 4320000. This ratio produces 13; 15, 20, 56, 39 \approx 13; 15, 21 anomalistic months per year. This is converted to tithis by computing the ratio of tithis to anomalistic months in a yuga: 1603000080/57265133 = 27; 59, 34 tithis per anomalistic month, which is rounded up to 28; and then multiplying it by the yearly fraction of an anomalistic month to give 0; 15, 21 \times 28 = 7; 9, 48 tithis per year in excess of integer anomalistic months. The integer part of this annual increment will produce $7 \times 4 = 28$ tithis or one anomalistic month every 4 years; hence the factor of y modulo 4 in the first term.²¹ The fractional part may be rewritten as $588/3600 = (600 - 12)/3600$ tithis per year, which ought to give exactly $y/6 - y/300$ for the second and third terms, instead of $y/6 - y/321$ as stated.²²

The epoch constant results, as usual, from a ratio involving the yuga parameters and the number of years elapsed of the kalpa: 1955884626 lapsed years times 57265133 anomalistic months per yuga, divided by 4320000 years per yuga, gives integer anomalistic months plus 0; 9, 47, 42, 54. When this is multiplied by 28,

²¹Mathematically $(y \pmod{4}) \times 7 = 7y \pmod{28}$.

²²Can Gaṇeśa possibly have made an error in computation here? The fraction $1/6 - 1/321$ corresponds to 0; 9, 48, 47 rather than 0; 9, 48 as the fractional part of the annual increment. If the number of rotations of the lunar apogee in a yuga were taken to be 488106 (approximately in accordance with the Brāhmapakṣa), the corresponding amount would be 0; 9, 48, 42 or about $1/6 - 1/319$ tithis per year. This change of parameter, besides producing a result that still falls short of Gaṇeśa's, would produce a wrong value for the kṣepaka and undermine Gaṇeśa's assertion (TC 18) that the lunar apogee according to the Saurapakṣa is correct. We can think of no other deliberate modification of this method that would come close to producing Gaṇeśa's actual results.

there results 4;34,16,1,12, a quantity that can be adjusted to Gaṇeśa's given constant by judicious use of truncation and rounding in the calculation. (E.g., 0;9,47,42 \times 28 = 4;34,15,36.) This, then, represents the tithis accumulated at the epoch date (MS of Śaka 1447) since the end of the preceding mean anomalistic month. The sum of that and the fractions of anomalistic months accumulated in the y years since epoch, modulo 28, is the number of tithis between MS and AA, or \bar{K}_t . The end of that month, however, does not actually coincide with the end of a tithi, so the difference between the kendra \bar{K}_t and the śuddhi S will not be an integer number of tithis.

III.7 Verse 7: Mean Kendras for Nakṣatra and Yoga; Corrected Kendras

*taddvidhā svarasarāmalavonaṃ
svāśvidasralavayuk kramaśaḥ staḥ |
kendraḥ uḍuyujor bhavakendraṃ
syāt sphuṭaṃ dhruvaghāṭīmukhayuktam || 7*

Write it (the tithikendra) down in two places. (That amount,) diminished by its own 36th part and increased by its own 22nd part, is the (mean) kendra of the nakṣatra and yoga respectively. These kendras become true when increased by the ghaṭikās of the dhruva.

This verse includes two calculations:

$$1) \quad \bar{K}_n = \bar{K}_t \cdot \left(1 - \frac{1}{36}\right), \quad 2) \quad K = \bar{K} + D^g.$$

$$\bar{K}_y = \bar{K}_t \cdot \left(1 + \frac{1}{22}\right).$$

1) The mean nakṣatrakendra \bar{K}_n and mean yogakendra \bar{K}_y are simply converted from the mean tithikendra \bar{K}_t using the yuga parameters. The number of nakṣatras in a yuga is just 27 times the number of lunar rotations: $27 \times 57753336 = 1559340072$. The corresponding number of yogas is 27 times the sum of solar and lunar rotations: $27 \times (57753336 + 4320000) = 1675980072$. Then the ratio of these to the corresponding number of tithis (1603000080) can be found from continued fractions:

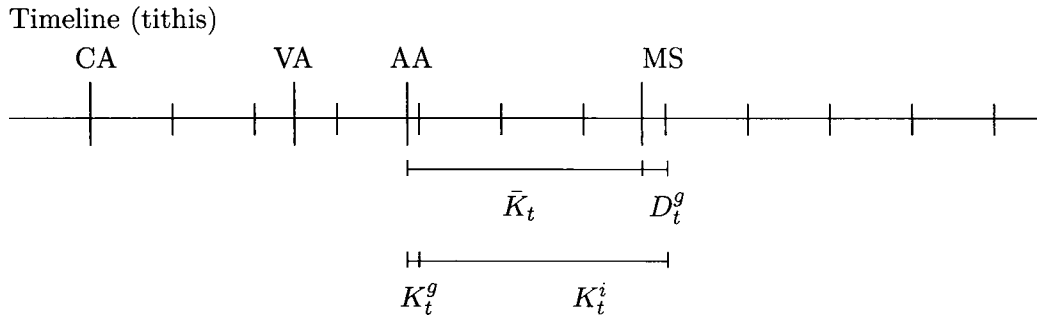
$$\frac{\text{nakṣatras}}{\text{tithis}} = \frac{1559340072}{1603000080} = \frac{1}{1 + \frac{1}{35 + \frac{1}{31239792/43660008}}} \approx \frac{1}{1 + \frac{1}{35}} = \frac{35}{36},$$

(although 36/37 would be a slightly better approximation). Similarly,

$$\frac{\text{yogas}}{\text{tithis}} = \frac{1675980072}{1603000080} = 1 + \frac{1}{21 + \frac{70420122}{72979998}} \approx 1 + \frac{1}{22} = \frac{23}{22}.$$

2) One is then expected to correct each mean kendra \bar{K} to the true kendra K by adding the fractional part of its corresponding dhruva, D^g .²³ In the case of the tithi, since D^g is assumed to be the complement of the śuddhi (see section III.5), K_t therefore is the time interval in tithis between the beginning of the first mean tithi after MS (or the end of mean tithi 0) and the end of the preceding anomalistic month.

Figure 9. Corrected Tithi Kendra



The mean kendras \bar{K}_n and \bar{K}_y of nakṣatra and yoga are just \bar{K}_t converted to different units, and thus represent the amount of time from the end of the anomalistic month to MS. Adding the corresponding D_n^g or D_y^g to them then is supposed to give the time K_n or K_y from the end of the preceding anomalistic month (which does *not* necessarily correspond to the end of a nakṣatra or yoga) to the beginning of the first nakṣatra or yoga following MS (or the end of mean nakṣatra 0 or mean yoga 0).

III.8 Verse 8: Bhogas

*tither dhruvaghaṭīmukhaṃ svayugaṣaḍkabhāgonitaṃ hy
 uḍor bhavati yan nijāmbudhigajāṃśakenānṅvitaṃ |
 yuter²⁴ nijaśiloccayakṣitilavonitaṃ tadyutaḥ
 pṛthak pṛthag ihābdapo nijanijaś ca bhogo bhavet || 8*

²³“Mean” and “true,” though they echo Gaṇeśa’s “madhya” and “sphuṭa,” are somewhat misleading terms here: this procedure does not actually apply a correction for accuracy but rather produces a result corresponding to a different time interval.

²⁴Published text: *yute*.

Add to the lord of the year the ghaṭikās of the dhruva of the tithi diminished by its own 64th part, that of the nakṣatra increased by its own 84th part, and that of the yoga diminished by its own 17th part, separately. (The sums are) their bhogas (B) respectively.

The bhoga B (see figures 1 and 3) in days is found by (i) converting to days the interval D^g representing (inaccurately, in the case of the nakṣatra and yoga) the time between mean Meṣasaṅkrānti and the beginning of the following tithi, nakṣatra or yoga; and (ii) adding to that l in days, representing (as explained in section III.3) the lord of year modified by the local longitudinal difference.

$$\begin{aligned} B_t &= l + D_t^g \cdot \left(1 - \frac{1}{64}\right), \\ \bar{B}_n &= l + D_n^g \cdot \left(1 + \frac{1}{84}\right), \\ B_y &= l + D_y^g \cdot \left(1 - \frac{1}{17}\right). \end{aligned}$$

Thus B is the interval in days between VA and the end of the tithi, nakṣatra or yoga following MS. The initial nakṣatrabhoga \bar{B}_n is further modified in section III.9.

As in the case of conversion to tithis, the factors for conversion to days are obtained quite simply from continued fractions involving the yuga parameters, to wit:

$$\begin{aligned} \frac{\text{days}}{\text{tithis}} &= \frac{1577917828}{1603000080} = \frac{1}{1 + \frac{1}{62 + \frac{1}{22818204 + \frac{1}{25082252}}}} \approx \frac{1}{1 + \frac{1}{63}} = \frac{63}{64}, \\ \frac{\text{days}}{\text{nakṣatras}} &= \frac{1577917828}{1559340072} = 1 + \frac{1}{83 + \frac{1}{17396324 + \frac{1}{18577756}}} \approx 1 + \frac{1}{84} = \frac{85}{84}, \\ \frac{\text{days}}{\text{yogas}} &= \frac{1577917828}{1675980072} = \frac{1}{1 + \frac{1}{16 + \frac{1}{8121924 + \frac{1}{98062244}}}} \approx \frac{1}{1 + \frac{1}{16}} = \frac{16}{17}. \end{aligned}$$

III.9 Verse 9: Modified Bhoga for Nakṣatra

bhaspaṣṭakendramūrdhāṅkaḥ svavedāṣṭāṃśasamyutaḥ |
saptataṣṭas tadūno 'harmukhaḥ kāryo bhabhogakaḥ || 9

Increase the number in the first place of the true kendra of the nakṣatra by its own 84th part and divide by 7. Subtract the remainder from the bhoga of the nakṣatra beginning with days. (The result is the rectified bhoga of the nakṣatra.)

$$B_n = \bar{B}_n - K_n^i \cdot \left(1 + \frac{1}{84}\right) \pmod{7}.$$

The nakṣatrabhoga \bar{B}_n (see figure 3) given above is modified by diminishing it by the integer part of the nakṣatrakendra K_n converted to days, modulo 7. (The conversion factor is the same as that explained in section III.8). The purpose seems to be to redefine the bhoga to be the weekday and time not of the end of nakṣatra 0 but of AA (the reason for which is explained in section III.13). If the kendra is less than the bhoga—that is, if AA falls between VA and MS, as depicted in the above figures—the subtraction will accomplish this purpose. If not, according to Viśvanātha, the bhoga is increased by 7.

III.10 Verse 10. Koṣṭhaka, or First Tabular Argument

*caitrāder gatatithayo dhruvasya tithyā²⁵
hīnāḥ syus tithidinaḥkoṣṭhakā²⁶ dvidhā te |
svāṅgāgnyamaśakarahitāḥ kramāt svadr̥gdṛg-
bhāgādhyā²⁷ udvyutijās tyajet tadagram || 10*

The tithis passed since the beginning of the month Caitra (t) are diminished by the tithi of the dhruva (D_t^i). (The remainder) is the argument for the weekdays of the tithi (N_t). Write it down in two places. (That,) diminished by its own 36th part and increased by its own 22nd part, is (the amount) produced from the nakṣatra and the yoga respectively. Subtract (the numbers of) their initial (places).²⁸ (The results are the arguments of nakṣatra and yoga, N_n and N_y .)

This verse gives the arguments (N) for using the varādi or weekday tables of tithi, nakṣatra, and yoga.

$$N_t = t - D_t^i, \quad N_n = \left[N_t \cdot \left(1 - \frac{1}{36}\right) \right], \quad N_y = \left[N_t \cdot \left(1 + \frac{1}{22}\right) \right].$$

²⁵Published text: tithyāḥ.

²⁶Published text: °koṣṭakā.

²⁷Published text: svadr̥k°.

²⁸The meaning of this sentence is not clear.

See figures 1 and 3. The quantity t represents the integer number of tithis in the interval from CA of the current year to the beginning of the tithi for whose end the corresponding weekday and time of day are to be found. This is diminished by the integer part of the tithidhruva D_t , which is equal to the integer part of the śuddhi S (see section III.5). So the resulting difference N_t is the integer number of tithis from the beginning or end of the tithi within which MS falls (called tithi 0 in the table heading) to the beginning or end, respectively, of the given tithi. N_n and N_y represent the same time interval converted to nakṣatras and yogas, respectively (using the conversion factors explained in section III.7), and truncated to the nearest integer. It appears that they are meant to represent the number of time-units from the end of the nakṣatra or yoga including MS to the end of the given nakṣatra or yoga (which is the one closest to the end of the given tithi).

III.11 Verse 11: Parākhyakoṣṭhaka, or Second Tabular Argument; Use of Tables

*svakendramūrdhāṅkayutāḥ*²⁹ *svakoṣṭhakāḥ*
*syus te parākhyā atha yattithīyujoh*³⁰ |
*dyukoṣṭhakādhodyumukhaṃ*³¹ *svabhogayuk*
parākhyakoṣṭhasthaghaṭīyutonitam || 11

Their arguments are increased by the number in the initial (place) of their own kendras. Those are (the arguments of) the parākhyas (P). Then, increase (the entries) beginning with days, under the arguments for the weekdays of the tithi and the yoga, by their own bhogas and increase or decrease them by the ghaṭīs in the column of the parākhyā.

See figures 2 and 3. The parākhyakoṣṭhaka (P) or second tabular argument, with which we enter into the parākhyā and hāra tables, is the sum of the first argument N and the integer part K^i of the kendra:

$$P_t = N_t + K_t^i, \quad P_n = N_n + K_n^i, \quad P_y = N_y + K_y^i.$$

It therefore represents the time interval between AA and the end of the given tithi, nakṣatra or yoga—minus the small interval of the fractional part K^g of the kendra. The result in the table is apparently the time difference in ghaṭīs produced by the effect of the lunar anomaly upon P . As explained in the preceding “Overview of the *Tithicintāmaṇi*” (section II), for the tithi and yoga, entering the vārādi table with

²⁹Published text: °mūrdhvā°.

³⁰Published text: *yattithiyujoh*.

³¹Published text: °ādho dyumukhaṃ.

the koṣṭhaka N yields the difference, in weekdays and fractions of a day, between the end of time-unit 0 and the end of the given time-unit; increasing this by the bhoga gives the weekday and time at the end of time-unit 0, taking into account only *true* solar and *mean* lunar motion; and adjusting this by the result of P in the parākhyā table corrects it for *true* lunar motion (exclusive of the small interval K^g).

III.12 Verse 12: Use of Hāra Tables for Tithi and Yoga

adhahsthahāroddhṛtakendranāḍikāḥ
*susamskṛtaṃ syāt tapanodayāt*³² *sphuṭam |*
tathaiiva caikaikasametakoṣṭhataḥ
*tadagratithyādidinādikam*³³ *bhavet || 12*

Divide the nāḍīs of the kendra by the divisor (hāra) placed under (the parākhyā). What is corrected (by that result) is the true (weekdays) from sunrise. In the same manner, the weekdays of the tithi etc. beginning with that are produced (successively) from the arguments successively increased by one.

As described above in the “Overview,” the hāra table produces an amount in ghaṭīs etc. to be applied to the previous result, and apparently intended to account for the effect of lunar anomaly upon the small interval K^g omitted earlier. The argument P produces the resulting divisor by which K^g is divided to give an increment or decrement that provides the final correction to the true weekday and the time of day at the end of the given tithi or yoga. Gaṇeśa notes that the procedure can be repeated for each succeeding tithi or yoga.

III.13 Verse 13: Use of Nakṣatra Tables

parākhyakoṣṭhe dyumukhaṃ tu bhasya
svahārahṛtkendraghaṭīmukhena |
susamskṛtaṃ tac ca parisphuṭam syāt
svabhogayuktaṃ tapanodayāt syāt || 13

(The number of) week-days of the nakṣatra in the column of the parākhyā is corrected by the ghaṭīs etc. of the kendra divided by its own divisor. That increased by its own bhoga is the very accurate (number of weekdays) from sunrise.

³²Published text: *tāpano*°.

³³Published text: °*tithyādi dinādikam*.

Since the calculation of the nakṣatra does not involve solar motion, it can be accomplished by the use of only two tables: one to account for the effect of the lunar anomaly upon the integer parākhyakoṣṭhaka P_n , and one to account for the effect of the lunar anomaly upon the fractional kendra K_n^g . The combined result gives the accurate offset in days etc. from AA to the end of the given nakṣatra. To produce the true vārādi for that time, this result must be augmented (modulo 7) by the weekday and time of AA itself. Hence the modification of the nakṣatrabhoga B_n , described in section III.9, to produce that quantity.

IV Tips for Calendar-Makers

The remaining verses 14–18 are intended as helpful hints for the construction of pañcāṅgas.

IV.1 Verse 14: Excess and Deficiency of a Day

*ullaṅghya ced divasam eti tithiḥ parāhaṃ
pūrve 'hni ṣaṣṭighaṭikāḥ syur iyaṃ dinarddhīḥ |
ekāhni cet tithiyugaṃ paranāḍikās tāḥ
pūrvonitāḥ syur iha pūrvagamāt kṣayo 'yam || 14*

If (the end of) a tithi, passing over a day, falls into the following day, (there are) 60 ghaṭikās in the previous day: this is the “excess day” (in ghaṭikās). If two (ends of) tithis (occur) in one day, the nāḍikās (in the day at the end) of the second (tithi are) diminished by (those of the end of) the first (tithi): this is (the length in nāḍikās of) the “deficient (day)” from (the end of) the first (tithi).

Here Gaṇeśa seems to state the definitions of the length in ghaṭikās (=nāḍikās) of excess and deficient days. If a day does not include the end of any tithi, the day is an “excess” day and its length is 60 ghaṭikās. On the other hand, if a tithi is completely included in a day and therefore includes no ends of days, that tithi converted into day-nāḍīs is called a “deficient” day in this verse.

IV.2 Verse 15: Identification of Nakṣatra and Yoga

*nijadhruvordhvāṅkayutā dyukoṣṭhakāḥ
bhaśeṣitās te bhayutī ca te stah |
ekaikayuktau pratikoṣṭham atra
kṣaye 'pi vṛddhau tithivad vidhiḥ syāt || 15*

The argument of weekdays (of the nakṣatra and the yoga) increased by the argument of their own dhruva, modulo 27, is the nakṣatra and yoga. Add 1 (to the nakṣatra and yoga) successively for each successive argument. As for deficiency and excess, the same rule as for tithis is (applied).

According to this, the sum of the koṣṭhaka N_n or N_y (integer number of time-units following MS) and the integer dhruva D_n^i , D_y^i (integer number of time-units between CA and MS), modulo 27, gives the number of the given nakṣatra or yoga. This makes use of the definition in section III.5 of the integer nakṣatra- and yoga-dhruva as equivalent to the number of the nakṣatra or yoga at MS.

IV.3 Verse 16: Entrances of the Sun into Zodiacal Signs and Nakṣatras

*sābdeśakāḥ saṅkramaṇapraveśā
vṛṣād yamaṅkṣāc ca parisphuṭāḥ syuḥ |
tithidyukoṣṭhe likhitaṃ svanāma
yasmin tadāsannadine 'thavā syuḥ || 16*

(The time offsets) of the entrances of the sun (into) Taurus etc. or (the constellation) Bhāraṇī etc., increased by the lord of the year, are the true (weekdays of the entrance). Or, (the entrance) occurs on the day (in the cell) of the table of the weekdays of the tithi in which its own name is written.

If the calendar-maker wishes to include in his work the times of entrance of the sun into zodiacal signs and nakṣatra-constellations besides MS, all he has to do is add the lord of the year to the time offset from MS to that entrance.³⁴

IV.4 Verse 17: Intercalary and Omitted Months; Tithis within the Śuddhi

*meṣād dvādaśasaṅkramaiś ca sahitāś caitrādimāsā amā-
nto māsaḥ sa visaṅkramo 'dhika iti dviḥ saṅkramaḥ sa kṣayaḥ |*

³⁴Tables 4 and 5 of SATIUS pp.49–50 are the tables of the times of entrance into nakṣatra-constellations and zodiacal signs respectively, though it is not clear why the tabulated values of the times of entrance into Aries and Aśvinī are not 0;0,0 but 4;49,34 (or 6;4,34). These values are presumably calculated by computing the time corresponding to the longitude difference between that entrance and Aries 0°; if it is a *true* longitude difference that is required, some sort of iterative rule will be necessary for this. Cf. *Brāhmasphuṭasiddhānta* 3, 61–62.

*syāt spaṣṭājābhasaṅkramaḥ prathamataḥ śuddhes tṛtīye dine
śuddheḥ prāktithayo madhupratipadaḥ syuḥ pūrvavarṣadhruvaiḥ || 17*

The months beginning with Caitra include 12 entrances (of the sun into the zodiacal signs) beginning with Aries (respectively). The month ends (and begins) at the instant of new moon. (A month) without an entrance is an intercalary month. (A month) with two entrances is an omitted month. The true entrance (of the sun) into Aries occurs on the third day of the śuddhi from the beginning (MS).

(Any) tithis from the beginning of Caitra (that are) before the śuddhi are treated with the constants of the previous year.

Intercalary and omitted months are defined. Gaṇeśa adds that for calculating the times of the tithis of the current year preceding MS, the dhruva (and, presumably, the other required quantities) should be that determined for the preceding year.

He also points out that the *true* entrance of the sun into Aries falls on the third day before the *mean* one, which we call MS, since the solar equation near Aries 0° is 2+°.

Finally he instructs that if the number of tithis counted from CA is less than the śuddhi, one should use the constants of the previous year.

IV.5 Verse 18: Parameters

*sauro 'rko 'pi vidhūccam aṅkalkikonābjas tamas tv āryajas
tebhyaḥ syād grahaṇādi dṛksamam³⁵ iyaṃ proktā mayā sā tithiḥ |
grāhyā maṅgaladharmanirṇayavidhāv eṣā yato dṛksamā
'thāpekṣā yadi cālītopakaraṇais tatpakṣajā syāt tithiḥ || 18³⁶*

(The parameters of) the sun and lunar apogee are (from the) Saura (school). (That of) the moon diminished by 9 minutes is (the Saura value). The node (of the moon's orbit) is, however, (from the) Ārya (school). From those, the agreement of (the calculation of) eclipses with observation occurs. What is stated by me is such a tithi.

This (tithi) should be accepted in the determination and performance of auspicious ceremonies and duties because of (its) agreement with observation.

(Though the parameters of some schools which do not agree with observations are useless,) it may be possible to get a tithi produced by such schools by means of adjusted auxiliary tables, if it is desired.

³⁵Published text: ° dṛg°.

³⁶The first quarter of this verse is similar to that of *Grahalāghava* 1, 16.

Besides the peroration, verse 18 states that the positions for the sun and lunar apogee as given by the Saurapakṣa are correct, as is that of the moon diminished by nine minutes. This alteration would change the number of rotations of the moon in a yuga from the Saura school's 4, 27, 22, 35, 36 to 4, 27, 22, 35, 35; 59, 59, 59, 48, so it seems reasonable to suppose that for simplicity Gaṇeśa retained the Saura yuga parameters in his calculations. He also mentions that the Ārya school gives the correct parameter of the lunar node, yet he never uses it in the TC.

V *Tithicintāmaṇi* Table Reconstruction

To date, our reconstructions of the *Tithicintāmaṇi*'s tables are tentative (no hints as to his methods being given by Gaṇeśa himself) and only moderately successful.

V.1 Parameters

The reconstructions rely upon the following yuga parameters from the Saurapakṣa³⁷ which seem to be the same as those used to calculate the epoch constants:

Years	4320000	Sidereal months	57753336
Synodic months	53433336	“Yoga months”	62073336
Apogee rotations	488203	Anomalistic months	57265133
Tithis	1603000080	Nakṣatras	1559340072
Yogas	1675980072	Civil days	1577917828

This produces the following mean motions, \bar{v} , for the sun, moon, and lunar apogee (denoted by subscripts S , M , and A respectively) in a mean day, tithi, nakṣatra, or yoga (denoted by superscripts d , t , n , and y):

\bar{v}_S^d	0;59, 8, 10, 10	\bar{v}_M^d	13;10, 34, 52, 4	\bar{v}_A^d	0;6, 40, 58, 43
\bar{v}_S^t	0;58, 12, 39, 4	\bar{v}_M^t	12;58, 12, 39, 4	\bar{v}_A^t	0;6, 34, 42, 16
		\bar{v}_M^n	13;20	\bar{v}_A^n	0;6, 45, 45, 20
\bar{v}_S^y	0;55, 40, 33, 53	\bar{v}_M^y	12;24, 19, 26, 7	\bar{v}_A^y	0;6, 17, 31, 1

Since the end of the period of quiescence at the beginning of the kalpa there have elapsed 1955884626 years up to the epoch, mean Meṣasankrānti of Śaka 1447 (MS1447).

³⁷These parameters are given, for example, in SS 1, 29–44.

V.2 Required Quantities

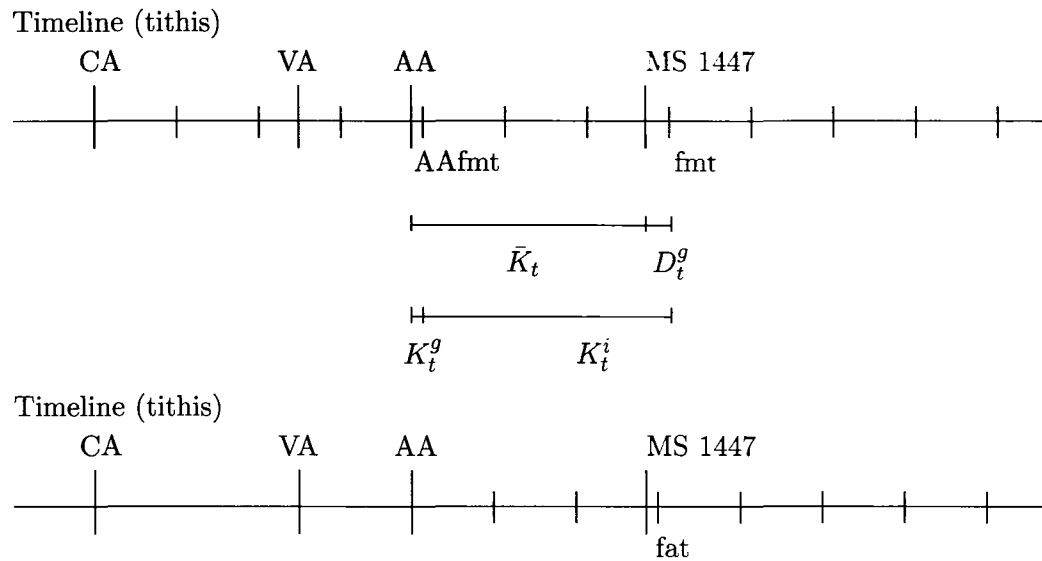
The yuga ratios give the epoch mean longitudes (mean longitudes at MS1447) $\bar{\lambda}_S^e (= 0^\circ)$, $\bar{\lambda}_M^e$, $\bar{\lambda}_A^e$ for each of the astronomical bodies in question.³⁸ The fractional dhruvas D_t^g , D_n^g , D_y^g and the kendras K_t , K_n , K_y , for tithi, nakṣatra and yoga respectively, are computed as specified in the text. From these we can compute the mean longitudes of the sun, the moon, and apogee at the end of the mean tithi, nakṣatra and yoga in which MS falls (denoted by the subscript 0 or by “fmt,” “fmn,” “fmy” for “first mean tithi,” etc.):

$$\bar{\lambda}_S^{\bar{t}_0} = D_t^g \cdot \bar{v}_S^t + \bar{\lambda}_S^e, \text{ etc.}$$

In addition, we compute their mean longitudes at the end of the mean tithi, nakṣatra and yoga in which MS falls if these units are counted from the end of the preceding anomalistic month, AA (which ordinarily does *not* coincide with the end of a tithi, nakṣatra or yoga). These are denoted by the subscript 00, or by “fat”, “fan”, “fay” for “first anomalistic tithi”, e.g., $\bar{\lambda}_S^{\bar{t}_{00}}$ means the mean solar longitude at “fat”. These reference points are illustrated below by an example using tithis.

We demonstrate here only the procedure for calculating the tables for the tithi because the algorithms for calculating the nakṣatra and yoga are almost identical to that for the tithi, except that no corrections for solar motion are involved for the nakṣatra.

Figure 10. fmt and fat



³⁸At present, $\bar{\lambda}_M^e$ omits Gaṇeśa’s nine-minute correction to the lunar longitude given in verse 18. When this correction is included, the resulting difference is negligible.

V.3 Recalculation of Tables for the Tithi

The basic procedure for recomputing the tabular values is this:

- (step 1) Knowing the mean longitudes at “fmt” ($\bar{\lambda} \bar{t}_0$) as well as the number of mean tithis (K_t^i) between that time and the end of the mean tithi in which AA falls (“AAfmt”), and possessing also the mean motions per tithi as shown above, we can compute the mean longitudes for the sun, the moon, and apogee at the end of every mean tithi \bar{t}_j from “AAfmt” to the end of the table.
- (step 2) We do the same for every mean tithi \bar{t}_{jj} as computed from the reference point “fat,” and get the mean longitudes at the end of every mean tithi \bar{t}_{jj} from AA.
- (step 3) For each of these mean longitudes the corresponding solar and lunar anomalies and equations are calculated. The mean longitude and anomaly of the sun for \bar{t}_{jj} are only used to compute solar velocities. We have no need of the equation of the sun here.
- (step 4) From these, using Gaṇeśa’s formulas, we compute the corrected solar and lunar velocities.
- (step 5) And hence we find the “mean-true” elongation during this tithi between the mean moon and true sun, and the “true” elongation between the true moon and true sun.
- (step 6) Then the time difference is calculated between the end of this “mean-true” tithi (\check{t}_j), the conjunction of the mean moon and true sun, and that of the corresponding mean tithi from the solar equation and the “mean-true” elongation. This time difference, converted into days and added (modulo 7) to the number of days in j mean tithis, is the vārādi or “mean-true” weekday between fmt and the end of \check{t}_j .
- (step 7) The parākhyā, or correction to the vārādi due to lunar anomaly, is computed from “fat” parameters, to reflect the underlying assumption that AA is the end of a tithi. The lunar equation divided by the true elongation and converted to days is the difference between the “mean-true” and the true current tithi, i.e., the parākhyā.
- (step 8) The fractional kendra divided by the hāra, or “hāra-difference,” is the correction applied to compensate for the error introduced by the assumption that AA is the end of a tithi. In our program, the parākhyā is recalculated as above but with “fmt” parameters, and the difference between the “fmt” and “fat” parākhyās is then the hāra-difference, which when divided into the epoch fractional kendra is the recomputed hāra. This recomputed hāra, however, is apparently not the one that Gaṇeśa calculated, because if he had obtained “fmt” parākhyās or the *true* correction to the vārādi due to lunar anomaly, he would have had no need to calculate and tabulate hāras. He seems to have used a linear interpolation to get hāras from two successive “fat” parākhyās.

as we will show below.

The specific algorithms used in this procedure are as follows:

- (step 1) Computations are carried out for each successive mean tithi \bar{t}_j (for $-K_t^i \leq j \leq$ the end of the table, so that the first of these \bar{t}_j , when $j = -K_t^i$, is the first tithi after AA). To calculate the mean longitude for each body at the corresponding mean tithi ($\bar{\lambda}^{\bar{t}_j}$)³⁹:

$$\begin{aligned}\bar{\lambda}_S^{\bar{t}_j} &= \bar{\lambda}_S^{\bar{t}_0} + j \cdot \bar{v}_S^t, \\ \bar{\lambda}_M^{\bar{t}_j} &= \bar{\lambda}_M^{\bar{t}_0} + j \cdot \bar{v}_M^t, \\ \bar{\lambda}_A^{\bar{t}_j} &= \bar{\lambda}_A^{\bar{t}_0} + j \cdot \bar{v}_A^t.\end{aligned}$$

- (step 2) In addition, we calculate the mean longitude ($\bar{\lambda}^{\bar{t}_{jj}}$):

$$\begin{aligned}\bar{\lambda}_S^{\bar{t}_{jj}} &= \bar{\lambda}_S^{\bar{t}_{00}} + j \cdot \bar{v}_S^t, \\ \bar{\lambda}_M^{\bar{t}_{jj}} &= \bar{\lambda}_M^{\bar{t}_{00}} + j \cdot \bar{v}_M^t, \\ \bar{\lambda}_A^{\bar{t}_{jj}} &= \bar{\lambda}_A^{\bar{t}_{00}} + j \cdot \bar{v}_A^t.\end{aligned}$$

- (step 3) The solar and lunar anomalies κ_S and κ_M at this time, starting from each of these reference points, are found (taking the longitude of the solar apogee to be 78° , as in the Saurapakṣa):

$$\begin{aligned}\kappa_S^{\bar{t}_j} &= \bar{\lambda}_S^{\bar{t}_j} - 78, \\ \kappa_M^{\bar{t}_j} &= \bar{\lambda}_M^{\bar{t}_j} - \bar{\lambda}_A^{\bar{t}_j}, \\ \kappa_S^{\bar{t}_{jj}} &= \bar{\lambda}_S^{\bar{t}_{jj}} - 78, \\ \kappa_M^{\bar{t}_{jj}} &= \bar{\lambda}_M^{\bar{t}_{jj}} - \bar{\lambda}_A^{\bar{t}_{jj}}.\end{aligned}$$

Then Gaṇeśa's formulas from *Grahalāghava* 2. 2–3 are applied to produce the

³⁹For simplicity's sake, no conditions for determining sign or result modulo 360 are indicated in these or any of the following formulas.

corresponding equations $\mu_S^{\bar{t}_j}$, $\mu_M^{\bar{t}_j}$, $\mu_M^{\bar{t}_{jj}}$. The formulas are as follows:⁴⁰

$$\mu_S^{(o)} = \frac{\left(20 - \frac{\kappa_S}{9}\right) \cdot \frac{\kappa_S}{9}}{57 - \frac{\left(20 - \frac{\kappa_S}{9}\right) \cdot \frac{\kappa_S}{9}}{9}}$$

$$\mu_M^{(o)} = \frac{\left(30 - \frac{\kappa_M}{6}\right) \cdot \frac{\kappa_M}{6}}{56 - \frac{\left(30 - \frac{\kappa_M}{6}\right) \cdot \frac{\kappa_M}{6}}{20}}$$

(step 4) Now we compute for this mean tithi \bar{t}_j (or \bar{t}_{jj} , depending on whether we count it from “fmt” or from “fat”) the “daily velocity of elongation,” that is, the difference accruing in the day corresponding to this tithi between the longitudes of sun and moon.

The true velocity formulas for the luminaries are taken from *Grahalāghava* 2, 4.⁴¹ They are slightly modified to represent daily motions at the given tithi,

⁴⁰There is no evidence to show that Gaṇeśa used them to compute these tables. These rules can be derived by starting with a simplified procedure from the *Sūryasiddhānta* (*SS* 2, 34–39), namely

$$\mu^{(')} = \text{Sin } \kappa \cdot \frac{c}{360}$$

where c is the (maximum) circumference of the epicycle, and combining it with the well-known approximation

$$\text{Sin } \kappa \approx \frac{4R\kappa(180 - \kappa)}{40500 - \kappa(180 - \kappa)}$$

If R is taken as 3438, c for the sun as 14, and for the moon as 32, the resulting constants are close to Gaṇeśa’s.

⁴¹It is not yet apparent how Gaṇeśa derived these formulas for daily motions, and, again, whether indeed he used them to compute these tables.

in degrees:⁴²

$$v_S^{\bar{t}_j^{(o)}} = \bar{v}_S^{(o)} \pm \frac{\left(11 - \frac{90 - \kappa_S^{\bar{t}_j}}{20}\right) \left(\frac{90 - \kappa_S^{\bar{t}_j}}{20}\right)}{13 \cdot 60},$$

$$v_M^{\bar{t}_j^{(o)}} = \bar{v}_M^{(o)} \pm \frac{7 \cdot \left(11 - \frac{90 - \kappa_M^{\bar{t}_j}}{20}\right) \left(\frac{90 - \kappa_M^{\bar{t}_j}}{20}\right)}{3 \cdot 60},$$

$$v_S^{\bar{t}_{jj}^{(o)}} = \bar{v}_S^{(o)} \pm \frac{\left(11 - \frac{90 - \kappa_S^{\bar{t}_{jj}}}{20}\right) \left(\frac{90 - \kappa_S^{\bar{t}_{jj}}}{20}\right)}{13 \cdot 60},$$

$$v_M^{\bar{t}_{jj}^{(o)}} = \bar{v}_M^{(o)} \pm \frac{7 \cdot \left(11 - \frac{90 - \kappa_M^{\bar{t}_{jj}}}{20}\right) \left(\frac{90 - \kappa_M^{\bar{t}_{jj}}}{20}\right)}{3 \cdot 60}.$$

(step 5) Combining these to produce the daily velocity of elongation is fairly straightforward. For the *vārādi*, this is computed for the mean moon and true sun, producing the “mean-true” velocity of elongation $\check{v}_\Delta^{\bar{t}_j}$; for the *parākhya*, both moon and sun are true, giving the true velocity of elongation $v_\Delta^{\bar{t}_j}$ or $v_\Delta^{\bar{t}_{jj}}$.

$$\begin{aligned}\check{v}_\Delta^{\bar{t}_j} &= \bar{v}_M - v_S^{\bar{t}_j}, \\ v_\Delta^{\bar{t}_j} &= v_M^{\bar{t}_j} - v_S^{\bar{t}_j}, \\ v_\Delta^{\bar{t}_{jj}} &= v_M^{\bar{t}_{jj}} - v_S^{\bar{t}_{jj}}.\end{aligned}$$

⁴²The original formulas in the *Grahalāghava* are:

$$v_S^{(')} = \bar{v}_S \pm \frac{\left(11 - \frac{90 - \kappa_S}{20}\right) \left(\frac{90 - \kappa_S}{20}\right)}{13},$$

$$v_M^{(')} = \bar{v}_M \pm \frac{7 \cdot \left(11 - \frac{90 - \kappa_M}{20}\right) \left(\frac{90 - \kappa_M}{20}\right)}{3}.$$

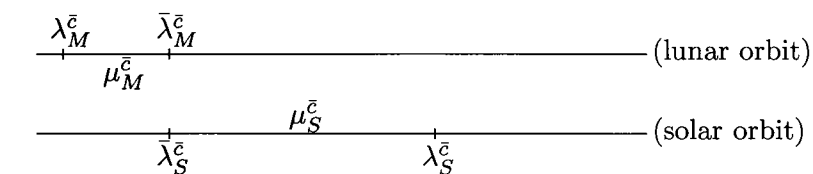
(step 6) Then the resulting difference $\delta_{\bar{t}_j}$ in days between the end of mean-true tithi \check{t}_j and that of the mean tithi \bar{t}_j is computed:

$$\delta_{\bar{t}_j} = \frac{\mu_S^{\bar{t}_j}}{\check{v}_{\Delta}^{\bar{t}_j}}$$

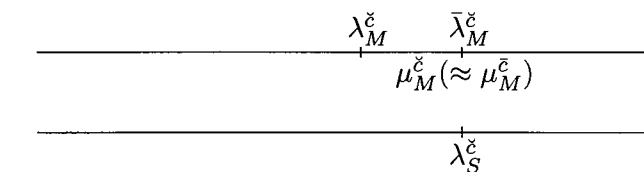
This formula is derived as follows: In the following explanation, we use the conjunction of the sun and moon as the end of tithi for the sake of simplicity; in figure 11, \bar{c} , \check{c} , and c represent the moment of the conjunction of the mean moon and mean sun, that of the mean moon and true sun, and that of the true moon and true sun, respectively.

Figure 11. Calculation of Time Difference

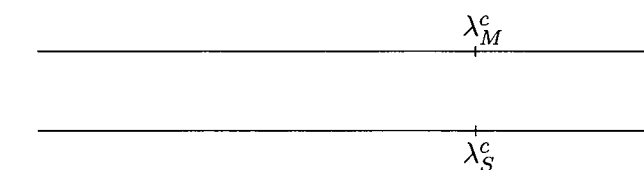
11, 1 Positions of the Sun and Moon at \bar{c}



11, 2 at \check{c}



11, 3 at c



At \bar{c} (figure 11, 1), the mean sun and moon conjoin. The true sun and true moon locates at distances of $\mu_S^{\bar{c}}$ and $\mu_M^{\bar{c}}$. Then, assuming that at \check{c} , $\delta_{\bar{c}}$ after \bar{c} , the mean moon catches up with the true sun (figure 11, 2),

$$\begin{aligned} \delta_{\bar{e}} \cdot \bar{v}_M &= \delta_{\bar{e}} \cdot v_S^{\bar{c}} + \mu_S^{\bar{c}} \\ \delta_{\bar{e}} &= \frac{\mu_S^{\bar{c}}}{\bar{v}_M - v_S^{\bar{c}}} \\ &= \frac{\mu_S^{\bar{c}}}{\check{v}_{\Delta}^{\bar{c}}} \end{aligned}$$

The *vārādi* $V^{\bar{t}_j}$ is the sum (modulo 7) of this difference and the number of days in j mean tithis:

$$V^{\bar{t}_j} = \left(j \cdot \frac{d}{t} + \delta_{\bar{t}_j} \right) \pmod{7},$$

where d/t is the ratio of the length of a day to a tithi for converting j in tithis to days.

(step 7) The *parākhyā* $p^{\bar{t}_{jj}}$ is the difference in days (due to lunar anomaly) between the end of the mean-true tithi \check{t}_{jj} and that of the true tithi t_{jj} :

$$p^{\bar{t}_{jj}} = \frac{\mu_M^{\check{t}_{jj}}}{v_{\Delta}^{\check{t}_{jj}}}.$$

Theoretically, $\mu_M^{\check{t}_{jj}}$ should be used instead of $\mu_M^{\bar{t}_{jj}}$ as the numerator, and $v_{\Delta}^{\check{t}_{jj}} = v_M^{\check{t}_{jj}} - v_S^{\check{t}_{jj}}$ as denominator in this formula (see figure 11, 2 and 3). It is, however, extremely complicated to compute these values and we assumed that Gaṇeśa used approximations.

(step 8) Finally, the correction derived from *hāra*, the “*hāra*-difference” $h^{\bar{t}_j}$, is the difference between the above *parākhyā* $p^{\bar{t}_{jj}}$ and the *parākhyā* as computed from “*fmt*,” $p^{\bar{t}_j}$:

$$\begin{aligned} p^{\bar{t}_j} &= \frac{\mu_M^{\bar{t}_j}}{v_{\Delta}^{\bar{t}_j}}, \\ h^{\bar{t}_j} &= p^{\bar{t}_j} - p^{\bar{t}_{jj}}. \end{aligned}$$

Then, the *hāra* itself (which is tabulated) should be

$$\text{hāra} = \frac{K_t^g}{h^{\bar{t}_j}} = \frac{K_t^g}{P^{\bar{t}_j} - p^{\bar{t}_{jj}}}.$$

Gaṇeśa, however, seems to use a simple linear interpolation for $h^{\bar{t}_j}$ using two successive “*fat*” *parākhyas* ($p^{\bar{t}_{jj}}$) instead of $p^{\bar{t}_j}$ and $p^{\bar{t}_{jj}}$: the ratio of $p^{\bar{t}_{j+1j+1}} - p^{\bar{t}_{jj}}$ to 1 tithi is equal to that of ($h^{\bar{t}_j}$) to $K_t^g (< 1)$.

$$h^{\bar{t}_j} = (p^{\bar{t}_{j+1j+1}} - p^{\bar{t}_{jj}}) \cdot K_t^g.$$

From this, we arrive at the formula

$$\text{hāra} = \frac{1}{p^{\tilde{t}_{j+1j+1}} - p^{\tilde{t}_{jj}}}.$$

This formula appears previously in the *Śighrasiddhi* of Lakṣmīdhara (A.D. 1278), in verse 4, 34, and gives values closer to those in the table than the theoretical formula above.

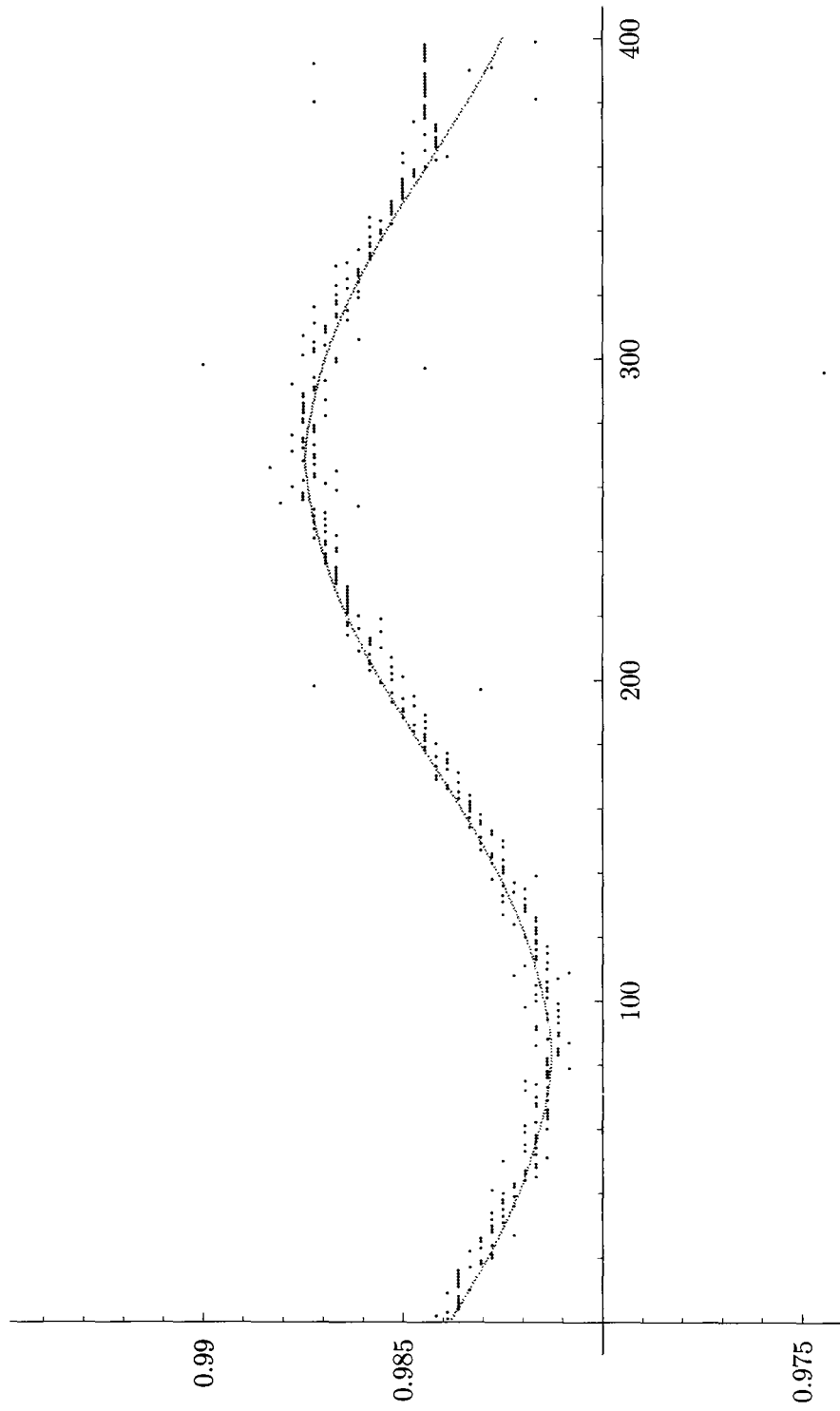
We give in the appendix the results of our recomputation of the tithivārādi and tithiparākhya according to the algorithms described above.

In the following graphs (graphs 1, 2, and 3) drawn with *Mathematica*,⁴³ gray lines or dots are the results from the algorithms of our reconstruction, and black dots are original data in the table of the published TC after correction of apparent errors. The graph for vārādi displays not tabulated or calculated vārādis themselves but the differences between two successive ones. This means that the graph shows the variation of the length of “mean-true” tithis throughout approximately one solar year.

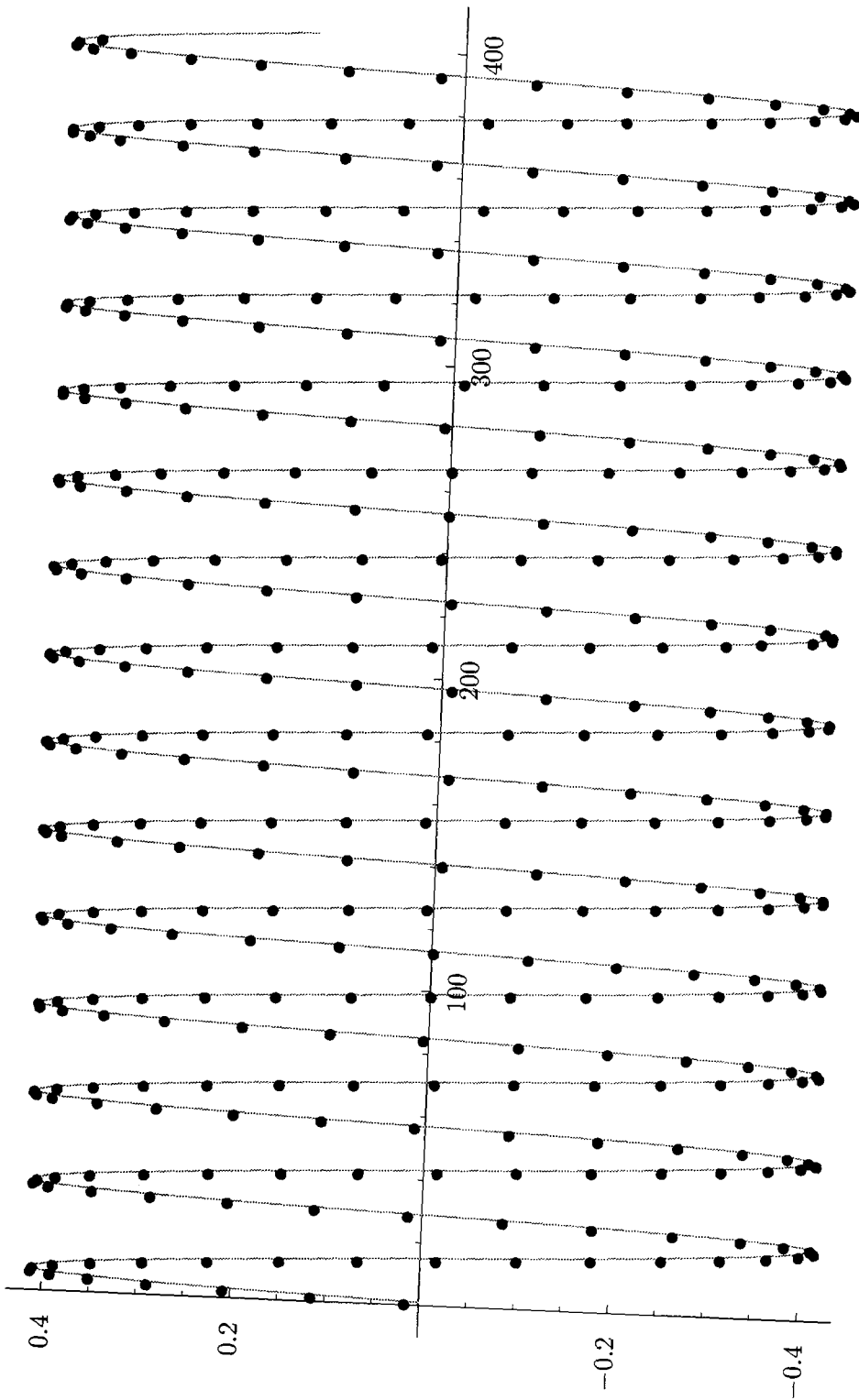
These comparisons indicate that our reconstruction does not fully succeed yet in reproducing the tabulated data, but seems to be proceeding along the right lines to a correct understanding of Gaṇeśa’s techniques.

⁴³Version 2, Wolfram Research, 1991.

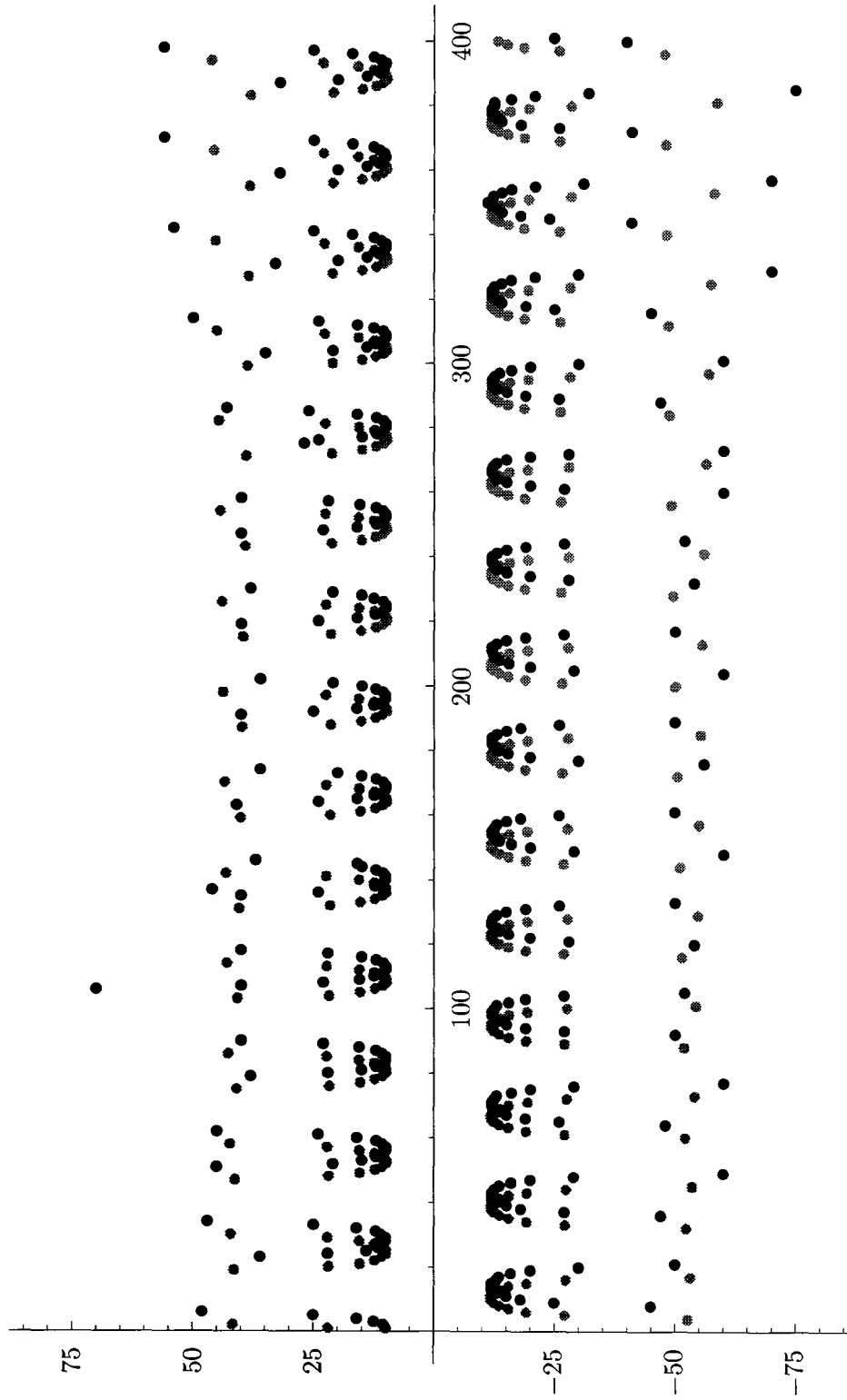
Graph 1: Tithivārādi



Graph 2: Tithiparakhya



Graph 3: Tithihāra



Appendix

Sample of the Tables of TC

arg	Tithi			Nakṣatra		Yoga		
	Vārādi	Parākhyā	Hāra	Parākhyā	Hāra	Vārādi	Parākhyā	Hāra
0	0; 10, 43	0; 55	10	6; 59, 44	12; 30	6; 51, 1	-0; 46	12
1	1; 9, 45	6; 53	10; 30	1; 6, 12	11	0; 47, 32	4; 15	12; 30
2	2; 8, 48	12; 30	12; 30	2; 12, 16	13	1; 44, 4	9; 0	14
3	3; 7, 50	17; 20	16	3; 17, 36	16	2; 40, 37	13; 10	17
4	4; 6, 51	21; 4	25	4; 22, 0	24	3; 37, 11	16; 42	24
5	5; 5; 52	23; 30	48	5; 25, 10	50	4; 33, 43	19; 12	38
6	6; 4, 53	24; 44	-600	6; 27, 4	∞	5; 30, 18	20; 41	400
7	7; 3, 54	24; 38	-45	0; 27, 50	-46	6; 26, 51	21; 25	-160
8	1; 2, 55	23; 19	-25	1; 27, 14	-25	0; 23, 24	21; 2	-47
9	2; 1, 57	20; 56	-18	2; 25, 37	-18; 30	1; 19, 58	19; 44	-28
10	3; 0, 57	17; 38	-15	3; 23, 10	-15	2; 16, 32	17; 35	-21
11	3; 59, 58	13; 32	-13	4; 20, 0	-13; 30	3; 13, 6	14; 47	-18
12	4; 58, 59	8; 58	-12	5; 16, 18	-12; 30	4; 9, 40	11; 22	-16
13	5; 58, 0	4; 0	-12	6; 12, 18	-12; 15	5; 6, 15	7; 33	-15
14	6; 57, 1	-1; 3	-12	0; 8, 10	-12; 30	6; 2, 49	3; 26	-14
15	0; 56, 2	-6; 5	-13	1; 4, 6	-13; 15	6; 59, 24	-0; 48	-14
16	1; 55, 3	-10; 48	-13; 30	2; 0, 21	-15; 30	0; 55, 57	-4; 56	-14; 30
17	2; 54, 3	-15; 17	-16	2; 57, 3	-18	1; 52, 32	-9; 4	-16
18	3; 53, 2	-19; 1	-20	3; 54, 26	-23	2; 49, 7	-12; 45	-19
19	4; 52, 1	-21; 58	-30	4; 52, 35	-40	3; 45, 41	-15; 56	-22
20	5; 50, 59	-24; 0	-50	5; 51, 49	-160	4; 42, 16	-18; 35	-35

Results of Our Recomputation

arg	Tithivārādi	Parākhyā	arg	Tithivārādi	Parākhyā
0	0; 10, 33, 8, 53	-0; 0, 20, 50	11	3; 59, 41, 14, 20	14; 22, 14, 14
1	1; 9, 34, 46, 34	6; 4, 43, 18	12	4; 58, 40, 55, 33	9; 55, 4, 4
2	2; 8, 36, 13, 22	11; 41, 0, 4	13	5; 57, 40, 26, 35	5; 4, 13, 38
3	3; 7, 37, 29, 21	16; 33, 21, 26	14	6; 56, 39, 47, 31	-0; 0, 51, 26
4	4; 6, 38, 34, 35	20; 27, 32, 11	15	0; 55, 38, 58, 25	-5; 5, 51, 36
5	5; 5, 39, 29, 5	23; 11, 43, 7	16	1; 54, 37, 59, 24	-9; 56, 31, 8
6	6; 4, 40, 12, 55	24; 38, 2, 11	17	2; 53, 36, 50, 33	-14; 23, 24, 17
7	0; 3, 40, 46, 9	24; 43, 43, 48	18	3; 52, 35, 31, 56	-18; 15, 57, 37
8	1; 2, 41, 8, 50	23; 35, 20, 55	19	4; 51, 34, 3, 40	-21; 23, 17, 3
9	2; 1, 41, 21, 3	21; 22, 58, 10	20	5; 50, 32, 25, 51	-23; 35, 7, 42
10	3; 0, 41, 22, 52	18; 15, 10, 33			

References

Sources

- BSS *Brāhmasphuṭasiddhānta* of Brahmagupta. Composed in 628. Edited with his own Sanskrit *Ṭīkā* by S. Dvivedin, Benares 1902.
- GL *Grahalāghava* of Gaṇeśa. Composed in 1520. Published several times, for example, edited with the commentaries of Mallāri and Viśvanātha by Sudhākara Dvivedin, Benares 1904. reprinted Bombay 1925.
- KK *Karaṇakutūhala* of Bhāskara. Composed in 1183. Edited with the commentary of Sumatiharṣa by M. S. Purohita, Munbaī 1901.
- PS *Pañcasiddhāntikā* of Varāhamihira. Composed in the sixth century. Edited with an English translation and an English commentary by O. Neugebauer and D. Pingree, 2vols., Cøbenhavn 1970–1971.
- ŚS *Śīghrasiddhi* of Lakṣmīdhara. Composed in 1278. Edited by D. Pingree, *Journal of the Oriental Institute* Vol. 37, Nos. 1–2 (1987) pp. 65–81.
- SS *Sūryasiddhānta*. Revised in ca. 800. Published several times, for example, edited with his own *Bhāṣya. Tattvāmṛta*, by Kapileśvara Śāstrin, Kāśī Sanskrit Series 144, Banārasa 1946. Translated into English by E. Burgess, *Journal of the American Oriental Society* 6.2 (1860) pp. 141–498 (reprinted many times).
- TC *Tithicintāmaṇi* of Gaṇeśa. Composed in 1525. Edited with the *Udāharaṇa* of Viśvanātha by Dattātreyā Āpaṭe, Ānandāśrama Sanskrit Series 120, Poona 1942, pp. 1–28.

Studies

- CESS David Pingree, *Census of the Exact Sciences in Sanskrit*, Series A, vols. 1–5, American Philosophical Society, Philadelphia 1970–1994.
- DSB Gillispie ed. *Dictionary of Scientific Biography*, 16 vols., New York 1970–1980.
- JYOTIḤ David Pingree, *Jyotiḥśāstra*, A History of Indian Literature, IV, Fasc. 4, Otto Harrassowitz, Wiesbaden 1981.
- SATE David Pingree, *Sanskrit Astronomical Tables in England*, Madras 1973.
- SATIUS David Pingree, *Sanskrit Astronomical Tables in the United States*, Philadelphia 1968.
- IAS B. V. Subbarayappa and K. V. Sarma, *Indian Astronomy: A Source-Book*, Nehru Centre, Bombay 1985.
- YANO Michio Yano, *Senseijyutsushi tachi no Indo* (Japanese), Chūōkōronsha (chū-kōshinsho 1084), Tokyo 1992.