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Notes on the Differences between the Two Recensions of the *Līlāvatī* of Bhāskara II

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Introduction

The *Līlāvatī* (abbr. L) is a Sanskrit work on arithmetic and mensuration (computational geometry) composed by Bhāskara II in or a little before A. D. 1150. The number of the manuscripts (more than six hundred) of the work listed in the CESS,[1] as well as the number of the commentaries on it (more than thirty),[2] proves that it has been the most popular mathematical textbook in India used by a number of people of the entire subcontinent. It is regrettable that virtually no study based on those manuscripts has been done yet on the history of the transmission of such an influential work. So far, we only know that there are some differences between the northern and the southern recensions of the work,[3] represented respectively by the Ānandāśrama edition (I call it ASS)[4] and by the Hośiarapura edition (VIS).[5]

The ASS is accompanied by two commentaries, one by Gaṇeśa and the other by Mahīdhara. Gaṇeśa, son of Lakṣmī and Keśava of the Kauśikagotra, composed

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[1]CESS: David Pingree, *Census of the Exact Sciences in Sanskrit*, Ser. A, Vols. 1–5, Philadelphia: American Philosophical Society, 1970–1994. For the manuscripts of the *Līlāvatī*, see A4, pp. 300–308, and A5, pp. 254–257. The number of manuscripts may be multiplied if we take into account those which are left and forgotten at corners of a number of family libraries.

[2]CESS, A4, pp. 299–300.

[3]See R. C. Gupta's article cited in III.15. See also fn. 5 below.

[4]ASS: The *Līlāvatī* edited with Gaṇeśa's *Buddhivilāsinī* and Mahīdhara's *Līlāvatīvivaraṇa* by Dattātreya Āpaṭe, *et al.*, 2 parts, Anandashrama Sanskrit Series 107, Poona: Anandashrama Press, 1937.

[5]VIS: The *Līlāvatī* edited with Śaṅkara and Nārāyaṇa's *Kriyākramakarī* by K. Venkateswara Sarma, Vishveshvaranand Indological Series 66, Hoshiarpur: Vishveshvaranand Vedic Research Institute, 1975. Yano and I have pointed out some of the variant readings found in the VIS in our Japanese translation of the ASS edition, which has been included in: *Collection of Astronomical and Mathematical Works in India*, edited by Michio Yano, Kagaku no Meicho Series 1, Tokyo: Asahi Press, 1980, pp. 139–372.

his commentary, *Buddhivilāsinī*, in Nandigrāma[6] in A. D. 1545. Son of Rāmadāsa, Mahīdhara was born in Ahicchatra and went to Vārāṇasī, where he composed his commentary, *Līlāvatīvivaraṇa*, in A. D. 1587.

According to the editor's description of the manuscripts,[7] the *Buddhivilāsinī* accompanying the ASS has been edited on the basis of a manuscript belonging to the manuscripts library of the Ānandāśrama with the help of another manuscript acquired from Rājāpura (Mahārāṣṭra). Three manuscripts of the same work were acquired from England as well, but they have been used only partially. The *Līlāvatīvivaraṇa*, on the other hand, has been edited on the basis of a manuscript belonging to the Bhāratīya Itihāsa Saṃśodhana Maṇḍala of Puṇyapattana with the help of an Ānandāśrama manuscript. For the text of the *Līlāvatī* itself including the *vāsanābhāṣya* (a prose commentary which explains how to apply a rule to an example, hereafter *vāsanā*) by Bhāskara II himself, many printed texts are said to have been consulted, and suitable readings determined after examination.[8] The editor does not specify the printed texts he used, and rarely gives variants from them, but the text edited agrees very well with the commentaries of Gaṇeśa and Mahīdhara.[9] So, I hypothetically assume, for the time being (unless and until it is denied by a study based on the manuscripts), that the ASS represents a northern recension of the time of Gaṇeśa and Mahīdhara, that is, in the sixteenth century.

The VIS is accompanied by the commentary, *Kriyākramakarī*. It was composed first by Śaṅkara Vāriyar in Kerala ca. A. D. 1540 up to verse 199 of the *Līlāvatī* but left unfinished, and later, ca. A. D. 1560, completed by Mahiṣamaṅgala Nārāyaṇa.

The VIS has been edited on the basis of four manuscripts of south Indian origin. Two of them contain Śaṅkara's commentary and the last portion (on verses 268 and 269) of Nārāyaṇa's, while the other two contain both Śaṅkara's and Nārāyaṇa's commentaries. The VIS may, therefore, be hypothetically regarded as representing a southern recension of the sixteenth century.

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[6]Nandigrāma has been identified with Nāndgāon in Janjīrā State (about 40 miles to the south of Mumbāī) by Sankar Balakrishna Dikshit (*Bharatiya Jyotish Sastra (History of Indian Astronomy)*, translated in 2 parts by R. V. Vaidya from the Marathi work, *Bharatiya Jyotish Sastracha Prachin Ani Arvachin Itihas* (A. D. 1896), Delhi: The Manager of Publications 1969/1981, Part 2, p. 128), and with Nandod in Gujarat by David Pingree (CESS A2, pp. 65 and 94). Yukio Ôhashi argues for the former on the basis of the latitude of the observer, 18°N, employed in one of Gaṇeśa's works (see Ôhashi's "The Cylindrical Sundials in India", *Indian Journal for History of Science* 33(4), 1998, Supplement, p. S175).

[7]ASS, on the page next to p. 11 or on the opposite side of the table of contents.

[8]vāsanāsahitalīlāvatyāḥ pustakāni tu prāgbahutra mudritāni / teṣu grāhyāgrāhyavicārapūrvakaṃ pāṭhānāṃ grahaṇamakāri // (ASS, on the page next to p. 11 or on the opposite side of the table of contents)

[9]See, however, verse 8^a (II.1), which occurs in Mahīdhara's text but not in Gaṇeśa's.

The present paper mainly consists of notes on the differences between these two hypothetical recensions of the *Līlāvatī*, but I have also added notes on Parameśvara's (fl. ca. 1380/1460) commentary,[10] which is the oldest among the datable commentaries on the *Līlāvatī*.[11]

Parameśvara usually cites only the first few words of each verse and ignores about half of the examples attached to the rules and all of the *vāsanā*,[12] but we can gather a not inconsiderable amount of information about the text (*mūla*) of the *Līlāvatī* used by him from his paraphrases of, and comments on, each verse. As will be seen below, Parameśvara's text agrees with the VIS in most cases. Moreover, some of his verses, which he gives in his commentary, have been included in the VIS.[13] In some other cases, however, his text agrees with the ASS.[14] It is interesting that he himself refers to a suspicion of the interpolation of several verses.[15]

In the four sections that follow, I will take up differences between the ASS and the VIS, and refer to the degree of their affinities with Parameśvara's text, in four categories, namely, transposition of verses, additions and omissions of verses, changes and alterations of verses, and differences in the *vāsanā*. In each section, the notes are arranged in the order of the verse-number. I hope this study, though based on only a part of the available evidence, will be a preliminary to future, more extensive studies of the transmission of the *Līlāvatī* based on the existing manuscripts.

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[10]The manuscripts I have used are: R 338, R 5160, and R 5231 (b) of the Government Oriental Manuscripts Library, Madras; 40 C 20 of the Adyar Library and Research Centre, Madras; and T 295, 5783, 10614B, 18255, 19963 and 22117 of the Oriental Research Institute and Manuscripts Library, University of Kerala, Trivandrum.

[11]CESS, A4, p. 299.

[12]The verses cited in full by him are: 182, 188 (which is a quotation from BSS 12.28), 250, 256, 261, 264, 266, 268, and 270. All these, except the first two that may be interpolations (see III.11), are the rules of the last two chapters called "Pulverizer" and "Chain of Digits." One of the ten manuscripts I used, R 338 in the Telugu script, contains the full verses and the *vāsanā* of the *Līlāvatī* interspersed in the commentary in addition to the quotations of the first few words of each verse, but they seem to be a later addition. BSS = *Brāhmasphuṭasiddhānta* of Brahmagupta, edited with his own commentary by Sudhākara Dvivedin, Benares: Medical Hall Press, 1902.

[13]See II.6, II.8, and III.9.

[14]See I.1, II.1, II.2, II.7, and III.16.

[15]See III.11.

Table 1: Transposition of Verses 112–134

Verse Nos.		Contents	
ASS	VIS	ASS	VIS
90	90	Miśra-vyavahāra begins.	Miśra-vyavahāra begins.
⋮	⋮		
111	111		(Miśra-vy. ends.)
112	130	chandas-citi-ādi begins.	
113	131		
114	132		
115	133		
116	134	Miśra-vy. and chandas- end.	chandas- ends.
117	112	Śreḍhī-vyavahāra begins.	Śreḍhī-vyavahāra begins.
118	113		
⋮	⋮		
131	126		
132	127		(Śreḍhī-vy. ends.)
133	128		chandas-citi-ādi begins.
134	129	Śreḍhī-vy. ends.	

I Transposition of Verses

I.1 Verses 112–134

The order of verses in the VIS differs greatly at one place from that in the ASS. This is due to different locations of the section on the *chandas-citi-ādi* ("accumulation of meters, etc.") in these texts. See Table 1.

In the ASS, the section on the *chandas-citi-ādi* occupies the last part of the chapter on *miśra-vyavahāra* (practical mathematics on mixture), while in the VIS it is placed after the chapter on *śreḍhī-vyavahāra* (practical mathematics on series), which comes after the *miśra-vy.* In fact, in the VIS, it is not very certain whether that section is independent of the *śreḍhī-vy.* or included in it, since the end of the *śreḍhī-vy.*, as well as that of the *miśra-vy.*, is not indicated in the four manuscripts used for the VIS.

The two verses, ASS 133 and 134 (= VIS 128 and 129), are placed at the end of the *śreḍhī-vy.* in the ASS but at the beginning of the *chandas-citi-ādi* in the VIS. ASS 133 is a rule for the number of meters, which is obtained as the sum of a geometric progression, and ASS 134 is an example for it. These two verses, therefore, may be included in either the "series" (*śreḍhī*) or the "accumulation of meters, etc." (*chandas-citi-ādi*).

In Parameśvara's commentary, the location of the verses in question agrees with

Table 2: Transposition of Verses 137 and 138

Verse Nos.		Contents
ASS	VIS	(Let a, b, c be the three sides of a right-angled triangle.)
136	136	Formulas: $c = \sqrt{a^2 + b^2}$, $a = \sqrt{c^2 - b^2}$, $b = \sqrt{c^2 - a^2}$.
137	138	Example for the above computation: $(a, b, c) = (3, 4, 5)$.
138	137	Formulas: $p^2 + q^2 = (p - q)^2 + 2pq$, $p^2 - q^2 = (p + q)(p - q)$.

Table 3: Transposition of Verses 269 and 270

Verse Nos.		Contents
ASS	VIS	
268	268	Formula: $N = \frac{s-1}{1} \cdot \frac{s-2}{2} \cdot \frac{s-3}{3} \cdots \frac{s-n+1}{n-1}$.
269.1	270.1	Condition for the above formula: $s < n + 9$.
269.2	270.2	Concluding remark.
270	269	Example: $n = 5$, $s = 13$.

that of the ASS.

I.2 Verses 137 and 138

A minor change in the verse order occurs at verses 137–138. This case concerns the application of the Pythagorean Theorem. See Table 2.

Parameśvara does not refer to ASS 137 (= VIS 138) as it is an example.

I.3 Verses 269 and 270

Another case of a minor change in the verse order occurs at verses 269–270. This case concerns the last rule of the last topic of the *Līlāvatī*, which prescribes a formula for the number (N) of combinations of n digits to be put in n decimal places when the sum (s) of those n digits is given. See Table 3.

In the VIS, 270.1 (condition for the formula) has been supplied by the editor since it is missing in the two manuscripts used for this part of the VIS. VIS 270.2 (concluding remark) is different from ASS 269.2.

> ASS 269.2: A short ⟨account⟩ has been told ⟨here by me⟩ for fear of prolixity since there is no limit to the ocean of mathematics.[16]
>
> VIS 270.2: The seed of various works of the ocean of mathematics, which is desired by intelligent people, has been told concisely ⟨here by me⟩.[17]

[16] saṃkṣiptamuktaṃ pṛthutābhayena nānto sti yasmādgaṇitārṇavasya // ASS 269.2 // Hereafter, I do not use the *avagraha*.

[17] saṃkṣiptamuktaṃ pṛthukāryabījaṃ vidvajjaneṣṭaṃ gaṇitārṇavasya // VIS 270.2 //

Table 4: Addition of Verse 20.1^a in VIS

Verse Nos.		Contents
ASS	VIS	
20.1	20.1	Formula: $p^2 = 2ab + (a^2 + b^2)$ where $p = a + b$.
	20.1^a	Formula: $p^2 = 4ab + (a - b)^2$ where $p = a + b$.
20.2	20.2	Formula: $p^2 = (p + a)(p - a) + a^2$.

In Parameśvara's commentary, the order of the two verses, 269 and 270, agrees with that of the VIS edition and its concluding remark is identical with VIS 270.2.

II Additions and Omissions of Verses

II.1 Verse 8^a

Verses 7–8 at the end of the section on weights and measures (*paribhāṣā*) give volume measures and their conversion ratios: 1 cubic *hasta* = 1 *khārikā* = 16 *droṇas*, 1 *droṇa* = 4 *āḍhakas*, 1 *āḍhaka* = 4 *prasthas*, 1 *prastha* = 4 *kuḍavas*.

After verse 8, the ASS has an additional verse, numbered 1 (I designate it ASS 8^a) and bracketed, which provides "western terminology[18] for the measurement of grains, etc.", that is, $\frac{3}{4}$ *gadyāṇaka* = 1 *ṭaṅka*, 72 *ṭaṅkas* = 1 *sera*, 40 *seras* = 1 *maṇa*.[19]

Mahīdhara comments on this verse, but Gaṇeśa does not refer to it. It occurs neither in the VIS nor in Parameśvara's commentary.

II.2 Verse 20

The two verse lines of ASS 20 each give a formula for squaring a number based on an algebraic identity.[20] The VIS has another line for another formula in between the two lines.[21] I designate it 20.1^a.[22] See Table 4.

Parameśvara does not refer to this line while commenting on 20.1 and 20.2.

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[18] *turuṣka-saṃjñā*, which literally means "the Turkish terminology". Mahīdhara explains it with the words, *yavanānāṃ saṃjñā* (lit. "terminology of the Yavanas or the Muslims").

[19] pādonagadyāṇakatulyaṭaṅkairdvisaptatulyaiḥ kathito tra seraḥ /
maṇābhidhānaṃ khayugaiśca serairdhānyāditaulyeṣu turuṣkasaṃjñā // ASS 8^a //

[20] khaṇḍadvayasyābhihatirdvinighnī tatkhaṇḍavargaikyayutā kṛtirvā /
iṣṭonayugrāśivadhaḥ kṛtiḥ syādiṣṭasya vargeṇa samanvito vā // L 20 //

[21] khaṇḍadvayasyābhihatiścaturghnī tatkhaṇḍayorantaravargayuk / VIS 20.1^a /

[22] For an example for this formula, see IV.2.

Table 5: Omission of the latter half of Verse 42 in VIS

Verse Nos.		Contents
ASS	VIS	
42.1	42	Examples: $5 \div 2\frac{1}{3}$, $\frac{1}{6} \div \frac{1}{3}$.
42.2		"if your intelligence, which is very sharp like the tip of a *darbha* leaf, has ability in division of fractions."

II.3 Verse 42

VIS 42 is identical with ASS 42.1, which gives two examples for the division of fractions, and the VIS does not contain ASS 42.2,[23] which consists of an if-clause for the ordinary style of question, "Tell me . . . , if you know etc." See Table 5.

Parameśvara does not refer to this example.

II.4 Verse 64

ASS 64 in four verse-lines,[24] which briefly states characteristic features of the two fields of Indian mathematics, *pāṭī-gaṇita* (mathematics of procedures or of algorithms) and *bīja-gaṇita* (mathematics of seeds or of algebraic equations), exists neither in the four manuscripts used for the VIS nor in Parameśvara's commentary. In the VIS, they have been supplied by the editor from unspecified sources, and designated 64^a and 64^b.

II.5 Verse 77

VIS 77, which gives a definition of the inverse three-quantity operation, is slightly different from ASS 77. That is, the former reads *phalasya tu* ("of the result") instead of *ca jāyate* ("is produced") at the end of the first line. But it does not cause any substantial change in the definition itself. What is noteworthy here is that none of the four manuscripts used for the VIS contains verse 77 and that it has been supplied by the editor from somewhere else. The four manuscripts, however, contain Bhāskara II's own *vāsanā*, which is almost the same as verse 77.

The situation is the same in the commentary of Parameśvara, who omits verse 77 and simply paraphrases Bhāskara II's *vāsanā* upon it. The *vāsanā* cited in the VIS is closer to Parameśvara's paraphrase than to the *vāsanā* itself in the ASS. Compare

[23] satryaṃśarūpadvitayena pañca tryaṃśena ṣaṣṭhaṃ vada me vibhajya /
darbhīyagarbhāgrasutīkṣṇabuddhiścedasti te bhinnahṛtau samarthā // ASS 42 //

[24] pāṭīsūtropamaṃ bījaṃ gūḍhamityavabhāsate /
nāsti gūḍhamamūḍhānāṃ naiva ṣoḍhetyanekadhā //
asti trairāśikaṃ pāṭī bījaṃ ca vimalā matiḥ /
kimajñātaṃ subuddhīnāmato mandārthamucyate // ASS 64 //

Table 6: Addition of Verse 161^a in VIS

Verse Nos.		Contents
ASS	VIS	
161	161	Formula: $h = (H_1 H_2) \div (H_1 + H_2)$.
	161^a	Formula: $h = H_2 \times \frac{H_1}{H_2} \div \left(\frac{H_1}{H_2} + 1\right)$.
162	162	Example: $H_1 = 15$, $H_2 = 10$. Answer: $h = 6$.

the following three passages corresponding to one another (here, the italics indicate the text of the L).

ASS:
atha vyastatrairāśike karaṇasūtram —
icchāvṛddhau phale hrāso hrāse vṛddhiśca jāyate /
vyastaṃ trairāśikaṃ tatra jñeyaṃ gaṇitakovidaiḥ // 77 //
yatrecchāvṛddhau phale hrāso hrāse vā phalavṛddhistatra vyastatrairāśikam / tadyathā — (Verse 78 follows.)

Parameśvara:
... iti kalpyam // (End of the comm. on verse 76) yatrecchāyā vṛddhau phalasya hrāsaḥ syādicchāyā hrāse phalasya vṛddhirvā tatra vyastatrairāśikaṃ kāryam / tadviṣayaṃ pradarśayati *jīvānāṃ vayasa*[25] ityādinā / (Verse 78 follows.)

VIS:
atha *vyastavidhirviloma*[26] ityuktaṃ vyastatrairāśikamudāhartumāha /
atha vyastatrairāśikam[27] —
⟨*icchāvṛddhau phale hrāso hrāse vṛddhiḥ phalasya tu /*
vyastaṃ trairāśikaṃ tatra jñeyaṃ gaṇitakovidaiḥ // 77 //⟩[28]
vyastatrairāśikasya viṣayaṃ pradarśayati /
yatrecchāyā vṛddhau phalasya hrāsa icchāyā hrāse vā phalasya vṛddhistatra vyastatrairāśikam / tadyathā —
*tadyathe*ti tadvyastatrairāśikaṃ yathā spaṣṭaṃ bhavati tathā vibhajyocyata ityarthaḥ / (Verse 78 follows.)

II.6 Verse 161^a

Verse 161 gives a formula for the height (h) of the perpendicular drawn from the intersection of the two straight lines extending from the tops of two bamboos (height H_1, H_2) standing on a flat ground to their mutually opposite feet. Verse 162 gives an example for it. See Table 6.

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[25] The first two words of verse 78.

[26] Cited from verse 73.

In between verses 161 and 162, the VIS has an additional verse (designated 161^a), which prescribes another formula equivalent to the above.

> VIS 161^a: The longer bamboo divided by the shorter is the multiplier, and the divisor is that increased by unity. By means of these, the multiplier and the divisor, from the shorter bamboo the perpendicular is alternatively ⟨obtained⟩.[29]

That is to say,

$$h = \frac{H_2 \cdot \frac{H_1}{H_2}}{\frac{H_1}{H_2} + 1}.$$

A similar formula has been given by Parameśvara as an alternative rule:

> PL 161.1: The other bamboo divided by the shorter one is the multiplier, and the divisor is that increased by unity. The shorter bamboo divided by the divisor and multiplied by the multiplier will be the perpendicular from the intersection of the ⟨two⟩ threads of two bamboos.[30]

That is to say,

$$h = \frac{H_2}{\frac{H_1}{H_2} + 1} \cdot \frac{H_1}{H_2}.$$

It is noteworthy that their introductions are similar to each other; that is, VIS 161^a is introduced with the words *evaṃ vā sūtram* ("Likewise, an alternative rule —"), while Parameśvara's verse with the words *atha vā sūtram* ("Now, an alternative rule —").[31]

The only example (in verse 162) is solved by Śaṅkara as well as by Bhāskara II by means of the algorithm, not of verse 161^a, but of verse 161.

II.7 Verses 170, 171, 171^a, 171^b

In both ASS and VIS, verse 169 provides a formula for calculating the area (A) of a quadrilateral and a trilateral from their sides (a, b, c, d; for a trilateral, $d = 0$), that is, $A = \sqrt{(s-a)(s-b)(s-c)(s-d)}$, where $s = (a+b+c+d)/2$. In the ASS, verse 170 gives an example for the formula,[32] and verse 171 criticizes the formula

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[29] vaṃśaḥ svalpoddhṛto nalpo guṇo hāraḥ svarūpayuk /
tābhyāṃ vā guṇahārābhyāṃ svalpādvaṃśāttu lambakaḥ // VIS 161^a //
Read *sa rūpayuk* for *svarūpayuk* at the end of the first line, as in two of the four mss.

[30] svalpena vaṃśena hṛto nyavaṃśo guṇo haro rūpayutaḥ sa eva /
vaṃśo laghurhārabhakto guṇaghno lambo bhavedvaṃśayoḥ sūtrayogāt // PL 161.1 //

[31] See III.9 (verse 160) for a similar introduction by Parameśvara.

[32] bhūmiścaturdaśamitā mukhamaṅkasaṃkhyaṃ
bāhū trayodaśadivākarasaṃmitau ca /

because it produces only a crude result for a quadrilateral.

> ASS 171: Because the two diagonals of a quadrilateral are indeterminate ⟨when its four sides are given⟩, therefore how can the area be determinate in that case? ⟨Even⟩ when its two diagonals have been determined by ⟨our⟩ predecessors, those two, which have been assumed by each, are not ⟨as such⟩ elsewhere. When the sides are the same, the diagonals ⟨can be⟩ otherwise, and therefore the field-fruit (i.e., the area) is ⟨obtained⟩ variously.[33]

Then follow, in the ASS, two unnumbered stanzas,[34] which criticize an inquirer (*pṛcchaka*) who asks for the area of a quadrilateral without giving any of its diagonals and perpendiculars.

In the VIS, these four stanzas, 170, 171, and the subsequent two, have been supplied by the editor from unspecified Kerala manuscripts in Malayalam script (the last two stanzas are designated 171^a and 171^b in the VIS) as none of the four manuscripts used for the VIS contains them. They have several minor variants.[35]

Parameśvara comments on each of these four verses. As he only cites part of the text, we do not know exactly and entirely the verses he had in hand, but his citations[36] show that his 171^a is closer to the corresponding verse of the ASS (see *vaikamanirdiśya*) than to the 171^a supplied in the VIS, but that his 170 and 171^b are closer to VIS 170 and 171^b (see *tadādyaiḥ* and *gaṇako*, respectively). His 171 is, however, different from both the ASS and VIS, since its third line begins with *tathaiva bāhuṣu* while those of the latter two with *teṣveva bāhuṣu*.

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lambo pi yatra ravisaṃkhyaka eva tatra
kṣetre phalaṃ kathaya tatkathitaṃ yadādyaiḥ // ASS 170 //

[33]caturbhujasyāniyatau hi karṇau kathaṃ tato sminniyataṃ phalaṃ syāt /
prasādhitau tacchravaṇau yadādyaiḥ svakalpitau tāvitaratra na staḥ /
teṣveva bāhuṣvaparau ca karṇāvanekadhā kṣetraphalaṃ tataśca // ASS 171 //

[34]lambayoḥ karṇayorvaikamanirdiśyāparaṃ katham /
pṛcchatyaniyatatve pi niyataṃ cāpi tatphalam // 171^a // (unnumbered in the ASS)
sa pṛcchakaḥ piśāco vā vaktā nitarāṃ tataḥ /
yo na vetti caturbāhukṣetrasyāniyatāṃ sthitim // 171^b // (unnumbered in the ASS)

[35]In 170: *ced* for *ca*; *tadādyaiḥ* for *yadādyaiḥ*. In 171: *yadā dvau* for *yadādyaiḥ*; *svakalpitatvāt* for *svakalpitau tau*. In 171^a: *naikaṃ samuddiśyāparān* for *vaikamanirdiśyāparaṃ*. In 171^b: *gaṇako* for *vaktā*; *caturbāhau* for *caturbāhu-*.

[36]From 170: *bhūmiścaturdaśa*, *kathitaṃ tadādyaiḥ*. From 171: *caturbhujasyāniyatau*, *svakalpitāt*, *tathaiva bāhuṣu*. From 171^a: *lambayoḥ karṇayorvā*, *ekam*, *anirdiśya*. From 171^b: *pṛcchakaḥ*, *piśācaḥ*, *gaṇakaḥ*.

II.8 Verses 190^a, 190^b

Two of the four manuscripts used for the VIS have two additional verses after verse 190. Neither the ASS nor the other two manuscripts used for the VIS contain them.

Verse 190 gives a computational rule for the diagonals of a cyclic quadrilateral (see III.12), while the two verses in question give a formula for the radius of the circumscribing circle of a cyclic quadrilateral.

> VIS 190^a–190^b: When the product of the three sums of the products of the sides taken two at a time is divided by the tetrad of the sums of three ⟨sides⟩ decreased by the other, the entire quadrilateral field exists within that circle which is constructed with the radius equal to the square root of the quotient.[37]

That is to say,

$$r = \sqrt{\frac{(ab+cd)(ac+bd)(ad+bc)}{(b+c+d-a)(c+d+a-b)(d+a+b-c)(a+b+c-d)}},$$

where a, b, c, and d are the four sides of a cyclic quadrilateral.[38]

Exactly the same verses with a minor variant,[39] together with another verse, occur in Parameśvara's commentary immediately after verse 198, i.e., at the end of the section on rectilinear figures. The third verse reads as follows.

> L 198^c: One fourth of the square root of the divisor told above ⟨in L 198^a–198^b⟩ is its area. When one has regarded a fourth side as zero, this ⟨rule⟩ shall be the same in the case of a trilateral as well.[40]

That is to say, the area (A) of the cyclic quadrilateral is calculated by:

$$A = \frac{\sqrt{(b+c+d-a)(c+d+a-b)(d+a+b-c)(a+b+c-d)}}{4},$$

[37]doṣṇāṃ dvayordvayorghātayutīnāṃ tisṛṇāṃ vadhe /
ekaikonetaratryaikyacatuṣkeṇa vibhājite // VIS 190^a //
labdhamūlena yadvṛttaṃ viṣkambhārdhena nirmitam /
sarvaṃ caturbhujaṃ kṣetraṃ tasminnevāvatiṣṭhate // VIS 190^b //

[38]For Śaṅkara's proof of this formula, see Radha Charan Gupta, "Parameśvara's Rule for the Circumradius of a Cyclic Quadrilateral", *Historia Mathematica* 4, 1977, 67–74; and T. A. Sarasvati, *Geometry in Ancient & Medieval India*, Delhi: Motilal, 1979, 108–109.

[39]*-catuṣkavadhabhājite* ("is divided by the product of the tetrad of ...") for *-catuṣkeṇa vibhājite* ("is divided by the tetrad of ...") in 190^a. Śaṅkara, too, has recorded this variant.

[40]proktabhājakamūlābdhibhāgo bhavati tatphalam /
śūnyaṃ prakalpya turyaṃ dostryaśre pyetatsamaṃ bhavet // L 198^c //

Table 7: Weight measures in ASS 4 and VIS 4

ASS 4	VIS 4
5 *guñjās* = 1 *māṣa*	$10\frac{1}{2}$ *guñjās* = 1 *māṣa*
16 *māṣas* = 1 *karṣa*	16 *māṣas* = 1 *karṣa*
4 *karṣas* = 1 *pala*	4 *karṣas* = 1 *pala*
	100 *palas* = 1 *tulā*
1 *karṣa* of gold = 1 *suvarṇa*	100 *palas* of gold = 1 *suvarṇa*

and for a trilateral:

$$r = \frac{abc}{\sqrt{(b+c-a)(c+a-b)(a+b-c)(a+b+c)}},$$

$$A = \frac{\sqrt{(b+c-a)(c+a-b)(a+b-c)(a+b+c)}}{4}.$$

III Changes and Alterations of Verses

As a rule, I do not include here alterations of words which do not affect the mathematical contents.

III.1 Verse 4

Verses 3–4 give weight measures and their conversion ratios. VIS 4 includes the relation, 100 *palas* = 1 *tulā*,[41] which ASS 4 does not have. See Table 7. Consequently, the *suvarṇa* of the VIS is 400 times that of the ASS. Moreover, according to the commentator Śaṅkara (in VIS),[42] we have to read the compound, *daśārdha-guñjaṃ*, as "ten and a half *guñjās*", while it is "half of ten *guñjās*" acccording to the commentators Gaṇeśa and Mahīdhara (in ASS).[43]

> ASS 4: Experts in balance-beam say that a *māṣa* comprises half of ten *guñjās*, that a *karṣa* is ⟨measured⟩ by sixteen ⟨weights⟩ called *māṣa*, that a *pala* is ⟨measured⟩ by four

[41] For different values of *tulā*, including the present one, see Saradha Srinivasan, *Mensuration in Ancient India*, Delhi: Ajanta Publications, 1979, p. 99.

[42] daśa cārdhaṃ ceti daśārdham / ardhottaradaśakamityarthaḥ / yādṛśībhistisṛbhirguñjābhireko vallastādṛśībhiḥ sārdhadaśakamitābhirekaṃ māṣaṃ pravadanti / tairmāṣaiḥ ṣoḍaśabhirekaḥ karṣaḥ / guñjānāmaṣṭaṣaṣṭyuttaraśatena vā karṣo bhavati / karṣaiścaturbhirekaṃ palam / tacchataṃ palaśatamekā tulā ca syāt / tatra suvarṇasya yatpalaśataṃ tatsuvarṇasaṃjñitaṃ bhavatītyapi boddhavyam / (part of Śaṅkara's commentary on VIS 4)

[43] daśārdhaṃ pañca / tanmitā guñjā mānaṃ yasyāsau daśārdhaguñjaḥ / taṃ māṣaṃ vadanti tulājñāḥ / (part of Gaṇeśa's commentary on ASS 4) pañca guñjā māṣaḥ / (part of Mahīdhara's commentary on ASS 4)

karṣas, and that a *karṣa* of gold is called a *suvarṇa*.[44]

VIS 4: They say that a *māṣa* comprises ten and a half *guñjās*, that a *karṣa* is ⟨measured⟩ by sixteen ⟨weights⟩ called *māṣa*, that a *pala* is ⟨measured⟩ by four *karṣas*, that a *tulā* is hundred of it (*pala*), and that ⟨the same amount⟩ of gold is called a *suvarṇa*.[45]

The relationship, 5 *guñjās* = 1 *māṣa*, is common and has been noted in Hindu law books such as those of Manu and Yājñavalkya, while the other one, $10\frac{1}{2}$ *guñjās* = 1 *māṣa*, has not been attested anywhere else, although a similar relationship, namely 10 *guñjās* = 1 *māṣa*, occurs in the *Śukranīti* (2.387–388), etc.[46]

Verse 4 in Parameśvara's text must have been closer to VIS 4, since he cites the phrase, "a *tulā* is a hundred of it" (*tulā tacchataṃ*). Moreover, Śaṅkara's interpretation of *daśārdha* seems to be based on Parameśvara's.[47]

III.2 Verses 10–11

Verses 10–11 give the names of the first 18 decimal places, and at two places the VIS difers from the ASS.[48] That is, 10^9 is called *abja* (lotus) in the ASS but *abda* (cloud) in the VIS, and 10^{13} is called *śaṅku* (a spear or a peg) in the ASS but *śaṅkhu* (employed in the plural form, *-śaṅkhavas*) in the VIS. The *abda* for 10^9 and the *śaṅkha* (a conch shell) for 10^{13} occur also in Śaṅkara's quotation from PG 7–8 (= Tr paribhāṣā 2–3), although the published texts of the PG and Tr, just like the ASS, read *abja* and *śaṅku*.[49] It is, however, noteworthy that the unique manuscript of the PG has *śaṅkha* and not *śaṅku*. See Table 8. Mahāvīra assigns 10^{18} to *śaṅkha*

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[44] daśārdhaguñjaṃ pravadanti māṣaṃ māṣāhvayaiḥ ṣoḍaśabhiśca karṣam /
karṣaiścaturbhiśca palaṃ tulājñāḥ karṣaṃ suvarṇasya suvarṇasaṃjñam // ASS 4 //

[45] VIS 4 has *tulā tacchataṃ* for *tulājñāḥ karṣaṃ* in ASS 4.

[46] *Śukranīti*, edited by Paṇḍita Brahmaśaṅkara Miśra, Kāśī Sanskrit Series 185, 2nd edition, Vārāṇasī: Chowkhamba Sanskrit Series Office, 1987. The passage runs: guñjā māṣastathā karṣaḥ padārdhaḥ prastha eva hi // 387 // yathottarā daśaguṇāḥ ... / See also S. Srinivasan, *op. cit.*, p. 103.

[47] daśārdhaguñjamiti / daśa cārdhañca daśārdham / guñjānāmaṣṭaṣaṣṭyuttaraśatena karṣo bhavati / tulā tacchatamiti / palānāṃ śataṃ tulāsaṃjñitaṃ bhavati // (Parameśvara's commentary on L 4)

[48] ekadaśaśatasahasrāyutalakṣaprayutakoṭayaḥ kramaśaḥ /
arbudamabjaṃ kharvanikharvamahāpadmaśaṅkavastasmāt // ASS 10 //
jaladhiścāntyaṃ madhyaṃ parārdhamiti daśaguṇottarāḥ saṃjñāḥ /
saṃkhyāyāḥ sthānānāṃ vyavahārārthaṃ kṛtāḥ pūrvaiḥ // ASS 11 //

[49] PG = *Pāṭīgaṇita* of Śrīdhara, edited with an old anonymous commentary by Kripa Shankar Shukla, Lucknow: Lucknow University, 1959. Tr = *Triśatikā* of Śrīdhara, edited by Sudhākara Dvivedin, Benares: Chandraprabha Press, 1899.

Table 8: Terms for 10^9 and 10^{13} in ASS, VIS and Śrīdhara's works

	ASS	VIS	PG/Tr (cited in SL)	PG/Tr (published)	PG (ms.)
10^9	abja	abda	abda	abja	abja
10^{13}	śaṅku	śaṅkhu	śaṅkha	śaṅku	śaṅkha

Table 9: Formulas for cubing

Verse Nos.		Contents
ASS	VIS	
26.1	26.1	Formula: $p^3 = 3abp + (a^3 + b^3)$ where $p = a + b$.
26.2		Formula: $(p^2)^3 = (p^3)^2$.
	26.2	Formula: $p^3 = (p + a)(p - a)p + a^2p$.

in his list of 24 decimal places (GSS 1.67).[50]

The words employed for these values in Parameśvara's text are not known since only part of the verses is cited in his commentary.

III.3 Verse 26

Verse 26 is concerned with computational rules for the cube of a number based on algebraic identities. The formula given in VIS 26.2 is different from that of ASS 26.2.[51] See Table 9.

Parameśvara comments on the rule, not of ASS 26.2, but of VIS 26.2.[52]

III.4 Verse 71

Verse 71 is one of the six verses, 67–72, which give examples for the "multiplier computation" (*guṇakarman*), two rules for which are given in verses 65–66.[53]

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[50] GSS = *Gaṇitasārasaṃgraha* of Mahāvīra, edited by M. Raṅgācārya, Madras: Government Press, 1912.

[51] khaṇḍābhyāṃ vāhato rāśistrighnaḥ khaṇḍaghanaikyayuk /
vargamūlaghanaḥ svaghno vargarāśerghano bhavet // ASS 26 //
iṣṭonayugrāśihato veṣṭavargaghnarāśiyuk // VIS 26.2 //

[52] iṣṭarāśiṃ khaṇḍīkṛtya tābhyāmiṣṭarāśiṃ nihatya punastribhiśca nihatya tayoḥ khaṇḍayorghanaikyaṃ ca prakṣipet / tadghanaphalaṃ bhavati // athaveṣṭonarāśiriṣṭayutarāśiḥ kevalarāśiśceti trayāṇāmeṣāṃ rāśīnāṃ vadha iṣṭavarganihataṃ kevalarāśiṃ prakṣipet / tadghanaṃ bhavati // evaṃ ghanakarma // (Parameśvara's commentary on L 26)

[53] guṇaghnamūlonayutasya rāśerdṛṣṭasya yuktasya guṇārdhakṛtyā /
mūlaṃ guṇārdhena yutaṃ vihīnaṃ vargīkṛtaṃ praṣṭurabhīṣṭarāśiḥ // ASS 65 //
yadā lavaiśconayutassa rāśirekena bhāgonayutena bhaktvā /
dṛśyaṃ tathā mūlaguṇaṃ ca tābhyāṃ sādhyastataḥ proktavadeva rāśiḥ // ASS 66 //

Rule 1: When $x \mp a\sqrt{x} = b$ (where a is called "the multiplier" and b "the visible"),

$$x = \left(\sqrt{b + \left(\frac{a}{2}\right)^2} \pm \frac{a}{2}\right)^2.$$

Rule 2: When $x \mp a\sqrt{x} \mp mx = b$, first rewrite the given equation as

$$x \mp \frac{a}{1 \mp m} \cdot \sqrt{x} = \frac{b}{1 \mp m},$$

and then apply Rule 1 to the multiplier and the visible thus obtained.

The example given in verse 71, which is concerned with a cluster of bees, is,

$$x - \sqrt{\frac{x}{2}} - 8 \times \frac{x}{9} = 2,$$

according to the ASS,[54] but,

$$x - \frac{x}{2} - 8 \times \frac{x/2}{9} - \frac{\sqrt{x/2}}{2} = 1,$$

according to the VIS.[55] These equations are, however, equivalent to each other, and the answer is 72 in both cases.

Parameśvara comments on verse 67 but ignores 68–72.

III.5 Verse 84

Verse 84 gives an example for the five-quantity operation, an algorithm for which is given in verse 82 (cf. IV.3). The example is concerned with the interest on loaned money or capital (*mūla-dhana*), and the capital is "sixty-two and a half" (*sārdhadviṣaṣṭi*), $62\frac{1}{2}$, in the ASS but "two six's and a half" (*sārdhadviṣaṭka*), $12\frac{1}{2}$, in the VIS.

> ASS 84: If the interest on one hundred for one and a third months is five and a fifth, then what is the fruit (interest) on sixty-two and a half for three and a third months?

[54] alikuladalamūlaṃ mālatīṃ yātamaṣṭau
nikhilanavamabhāgāścālinī bhṛṅgamekam /
niśi parimalalubdhaṃ padmamadhye niruddhaṃ
pratiraṇati raṇantaṃ brūhi kānte lisaṃkhyām // ASS 71 //

[55] alikuladalamabdhestīrayātaṃ tathāṣṭau
dalitanavamabhāgāścālinī bhṛṅgamekam /
niśi padadalahīnaṃ padmamadhye niruddhaṃ
pratiraṇati raṇantaṃ brūhi kānte lisaṃkhyām // VIS 71 //

Let it be told.[56]

VIS 84: If the interest on one hundred for one and a third months is five and a fifth, then what is the fruit (interest) with two six's and a half for three and a third months? Let it be told.[57]

It naturally follows that the answer, $7\frac{4}{5}$, in the ASS is different from the one, $1\frac{14}{25}$, in the VIS.

Parameśvara's text agrees with the VIS in this regard.[58]

III.6 Verse 88

Verse 88 is concerned with barter. In the ASS, it prescribes an algorithm, which is an extension of the algorithm for the $(2n+1)$-quantity operation prescribed in verse 82 (cf. IV.3). That is, ASS 88 requires one to exchange the prices, in addition to the denominators of the fractions and the "fruits", of both "sides". VIS 88, on the other hand, only refers to the $(2n+1)$-quantity operation and does not mention the mutual exchange of prices.

ASS 88: In the barter, too, ⟨the procedure is⟩ the same ⟨as in the five-quantity operation, etc.⟩, but there is always mutual exchange in the price.[59]

VIS 88: In the barter, too, which involves denominators, numerators and prices, the procedure for that (i.e., the five-quantity operation, etc.) should be performed in the like manner.[60]

The problem treated here is: When the price for a_2 of a certain thing (A) is a_1 and that for b_2 of another thing (B) is b_1, how much of B is obtained for p of A?

According to the ASS, this is solved as follows ($v.$ = verse).

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ p & \end{bmatrix} \xrightarrow{v.82} \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ & p \end{bmatrix} \xrightarrow{v.88} \begin{bmatrix} b_1 & a_1 \\ a_2 & b_2 \\ & p \end{bmatrix} \xrightarrow{v.82} \frac{a_1 b_2 p}{a_2 b_1}.$$

According to VIS 88 and the *vāsanā* on VIS 89, which gives an example of barter, on the other hand, the barter is apparently reduced to a single three-quantity oper-

[56] satryaṃśamāsena śatasya cetsyātkalāntaraṃ pañca sapañcamāṃśāḥ / māsaistribhiḥ pañcalavādhikaistatsārdhadviṣaṣṭeḥ phalamucyatāṃ kim // ASS 84 //

[57] *taiḥ sārdhadviṣaṭkaiḥ* in VIS 84 for *tatsārdhadviṣaṣṭeḥ* in ASS 84.

[58] atra satryaṃśo māsaḥ śataṃ ca pramāṇarāśiḥ / pañcalavādhikāstrayaḥ sārdhadviṣaṭkaṃ cecchārāśiḥ / (part of Parameśvara's commentary on L 84)

[59] tathaiva bhāṇḍapratibhāṇḍake pi viparyayastatra sadā hi mūlye // ASS 88 //

[60] tathaiva bhāṇḍapratibhāṇḍake pi vidhirvidheyo sya harāṃśamūlye // VIS 88 //

ation as in the case of the $(2n+1)$-quantity operation (cf. IV.3).

$$\begin{bmatrix} a_1 \; b_1 \; p \\ a_2 \; b_2 \end{bmatrix} \quad \longrightarrow \quad \begin{bmatrix} a_2 b_1 \;\; a_1 b_2 \;\; p \end{bmatrix} \quad \longrightarrow \quad \frac{a_1 b_2 p}{a_2 b_1}.$$

In Śaṅkara's solutions to some of the examples cited by him after his commentary on VIS 89, the initial arrangement of the five factors is:

$$\begin{bmatrix} a_1 \; b_1 \\ a_2 \; b_2 \; p \end{bmatrix}$$

Verse 88 in Parameśvara's text must have been closer to VIS 88 than to ASS 88 as his paraphrase of the verse shows.[61]

III.7 Verse 142

Verses 141 and 142 each provide a set of rational solutions to $x^2 + y^2 = z^2$.

$$\text{Verse 141:} \quad x = a, \quad y = \frac{2b \cdot a}{b^2 - 1}, \quad z = by - x \left\langle = \frac{(b^2+1)a}{b^2-1} \right\rangle.$$

$$\text{Verse 142:} \quad x = a, \quad y = \frac{1}{2}\left(\frac{a^2}{b} - b\right), \quad z = \frac{1}{2}\left(\frac{a^2}{b} + b\right).$$

There is no substantial difference, both in the contents and in the expressions, between ASS 141–142[62] and VIS 141–142,[63] but Śaṅkara, the commentator on the VIS, has recorded a variant for the first line of verse 142. The difference, however, lies only in their expressions and not in the contents.

> ASS 142.1 = VIS 142.1: Or, the arm (i.e., side, x) is an optional number (a). Its square is divided by ⟨another⟩ optional number (b), and put down twice (i.e., in two places). Decreased ⟨in one place⟩ and increased ⟨in the other⟩ by the optional number and halved, ⟨they are respectively the upright (y) and the ear (i.e., hypotenuse, z)⟩.
> Variant for VIS 142.1: Or else, the square of any optional arm (i.e., side, $x = a$) is divided by ⟨another⟩ optional number (b). The halves of the difference and of the sum of the

[61] bhāṇḍapratibhāṇḍake ye harā aṃśāśca mūlyadravyaṃ ca bhavanti tatrāpyasya pañcādirāśikasya vidhirvidheyaḥ // (part of Parameśvara's commentary on L 88)

[62] iṣṭo bhujo smāddviguṇeṣṭanighnādiṣṭasya kṛtyaikaviyuktayāptam /
koṭiḥ pṛthakseṣṭaguṇā bhujonā karṇo bhavettryasramidaṃ tu jātyam // ASS 141 //
iṣṭo bhujastatkṛtiriṣṭabhaktā dviḥsthāpiteṣṭonayutārdhitā vā /
tau koṭikarṇāviti koṭito vā bāhuśrutī vākaraṇīgate staḥ // ASS 142 //

[63] Variants in VIS. In 141: *pṛthaksthesṭa-* for *pṛthakseṣṭa-*, *bhujābhyām* for *tu jātyam*. In 142: *icchā* for *iṣṭo*, *-ārdhitā yā* for *-ārdhitā vā*, *-śrutī cā-* for *-śrutī vā-*.

Table 10: Contents of Verse 160

Verse Nos.		Contents
ASS	VIS	(x, y, z = side, upright, hypotenuse of a right triangle.)
158	158	Formula: $x,\, y = \left(a \pm \sqrt{2c^2 - a^2}\right)/2$ where $x + y = a$, $z = c$.
159	159	Example: $a = 23$, $c = 17$. Answer: $x = 8$, $y = 15$.
160		Example: $b = x - y = 7$, $c = 13$. Answer: $x = 12$, $y = 5$.
	160	Formula: $x,\, y = a/2 \pm \sqrt{c^2/2 - (a/2)^2}$ where $x + y = a$, $z = c$.

divisor and the quotient ⟨are respectively the upright (y) and the ear (i.e., hypotenuse, z)⟩.[64]

Parameśvara's text of the present passage is closer to the VIS as he quotes the phrase, *tryaśramidaṃ bhujābhyām*, which occurs at the end of VIS 141 and not in the ASS, but he does not refer to the variant for VIS 142.1 recorded by Śaṅkara.

III.8 Verses 150 and 152

Different expressions are employed for the same numbers in ASS 150 and VIS 150, and so also in ASS 152 and VIS 152.

In verse 150. The number 32 in the phrase, "a bamboo thirty-two *hastas* long", is expressed as *dvi-tri* ("two-three") in the ASS but as *danta* ("tooth") in the VIS. The latter is a word numeral (so-called *bhūtasaṃkhyā*) for 32.

In verse 152. The height, 9 *hastas*, in the phrase, "a post nine *hastas* high", is expressed as *hasta-nava-ucchrita* in the ASS but as *nanda-kara-ucchrita* in the VIS. *Nanda* is a word numeral for 9.

Commenting on 150 and 152, Parameśvara uses the ordinary numerals, *dvātriṃśat-* and *nava-*, for 32 and 9 respectively. It is, therefore, not certain which text is closer to his.

III.9 Verse 160

In the ASS verse 160 gives the second example for the formula given in verse 158, while in the VIS it gives another form of the same formula. See Table 10.

VIS 160 is, however, identical with an alternative rule given by Parameśvara in his commentary on verse 158. It is noteworthy that VIS 160 is introduced with the words, *atraivaṃ vā sūtram* ("In this regard, there is an alternative rule as follows"), while exactly the same verse is introduced with almost the same words, *evaṃ vā sūtram* ("There is an alternative rule as follows"), in Parameśvara's commentary.[65]

[64] yadveṣṭabāhoḥ kṛtiriṣṭabhaktā hārāptayorantarayugdale ye / (Variant for VIS 142.1)

[65] See II.6 (Verse 161^a) for Parameśvara's similar introduction.

The example in ASS 160 is not exactly meant for the formula in verse 158, since it is concerned with the case where $(x-y)$ and z are given while in that formula $(x+y)$ and z are presupposed. This fact seems to have troubled the commentators of the ASS. Gaṇeśa forcibly interprets verse 158 and states that it includes the formula, $x,\, y = \left(\sqrt{2c^2 - b^2} \pm b\right)/2$, as well. Mahīdhara, on the other hand, adds another verse for it immediately before his comment on ASS 160. The provenance of this verse is not known.

> ML 159.1: The square root from twice the square of the hypotenuse minus the difference of the side and the upright multiplied by itself, decreased and increased by the difference and halved at two places (i.e., separately), become measured each by the side and the upright.[66]

III.10 Verse 163

Verse 163 gives a definition (*lakṣaṇa*) of "a non-field" (*akṣetra*) or an impossible geometric figure. Its expression in the ASS differs from that in the VIS, although both definitions are substantially the same.

> ASS 163: A field with straight sides pointed out by a bold man, where the sum of the sides except one is smaller than, or equal to, that side, should be known as a non-field.[67]
> VIS 163: If the sum of the sides except one, which is smaller than, or equal to, the size of that side, is pointed out due to ignorance, it is a non-field since such a field does not exist.[68]

Śaṅkara records a variant reading for this verse, which restricts the non-figure to a trilateral and a quadrilateral.

> SL 163.1: When, in a trilateral or a quadrilateral field, the sum of the sides except one is smaller than, or equal to, that side, it should be known as a non-field.[69]

This verse is identical with ASS 163 except for the first quarter.

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[66] nijaguṇātkila koṭibhujāntarāddviguṇakarṇakṛteḥ patitātpadam /
vivarahīnayutaṃ dalitaṃ dvidhā prabhavato bhujakoṭimite pṛthak // ML 159.1 //

[67] dhṛṣṭoddiṣṭamṛjubhujakṣetraṃ yatraikabāhutaḥ svalpā /
taditarabhujayutirathavā tulyā jñeyaṃ tadakṣetram // ASS 163 //

[68] svalpā taditarabhujayutirathavā tulyaikabhujamānāt /
uddiṣṭā yadi mohānnedṛkkṣetraṃ bhavatyato kṣetram // VIS 163 //

[69] caturaśre tryaśre vā kṣetre yatraikabāhutaḥ svalpā /
taditarabhujayutirathavā tulyā jñeyaṃ tadakṣetram // SL 163.1 //

The verse for the definition of "non-field" which Parameśvara comments on begins with the word, *svalpā* ("smaller"), and the words he uses in his explanation are closer to VIS 163 than to ASS 163.[70] Therefore, the verse Parameśvara had in hand must have been nearer to VIS 163 than to ASS 163, although, like the variant Śaṅkara records, he restricts the figure to a trilateral and a quadrilataral.

III.11 Verse 182

Verse 182 gives the ranges for the lengths of the diagonals, d_1 and d_2, of a quadrilateral when its four sides, a, b, c, and d, are given. VIS 182 is substantially the same as ASS 182, although their expressions are not exactly the same. What is interesting is that the variant Śaṅkara records gives slightly different ranges for them.

> ASS 182: When one has assumed the smaller of the sums of the sides resting on one ear (diagonal) to be the ground (base) and the remaining sides to be the arms (flank sides) ⟨of a triangle⟩, ⟨its⟩ perpendicular should be determined. In the same way, ⟨the perpendicular is determined for⟩ the other ear. ⟨Then,⟩ a diagonal is never longer than its own ground and is not shorter than the perpendicular for the other. Having recognized this in that way, the intelligent one should assume optional ears.[71]
> Variant (for the 2nd line and the first 7 syllables of the 3rd line): ⟨its⟩ perpendicular should be determined. This ear is smaller than the smaller sum of the sides and is neither equal to nor longer than ⟨that⟩. The other ⟨ear⟩ is greater than the perpendicular.[72]

Let us assume that the first diagonal, d_1, goes from the intersection of a and b to that of c and d, and the second diagonal, d_2, from the intersection of a and d to that of b and c. We also assume that $a + b < c + d$ and $a + d < b + c$. Let h_1 be the perpendicular from the vertex of the triangle, $(b, c, a + d)$, to its base $a + d$, and h_2 the perpendicular from the vertex of the triangle, $(c, d, a + b)$, to its base $a + b$.

[70] athākṣetralakṣaṇamāha / svalpetyādinā / tryaśre caturaśre voddiṣṭe kṣetre yadyekabhujāmānād-itareṣāṃ bhujānāṃ yutiḥ svalpā bhavatyathavā tulyā bhavati tadā tatkṣetramakṣetramiti vācyaṃ yatastādṛgvidhaṃ kṣetraṃ na saṃbhavati // (Parameśvara's commentary on L 163)

[71] karṇāśritasvalpabhujaikyamurvīṃ prakalpya taccheṣabhujau ca bāhū /
sādhyo valambo tha tathānyakarṇaḥ svorvyāḥ kathaṃcicchravaṇo na dīrghaḥ /
tadanyalambānna laghustathedaṃ jñātveṣṭakarṇaḥ sudhiyā prakalpyaḥ // ASS 182 //
VIS 182 reads: *sādhyo valambaśca* for *sādhyo valambo tha*, *tathānyakarṇasyorvyāḥ* for *tathānya-karṇaḥ svorvyāḥ*, and *tadanyalambācca* for *tadanyalambānna*. The second of these variants requires the following changes in the above translation: "⟨the perpendicular is determined for⟩ the other ear" ⟶ "⟨the perpendicular is determined⟩ for the other ear"; "its own ground" ⟶ "⟨its own⟩ ground".

[72] sādhyo valambo laghudoḥsamāsādūno tra karṇo na samo na dīrghaḥ / anyastu lambādhikaḥ / iti keṣucitpustakeṣu pāṭho dṛśyate / (part of Śaṅkara's commentary on VIS 182)

Then, according to ASS 182 and VIS 182,

$$h_2 \leq d_1 \leq a+b, \qquad h_1 \leq d_2 \leq a+d.$$

According to the variant, on the other hand, the ranges do not include the equal signs, that is to say,

$$h_2 < d_1 < a+b, \qquad h_1 < d_2 < a+d.$$

This is true with "proper" quadrilaterals, and in that sense the author of the variant seems to have tried to correct the "fault" of the original verse. The original verse, however, seems to have been meant for "extended" quadrilaterals, which include trilaterals, a fourth side of which is regarded as zero. This is a traditional concept of triangles.[73]

It is noteworthy that Parameśvara cites verse 182 in full, since usually he cites only the first few words of each verse.[74] Verse 182 cited by him agrees with VIS 182, but a doubt about the authenticity of the present verse seems to have existed in his time, as he remarks as follows at the end of his commentary on it:[75] "This ⟨stanza⟩ (i.e., verse 182) has been interpolated, according to some people." He has made the same remark at the end of his commentary on L 188, which is a verbatim quotation from BSS 12.28.

III.12 Verses 189–190

Verses 189–190 deal with the lengths of the diagonals of a cyclic quadrilateral constructed by a certain prescribed method.

> ASS 189: In an odd quadrilateral[76] assumed with those sides which are the arms and uprights of any two arbitrary right-angled triangles multiplied by the reciprocal ear, the ears are ⟨obtained⟩ from those two triangles.
>
> ASS 190: ⟨That is⟩, the product of the arms increased by the product of the uprights is

[73] See, for example, *Brāhmasphuṭasiddhānta* 12.21. Cf. II.8 above.

[74] See Introduction above.

[75] iṣṭakarṇakalpanāyāṃ viśeṣapradarśanāya sūtram / karṇāśrita- ... prakalpyaḥ // ⟨L 182 //⟩ iti / karṇasyobhayapārśvagate ye bhujadvandve tayoryasya dvandvasyaikyamalpasaṃkhyaṃ syāttasya dvandvasyaikyamurvīṃ prakalpya karṇasyānyapārśvagatau bhujau bāhū prakalpya taistribhistribhujoktaṃ lambaṃ sādhayet / evamanyakarṇasya pārśvagatairbhujairapi lambaḥ sādhyaḥ / tatra tathākalpitabhūmeḥ kathañcidapi dīrghaḥ karṇaḥ kadācidapi na syāt / tathākalpitādanyakarṇāśritalambādūnaśca na syāt / etajjñātveṣṭakarṇaḥ prakalpyaḥ / etatprakṣiptamiti kecit // (Parameśvara's commentary on L 182)

[76] An "odd quadrilateral" (*viṣama-catur-bhuja*) means a figure formed by four straight sides (*bhujas*) of different (*viṣama*) lengths.

> one ear, and the sum of the products of the arms and the uprights is the other. We do not understand why such a difficult means was employed by our predecessors when this easy one exists.[77]

Let (a_1, b_1, c_1) and (a_2, b_2, c_2) stand for the arm (side), upright and ear (hypotenuse) of two arbitrary chosen right-angled triangles, and let a quadrilateral be constructed with the four sides, $(a, b, c, d) = (a_2c_1, a_1c_2, b_2c_1, b_1c_2)$, and with two diagonals (d_1, d_2) orthogonal to each other (hence it is a cyclic quadrilateral). Then, ASS 190 gives the relationships,

$$d_1 = a_1a_2 + b_1b_2, \qquad d_2 = a_1b_2 + a_2b_1.$$

The criticism made at the end of verse 190 is to Brahmagupta's formula cited in the previous verse,[78]

$$d_1 = \sqrt{\frac{ab+cd}{ad+bc}\cdot(ac+bd)}, \quad d_2 = \sqrt{\frac{ad+bc}{ab+cd}\cdot(ac+bd)},$$

where (a, b, c, d) are the sides of a cyclic quadrilateral.

VIS 189–190 are the same as ASS 189–190 except for the last quarter of verse 189, which reads, "The two ears are like that, and two triangles are ⟨produced⟩ from it (i.e., on both sides of each diagonal)",[79] in place of, "The ears are ⟨obtained⟩ from those two triangles". The meaning of the first half of this quarter, "The two ears are like that", is not clear.

Śaṅkara's reading of the same passage, which is slightly different from that of the VIS itself, is interesting as it refers to the "third" ("first", according to Śaṅkara) diagonal of a cyclic quadrilateral.[80] He says: "⟨The text reads:⟩ 'A ear is ⟨computed⟩ like that.' When this quadrilateral figure is assumed (i.e., constructed), what[81] is the ear? ⟨It is⟩, indeed, like that. This means that the ear of one of the right triangles multiplied by the ear of the other is the first ear ⟨of the quadrilateral⟩."[82]

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[77] abhīṣṭajātyadvayabāhukoṭayaḥ parasparaṃ karṇahatā bhujā iti /
caturbhujaṃ yadviṣamamaṃ prakalpitaṃ śrutī tu tatra tribhujadvayāttataḥ // ASS 189 //
bāhvorvadhaḥ koṭivadhena yuksyādekā śrutiḥ koṭibhujāvadhaikyam /
anyā laghau satyapi sādhane sminpūrvaiḥ kṛtaṃ yadguru tanna vidmaḥ // ASS 190 //

[78] karṇāśritabhujaghātaikyamubhayathānyonyabhājitaṃ guṇayet /
yogena bhujapratibhujavadhayoḥ karṇau pade viṣame // L 188 = BSS 12.28 //

[79] śrutī tu tadvattribhujadvayaṃ tataḥ // (VIS 189d)

[80] For a discussion of the third diagonal, see V. Mishra and S. L. Singh, "Incircumscribing Triangles and Cyclic Quadrilaterlas in Ancient and Medieval Indian Geometry," *Sūgakushi Kenkyū* 161, 1999, 1–11.

[81] Read *kā*, instead of *yā*, as in two of the four mss. used for the VIS.

[82] śrutistu tadvaditi / kalpite smiṃścaturaśrakṣetre hi yā śrutistadvadeva / ekasya jātyatryaśrasya karṇastaditarakarṇaguṇitaḥ prathamaḥ karṇo bhavatītyarthaḥ // (part of Śaṅkara's commentary

That is, the third diagonal (d_3), which is produced when two adjacent sides of the cyclic quadrilateral are interchanged, is the product of the hypotenuses of the two right triangles,

$$d_3 = c_1 c_2.$$

In the ASS, this third diagonal is briefly touched upon at the end of the *vāsanā* on verses 189–190: "Now, if the ⟨quadrilateral⟩ figure is placed after one has exchanged a flank side and the top, then the product of the ears of the two right triangles is a second ear."[83] The VIS does not have this passage.

Śaṅkara's reading of the last quarter of verse 189 seems to be based on that of Parameśvara, who says: "⟨The text reads:⟩ 'A ear is ⟨computed⟩ like that.' In that ⟨quadrilateral⟩ figure one ear should be made (i.e., calculated) like that. This means that the product of the ears of the two right triangles is one ear ⟨of the quadrilateral⟩."[84]

III.13 Verse 192

Verses 191–192 give an examle of a "needle-figure" (*sūcīkṣetra*), the lengths of whose various parts are to be calculated. The wording of VIS 192 is different from that of ASS 192, and does not refer to "the lengths of the two arms of the needle⟨-figure thus produced⟩". Śaṅkara has recorded a variant for two quarters in verse 192.[85]

> ASS 191: In a ⟨quadrilateral⟩ figure, where the length of the ground (base) is three hundred, the mouth (top) is equal to [primary-substances, moon] (125), and the two arms (flanks) are equal to [space, Utkṛti-meter] (260) and [arrows, Atidhṛti-meter] (195), one of the two ears (diagonals) is equal to [space, eight, twins] (280) and the other to [lunar-days, natural-properties] (315), and its two perpendiculars are equal to [cows, Dhṛti-meter] (189) and [Jina, twins] (224).[86]

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on VIS 189d)

[83]atha yadi pārśvabhujamukhayorvyatyayaṃ kṛtvā nyastaṃ kṣetraṃ tadā jātyadvayakarṇayorvadho dvitīyakarṇaḥ // (part of the *vāsanā* on ASS 189–190)

[84]śrutistu tadvaditi / tasmin kṣetra ekā śrutistadvatprakalpyā / jātyadvayasya karṇayorāhatirekaḥ karṇa ityarthaḥ // (part of Parameśvara's commentary on L 189d)

[85]The variant cited by Śaṅkara is one quarter line but its contents correspond to those of the second and the third quarters of VIS 192. Here and hereafter, a pair of square brackets, [A], in the translations indicates that A is a number expressed by word-numerals.

[86]kṣetre yatra śatatrayaṃ kṣitimitistattvendutulyaṃ mukhaṃ
bāhū khotkṛtibhiḥ śarātidhṛtibhistulyau ca tatra śrutī /
ekā khāṣṭayamaiḥ samā tithiguṇairanyātha tallambakau
tulyau godhṛtibhistathā jinayamairyogācchravolambayoḥ // ASS 191 //

ASS 192: Tell the parts of the ears and perpendiculars below their intersection,[87] and the perpendicular and projections from the intersection of the two ears. Its needle⟨-figure⟩ will be ⟨constructed⟩ from the intersection of the two arms when extended along their own ways; tell the perpendicular from it and the corresponding projections; and what are the lengths of the two arms of the needle⟨-figure thus produced⟩? Tell everything, calculator, if you are well versed in this figure.[88]

VIS 192: Tell the parts of the ears and perpendiculars below their intersection,[89] the perpendicular and projections from the intersection of the two ears, the needle, which will be ⟨made⟩ by the two arms extended along their own ways, together with its projections and its perpendicular as well when the two arms ⟨of the needle⟩ are unknown.[90] What is ⟨the size of⟩ everything? Tell ⟨it⟩, calculator, if you are well versed in this figure.[91]

Variant[92]: ⟨Tell⟩ the size of the two arms extending up to the tip of the needle, and of the two projections as well.[93]

Parameśvara's text of the present passage is closer to the VIS as he cites the phrase, *śūnyapramāṇena*, which occurs in VIS 192 and not in ASS 192. Explaining the phrase, he says: "This means: 'Without knowing the size of the perpendicular and the arms of the needle⟨-figure⟩.' "[94] Śaṅkara, on the other hand, takes it to have an affirmative sense: "This means: 'Having known the size of the perpendicular and

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[87]Part of this sentence is stated at the end of verse 191.

[88]tatkhaṇḍe kathayādhare śravaṇayoryogācca lambābadhe
tatsūcī nijamārgavṛddhabhujayoryogādyathā syāttataḥ /
sābādhaṃ vada lambakaṃ ca bhujayoḥ sūcyāḥ pramāṇe ca ke
sarvaṃ gāṇitika pracakṣva nitarāṃ kṣetre tra dakṣo si cet // ASS 192 //

[89]Part of this sentence is stated at the end of verse 191.

[90]Literally, "with empty sizes of the two arms ⟨of the needle⟩." According to Parameśvara, "without knowing the sizes of the perpendicular and two arms of the needle". See below.

[91]tatkhaṇḍe kathayādhare śravaṇayoryogāvalambābadhāḥ
tatsūciṃ nijamārgavṛddhabhujayoryogena yā syāttataḥ /
sābādhāṃ tvavalambakaṃ ca bhujayoḥ śūnyapramāṇena kiṃ
sarvaṃ gāṇitika pracakṣva nitarāṃ kṣetre tra dakṣo si cet // VIS 192 //
Śaṅkara however reads, as in ASS 192, *tatsūcī* instead of *tatsūciṃ*, and *sābādhaṃ* instead of *sābādhāṃ*. This is known from his paraphrase of the passage: tathā nijamārgavṛddhayoḥ pārśvabhujayoryoganiṣpannasūcyā lambo pi vaktavyaḥ / so pyābādhāsahito vācyaḥ / (part of Śaṅkara's comemntary on VIS 191–192)

[92]For the phrase, "the needle, . . . the two arms ⟨of the needle⟩ are unknown."

[93]sūcyagrāvadhivardhamānabhujayormānaṃ tathābādhāṃ tayoḥ iti vā pāṭhaḥ. Read *tathābādhayoḥ* instead of *tathābādhāṃ tayoḥ* to meet the meter (Śārdūlavikrīḍita).

[94]śūnyapramāṇeneti / sūcīlambabhujayoḥ pramāṇamajñātvetyarthaḥ // (part of Parameśvara's commentary on L 191–192)

the arms of the needle⟨-figure⟩.' "[95] Since, however, this interpretation does not fit in the context, *jñātvā* in Śaṅkara's commentary may be a misprint or a miscopy of *ajñātvā*.

III.14 Verse 199

Verse 199 gives two approximate values of π. There is only a minor difference in wording between ASS 199 and VIS 199, which does not affect the mathematical contents. Noteworthy is that Śaṅkara records a variant for the first line of the verse, which gives a much better approximation. According to him, "some people have laid down another reading for the sake of a more accurate circumference," and "this is the reading accepted by the reasoning-knowers" (see the footnote for the translation below). Parameśvara is silent about this variant.

> ASS 199: When the diameter is multiplied by [zodiac, Nandas, fires] (3927) and divided by [space, arrows, sun] (1250), it (the result) is an accurate circumference. Or else, when ⟨the diameter is⟩ multiplied by twenty-two and divided by [mountains] (7), ⟨the result⟩ will be a rough ⟨circumference⟩ to be employed for worldy business.[96]
> Variant (for the first line): When the diameter is multiplied by [arrows, arrows, fires] (355) and divided by [Rāmas, moon, unity] (113), ⟨the result is⟩ a very accurate circumference.[97]

That is to say, according to ASS 199 and VIS 199,

$$C = \frac{3927d}{1250}, \qquad C = \frac{22d}{7},$$

and the variant replaces the former with:

$$C = \frac{355d}{113}.$$

After mentioning this variant, Śaṅkara cites two verses for "most accurate" (*āsannatama/sūkṣmatama*) values. One of them, which Śaṅkara explicitly ascribes

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[95] bhujayoḥ śūnyapramāṇeneti / sūcīlambabhujayoḥ pramāṇaṃ jñātvetyarthaḥ // (part of Śaṅkara's commentary on VIS 191–192)

[96] vyāse bhanandāgnihate vibhakte khabāṇasūryaiḥ paridhiḥ sa sūkṣmaḥ /
dvāviṃśatighne vihṛte tha śailaiḥ sthūlo thavā syādvyavahārayogyaḥ // ASS 199 //
VIS 199 reads: *susūkṣmaḥ* for *sa sūkṣmaḥ*, which would require the following change in the above translation: "it is an accurate circumference" ⟶ "⟨the result is⟩ a very accurate circumference."

[97] kecitpunaratraivāsannataraṃ paridhimuddiśya pāṭhāntaraṃ vyadhuḥ — vyāse śareṣvagnihate vibhakte rāmendurūpaiḥ paridhiḥ susūkṣmaḥ / iti / ayameva pāṭho yuktividāmabhimataḥ // (part of Śaṅkara's commentary on VIS 199)

Table 11: Coefficients for the sides of regular polygons

	ASS	VIS	Modern
k_3	10392$\underline{3}$	10392$\underline{2}$	103923.05
k_4	84853	84853	84852.81
k_5	70534	70534	70534.23
k_6	60000	60000	60000
k_7	520$\underline{55}$	520$\underline{67}$	52066.05
k_8	45922	45922	45922.01
k_9	410$\underline{31}$	410$\underline{43}$	41042.42

to Mādhavācārya, gives: $C/d = 2827433388233/10^{11}$; and the other, which is anonymous, prescribes: $C = 104348d/33215$.[98]

III.15 Verses 206–208

Verses 206–208 give the formula,

$$s_n = \frac{k_n d}{120000},$$

for calculating the side (s_n) of a regular n-gon, for $n = 3, 4, 5, 6, 7, 8, 9$, inscribed in a circle of diameter d.[99]

The values of the multipliers (or coefficients), k_n, are given in the same verses but those for k_3, k_7, and k_9 in the VIS are different from those in the ASS. See Table 11, where the underlines indicate the differences. Since k_7 and k_9 in the ASS are very crude, those in the VIS may have been meant to be corrections, although they are greater than the nearest integers by one, and k_3 in the VIS is worse than that in the ASS, which is indeed the nearest integer. The origins of these values have not been fully explained.[100]

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[98]For these approximations, see Takao Hayashi, Takanori Kusuba, Michio Yano, "Indian Values for π Derived from Āryabhaṭa's Value", *Historia Scientiarum* 37, 1989, 1–16 (esp. p. 6).

[99]tridvyaṅkāgninabhaścandraistribāṇāṣṭayugāṣṭabhiḥ /
vedāgnibāṇakhāśvaiśca khakhābhrarasaiḥ kramāt // ASS 206 //
bāṇeṣunakhabāṇaiśca dvidvinandeṣusāgaraiḥ /
kurāmadaśavedaiśca vṛttavyāse samāhate // ASS 207 //
khakhakhābhrārkasaṃbhakte labhyante kramaśo bhujāḥ /
vṛttāntastryasrapūrvāṇāṃ navāsrāntaṃ pṛthakpṛthak // ASS 208 //
The VIS reads, in 206, *dvidvy-* (corrected to *tridvy-* in one of the four manuscripts used for the VIS) for *tridvy-*, *-agnipañca-* for *-agnibāṇa-*; in 207, *śailartu-* for *bāṇeṣu-*, *triveda-* for *kurāma-*; and, in 208, *vṛttatattryaśra-* for *vṛttāntastryasra-*, and *navāntānāṃ* for *navāsrāntaṃ*.

[100]For a discussion in this regard, see Radha Charan Gupta, "The *Līlāvatī* Rule for Computing Sides of Regular Polygons." *The Mathematics Education* 9 (2), 1975, B, 25–29.

Parameśvara quotes the beginning of verse 206, which reads *dvidvyaṅka-* (...922). His text, therefore, must have been closer to the VIS.

III.16 Verse 240

Verses 239–240 deal with a problem of two shadows cast by a gnomon placed at two places in such a way that the lamp post and the two places for the gnomon make a straight line on the ground. In both ASS and VIS, verse 239 provides the formulas for the distance (x_i) from the foot of the lamp post to the tip of each shadow and for the height (y) of the lamp post:

$$x_i = \frac{as_i}{s_2 - s_1}, \quad y = \frac{gx_i}{s_i} \quad (i = 1, 2),$$

where s_i is the length of each shadow, a the distance between the tips of the two shadows, and g the height of the gnomon.

Verse 240 gives an example for this rule, but the given quantities in the VIS are different from those in the ASS. That is, they are g, s_i, and a in the VIS, but g, s_i, and b in the ASS, where b is the distance between the two places where the gnomon is fixed. In ASS 240, therefore, a is to be obtained by means of the relationship, $s_1 + a = s_2 + b$, before the application of the formulas prescribed in verse 239.

> ASS 240: O intelligent man, the shadow of a gnomon measured by [sun] (12) *aṅgulas* is seen to be eight *aṅgulas*. If, further, ⟨the shadow⟩ of the same ⟨gnomon⟩ put down at the place two *hastas* distant ⟨from the first place⟩ towards the tip of the ⟨first⟩ shadow is measured by [sun] (12) *aṅgulas*, how much is the distance between ⟨the tips of⟩ the shadows and ⟨the foot of⟩ the lamp, and the height of the lamp? Say if you know the practical mathematics called shadow.[101]
>
> $g = 12$, $s_1 = 8$, $s_2 = 12$ *aṅgulas*, $b = 2$ *hastas* ($= 48$ *aṅgulas*).
>
> VIS 240: The shadow of a gnomon measured by [sun] (12) *aṅgulas* is equal to [Vasus] (8), and to [sun] (12) ⟨when the gnomon is placed⟩ at another place on the same line. The distance between the tips of the shadows is [twins, arrows] (52). In that case, say the two distances lying from the foot of the lamp to the tips of the shadows. Also, if you know the practical mathematics called shadow, O friend, how much is the height of the lamp?[102]
>
> $g = 12$, $s_1 = 8$, $s_2 = 12$, $a = 52$ *aṅgulas*.

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[101] śaṅkorbhārkamitāṅgulasya sumate dṛṣṭā kilāṣṭāṅgulā
chāyāgrābhimukhe karadvayamite nyastasya deśe punaḥ /
tasyaivārkamitāṅgulā yadi tadā chāyāpradīpāntaraṃ
dīpauccyaṃ ca kiyadvada vyavahṛtiṃ chāyābhidhāṃ vetsi cet // ASS 240 //

[102] śaṅkorbhārkamitāṅgulasya vasubhistulyā tathā bhāskarair-
anyatrāpi ca tatpathe yamaśarāśchāyāgrayorantaram /
bhūmāne vada tatra dīpatalataśchāyāgrayormadhyage

The answer is, of course, the same in both cases.

Parameśvara does not refer to verse 240 but gives an example probably composed by himself, and the quantities given in it are g, s_i, and b just as in ASS 240.

PL 239.1: In the northern[103] area of the river Nilā, on the seashore, there is a region called Vṛṣabhavana. Suppose that, on a spot in there, the shadow of a gnomon of [sun] (12) *aṅgulas* caused by a lamp shining on the top of a flag of a ship is equal to [two, arrows, mountains] (752), and that, at ⟨another spot⟩ seven *hastas* distant ⟨from that spot⟩, it is that (752 *aṅgulas*) minus two. O friend, say in this case the length of the way (i.e., distance) to the ship and the elevation (i.e., height) of the lamp.[104]

$g = 12$, $s_1 = 750$, $s_2 = 752$ *aṅgulas*, $b = 7$ *hastas* (= 168 *aṅgulas*).

III.17 ASS 269.2 = VIS 270.2

See I.3 Verses 269, 270.

III.18 Verse 271

The two mss. used for the latter part of the VIS lack the last two verses, 271 and 272, which conclude respectively the chapter on the "Chain of Digits" or combinatorics and the entire *Līlāvatī*, and they have been supplied by the editor from other, unspecified Kerala manuscripts. VIS 272 thus supplied agrees with ASS 272, but VIS 271 does not agree with ASS 271.

ASS 271: Neither a multilpier, nor a divisor, nor the square, nor the cube is asked for ⟨here⟩, but defective, haughty calculators will inevitably fall in this chain (or net) of digits.[105]

VIS 271: Neither a multiplier, nor a divisor, nor the square, nor the cube is wide (i.e., important), but the mind of defective, haughty calculators does not shine at all in this ⟨topic, the chain of digits⟩.[106]

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dīpoccaṃ ca kiyatsakhe vyavahṛtiṃ chāyābhidhāṃ vetsi cet // VIS 240 //

[103] *saumya*, which also means "auspicious".

[104] saumye bhāge nilāyā vṛṣabhavanamiti kṣetramabdhestaṭe syāt
tasminnekatra potadhvajaśirasi lasaddīpajātā prabhā tu /
śaṅkorarkāṅgulasya dviśaranagasamā saptahastāntare smāt
dvyūnā sā potamārgapramitimiha sakhe brūhi dīponnatiṃ ca // PL 239.1 //

[105] na guṇo na haro na kṛtirna ghanaḥ pṛṣṭastathāpi duṣṭānām /
garvitagaṇakabaṭūnāṃ syātpāto vaśyamaṅkapāśe smin // ASS 271 //

[106] na guṇo na haro na kṛtirna ghanaḥ pṛthulastathātiduṣṭānām /
garvitagaṇakānāṃ yacceto vaśyaṃ na vai cakāstyasmin // VIS 271 //

In Parameśvara's commentary, the corresponding verse mostly agrees with VIS 271.[107] He takes the last word, "in this" (*asmin*), to refer to the entire text.[108]

IV Differences in the *vāsanā*

As a rule, I do not include here alterations of words which do not affect the mathematical contents.

IV.1 *Vāsanā* on verse 13

Verse 13 gives one example each for addition and subtraction by place-value notation,[109] but the *vāsanā* of the VIS does not agree with that of the ASS at two points. First, the compound, -*trinavatiśata*-, in verse 13 means two numbers, 93 and 100, according to the former, but one number, 193, according to the latter. Second, to be subtracted from 10000 in the example for subtraction is either each of the numbers given in the example of addition or the sum (360) of them according to the former, but the sum (360) of them only according to the latter.

Addition:
ASS: $2 + 5 + 32 + \underline{193} + 18 + 10 + 10$ (= 360).
VIS: $2 + 5 + 32 + \underline{93 + 100} + 18 + 10 + 10$ (= 360).
Subtraction:
ASS: $10000 - 360$ (= 9640).
VIS: $10000 - 2 - 5 - 32 - 93 - 100 - 18 - 10 - 100$; $10000 - 360$ (= 9640).

IV.2 *Vāsanā* on verse 21

Verse 21 gives examples for squaring, several rules for which are given in verses 19–20. Verse 19 gives a definition of the square and prescribes an algorithm for squaring a number by place-value notation, and verse 20 several alternative methods based on algebraic identities (cf. II.2). Then, verse 21 requires one to calculate the squares of 9, 14, 297, and 10005. The *vāsanā* on it gives answers by means of these methods. Since the VIS contains an additional formula (see II.2), the *vāsanā* in the VIS contains an additional paragraph which works out the computation of 14^2 by

[107] It reads *tathāpi* and *syāc* instead of *tathāti*- and *yac*, respectively, of VIS 271.

[108] guṇaharādīnāṃ pṛthutvābhāve pyasmiñśāstre duṣṭagaṇakānāṃ na praveśa ityarthaḥ // (Parameśvara's commentary on L 271)

[109] aye bāle līlāvati matimati brūhi sahitāndvipañcadvātriṃśattrinavatiśatāṣṭādaśa daśa / śatopetānetānayutaviyutāñcāpi vada me yadi vyakte yuktivyavakalanamārge si kuśalā // L 13 //

means of that formula:[110]

$$14^2 = (6+8)^2 = 4 \times 6 \times 8 + (8-6)^2 = 192 + 4 = 196.$$

This paragraph does not exist in the ASS.

IV.3 *Vāsanā* on verses 83–87

Verses 83–87 give examples for the $(2n+1)$-quantity operation, an algorithm for which is given in verse 82. The $(2n+1)$-quantity operation, or the $(2n+1)$-*rāśika*, is a compound computation, which consists of n three-quantity operations. That is to say, when $(2n+1)$ quantities ($a_1, \ldots, a_n; b_1, \ldots, b_n$ and p) are given and they satisfy the condition:

$$\begin{aligned} a_1 : p &= b_1 : x_1, \\ a_2 : x_1 &= b_2 : x_2, \\ a_3 : x_2 &= b_3 : x_3, \\ \vdots \quad & \quad \vdots \quad \vdots \\ a_n : x_{n-1} &= b_n : x, \end{aligned}$$

where a_i's are called "the standard quantities" (*pramāṇa-rāśis*), p "the fruit" (*phala*), and b_i's "the requisite quantities" (*icchā-rāśis*), an algorithm leading to the solution for x,

$$x = \frac{b_1 \cdot b_2 \cdots b_n}{a_1 \cdot a_2 \cdots a_n} \cdot p,$$

is called "the $(2n+1)$-quantity operation".

Verse 82 says:

> L 82: In the case of the five-quantity operation, seven-quantity operation, nine-quantity operation, etc., when one has performed the transposition of the fruit and the denominators ⟨of fractions, if any⟩, and when the product produced from the greater number of quantities is divided by the product of the smaller number of quantities, the result ⟨of the multi-quantity operation is obtained as the quotient⟩.[111]

In the *vāsanā* of the ASS, the $(2n+1)$ given quantities are first arranged vertically in two columns (see below), where the a_i-column, including p, is called "the standard side" (*pramāṇa-pakṣa*) and the b_i-column "the requisite side" (*icchā-pakṣa*), and then, according to verse 82, "the fruit", p, and the denominators of fractions, if any,

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[110] athavā khaṇḍe 6 / 8 / anayorhatiścaturghnī 192 / tayorantaravargeṇa 4 yutā jātā saiva kṛtiḥ 196 // (4th paragraph of the *vāsanā* on verse 21, VIS)

[111] pañcasaptanavarāśikādike nyonyapakṣanayanaṃ phalacchidām /
saṃvidhāya bahurāśije vadhe svalparāśivadhabhājite phalam // L 82 //

are transposed to the mutually opposite sides, and finally the product of the elements of the longer side (*bahurāśi-pakṣa*, lit. "the side which contains the greater number of quantities") is divided by the product of those of the shorter side (*svalparāśi-pakṣa*, lit. "the side which contains the smaller number of quantities") . The result is equivalent to the above solution.

$$\begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ \vdots & \vdots \\ a_n & b_n \\ p & \end{bmatrix} \longrightarrow \begin{bmatrix} a_1 & b_1 \\ a_2 & b_2 \\ a_3 & b_3 \\ \vdots & \vdots \\ a_n & b_n \\ & p \end{bmatrix} \longrightarrow \frac{b_1 \cdot b_2 \cdots b_n \cdot p}{a_1 \cdot a_2 \cdots a_n}.$$

This is the way the *vāsanā* of ASS 83–87 performs the $(2n + 1)$-quantity operation.

In the *vāsanā* of VIS 83–87, on the other hand, the given quantities are arranged in three columns, "the fruit", p, being put in the third column. Then, the product of the elements of "the requisite compartment" (called *icchā-kakṣyā* by Śaṅkara), b_i's, and of "the fruit" is divided by the product of the elements of "the standard compartment" (*pramāṇa-kakṣyā*), a_i's.

$$\begin{bmatrix} a_1 & b_1 & p \\ a_2 & b_2 & \\ a_3 & b_3 & \\ \vdots & \vdots & \\ a_n & b_n & \end{bmatrix} \longrightarrow \frac{b_1 \cdot b_2 \cdots b_n \cdot p}{a_1 \cdot a_2 \cdots a_n}.$$

In his solutions to some of the examples that he quotes from the works of Śrīdhara and others, Śaṅkara puts "the fruit", p, in between the "standard" and the "requisite compartments".

$$\begin{bmatrix} a_1 & p & b_1 \\ a_2 & & b_2 \\ a_3 & & b_3 \\ \vdots & & \vdots \\ a_n & & b_n \end{bmatrix} \longrightarrow \frac{b_1 \cdot b_2 \cdots b_n \cdot p}{a_1 \cdot a_2 \cdots a_n}.$$

In this last form of the initial arrangement, therefore, the three "compartments" are assigned, in order, to *pramāṇa*, *phala* and *icchā* just as in the case of a regular three-quantity operation. The $(2n+1)$-quantity operation is thus reduced to a single three-quantity operation (cf. III.6).

Certainly verse 84 does not say a word about the arrangement of the given quantities, but the expression in it, "the transposition of the fruit and the denominators ⟨of fractions⟩ to the mutually opposite sides (*pakṣa*)", implies that the arrangement originally consisted of two columns as in the ASS, and not of three.

IV.4 *Vāsanā* on verse 99

In the *vāsanā* of one example given in ASS 99, the price of a certain quantity of rice obtained is expressed as "2 *paṇas*, 2 *kākiṇīs*, and $13\frac{1}{3}$ *varāṭakas*". In the *vāsanā* of VIS 99, on the other hand, the same price is expressed as "10 *kākiṇīs* and $13\frac{1}{3}$ *varāṭakas*". The conversion ratio, 1 *paṇa* = 4 *kākiṇīs*, is given in L 2.

IV.5 *Vāsanā* on ASS 120 = VIS 115

ASS 120 (= VIS 115) gives an example each for the square series and the cubic series, that is, $S^2(n) = 1^2 + 2^2 + \cdots + n^2$, and $S^3(n) = 1^3 + 2^3 + \cdots + n^3$, formulas for which are given in ASS 119 (= VIS 114) .

> ASS 120: Say quickly the sum of the squares and that of the cubes of the same ⟨numbers⟩, if your intelligence is proficient enough for the practice of the summation of squares ⟨and of cubes⟩.[112]

The expression, "the same ⟨numbers⟩", in this problem refers to the nine numbers, 1 to 9, stated in the previous example (ASS 118 = VIS 113). In the *vāsanā* of the ASS, $S^2(1) = 1, S^2(2) = 5, \ldots, S^2(9) = 285$ and $S^3(1) = 1, S^3(2) = 9, \ldots, S^3(9) = 2025$ are tabulated as answers to the question. In the *vāsanā* of the VIS, on the other hand, only $S^2(9) = 285$ and $S^3(9) = 2025$ are given as answers, although Śaṅkara, the commentator, calculates all the nine cases each for the square and the cubic series.

IV.6 *Vāsanā* on verses 186–187

Verses 186–187 require one to illustrate, by means of the traditional quadrilateral, whose sides are 25, 52, 60, and 39, and whose diagonals are usually taken to be 56 and 63, that a quadrilateral is not determined by its four sides. In the course of the calculation of different diagonals for the four sides, $\sqrt{5049}$ is approximated by $71\frac{1}{20}$ in the *vāsanā* of the ASS and by $71\frac{3}{50}$ in the *vāsanā* of the VIS.

The calculation of these approximate square roots, which is not worked out in the *vāsanā*, must have been made by means of the formula prescribed by Bhāskara

[112] teṣāmeva ca vargaikyaṃ ghanaikyaṃ ca vada drutam /
kṛtisaṃkalanāmārge kuśalā yadi te matiḥ // ASS 120 //
VIS 115, which corresponds to ASS 120, reads *padaṃ* (which seems to be a misprint) for *vada*, *iti* for *kṛti*, and ⟨*a*⟩*saṅkulā* for *kuśalā*. See I.1 for the different numbering of the verses.

himself in verse 140:[113]

$$\sqrt{\frac{b}{a}} = \frac{\sqrt{abp^2}}{ap},$$

where the integer part of the $\sqrt{abp^2}$ is calculated by the popular algorithm given in verse 22, which utilizes the place-value notation.

By taking $p = 20$, we can obtain the approximation in the ASS:

$$\sqrt{5049} = \frac{\sqrt{5049 \times 20^2}}{20} = \frac{\sqrt{2019600}}{20} = \frac{\sqrt{1421^2 + 359}}{20} \approx \frac{1421}{20} = 71\frac{1}{20}.$$

By taking $p = 50$, we can obtain the approximation in the VIS:

$$\sqrt{5049} = \frac{\sqrt{5049 \times 50^2}}{50} = \frac{\sqrt{12622500}}{50} = \frac{\sqrt{3552^2 + 5796}}{50} \approx \frac{3553}{50} = 71\frac{3}{50}.$$

Here, the VIS took not the 3552 but 3553 as an approximation, presumably by comparing the following values:

$$3552\frac{1}{2} < \sqrt{12622500} < \sqrt{12623809} = 3553,$$

where the first inequality is known from the computation, $\left(3552\frac{1}{2}\right)^2 = 3552^2 + 3552 + \frac{1}{4} = 1262056\frac{1}{4}$. Śaṅkara, too, mentions the 50 for p, although he does not work out the computation either.

IV.7 *Vāsanā* on verse 249

Verse 249 gives an example for the *kuṭṭaka* or "pulverizer":

$$y = \frac{100x \pm 90}{63}.$$

The *kuṭṭaka* is the name given to a procedure for solving linear indeterminate equations of the type,

$$y = \frac{ax + c}{b},$$

an algorithm for which is prescribed in verses 242–246.

The *vāsanā* on verse 249 first solves the problem by means of the original algorithm, and then by means of an additional rule given in verse 248, which says: If $x = \alpha$ is a solution to the equation,

$$y = \frac{a'x + c'}{b},$$

[113] vargeṇa mahateṣṭena hatācchedāṃśayorvadhāt /
padaṃ guṇapadakṣṇṇacchidbhaktaṃ nikaṭaṃ bhavet // L 140 //

where $a = a'd$ and $c = c'd$, then it is also a solution to the original equation. In fact, this rule can be extended to: If $(y, x) = (\beta, \alpha)$ is a pair of solutions to the reduced equation, then $(y, x) = (d\beta, \alpha)$ is a solution to the original equation.

In the ASS, the *vāsanā*[114] reduces the above equation (with the plus sign) to

$$y = \frac{10x + 9}{63},$$

silently[115] obtains the solution, $x = 171$, to this reduced equation according to the original algorithm, and then, from it, calculates the least solution, $x = 18$, to the original equation by the procedure:

$$171 = 63 \times 2 + 45, \quad 63 - 45 = 18,$$

since $x = 171$ is a solution to the original equation as well according to the additional rule. Then, the corresponding least solution, $y = 30$, is obtained by substituting $x = 18$ in the original equation. Hence follows the solution, $(y, x) = (30, 18)$.

In the VIS, on the other hand, the *vāsanā*[116] silently obtains a pair of solutions, $(y, x) = (27, 171)$, to the reduced equation according to the original algorithm, calculates from it the least solutions, $(y, x) = (3, 18)$, to the reduced equation by the procedures:

$$171 = 63 \times 2 + 45, \quad 63 - 45 = 18,$$

$$27 = 10 \times 2 + 7, \quad 10 - 7 = 3,$$

and then obtains the least solutions to the original equation according to the extended additional rule: $(y, x) = (10 \times 3, 18) = (30, 18)$.

IV.8 *Vāsanā* on verse 258

Verse 258 provides a guide for applying the *kuṭṭaka* procedures to the "computation of planets" (*graha-gaṇita*).

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[114]pūrvavallabdho guṇaḥ 45 / atra labdhirna grāhyā / yato labdhayo viṣamā jātāḥ / ato guṇe 45 svatakṣaṇādasmād 63 viśodhite jāto guṇaḥ sa eva 18 / guṇaghnabhājye kṣepa-90-yute hara-63-taṣṭe labdhiśca 30 // (part of the *vāsanā* on verse 249, ASS) Read *jātā ato* instead of *jātāḥ / ato*.

[115]This step is not stated in the *vāsanā*.

[116]pūrvavallabdho guṇaḥ 45 / ⟨atra labdhirna grāhyā yato labdhayo viṣa⟩mā vallyāṃ jātāḥ / ato guṇaḥ 45 / svatakṣaṇādasmād 63 viśodhite jāto guṇaḥ sa eva 18 / labdhiḥ 7 / svatakṣaṇāc 10 chodhitā 3 / ⟨apavartanena guṇitā⟩ jātā sphuṭā labdhiḥ 30 // (part of the *vāsanā* on verse 249, VIS) The words within ⟨...⟩ have been supplied by the editor since the two manuscripts used for this part of the VIS have gaps here. However, the first four words supplied by the editor, that is, *atra labdhirna grāhyā* ("Here, the quotient is not to be taken"), are inappropriate because "the quotient", 7, obtained here is actually employed in the next passage. Read *labdhiśca 7 /* for *labdhirna grāhyā*; *jātā ato guṇe 45* for *jātāḥ / ato guṇaḥ 45 /*; and delete / after *labdhiḥ 7*.

Table 12: Application of *kuṭṭaka* to the "computation of planets"

Dividend a	Divisor b	Subtrahend c	Quotient y	Multiplier x
60	civil days	rem. of sec.	sec. elps.	rem. of min.
60	civil days	rem. of min.	min. elps.	rem. of deg.
30	civil days	rem. of deg.	deg. elps.	rem. of *rāśi*
12	civil days	rem. of *rāśi*	*rāśis* elps.	rem. of rotation
a_0	civil days	rem. of rotation	rotations elps.	x_0
a_1	b_1	rem. of ic.mon.	ic.mon. elps.	sol. days elps. (x_1)
a_2	lunar days	rem. of om. days	om. days elps.	lun. days elps.

(sec. = seconds, min. = minutes, deg. = degrees, *rāśi* = 30 deg., mon. = months, rem. = remainder, elps. = elapsed, ic. = intercalary, om. = omitted)

L 258: The remainder of the *vikalās* (seconds) ⟨for a certain mean longitude of a planet⟩ should be regarded as the subtrahend, sixty as the dividend, and the earth-days (i.e., civil days) as the divisor ⟨for the *kuṭṭaka* procedures⟩. The "quotient" produced from them ⟨by means of the *kuṭṭaka*⟩ shall be the *vikalās* (seconds) and the "multiplier" the remainder of the *liptās* (minutes). From this ⟨remainder⟩ also, the *kalās* (minutes) and the remainder of the *lavas* (degrees) ⟨for the mean planet are obtained in the same way by the *kuṭṭaka*⟩, and likewise the upper ⟨units, i.e., *rāśis* and rotations⟩ too ⟨are obtained⟩. In the same way, from the remainders of intercalary months and of omitted ⟨days respectively⟩, the solar and the lunar days ⟨elapsed are obtained⟩.[117]

That is to say, the dividend (a), the divisor (b), and the subtrahend (c) of the linear indeterminate equation,

$$y = \frac{ax - c}{b},$$

are taken to be such quantities as shown in Table 12, and the corresponding "quotient" (y) and "multiplier" (x) are obtained by means of the *kuṭṭaka* procedures.

The quantities underlined in the table have been explicitly mentioned in verse 258. As for the quantities expressed by the symbols, a_0, etc., the explanation given

[117] kalpyātha śuddhirvikalāvaśeṣaḥ ṣaṣṭiśca bhājyaḥ kudināni hāraḥ /
tajjaṃ phalaṃ syurvikalā guṇastu liptāgramasmācca kalā lavāgram /
evaṃ tadūrdhvaṃ ca tathādhimāsāvamāgrakābhyāṃ divasā ravīndvoḥ // L 258 //

Table 13: Differences in the application of *kuṭṭaka*

	ASS	VIS	Parameśvara
a_0	rotations in the *kalpa*	rotations	rotations
x_0	civil days elps.	ø	civil days elps.
a_1	ic.mon. in the *kalpa*	ø	ic.mon. in the *yuga*
b_1	sol.days	sol.mon. in the *yuga*	sol.mon. in the *yuga*
x_1	sol.days elps.	sol.mon. elps.	sol.mon. elps.
a_2	om. days in the *kalpa*	om. days	om. days in the *yuga*

(The symbol, ø, indicates that the text is silent about it.)

in the *vāsanā* of the VIS[118] differs from that in the ASS,[119] as Table 13 shows. There are two major differences here, namely, that the b_1 and x_1 are taken to be solar days in the ASS but solar months in the VIS in spite of the fact that the x_1 is explicitly stated to be solar days in verse 258, and that the standard period for calculation is the *kalpa* in the ASS but the *yuga* in the VIS.[120] Verse 258 itself is not specific about this last point.

[118] bhagaṇo bhājyaḥ kudināni hāro bhagaṇaśeṣaśśuddhiḥ / phalaṃ gatabhagaṇaḥ // asyodāharaṇāni praśnādhyāye draṣṭavyāni // adhimāsaśeṣaśśuddhiḥ / yugaravimāsā hāraḥ / labdhiradhimāsāḥ / guṇo gatā ravimāsāḥ / evamadhimāsaśeṣeṇa // avamaśeṣeṇa tvavamāni bhājyaḥ / cāndradi⟨nāni hāraḥ / puna⟩ravamaśeṣaśśuddhiḥ / phalaṃ gatāvamāni / guṇo gatāścāndradivasāḥ // (part of the *vāsanā* on L 258, VIS) The passage from *adhimāsaśeṣaś* to the end is separated from the preceding passage by a long commentary of Śaṅkara placed in between them, and the editor of the VIS does not regard it as part of the *vāsanā*. The passage in question is introduced with the words, "He has told an explanation of this ⟨last line of the verse⟩, 'In the same way, ... the solar and the lunar days ⟨are obtained⟩,' ⟨as follows⟩ —" (*tathā ... ravīndvorityasya vyākhyānamāha* —), and is followed by the words, "The meaning ⟨of this passage⟩ is clear" (*spaṣṭo rthaḥ*).

[119] kalpabhagaṇo bhājyaḥ kudināni hāro bhagaṇaśeṣaṃ śuddhiḥ / phalaṃ gatabhagaṇo guṇo hargaṇassyāditi // asyodāharaṇāni tripraśnādhyāye // evaṃ kalpādhimāsā bhājyo ravidināni hāro dhimāsaśeṣaṃ śuddhiḥ / phalaṃ gatādhimāsā guṇo gataravidivasāḥ // evaṃ kalpāvamāni bhājyaścandradivasā hāro vamaśeṣaṃ śuddhiḥ / phalaṃ gatāvamāni guṇo gatacāndradivasā iti // (part of the *vāsanā* on L 258, ASS)

[120] According to the Brāhma school of Indian astronomy, to which Bhāskara II belonged, 1 kalpa = 1000 ⟨mahā-⟩yugas = 432×10^7 years. Cf. David Pingree, "History of Mathematical Astronomy in India", *Dictionary of Scientific Biography*, Vol. 15, edited by C. C. Gillispie, New York: Charles Scribners's Sons, 1978, p. 555.

Parameśvara[121] perfectly agrees with the VIS as far as the available evidence shows (see Table 13).

IV.9 *Vāsanā* on verse 262

Verse 262 gives three examples of permutations of digits. One of them is a problem of the sum of all the possible numbers in eight decimal places made of the eight digits, 2, 3, 4, 5, 6, 7, 8, and 9. The answer given is 2,463,999,975,360 (or 24,63,99,99,75,360 according to the Indian way), which is expressed as "twenty-four *nikharvas*, sixty-three *padmas*, ninety-nine *koṭis*, ninety-nine *lakṣas*, seventy-five *sahasras* (thousand), three *śatas* (hundred), and sixty" in the ASS. The VIS, on the other hand, replaces the *padma* ("a lotus") for 10^9 with the word, *vṛnda* ("a multitude"). The latter word occurs as the last term in Āryabhaṭa's list of the names of the first ten decimal places (AB 2.2).[122]

It would, however, not be correct to say that the numerals used in the present *vāsanā* of the VIS have been taken from the Āryabhaṭa's list, since Āryabhaṭa used *niyuta* instead of *lakṣa* for 10^5, although the latter has been mentioned by Bhāskara I, one of the earliest commentators of the *Āryabhaṭīya*.[123]

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[121] punarbhagaṇaśeṣaśśuddhiḥ / grahasya bhagaṇo bhājyaḥ / tatra labdhiriti bhagaṇā guṇo hargaṇaḥ / sarvatra bhūdinānyeva hāraḥ // tathādhimāsāvamāgrakābhyāṃ divasā ravīndvoriti / adhimāsaśeṣaṃ śuddhiṃ prakalpya yugādhimāsānbhājyaṃ prakalpya yugaravimāsānhārañca prakalpya kṛte kuṭṭake labdhirgatādhimāsā bhavanti / guṇo gataravimāsāḥ / evamatītaravimāsās-sādhyāḥ // punaravamaśeṣaḥ śuddhiḥ / yugāvamāni bhājyaḥ / yugacāndradivasā haraḥ / tatra kṛte kuṭṭake labdhirgatāvamāni / guṇo tītacāndradivasāḥ / evamatītacāndradivasāśca sādhyāḥ // (part of Parameśvara's commentary on L 258)

[122] AB = *Āryabhaṭīya* of Āryabhaṭa, edited with the commentary (*bhāṣya*) of Bhāskara I by Kripa Shankar Shukla, New Delhi: Indian National Science Academy, 1976.

[123] See his commentary on AB 2.2, p. 46.

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