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TU 11
A Collection of Rules for the Prediction of Lunar Phases and of Month Lengths

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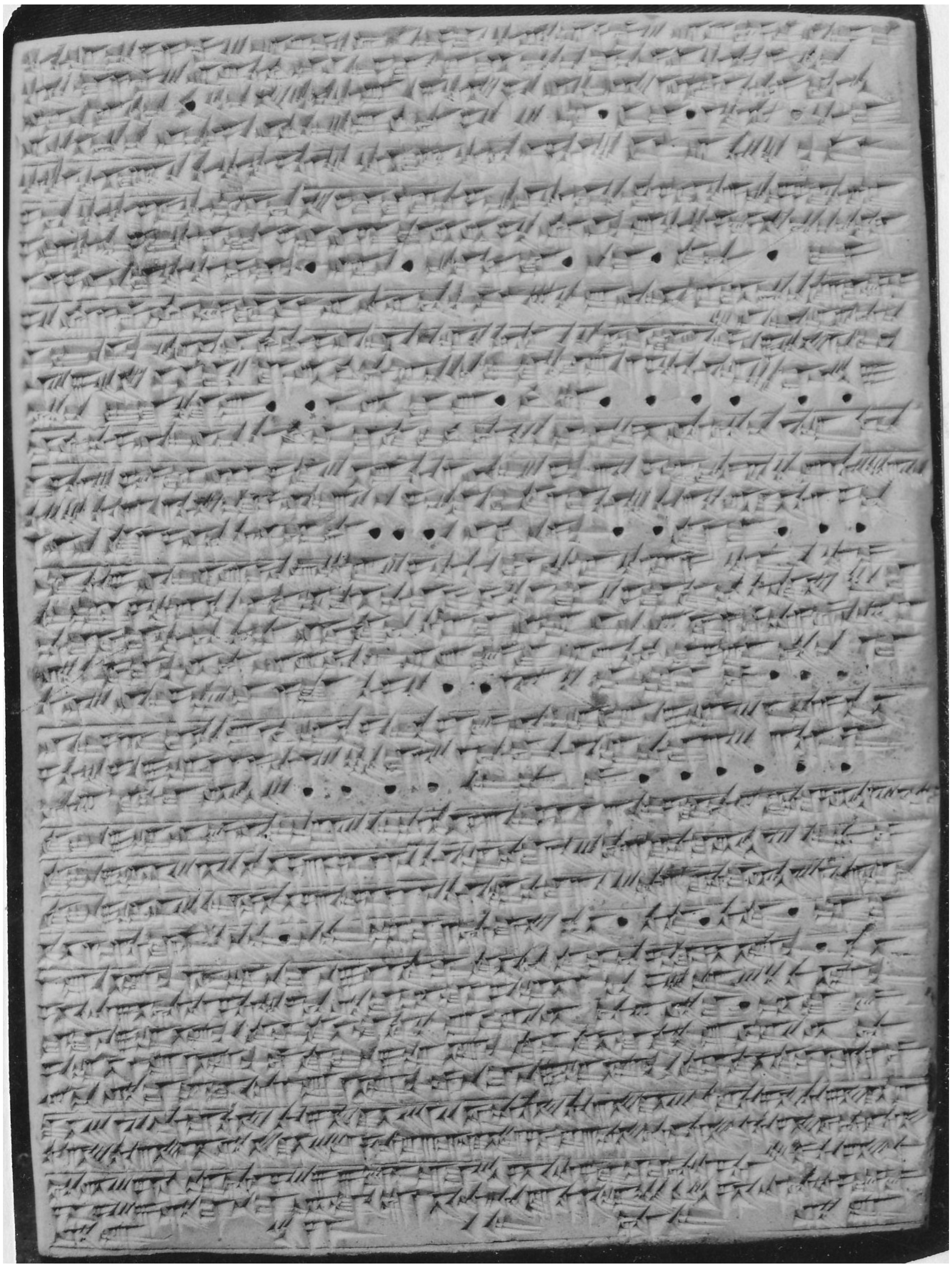
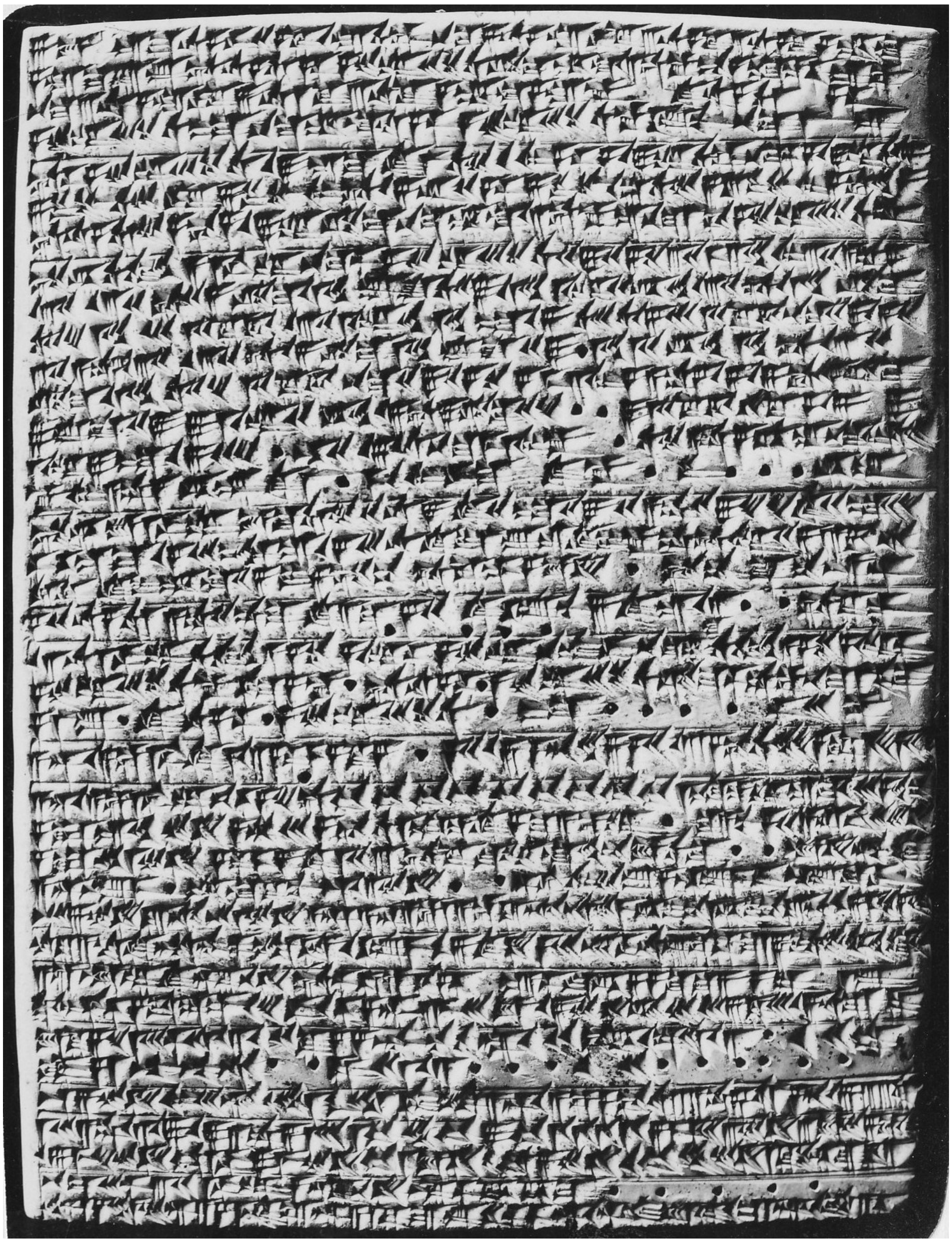


TU 11 (Obverse)



TU 11 (Reverse)

Introduction

The tablet AO 6455 is perfectly preserved and was published in an excellent copy by F. Thureau-Dangin as No. 11 in "Tablettes d'Uruk" in 1922. Nobody ever offered a complete edition of the text. Only short sections were understood and discussed by Thureau-Dangin, van der Waerden, Schaumberger and Neugebauer[1]. This is not surprising once one tries to read the entire tablet.

When Lis Brack-Bernsen became interested in the astronomical rules contained in the text, she convinced Hermann Hunger that a complete edition, including the non-astronomical (and mostly incomprehensible) parts, was desirable. We present in the following our attempt at such an edition. In this we were considerably helped by parallel texts in the British Museum identified by Irving Finkel and Christopher Walker. These showed that TU 11 is not free of errors. The parallel texts will be quoted here occasionally, but a complete edition will be published by Walker and Brack-Bernsen.

The tablet was written in Uruk towards the end of the 3rd century B.C., as can be seen from the colophon (rev. 38): The same scribe Anu-uballiṭ is attested on tablets dated to years 97 and 98 of the Seleucid era.[2] The tablet is a copy of an original which was damaged, as is evident from the remark *he-pí* "broken" in several places. We have been able to use a photograph of the tablet.

There is no indication when the text was composed, except for the possible reference to year 36 of the Seleucid era in rev. 31 (see below). Certainly some of the rules are more primitive than those used in the ACT texts, but this need not mean that they are older. Less sophisticated procedures can very well co-exist with more developed ones. In addition, the scribe of the tablet TU 11 seems to have had problems in understanding what he was copying; errors can be corrected with the help of partial duplicates. Some of the rules we found on TU 11 seemed so strange to us, that we considered two possibilities: Are they just inventions and speculations by some Seleucid scribe, or are they a collection of rules which had really been used by earlier Babylonian astronomers? The existence of numerous earlier duplicates shows that the second possibility is most probable. For further arguments see Brack-Bernsen [2002].

Only part of the text is concerned with what we today would call astronomical procedures, while the rest deals with astrology in the widest sense. Of course, this difference had no meaning for the users of the text. As far as we know, the same persons were occupied with celestial omens and with astronomy. This can be shown for the Seleucid period as well as for the scribes at the Assyrian court

[1] Neugebauer called the text "extremely difficult" in Isis 37 (1947) 37 n. 4.

[2] TU 16 rev. 55 (celestial omens, edited by R. Largement, ZA 52, 235ff.); BRM 4, 12:84 (extispicy).

in the 7th century. MUL.APIN[3] and Enūma Anu Enlil XIV[4] both have roots in observation. We consider them as early attempts to a mathematical modeling of natural phenomena i.e., an early attempt to formation of theory. But it would be ahistorical to try to separate astronomy and astrology in these texts. We have no tablets from the time between 700 BC and about 1100 BC, when MUL.APIN and Enūma Anu Enlil XIV were probably composed. But we assume that both were used for divination as well as astronomy from the beginning. We have here concentrated on the astronomical aspects.

Acknowledgements:

At this place we express our thanks to Irving Finkel and Christopher Walker for identifying parallel texts in the British Museum. We warmly appreciate a careful reading of the manuscript by John M. Steele. We thank Peter Huber and S.L. Moshier cordially for making their astronomical computer codes available to us, and we thank M. Brack warmly for his help in computing the Lunar Six and in producing the corresponding figures. We also thank the two referees for valuable suggestions. Finally we thank the Deutsche Forschungsgemeinschaft for supporting this work. The manuscript is written in LaTeX.

I Text

I.1 Transliteration

Upper edge

1) *ina a-mat* $^{\text{d}}$1 *u An-tum liš-lim*

Obverse

1) $^{\text{múl}}$UD-KA-DUH-A *a-dir sin ina* EN-NUN USAN AN-KU$_{10}$ GAR-*ma* : § 1
$^{\text{múl}}$SIM-MAH *a-dir sin ina* EN-NUN MURUB$_4$-BA AN-KU$_{10}$ GAR-*ma*
2) $^{\text{múl}}$GU-LA *a-dir sin ina* EN-NUN U$_4$-ZAL-LA AN-KU$_{10}$ GAR-*ma* : *lu-ma-šú gab-bi* ŠU-BI-AŠ-ÀM $^{\text{múl}}$ALLA *a he-pí*
3) $^{\text{d}}$UDU-IDIM LU *he-pí he-pí* MEŠ *he-pí* TE-*ú* 15 MÚL$^{\text{meš}}$ *šá-niš* $^{\text{d}}$*Be-let* DINGIR$^{\text{meš}}$ *ana* $^{\text{d}}$Šul-pa-è-a *iṭ-hi*
4) EN TI-*šú gab-bi* $^{\text{d}}$IZI-GAR *u* $^{\text{d}}$*ṣal-lum-mu-ú* DÙ-*uš šá-niš sin ina* DU$_6$ IGI-*ma ana* $^{\text{múl}}$ALLA TE-*ma*

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5) MÚL-MÚL $^{\text{múl}}$ŠUDUN KUR-*ud* MÚL-MÚL *šá* UD-1-KAM $^{\text{múl}}$ŠUDUN § 2

[3] Edited in Hunger and Pingree [1989].

[4] Abbreviated EAE XIV. For an edition and commentary see Al-Rawi and George [1991].

$^{\text{múl}}$GU$_4$-AN *šá* UD-1-KAM IGI-LÁ EN UD-20-KAM NU IGI-LÁ

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6) *šá* GU$_4$ MÚL-MÚL *šá* SIG $^{\text{múl}}$UG$_5$-GA$^{\text{meš}}$ *šá* ŠU *šu-ku-du šá* IZI **§ 3**
$^{\text{múl}}$MAR-GÍD-DA *šá* KIN $^{\text{múl}}$E$_4$-RU$_6$ *šá* DU$_6$ $^{\text{múl}}$UR-GI$_7$
7) *šá* APIN $^{\text{múl}}$GÍR-TAB *šá* GAN $^{\text{múl}}$MÁ-GUR$_8$ *šá* AB AN *šá* ZÍZ *har-ri-ri šá* ŠE AŠ.IKU *an-nu-ú* MÚL-MÚL *šá* ITU$^{\text{meš}}$ *gab-bi* IZI-ŠUB
8) ME-*a* GAR-*an* BE-*ma* MÚL-MÚL *e-la-nu* $^{\text{d}}$UDU-IDIM IZI-ŠUB RA BE-*ma* KI-TA *i-ṣa*

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9) *šá-niš* AŠ.IKU *a-dir* AŠ.IKU *ina* $^{\text{itu}}$BÁR IGI-LÁ *en-na* EN GU$_4$ NU IGI-LÁ **§ 4**
lìb-bu-ú AN-KU$_{10}$-*šú u lu-ma-šú gab-bi* KI-MIN-*ma ina* ITU DIR AN-KU$_{10}$ *ana* UGU *iq-ta-bi*

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10) PA GAN BE PA NU *im-haṣ pi-ir-du šá* AN-KU$_{10}$ *šú-u ina* UGU KUR ŠUB-*di* **§ 5**
u KUR TUŠ-*ab ina* BÁR AN-KU$_{10}$ GAR *ina* 18 MU$^{\text{meš}}$ *ana* GU$_4$ *i-tar*
11) *ina* 1,36 MU$^{\text{meš}}$ BAR BAR-*ma* LÚ ŠU *šip-ri* DÙ-*uš dib-bi ul i-qat-tu-ú* 12 *šá* MÚL-BABBAR *ana* UGU DAH-*ma ina* 1,48
12) *gab-bi i-qat-tu-ú šip-ri* GAL-*ú* DÙ-*uš*

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13) *pi-ir-du šá* AN-KU$_{10}$ $^{\text{d}}$AMAR.UD *né-pe-šu ki-i* DÙ-*uš* $^{\text{lú}}$KÚR *is-si-kip* ŠÚ **§ 6**
$^{\text{d}}$UTU *gab-bi ana* UGU *qa-bi*

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14) *áš-šá* UD-1-KAM *šá* $^{\text{itu}}$BÁR $^{\text{d}}$SÀG-ME-GAR KI *sin* ŠÚ-*ú* UD-12 AN-KU$_{10}$ **§ 7**
sin GAR-*nu* 3,20 *šá ina* ÉŠ-GÀR *iq-bu-ú ana* UD-20-KAM 3,20 ÚŠ
15) UD-20-KAM UD 12 *ana* 12 UD$^{\text{meš}}$ EGIR AN-KU$_{10}$ 3,20 ÚŠ *šá* IZI *u* GAN KI-MIN-*ma* UD-1-KAM UD 13 KUR RI *áš-šú* UD 2 *šá* GU$_4$
16) *ana* $^{\text{d}}$*dele-bat* KI-MIN *šá* $^{\text{d}}$GU$_4$-UD $^{\text{d}}$SAG-UŠ *u* $^{\text{d}}$*ṣal-bat-a-nu* KI-MIN-*ma*

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17) *ina* DU$_6$ UD-14 AN-KU$_{10}$ EN-NUN USAN *ina* $^{\text{múl}}$LU *sin* GAR **§ 8**
MÚL-BABBAR *u* GENNA *ina* $^{\text{múl}}$LU $^{\text{múl}}$UR.A $^{\text{múl}}$PA *u* $^{\text{múl}}$RÍN
18) *mi-hir šá* $^{\text{múl}}$LU GU SI-SÁ DÚB-*šú* GUB-*ma* SIG SIG ŠE BAR RA ŠE GAR RA KI GU$_4$-UD *ina* KI$^{\text{meš}}$ ŠEŠ-*ma*
19) GUB-*ma* SIG-*ma* $^{\text{múl}}$GU$_4$-UD GUB AN SIG *ki* GU$_4$-UD KUR$_4$-*ma* AN MAH $^{\text{d}}$*dele-bat ina* ŠÀ SIG A.KAL GIN A.KAL *dele-bat ina* ŠÀ KUR$_4$
20) ZI KÚR SIG-*ma nu-hu-uš ina* EN-NUN USAN EN-NUN MURUB$_4$-BA EN-NUN U$_4$-ZAL-LA *šá lu-maš gab-bi* 1-*ma*
21) UD $^{\text{d}}$*sin* AN-KU$_{10}$ GAR *ziq-pi a-mur* UD *šá šámaš ana* ŠÀ KUR *pi-šìr* E

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22) AN-KU$_{10}$ *šá* MIN(?) IGI-*ma* GU-*ú ana* DÙ-*ka šal-šá* GE$_6$ *šal-šá* ME(text: 1) **§ 9**
ITU AN-KU$_{10}$-*ka ù ina* 18-*ka* 1,30 ME NIM-*a šal-šú*

23) *šá* u_4-*mu šá* 18-*ka* 1 1 *a-na* 1,30 DAH-*ma* 2,30 1 *ta-mar-tú ana* UGU DU-*ma* 3,30 3 u_4-*me šá* BAR(?, text: PA)
24) TA ŠÀ E$_{11}$-*ma re-hi* 30 GE$_6$ GIN E

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25) *ki-i ina* 18-*ka ina* BAR ITU AN-KU$_{10}$-*ka* 1,30 GE$_6$ GIN *šal-šú šá* 3 GE$_6$ *šá* **§10**
BAR 1 3 GE$_6$ *šá* BÁR TA ŠÀ E$_{11}$-*ma šá re-hi* 1,30 ME NIM E

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26) *ki-i ina* 18-*ka ina* BAR 30 *ana* ŠÚ *šámaš šal-šú šá* 3 ME *šá* BAR 1 1 BE UD **§11**
ana UGU DU-*ma* 2 : 30 TA 2 E$_{11}$-*ma re-hi* 1,30 GE$_6$ GIN E

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27) *ki-i ina* 18-*ka ina* BAR 30 GE$_6$ *ana* ZÁLAG *šal-šú šá* 3 GE$_6$ *šá* BAR 1 ≪1≫ **§12**
1 BE UD *ana* UGU DU-*ma* 2 : 30 TA 2 E$_{11}$-*ma re-hi* 1,30 ME NIM-*a* DUG$_4$-GA

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28) *šal-šú* 2-*ta* ŠU$^{\text{II me}}$: BE-*tim* BE-*tim um-ma* 2-*ta* ŠU$^{\text{II meš}}$ *šal-šú* UD DI UD **§13**
DI *um-ma*

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29) LAL-*ú u kun-nu ana* IGI-*ka* BE-*ma ina* 18 BAR 1 *ṭi-pi* KI-*šú ul ṭe-pi* GU$_4$ *šá* **§14**
EGIR-*šú kun-nu šal-šú*
30) *šá* ŠÚ *u* NA 6 *šú* TA NA *šá* UD-1-KAM *šá* GU$_4$ ZI-*ma al-la* BAR *šá eli-šú* LAL GU$_4$ *šá* MU-*ka eš-še-tú kun-nu*
31) *ma-la ina* 18-*ka kun-nu u ṭi-pi* KI-*šú la ṭe-pu-ú ù nis-hu* TA ŠÀ ZI-*ma al-la* ITU
32) *šá eli-šú* LAL-*ú ana kun-nu* E *ki-i ina* 18-*ka kun-nu ṭi-pi* KI-*šú la ṭe-pu-ú ù nis-hu* TA
33) *lìb-bi* ZI-*ma al-la* 10 UŠ LAL-*ú ana kun-nu* E BE-*ma ina* 18-*ka kun-nu u ṭi-pi* KI-*šú ṭe-pi ana* GUR-*ru* E

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34) *áš-šú sin ana šámaš* NIM GUR UD$^{\text{meš}}$ UD-15$^{\text{meš}}$ *áš-šú sin ana šámaš* SIG **§15**
GIN UD$^{\text{meš}}$ UD-13$^{\text{meš}}$ *áš-šú sin* NIM DIB UD-15$^{\text{meš}}$ UD-16$^{\text{meš}}$ *áš-šú* <SIG(?)> DIB
35) UD-12$^{\text{meš}}$ UD-13$^{\text{meš}}$ BAR SIG ZI$^{\text{meš}}$ *áš-šú sin* KASKAL$^{\text{II}}$ *šá* NIM DIB 3 GUR AN *áš-šú* KASKAL$^{\text{II}}$ *šá* SIG DIB 3 GIN$^{\text{meš}}$ TA BAR UD-1$^{\text{meš}}$ NIM UD-14$^{\text{meš}}$ SIG TA DU$_6$ UD-1$^{\text{meš}}$ SIG UD-14$^{\text{meš}}$ NIM

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36) GABA-RI 36(tablet erroneously: 34) *ana* DÙ-*ka* TA BAR *šá* 36 6 ITU BAR **§16**
GUR-*ma* 40 *šá* ŠÚ *u* NA *šá* DU$_6$ GIŠ-*ma* TA NA *šá* UD-1
37) *šá* BAR *šá* 36 ZI-*ma* BE-*ma al-la* 10 UŠ LAL ŠÚ *u* NA *bal-ṭu-ut ana* UGU DAH PAP 40 *šá* ŠÚ *u* NA TA NA *šá* MURUB$_4$ ITU
38) ZI-*ah* 40 *šá* ME *u* GE$_6$ TA GE$_6$ ZI-*ah*

Reverse

1) 6 ITU GUR-*ma* NA *šá* IGI *šámaš šá* DU_6 *u* APIN *tam-mar* BE-*ma* 6 NA *šá* APIN *ù* 3 NA *šá* DU_6 3 NA *šá* DU_6 *šá al-la* **§17**
2) *šá* APIN *ma-ṭu-ú* TA NA *šá* UD-1 *šá* BAR *šá* MU-*ka eš-še-tú* ZI-*ma al-la* 12 UŠ DIRI GU_4 *šá* EGIR-*šú* GUR E
3) BE-*ma* 3 NA *šá* APIN *ù* 6 NA *šá* DU_6 3 NA DU_6 *šá al-la šá* APIN DIRI KI NA *šá* UD-1 *šá* BAR
4) *šá* MU-*ka eš-še-tú* TAB-*ma* BE-*ma al-la* NA *šá* BAR *šá* MU-*ka eš-še-tú* DIRI *kun-nu* BE-*ma* LAL GUR

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5) TA BAR *šá* MU-*ka eš-še-tú* 18 GUR-*ma* KI-LAL u_4-*mu u* GE_6 *šá* BAR *šá* 18-*ka tam-mar* BE-*ma* BAR *šá* 18-*ka* UD-15 **§18**
6) 6 DANNA u_4-*mu ù* 6 DANNA GE_6 TA 15 EN 20 *šá* BAR 6 DANNA 10 UŠ u_4-*mu* 5 5/6 DANNA GE_6 5 UŠ 40 NINDA
7) TA NA *šá* UD-1-KAM *šá* GU_4 *šá* 18-*ka* ZI-*ma* BE-*ma al-la* 12 UŠ DIRI GUR BE-*ma* LAL GU_4 *šá* MU-*ka eš-še-tú* GIN

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8) GABA-RI U_4-NÁ-A *ana* DÙ-*ka* BE-*ma* BAR *šá* MU-*ka eš-še-tú* 27 25 KUR 3,20 ME 2,40 GE_6 3,20 A-RÁ 4 **§19**
9) 13,20 13,20 TA 25 ZI-*ma* 11,40 *uh-hur* UD 28 11,40 *ana* UGU *šámaš re-hi* 13,20 TA 11,40 ZI-*ma*
10) UD 29 1,40 *sin ana šámaš* DIB-*iq* 13,20 *ana muh-hi* 1,40 TAB-*ma* 15 UD-30 UD 15 *sin ana šámaš* DIB-*iq* BE-*ma* 27 15 KUR
11) 3,20 A-RÁ 4 13,20 13,20 TA 15 ZI-*ma* 1,40 *uh-hur* 28 1,40 *ana šámaš re-hi* 1,40 TA 13,20
12) ZI-*ma* 11,40 UD-29 *ana šámaš* DIB-*iq* BE-*ma* 27 24 KUR 4 ME 2 GE_6 4 A-RÁ 4 16 16 TA 24 ZI-*ma*
13) 8 *uh-hur* 28 8 *ana* UGU *šámaš re-hi* 16 TA 8 ZI-*ma* 29 8 *ana šámaš* DIB-*iq meš-lu šá* 16 8
14) *ana* UGU 8 TAB-*ma* 16 29 *ina* ŠÚ *šámaš* 16 NA BE-*ma* ME *ana* GE_6 DIRI u_4-*mu* A-RÁ 4 *tal-lak* BE-*ma* GE_6 *ana* u_4-*mu*
15) DIRI GE_6 A-RÁ 4 *tal-lak* KI-LAL ME *u* GE_6 IGI-*ma* KI DIRI KASKAL *tal-lak*

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16) GABA-RI ITU *ana* IGI-*ka* ^{itu}ŠU UD-1-KAM 20 NA 7 KI 20 TAB-*ma* 27 20 A-RÁ 27 9 9 KI 20 TAB-*ma* 29 **§20**
17) 1/2-*šú* GIŠ-*ma* 14,30 IZI GUR 14,30 NA *ki-i* LAL-*ú* GUR *ki-i* DIRI GIN TA UGU 25 *ana a-ha-ti*
18) ŠUB-*di ki-i al-la* 25 *i-ṣa gab-bi ta-ṭep-pi šá* ITU^{meš} *gab-bi* ŠU-BI-AŠ-ÀM

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19) 4 (read: *šá* ?) 20 NA *šá* UD-1 A-RÁ 20 DU-*ma* KI-MIN-*ma* 4 (read: *šá* ?) 20 **§21**
ki kun-nu 6 *ki* GUR 12 *ta-ṭep-pi*

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20) 12 NA *ana* IGI-*ka* U$_4$-NÁ-A *šá* ŠE *šá eli* BAR *šá* MU-*ka* TIL-*tum* IGI BE-*ma* **§22**
al-la U$_4$-NÁ-A *šá* ŠE *šá eli* BAR
21) *šá* MU-*ka eš-še-tú* DIRI *mim-ma šá* DIRI TA NA *šá* BAR *šá* MU-*ka eš-še-tú*
ZI-*ma* BAR GIN E BE-*ma* LAL *mim-ma*
22) *šá* LAL-*ú* KI NA *šá* BAR *šá* MU-*ka eš-še-tú* TAB-*ma* BAR 30 E

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23) ŠÈG *u* A.KAL *ana* DÙ-*ki* 1,12 *šá* SÀG-ME-GAR 1 *šu* 4 *šá-niš* 16 *šá dele-bat* **§23**
46 *šá-niš* 13 *šá* GU$_4$-UD 59 *šá* GENNA 1,19 *šá-niš* 47 *šá* AN

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24) ŠÈG *u* A.KAL *ana* DÙ-*ka* TAR-*šú šá* 9 UŠ ŠÁR ŠÁR ŠÁR ŠÁR ŠÁR ŠÁR **§24**
ŠÁR ŠÁR ŠÁR *ana* EGIR-*ka* GUR-*ma* 10, 30 *šá he-pí*
25) *šá* GENNA GUR 40 MU$^{\text{meš}}$ *šá dele-bat* 30 *šá* GU$_4$-UD *ana* EGIR-*ki* GAR-*ma*
A.KAL *ka-ma*(?)-*a-nu šá-niš* ŠÈG *ana* DÙ 30 MU$^{\text{meš}}$
26) *šá* GENNA 41 MU$^{\text{meš}}$ *šá* SÀG-ME-GAR DI 1,23 *ta-ṭeb-ba il-la- he-pí* ITU
UD BA *ina* ITU$^{\text{meš}}$ *an-nu-tu*
27) ŠÈG *ina šam-me il-la-ku-nu ina* 1,12 MU$^{\text{meš}}$ $^{\text{d}}$*ṣal-lum-mu-ú šá ina* KUN$^{\text{me}}$
IGI *ina* KUN-*ma* IGI
28) *ina* 36 MU$^{\text{meš}}$ *ip-pal ina* 21 MU$^{\text{meš}}$ ŠÈG *ana* ŠÈG A.KAL *ana* A.KAL *ip-pal*
ina 21 MU$^{\text{me}}$ *ri-i-bi ana ri-i-bi ip-pal ina* 6 *me* 54 MU$^{\text{meš}}$ *he-pí*

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29) $^{\text{múl}}$UR.A $^{\text{múl}}$ABSIN $^{\text{múl}}$GÍR-TAB PA-BIL MÁŠ *u* KUN$^{\text{me}}$ PAP 6 KI$^{\text{meš}}$ *šá* **§25**
sin ina ŠÀ-*ši-na* TÙR NÍGIN-*ma ana* 8-*i u*$_4$-*mu* IM ŠÁR-*tu-u* ZI-*a*

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30) A.KAL *ina* ITU-*ka* KI $^{\text{d}}$UDU-IDIM$^{\text{meš}}$ *ta-maš-šah šal-šá-šú* DIRI BAR **§26**
ba-ri-ṣa šá ŠU *ba-<ri->ṣa šá* AB 1 KÙŠ
31) GIŠ.MI *šal-šú šá da-ap-pi* 19 IGI *u* ŠÚ GAR-*an* 2 *me* 30 ITU 18 MU$^{\text{meš}}$ GI
MU 36 SIG 30 15 NA *ma-diš* SIG

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32) $^{\text{itu}}$DU$_6$ 5(?) 9 1,30 BAR 21 12 *u* 20 DU$_6$ 4 4 GÍR GÍR : APIN 10 20 GU$_4$ KIN **§27**
u AB 21 12 *u* 20 NIM
33) BÁR *šá ana* UB UB ti(?) SIG 21 12 *u* 20

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34) *ina* 1,5 MU$^{\text{meš}}$ GE$_6$ 30 GI(?) : 50 1 PAP 40 1 ZUM NE ŠÈG *u* A.KAL *šá* **§28**
GU$_4$-UD 2 A-RÁ 5 UD LAL *šá dele-bat* 16 3 3 UD LAL

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35) 1,30 A-RÁ 12 18 18 A-RÁ 5 1,30 1 ŠÁR 50 ŠÁR 8 ŠÁR DIŠ+U DIŠ+U **§29**
DIŠ+U DIŠ+U 5 *me* 1,33 MU$^{\text{meš}}$ 3 ITU MU 5 ITU 10 UD-DA-ZAL

36) 16 *me* 9 MU$^{\text{meš}}$ NIM 1 *lim* 8 *me* MU$^{\text{meš}}$ ŠÁR(?) *ana* ITU$^{\text{meš}}$: NIM 5 MU$^{\text{meš}}$ 10 ŠÁR 50 MU$^{\text{meš}}$ *ana ku-us-su* ZÁH

==

37) BE-*ma* EŠ-BAR 3,20 *ana* IGI-*ka šá* $^{\text{d}}$UDU-IDIM$^{\text{meš}}$ *ina lu-maš* KIN-KIN-*ma*
38) IM $^{\text{I}}$NÍG-SUM-MU- $^{\text{d}}$1 A *šá* $^{\text{I}}$*Ina-qí-bit-*$^{\text{d}}$1 A $^{\text{I}}$*Hun-zu-ú* $^{\text{lú}}$MAŠ-MAŠ $^{\text{d}}$1 *u An-tum Uruk*$^{\text{ki}}$*-u qàt* $^{\text{I}}$ $^{\text{d}}$1-TIN-*iṭ* A-*šú*

I.2 Translation

Upper edge

1) At the command of Anu and Antu may it go well!

Obverse

1) The Panther is dark: i.e., the moon makes an eclipse in the evening watch; the Swallow is dark: i.e., the moon makes an eclipse in the middle watch; **§ 1**
2) the Great One is dark: i.e., the moon makes an eclipse in the morning watch; for all zodiacal constellations, it is the same. The Crab - broken
3) planet ... - broken, broken - ....s - broken - comes close(?); Ištar of the stars, variant, Belet-ili came close to Jupiter.
4) until all of its ...., it will make a meteor and/or comet; variant, i.e., the moon is seen in month VII(?) and comes close to the Crab.

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5) The Stars reached the Yoke. The Stars on the 1st day (are?) the Yoke. The Bull of Heaven which was seen on the 1st day, is (ideally?) not seen until the 20th day. **§ 2**

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6) Of month II, the Stars; of month III, the Raven(?); of month IV, the Arrow; of month V, the Wagon; of month VI, Erua; of month VII, the Dog; **§ 3**
7) of month VIII, the Scorpion; of month IX, the Boat; of month X, Mars; of month XI, the Harrow; of month XII, the Field. This (constellation of the) Stars (means?) "fall of fire" in(?) all the months;
8) you make a prediction: if the Stars are above the planet, the "fall of fire" will be ...., if below, little.

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9) Variant: The Field is dark. The Field becomes visible in month I. Now, it does not become visible until month II. I.e., therein(?), its eclipse and all the zodiacal constellations, ditto. In the intercalary month, he predicted an eclipse for it. **§ 4**

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10) .... It is the *pirdu* of an eclipse. On account of it, a country will be abandoned, and a country will be settled. In month I it makes an eclipse, in 18 years it returns to month II. **§ 5**

11) In 96 years month I (is again) month I(?), and this(?) man performs work; words(?) do not finish. 12 (years) of Jupiter you add to it, and in 108 (years)
12) they(?) all finish; he(?) performs a great work.

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13) The *pirdu* of an eclipse. When Marduk performed his procedure, he overthrew the enemy. All the settings of the sun, is said about it. **§ 6**

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14) Because Jupiter on the 1st day of month I set with the Moon, (and) the Moon made an eclipse on the 12th day, the king, as it says in the series, the king will die on the 20th day. **§ 7**
15) The 20th day (is) the 12th day, in 12 days after the eclipse the king will die. Of month V and IX, it is the same; the 1st day is the 13th .... because of day 2 of month II.
16) For Venus, ditto; of Mercury, Saturn and Mars, also ditto.

---

17) (If) the Moon makes an eclipse in the evening watch in month VII the 14th day in Aries, Jupiter and Saturn are in Aries, Leo, Sagittarius; and Libra (which is) **§ 8**
18) opposite of Aries, a straight string you(?) tighten, and ...., and .... you observe the place of Mercury in the “places”, and
19) .... and Mercury stands(?), weak rain; when Mercury is bright, much rain; (if) Venus is faint in it, the flood will come, the flood(?); (if) Venus is bright in it,
20) an attack of the enemy will become weak(?); abundance. In the evening watch, middle watch, morning watch of all the (zodiacal) constellations, it is the same.
21) When the Moon makes an eclipse, look at the *ziqpu*; when it is of the Sun(?), give an interpretation for the land(?).

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22) (If?) you see an eclipse of the same(?), and in order for you to make ....: one-third of the night one-third of the day(?). In(?) the month of your eclipse and in your 18(th year preceding) 1,30 is “after sunrise”, one-third **§ 9**
23) of the day of your 18(th year preceding) is 1. 1 add to 1,30, and (it is) 2,30. 1, the visibility, add(?) to it, and (it is) 3,30. 3, the length of daylight of month I(?)
24) you subtract from it, and there remains 30; you call it “after sunset”.

---

25) If in your 18(th year preceding) in month I, the month of your eclipse, 1,30 is “after sunset”, one-third of 3, the night of month I, is 1. <....> 3, the night of month I, you subtract from it, and what remains, 1,30, you call “after sunrise”. **§10**

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26) If in your 18(th year preceding) in month I, 30 is “before sunset”: one-third of 3, the day of month I is 1. 1 .... you add to it, and it is 2. 30 you subtract from 2, and there remains 1,30; you call it “after sunset”. **§11**

---

27) If in your 18(th year preceding) in month I, 30 is “before sunrise”: one-third of **§12**
3, the night of month I, is 1. 1 .... you add to it, and it is 2. 30 you subtract from 2, and there remains 1,30; you call it “after sunrise”.

---

28) One-third; two-thirds; the whole(?); the whole(?) again(?); two-thirds; **§13**
one-third; ...., ... again(?).

---

29) In order for you to see a hollow or full (month). If in the 18(th year preceding) **§14**
month I (begins on) the 1st (day), and an addition is not added to it, month II, which is after it, is full. One-third
30) of ŠÚ+NA is 6: you subtract this(?) from NA of the 1st day of month II, and (if) it is less than in month I, which is before it, then month II of your new year is full.
31) Whatever (month) in your 18(th year preceding) is full, and to which there is no addition added, and a subtraction is subtracted from it, and which is
32) less than the month preceding it, you declare as full. If in your 18(th year preceding) (a month) is full, and an addition is not added to it, and a subtraction
33) is subtracted from it, and it is less than 10 UŠ, you declare (your month) as full. If in your 18(th year preceding) (a month) is full, and an addition is added to it, you declare (your month) as hollow.

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34) When the Moon is high to the Sun, hollow; the days are fifteenth days; when **§15**
the Moon is low to the Sun, full; the days are thirteenth days; when the Moon takes a high (position), fifteenth and sixteenth days; when it takes <a low (position)>,
35) twelfth and thirteenth days. Month I .... When the Moon takes a path of height, three hollow ones; when it takes a path of depth, three full ones. From month I on the first days are high, the fourteenth days are low; from month VII on, the first days are low, the fourteenth days are high.

§16

---

36) In order for you to calculate (lit., make) the equivalent for 36 (tablet erroneously: 34) (years). From month I of the 36(th year preceding) you return 6 months, and 0;40 (= two thirds) of ŠÚ+NA of month VII you take, and from NA of the 1st day
37) of month I of the 36(th year preceding) you subtract, and if it is less than 10 UŠ, you add ŠÚ+NA entirely.... 0;40 of ŠÚ+NA from NA in the middle of the month
38) you subtract. 0;40 of ME+GE_6 you subtract from GE_6.

Reverse

1) You go back six months, and you look at NA before the Sun of months VII and **§17**

VIII. If NA of month VIII is 6, and NA of month VII is 3, you subtract 3, (by which) the NA of month VII
2) is less than (that of) month VIII, from NA of the first day of month I of your new year; and (if) it exceeds 12 UŠ, you declare the month II, which follows it, as hollow.
3) If NA of month VIII is 3, and NA of month VII is 6, you add 3, (by which) the NA of month VII exceeds that of month VIII, to the NA of the first day of month I
4) of your new year, and if it exceeds the NA of month I of your new year, (the month) is full; if it is less, (the month) is hollow.

---

5) From month I of your new year you go back 18 (years), and you look at the weight of day and night of month I of your 18(th year preceding). If (in) month I of your 18(th year preceding) on the fifteenth day **§18**
6) 6 *bēru* are daylight and 6 *bēru* are night, from the 15th to the 20th of month I daylight is 6 *bēru* 10 UŠ, night is 5 5/6 *bēru*: you subtract 5 UŠ 40 NINDA
7) from the NA of the first day of month II of your 18(th year preceding), and if it exceeds 12 UŠ, (the month) is hollow; if it is less, month II of your new year is full.

---

8) In order for you to calculate (lit., make) the equivalent of the day of invisibility of the moon. If (in) month I of your new year, the 27th, KUR is 25, daylight is 3,20, night is 2,40: 3,20 times 4 **§19**
9) is 13,20. You subtract 13,20 from 25, and 11,40 (the moon) will be delayed. The 28th, 11,40 is remaining for the sun. You subtract 13,20 from 11,40, and
10) on the 29th day the moon will have passed the sun by 1,40. You add 13,20 to 1,40, and (it is) 15. (On) the 30th day, the moon will have passed the sun by 15. If (on) the 27th KUR is 15:
11) 3,20 times 4 is 13,20. You subtract 13,20 from 15, and 1,40 (the moon) will be delayed. The 28th, 1,40 is remaining for the sun. You subtract 1,40 from 13,20,
12) and (the moon) will have passed the sun by 11,40 on the 29th. If (on) the 27th KUR is 24, daylight is 4, night is 2: 4 times 4 is 16. You subtract 16 from 24, and
13) 8 (the moon) will be delayed. The 28th, 8 is remaining for the sun. You subtract 16 from 8, and (on) the 29th (the moon) will have passed the sun by 8. Half of 16 is 8;
14) add it to 8, and (it is) 16. (On) the 29th, at sunset, NA is 16. If daylight is longer than night, you multiply daylight by 4. If night is longer than daylight,
15) you multiply night by 4. You look at the weight of daylight and night, and you go the way with the excess.

---

16) In order for you to see the equivalent of the month. Month IV, the 1st day, "sunset to moonset"(NA): 20°. You add 7 to 20, and (it is) 27. 20 times 27 is 9. You add 20 to 9, and (it is) 29. **§20**

17) You take its half, and (it is) 14,30. Month V is hollow. If 14,30 is less (than) NA, (the month) is hollow; if it is more, (the month) is full. From above 25 you place aside(?);
18) when it is less than 25, you add all. For all months it is the same (procedure).

---

19) Of(?) 20: NA of day 1 you multiply by 20, and it is the same(?); of(?) 20: if (the month) is full, you add 6, if it is hollow, 12. **§21**

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20) In order for you to see 12, the NA: look at the invisibility of month XII which is above month I of your old year. If it is more than the invisibility of month XII which is above month I **§22**
21) of your new year, you subtract whatever it is more from NA of month I of your new year, and you declare month I as full. If it is less,
22) you add whatever is less to NA of month I of your new year, and you declare month I as having 29 days.

---

23) In order for you to calculate (lit., "make") rain and flood: 72 of Jupiter; 64, variant: 16, of Venus; 46, variant: 13, of Mercury; 59 of Saturn; 79, variant: 47, of Mars. **§23**

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24) In order for you to calculate (lit., "make") rain and flood: you return behind you .... of 9 (times?) 60, 3600, 3600, 3600, 3600, 3600, 3600, 3600, 3600, 3600, and 10,30 which - broken - **§24**
25) of Saturn you return. 40 years of Venus, 30 of Mercury you set(?) behind you, and (there will be) regular(?) flood. Secondly, to calculate (lit. "make") rain: 30 years
26) of Saturn, 41 years of Jupiter ... you add(?) 83 ... - broken - ... In these months
27) rain will come from the sky. In 72 years a comet which had appeared in the Tails, will appear (again) in the Tails,
28) in 36 years it will correspond (lit., answer). In 21 years rain will correspond to rain, flood to flood. In 21 years an earthquake will correspond to an earthquake. In 654 years - broken -

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29) Leo, Virgo, Scorpius, Sagittarius, Capricorn and Pisces: together six places, where, if the moon will be surrounded in them by a halo, a .... wind will rise on the eighth day. **§25**

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30) .... in your month, you measure the position(?) of the planets. A third of it .... *bariṣu* of month IV, *bariṣu*(?) of month X, 1 cubit **§26**
31) of shadow, a third of the *dappu*, 19(?), appearance and disappearance (or: rising and setting) you set. 230 months complete(?) 18 years. Year 36, month III,

(the first day of which was equivalent to) the 30th (day of the preceding month), sunset to moonset: 15°; (the moon) was very low.

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32) Month VII, 5(?) 9 1,30. Month I(?), 21, 12 and 20. Month (?) VII, 4, 4, ... **§27**
Month VIII, 10, 20. Month II, VI, and X, 21, 12, and 20 ....
33) Month I which .... Month III, 21, 12, and 20.

---

34) In 65 years .... rain and flood; of Mercury, 2 times 5 days(?) **§28**
less(?); of Venus, 16 3 3 days(?) less(?).

---

35) 1,30 times 12 is 18; 18 times 5 is 1,30. 1 3600, 50 3600, 8 3600, 600, 600, 600, **§29**
600, 5 hundred 93 years 3 month. Year 5 month 10 coefficient.
36) 16 hundred 9 years high(?), 1 thousand 8 hundred years high(?). For(?) months : high(?). 5 years 10 3600 50 years .... will be lost(?).

==

37) In order for you to see an ominous decision about the king, you seek (the position) of the planets within the (zodiacal) constellations, and.
38) Tablet of Nidinti-Ani, son of Ina-qibīt-Ani, descendant of Hunzû, incantation priest of Anu and Antu, from Uruk. Hand of Anu-uballiṭ, his son.

I.3 Comments

The star names are more or less literal translations of Babylonian words, if possible, and are the same as those used in the edition of MUL.APIN[5].

Upper edge

1: For this invocation, common in Late Babylonian texts, see most recently M. Roth, *Journal of Semitic Studies* 33 (1988) pp. 1-9.

Obverse

1: the "darkening" of the constellations named in this and the following line cannot simply be explained as lunar eclipses taking place in them; while the moon can stand in the Swallow (Pisces) and in the Great One (Aquarius), the Panther (Cygnus) is too far to the north.

2f.: lines 2 and 3 were damaged already on the original from which TU 11 was copied, and cannot be reconstructed.

4: IZI-GAR occurs here next to *ṣallummû* "comet". It therefore is probably to be read *dipāru* "torch", which is used as a word for "meteor" in the Diaries[6]: No. -165 A r.11: IZI-GAR TA ULÙ *ana* SI DIB "a torch crossed (the sky) from south

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[5]Hunger and Pingree 1989.

[6]Sachs and Hunger, 1988-1996.

to north"; No. -418 A 7: AN-BAR$_7$ MÚL GAL *šá* GIM *di-par* TA ULÙ *ana* SI ŠUR-*ma* "at noon, a big star which was like a torch flashed from south to north".

In other contexts, EN TI-*šú* could be read *bēl balāṭišu* or *adi balāṭišu*; neither seems to fit here.

5: for the omen at the beginning, see F. Gössmann, Planetarium Babylonicum p. 111. The second part of the line may refer to a first visibility of Taurus at too early a date; in MUL.APIN, the 20th day of month II is given as the ideal date. Otherwise, this line is obscure to me.

6f.: some of the constellations assigned here to certain months (Stars, Arrow, Dog, Scorpion) have their heliacal risings in those months, according to MUL.APIN[7]; but this explanation does not apply to all of them.

8: the interpretation of IZI-ŠUB as "fall of fire", i.e., conflagration or stroke of lightning, is somewhat uncertain here and in the preceding line. In line 8, this interpretation makes some sense. RA could be a logogram for *mahāṣu* "to hit", but this verb is not used with "lightning", so I am uncertain about its reading here. One could combine RA with the following BE-*ma*, but *ra-be* would be a very unusual writing for *rabi* "is great". Admittedly, it would fit as opposition to *iṣa* "little".

9: the Field is supposed to rise in month I according to "Astrolabe B"[8], but MUL.APIN (Tablet I iii 10) places its rising in month XI.

10: the beginning of the line is obscure to me; *im-haṣ* could as well be read *im-qut*. *pirdu ša attalî* occurs again in line 13. *parādu* means "to be frightened", and *pirdu* could be a noun "fright"; however, such a noun is not used in Akkadian, but is replaced by its feminine form *pirittu*. A root *prd occurs elsewhere in astronomical context: ACT 200g ii 7 reads ^{d}*sin i-par-rid-ma šal-šú* HAB-*rat sin al* MURUB$_4$ ... (rest broken). Unfortunately, *parādu* in this passage may mean "becomes afraid", said of the moon in some transferred meaning, or may have some entirely unknown meaning; so it is not helpful in determining the meaning of *pirdu*.

11: or is DIB-BI to be understood as *etēq-šu* "its passing" ? But to what would it refer?

While 12 years are a well-known period of Jupiter, 96 years is not, to my knowledge, attested as a period (and would not make sense).

13: note the Assyrian form *issikip*. It could also be translated "(the enemy) was overthrown"; but a N-stem of *sakāpu* is very rare.

15: the equivalence of 20th and 12th day is here supposed to be found "in the series", i.e., in Enūma Anu Enlil, the collection of celestial omens. While I could not find this particular "equivalence", similar interchanges of numbers are used in K. 2164:14ff.[9]. - Since the same explanation, according to the text, applies not only

[7] Hunger and Pingree 1989 p. 140f.

[8] KAV 218, see the explanation in Hunger and Pingree 1999, p. 52.

[9] A. Livingstone, Mystical and Mythological Explanatory Works of Assyrian and Babylonian Schol-

to month I, but also to months V and IX, we have another example of "triplicity"[10].

16: the sequence of planet names is that used in late astronomical texts, like the Diaries etc.

18: A "string" is also found in BM 78161[11] and in the so-called Dalbanna-Text[12], both of which combine several stars into a "string". The latter text also mentions "straight, taut strings" (GU SI-SÁ DÚB-BA). While it is obvious that the "Dalbanna-Text" deals with alignments, the terminology used is so far obscure to a large extent[13].

For *mehru* meaning "opposite" see CAD s.v. *mihru* mng. 3. ŠE.BAR.RA could be read as *uṭṭatu* "barley", but I don't see what that could mean here. The last words of the line are understandable: the "places" refer to positions in the ecliptic, so a modern expression would be "you observe the position of Mercury in longitude".

19: The beginning of the line is unclear to me, even whether GUB and SIG are verbs.

AN SIG is here opposite to AN MAH; AN is therefore most likely "rain", and SIG means "little", whereas MAH is "much". These logographic writings are common in the Diaries. Their Akkadian equivalents are not all certain: AN is *zunnu*; MAH is most likely to be read *gapšu* "massive" on the basis of TU 19:28 ŠÈG *u* A.KAL *gapšu-tu* compared to TU 19:33 ŠÈG *u* A.KAL MAH; SIG may be *qatnu* "thin". The corresponding opposition is between SIG-*ma* (here SIG stands for *unnutu* !) "faint" and KUR_4-*ma* "bright", both referring to Mercury, although the syntax in the first part of the line is not clear.

Another opposition in this line is between Venus being faint (SIG) or bright (KUR_4) together with the previously mentioned appearance of Mercury. I do not know why A.KAL (= *mīlu* "flood") is repeated.

20: *nu-hu-uš* is an unusual writing for *nuhšu* "abundance".

The rest of the line explains that the apodoses derived from the phenomena described earlier are independent of where in the zodiac and at which time of night the phenomena happen.

21: If one reads *šá šámaš*, the second half of this line would refer to a possible solar eclipse. I do not see why this implies a prediction for the land more than a lunar eclipse. A reading *šá-niš* "secondly, otherwise" does not yield a better sense.

22: in the beginning of the line, after $\mathrm{AN\text{-}KU}_{10}$ *šá*, there is one vertical wedge followed by another vertical wedge which seems to have a weak horizontal running

ars (Oxford 1986) 22.

[10] F. Rochberg-Halton, JNES 43 (1984) pp. 115-140.

[11] D. Pingree and C. B. F. Walker, in: A Scientific Humanist. Studies in Memory of Abraham Sachs, 313-322.

[12] C. B. F. Walker, WO 26 (1995) 27-42; J. Koch, WO 26 (1995) 43-85.

[13] Cf. Hunger and Pingree 1999, 100-111.

through it, which would make it “1/2” or “month I”. However, in calendar dates usually the month precedes the day number. An “eclipse of 1 1/2” would appear to be one of 6 hours duration (assuming a unit of mina, or 60 UŠ), which is impossible. I therefore disregard the small horizontal wedge and read both verticals together as the sign “MIN”.

I do not know what GU-*ú* could be. “String” is possible, but I do not see what could be made of it. *šal-šá* probably means “one-third”. In other cases, “one-third” is written with the signs *šal-šú* on this tablet (e.g., at the end of this very line). If one separates SAL and *šá*, the result is not better. To produce some meaning, we emend 1 ITU to ME ITU, leading to: “one-third of the night, one-third of the daylight, (in) the month of your eclipse ...”; but this is still not very clear. In the following, the unit for measuring time intervals is not specified; it corresponds to 1/6 of a nychthemeron, i.e., to 1 mina or 60 time degrees.

23: DU here and in lines 26f. must be an expression for adding. The similar idiom *x ana y wabālu*, written with the same sign DU (read TÚM) for *wabālu*, is attested in mathematical texts with the meaning “to multiply x and y”.

The last sign of the line has two horizontal wedges, so one would read it PA. It is nevertheless very likely that this is a mistake for BAR = month I.

25: from the numbers it appears that an addition has been omitted here. If the result of subtracting 3 is 1,30, the number from which it was subtracted cannot be 1.

26: in this line and the following, between the signs BAR and BE-UD there are two single verticals in line 26 and three in line 27. Probably, both lines ought to contain the same number. If there are two verticals, each one is to be read “1”. If there are three verticals, the first of them can only be understood as belonging with the month name BAR “month I”, and indicating day 1. This seems to be superfluous, and we consider one vertical as erroneous.

To one-third of the length of daylight, 1 BE-UD is added. 1 seems to be a time interval, as it is added to other time intervals. Then BE-UD will be name of this time interval. Its reading and meaning is, however, unclear to me. From the parallel in line 23 one expects BE-UD (or BE-*tú*) to be a writing for *tāmartu*; but I could not find a suitable reading (like *amāru* for BE). What is expected here is an expression for one-third (of the night or of the daylight); but see the astronomical comments below.

28: BE-*tim* occurs as a word for “totality” in records of eclipses. If the same is meant here, I do not know how it fits the rest of the line. It is more likely that it refers to the preceding and following paragraphs. For each Saros, there is a shift of one-third of a certain quantity; 2 Saroi will correspond to two-thirds of the same quantity.

In the first part of the line, the sign me and the separation sign are written over an erasure.

29: At the beginning of this line, the translations "hollow" and "full" mean months of 29 and 30 days, respectively. Literally, the Babylonian terms *maṭû* and *kunnu* mean "being small (in quantity), deficient" and "confirmed, established". The term *kunnu* "confirmed" for a 30-day month is frequent[14]. For a 29-day month, the usual designation in other texts is *turru* "returned, rejected". The length of a month was important also in everyday administration, as can be seen in the Neo-Babylonian letter CT 22 167:8: *kî ūmu kunnu u kî turru kapda ṭēmu ša bēlija lušmu* "let me hear quickly an order of my lord whether the day is confirmed or rejected".[15]

The words *ṭīpu* "addition" and *nishu* "subtraction" probably correspond to the logograms TAB and LAL, found frequently in astronomical texts.

30: in most cases, the time intervals have not been translated, but rendered by their transliterations. For their meaning, see chapter IV in the astronomical commentary.

We take *šú* to mean "this" because only then does the text make sense. To understand the sign as the logogram for "moonset to sunrise", as in other parts of this section, would not fit the procedure.

33: "less than 10 UŠ" must be an error for "more than 10 UŠ"; see the astronomical commentary.

36: GABA-RI is "counterpart, equivalent, corresponding item" or "opponent". Here, and in rev. 8 and 16, it is followed by a word for a time interval. This is reminiscent of the use of GABA-RI MU-AN-NA in ACT p.70, where it is something like "epact".

The erroneous number 34 is corrected to 36 by the duplicate BM 42282:14.

37: *balṭūtu* literally means "state of being alive, healthy". Since the context suggests that the *whole* sum of ŠÚ and NA is added, I have translated it rather freely as "entirely". Unfortunately, the duplicate BM 42282 is broken at this point.

After DAH "you add" we find the sign PAP whose reading here is uncertain. In continuation of the preceding sentence, one expects something like "otherwise, or else", referring to the case if the number just calculated exceeds 10 UŠ.

Reverse

1: "before the Sun" can also be translated "opposite the Sun"; this expression serves to identify NA as occurring in the middle of the month, as opposed to NA on the first day of the month. In obv. 37 we find the expression "NA in the middle of the month", which is clear; unfortunately, the text has varying expressions for the same concept also in other places.

5: KI-LAL (akkad. *šuqultu*) "weight" recalls the measuring of time by weight in

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[14] Neugebauer, ACT p. 479.

[15] S. Parpola, Letters from Assyrian Scholars to the Kings Esarhaddon and Assurbanipal, vol. II, p. 88.

a water clock. But the measures used in the following are those derived from the measures of length.

6f.: something is wrong here with the numbers. 6 *bēru* daytime and 6 *bēru* night is the ratio at the equinox. Then the text proceeds five days to the 20th day. Assuming a linear zigzag function and a ratio of 2 : 1 between maximum and minimum, the increase in three months (up to the solstice) is 2 *bēru* = 60 UŠ. This implies 3;20 UŠ for 5 days, not 10 UŠ, as the text seems to say. 5 5/6 *bēru* not only does not fit 6 *bēru* 10 UŠ (which would be 6 1/3 *bēru*), but is not correct either. An emendation of 5 5/6 to 5 2/3 (5/6 and 2/3 are very similar signs) and of "20th day" to "30th day" would solve the problem. On the photo, "20" seems to consist of three wedges, of which the first two are written on top of each other.

5 UŠ 40 NINDA are 5 2/3 UŠ; if one emends 5 5/6 *bēru* to 5 2/3 *bēru* as proposed, then 5 UŠ 40 NINDA is one-thirtieth of 5 2/3 *bēru*, or one-thirtieth of the length of the night; this has to be subtracted from NA at the beginning of the following month.

8-15: these lines were discussed by F. Thureau-Dangin, RA 37 (1940/41) 6-8, Neugebauer [1947], and van der Waerden [1950].

17: GIŠ is probably to be read *našû*.

19: The beginning of this line seems to be the number 4,20, which recurs in the middle of the line as well. I could not find any significance of this number in astronomical context; 4 may be an error for *šá*, but that does not make the text clearer. Then the question is, what is the purpose of multiplying NA at the beginning of the month ("sunset to moonset") by 20 or, equivalently, by one-third? See the astronomical commentary.

20: "invisibility" translates the logogram U_4-NÁ-A "day of sleeping" which means the day(s) around conjunction with the sun when the moon is invisible. It seems that this term corresponds to KUR, the time interval between moonrise and sunrise.

23: this line lists planetary periods, to be applied here for the purpose of predicting rain and high water in the rivers. The order of planets is that of late astronomical texts. A 72-year period of Jupiter does not exist; maybe it was chosen as a multiple of 12 years which is an acceptable period. The better periods used in Goal-year texts are 71 and 83 years. A good period for Venus is 8 years; its multiples 64 and 16 are no improvements. The remaining periods of 46 years for Mercury, 59 for Saturn, and 79 or 47 years for Mars, are the same as those found in the Goal-year texts.

This line and the following paragraph show again that the Babylonians tried to find periods for the return not only of astronomical phenomena, but also of weather and other events in nature[16].

24: The sign ŠÁR is written archaisingly. Why it is repeated nine times is obscure to me; if the whole string is a number, it could have been written in the

[16]See the texts TU 19 and 20, edited in ZA 66 (1976) 234-260.

normal sexagesimal style, which is capable of writing all numbers, however large they might be.

25 f.: the numbers of years do not correspond to any known periods relating to the planets mentioned.

I take *ka-ma-a-nu* to be an unusual writing for *kajjamānu* "regular", but note that *ka* is separated from *ma-a-nu* by a wide space.

26: 83 is a period of Jupiter known from Goal-year texts, but whether this was intended remains doubtful since the other numbers in this section have no astronomical significance.

UD-BA could be a logogram for *adannu* "period", but the context was damaged already on the original.

27: the writing *šam-me* for *šamû* "heaven" is highly unusual.

29: the word following "wind" is unclear to me. Is it related to ŠÁR, frequently attested in Diaries as describing a wind?

30: Since I don't understand the purpose of this section, I am not sure whether A.KAL in the beginning of the line is *mīlu* "flood".

The sign *ba* in the two words read *bariṣu* is written quite differently.

bariṣu occurs in the texts LBAT 1494 and 1495, "instructions for making a gnomon"; so does *dappu* in line 31. Both are of unknown meaning.

31: 19 could also be *u* 9 "and 9"; I don't understand the purpose of the numbers.

The understanding of GI as "complete" is uncertain; 230 months do not fit well to 18 years: without intercalary months it ought to be 216, with them 223. In contrast, 19 years contain 235 months (intercalations included). The end of the line clearly looks like the beginning of a section of an astronomical Diary. Unfortunately, no Diary is preserved for 36 Seleucid era.

32f.: I note the "triangular" distribution of months II, VI, and X. The repeated occurrence of 21 12 and 20 is obscure; they do not seem to be day numbers.

35: only the two multiplications in the beginning are understandable, although their meaning is unknown. The remaining numbers are entirely obscure to me. The signs for 3600 (ŠÁR) are written in an archaising manner, as in rev. 24. Again, why are some numbers not put together into one?

36: even the readings of the numbers are open to doubt. Why is 1600 written with the number 16, whereas 1800 is written 1 $\times$ 1000 and 8 $\times$ 100 ?

The sign before *ana* ITU$^{\text{meš}}$ may be NIM (with a very faint final vertical wedge) or ŠÁR. Neither reading is meaningful to me.

The end of the line seems to mention "loss" or "flight" (*halāqu*). Does *kussu* mean the throne?

37: This is a catch-line, containing the beginning of a continuation tablet. I don't know a text beginning with these words.

II Introduction to the Astronomical Commentaries

The text TU 11 contains a mixture of what we would classify as astrological, meteorological, and astronomical prediction rules. It is a very difficult text with respect to both language and interpretation. We have, however, been able to understand most of the predicting rules written in the astronomical sections. By "understanding" such a rule we mean the following: we are able to identify all the quantities involved in the sections concerned and to establish their mutual relation. We try to investigate the inner structure of these rules by answering the following questions: what is the starting point, what is predicted, and how is it predicted. We want to find both the empirical and the theoretical basis of the method. The empirical basis of a method can, e.g., be the regular behavior of the observable alluded to, or a systematic and regular connection between different quantities which the Babylonians could have found empirically through analyses of observational data.

Some of the astronomical rules appeared so strange that we initially considered the possibility that they were just the speculations of a late scribe. We are, however, sure now that the astronomical rules on TU 11 form a collection of genuine practices, i.e., rules that were really used by Babylonian astronomers.[17] Collecting primitive and more advanced rules reveals concepts and methods used in the intermediate astronomy. Therefore, even if we have not been able to understand all the rules, we are convinced that the text is important for everyone who tries to understand the development of Babylonian lunar theory. The reason for this conviction is summarized below through three points.

We see the importance of TU 11 as:

- a connection between observation and theory,
- a collection of different methods of older and newer origin, and
- a link between different types of texts (connecting Goal-Year Tablets,[18]the Diviner's manual,[19] Enūma Anu Enlil Tablet XIV, and MUL.APIN).

One central problem within the study of Babylonian astronomy is to explain the development of the ACT astronomy. In this connection TU 11 is especially interesting. It contains empirical rules that form "bridges" between observation and theory. The eight different methods for predicting a month's length bear witness of the development of concepts and refinements of these methods through the ages. Furthermore the calculations in Section 19 demonstrate how astronomical schemes

[17]See Brack-Bernsen [2002]

[18]These tablets collect for some year observations which are earlier by certain periods (different for each planet and the moon). See Sachs [1948], 282-285; Hunger and Pingree [1999], 167-173.

[19]Edited by A. L. Oppenheim, JNES 33 [1974] 197-220.

from EAE XIV and MUL.APIN could be used for prediction.

For a long time, Section 19 was the only part of TU 11 to be translated and edited. This was by O. Neugebauer [1947]. In his paper, Neugebauer just states that TU 11 contains a collection of rules for lunar and planetary phenomena and that it would lead far beyond the scope of his article to analyze all the relevant passages of the text. He writes that the translation of Section 19 will show the general direction of these rules. Van der Waerden[20] using Neugebauer's translation, explained the rules of Section 19 and connected them to the earlier texts EAE and MUL.APIN. Van der Waerden[21] must also have been in the possession of at least a partial translation of other parts of TU 11. He surmises that in this text some rules are given for calculating risings and settings of the moon from values observed either a few days or 18, 36, or 54 years earlier. How these rules worked he does not say - only that some indications about the methods might be drawn from TU 11. The present edition confirms some of his surmises and demonstrates in detail how some of the rules worked.

Basic Working Principle

Whenever we try to understand or reconstruct an empirical rule, we start with one basic assumption: it is meaningful what is written in the text, something that reflects nature. Therefore, knowing the quantities used for predicting from the cuneiform text, it should be possible to find the inherent empirical rule, through analyzing computed lunar data and their dependence on the quantities mentioned in the text. Often, the hints given in the cuneiform text have guided us to rediscover useful empirical rules (see for instance Section VII.4 or VII.5); sometimes, however, it has not been possible to find any systematic dependence so that we must conclude either that we have failed to understand a point, or that the rule has no empirical foundation, or that the text is corrupt. (This is for instance the case with our inquiries into Section 22.)

III Survey of Astronomical Content

TU 11 is arranged in 29 Sections, separated from each other by dividing lines. At the end a colophon follows after two separating lines. Sections 9 to 22 are all concerned with astronomical predicting rules, while the content of the remaining sections is astrological. But also in Section 8, one of these astrological sections, we find important information on observational practices.

The times of future lunar eclipses are predicted: Rules for predicting the

[20] Van der Waerden [1950], p. 307 ff.

[21] van der Waerden, [1951], p. 29.

time of lunar eclipses are demonstrated through 4 examples in Sections 9 through 12 (Obv 22-24, 25, 26, and 27). Starting out with the supposedly known time of a (lunar) eclipse, the time of the eclipse which takes place a Saros later is calculated. Since the method is rather primitive and not very accurate, it is presumably quite old; the same is true for some of the other sections.

The movement of the invisible moon: Section 19 is very important. Firstly, it reveals to us how the Babylonians surveyed the movement of the moon during the days of invisibility around conjunction, i.e., through extrapolation from the time between moonrise and sunrise, observed on the last morning before the moon became invisible. And secondly, the extrapolation utilized in Section 19 seems to apply to astronomical schemes from Enūma Anu Enlil and MUL.APIN. Indeed these calculated examples demonstrate *that* and *how* these tables were used.

The "Goal-Year Method" for predicting the Lunar Six: Rules for predicting the Lunar Six[22] for a specific month(i) are based on their known values in the months ($i - 223$) and ($i - 229$) situated 1 Saros and 1 Saros plus 6 months earlier, respectively. They are more or less implicitly given in Sections 14 and 16 (see VII.4 and VII.6). A clearer formulation of these prediction rules has recently been found on fragments (BM 42282+42294) in the British Museum.

Terminology of the text: what we call month(i) is called "the month in your new year", or "the month you hold in your hand", whereas month($i-223$) is called "the month in your old year", or "the month 18 years back". (Section 22 also mentions an "old" year; but without specifying which year is meant.)

It should be pointed out that these predicting rules are very easy, elegant, and astonishingly precise. It was a great surprise to discover that the Babylonians could cope with the complicated quantities, which we call the Lunar Six, and that they were able to find a method for their empirical prediction at all, let alone one that is so accurate.

Rules for Predicting Full or Hollow Month

The first visibility of the new moon defines the beginning of a new month. If the new crescent is seen at the beginning of day 30 of a month M, this day will become the first of the next month M+1, so that month M only had 29 days; it was hollow. If the moon is not seen until the 31$^{\text{st}}$ day of a month, this month will have 30 days and is long (full).

Figure 1 shows the new moon at its first visibility after a conjunction. (This first

[22] See Chapter IV and V.

visibility of the thin new moon is called new crescent — it indicates the beginning of a new Babylonian month.) One of the Lunar Six, NA_N, is indicated on the figure: it is the time from sunset to the setting of the new crescent.

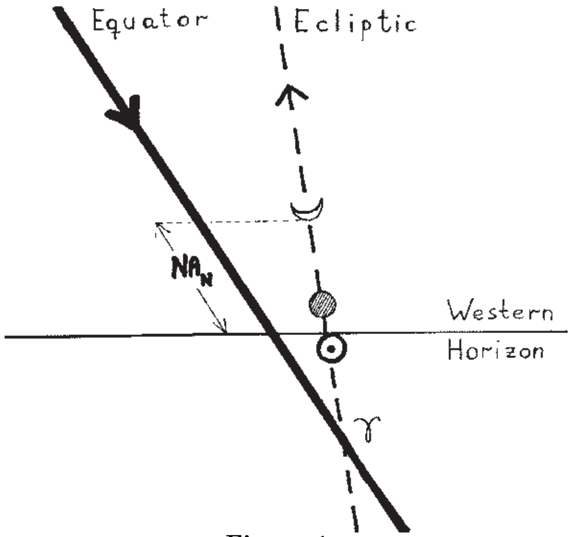

Figure 1

The situation at the western horizon on the evening when the new crescent is visible for the first time after conjunction, announcing the begin of month I. The dashed line depicts the ecliptic, the path along which, the sun and moon move. The direction of motion is indicated by the arrow, and ⊙, the sun, shows where the conjunction took place some 1 1/2 days earlier. The moon, moving faster than the sun, has on this evening reached a position far enough from the sun, that it will be visible at sunset. On the evening before it might have been in Position ● at sunset, and still too near to the sun to be seen. The fat line is the equator, it shows the direction along which all luminaries set. The time NA_N from sunset until moonset is measured by the arc of the equator, which sets simultaneously with arc (⊙ ☽).

The most important topic on TU 11 seems to be the determination of the length of a month to come. Of the 29 Sections on the tablet, 8 Sections are concerned with this problem: the seven Sections 14, 15, 17, 18, 20, 21, and 22 all mention the words usually translated as "full" or "hollow" and give different rules for the prediction of a full or hollow month. Section 19 does not mention these words. But the calculations performed in this Section reproduce in a schematic way the movement of the moon relative to the sun on the days around new moon. As we shall see, the calculations always stop on the day after conjunction, when it is clear that the estimated time from sunset to moonset, NA_N, will be so large that the new moon will be visible at dusk and the new month will begin. It is therefore clear that the calculations in Section 19 aim at determining the day (30 or 31) on which the new month begins, delivering the length of the actual month. It will have only 29 days and be called "hollow" if the new month is expected to begin on its 30th day, and it will have 30 days, being "full", if the new month starts on the day thereafter.

The best method for the prediction of the month length is given in Section 14. Here the Goal-Year method is used to calculate NA_N. One little detail in the calculations (is a correction needed or not) decides the month length. The rules given in the other sections are not that good; but most of them too are based on the quantity NA_N, which is found by different methods.

Lunar Latitude ?

At a first glance, Section 15 seems to be concerned with the lunar latitude. The text connects the expressions "high" and "low" to the duration of the months. These terms are normally understood as referring to positive and negative lunar latitude, but as we shall see in Chapter VII.5, when used for predicting the length of the month, "high" and "low to the sun" must refer to the altitude of the new crescent, not its latitude. This example will illustrate some of the difficulties one is faced with when trying to understand or reconstruct ancient predicting rules.

General Problems with the Texts. How to Understand the Terminology:

The text uses the following expressions for long and short months: *kun-nu* or GIN, and 1 are used in connection with a long, "full", month; LAL-*ú*, *tur-ru* or GUR, and 30 are used in connection with a short, "hollow", month.[23]

But here we have a problem: which month is meant to be full or hollow? The one mentioned in the text or the month just before? For the Diaries, we know the convention used: the length of the previous month was indicated on the first day of a new month. A remark like "month M 30 NA 17" is understood as follows: Month M began on the 30^{th} day of the preceding month (M-1), the new crescent set 17 time degrees after sunset. Hence "month M 30" tells us that month (M-1) had only 29 days, i.e., it was hollow. And correspondingly, "month M 1" means that month (M-1) was full (had 30 days). Neugebauer used the terms: post-hollow and post-full when referring to texts using this convention. But as we shall see, another convention is used in at least some of the Sections of TU 11. We have been

[23]The expressions "full" and "hollow" are taken over from Greek astronomical texts. The Babylonian words should rather be translated "confirmed" and "rejected" or "turned back"; expressions which reflect their practices. The Babylonian day starts at sunset. If the new crescent, which indicates the beginning of a new month, is seen shortly after sunset on day 30, this day is rejected as the last of the month and identified to be the first of the next month. In the case where the new crescent is not seen at the beginning of day 30, this day is confirmed as the last of the month; the next day will be the first of the new month.

able to reconstruct the prediction rule of Section 14 and therefore we know how to understand the terminology used in that Section. When month M is said to be hollow it must mean that month M has 29 days; if it is said to be full, then month M has 30 days.

TU 11 is not consistent in its way of denoting full and hollow month. There is in TU 11 apparently no system in use which differentiates when full and hollow are used for the current or the previous month. It seems as if the signs are interchangeable: Section 14 uses both expressions LAL-*ú* and GUR-*ru* to indicate hollow, while *kun-nu* and 1 indicates full. In Section 15, the signs GUR and GIN are used for hollow and full; while Section 22 uses the signs GIN and 30 for full and hollow.

Another problem in the text is the terminology "NIM" and "SIG" used in Section 15. In ACT "NIM" and "SIG" are related to the latitude of the moon; but if the rule given in Section 15 shall make any sense, "NIM" and "SIG" can in this connection only refer to the altitude of the moon above the horizon on the evening of the new crescent. (We refer to the introduction where our approach for understanding and finding an interpretation of the text is formulated as "basic working principle".)

Concluding this Chapter we present the content of the astronomical Sections (as far as we have succeeded in understanding them) below in a condensed and schematic way. The investigations and arguments leading to these interpretations are given in the later Chapters.

The astronomical predicting rules on TU 11 presented in a schematic form:

Section 9 – 12: $\left\{ \begin{array}{l} \text{time of ``old eclipse''} \\ \text{time shift after 1 Saros} \end{array} \right\} \longrightarrow$ time of "new eclipse"

Section 14: Goal-Year Method $\longrightarrow$ $\mathrm{NA}_N \left\{ \begin{array}{l} \text{addition: month hollow} \\ \text{subtraction: month full} \end{array} \right.$

Formulated (and valid) in case of a full old month: $\mathrm{NA}_N(\text{i-223}) \longrightarrow \mathrm{NA}_N(\text{i})$

Section 15: New crescent seen $\left\{ \begin{array}{l} \text{``high to the sun'' } (h_{☾} \text{ large): month hollow} \\ \text{``low to the sun'' } (h_{☾} \text{ small): month full} \end{array} \right.$

Section 16: Goal-Year Method

Section 17: $\mathrm{NA}_N(\mathrm{I}) \longrightarrow \mathrm{NA}_N(\mathrm{II}) \left\{ \begin{array}{l} \mathrm{NA}_N(\mathrm{II}) < \mathrm{NA}_N(\mathrm{I}) : \text{hollow} \\ \mathrm{NA}_N(\mathrm{II}) > \mathrm{NA}_N(\mathrm{I}) : \text{full} \end{array} \right.$

Difference $\mathrm{NA}_N(\mathrm{I}) - \mathrm{NA}_N(\mathrm{II})$ approximated by $\pm\,|\mathrm{NA(VII)} - \mathrm{NA(VIII)}|$

Section 18: $\mathrm{NA}_N(\mathrm{II})$ [– 18 years] $\longrightarrow \mathrm{NA}_N(\mathrm{II})$ $\begin{cases} \text{if larger than 12: hollow} \\ \text{if smaller than 12: full} \end{cases}$

Section 19: KUR(i) extrapolated $\longrightarrow$ day of new crescent

Section 22: $\mathrm{NA}_N(\text{new})$ found through $\begin{cases} \text{addition} & : \text{hollow} \\ \text{subtraction} & : \text{full} \end{cases}$

Difference $\mathrm{NA}_N(\text{new}) - \mathrm{NA}_N(\text{old})$ approximated by $\pm|\mathrm{KUR}(\text{new}) - \mathrm{KUR}(\text{old})|$

IV The Lunar Six

When observing the Moon or surveying its movement through calculation, there are six time intervals, called the "Lunar Six" by Sachs,[24] which were given special attention by the Babylonians. They were measured in UŠ i.e., degrees of time. One is to be observed on the evening when the new crescent became visible for the first time after conjunction, indicating the beginning of a new month:[25]

NA_N = time between setting of the sun and of the new crescent.[26]

Around the middle of the month, the four intervals relating to the full moon, which we call the "Lunar Four", are the following:

ŠÚ = time from moonset to sunrise, measured at last moonset before sunrise.
NA = time from sunrise to moonset, measured at first moonset after sunrise.
ME = time from moonrise to sunset, measured at last moonrise before sunset.
GE_6 = time from sunset to moonrise, measured at first moonrise after sunset.

At the end of the month:

KUR = the time from last visible moonrise before conjunction to sunrise.

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[24]Sachs [1948].

[25]See Figure 1, in which we have however neglected the lunar latitude.

[26]In the texts with which we are working, this interval is called NA, but it occurs always together with an indication that it is the NA of the first day or the NA at the beginning of the month. We put this identification into the name, calling it NA(of the new crescent), or NA_N. We do this in order to be as precise as the Babylonian texts. There the term NA is also used for a time interval in the middle of the month, but always identified by calling it the NA of day 14 or the NA opposite the sun.

It is evident from their definitions that these time intervals are obvious and easy to observe, that they occur around conjunction and opposition, and that they somehow contain information on these two events.[27] From a theoretical point of view however, these intervals are very complicated quantities. They depend on the time when in comparison to sunset or sunrise the conjunction (or the opposition) takes place. They also depend on the position of the full or new moon in the ecliptic, and on the lunar velocity and latitude (e.g., NA = NA(Δt, $\lambda_☾$, $\beta_☾$, $v_☾$)).

When modern scholars started to investigate the structure of Babylonian astronomy, it was noticed that the Babylonians observed the Lunar Six regularly, and that they somehow were able to predict their numerical values and thus fill gaps in their Diaries. And it was realized that Babylonian astronomers, during the Seleucid period, were able to calculate lunar and planetary phases by elegant and efficient numerical methods. But how the Lunar Six were utilized and how the ACT methods were developed, was completely unknown. Let us quote O. Neugebauer [1975, p.348]: "For the cuneiform ephemerides we can penetrate the astronomical significance of the individual steps, as one may expect with any sufficiently complex mathematical structure. But we have practically no concept of the arguments, mathematical as well as astronomical, which guided the inventors of these procedures."

In this respect, astronomical texts from the time before the Seleucid Era are of special interest, since they are witness of earlier stages which might have led to the ACT astronomy. Together with Sachs, Neugebauer has edited and commented on some "Atypical Texts".[28] From these we know about alternative (early?) approaches to visibility problems: e.g., to use the time intervals NA_N and KUR for a month to find these quantities for the next month. But here, as in other tablets from the intermediate period, we find methods which belong to the most advanced lunar theory associated with a primitive attempt to deal with visibility problems or solar positions. It is the occurrence of column Φ in early texts, giving the momentaneous position of the moon in its anomalistic cycle, next to primitive schemes, which puzzled Neugebauer [1975, pp. 552-3] and still puzzles us, today.

We still know quite little about the development of the Babylonian lunar (and planetary) calculation schemes. Some progress has been made, and we hope that the edition of TU 11 and related texts will help us to get an insight into some of the methods used before the Seleucid Era for predicting lunar phenomena. Early concepts and theoretical arguments, implicitly given in these texts, may eventually help us to get a deeper understanding of the development of this first and very sophisticated mathematical astronomy.

One idea which turned out to be fruitful was the use of the Lunar Six for con-

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[27] Normally, ŠÚ and ME take place before while NA and GE_6 occur after opposition. Sometimes, however, an extreme lunar latitude can cause the events to occur one day earlier or one day later.

[28] O. Neugebauer and A.Sachs [1967 and 1969].

structing column Φ.[29] A proposal was made for utilizing the huge amount of lunar data, which had been observed continuously since the beginning of the regular watching (around 750 B.C.). It could be shown that the simple sum of the Lunar Four, calculated for consecutive full moons, can be fitted very well to a linear zigzag function which has the *same period, amplitude and phase* as Column Φ.[30] Only the mean value differs by a still unexplained 100 UŠ. It is thus quite probable, but not proven, that column Φ was constructed by means of the Lunar Four. See Britton [1999] for an alternative derivation of Column Φ. We have since studied the Lunar Six more closely and deepened our understanding of their astronomical significance[31] and utilization.

Systematic analyses of computer-simulated Lunar Four data have resulted in a proposal of how such data as collected on the Goal-Year tablets could have been used for predicting the future Lunar Four. Sections 14 and 16 of TU 11 confirm that the Babylonians really knew and used the proposed method.[32] Therefore, we now know exactly how the Goal-Year tablets were used for predicting the Lunar Six. And we have strong evidence that this empirical predicting method was known already before 500 B.C.

We shall emphasize that these new findings stay within the framework of early Babylonian astronomy. The same quantities and ideas are dealt with in older and newer texts: visibility times of the moon and its daily retardation. In EAE XIV and MUL.APIN visibility times and the daily retardation of the moon were deduced from the length of the night. The Goal-Year method for finding visibility times is based on empirical knowledge and finds the Lunar Six by means of their values determined a Saros earlier in combination with a very good approximation for the moon's daily retardation. The sum ŠÚ+NA was used for the daily delay of moonset around full moon and the sum ME+GE_6 for the daily delay of moonrise at full moon, both measured a Saros earlier. Obviously the Babylonians had added ŠÚ and NA (or ME and GE_6) in order to get the moon's delay at the western (or eastern respectively) horizon. And they had realized and utilized the fact that these values (ŠÚ+NA and ME+GE_6) repeat after one Saros.

Now, ŠÚ+NA is the setting time of the moon's movement relative to the sun at the day of opposition, and ME+GE_6 is its rising time. The sum of these two, ŠÚ+NA+ME+GE_6 is a good measure for the moon's velocity (more exactly it approximates its movement relative to the sun over two days). The disturbing effect of the oblique ascension has been largely reduced by the addition of the times observed at the western and eastern horizons. It is this sum of the Lunar Four which

[29] Brack-Bernsen, [1990].

[30] Brack-Bernsen, [1994] and [1997].

[31] Brack-Bernsen and Schmidt, [1994].

[32] See Brack-Bernsen, [1997] and [1999] with Hunger's translation of Section 14 and 16.

we had proposed to be the observational basis for column Φ. Section 19 uses 1/15 of the day's length for the daily retardation of the moon. But clearly, as soon as the Babylonians had realized that the retardation of the moon was different if measured at the eastern (ME+GE$_6$) and western (ŠÚ+NA) horizon, then it would be quite obvious to combine these two different values. And hence somehow their sum ŠÚ+NA+ME+GE$_6$ will show up.

Also in another respect, the Lunar Six data have become interesting: they belong to the most accurate of all astronomical data which are recorded in Babylonian texts. Therefore the numerous data bases of Lunar Six measurements have been analyzed statistically in different ways in order to get information on time measurement in Mesopotamia.[33]

V The "Goal-Year" Method for Predicting Lunar Six

An elaborate description of the Goal-Year method and its empirical foundation is given by Brack-Bernsen [1999]. We refer to that paper and shall at this place give only an overview of the method.

The Babylonians knew that the period of 223 synodic months is useful for lunar predictions. We call this period a Saros, they called it 18, presumably because 223 synodic months approximates 18 years quite well (it equals 18 solar years plus 11 days). For the moon, it is an important time interval, because in a good approximation it also equals an integer number of anomalistic, sidereal or draconitic months:

$$223 \text{ syn.m.} \approx 239 \text{ anom.m.} \approx 241 \text{ sid.m.} \approx 242 \text{ drac.m.} \approx 18 \text{ years.}$$

Therefore the three variables $v_☾$, $\lambda_☾$, and $\beta_☾$ (i.e., the velocity, the longitude and the latitude of the moon) will have approximately the same magnitudes at oppositions or conjunctions, respectively, situated one Saros apart. This is the reason for its usefulness for the prediction of lunar phases and as an eclipse cycle.

The Saros was indeed used for constructing eclipse tables, where the years and months were recorded in which the moon (or the sun) was expected to become eclipsed.[34] The earliest Eclipse Table probably starts in 747 BC. They might have been constructed later and projected backwards; but according to J. Steele,[35] the Saros cycle was identified by the Babylonian astronomers by at least the seventh century BC, and probably earlier.

A type of tablet, called the Goal-Year tablets by A. Sachs [1948], collects specific data on moon and planets to be used for predicting the lunar and planetary phases

[33]See Stephenson [1974], Fermor and Steele [2000], P.Huber [2000], and Brack-Bernsen [1999a].

[34]See Hunger [2001], Appendix by J. Steele.

[35]See Steele [2000], p. 78 and [2000a].

which are expected to occur in a special year, the goal year. The Lunar data recorded on a Goal-Year tablet are eclipses of sun and moon which took place in the year one Saros earlier than the goal year, all the Lunar Six for that year, plus the sums ŠÚ+NA and ME+GE_6 for the last six months of the year before (i.e., of the year coming 19 years before the Goal Year).

We know how the Babylonian astronomers used such data for the prediction of lunar phases, and that the method was easy and very precise. The procedures used by the Babylonians are reproduced below in mathematical formulas:[36] equations (V.1) – (V.6). A remark on the notation used here: We consider a series $(0, 1, 2, \ldots, i, \ldots)$ of consecutive lunar months, identifying them by their lunation number, i. The Lunar Six occurring in month (i) shall be identified by the index i: For instance ŠÚ_i or $\text{ŠÚ}(i)$ is the time from last moonset before opposition to sunrise. It can be predicted by means of ŠÚ_{i-223} and NA_{i-223} from month$(i-223)$ by equation (V.2).

$$(\text{NA}_N)_i = (\text{NA}_N)_{i-223} - 1/3\,(\text{ŠÚ} + \text{NA})_{i-229} \qquad \text{(V.1)}$$

$$\text{ŠÚ}_i = \text{ŠÚ}_{i-223} + 1/3\,(\text{ŠÚ} + \text{NA})_{i-223}, \qquad \text{(V.2)}$$

$$\text{NA}_i = \text{NA}_{i-223} - 1/3\,(\text{ŠÚ} + \text{NA})_{i-223} \qquad \text{(V.3)}$$

$$\text{ME}_i = \text{ME}_{i-223} + 1/3\,(\text{ME} + \text{GE})_{i-223} \qquad \text{(V.4)}$$

$$\text{GE}_i = \text{GE}_{i-223} - 1/3\,(\text{ME} + \text{GE})_{i-223} \qquad \text{(V.5)}$$

TU 11 has traces of these 5 formulas, but nothing on KUR. However, corresponding to (V.1), the reconstructed formula for calculating KUR must be:[37]

$$\text{KUR}_i = \text{KUR}_{i-223} + 1/3(\text{ME} + \text{GE})_{i-229} \qquad \text{(V.6)}$$

We understand why this works: at syzygies which are separated by one Saros, the lunar longitude, latitude, and velocity, will be almost the same. And the daily delay of the moon (ŠÚ+NA for its setting, and ME+GE_6 for its rising) will repeat:

$$(\text{ŠÚ+NA})_i \simeq (\text{ŠÚ+NA})_{i-223} \;\text{ and }\; (\text{ME+GE}_6)_i \simeq (\text{ME+GE}_6)_{i-223}$$

Of all quantities determining ŠÚ and NA, only the time from opposition to sunset has changed by 1/3 day (223 synodic months $\simeq$ 6585 1/3 day). As a consequence, the new ŠÚ will become larger by 1/3 $(\text{ŠÚ+NA})_{i-223}$ while the new NA will be reduced by the same amount.[38]

For the prediction of $\text{NA}_N(i)$, the visibility time of the new crescent, the quantity $(\text{ŠÚ+NA})_{i-229} \simeq (\text{ŠÚ+NA})_{i-6}$ was used for the daily delay of the moon in month

[36] In the following we drop the subscript from GE_6 for clarity.

[37] In the mean time we found textual evidence also for this case: according to our understanding, one section of fragment BM 37110 is, indeed, concerned with this rule.

[38] For further details, see Brack-Bernsen [1999].

i. Indeed this is a very good approximation: the daily retardation of the new moon equals almost exactly the daily delay of the full moon measured 5 1/2 month earlier. On TU 11 these rules were expressed only indirectly (in Section 14 and 16) – some of them were reconstructed; on the parallel tablet BM 42282 + 42294 [39] the rules are formulated more clearly.

Corrections to the Goal-Year Formulae

Sometimes, the results found by these calculations are preliminary. If NA_N happens to become too small (the text says smaller than 10 UŠ), then the moon will only become visible the next day, and hence the value of NA_N will become (ŠÚ+NA) larger: corrected $(\mathrm{NA}_N)_i$ = preliminary $(\mathrm{NA}_N)_i + (\text{ŠÚ+NA})_{i-229}$.

$$(\mathrm{NA}_N)_i = (\mathrm{NA}_N)_{i-223} + 2/3\,(\text{ŠÚ} + \mathrm{NA})_{i-229} \qquad \text{(V.1c)}$$

(reference: TU 11 obv. 37.) As we understood this passage of TU 11, we knew what the text meant by “addition”. It signifies the correcting addition of $(\text{ŠÚ+NA})_{i-229}$ to (V.1) resulting in (V.1c), to be applied in those cases, where the (preliminary) predicted NA_N happened to be so small, that the new crescent was not yet visible. This knowledge was the key for understanding how the text predicts full or hollow month by means of the Goal-Year method.

The Goal-Year Method for Finding NA_N

$$(\mathrm{NA}_N)_i = \begin{cases} (\mathrm{NA}_N)_{i-223} - 1/3\,(\text{ŠÚ} + \mathrm{NA})_{i-229} & \text{if} \geq 10\text{UŠ} \qquad \text{(V.1)} \\ (\mathrm{NA}_N)_{i-223} + 2/3\,(\text{ŠÚ} + \mathrm{NA})_{i-229} & \text{otherwise} \qquad \text{(V.1c)} \end{cases}$$

Sometimes similarly, the calculated Lunar Four will need a correction:[40] Text TU 11 does not say anything about it; however BM 42282+42294[41] gives the correct rules. According to I. Finkel, this tablet is to be dated to the 5th century BC; but the rules must have been known even earlier: The tablet Cambyses 400 indicates that the Goal-Year method was already known around 523 BC. This tablet gives all Lunar Six for a whole year. Control calculations by modern computer programs have shown that the Lunar Four are surprisingly accurate.[42] Since they clearly cannot all have been observed, at least some must have been found by a precise empirical method. The only precise one known is the Goal-Year method; so we conclude that the non-observed data on Cambyses 400 were established by means of the Goal-Year method.

[39] The two joining fragments have been found in the British Museum by I. Finkel and Chr. Walker.

[40] Proposed by Brack-Bernsen [1997], p.121

[41] To be published by Walker and Brack-Bernsen

[42] See Brack-Bernsen 1999a, Figure 2, p. 18.

VI Other Methods for Predicting the Lunar Six

As mentioned above, Section 14 and 16 contain short passages which clearly show that the text uses the "Goal-Year" method for predicting Lunar Sixes. But this is not the only method to be found on TU 11 for determining some of the Lunar Six. All the Sections 17 through 22 seem to use different methods for determining the quantity NA_N for some month, whereby some details, (mostly inequalities) which turn up in the procedures, are used to decide if a month will become full or hollow.

Except for Section 19, which is analyzed more closely below, we refer to Chapter VII and shall at this place only give a survey of the quantities used for predicting NA_N. Finally, the method found in the atypical astronomical cuneiform text K for predicting the lunar six shall be mentioned.

Methods Using the Quantity NA_N for Predicting the Month's Length

- NA_N is found by means of the Goal-Year method (using Lunar Six data from lunations 18 years earlier) and a little detail within this calculation will determine the length of the new month: Is an addition needed or not (Section 14).

- NA_N for a month II is found from its value, NA_N(I) one month earlier by means of the monthly change in NA_N. Here, the monthly change in NA_N is derived from the monthly change in NA, measured at full moon half a year earlier:

$$\mathrm{NA}_N(\mathrm{II}) = \mathrm{NA}_N(\mathrm{I}) \pm |\mathrm{NA}(\mathrm{VII}) - \mathrm{NA}(\mathrm{VIII})|;$$

the sign of NA(VII) − NA(VIII) indicates the length of the month (Section 17).

- NA_N is found from its value 1 Saros earlier. One thirtieth of the schematic length of the night is used to approximate the amount by which NA_N will have changed after one Saros: $[(\mathrm{NA}_N(\text{i-223}) - \mathrm{NA}_N(\text{i})) \simeq 1/30 \text{ night}]$ (Section 18).

- NA_N is found from KUR through extrapolation (see below). The same value for the daily retardation of the moon is used at the eastern and western horizon, namely a fifteenth of the day. The size of NA_N decides the day on which the crescent is expected to be visible. This is a short term determination: KUR(i) is measured at the last visible moonrise before conjunction toward the end of month(i), and NA_N(i+1) is to be measured a few days later, at the first visibility after conjunction (Section 19).

- NA_N and KUR of some old and new year are mentioned, however it is not clear what is meant by the old year: is it 1 year earlier, or 18 or 19 or..? The text seems to calculate NA_N(new) from NA_N(old) by adding or subtracting (KUR(new) − KUR(old)) (Section 22).

A Close Analysis of Section 19:

Section 19 demonstrates through three calculated examples a method for surveying the movement of the (invisible) moon around conjunction. Each example starts with a hypothetic value of the observable KUR, which is the time from last visible moonrise before conjunction to sunrise. Then the time between moonrise and sunrise is calculated for the following mornings, where the moon is invisible. We concentrate on the first example and quote the corresponding passage:

> *In order for you to calculate the equivalent of the day of invisibility of the moon. If (in) month I of your new year, the 27th, KUR is 25, daylight is 3,20, night is 2,40: 3,20 times 4 is 13,20. You subtract 13,20 from 25, and 11,40 (the moon) will be delayed. The 28th, 11,40 is remaining for the sun. You subtract 13,20 from 11,40, and on the 29th day the moon will have passed the sun by 1,40. You add 13,20 to 1,40, and (it is) 15. (On) the 30th day, the moon will have passed the sun by 15.*

The calculations from this example I can be reproduced in a schematic way, calling "KUR(m)" the time from moonrise to sunrise on morning m:

$$\begin{aligned}
\text{“KUR(27)”} &= \text{KUR} & &= 25\\
\text{“KUR(28)”} &= \text{KUR} & -13;20 &= 11;40\\
\text{“KUR(29)”} &= \text{KUR(28)} & -13;20 &= -1;40\\
\text{“KUR(30)”} &= \text{KUR(29)} & -13;20 &= -15
\end{aligned}$$

Only KUR = "KUR(27)" is observable, the others are not. The calculations reflect the movement of the invisible moon relative to the sun in the days around conjunction: in comparison to sunrise, the moon will rise later day after day. On morning 27 and 28 it rises before the sun while on morning 29 and 30 it rises after the sun. We see, that the same value 13;20 [UŠ] is used as the daily retardation of the moon for all the days concerned. The text found this daily retardation, ΔKUR, from the (schematic) length of daylight: 3;20 [mana] × 4 = 13;20 [UŠ], or, if we think of the daylength as measured in UŠ: retardation = 3,20 [UŠ] / 15 = 13;20 [UŠ]. Thus the value for ΔKUR equals 1/15 of the daylight. This reminds us of schemes from EAE and from MUL.APIN, in which the daily retardation of the moon is calculated as 1/15 of the schematic length of the night. Therefore TU 11 might give us information on how the older schemes from EAE and MUL.APIN were used.

Figure 2 illustrates example I. On the celestial sphere for Babylon, the position of the moon at the moment of sunrise is drawn for the days 27, 28, and 29 in the Babylonian month. As assumed in the calculated example, the last visibility took place on day 27, and KUR is 25 UŠ. The position of the moon at that morning is called $☾_{KUR}$. That KUR is 25 UŠ means that the moon rose $25° = 100^{min} = 1^h\ 40^{min}$ before sunrise. KUR is the time it takes the elongation arc($☾_{KUR}, \odot$) to rise.

Being a time difference, it is measured along the equator and given by the length of arc(A,O). Here O is the point of the equator which rises simultaneously with the sun, while point A rises at the same time as the moon. On the next morning 28, the moon, now invisible, is at position ☾$_{28}$ and still rises before the sun. On morning 29, the moon at position ☾$_{29}$ has passed the sun. The differences in time between the risings of sun and moon on these days, arc(B,O) and arc(C,O), respectively, are calculated in the example.

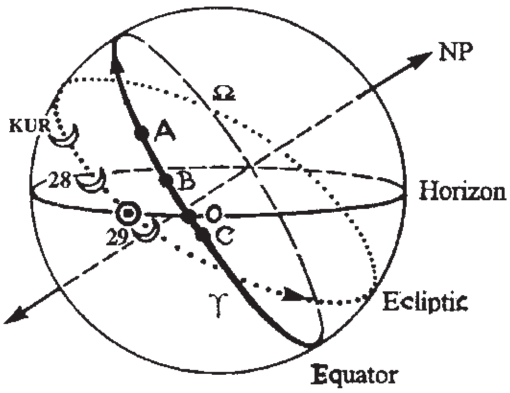

Figure 2

The celestial sphere for Babylon. Position of the moon near the eastern horizon at the moment of *sunrise*, shown on three consecutive mornings around conjunction. In the position 28 and 29, the moon is too close to the sun to be observable.

We want to stress that in our geocentric model, differences in position are measured along the ecliptic, while differences in time are measured along the equator. One of the most important and difficult problems in ancient astronomy was to find a connection between these two types of arcs – i.e., to find the rising–arc of an arbitrary arc of the ecliptic, which is the arc of the equator which rises during the same time as the ecliptic arc in question.

The text here seems to identify time differences with the movement of the moon relative to the sun. For instance, instead of saying something like "the moon will rise 1,40 [UŠ] after the sun", it says (rev. 10) "the moon will have passed the sun by 1,40". The "theoretical basis" of the calculations in section 19 is thus an early one, in which the position of the moon relative to the sun is inferred directly from time differences of their risings. Consistently, the calculations in example III find the time (NA_N) between sunset and moonset (at the very beginning of day 30) just by adding half the daily retardation of the moon to KUR(29), which is the time from sunrise to moonrise calculated for the middle of day 29. This is a very rough procedure, KUR being measured at the eastern horizon while NA_N is measured at the western horizon. In our geocentric description, it corresponds to neglecting the difference between the ecliptic and the equator. Obviously the text has not yet reached a level where it differentiates between the equator and the path of sun and moon. Or to put it in another way – but again expressed in terms of our geometrical model: in this theory the rising arc (O,B) of an ecliptic arc ($\odot$, ☾) was taken to be the arc($\odot$, ☾).

But an important point is that we have here a (admittedly rather primitive) method for finding NA_N from KUR.

In the so-called Atypical Astronomical Text K, Neugebauer and Sachs identified a method for determining Lunar Sixes.[43] For each of the Lunar Six some parameters were given in tables, and the method for calculating a Lunar Six from its value the month before was demonstrated through examples. The tablet was broken and the text was not always visible, so that only for the quantities NA_N and KUR was it possible to reconstruct the method. In a first approximation, the quantities NA_N or KUR for a future month were found from their values the month before by the addition of a number t:

$$\begin{aligned} &\mathrm{NA}_N(i+1)_0 = \mathrm{NA}_N(i) + t_N, \\ \text{and}\quad &\mathrm{KUR}(i+1)_0 = \mathrm{KUR}(i) + t_K, \end{aligned}$$

where the numbers $t = t(\lambda)$ were functions of the lunar longitude and found by interpolation between values given in a table. If $\mathrm{NA}_N(i+1)_0$ or $\mathrm{KUR}(i+1)_0$ happened to become too large, they were corrected by subtracting s_N or s_K from $\mathrm{NA}_N(i+1)_0$ or $\mathrm{KUR}(i+1)_0$, respectively. This corresponds to the astronomical situation where NA_N could have been observed one day earlier or where KUR was visible one day later. Hence, the quantities s_N or s_K are used as the daily change of NA_N and KUR. Therefore, if astronomically correct, they should approximate the rising arc and setting arc of the daily movement of the moon relative to the sun. But they don't. The values of s_N or s_K used in text K are given in a table as functions of the lunar longitude. But as pointed out by Neugebauer, the numbers do not show certain symmetries with respect to the equinoxes and solstices, which are inherent to the trigonometric problem of transforming an increment of elongation into right ascension. Neither are the numbers chosen such that $t \simeq 1/2s$, which also should have been the case.

We have here an example of a theoretical method (involving the position of the moon in the ecliptic) which is much less accurate than an (earlier?) empirical method. From the Goal-Year method we know the corrections to be applied, if the calculated Lunar Six happen to be too large or small, so that the phenomenon would occur one day earlier or later. It uses ŠÚ+NA and ME+GE_6 for the daily

[43] See Neugebauer and Sachs, [1969] pp.96–113.

retardation of the moon at the western and eastern horizon, respectively. And we have demonstrated that these quantities, if correctly observed, approximate the daily retardation surprisingly well.[44] Indeed the Goal-Year method is very precise. Therefore NA_N or KUR determined by this method will always be reasonably accurate, provided that the data used for the calculations are sufficiently good.[45] This is not the case for the method described in Text K. The tabulated values for s do not approximate the mean values of ŠÚ+NA and $\mathrm{ME}+\mathrm{GE}_6$ well, sometimes the value of s lies even outside the total variation range of these quantities. We conclude: the empirical Goal-Year method displays a deeper understanding of the Lunar Six and results in a much better prediction than the theoretical method in the Atypical Text K.

VII Duration of the Babylonian Month

VII.1 Quantities Used in TU 11 for Determining the Month's Length

TU 11 collects a variety of practices used to determine if a future month will become short or long. Let us list the quantities used to indicate Full or Hollow, starting with the Sections which we have been able to understand, and continuing with the more problematical ones:

Section 14 calculates NA_N according to the "Goal-Year" method; subsequently one detail of the procedure (is an addition necessary or not) decides the month's length; but *only* in the cases where the basic month in "your old year" is full. Of all rules on TU 11, this is the most advanced and precise, so we shall start our comments on TU 11 by analyzing and explaining section 14.

Section 15 uses the altitude of the new crescent above the Horizon (as observed at the beginning of a month) for predicting the length of that month. It also connects the month length to "the day of NA" (i.e., the day within the Babylonian month on which the moon, for the first time, sets after sunrise).

Section 17 involves NA (full moon) established for months VI and VII for the prediction of NA_N and hence of the lengths of months I or II.

Section 18 relies on older practices using the length of the night to determine how much, in comparison to sunset, the moon is delayed after 1 Saros. This delay and the value of NA_N 18 years back delivers the magnitude of the new NA_N, which then decides the duration of the month to come.

Section 19 demonstrates through examples how some of the astronomical tables

[44] See Brack-Bernsen [1999], Figure 7, p. 171.

[45] An analysis of the data extracted from the Goal-Year tablets has shown that this was the case. See Brack-Bernsen 1999a.

from Enūma Anu Enlil and MUL.APIN may have been used. Starting with a supposedly known KUR, and using the daily retardation of the moon found as 1/15 of the schematic day length, the time difference between risings of Sun and (invisible) Moon is found through extrapolation. It must be remarked that the extrapolation always (i.e., in all 3 examples) stops at the very moment when it becomes clear at which day the new moon will become visible. Hence Section 19 also demonstrates a method for determining if a coming month will become Full or Hollow.

Section 20 and 21 show some strange calculations (containing errors), but they seem similar to the method found in Atypical texts K.

Section 22 is concerned with the quantities NA_N of month I and KUR of month XII measured some days before. It seems to use NA_N(I) and KUR(XII) from some "old" year together with some "new" KUR(XII) for calculating NA_N(I) of the "new" year. If NA_N(I)(new) is found by addition, the month will be hollow, and it will become full if a subtraction was used.

We remark: Except for section 15, all rules mentioned here seem to determine NA_N (by different methods), and then predict the month's length.

Before we look closer into the Babylonian practices, let us figure out if we can predict the length of the Babylonian month. After several attempts, we have finally found an extremely simple method:

VII.2 How We Can Determine the Month's Length

It is the magnitude or size of consecutive NA_N which decides the month's length. In 96 cases out of hundred, the following **rule R** works:

$$\text{Rule R:} \quad \begin{cases} \text{If} \quad \mathrm{NA}_N(i) < \mathrm{NA}_N(i+1), \text{ then month}(i) \text{ is full} \\ \text{If} \quad \mathrm{NA}_N(i) > \mathrm{NA}_N(i+1), \text{ then month}(i) \text{ is hollow} \end{cases}$$

The two following figures, 3 and 4, illustrate this rule.

For our investigations of the length of the Babylonian months and for the construction of these and the following figures, we have used the computer program of P. Huber for calculating the Lunar Six. The data output of these calculations lists in addition to the size of NA_N for consecutive lunations among other things also the duration of the month. This is done in the same way as in the Babylonian Diaries: the number 1 after the date of new crescent indicates that the preceding month did have 30 days, while the number 30 indicates a preceding month of 29 days. Contrary to this convention, in our figures we indicate the length of the actual month. This is done in order to be in agreement with (most of) the rules of TU 11 in which, as far as we understand the text, the length of the months concerned is predicted.

In Figure 3 and 4, the magnitude of NA_N is plotted as a function of the lunation number. Underneath the curve, a black circle indicates that the current month (not the preceding one as in the Diaries) is full, and a triangle at a lunation i indicates, that the NA_N of the following lunation $(i+1)$ is bigger than $NA_N(i)$.

In Figure 3, all 70 lunations observe our rule that a month is long when the visibility time of its crescent is shorter than that of the next month: the circles and triangles always occur together. In other words, the long months are exactly those for which the observable $NA_N(i)$ is smaller than the $NA_N(i+1)$ of the next month. And the short months are exactly those where $NA_N(i)$ is greater than the $NA_N(i+1)$ to be measured at the beginning of the next month$(i+1)$.

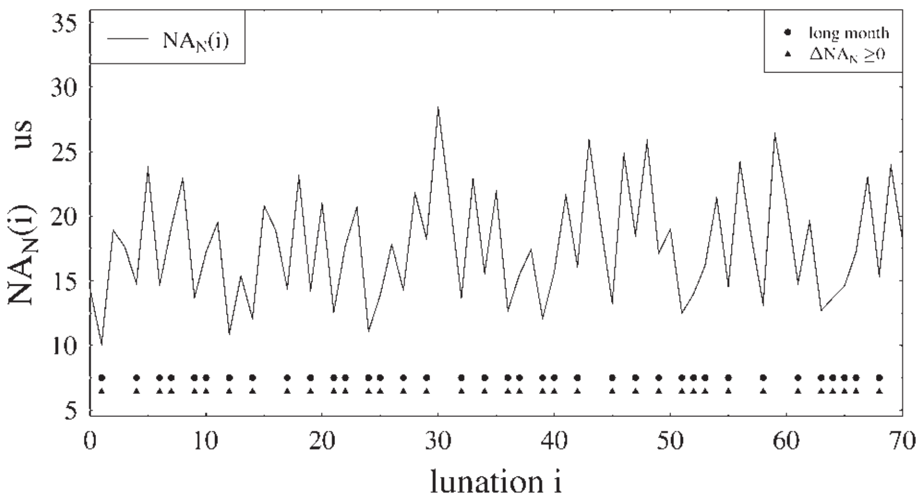

Figure 3

For consecutive months $i = 0, 1, 2, \ldots, 70$, the time $NA_N(i)$ from sunset to the setting of the new crescent is plotted as function of the lunation number i. A circle at a lunation marks a long month, while a triangle tells that at the first day of the next month, the new crescent will be visible for a longer time before setting. (Month 0 equals the Babylonian month, which began August 1 759 BC (= JD 1444411). In this and all computer generated figures the unit UŠ is written as us, because the program has no special characters.

Figure 4 illustrates some deviations from our rule. In most cases, the circles and triangles occur at the same lunations; however, at lunation 123, 135, and 136 there is a circle but no triangle. This indicates that these three months are full in spite of the fact that NA_N measured at the beginning of the months is larger than the NA_N to be measured at the beginning of the next month. Lunation 127 exemplifies the other type of deviation: $NA_N(127) < NA_N(128)$, and still month 127 has only 29 days.

We have controlled our rule for over 669 lunations = 3 Saroi, and we found only

29 exceptions. This shows, that our rule works in 95.7 per cent of the cases; a rather good empirical rule.[46] An even rougher version of this rule would be: NA_N large, the month will become hollow, while it will become full if NA_N is small.

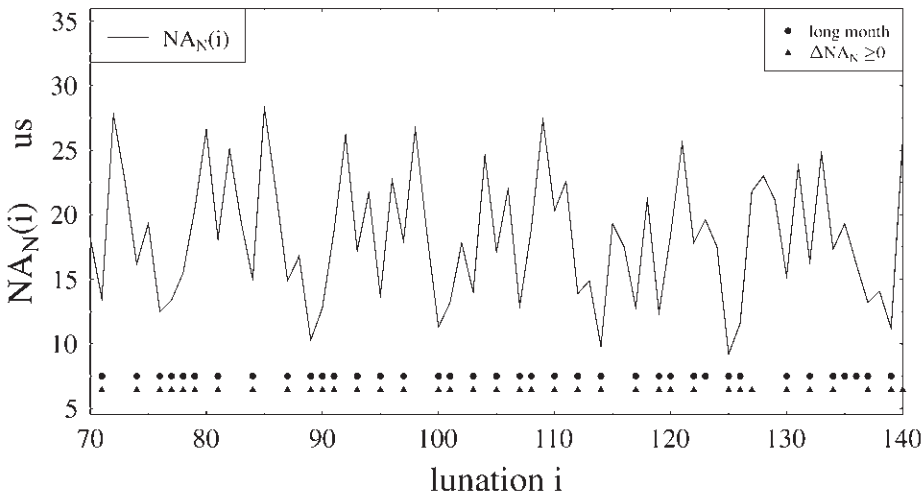

Figure 4

The same as in Figure 3 - here for the lunations $i = 70, 71, \ldots, 140$. We point at the anomalies at the lunations $i = 123, 127, 135, \text{and} 136$.

VII.3 How to Predict the Month's Length with Babylonian Tools

Our rule R in combination with precise predictions of consecutive NA_N would enable us to predict the length of consecutive Babylonian months. Indeed, the Babylonians did have the elegant and precise Goal-Year method for predicting the Lunar Six. A presumed Goal-Year tablet for year Y contains all information needed for establishing all Lunar Six of year Y. In particular, the observable NA_N (the time between the settings of sun and the new crescent) can be calculated for each month in year Y by means of formula (V.1) and (V.1c) given in Chapter V:

$$(\mathrm{NA}_N)_i = \begin{cases} (\mathrm{NA}_N)_{i-223} - 1/3\,(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA})_{i-229} & \text{if} \geq 10\text{UŠ} \qquad (\text{V.1 }) \\ (\mathrm{NA}_N)_{i-223} + 2/3\,(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA})_{i-229} & \text{otherwise} \qquad (\text{V.1c}) \end{cases}$$

[46] In order to get a realistic estimate of the validity of our rule, we must check it over a period of time after which all the parameters crucial for new crescent repeat. 669 synodic months is such a time period, it equals quite exactly a whole number of draconitic and anomalistic months. As P. Huber has shown [1982, p.25], the sequences of full and hollow months have the highest tendency to repeat after 3 Saroi.

We can now assume to know the value of NA_N for month I through XII of year Y. Then rule R can be used to predict the duration of month I through XI. (In order to know if month XII will become full or hollow, of course we also need to know NA_N of the first month in year $Y+1$.)

In principle the Babylonian astronomers could have worked in the same way, but it seems to us that they did not. As we shall see, all the practices on TU 11 utilize quantities which will occur *before* the month in order to predict its length. The method sketched above uses the NA_N of the month in question together with NA_N of the *next* month. In the following paragraphs VII.4 – VII.11, all the methods collected on TU 11 for the prediction of the month's length shall be presented.

VII.4 One Babylonian Method, TU 11, Section 14.

Let us quote the relevant Section 14, obv. 29-33:

> [**29)**] *In order for you to see a hollow or full (month). If in the 18(th year preceding) month I (begins on) the 1st (day), and an addition is not added to it, month II, which is after it, is full. One-third* [**30)**] *of ŠÚ+NA is 6: you subtract this(?) from NA of the 1st day of month II, and (if) it is less [sic!] than in month I, which is before it, then month II of your new year is full.* [**31)**] *Whatever (month) in your 18(th year preceding) is full, and to which there is no addition added, and a subtraction is subtracted from it, and which is* [**32)**] *less than the month preceding it, you declare as full. If in your 18(th year preceding) (a month) is full, and an addition is not added to it, and a subtraction* [**33)**] *is subtracted from it, and it is less than 10 UŠ, you declare (your month) as full. If in your 18(th year preceding) (a month) is full, and an addition is added to it, you declare (your month) as hollow.*

In this text, a method is first demonstrated by the example of month II, then the method is formulated generally in the last two lines of the Section. As far as we understand the text, the quantity NA_N for some month of the new year is calculated according to the Goal-Year method. The lines obv. 29 and 30 confirm for us that the procedure expressed in Equation (V.1) really was in use by the Babylonians of the Seleucid era. The concern in these lines is to predict whether a month has 29 or 30 days. We shall, however, first concentrate on the calculation of NA_N and afterwards look at the method for predicting the month's length. The 18-year period (*one* Saros) is mentioned, and so is NA_N of a month II, and one–third of ŠÚ+NA is said to be 6. At spring time (month II), ŠÚ+NA always is minimum, ranging between 8 and 10 UŠ; but here is ŠÚ+NA indirectly said to be 18. We hence know that this value must stem from fall time, where ŠÚ+NA assumes its maximum value.[47] The

[47] A parallel passage in section 16 does at the corresponding place explicitly say "return 6 months".

whole text is consistent, and that brief remark makes sense if we read it as follows: "Subtract one–third of ŠÚ+NA (six months earlier than II) from NA_N (of month II)". But this is exactly the rule contained in Equation (V.1).

For the determination of the length of a month in the new year, the text is among other criteria concerned with the question if an addition is needed or not in the procedure for finding NA_N. (This is best seen in the last part of the text (*If in your 18...*), where the rule is summarized.) We must, however, propose a little correction to the text: where it writes "less than 10 UŠ" we correct it to "more than 10 UŠ". Otherwise, the procedure is not consistent.[48] As we have seen in the discussion in Chapter V and VII.3 above, the visibility limit of NA_N was taken to be 10 UŠ. This means, that a result NA_N found by a subtraction is only accepted if it is greater than 10 UŠ. Should it happen to become too small, i.e., smaller than 10 UŠ, the new crescent will only become visible a day later, and the value of NA_N must be corrected by an addition. The month length is clearly connected to the question of whether an addition is necessary or not. With this correction we render in modern terms the rule mentioned in the two last lines (32 and 33 of Section 14) as follows:

If the "old month" $(i-223)$ is full, and if $\mathrm{NA}_N(i)$ is found by a subtraction alone (i.e., if $\mathrm{NA}_N(i)$, predicted by means of formula (V.1), is larger than 10 UŠ), then the new month(i) will be full. Is the predicted $\mathrm{NA}_N(i)$ smaller than 10 UŠ, an addition is needed (formula V.1c) and month (i) will become hollow.

As we shall see below, this rule works in 95 per cent of all cases - it is almost as good as our rule R. (Of course, there is a connection between these rules: a NA_N found by formula (V.1c) will be large, normally larger than the NA_N of the next month, so that rule R would in such a case also predict a hollow month.) Below we have analyzed computer simulated lunar data and found a connection between the length of the Babylonian month and the necessity of an addition. The advices given in the text have led us to (re)discover an elegant way of determining the month's length by means of the Goal-Year method. Therefore we are confident, that our correction of ''less" to "more" is justified, and that we have understood the rule alluded to in Section 14.

It can be shown (see below, VII.4b), that a similar rule does exist for the case where the old month is hollow. If month$(i-223)$ is hollow, then a correcting addition is never needed for finding $\mathrm{NA}_N(i)$; formula (V.1) does always apply. In this case, the crucial question is whether an addition is needed for calculating $\mathrm{NA}_N(i+1)$. The procedure used for calculating $\mathrm{NA}_N(i+1)$ decides the length of month i. TU 11 has however no traces of this rule. The empirical basis of these rules shall be presented below. The length of the "old" month is crucial, it decides which procedure to apply. Therefore, in the following analyses, we give the old month the index L, and change the formula correspondingly.

[48] There are several other obvious errors in TU 11.

The Goal-Year method for predicting NA_N, formulated differently:

We will formulate the Goal-Year rule slightly differently and use another index for our lunations: the elements which determine the length of a month(i) all stem from "old" lunations, namely from the quantity NA_N measured in month($i-223$) i.e., 1 Saros earlier than month(i), and from the sum ŠÚ+NA measured 6 months further back. Each prediction starts out with an old month which we will now call month (L). Hence, the "new month" whose length is to be predicted will become month ($L+223$). Using this notation, NA_N found by the "Goal-Year" method is given in equations (VII) and (VIIc).

$$(\mathrm{NA}_N)_{L+223} = (\mathrm{NA}_N)_L - 1/3\,(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA})_{L-6}. \tag{VII}$$

The Babylonian rule says: if L is full and a subtraction but no addition has taken place, then the new month $(L+223)$ will be full.

If $\mathrm{NA}_N(L+223)$ becomes smaller than 10 UŠ,[49] then month $(L+223)$ will begin 1 day later. The preliminary value of $\mathrm{NA}_N(L+223)$ shall be corrected by the addition of (ŠÚ+NA)$(L-6)$ to (VII), resulting in (VIIc).

$$(\mathrm{NA}_N)_{L+223} = (\mathrm{NA}_N)_L + 2/3\,(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA})_{L-6}. \tag{VIIc}$$

In words close to the text: if an addition was necessary, month $L+223$ will be hollow. We see, the crucial question is whether NA_N of the new year was found with or without an addition.

Inspired by the text, and in order to understand it, we have constructed figures which can help us to check and understand this procedure. In the following figures 5 to 7, the value NA_N at lunation L is compared with its value at lunation $L+223$: the curve introduced in figure 3 is overlayed with the curve of the lunations to occur one Saros later. A dot at a lunation L indicates a full month L while a star indicates that month $(L+223)$ is full. The study of these figures leads us to understand the connection between the behavior of the quantity NA_N, its prediction by means of the Goal-Year method, and the month length: From Figure 5 it can be decided which of the procedures (VII) or (VIIc) has to be used for finding $(\mathrm{NA}_N)_{L+223}$. And in Figure 6 this knowledge is combined with indications of full and hollow months.

Addition Needed or not for the Prediction of NA_N?

Comments on how Figure 5 can inform us, if an addition was needed by the "Goal-Year" prediction of NA_N(L+223): For the "old" lunations $L = 0, 1, 2, ..., 57$, the observable $\mathrm{NA}_N(L)$ has been drawn as a function of L (points connected by unbroken lines). The values of NA_N one Saros later, $\mathrm{NA}_N(L+223)$, are drawn into the same

[49] We remind the reader that this visibility criterion is mentioned in TU 11 obv. 37.

figure (points connected with a dashed line).[50] At those lunations L where the new (dashed) curve is situated under the old curve, $\mathrm{NA}_N(L+223)$ will be found from $\mathrm{NA}_N(L)$ through a subtraction, rule (VII) applies. In the cases where a correcting addition is necessary, rule (VIIc), the new curve is above the old curve. Of course, $\mathrm{NA}_N(L+223)$ in this figure is calculated directly, and hence represents "observed" quantities. However, since the Goal-Year method is known to be very accurate, we can also interpret the computer simulated data $\mathrm{NA}_N(L+223)$ as being predicted by means of the Goal-Year method. In Brack-Bernsen [1997, p. 129] the Goal-Year method for the prediction of NA_N was controlled numerically: There, Figure 15.2 shows, that there is almost no difference between an "observed" value of NA_N and its value found by the Goal-Year method.

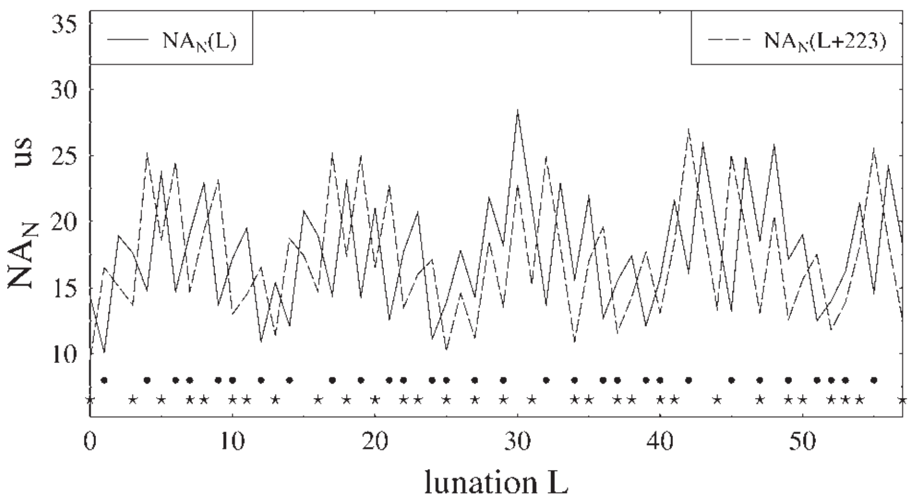

Figure 5

The time NA_N from sunset to setting of the new crescent is plotted by a solid line as a function of the lunation number L. Underneath, a black circle • indicates which months L are full. In the same Figure, the dashed line gives this time interval, $\mathrm{NA}_N(L+223)$, for the months $(L+223)$ to come 1 Saros later. A star $\star$ indicates that month $(L+223)$ is full. (In this figure Month 0 began on JD 1444411.)

From figure 5, we have found the lunations L for which no addition takes place, i.e., where rule (VII) is used: $L = 0$, 2, 3, 5, 7, 8, 10, 11, 13, 15, 16, 18, 20, 22, 23, 25, 26, 27, 28, 29, 30, 31,.... For $L = 1$, 4, 6, 9, 12, 14, 17, 19, 21, 24, 32, ... an addition is necessary, rule (VIIc) applies.

This information can now be combined with the information on the month's

[50] Again all lunar data have been calculated by means of P. Huber's computer code.

length: the black circle • indicates that month L is full, and the star $\star$ indicates that month $L + 223$ is full. We can now examine the correctness of the Babylonian rule through figures.

VII.4a Investigation for the Cases Where the "Old" Month is Full

The Babylonian rule starts out with a full old lunation, L. We are therefore presently only interested in such cases and investigate the mutual behavior of curves $\mathrm{NA}_N(L)$ and $\mathrm{NA}_N(L + 223)$ at the full lunations L.

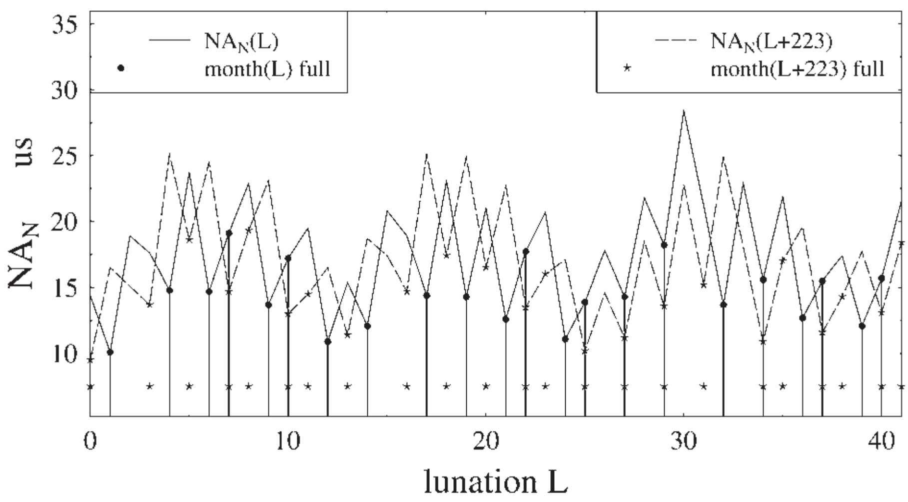

Figure 6

As in Figure 5. In addition to the black circles, here vertical lines too mark the full lunations L, while full lunations $(L + 223)$ are marked by stars.

In Figure 6 we have marked by vertical lines all the lunations for which L is full. The black circles also indicating L to be a full month have been moved up to the curve. A star at lunation number L will indicate that lunation $L + 223$ is also full.

In Figure 6 we see that along all vertical lines, which are marked with a star, the new dashed curve is situated underneath the old solid curve. Hence, for all lunations where L as well as $L+223$ are full, $\mathrm{NA}_N(L+223)$ will have to be predicted without an addition, i.e., by means of rule (VII).

The other case: In our figure, along all the vertical lines which have no star, the old solid curve is situated underneath the new dashed curve. For these lunations $\mathrm{NA}_N(L + 223)$ is larger than $\mathrm{NA}_N(L)$. This is in accordance with the Babylonian rule: all these $\mathrm{NA}_N(L + 223)$ predicted by means of the Goal-Year method would have to use formula (VIIc), and the new months $(L + 223)$ are all hollow.

We conclude: For all lunations (L from 0 to 56) depicted in Figure 6 the Babylonian rule works, the full lunations L for which $L + 223$ is hollow are exactly those for which an addition was needed. We are now confident that the text really talks about the following **"rule Full"**:

$$\text{month(L) full}\begin{cases} \text{NA}_N(L+223) \text{ found by rule (VII):} & \text{month}(L+223) \text{ full} \\ \text{NA}_N(L+223) \text{ found by rule (VIIc):} & \text{month}(L+223) \text{ hollow} \end{cases}$$

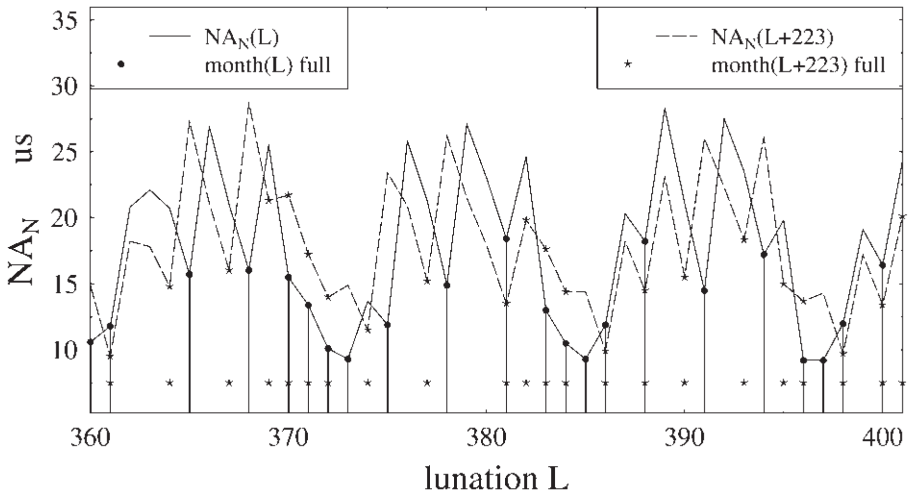

Figure 7

As in Figure 6 but now for the lunations $L = 360, 361, \ldots, 400$.

In a similar way, by means of figures and controlled by the computer program, all lunations L during three Saroi (669 months = 54 Years) have been studied together with the lunation $(L + 223)$ to occur one Saros later. Only 19 exceptions from our rule were found. At the lunations L = 89, 275, 287, 288, 300, 335, 358, 359, 370, 371, 372, 383, 384, 396, 457, 469, 570, 571, and 581, the old month was full and so was the new month, in spite of the fact that there had been an addition.[51] It should be remarked that in all these exceptional cases, one and the same anomaly occurs:

[51]For his calculations, P. Huber has used Schoch's astronomical criteria for the visibility of the new crescent - and not just the 10 UŠ as mentioned in our text TU 11. According to Schoch's visibility criteria, month 580 is hollow while month 581 will have 31 days. It is reasonable to put both months 580 and 581 equal to 30 days, i.e., to assume both to be full. This corresponds to observing $\text{NA}_N(581)$ 1 day too late resulting in a larger value of $\text{NA}_N(581)$. As a consequence, lunation 357 (= 580 - 223) will represent a new exception from our rule - while the exception for L = 581 will disappear. Again there are 19 exceptions from "rule full". With these corrections, there are 356 full months during 3 Saroi, the Babylonian "rule Full" is hence correct in 94.7 per cent of the cases.

additions on two consecutive months, namely L and $L+1$, were needed in all cases. (In most cases, if an addition is needed by a month, for the next month no addition is needed.) Figure 7 shows the exceptional behavior of the two curves at lunation L = 370, 371, 372, 383, and 384. For each of these months, an addition is needed for the prediction of $\mathrm{NA}_N(L+223)$; but against the "rule Full" these months $(L+223)$ are all full. In all cases, an addition is also needed for the next month.

A similar control based on data computed by Moshier's program, using the visibility criterion 10 UŠ, also comfirms the "rule Full" to within 96 per cent. The text says nothing about the case where lunation L is hollow, but for the sake of completeness we shall investigate this case, too.

VII.4b Detailed Study for the Case Month L Being Hollow

Figure 8, analogous to Figure 6, has been studied; now concentrating on the hollow lunations L. The text says nothing about this case. However, inspired by the rule above, and using the same methods as just described, we have investigated if the necessity of an addition when predicting the NA_N of the new year had any connection to the length of the month $L+223$.

The result of this investigation is shown in Table 1. All hollow lunations L

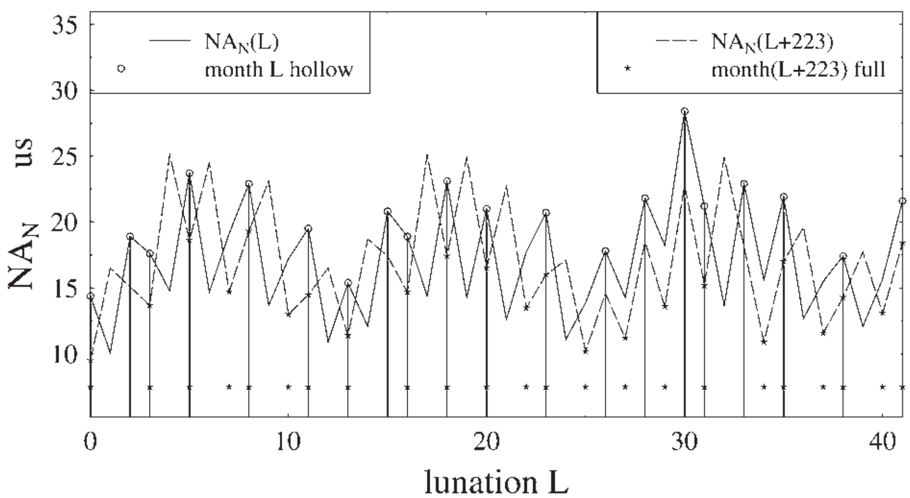

Figure 8

As in Figure 6, the quantity NA_N as well as its value one Saros later has been printed as a function of the lunation number L. Here, the vertical lines and hollow circles indicate that month L is hollow - while a star $\star$ marks all lunations L for which $L+223$ is full. (Again lunation L starts on JD 1444411; our lunation numbers L are just by 3000 smaller than the numbers in Goldstine's tables.)

Month L hollow				
month$(L+223)$ full	month$(L+223)$ hollow	addition by month$(L+222)$	addition by month$(L+223)$	addition by month$(L+224)$
	2	$+$	$-$	$-$
3		$-$	$-$	$+$
5		$+$	$-$	$+$
8		$-$	$-$	$+$
11		$-$	$-$	$+$
13		$+$	$-$	$+$
	15	$+$	$-$	$-$
16		$-$	$-$	$+$
18		$+$	$-$	$+$
20		$+$	$-$	$+$
23		$-$	$-$	$+$
	26	$-$	$-$	$-$
	28	$-$	$-$	$-$
	30	$-$	$-$	$-$
31		$-$	$-$	$+$
	33	$+$	$-$	$-$
35		$-$	$-$	$+$
38		$-$	$-$	$+$
41		$-$	$-$	$+$
	43	$+$	$-$	$-$
44		$-$	$-$	$+$
	46	$+$	$-$	$-$
	48	$-$	$-$	$-$
50		$-$	$-$	$+$
54		$-$	$-$	$+$
	56	$+$	$-$	$-$
57		$-$	$-$	$+$

Table 1

between 0 and 57 have been investigated. The table lists these hollow lunations L together with information on the lengths of the months $L + 223$. The numbers L for which $L + 223$ is full are listed in the first column. The next column lists the numbers L for which month$(L + 223)$ is hollow. The following three columns 3 – 5 indicate if a correcting addition was necessary by the prediction of NA_N for the three consecutive months $L + 222$, $L + 223$, and $L + 224$.

Cases for which no addition is needed, are marked by a '$-$'. A necessary addition is marked by a '+'. (For instance: Line one of our table is concerned with (the hollow) lunation $L = 2$. It informs us that $L + 223(= 225)$ was hollow and that for the calculation of $\mathrm{NA}_N(224)$ an addition was needed, while NA_N in month (225) and (226) is found without additions.)

Column 4 demonstrates that in case of L being hollow, the quantity NA_N one Saros later is always found by procedure (VII); an addition is never needed. The length of that month cannot be predicted on the basis of procedures for calculating $\mathrm{NA}_N(L+223)$, neither can it be predicted from procedures for finding $\mathrm{NA}_N(L+222)$. There is no correlation between the '+' and '$-$' in column 3 and the length of month $(L+223)$. However, in column 5 the '+' occur exactly at those lunations L for which month $(L+223)$ is full, and '$-$' occurs at those lunations for which month $(L+223)$ is hollow. Crucial for determining the length of a month coming 1 Saros after a hollow month is apparently the question if an addition is needed or not for the prediction of $\mathrm{NA}_N(L + 224)$. Let us formulate the rule for determining the month's length for lunations coming 1 Saros after a hollow month: **"Rule Hollow"**

$$\text{month } (L) \text{ hollow } \begin{cases} \mathrm{NA}_N(L+224) \text{ found by rule (VII): month } (L+223) \text{ hollow} \\ \mathrm{NA}_N(L+224) \text{ found by rule (VIIc): month } (L+223) \text{ full} \end{cases}$$

We remind the reader that this rule has not yet been found in cuneiform texts. It will be of great interest to look for traces of it. Until now, all the predictions made in Babylonian texts known so far, use quantities which occur before the event for which the prediction is made. This is also true for the predictions made for the lengths of all months coming 1 Saros after a full lunation. Here, however, the predicted value of the quantity NA_N to be measured at the beginning of the next month $L + 224$ is used for finding the length of month $L + 223$. If this rule were found, we wonder why the simplest rule R was not found. This rule just compares the predicted values of $\mathrm{NA}_N(L+223)$ and $\mathrm{NA}_N(L+224)$ for determining the length of month $(L + 223)$.

In any case, this second rule (Hollow) is closely connected with our old rule R: It is seldom that an addition is needed for the prediction of two consecutive months. Therefore, generally, if an addition is needed for finding $\mathrm{NA}_N(L + 224)$, then none is needed for finding $\mathrm{NA}_N(L + 223)$ which, consequently, will become smaller than the $\mathrm{NA}_N(L+224)$ of the next month. Hence, according to our very first rule, month $L + 223$ will be full.

The "Rule Hollow" has been checked through 3 Saroi, i.e., for 669 lunations, out of which 313 were hollow. It turns out that there are only 4 exceptions from our rule. This means that the rule is very accurate — it works in 98.6 per cent of all cases. An astonishingly good empirical rule.

Concluding Remarks

Let us combine the two rules, using words closer to the cuneiform texts:

$$\text{month(L) is full} \begin{cases} \mathrm{NA}_N(L+223) \text{ found by subtraction: month}(L+223) \text{ full} \\ \mathrm{NA}_N(L+223) \text{ an addition is needed: month}(L+223) \text{ hollow} \end{cases}$$

$$\text{month(L) hollow} \begin{cases} \mathrm{NA}_N(L+224) \text{ found by subtraction: month}(L+223) \text{ hollow} \\ \mathrm{NA}_N(L+224) \text{ an addition is necessary: month}(L+223) \text{ full} \end{cases}$$

Out of curiosity, we have analysed the circumstances in connection with full lunations L once more. Now in the same way as done in Paragraph VII.4b for hollow "old" lunations L. The result of investigating the first 30 full old lunations L is shown in Table 2: Column 4 demonstrates once more the well–known rule: '$-$' (no addition necessary for finding $\mathrm{NA}_N(L+223)$) occur exactly at those lunations L for which month $(L+223)$ is full, and '+' at those lunations for which month $(L+223)$ is hollow. The procedures for predicting NA_N of month $(L+222)$ or $(L+224)$ have no influence: as we can see from Table 2, there is no connection between the length of month$(L+223)$ and the '+' and '$-$' in column 3 and 5, respectively. The crucial question for determining the length of a month coming 1 Saros after a full month is, apparently, whether an addition is needed or not for the prediction of $\mathrm{NA}_N(L+223)$.

The first rule ("full") workes in 337 out of 356 cases, while the second ("hollow") works in 309 out of 313 cases. The combined use of these two rules is hence an easy and trustworthy method for predicting the length of coming months. It will have only 19+4 = 23 exceptions over 669 months = 3 Saroi and therefore deliver the right answer in 97 per cent of all cases. — And this is a better predicting rule than our "old" rule R. Once again, the instructions on ancient cuneiform tablets have led us to find unknown but good and useful empirical rules.

We want to point at the fact that many different figures helped us to understand the connection between month length and predicting procedures for finding NA_N — and for demonstrating the rules. Of course, figures are our tool; the Babylonians did not argue using figures. They must have found their rules by other means.

Month L full				
month$(L+223)$ full	month$(L+223)$ hollow	addition by month$(L+222)$	addition by month$(L+223)$	addition by month$(L+224)$
	1	−	+	−
	4	−	+	−
	6	−	+	−
7		+	−	−
	9	−	+	−
10		+	−	−
	12	−	+	−
	14	−	+	−
	17	−	+	−
	19	−	+	−
	21	−	+	−
22		+	−	−
	24	−	+	−
25		+	−	−
27		−	−	−
29		−	−	−
	32	−	+	−
34		−	−	−
	36	−	+	−
37		+	−	−
	39	−	+	−
40		+	−	−
	42	−	+	−
	45	−	+	−
47		−	−	−
49		−	−	−
	51	−	+	−
52		+	−	−
53		−	−	−
	55	−	+	−

Table 2

VII.5 Comments on Section 15 (TU 11 obv. 34 – 35)

> *If the Moon is high to the Sun, hollow; the days are fifteenth days; if the Moon is low to the Sun, full; the days are thirteenth days; if the Moon takes a high (position), fifteenth and sixteenth days; if it takes <a low (position)>, twelfth and thirteenth days. Month I .... If the Moon takes a path of height, three hollow ones; if it takes a path of depth, three full ones. From month I on the first days are high, the fourteenth days are low; from month VII on, the first days are low, the fourteenth days are high.*

In this section, "high" and "low" are used for two purposes: to foretell the length of lunar months, and to determine some day in the middle of the month. We shall concentrate on the first part of the rule and try to find a connection between the moon being high or low and the month's length.

The first question to ask of the text is: what do high (NIM) and low (SIG) to the sun mean in this connection? According to Neugebauer's Vocabulary (ACT II, p.485), in the ACT astronomical texts, NIM and SIG refer to high and low lunar latitude.

Over a period of 47 synodic months $\simeq$ 51 draconitic months, the situation of the new crescent, which indicates the first evening of the Babylonian month, has been checked: At the moment of sunset, the latitude of the moon $\beta_{\leftmoon}$ as well as its altitude $h_{\leftmoon}$ above the western horizon have been determined.[52] It turns out that the lunar latitude shows no correlation to the duration of the Babylonian month, but the lunar altitude does. Therefore, if the rule quoted here is worth anything, NIM and SIG must mean that the new moon is high or low, respectively, over the horizon at sunset.

This is shown as follows: in table 3, for the lunations $L = 0$ through 46, we have collected data of the hollow months (in the left columns), while the corresponding data for the full months are collected in the right columns. Of the 47 consecutive Babylonian months concerned, 23 were hollow and 24 full. Columns 2 and 4, respectively, give the altitude $h_{\leftmoon}$ of the new crescent. Column 3 and 5 give the latitude $\beta_{\leftmoon}$ of the new crescent. The latitude of the new crescent $\beta_{\leftmoon}$ shows a variation of 10.04°, between 5.02° and -5.02°, while the altitude $h_{\leftmoon}$ varies between 20.7° and 9.2°. Their total variation of 11.5° is a bit larger than the variation of the latitudes. Apparently, the lunar latitude, $\beta_{\leftmoon}$, exhibits its whole variation in both hollow and full months. However the altitude in hollow months tends to be larger than in full months.

[52] These data have been calculated with Moshier's computer code.

Lunation	Hollow months		Full months	
number L	altitude $h_☾$	latitude $\beta_☾$	altitude $h_☾$	latitude $\beta_☾$
0	12.6	− 2.85		
1			9.2	− 0.86
2	15.9	+ 2.58		
3	14.3	+ 4.13		
4			11.8	+ 4.93
5	17.7	+ 4.44		
6			11.9	+ 3.14
7			15.9	+ 0.19
8	19.5	− 2.74		
9			11.9	− 4.08
10			14.4	− 5.02
11	16.2	− 4.16		
12			9.5	− 2.74
13	13.6	+ 0.38		
14			10.9	+ 2.38
15	17.0	+ 4.68		
16	14.8	+ 5.02		
17			11.3	+ 4.46
18	17.5	+ 1.90		
19			11.9	− 0.12
20	17.8	− 3.18		
21			11.2	− 4.40
22			14.9	− 4.99
23	17.0	− 3.77		
24			9.5	− 2.26
25			12.2	+ 0.7

Table 3, first half.

Lunation number L	Hollow months altitude $h_☾$	Hollow months latitude $\beta_☾$	Full months altitude $h_☾$	Full months latitude $\beta_☾$
26	15.6	+ 3.46		
27			12.6	+ 4.61
28	17.0	+ 4.87		
29			13.8	+ 3.86
30	20.3	+ 0.84		
31	16.5	– 1.33		
32			11.7	– 3.22
33	19.5	– 4.95		
34			13.4	– 4.92
35	17.8	– 3.35		
36			10.6	– 1.68
37			13.1	+ 1.28
38	15.5	+ 3.79		
39			11.0	+ 4.73
40			13.2	+ 4.82
41	16.1	+ 2.99		
42			12.0	+ 1.06
43	19.1	– 2.37		
44	15.7	– 4.03		
45			11.6	– 4.92
46	20.7	– 4.43		
Mean values over these 47 lunations	Hollow months $< h_☾ >$	Hollow months $< \beta_☾ >$	Full months $< h_☾ >$	Full months $< \beta_☾ >$
	16.86	+ 0.08	12.05	– 0.12

Table 3, second half.

Therefore we look at the mean values $\langle\ ..\ \rangle$ of latitude and altitude, respectively, taken over the full months and over the hollow months. We find a significant difference:

$$\text{Mean values of Lunar latitude } \beta_☾ \text{ at new crescent:} \quad \begin{cases} \langle\ \beta_☾\ (\text{hollow})\ \rangle = \ \ 0.08^\circ \\ \langle\ \beta_☾\ (\text{full})\ \rangle \quad = -\ 0.12^\circ \end{cases}$$

$$\text{The difference: } \langle \beta_☾(\text{hollow})\rangle - \langle \beta_☾(\text{full})\rangle \simeq 0.2^\circ.$$

$$\text{Mean values of Lunar altitude } h_☾ \text{ at new crescent:} \quad \begin{cases} \langle\ h_☾\ (\text{hollow})\ \rangle = 16.86\ ^\circ \\ \langle\ h_☾\ (\text{full})\ \rangle \quad = 12.05\ ^\circ \end{cases}$$

$$\text{Difference: } \langle h_☾(\text{hollow})\rangle - \langle h_☾(\text{full})\rangle = 4.8^\circ$$

We conclude: the difference of 0.2° is negligible in comparison to the total variation of $\beta_☾$, of 10°. However, for the altitude we find the significant difference 4.8° between the mean values, the total variation of $h_☾$ being 11.5°. We see that the lunar latitude does not determine the length of Babylonian months. The situation is even more pronounced when we take the mean values of all full and hollow months within a Saros.

Mean values taken over all hollow and full months within a Saros:

$$\text{Lunar latitude } \beta_☾ \text{ at new crescent:} \quad \begin{cases} \langle\ \beta_☾\ (\text{hollow})\rangle = \ \ 0.01^\circ \\ \langle\ \beta_☾\ (\text{full})\rangle \quad = -\ 0.02^\circ \end{cases}$$

$$\text{Lunar altitude } h_☾ \text{ at new crescent:} \quad \begin{cases} \langle\ h_☾\ (\text{hollow})\rangle = 17.2\ ^\circ \\ \langle\ h_☾\ (\text{full})\rangle \quad = 12.0\ ^\circ \end{cases}$$

For these data covering a Saros, difference of mean values:

$$\langle \beta_☾(\text{hollow})\rangle - \langle \beta_☾(\text{full})\rangle = 0.01^\circ \simeq 0$$
$$\langle h_☾(\text{hollow})\rangle - \langle h_☾(\text{full})\rangle = 5.2^\circ$$

We must conclude: the latitude of the moon cannot be used for the prediction of the month's length; the altitude above the horizon at new crescent, however, can be useful for that purpose. In our opinion, this is what the text was concerned with.

If our interpretation is correct, we have here a primitive rule for predicting the month's length. Being simple and rather good, a rule of this kind is exactly what one would expect to be the first thing to be found by early astronomers.

The Day Numbers

The text combines the prediction of the length of the month with another indication: *...; the days are fifteenth (respectively thirteenth) days.*

Presumably the day number indicates when NA is expected to occur within the Babylonian month. We base this interpretation on the fact that the days fifteen and thirteen are in the middle of the month. Therefore, for the phenomenon referred to, one can think of the Lunar Four or opposition. But ME and GE_6 are measured at night and thus excluded.[53] In the Reports[54], the day on which the moon was seen with the sun (which is the day of NA) was of special interest. In the "ideal case" it occurs on day 14 (in some texts, NA is identified as the NA of the 14th day); 13 and 15 having 14 in the middle, makes us also think of NA rather than of ŠÚ.

In the Reports, it seems as if the day on which moon and sun were seen together was used to foretell the month's length: if it is seen early, the month will become short, while it will become long when this event occurs late (day 15 or 16). Such a rule is rough and primitive, but a check has shown that it will work in ca. 75 percent of the cases.[55] But the day numbers given in our text are the other way around. A high day number is connected to a short month, while a low day number is connected to a long month. The text is clearly corrupt.

Maybe the text has compressed two rules into one: If the Moon is high to the Sun [at new crescent] hollow; If the Moon is high [at full moon] the days are fifteenth days? In which case high and low [at full moon] could possibly refer to high and low lunar latitude. But since the rule is corrupt, it is hard to tell what is meant at this place.

One part of section 15: *If the Moon takes a path of height, three hollow ones; if it takes a path of depth, three full ones*, reminds us of passages of the Atypical Astronomical Cuneiform Text E.[56] Sections 1 and 4 of this text are clearly concerned with lunar latitude. Just after the statement "If the moon passed by a Normal Star high, or if it passed by low, it will repeat it 19 (or 18) years later", the text continues: *Determine the full and hollow months.* This could give the impression that the lunar latitude was used for determining the month's length. We have not been able to find any connection between the two. But if our text here really is talking about lunar latitude, we would have the strange situation that a cuneiform tablet within

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[53]In the Diaries, the observed values of ME and GE_6 are explicitly recorded under the label "night XY"

[54]H.Hunger [1992].

[55]Beaulieu has come to the same conclusion, 1993, pp. 72–73. He writes that at the Assyrian court, most of the predictions of month length were made only fourteen days in advance, after the astronomers had determined the day of full moon.

[56]BM 41004, published by Neugebauer and Sachs [1967] pp. 200–208.

two lines uses the same words NIM and SIG in two different meanings: namely for high and low altitude, as well as for high and low latitude.

We shall now turn to the last half of obv. 35:

> *... From month I on the first days are high, the fourteenth days are low; from month VII on, the first days are low, the fourteenth days are high.*

We are convinced that here, again, high and low (NIM and SIG) must refer to the altitude of the moon above the horizon, and not to its latitude. Certainly, if the lunar latitude is high at the first day of a month, it will be low on day fourteen, and vice versa; but this does not explain why it is said that the first days from month I on are high, and low from month VII on. The moon can have low as well as high latitude at the beginning of every month.

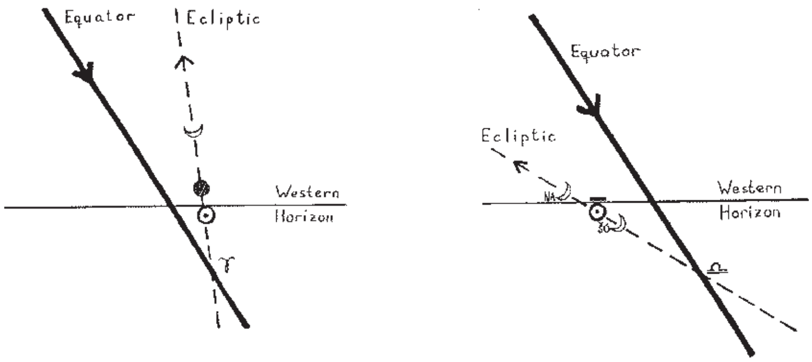

Figure 9

The situation at the western horizon; left: the new crescent announces the beginning of month I; right half: the position of the moon at sunrise on the two consecutive mornings where ŠÚ and NA are observed. For the sake of simplicity we have introduced the symbol $\overline{\odot}$ for the "anti-sun", which we define as the point on the ecliptic situated directly opposite the sun. At the very moment when the sun rises, $\overline{\odot}$ sets, and vice versa. It marks the point of the ecliptic at which the opposition takes place.

The whole sentence gives important and coherent information, if we remember that month I is near spring equinox. Let us imagine the situation at the western horizon (see Figure 9, left half). At sunset on its first day, the ecliptic stays very steep, so that the new crescent will be seen high to the sun. In the middle of that month ("the fourteenth day") the path of the moon will stand flat at sunrise, i.e., at the moment where ŠÚ and NA is observed. The moon will be seen low and south of the sun's setting point, as shown in the right half of Figure 9. We remind the reader that the point of sunset in the middle of month I will be a bit north of straight west; i.e., symmetric to the setting point of $\overline{\odot}$ with respect to the west point.

Month VII always falls near fall equinox, and hence the situation at the western horizon will be reversed (Figure 10): the moon will be seen low to the sun on the first day (NA_N), and around opposition the ecliptic will be steep, so that the moon will be seen high above the horizon (at NA) and set north of the sun's setting point — which, in the middle of month VII, is south of the west point.

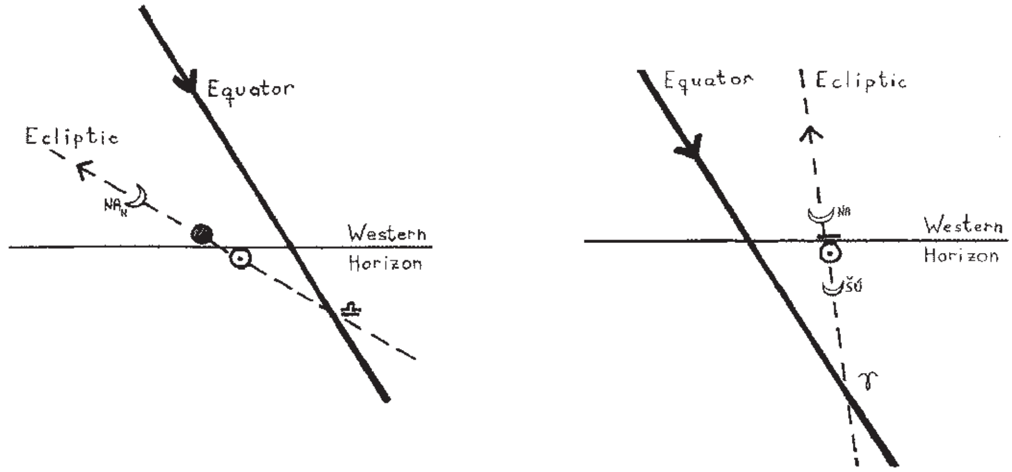

Figure 10

The situation at the western horizon in month VIII; left half on day 1 and right half around opposition, half a month later.

We are convinced that it is exactly these situations which our text describes (admittedly in very short terms). But this is very remarkable, since it is the first time we have found (more or less indirect) textual evidence that the Babylonian astronomers knew about the changing obliquity of the ecliptic at the western horizon.

Before we continue with the more advanced or theoretical rules in the following sections, we shall in a schematic form reproduce these rather simple methods for the determination of the length of a Babylonian month, which we have commented on in this paragraph. We include a third rule, the validity of which is shown below. Expressed in words, the third primitive rule says: If NA_N, the time from sunset to the setting of the first crescent is small, in most cases, the month will become long; is NA_N large, the month will become short. That this is a reasonably good empirical rule shall be illustrated in figure 11. Here for a series of consecutive Babylonian months, the time between setting of the sun and the first crescent is depicted. The full months are marked with a black dot. We note: all minima of the curve have a dot, but none of the maxima have one.

$$\text{“Primitive Rules”:} \quad \left\{ \begin{array}{l} \text{NA, occurs} \left\{ \begin{array}{l} \text{early: month short} \\ \text{late: month long} \end{array} \right. \\[2em] \text{Crescent} \left\{ \begin{array}{l} \text{high to the sun: month short} \\ \text{low to the sun: month long} \end{array} \right. \\[2em] \text{NA}_N \left\{ \begin{array}{l} \text{large: month short} \\ \text{small: month long} \end{array} \right. \end{array} \right.$$

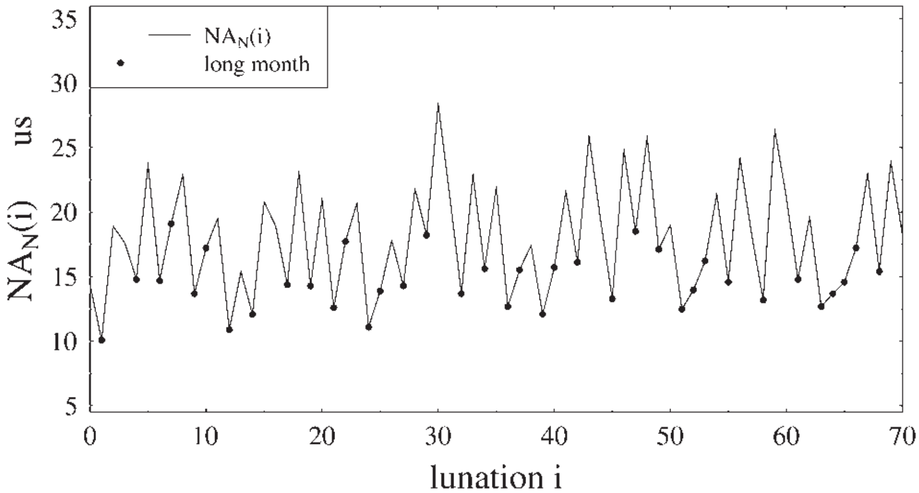

Figure 11

For consecutive months $i = 0, 1, 2, \ldots, 70$, the time NA_N(i) from sunset to the setting of the new crescent is plotted as function of the lunation number i. A circle at a lunation i indicates that month(i) will be long.

Comments to the last rule: The size of NA_N can be used as indicator for the month's length. The sections 17, 18, 19, and 22 all seem to give different methods for finding the value of a NA_N of a month to come, and then depending on its magnitude to predict the length of a month. We therefore point at the following simple feature. The first crescent announces the new month, and by so doing it decides the length of the former month (this is done in Section 19). But the first crescent also contains information on the length of the month just started: the size of NA_N, measured (or calculated) at the beginning of a month, is connected to the length of the current month. As shown in section VII.2, a month tends to become hollow when its first crescent is seen for a longer time than the first crescent of the next month (Sections 17 and 22 seem to utilize this). The simplest (but, of course, not so good) empirical rule is the one mentioned above: If NA_N is small, the month will become long; is NA_N large, the month will become short (and this rule seems to be the basis for prediction in Section 18).

VII.6 Short Remarks on Section 16 (TU 11 obv. 36 – 38)

We have already in Chapter V presented the Goal-Year method for the prediction of the Lunar Six. For completeness, we shall here quote Section 16 and show that its rules are in agreement with the Goal-Year formula from Chapter V.

36) *In order for you to calculate (lit., make) the equivalent for 36 (tablet erroneously: 34) (years). From month I of the 36(th year preceding) you return 6 months, and 0;40 (= two thirds) of ŠÚ+NA of month VII you take, and from NA of the 1st day*
37) *of month I of the 36(th year preceding) you subtract, and if it is less than 10 UŠ, you add ŠÚ+NA entirely.... 0;40 of ŠÚ+NA from NA in the middle of the month*
38) *you subtract. 0;40 of ME+GE$_6$ you subtract from GE$_6$.*

These lines give rules for calculating NA_N (explicitly called "NA of the first day"), NA (identified as "NA in the middle of the month"), and GE_6; the sums ŠÚ+NA and ME+GE_6 are used, and so is a 36-year period plus the coefficient 0;40 = 2/3. The Saros is, as often in cuneiform texts, just called "18". This justifies the reading: "36 [year period] as 2 Saroi = 446 synodic months".

We first concentrate on the comments on the Lunar Four. Having mentioned 2 Saroi, the text goes on and requests the reader in the last third of line 37 and in line 38 to calculate the following difference:

$$\mathrm{NA} - 2/3\,(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA}) \qquad \text{and} \qquad \mathrm{GE}_6 - 2/3\,(\mathrm{ME} + \mathrm{GE}_6).$$

The text does not tell clearly from which month or opposition these values shall be taken. But the passages make sense when we read them as: "In order to find NA one has to go 2 Saroi back and then to subtract $\frac{2}{3}$ of ŠÚ+NA from NA" (both values stemming from the lunation two Saroi earlier than the one, say number i, we are concerned with):

$$\mathrm{NA}_i = \mathrm{NA}_{i-446} - 2/3\,(\check{\mathrm{S}}\acute{\mathrm{U}} + \mathrm{NA})_{i-446},$$

and analogously for GE_6:

$$\mathrm{GE}_i = \mathrm{GE}_{i-446} - 2/3\,(\mathrm{ME} + \mathrm{GE})_{i-446}\ ;$$

but these formulae are simply the equations (V.3) and (V.5) used twice, namely for two consecutive Saroi. We consider this as a strong support for our reading.

Furthermore, it is a clear confirmation that the Babylonians really did know the procedure which we have written in form of the mathematical formulae (V.3) and (V.5).

Lines 36 and 37 are dealing with NA of the first day which we call NA_N. The text, here more clearly formulated than the passages mentioned above, gives a procedure how to estimate (calculate) NA_N for the first month (I) in a year Y.

We understand the text, aiming at finding $(\mathrm{NA}_N)_I$, as follows: "From month I [of your] 36 [year period] (i.e., month $I{-}446$) you go 6 months backwards (to month $I{-}452$) and 0;40 of ŠÚ+NA (i.e., 2/3 of $(\text{ŠÚ+NA})_{I-452}$) you subtract from NA of the first day of the 36 [year period]" (i.e., from $(\mathrm{NA}_N)_{I-446}$).[57]

We write this instruction as an equation:

$$(\mathrm{NA}_N)_I = (\mathrm{NA}_N)_{I-446} - 2/3\,(\text{ŠÚ} + \mathrm{NA})_{I-452}.$$

This formula calculates NA_N from the NA_N observed 2 Saroi earlier and uses for the daily change of NA_N (which we shall call $\Delta\mathrm{NA}_N$) the quantity ŠÚ+NA stemming from a full moon 5 1/2 months further back in time. The daily change of NA_N cannot be determined by observation, since the setting moon is invisible before conjunction. Using ŠÚ+NA for $\Delta\mathrm{NA}_N$ is a very clever and, as we have shown, also a very precise method.

The text continues: "... and if it is less than 10 UŠ, you add the whole ŠÚ+NA to it". The Babylonians knew and utilized that NA_I, when not observable on the first evening after conjunction, would be equal to the expected amount plus the sum ŠÚ+NA observed 6 months earlier. We know, and they evidently knew too, that the tiny new moon cannot be seen if it is too close to the sun and sets less than, say, 10 UŠ = 40 minutes after sunset. The new moon will in that case first be visible on the next evening, and its amount will then be enlarged by $\Delta\mathrm{NA}_N$. The text really tells us to add the whole ŠÚ+NA. The Babylonians evidently used ŠÚ+NA as the daily change of NA_N.

We derived the above formula for the specific case of the first month (I) of a year, as it occurs in the text TU 11. Obviously, the procedure works equally well for an arbitrary month of the year. We shall therefore continue using the general index i for a month.

If we now instead of 2 Saroi only go back by 1 Saros, the formula would be:

$$(\mathrm{NA}_N)_i = (\mathrm{NA}_N)_{i-223} - 1/3\,(\text{ŠÚ} + \mathrm{NA})_{i-229},$$

which is the equation (V.1) for determining NA_N, which we introduced in Chapter V.

[57] Note: the indices i, I and $I{-}452$ refer to the Babylonian months starting on the evening when NA_N is observed. The magnitude of $(\mathrm{NA}_N)_{I-446}$ is measured on the first day of month $I{-}446$, whereas $(\text{ŠÚ+NA})_{I-452}$ is measured 5 1/2 months earlier, namely in the middle of month $I{-}452$.

VII.7 Comments on Section 17 (TU 11 rev. 1 – 4)

You go back six months, and you look at NA before the Sun of months VII and VIII. If NA of month VIII is 6, and NA of month VII is 3, you subtract 3, (by which) the NA of month VII is less than (that of) month VIII, from NA of the first day of month I of your new year; and (if) it exceeds 12 UŠ, you declare the month II, which follows it, as hollow. If NA of month VIII is 3, and NA of month VII is 6, you add 3, (by which) the NA of month VII exceeds that of month VIII, to the NA of the first day of month I of your new year, and if it exceeds the NA of month I of your new year, (the month) is full; if it is less, (the month) is hollow.

The text seems to connect NA_N of month I and II with the quantity of NA, measured around full moon 6 months earlier, in the middle of month VII and VIII. We have tried to find a connection between these quantities, and got two different answers depending upon what we were looking for. Accordingly we shall present two different ways to interpret the text. First we read the text as giving a general rule, to be applied for all months, but demonstrated by month I. (TU 11 does present methods in this way: in Section 14 a general rule is only demonstrated for month I, and Section 20 ends with the remark: For all months it is the same procedure.) We therefore looked in the first place for a rule which would apply for all months, and found none: There is no significant connection between the length of month$(i+1)$ (or month i) and the size of $\mathrm{NA}(i-6)$ compared to $\mathrm{NA}(i-5)$. Our first interpretation is thus not empirically based, but it leaves the text as it is.

Then we investigated if month I and II are special in the sense that their durations are connected to the relative size of NA(VII) and NA(VIII), measured half a year earlier. And, indeed, they are: we found a good empirical rule involving the quantities mentioned in the text. But if the text intended to give that rule, then it mixed up addition and subtraction, and we will have to correct the text accordingly.

First Approach — the Search for a General Rule:

The text tells us to calculate

$$\begin{array}{lll} \mathrm{NA}_N(\mathrm{I}) - (\mathrm{NA}(\mathrm{VIII}) - \mathrm{NA}(\mathrm{VII})) & \text{when} & \mathrm{NA}(\mathrm{VIII}) > \mathrm{NA}(\mathrm{VII}), \text{ and} \\ \mathrm{NA}_N(\mathrm{I}) + (\mathrm{NA}(\mathrm{VII}) - \mathrm{NA}(\mathrm{VIII})) & \text{when} & \mathrm{NA}(\mathrm{VIII}) < \mathrm{NA}(\mathrm{VII}). \end{array}$$

It differentiates between two cases, but we would reproduce the computations given in the text in one formula, knowing that the difference NA(VII) − NA(VIII) might become negative: Section 17 calculates $\mathrm{NA}_N(\mathrm{I})$ + (NA(VII) − NA(VIII)); it does however not say what it calculates, but to us it must be NA_N of month II:

$$\mathrm{NA}_N(\mathrm{II}) = \mathrm{NA}_N(\mathrm{I}) + (\mathrm{NA}(\mathrm{VII}) - \mathrm{NA}(\mathrm{VIII})).$$

The criterion for hollow or full month is connected to the sign of (NA(VII) − NA(VIII)) which decides if a subtraction or an addition is used. The text says: if subtraction is used, the month is hollow; if an addition is used, the month is full. And accordingly the last passage summarizes: if the result [NA_N(II)] exceeds NA_N(I), the month is full; if it is less, the month is hollow. We express this rule with our symbols:

If $\mathrm{NA}_N(\mathrm{II}) < \mathrm{NA}_N(\mathrm{I})$ [i.e., when (NA(VIII) > NA(VII)] then the month is hollow.
If $\mathrm{NA}_N(\mathrm{II}) > \mathrm{NA}_N(\mathrm{I})$ [i.e., when (NA(VIII) < NA(VII)] then the month is full.

According to our rule R, it must be month I which is predicted to become full or hollow. To summarize: If this understanding of the text is correct, then the procedure is based on something equivalent to the incorrect theoretical assumption that

$$\mathrm{NA}_N(\mathrm{II}) + \mathrm{NA}(\mathrm{VIII}) = \mathrm{NA}_N(\mathrm{I}) + \mathrm{NA}(\mathrm{VII}),$$

and on the correct empirical rule R (if $\mathrm{NA}_N(i) > \mathrm{NA}_N(i+1)$ then month i will be hollow, if $\mathrm{NA}_N(i) < \mathrm{NA}_N(i+1)$ then month i will be full).

The remark in the last half of Rev.2 *"You pronounce the month II, which follows it as hollow",* seems to contradict this understanding. The text might be corrupt here, but it seems more likely that in section 17 the alternative convention (known from the Diaries) was used, i.e., to pronounce the length of a month at the beginning of the next one.

The search for an empirical foundation of this procedure ended without result: Figure 12 compares for consecutive months the behavior of NA_N to that of NA, measured 5 1/2 month earlier. But as far as we can see[58], there is no systematic connection which can be used for predictions as found in this text. This was confirmed by investigating the sign of (NA($i-6$) - NA($i-5$)) in connection with the length of month (i) and ($i+1$); no significant dependence was found. We therefore tend to classify the rule here as based on theoretical rather than empirical arguments. Of course one basic empirical experience is used, namely, that the size of NA_N can be used for determining the length of a month.

[58] We have analyzed a series of figures like Figure 12, but without result.

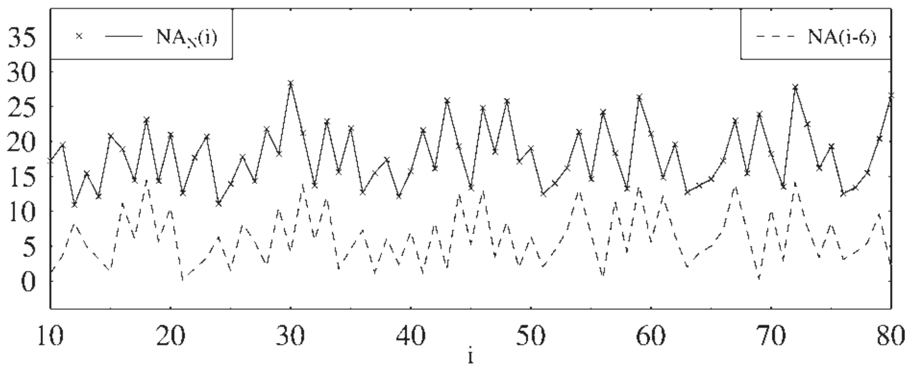

Figure 12

The time NA_N(i) measured in UŠ is plotted by a solid line as function of the lunation number i. Underneath, in the same Figure, the dashed line gives the time interval $\mathrm{NA}(i-6)$ stemming from the middle of month$(i-6)$.

Since we, in this first approach, understand the procedure as presenting a general rule by the example of month I, we repeat its content in a general form:

$$\mathrm{NA}_N(i+1) = \mathrm{NA}_N(i) + (\mathrm{NA}(i-6) - \mathrm{NA}(i-5))$$

If $\mathrm{NA}_N(i+1) < \mathrm{NA}_N(i)$ [i.e., if$(\mathrm{NA}(i-5) > \mathrm{NA}(i-6)$] then month i is hollow.
If $\mathrm{NA}_N(i+1) > \mathrm{NA}_N(i)$ [i.e., when$(\mathrm{NA}(i-5) < \mathrm{NA}(i-6)$] then month i is full.

The (incorrect) theoretical assumption behind this rule could be:

$$\mathrm{NA}_N(i+1) + \mathrm{NA}(i-5) = \mathrm{NA}_N(i) + \mathrm{NA}(i-6)$$

Second Approach — the Search for a Special Rule for Month I.

In this approach we take month I and II in the text to refer just to these special months, and not to be an example for every month. We looked at figures and data once more, now concentrating on month I and II of the Babylonian year:

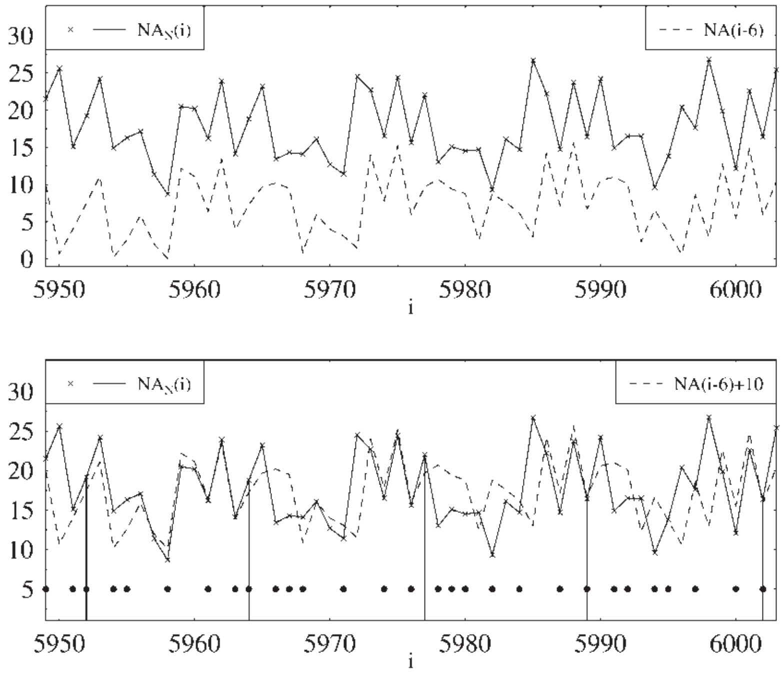

Figure 13

Upper half as Figure 12; at a date for which the connection between the lunation number and the Babylonian month is known. Lower half compares $\mathrm{NA}_N(i)$ with $(\mathrm{NA}(i-6)+10\mathrm{UŠ})$. A black dot indicates that month i is full. Each Babylonian Month I is marked by a vertical line, Lunation 5952 = Darius I, year 2 month I = −519, April 1.

Figure 13 (upper half) compares the magnitude of $\mathrm{NA}_N(i)$ and $\mathrm{NA}(i-6)$ for consecutive lunations i, in the same way as Figure 12. We note, that sometimes the curves are in phase, and sometimes not. For a better comparison, we have added 10UŠ $= k$ to $\mathrm{NA}(i-6)$ and hence compare $\mathrm{NA}_N(i)$ with $(\mathrm{NA}(i-6) + 10\text{UŠ})$ in the lower half of the figure. It turns out that it is always around the same time of the year that the curves have a similar shape. In the lower figure we have marked those lunations which correspond to month I in the Babylonian calendar by a vertical line. Note that around these lines the curves run parallel. Over shorter periods of time around the spring equinox, the two curves even come to cover each other. Having analyzed data over 3 Saroi, we conclude that for the time just before and around spring equinox, in most cases the following approximation is true:

$$\mathrm{NA}_N(i) \simeq \mathrm{NA}(i-6) + k$$

Concretely for month I and II mentioned in our text we have:

$$\mathrm{NA}_N(\mathrm{I}) \simeq NA(VII) + k \quad \text{and} \quad \mathrm{NA}_N(\mathrm{II}) \simeq \mathrm{NA}(\mathrm{VIII}) + k.$$

As a consequence we get: $\mathrm{NA}_N(\mathrm{II}) \simeq \mathrm{NA}_N(\mathrm{I}) + (\mathrm{NA}(\mathrm{VIII}) - \mathrm{NA}(\mathrm{VII}))$

Working Hypothesis:

this connection was found by the Babylonians and constitutes the empirical basis for Section 17. The text however told us to calculate $\mathrm{NA}_N(\mathrm{I}) - (\mathrm{NA}(\mathrm{VIII}) - \mathrm{NA}(\mathrm{VII}))$. It has confused either plus and minus or the inequalities between NA(VII) and NA(VIII). We will have to correct the text; but how?

It is clear that the text tells us to start with $\mathrm{NA}_N(\mathrm{I})$, go 6 months back and compare NA(VIII) with NA(VII) and depending upon the sign of NA(VIII) − NA(VII) to add or subtract their numerical difference, $|\mathrm{NA}(\mathrm{VIII}) - \mathrm{NA}(\mathrm{VII})|$, to or from $\mathrm{NA}_N(\mathrm{I})$. "*If the result exceeds $NA_N(I)$* [i.e., if addition was applied] *the month is full, if it is less* [i.e., if something was subtracted] *the month is hollow*". Since we are convinced that the text wanted to calculate

$$\mathrm{NA}_N(\mathrm{II}) = \mathrm{NA}_N(\mathrm{I}) \pm |\mathrm{NA}(\mathrm{VIII}) - \mathrm{NA}(\mathrm{VII})|,$$

it must be the length of month I which is foretold. That $\mathrm{NA}_N(\mathrm{I}) < \mathrm{NA}_N(\mathrm{II})$ implies month I to become full, and $\mathrm{NA}_N(\mathrm{I}) > \mathrm{NA}(\mathrm{II}))$ predicts a hollow month I, is well known and in complete agreement with our rule R. Where we disagree with the text is in the decision when to add or subtract $|\mathrm{NA}(\mathrm{VIII}) - \mathrm{NA}(\mathrm{VII})|$ from $\mathrm{NA}_N(\mathrm{I})$.

The text should have said something like:

> If $\mathrm{NA}(\mathrm{VIII}) > \mathrm{NA}(\mathrm{VII})$ add the difference to $\mathrm{NA}_N(\mathrm{I})$ and predict month I to be full.
> If $\mathrm{NA}(\mathrm{VII}) < \mathrm{NA}(\mathrm{VIII})$ subtract the difference from $\mathrm{NA}_N(\mathrm{I})$ and predict month I to become hollow.

The empirical base of this rule is quite good: We have examined the relevant data over a period of 54 years (i.e., 3 Saroi) and found that in 85 percent of all cases, the procedure delivers the right month's length as well as a very accurate value for $\mathrm{NA}_N(\mathrm{II})$. The method is closely connected to the surprising fact mentioned above: During a few consecutive months near spring equinox, the visibility time of the new crescent, $\mathrm{NA}_N(i)$, can be found from $\mathrm{NA}(i-6)$, measured at full moon 5 1/2 months earlier, just by adding 10 UŠ. But so far we know, this rule has not (yet) been found in cuneiform texts.

We prefer this second interpretation and propose to correct the text accordingly. A duplicate of Section 17 is found on BM 36782; but only parts of the text are visible and not those parts which tell to add or subtract - so we have no textual support for our choice; except for the fact that we know that the text is corrupt at several other places.

One may ask why does there exist an empirical rule which is only valid for some special months, and how come that the Babylonians found it? One (astronomical) reason for the existence of a special rule for month I: The new crescent of month I and II becomes visible around spring equinox and the quantities $\mathrm{NA}_N(\mathrm{I})$ and $\mathrm{NA}_N(\mathrm{II})$ are measured at sunset. At these evenings, the ecliptic stands very steep at the western horizon. A consequence of such a situation is that the influence of the lunar latitude is very small. (The same is true for NA(VII) and NA(VIII) which are measured around full moon 5 1/2 months earlier.) Concentrating on NA_N of the months XII, I, and II is therefore very clever, since one of the variables, β☾, determining NA_N is practically eliminated.

In answer to the second question: Several tables from MUL.APIN use the Babylonian months as entry (or reference) for different observables (e.g., the day's length or rising stars). This provides evidence of the Babylonian practice of collecting data for each month separately. And that month I, the first month of the new year, had a prominent position among the months, in astronomical as well as in astrological connections, is obvious, and is well known from many early cultures.[59] It is therefore not too surprising that the Babylonians found a rule specially valid for month I.

VII.8 Comments on Section 18 (TU 11 rev. 5 – 7)

> *From month I of your new year you go back 18 (years), and you look at the weight of day and night of month I of your 18(th year preceding). If (in) month I of your 18(th year preceding) on the fifteenth day 6 bēru are daylight and 6 bēru are night, from the 15th to the 20th of month I daylight is 6 bēru 10 UŠ, night is 5* 5/6 *bēru: you subtract 5 UŠ 40 NINDA from the NA of the first day of month II of your 18(th year preceding), and if it exceeds 12 UŠ, (the month) is hollow; if it is less, month II of your new year is full.*

The text is corrupt and the numbers hard to read in this section. One possible way of reading the numbers consistently is as follows: from the 15th to the 30th of month I the daylight (has changed to) 6 bēru 10 UŠ, and the night to 5 2/3 bēru: you subtract 5 UŠ 40 NINDA (= 5 2/3 UŠ) from NA_N of the first day of month II of your "old year", and if it exceeds 12 UŠ, (the month) is hollow; if it is less, month II of your new year is full.

We had to change the reading of 20th (two wedges) to 30th (three wedges). Also we read 2/3 in stead of 5/6.[60] Both corrections are allowed and quite obvious, since in both cases the signs are similar and can easily be confused. The text puts daylight

[59] See also Beaulieu, 1993, p.74.

[60] The sum of day and night must equal 12 bēru: either 6 bēru 10UŠ or 5 5/6 must be corrected.

and night to be equal to 6 bēru on the 15th day of Month I. This is the ideal value also given in MUL.APIN. According to the zigzag function for duration of day and night, also (indirectly) given in MUL.APIN, after 15 days the daylight will have increased by 10 UŠ while the night will be reduced by the same amount, resulting in the values given in the (corrected) text. This accordance can also be demonstrated more directly: One table of MUL.APIN gives the length of the night on special days in the schematic year of 360 days:[61] Tablet II line ii 44 quotes the length of the night on the 15th of month I to be 3 mana, and line ii 45 records 2 5/6 mana to be the night's length at the beginning of month II.

$$\text{Month I, 15th night} = 3 \text{ mana} = 3{,}00 \text{ UŠ} = 6 \text{ bēru},$$

in agreement with our text.

$$\text{Month II, 1st night} = 2\ 5/6 \text{ mana} = 2{,}50 \text{ UŠ} = 5\ 2/3 \text{ bēru};$$

this value is in our text ascribed to the 30th of month I. Again we have agreement. Section 18 seems to quote the night's length at new moon under the entry last (30th) day of a month, in the same way as it is done in Table C of EAE XIV; while MUL.APIN quotes it under the first day of the next month.

We return to the text in Section 18. The next calculation reminds us of the Goal-Year method which finds each of the lunar six by means of their values 1 Saros earlier. In the Goal-Year method, the diminuation of NA_N after a Saros is derived from ŠÚ+NA, measured half a year earlier. Here in Section 18 the Sarosly change of NA_N is related to the night length (a similar practice is known from EAE and MUL.APIN, where the daily change of NA is derived from the length of the night). We understand the text to calculate NA_N of month II of the new year by subtracting 5 UŠ 40 NINDA from the value of NA_N in Month II of the old year (1 Saros back). $\mathrm{NA}_N(\mathrm{II})\ [-18 \text{ years}] - 5;40 \text{ UŠ} = \mathrm{NA}_N(\mathrm{II})$

The size of the resulting NA_N decides the length of month II: if it is larger than 12 UŠ, month II will be hollow, while it will become full in the case of NA_N being smaller than 12 UŠ. This is in agreement with the simple rule: a small NA_N indicates a full month while a large NA_N indicates a hollow month.[62]

[61] See Hunger and Pingree, [1989], pp. 101 – 107.

[62] In our opinion, it is not likely that the 12 UŠ mentioned in Section 18 is to be interpreted as the limit for visibility of the new lunar crescent. This was the case for the 10 UŠ mentioned in the Goal-Year method. Section 14 and 16, commenting on the Goal-Year method, explicitly say that if a NA_N found by subtraction is smaller than 10 UŠ, then one has to wait one day and correct the NA_N by an addition. A similar remark is missing in Section 18. Nor is there here any information on the length of the old month. And this information was vital for the prediction of month's lengths by means of a procedure (addition needed or not). Therefore we are convinced

The amount of 5 UŠ 40 NINDA (5 2/3 UŠ) equals 1/30 times the length 5 2/3 bēru of the night. And hence 5 UŠ 40 NINDA is half the daily retardation of the moon which in MUL.APIN (as well as in EAE) is assumed to be 1/15 of the night.

$$\mathrm{NA}_N(\mathrm{II})[-18\mathrm{years}] - 1/30\mathrm{night} = \mathrm{NAN(II)}$$

$$\mathrm{NA}_N(\mathrm{II})\ [\ -\ 18\ \mathrm{years}] \longrightarrow \mathrm{NA}_N(\mathrm{II}) \quad \begin{cases} \text{if larger that 12: hollow} \\ \text{if smaller than 12: full} \end{cases}$$

We see that the diminuation of NA_N after one Saros in this example is put to half the daily retardation of the moon, while the Goal-Year method uses a third of the moon's daily retardation. If our interpretation of Section 18 is correct, then it finds the diminuation of NA_N per Saros and the daily retardation of the moon by means of an ideal scheme like the one from MUL.APIN (II ii 43 - iii 15). Contrarily, Section 14 uses the sum ŠÚ+NA of real (observed) quantities.

VII.9 Comments on Section 19 (TU 11 rev. 8 – 15)

At this place, instead of repeating the translation from Chapter I, we have decided to reproduce the free and interpretative translation given by Neugebauer [1947, p 42]. This short, although very important, passage was the starting point of our project.

> *"in order for you to find the time of invisibility (of the moon). (A.) If in the month I of your new year (on the) 27th, the last visibility (is) 25°, 3;20 (mana) the daylight, 2;40 (mana) the night, (multiply) 3;20 by 4; (the result is) 13;20. Subtract 13;20 from 25, and 11;40 remains; on the 28th day (the moon) remains 11;40° behind the sun. Subtract 13;20 from 11;40; on the 29th day the moon passed the sun 1;40°. Add 13;20 to 1;40, and 15 (is the result); on the 30th day the moon passed the sun 15°. (B.) If on the 27th the last visibility is 15°, (multiply) 3;20 by 4; (the result is) 13;20. Subtract 13;20 from 15, and 1;40 remains; on the 28th (the moon) remains 1,40° behind the sun, subtract 13;20 from 1;40; on the 29th, (the moon) passed the sun 11;40°. (C.) If on the 27th the last visibility is 24°, 4 (mana) daylight, 2 (mana) night, (multiply) 4 by 4; (the result is) 16. Subtract 16 from 24, and 8 remains; on the 28th (the moon) remains 8° behind the sun, subtract 16 from 8; on the 29th, (the moon) passed the sun 8°. One-half of 16, namely, 8, add to (the preceding) 8, and (the result is) 16; on the 29th at sunset, the first visibility is 16°. (D.) If the daylight exceeds the night, (multiply) the daylight by 4 and you shall proceed (with this amount). If the night exceeds the daylight, (multiply) the night by 4, and you shall proceed (with this amount).[63] Consider (finally) equinox*

that the rule in Section 18 is not parallel to the rule in Section 14. The only similarity is that both methods calculate a NA_N by means of its value one Saros earlier.

[63] Already van der Waerden [1951, p. 29–30] pointed at the absurdity of this sentence. But see also

and proceed with the difference of the path (of sun and moon)."

In this section a procedure is demonstrated through calculations in 3 examples. The method is the same in all three cases; only some numerical values have changed. In Chapter VI we had concentrated on the first example and analysed the procedure. We saw that the method for finding rising times of the moon (in its invisible phase around conjunction) extrapolates from a KUR, measured at last visible moonrise before conjunction, and it uses "ΔKUR", the daily retardation of the rising moon. The numerical value of this retardation is found as 1/15 of the day's length. We also noted that the text identified the times between the risings of sun and moon with their relative position. This simplification corresponds to the rough approximation of identifying the ecliptic with the equator.

Obviously, we have here a rather primitive and presumably old method. This impression is supported by the fact that the length of daylight in example III is taken to be 4 [mana], a value which we know from EAE and MUL.APIN.[64]

Example I and II are both concerned with month I. The length of the day is taken to be 3,20. This is the duration of daylight on day 15 of month I according to the ideal scheme of table C of EAE XIV. Therefore it seems that the ideal scheme from EAE is used here, and not the one found in MUL.APIN. According to MUL.APIN the day's length is 2,50 on the first day of month I and 3,00 on its 15th day; a duration of daylight of 3,20 is given to month II day 15.[65] Example III does not mention the month, but only that the night equals 2 and the day 4. Therefore, if the text is consistent, it must be concerned with month III. According to table C in EAE the daylight amounts to 4 mana on day 15 in month III.

If we are right in our suspicion, that the length of daylight in these three cases is taken from the scheme of EAE, then we learn that it is the duration of day 15 which was used for finding ΔKUR, and not the duration of day 30.[66] We have here one more example of how the schemes of EAE (and MUL.APIN) were used.

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the discussion at the end of Chapter IX, where again it is argued that it could also show how scribes tried to model natural phenomena by means of the length of day or night.

[64] In the schemes of EAE and MUL.APIN, the longest day is given as 4 mana, and the shortest as 2 mana. Later astronomical texts use 3:2 as the proportion of longest to shortest day, which is an acceptable approximation for the latitude of Babylon; 4:2 being far off. From these two collections we also know the practice of finding the retardation of the setting moon as 1/15 of the night.

[65] The disagreement between the schemes for day length in MUL.APIN and EAE stems from the fact that the spring equinox according to EAE ideally takes place in the middle of Month XII, while according to MUL.APIN it shall take place in the middle of Month I.

[66] Similarly, we understand table D of EAE to give $\Delta\mathrm{NA}_N$, the daily change of NA_N, as 1/15 of the length of the first night in the month, and $\Delta\mathrm{GE}_6$ as 1/15 of the fifteenth night.

Now we shall inspect the calculations demonstrated in Section 19 more closely. We are convinced that these calculations all aim at determining the length of the Babylonian month. In all three cases, the calculations stop at the very moment when it is clear that the new moon will become visible for the first time in the evening. More precisely: the calculations stop as soon as the day 30 or 31 of the new crescent is known (found according to the theory applied in Section 19), and hence the length of the month is known. We start with the last example and comment on the calculations, rendering them in the same (schematic) way as in Chapter VI.

In Example III the last visible moonrise before conjunction is assumed to take place on day 27 [of month III], KUR is assumed to be 24 UŠ, and its daily change ΔKUR is 16 UŠ (= 1/15 × 4,00UŠ, the daylight). The text calculates the time from moonrise to sunrise on the subsequent mornings, and we identify these quantities by the day number n and call them "KUR(n)".

Example III:

"KUR(27)" = KUR = 24UŠ: the moon rises 24UŠ before the sun on day 27.
"KUR(28)" =24UŠ−16UŠ = 8UŠ: the moon rises 8UŠ before the sun on day 28.
"KUR(29)" = 8UŠ−16UŠ =−8UŠ: the moon rises 8UŠ after the sun on day 29.

The text says, at sunrise of day 29, the moon has passed the sun by 8 UŠ. It continues to calculate how far the moon has passed the sun 1/2 day later, namely at sunset of day 29, which defines the beginning of the next day 30. The text calls it (a bit misleadingly): *29 ina ŠÚ šámaš 16 NA:* "on the 29th before sunset, the first visibility is 16". But we reproduce it in accordance with the Diaries, as the beginning of day 30.

At sunrise on day 29, the moon has passed the sun by 8UŠ
At the beginning of day 30, the moon has passed the sun by 8UŠ + 8UŠ = 16UŠ.

According to the Babylonian theory, the moon will set 16UŠ after sunset. And it will set being visible, 16UŠ being much larger than 10UŠ, the limit for visibility. This tells us that the new month [IV] has just begun. According to the Babylonian theory, the new crescent will occur on day 30 of month III which is hence the first day of month IV. The 30th day is turned back, thus month III had 29 days.

In Example I all days refer to month I and ΔKUR= 13,20 UŠ:

day 27 "KUR(27)" = KUR = 25;00UŠ: moon rises 25;00UŠ before the sun.
day 28 "KUR(28)" = 25;00UŠ−13;20UŠ = 11;40UŠ: moonrise 11;40UŠ before sunrise.
day 29 "KUR(29)" = 11;40UŠ−13;20UŠ = −1;40UŠ: moonrise 1;40UŠ after sunrise.
day 30 "KUR(30)" =−1;40UŠ−13;20UŠ =−15;00UŠ: moonrise 15;00UŠ after sunrise.

Note that the text did not (as in example III) stop at day 29 but continues calculating KUR(30). Let us utilize the Babylonian idea (from example III) and use $1/2 \times \Delta$KUR = 6;40UŠ for finding how far the moon has passed the sun half a day later than morning 29 and 30, respectively, namely at the beginning of the day 30 and 31 of month I. (In order to make the argument simpler, we here use the notation "I day 31" instead of "II day 1".)

At sunrise on day 29, the moon has passed the sun by 1;40UŠ
At sunset begin of day 30, the moon has passed the sun by 1;40UŠ +6;40UŠ =8,20UŠ
At sunrise on day 30, the moon has passed the sun by 15UŠ
At sunset begin of day 31, the moon has passed the sun by 15UŠ +6;40UŠ =21;40UŠ

According to this theory we have found that "NA$_N$(30)" = 8;20UŠ, which is below the visibility limit:[67] the new moon will not be visible on that evening. Only on the next evening, NA$_N$(31) is sufficiently large to be seen. We therefore know now, that month II will start on the 31th day of month I, which hence had 30 days. Also in Example II, the calculation stops at the very moment when it becomes clear on which day the new crescent will be visible. This example is again concerned with month I, it uses the same value 13;20UŠ for the daily retardation of the moon as example I:

Example II:

day 27	"KUR(27)" =	KUR	= 15; 00UŠ:	moon rises 15; 00UŠ before the sun.
day 28	"KUR(28)"	=15; 00UŠ−13; 20UŠ	= 1; 40UŠ:	moon rises 1; 40UŠ before the sun.
day 29	"KUR(29)"	= 1; 40UŠ−13; 20UŠ	=−11; 40UŠ:	moon rises 11; 40UŠ after the sun.

The text stops here saying: the moon will have passed the sun by 11;40UŠ. It does not calculate KUR of the next day. The large negative value of KUR(29) shows clearly, that 1/2 day later, at the evening of day 29 which equals the beginning of day 30, the new crescent will be visible for the first time namely for 18;20UŠ (NA$_N$ = 11;40UŠ+ 6;40UŠ =18;20UŠ). In this example, day 30 of month I is turned back: so we know, month I will only have 29 days.

We repeat: in all three illustrating examples did the calculation stop at that very moment when the numbers indicated the evening on which the new month was expected to begin.

[67]Here we use the Babylonian limit 10 UŠ for visibility of NA$_N$, known from TU 11 obv. 37.

VII.10 Comments on Sections 20 and 21 (TU 11 rev. 16 – 19)

> **Section 20** *In order for you to see the equivalent of the month. Month IV, the 1st day, "sunset to moonset"(NA): 20°. You add 7 to 20, and (it is) 27. 20 times 27 is 9. You add 20 to 9, and (it is) 29. You take its half, and (it is) 14,30. Month V is hollow. If 14,30 is less (than) NA, (the month) is hollow; if it is more, (the month) is full. From above 25 you place aside(?); when it is less than 25, you add all. For all months it is the same (procedure).*

Let us reproduce the calculations in a schematic way and tentatively identify the quantities, knowing, that $NA_N = 20$:

calculations			interpretation		
$20 + 7$	=	27	27	=	$NA_N + 7$
20×27	=	9	9	=	$1/3\,(NA_N+7)$
$20 + 9$	=	29	29	=	$NA_N + 1/3(NA_N+7)$
$1/2 \times 29$	=	14,30	14;30	=	$1/2(NA_N + 1/3(NA_N+7))$
				=	$2/3 NA_N + 7/6$

The text continues:

if 14;30 $< NA_N$, the month is hollow	[i.e., if $2/3 NA_N + 7/6 < NA_N$ hollow month]
if 14;30 $> NA_N$, the month is full	[i.e., if $2/3 NA_N + 7/6 > NA_N$ full month]

Seemingly a method for determining the length of a month is demonstrated here starting out with the quantity NA_N. But we have a problem: The inequality "<" is always true, since 3 1/2 is always $< NA_N$, and therefore, $2/3\ NA_N + 7/6$ can never become greater than NA_N. Something must be wrong, so the text is corrupt; but still we see two possibilities for how to interpret some of the calculations:

1) we have here some strange arithmetical manipulations to be applied to NA_N in order to decide if the month will become full or hollow. Or

2) NA_N of month IV is used for calculating NA_N of the next month V.

The formulation *"In order for you to see the equivalent of the month."* at the beginning of the section seems to support this understanding. We prefer the second interpretation and comment on the text below under the assumption that 2) is correct.

The first addition: $20 + 7 = 27$ where $20 = NA_N$(IV) reminds us a little of the method from Atypical text K,[68] where in a first approximation, NA_N of a month is found by adding a number t to the value of NA_N of the month before.[69] What is

[68] See Neugebauer and Sachs [1969] pp. 96 – 113.

[69] In text K, the size of t, the number to be added, is given in a table as function of the lunar longitude. Of course our text is not concerned with the position of sun and moon in the zodiac.

going on in the following calculations escapes our understanding (to us it seems to be a confusion of two different methods). It also escapes our understanding why the half of 29 shall be calculated.

Of course also in this case 2) the calculations must be erroneous. But in spite of that, if our suspicion is right, that the (original or correct version of the) text aimed at calculating the value of NA_N for the next month V, then we understand a part of the text:

If $NA_N(V) < NA_N(IV)$ then month IV is hollow, and
if $NA_N(V) > NA_N(IV)$ then month IV is full,

as an indication of the fact, that the Babylonians knew and used our "rule R".

Section 21 *Of(?) 20: NA of day 1 you multiply by 20, and it is the same(?); of(?) 20: if (the month) is full, you add 6, if it is hollow, 12.*

The (cryptic) remarks here might repeat parts of the calculations in section 20, or they might refer to the Goal-Year method.

VII.11 Comments on Section 22 (TU 11 rev. 20 – 22)

In order for you to see 12, the NA: look at the invisibility of month XII which is above month I of your old year. If it is more than the invisibility of month XII which is above month I of your new year, you subtract whatever it is more from NA of month I of your new year, and you declare month I as full. If it is less, you add whatever is less to NA of month I of your new year, and you declare month I as having 29 days.

This section causes a lot of problems: not only is the terminology unclear, but the rule as it is written is also hopelessly wrong. We do not know what is meant by 12 NA nor do we know which years the text refers to by "old" and "new" year. Does "new year" mean the current year or the next year, and does "old year" refer to the year 1 Saros earlier, or to last year, or just to the present year which is about to end?

The text is concerned with the following quantities: $NA_N(I)$ of the new and old year and KUR(XII), new and old. We shall call these quantities NA_N(new), NA_N(old), KUR(new), and KUR(old). KUR(old) is the time from moonrise to sunrise measured on that morning toward the end of month XII, when the old moon

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Here the number to be added could be a function of the month in question. In any case, the value t = 7 used here correponds to the sun being somewhere between Gemini (t = 10) and Cancer (t = 5) , which is a realistic position for the sun at the beginning of month IV.

is visible for the last time; KUR(old) is therefore measured only a few days before the beginning of month I(old). The same remark holds for KUR(new) and NA_N(new).

The text tells us to calculate:

$$\mathrm{NA}_N(\mathrm{new}) - (\mathrm{KUR(old)} - \mathrm{KUR(new)}); \quad \text{if} \quad \mathrm{KUR(old)} > \mathrm{KUR(new)}$$
$$\mathrm{NA}_N(\mathrm{new}) + (\mathrm{KUR(new)} - \mathrm{KUR(old)}); \quad \text{if} \quad \mathrm{KUR(old)} < \mathrm{KUR(new)}.$$

Apparently, some NA_N is found by means of a NA_N, KUR(old) and KUR(new). If it is found by subtraction, month I will be full, while it will become hollow if it is found by addition. This last part of text determining the length of (the new?) month I, reminds us of the primitive rule: A small NA_N predicts a long month while a large one predicts a short month.

But which NA_N is found by the text? It makes no sense to find NA_N(I)(old) by means of NA_N(I)(new) – as the text seems to indicate. It should be the other way around. Therefore we have the impression, that the text is corrupt, mixing old and new. It must be the new NA_N which is found by means of NA_N(old) and KUR(old) and KUR(new).

We therefore propose, that the text meant to say: calculate NA_N(new):

$$\mathrm{NA}_N(\mathrm{I})(\mathrm{new}) = \mathrm{NA}_N(\mathrm{I})(\mathrm{old}) + (\mathrm{KUR(old)} - \mathrm{KUR(new)})$$

$$\text{If} \quad \mathrm{KUR(old)} > \mathrm{KUR(new)}, \text{ month I will become hollow,}$$
$$\text{if} \quad \mathrm{KUR(old)} < \mathrm{KUR(new)}, \text{ month I will become full.}$$

Nowadays (being used to handling negative numbers), we give the instructions in one formula whereas the text differentiates between the two cases.

According to our understanding, the text should have said subtract or add ... to NA_N of month I of your “old year” and not of your “new year”. We therefore propose to correct the text correspondingly[70]

We have the impression that this section is of rather early origin. The rule does not reflect nature (analysis of the quantities NA_N and KUR some 12, 13, or 223 months apart has not shown such a structure). We therefore assume that it is the result of a theoretical approach, maybe of the same type as found in the Atypical text K. Here in a first approximation, KUR(M) of a month was found by KUR(M-1) of the month before by a “pusher”: KUR(M) = KUR(M-1) + k. With some hesitation we propose that also NA_N was found from the preceding KUR by a pusher. In any case: the combination of NA_N(old) = KUR(old) + p with NA_N(new) = KUR(new) + p will result in the calculations given in the text.

[70]Some lines of Tablet BM42282 + 42294 duplicate this section 22 of TU 11. At one place the duplicate has “old” where TU 11 has “new”. This demonstrates that the texts sometimes did confuse old and new.

Another possibility would be that this rule is based on a "theoretical" assumption like:

$$\mathrm{NA}_N(\text{new}) + \mathrm{KUR}(\text{new}) = \mathrm{NA}_N(\text{old}) + \mathrm{KUR}(\text{old})$$

in connection with the experience that a small NA_N indicates a long month, while a large one predicts a short month. But in this case we would again have to correct the text and exchange addition and subtraction. Note: if this interpretation is correct, then the structure of this section 22 is very similar to the structure of section 17.

A third possibility would be that the text meant to calculate:

$$\mathrm{NA}_N(\mathrm{II})(\text{new}) = \mathrm{NA}_N(\mathrm{I})(\text{new}) + ((\mathrm{KUR}(\mathrm{XII})(\text{new}) - (\mathrm{KUR}(\mathrm{XII})(\text{old});$$

but also in this case we would have to change the text. A subtraction would lead to a hollow month I while an addition would indicate a full month I (according to the empirical rule R): therefore full and hollow in the text should be exchanged.

VIII Comments on Sections 8 and 9–12

VIII.1 Section 8 (TU 11 obv. 17 – 21)

> *(If) the Moon makes an eclipse in the evening watch in month VII the 14th day in Aries, Jupiter and Saturn are in Aries, Leo, Sagittarius; and Libra (which is) opposite of Aries, a straight string you (?) tighten, and ...., and ... you observe the place of Mercury in the "places", and ...*

This omen talks about a lunar eclipse which might take place after sunset in the middle of month VII. We hence know from the ideal schemes of MUL.APIN, that it is near fall equinox; the moon will be in Aries and the sun in Libra.

Jupiter and Saturn are mentioned together with some zodiacal signs: Aries, Leo, Sagittarius, and Libra. Within the circle of the Zodiac, Aries, Leo and Sagittarius constitute an equilateral triangle, well known for astrological purposes: Special importance is put on the planets if they are in the triangular aspect to the moon. Hence we understand the occurrence of the three signs Aries, Leo, and Sagittarius. Then follows Libra opposite Aries, and "GU", a straight string to be tightened, and the instruction to look for Mercury in the places. We understand this passage as follows. A straight string shall be tightened in the direction from East (Aries) to West (Libra) in which direction one shall look for Mercury. That would be astronomically reasonable.

Mercury is only visible near the sun. In case of this supposed lunar eclipse, Mercury might be visible shortly after sunset, near the direction in which the sun did set. We propose that the Babylonians referred to this direction by the word

"Libra". A similar practice has been found in other cuneiform tablets: In LBAT 1494 and 1495, the zodiacal signs Cancer and Capricorn are associated with directions toward the horizon. Only in this way can we understand that the texts mention "the morning shadow of Cancer", and "of Capricorn".[71]

In order to indicate the direction in which one shall look for Mercury, the text uses the term GU SI.SÁ DÚB. The same expression, which translates to "a straight string (shall be) tightened", is known from other texts. There the passage refers to the (still used) practice of marking the position on the ground where to build walls and buildings: a colored string is tightened leaving a coloured line on the ground. We are convinced that Section 8 refers to this practice and propose that the practice of "tightening a string" was also used to mark lines of sight for astronomical observations.

VIII.2 Section 9 – 12 (TU 11 obv. 22 – 27)

Section 9: *(If?) you see an eclipse of the same(?), and in order for you to make ....: one-third of the night one-third of the day(?). In(?) the month of your eclipse and in your 18(th year preceding) 1,30 is "after sunrise", one-third of the day of your 18(th year preceding) is 1. 1 add to 1,30, and (it is) 2,30. 1, the visibility, add(?) to it, and (it is) 3,30. 3, the length of daylight of month I(?) you subtract from it, and there remains 30; you call it "after sunset".*

Section 10: *If in your 18(th year preceding) in month I, the month of your eclipse, 1,30 is "after sunset", one-third of 3, the night of month I, is 1. <....> 3, the night of month I, you subtract from it, and what remains, 1,30, you call "after sunrise".*

Section 11: *If in your 18(th year preceding) in month I, 30 is "before sunset": one-third of 3, the day of month I is 1. 1 .... you add to it, and it is 2. 30 you subtract from 2, and there remains 1,30; you call it "after sunset".*

Section 12: *If in your 18(th year preceding) in month I, 30 is "before sunrise": one-third of 3, the night of month I, is 1. 1 .... you add to it, and it is 2. 30 you subtract from 2, and there remains 1,30; you call it "after sunrise".*

The method for finding the time of an eclipse to come is demonstrated through four examples. It is a rather primitive method which is closely related to concepts from MUL.APIN. The basis of the calculations is the time t_0 of an eclipse (or eclipse possibility) which took place 1 Saros = 18 years earlier than the eclipse to be predicted. The time t_0 of the "old eclipse" is given by its distance in time to the

[71] Namely as the direction of the rising sun's shadow, see Brack-Bernsen and Hunger [1999], pp. 281–282.

nearest sunset or sunrise (e.g., t_0 in Section 9 is given by 1,30 after sunrise).

The new eclipse is expected to take place 1/3 day plus 1/3 night later, i.e., at the time $t = t_0 + 1 + 1$. The time t is then reduced to the closest sunset or sunrise and given by its distance to that event. (All examples concern eclipses taking place in month I, so the length of day and night is assumed to be 3 in accordance with the MUL.APIN scheme[72], and as a consequence, 1/3 day = 1/3 night = 1.)

The text clearly utilizes the fact that (the mean value of) the Saros (= 6585.32 days) in a good approximation is one third of a day longer than an integer number of days. Therefore, after 1 Saros the time of full moon (and new moon) will on average be shifted by 1/3 of the nychthemeron (= day+night) in comparison to sunrise or sunset.

All four examples concern month I but they use different ways of calculation, depending on the moment when the old eclipse took place. We perceive the text as indicating a method valid for all Babylonian months, and render the general method in one formula:

$$t = t_0 + 1/3 \text{ day} + 1/3 \text{ night} = t_0 + 2.$$

The time of the expected eclipse, given by its distance to the nearest sunrise or sunset, can be determined from t by means of the appropriate length of day or night. The actual length of day and night can, e.g., be found in MUL.APIN: on tablet II ii 43 – iii 12 the length of the night at full moon is given for all the months I through XII.

The text says nothing about the units used. We tend to read times like 1,30 as given in UŠ rather than in mana, but because of the ambiguity of the cuneiform place value system, we cannot be sure. John Steele[73] reads it alike and refers to a zigzag function given in units of UŠ, a function which we suspect to be a simple and early Saros function.[74] But it is also possible that the numbers were meant to be interpreted in both ways as UŠ and as mana; 1 mana = 1,00 UŠ.

Below are the calculations and comments reproduced in a schematic way. In all cases, the "old eclipse" refers to an eclipse possibility in a month I, situated 18 years before the eclipse to be predicted. We have here tentatively identified the time intervals as given in UŠ.

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[72] On MUL.APIN tablet II i 19 – i 21, both the daylength and nightlength are given as 3 mana on the 15th of Nisannu. We interpret the 15th of Nisannu as full moon in Month I.

[73] Private communication.

[74] J. Steele [2002]

Section 9:

time of old eclipse	1,30	1,30 UŠ after sunrise
1/3 day = 1/3 × 3 =	1	1,00 UŠ
sum	2,30	2,30 UŠ
visibility (= 1/3 night?)	1	1,00 UŠ
sum = time of new eclipse	3,30	3,30 UŠ after sunrise
length of the day	3	3,00 UŠ
difference = time of new eclipse	30	30 UŠ after sunset

Section 10 (corrupt!):

time of old eclipse	1,30	1,30 UŠ after sunset
1/3 night = 1/3 × 3 =	1	1,00 UŠ
length of the night	3	3,00 UŠ
difference = time of new eclipse	1,30	1,30 UŠ after sunrise

Section 11:

time of old eclipse	− 30	30 UŠ before sunset
1/3 day = 1/3 × 3 =	1	1,00 UŠ
BE UD (= 1/3 night?)	1	1,00 UŠ
time of new eclipse	1,30	1,30 UŠ after sunset

Section 12:

time of old eclipse	− 30	30 UŠ before sunrise
1/3 night = 1/3 × 3 =	1	1,00 UŠ
BE UD (= 1/3 day?)	1	1,00 UŠ
time of new eclipse	1,30	1,30 UŠ after sunrise

The text in Section 10 is clearly corrupt. A passage is missing, and the result is wrong. Therefore, a version, corrected in analogy to section 9, is given below.

Section 10 corrected:

time of old eclipse	1,30	1,30 UŠ after sunset
1/3 night = 1/3 × 3 =	1	1,00 UŠ
visibility (= 1/3 day ?)	1	1,00 UŠ
sum = time of new eclipse	3,30	3,30 UŠ after sunset
length of the night	3	3,00 UŠ
difference = time of new eclipse	30	30 UŠ after sunrise

We noted above in our (first) interpretation that the examples on TU 11 determine the time of the predicted eclipse to be 2,00 UŠ later than the old eclipse. There

is, however, another way to interpret the four Sections 9 - 12. Let us now look more closely at the text and the calculations.
Note: the third of the day plus the third of the night always (i.e., for all months) equals 2 mana or 2,00 UŠ. Why didn't the text just say something like "add 2 or (1/3 day + 1/3 night) to the time of the old eclipse"?
Note also: in Section 9 and 11, the "old eclipse" took place during daytime (1,30 after sunrise and 30 before sunset, respectively). And both sections start by taking the third of daylight and then add 1, which is called *ta-mar-tú* (visibility) or BE UD. In the first approach we just interpreted it as 1/3 of the night, which in month I equals 1. But the text did not say "take the third of the night".

Similarly, Sections 12 and 10 start out by calculating a third of the night, which in month I equals 1. In these cases, the old eclipse took place during the night (1,30 after sunset and 30 before sunrise, respectively). Section 12 adds again 1, the "BE UD" and not 1 specified as 1/3 day.

All these particularities suggest to us to propose another interpretation. We read the 1 called "*ta-mar-tú*" or "BE UD" as the constant 1, and not as the third of day or night. If the old eclipse (possibility) takes place during daytime, then add the third of the actual daylength plus 1 in order to get the next eclipse. Does it take place during nighttime, then add a third of the night plus 1 to the time t_0, and reduce the result to its distance to the nearest sunrise or sunset. (This method is not a good approximation to nature – it can, however, be seen as a first approach to describe what we would call the varying duration of the Saros.)

In case of month I, in which day and night are both taken to be 3 mana or 3,00 *uš*, the two interpretations lead to the same result: in comparison to sunrise (and sunset), the new eclipse (possibility) will always take place 2,00 *uš* later than the old eclipse. But for the other months, except month VII, the different interpretations give different results.

We understand the examples calculated in Section 9 – 12 to demonstrate a method which is valid for all the Babylonian months. We therefore give two possible ways of understanding the calculations in formulas below, meant to apply for all months. Again we use t_0 to denote the time of the old eclipse (or eclipse possibility). This is used to find t which then, reduced to nearest sunset or sunrise, gives the time of the eclipse to be predicted.

- First interpretation: $\mathrm{t} = \mathrm{t}_0 + 1/3 \text{ day} + 1/3 \text{ night} = \mathrm{t}_0 + 2.$
- Alternative interpretation:

  $t = t_0 + 1/3 \text{ day} \quad + 1$ (if the time of the old eclipse was during daytime?).

  $t = t_0 + 1/3 \text{ night} \ + 1$ (if the time of the old eclipse was during nighttime?).

For month I and VII it is true that 2 = 1/3 day + 1/3 night = 1/3 day + 1 = 1/3 night + 1; but for the other months it is not the case. If the length of the night is taken from the ideal scheme of MUL.APIN, then the procedure: ($t = t_0 + 1/3$ night + 1) may result in a linear zigzag function with minimum 1,40 in month IV and maximum 2,20 in month X. This means, that the second interpretation may imply the use of a linear zigzag function for the length of the Saros. We sum up: what is modeled in Section 9 - 12 is the shift in time of eclipses occurring 1 Saros apart. According to the first interpretation, the time shift is taken to be constant an equal to 2 mana = 2,00 *uš*. (In reality, this time shift is not constant: it varies, reflecting the varying duration of the Saros.) According to the second interpretation, the time shift is variable and derived from the length of day or night.

A linear zigzag function like the one sketched above might have been the starting point for the construction of the linear zigzag function on BM 45861, which we tend to interpret as a simple function for the length of the Saros in Babylonian astronomy[75]

It seems that the Babylonians at early stages of the astronomical development tried to fit linear zigzag functions to nature. In Enūma Anu Enlil XIV and in MUL.APIN, the ideal length of day and night were given as linear zigzag functions, the month being the independent variable. Other quantities e.g., the daily retardation of the moon and its visibility time were derived from these functions: in the ideal case the full moon is visible all night. According to table A and B of EAE XIV, the full moon (on day 15) is expected to rise at sunset and set at sunrise – which is a sound first approximation. This was obtained by putting the daily retardation of the setting moon to 1/15 of the night. We see that it makes sense to approximate $\Delta\mathrm{NA}_N$ with 1/15 of the night. And the quantity NA_N is measured at sunset so it is natural to connect its daily change, $\Delta\mathrm{NA}_N$, with the length of the night. Similarly, Section 19 of TU 11 uses 1/15 of daylight as the daily retardation of the rising moon: KUR is measured at sunrise and ΔKUR its daily change is put to 1/15 day.

If the second interpretation of Section 9 - 12 is correct, then we are faced with a problem, namely that in reality the duration of the Saros does not depend on when the old eclipse took place: whether it was during day or night time. We must try to find out when the ancient method prescribed to take (1 + 1/3 day), and when to take (1 + 1/3 night) for the retardation after 1 Saros. Was the choice wrongly determined by the time of the old eclipse? Or was the daylength used for solar eclipses and the length of the night used for lunar eclipses? Or were other criteria used to determine if one had to choose the length of the day or of the night? Solar eclipses are evidently only visible during day time, and lunar eclipses are roughly speaking only visible at night time (if the full moon is above the horizon). Therefore, if the methods of Section 9 – 12 were only applied to old eclipses which had been

[75] See John Steele [2002].

observed, then the time of a new lunar eclipse (expected 1 Saros later) would always be determined to occur (1 + 1/3 night) later with respect to sunset.

The approach to use (1 + 1/3 night) as the shift in time of lunar eclipses after 1 Saros is not too bad: the shortest duration of the Saros is found by lunar eclipses taking place during the summer season, namely in (May), June or July. And this is the time when the nights are short. Maybe we have here some, admittedly very vague, indications to the fact that early stages of Babylonian astronomy had realized and used the fact that the length of the Saros varies concurrently with the solar year.[76]

In any case we see that the methods mentioned above are attempts to model nature arithmetically by means of linear zigzag functions derived from the length of day or night. The cryptic remarks at the end of Section 19 might illustrate the difficulties that are faced or may just show that the scribe did not understand the procedure.

[76] See also Brack-Bernsen [1980], where it was shown, that the duration of the Saros can be approximated by the sum of a solar and a lunar term, the solar term being the dominating one.

Abbreviations

ACT: Astronomical Cuneiform Texts, see Neugebauer [1955].

Diaries: see Sachs and Hunger (1988 – 1996).

EAE: Enūma Anu Enlil (for Tablet XIV, see Al-Rawi and George [1991]).

JCS: Journal of Cuneiform Studies.

JNES: Journal of Near Eastern Studies (Chicago)

LBAT: Late Babylonian Astronomical and Related Texts, see Pinches et al. (1955)

UOS: J.M. Steele and A. Imhausen (eds). *Under One Sky: Astronomy and Mathematics in the Ancient Near East* (Ugarit-Verlag, Münster) in press.

WO: Die Welt des Orients (Göttingen)

ZA: Zeitschrift für Assyriologie (Berlin)

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