

## Solstitial observations in thirteenth century Beijing

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### Introduction

The Yuan Shi is the Historical Classic of the Mongol period in China, the Yuan Dynasty (1264 - 1368). As in almost all the historical classics there is a section devoted to astronomy, since this was part of official government activity. In Chapter 52, the first of the chapters devoted to the calendar, there is a record of 98 solstitial observations carried out by Guo Shoujing (郭守敬, 1231-1316)<sup>1</sup>, the most distinguished astronomer of the time in China.<sup>2</sup> Although he composed a tract on the determination of the two solstices, *Studies on the gnomon shadows at the two solstices*,<sup>3</sup> listed among his works in the Biography, that work is lost, as are indeed all his other works.

The intention of this program of observations was to fix the time of the Summer and Winter solstices. The Chinese calendar may be viewed as a series of months anchored to a uniformly running mean solar time; each month begins with the New Moon, and has either 29 or 30 days. This time scale is divided into 24 equal intervals by 'solar terms', running from Winter Solstice to Winter Solstice. Since the length of the shadow reaches a maximum or minimum at a solstice it is difficult to fix the day and hour of the solstice with any precision if one were simply to observe the shadow from day to day. However Chinese astronomers proceeded by means of an interpolation using three observations on days preceding or following the time of the solstice. This was a time-honoured method, at least in China.<sup>4</sup>

One of the principal objects of this article is to establish the latitude of the site at which the observations were made. There have been earlier investigations of this matter by the Jesuit Father Antoine Gaubil in the early 18<sup>th</sup> century, and by Dong Zuobin and colleagues in 1939. Nevertheless there is still the common impression, even with some serious historians, that the solstitial shadow observations listed by Guo Shoujing were made at Yangcheng (陽城) in Henan province where there is extant an impressive tower, the 'Tower of Duke Zhou', with horizontal scale, apparently constructed by Guo

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<sup>1</sup> At the end of the Biography, Yuan Shi, Chapter 164, it is stated that his death was in the reign period Yan You, year 3, (1316), at age 86, but apparently it is customary to regard a person in his first year of life as being one year old, and according to that he would have been born in 1231.

The Yuan Shi is always consulted in the Beijing edition (Zhonghua Shuji, 1976), where Chapter 52 is on pp. 1119-1152, and the Biography in Chapter 164 is found on pp. 3845-3852.

<sup>2</sup> Guo was also distinguished as an engineer. Bouillard (1924), p. 1160, discusses an example of Guo's activity in laying canals to serve the new capital Dadu, to replace those of the Jin period.

<sup>3</sup> Er zhi gui jing kao (二至晷景考), in 20 chapters.

<sup>4</sup> It is curious that Edmund Halley was to rediscover the method, quite unaware of his distinguished ancient predecessors; Halley (1694).

Shoujing. Ironically, this impression is owing indeed to the work of Dong Zuobin who wrote the definitive monograph on the tower, although, like Gaubil before him, Dong Zuobin made it clear that the observations listed by Guo Shoujing were made in Beijing.

At a more popular level, Krupp presented a light account of the shadow measurements, where it is assumed that Guo Shoujing's measurements were made at the tower in Yang Cheng; this was based partly on Thurston (1994). Thurston, on the other hand, had an summary, but essentially correct, account in which he recognized that Beijing was the site.<sup>5</sup>

### The Observations

These records give the date of the observation and the length of the Noon shadow of a 40 foot gnomon. The gnomon is described in detail in the Yuan Shi, chap.48, translated by Needham as follows:

The gnomon (biao chang 表長) is made 50 feet long, 2 feet 4 inches wide and 1 foot 2 inches thick and is fixed in the stone base at the south end of the graduated scale (gui biao 圭表). Inserted to a depth of 14 feet in the earth, it rises 36 feet above the scale. At the top the gnomon divides into two dragons which sustain a cross-bar. From the centre of the cross-bar the measurement to the top of the gnomon is 4 feet and hence to the top of the scale it is 40 feet. The cross-bar is 6 feet long and 3 inches in diameter, and carries a water-channel on the top for the purpose of leveling. At its two ends and in the centre are transverse holes, 1/5 inch in diameter, through which are inserted rods 5 inches long, having plumb-lines attached to them so that the correct position can be ascertained and lateral deflection prevented.<sup>6</sup>

These observations are collected in Table 1, in which are listed the Chinese date, the length of the shadow, the Julian date and the Julian Day Number. The dates range from 1277 December 10 to 1280 February 2. They are given there in the same order as in the text. The same data are plotted in Fig 1, showing clearly how the observations were grouped around the five solstices in the period Winter 1277 to Winter 1279.

The list of observations begins<sup>7</sup>:

Investigation of Winter solstice of the year 14 of the Zhi Yuan reign period [1264-1294], (4,2) in year cycle. In month 11, day 14, (6,12), the shadow length is 7 zhang, 9 chi, 4 cun, 8 fen, 5 li, 5 hao.<sup>8</sup>

<sup>5</sup> Krupp (1994); Thurston (1994), 94-105.

<sup>6</sup> Beijing edition, p. 996-7; Needham (1959), p.298; Wylie (1897), p.16-8.

<sup>7</sup> In the Beijing edition, the observations are listed in pp. 1122-1130.

The year placed in the sexagenary cycle as stem 4 branch 2 of the reign-period Zhi Yuan (1264-1294), is year 14 in the cycle of 60, A.D. 1277.<sup>9</sup> In what follows such stem and branch notation for the 60 cycle is written for brevity as (4,2). The lunar month numbered 11 in the calendar in use at this time was that containing the winter Solstice. The day is given as the 14<sup>th</sup> of the month, and also by its position in the sexagenary cycle, stem 6 branch 12, in other words 36. From calendrical tables we know that the date in Western terms was 1277 Dec 10. The length of the shadow is expressed in terms of the foot (chi), and its multiple and sub-multiples, related to each other by the factor 10, as follows:

zhang (丈) = 10 chi (尺),  
 cun (寸) = chi/10,  
 fen (分) = cun/10,  
 li (釐 or 厘) = fen/10,  
 hao (毫) = li/10.

This li is to be distinguished from its homonym li (里) used in geographical measurement. Although the Chinese units are related by a factor a 10, this is not the same as a place value notation, since the unit is always expressed, and not left implied by its place. Nevertheless the decimal character means that we can succinctly record the shadow as 79.4855 feet. The length of the foot is not known exactly, but this is unimportant in the analysis of the shadow lengths since only ratios are involved. The Chinese foot length varied considerably over the centuries, but in the Yuan period appears to be roughly 30 cm.<sup>10</sup>

In the arrangement of his observations Guo gives three or more dates, followed by an application of the method that is about to be described, to derive therefrom the day and hour of the solstice. This is followed by another group of dates from which is derived another value for the time of Solstice, and so on. This continues up to 1280 February 2. In all there are 98 different observations. Of these, three (numbered 18, 59, 96 in Table 1), are given twice in the text, since they were used in two distinct interpolations. The text appears to be very well preserved, if one is to judge by the

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<sup>8</sup> Here, and in the rest of the article, the 10 stems and 12 branches of the sexagenary cycle are replaced by their serial numbers, the pair (stem, branch) being written, for example, as (4,2) or (6,12).

<sup>9</sup> The position in the 60-cycle is given by the formula 6·stem-5·branch (mod 60).

<sup>10</sup> Needham (1962), Fig. 287, shows a photograph of foot measures from the Chou to the Ming periods, but unfortunately without any metric scale for comparison. The length of the foot steadily decreases over this very long period.

numerical data, since the shadow lengths are doubtful in only two cases, where an emendation is easy<sup>11</sup>.

### Shadow interpolation

Briefly, let  $S_1$  and  $S_2$  be the Noon shadow lengths on two successive days  $T_1$  and  $T_2 (= T_1+1)$  before the solstice; a day  $T_3$  after the solstice is chosen so that the shadow  $S_3$  lies between  $S_1$  and  $S_2$ . Let  $S_3$  divide the interval between  $S_1$  and  $S_2$  in the proportion 1:a, so that

$$S_3 = aS_1 + (1-a)S_2, \text{ whence, } a = (S_3 - S_2) / (S_1 - S_2).$$

If we conceive of the Noon shadow as a continuous function of time, so interpolating through the points  $(T_n, S_n)$ , we have, to a good approximation, that at the time  $T_{\text{mid}} = aT_1 + (1-a)T_2$ , situated between  $T_1$  and  $T_2$ , the shadow is again equal to  $S_3$ . Assuming that the length of the shadow varies symmetrically about the time of the solstice, then the time of the Solstice lies midway between times of equal shadow length,  $T_{\text{mid}}$  and  $T_3$ ,

$$T_{\text{sol}} = (T_{\text{mid}} + T_3)/2 = (T_2 + T_3 - a)/2.$$

This analysis has been described as if  $S_3$  lay between  $S_1$  and  $S_2$ , making  $0 < a < 1$ , but this assumption is not essential, nor is it true in every case calculated by Guo. In all there were 38 triples, which are listed in Table 2.

In Fig. 2 the same points are plotted as in Fig. 1, but here the lines are added to join the points constituting the triple. On these lines the mid-points are marked, showing the time of the Solstice. Guo does not give the full details in every one of the 38 cases, often merely stating that there is good agreement with the previous result.

The time of solstice, fixed in this way, is evidently relative to true solar Noon, not mean solar Noon, whereas Guo's expression of the hour of the solstice is relative to midnight, assuming that Noon is in effect exactly 12 hours after midnight, so ignoring any question of the equation of time.

One might imagine that a more exact procedure of fitting a parabola to the three points would offer greater precision. However such a procedure reveals a discrepancy of a mere 0.003 day, compared with the Chinese method. The real source of error in this procedure lies in the assumption of symmetry before and after the minimum or maximum shadow; for this reason there is an error in the time of solstice in one direction at the Summer

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<sup>11</sup> These are 61 and 69, for which the shadow lengths may be conjectured as 21.855 and 38.5515, respectively.

solstice, and in the opposite direction at the Winter solstice. The asymmetry is a direct consequence of the non-uniform motion of the Sun.

### **Shadow Definer and Gnomon**

Guo gives the shadow with great precision, even down to the unit *hao*, which is 1/10,000 foot, or about 0.03 mm. While such precision in direct reading of the scale is altogether out of the question, he did in fact achieve a very high precision in the measurement of the length of the shadow, by means of a device which was apparently his own invention.

It should be explained to begin with that the shadow was not cast simply by the top of a pole. Rather Guo explains that at the top of the pole there was the figure of a dragon, and the dragon held aloft a horizontal rod. It was the shadow cast by the rod onto a long horizontal scale that was measured.

The position of that shadow was located by a device of extraordinary simplicity and ingenuity. This ‘shadow definer’, *yingfu* (景符), employed by Guo Shoujing, is described in two passages of the Yuan history.<sup>12</sup> Its true character was long misunderstood, until it was made clear to Western scholars by Maspero<sup>13</sup>, then by Needham<sup>14</sup> who provided a clear illustration<sup>15</sup>. However, correct drawings and descriptions are to be found in Japanese accounts from the 17<sup>th</sup> century, one of which is reproduced by Nakayama<sup>16</sup>. By its means it was possible to make very accurate measurements of the position of the shadow. This device was essentially a plate some 2 inches across, having a tiny hole cut in its middle, suspended just over the horizontal measuring scale. The sun’s light, passing through the hole, creates a bright spot on the scale, the size of a ‘grain of rice’, and if the horizontal bar suspended 40 feet above the ground is in line with the Sun and the hole, a tiny image of the bar appears in the spot of light, as a black bar across it. As the shadow definer is moved to and fro the image of the bar moves within the spot, so that by moving it one can find the best position where the image of the bar is in the centre of the bright spot; then the bright spot on the scale is in line with the rod and the centre of the Sun’s disk. Before the arrangement was properly explained, some people had supposed that this plate was situated at the top of the gnomon, near the bar.

Even with this device one could not have achieved the precision given in Guo’s measurements. Certainly the shadow lengths have been ‘dressed’ in some way, perhaps as a result of averaging over a number of measurements. The shadow of the Sun would pass

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<sup>12</sup> Beijing edition, Ch. 48, 997; Ch.52, 1121; also Ch. 164, p. 3847.

<sup>13</sup> Maspero (1939), 268-272.

<sup>14</sup> The character 景 is read *jing* when it means view, and *ying* when it means shadow. In Needham (1957), p. 299, the term is printed as 影符, although 景符 is found in the Yuan Shi. Both are found in the Ming Shi, 景 in Chapter 25, p. 357, and Chapter 31, p. 524; 影 is found in Chapter 25, p. 362-3.

<sup>15</sup> Needham (1986), p. 73.

<sup>16</sup> Nakayama (1962), p. 105; Prof. Nakayama tells me that this drawing was current in Japanese accounts from the 17<sup>th</sup> century.

over the scale in about 5 minutes, and in that time several measurements could have been made.

### Time

Time is indicated by the double hour shi (时), and by the so-called ‘quarter-hour’ ke (刻), which is in fact the hundredth part of the day. The double hours are indicated by the twelve branches, Zi, Chou, etc. The first double hour straddles midnight, running from 11 p.m. to 1 a.m.<sup>17</sup>

In Guo’s account the time of the solstice is expressed in shi and ke, and in a number of instances he gives the details. To take the first triple, for example, we have the times and shadows as follows. The three dates are in the Zhi Yuan year 14, month 11. The days are given by the number of the day in the month, and also the (stem, branch) reference.

	J.D.	Date	day	shadow
T <sub>3</sub>	2187826	1277 Dec 10	14 (6,12)	S <sub>3</sub> 79.4855
T <sub>1</sub>	2187833	1277 Dec 17	21 (3,7)	S <sub>1</sub> 79.5410
T <sub>2</sub>	2187834	1277 Dec 18	22 (4,8)	S <sub>2</sub> 79.4550

Guo calculates the differences  $S_3 - S_2 = 0.0305$ ,  $S_1 - S_2 = 0.0860$ , and then the ratio  $(S_3 - S_2) / (S_1 - S_2) = 0.35465$ , treated as 35 ke. This is therefore the interval from T<sub>2</sub> to the point T<sub>mid</sub>, within the interval T<sub>1</sub>T<sub>2</sub>, at which the shadow equals S<sub>3</sub>. The interval in days from T<sub>3</sub> to T<sub>2</sub> is 8 days, or 800 ke, so the interval from T<sub>3</sub> to T<sub>mid</sub> is  $800 - 35 = 765$  ke; thus the time of the solstice T<sub>sol</sub> is  $765/2 = 382\frac{1}{2}$  ke. When counted however from the midnight preceding T<sub>3</sub> it is necessary to add the half-day, or 50 ke, to make  $432\frac{1}{2}$  ke. This is divided by 100 to get 4 days and a remainder  $32\frac{1}{2}$  ke. This remainder  $32\frac{1}{2}$  is multiplied by 12 and divided by 100 to get 3 double hours, with remainder 90. Subtract 50 from this remainder to allow for the fact that from midnight only one half of the first double hour passes, so leaving  $90 - 50 = 40$ , and this is converted to ke by dividing by 12, to give 3 ke and a fraction. Thus 4 double hours are completed and the time of the solstice is at 3 ke into the fifth double hour, which is denoted Zhen (7 a.m. to 9 a.m.).

A more direct calculation in terms of J.D. yields 2187829.8227, that is 0.3227 after midnight 2187829.5, or 7<sup>h</sup>;44,41 after midnight.

### Solstices

In Table 2 are listed the 38 triples of observations from which Guo Shoujing determines the moments of the Winter and Summer solstice in the years 14-16, Dec 1277 to Dec

<sup>17</sup> For a practical summary, Needham (1960), p. 199.

1279. In the following table the average times of the solstices are listed, together with the time, expressed in ke, calculated from the modern parameters.

No.	JD of solstice	average	Modern
	1277 Dec 14		
1	2187830.3227		
2	2187830.3244		
3	2187830.3259		
4	2187830.3265		
5	2187830.3252		
6	2187830.3242	32.48 ke	35.79
	1278 June 15		
7	2188012.9510		
8	2188012.9513		
9	2188012.9555		
10	2188012.9513		
11	2188012.9519		
12	2188012.9636	95.41 ke	93.70
	1278 Dec 14		
13	2188195.5801		
14	2188195.5774		
15	2188195.5786		
16	2188195.5797		
17	2188195.5816		
18	2188195.5768		
19	2188195.5188	57.04 ke	59.62
	1279 Jun 15		
20	2188378.1941		
21	2188378.1986		
22	2188378.1881		
23	2188378.1892		
24	2188378.1882		
25	2188378.1965		
26	2188378.2472	18.68 ke	17.71

	1279 Dec 15		
27	2188560.8171		
28	2188560.8203		
29	2188560.8155		
30	2188560.8173		
31	2188560.8166		
32	2188560.8194		
33	2188560.8202		
34	2188560.8201		
35	2188560.8187		
36	2188560.8178		
37	2188560.8158		
38	2188560.8177	81.80 ke	83.55

It is clear that the Winter solstice found by Guo is earlier than that according to the modern calculation, while the Summer solstice is later. The reason, quite simply, is that in Guo's method it is assumed that the Sun moves at constant speed through the solstice, whereas in reality its rate of motion varies. Chinese astronomers knew of this variation in the speed, but Guo does not allow for it here. It is also clear that in calculating the time from midnight Guo treats the transit as midway between midnights.

At the end of this text Guo infers from his investigation the time of the Winter solstice in the year 1280. He gives 1280 Dec 14 6 ke after midnight, that is 0.1 day after midnight. While he does not give the details his result may easily be found by adding one whole year to the date 2188560, 19;37,59<sup>h</sup>. Guo gives elsewhere the length of the year as 365.2425 days, that is 365 days 5;49,12<sup>h</sup>; if this time is added to 19;37,59 the time of the solstice in 1280 results, 1;27,11, or 0.0605 day, that is 6 ke.

The year length 365.2425 is not in good agreement with the intervals that may be calculated from the above observations. These three 12-month intervals are 365.2456, 365.2462, 365.2476.

### Earlier use of the 3-day method

As Guo acknowledges, the method employed by him to fix the time of solstice had been used much earlier. He mentions the work of Zu Chong-zhi ( ) of the Liu Song dynasty<sup>18</sup>. According to the Historical Classic Song Shu, Zu found the following shadow lengths on three days in the Da Ming year 5 [(8,2) xin-chou], observed with an 8 foot gnomon at Nanjing.<sup>19</sup>

month 10, day 10 [(9,11) ren-xu, 461 Nov 27] shadow 10.775

<sup>18</sup> Yuan Shi, ch.52, p.1121.

<sup>19</sup> Song Shu, ch.13, p.313; Gaubil, *Connaissances des temps* (1809), 389.

month 11, day 25 [(4,8) ding-wei, 462 Jan 11] shadow 10.8175  
 month 11, day 26 [(5,9) wu-shen, 462 Jan 12] shadow 10.7508  
 from which the solstice was fixed at

month 11, day 3 [(2,10) yi-you , 461 Dec 20],  
 31 ke after midnight [7<sup>h</sup>;26,24].

When calculated from modern parameters the solstice is situated at 3 a.m. on day 3. The latitudes implied by these shadow lengths are approximately 31;50, consistent with measurements at Nanjing, the Liu Song capital.

The expressions of the shadow lengths illustrate a notation used occasionally for fractions. The shadow lengths are written as follows

一丈七寸七分半 = 1 chang 7 tsun 7 fen  $\frac{1}{2}$  fen = 10.775 feet

一丈八寸一分太 = 1 chang 8 tsun 1 fen  $\frac{3}{4}$  fen = 10.8175

一丈七寸五分强 = 1 chang 7 tsun 5 fen  $\frac{1}{12}$  fen = 10.7508333...

The fractional part of the fen in the shadow lengths is expressed by means of certain characters conventionally adopted for the purpose: ban 半, tai 太, qiang 强, representing 1/2, 3/4 and 1/12, respectively. Gaubil however read the last as '10.752 (or 3)', and here and elsewhere shows that he had not a perfectly clear grasp of this notation.<sup>20</sup>

### Height of Gnomon and Latitude of the site

The latitude of the site of observation may be calculated for each individual shadow observation. If the height of the gnomon is  $H$ , which is given by Guo as 40 feet, and if the length of the shadow is  $S$ , then the altitude  $a$  of the Sun at the time of transit is given by  $a = \arctan(H/S)$ . Since the altitude of the Sun above the celestial equator is its declination  $\delta$ , while the altitude of the celestial equator is  $90 - \phi$ , where  $\phi$  is the latitude of the site, then  $a = 90 - \phi + \delta$ . Therefore if the declination of the Sun at the time of transit is calculated, we can infer the latitude  $\phi = 90 + \delta - a$ .

In this investigation the modern astronomical details are computed according to the VSOP87 model created by Bretagnon and colleagues at the Paris Observatory. A version sufficiently precise for most historical research is available with the handbook of astronomical algorithms by Jean Meeus<sup>21</sup>.

The procedure employed by Guo did not require that the height be exactly 40 feet, since he was concerned only with the relative lengths of the shadows, whereas if we are to investigate the latitude of the site we actually need to determine the true height of the gnomon. Moreover while he is assumed to have measured the shadow from a point directly under the rod, there may have been some departure from this, which again did

<sup>20</sup> I am indebted to Christopher Cullen and Jean-Claude Mertzloff for discussions of these fractions, which remain the subject of investigation.

<sup>21</sup> Meeus (1991).

not matter for his purpose, since only differences of the shadows were required. The present analysis of the data allows for a range of heights (around 40 feet), and for a range of values of the bias in the shadow scale (either side of zero).

The main statistical analysis follows a standard least squares procedure, with three parameters controlling the range of gnomon height, scale bias, and latitude. The ranges are as follows.

- scale : in the range -0.06 to 0.04, in steps of 0.005
- height : in the range 39.5 to 40.5, in steps of 0.01
- latitude : in the range 39.8 to 40.0, in steps of 0.002

The latitude can be securely estimated in advance so that this range is known to be adequate. The least squares analysis is applied to the declination of the Sun. From the modern parameters the declination is computed at the moment of true meridian transit, and that moment is computed by an iterative procedure. The result is  $\delta_o(n)$ , calculated for each of the 98 dates of the recorded shadows. On the other hand the declination implied by the length of the shadow is found by taking the altitude of the Sun

$$\text{altitude}(p,q,n) = \arctan(\text{height}(p)/(\text{shadow}(n) + \text{scale}(q)))$$

where,

- scale(q) = -0.06 + 0.005q, where q runs from 0 to 40,
- height(p) = 39.5 + 0.01p, where p runs from 0 to 100,

for each of the values of height and scale bias in the range stated above. Each value of altitude(p,q,n) is then corrected for atmospheric refraction. This last quantity is taken according to a formula provided by Meeus<sup>22</sup>

$$R = \frac{1}{\tan\left(h + \frac{7.31}{h + 4.4}\right)} + 0.0013515,$$

where  $h = \text{altitude}(p,q,n)$  in degrees, and  $R$  is the measure of refraction in arc-minutes, to be subtracted from  $h$ , to obtain the true altitude  $h_{tr}(p,q,r,n)$  of the Sun. The implied declination is then

$$\delta(p,q,r,n) = h_{tr}(p,q,r,n) + \text{latitude}(r) - 90.$$

where,

- latitude(r) = 39.8 + 0.002r, where r runs from 0 to 100.

The quantity to which the least squares procedure is applied is the sum of squares of differences between this declination and the declination calculated from the theory  $\delta_o(n)$ ,

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<sup>22</sup> Meeus (1991), p. 102.

$$Q(p,q,r) = \sum_n (\delta(p,q,r,n) - \delta_o(n))^2 .$$

When the minimum of Q is sought as p, q, r vary, the result is

Scale =  $-0.0300 \pm 0.0043$

Height =  $40.0300 \pm 0.0209$

Latitude =  $39.8300 \pm 0.0140$  (39;49,48  $\pm 0;0,50$ )

Q = 0.0000097993 (radians squared).

The error estimates are derived in the usual way from the statistical theory of least squares analysis. From the value of Q at the minimum one can estimate the standard deviation in the declination, as

$$\Delta_\delta = \sqrt{Q/(N-R)}$$

where N = 98 is the number of observations, and R = 3 is the number of parameters. The result, converted to degrees, is  $0.0181 = 0;1$ . Thus the declinations implied by the shadow measurements are secure to about one minute of arc. Interpreted in statistical terms, the probability that the latitude lies within the margin  $\pm \Delta_\delta$  is 85% and that it lies within  $\pm 2\Delta_\delta$  is 98%.

These departures from the ideal situation (height = 40 and scale bias = 0) are indeed small, and testify to the care taken by Guo in creating the installation. It was not easy to fix the zero point of the scale with such precision, and involved perhaps dropping a 40 foot plumb line from the rod to the scale, and locating its point on the scale to within 0.03 foot, roughly one centimeter. A further analysis of the shadow values shows that the internal variance is about 0.048 foot, about 1.5 cm.

In particular, the accuracy of the values of the declination, with a standard deviation of one minute, shows that we are dealing with true observations here, and not, if any one were to doubt it, shadow lengths that were somehow manufactured in place of true observations.

## Observatories in Beijing

### Summary

The history of observatories in Beijing is rather involved. The observatory that is now found in the city is apparently the third of a series that began in the 12<sup>th</sup> century. The story begins with the Jin dynasty (1115-1234). The northern Song capital Kaifeng, in Henan province, was captured by the Jin army in 1127, and it is believed that astronomical instruments established there were transported to the new Jin capital Zhongdu, just adjacent to the future site of Dadu, the capital of the Yuan dynasty that succeeded the Jin. It is this indeed that appears to be the location of the observations made by Guo Shoujing from about 1276, in the Yuan period. Subsequently in 1279 the Yuan observatory proper was established, just within the South-East corner of the new

walls defining the Yuan capital Dadu. This in turn was replaced in the Ming period by another, the only one now extant in the city.

### **Jin dynasty**

The Jin dynasty (1115-1234) had established a capital Zhongdu 中都 near Beijing, indeed located just immediately adjacent to the southwestern corner of the area that was to become the large rectangle defining the Yuan capital Dadu; see Figs. 4, 5.<sup>23</sup> This square region Zhongdu overlaps the so-called Chinese city, a long rectangular region contiguous with the southern edge of Dadu. Each side of this square is about 4.5 km (0;1,15 degree), and is centred on the coordinates 39;53, 116;20, . In a recently published booklet written to mark the construction of a copy of Verbiest's star globe to be set up in the University of Leuven, Belgium, Yi Shitong of the Beijing Observatory, writes<sup>24</sup>

According to historical records, there were astronomical instruments and an observatory in Beijing as early as in the Jin dynasty. In 1127, when Jin troops occupied Bianjing (汴京, now Kaifeng 开封 in Henan province), capital of the Northern Song dynasty, they shipped Song astronomical instruments to Zhongdu, the newly built Jin capital in the southwest of modern Beijing. They were later installed on a platform set up by the Jin imperial court. When the Yuan dynasty built its new capital Dadu 大都 slightly northeast of Zhongdu, astronomical observations were at first made mostly through these Song instruments on the Jin platform. In 1279 an Imperial College of Astronomy and an Astronomical Observatory were set up at the southeastern corner of Dadu around the site of the present observatory .

While I can find no text to support the notion that instruments were transported by troops from Bianjing to the new Jin capital, it may be allowed as an inference from a passage in the Biography that the instruments were in fact moved. For there it is said that having found the instruments at Bianjing (Kaifeng) to be incorrect for the latitude of Dadu, which is four degrees further north, Guo Shoujing prepared for his observations on suitably elevated ground, installing wooden enclosures along with his various instruments, including the High Gnomon (gao biao 高表).<sup>25</sup> Since he was so concerned to find that the instruments designed for Bianjing were unsuitable, it is reasonable to infer that he actually did have the earlier instruments with him at Dadu.

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<sup>23</sup> The map in Fig. 5 is taken from Bouillard (1924).

<sup>24</sup> Yi Shitong (1989), p. 15. Cf remarks made in *History of Chinese Astronomy*, Beijing, 1981, p 214.

<sup>25</sup> Yuan Shi, Chapter 164, p. 3847.

Now, it is now reasonable to infer from the findings given above regarding the latitude of the site of the observations that they were made on this site already established in the Jin period, the site referred to in the Biography.

### **Yuan and Ming dynasties**

The most tangible sign of an observatory in Beijing is of course the one by the Eastern wall that now serves as a Museum, the Guanxiangtai (观象台). While people have in the past assumed that this extant observatory coincides with that created in Yuan capital Dadu, Chinese historians now believe that the latter was not exactly on the same site, but slightly to the north of it, on a site later occupied by the Examinations Hall. This view is now so far accepted that it is reflected even in the latest guide books, such as the Lonely Planet guide to China<sup>26</sup>.

In a recent critical review of the history of Chinese astronomy we have a discussion of the relation between the Yuan and Ming observatories:

The [third] Ming Emperor Cheng Zu [1403] moved the capital to Beijing, but the astronomical organization in Nanjing remained working. In the meantime, another astronomical bureau was set up in Beijing. Because of the lack of instruments, at first people had to make naked eye observations on the city wall near Qi Hua Gate (which was renamed as Chang Yang Gate later on). In the Yong-le year 22 (1424 A.D.), an observatory was built west of the Forbidden City. However, it seems that there was no instrument for use. It was not until the Zheng-tong year 2 (1437) that people were ordered to reproduce wooden copies of the instruments from Nanjing. Then from years 4 to 7 (1439 to 1442), several bronze instruments were made in Beijing and placed on the platform of a newly-built observatory, which is to the south of the one in the Yuan Dynasty. The body of the observatory is quite close to the city wall (only 3 or 4 chi/feet away), and about 5 zhang (50 chi/feet) in height, which is several chi/feet above the city wall. It was used in the Ming and Qing Dynasty, and still exists.

Why was the Ming observatory built so close to the city wall? Some people think there was originally a high platform at that place, and suppose that the observatory of the Ming Dynasty is just the small platform inside the Yuan Astronomical Bureau (Taishiyuan). But according to researchers of Beijing local history, the southern part of the Yuan city wall was along the Zhangan Street. Therefore, the Ming Observatory is outside the south of the Yuan city wall, and cannot be a remainder of the Yuan Astronomical Bureau, because Yan Huan clearly stated that the Yuan observatory was inside and just beside the eastern part of the Yuan city

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<sup>26</sup> *China* (2000), p. 208, 'The observatory dates back to Kublai Khan's days, when it was to the north of the present site.'

wall. Some other people believe that the Ming observatory is the remains of the Jin Observatory; however this opinion is not well accepted.

According to Xu Pingfang, the Yuan observatory was changed to Gong Yuan (Examinations Hall) in the Ming and Qing Dynasties, and it is right inside the court of present-day Chinese Academy of Social Sciences. Because of this re-function, the buildings inside the Yuan Observatory must have been destroyed. When Emperor Ming Cheng Zu [1403-1425] reconstructed the City of Beijing, the original city wall was moved south so far that the place where the later Ming observatory was built was then included [within the wall]. Because the Qi Hua Gate is near to the former Yuan observatory, the city wall nearby was easily to be chosen as an observational site by the Ming astronomers when there was no instrument for use in the beginning. Thus, it is also understandable that, the later Ming observatory was built near this part of city wall where an empty space was available and continuous observations could be easily carried on. This explanation of why the Ming Observatory was built very close to the city wall maybe is the most reasonable one.<sup>27</sup>

The Yuan observatory is described in detail in a precious text, the *Taishiyuan Ming (Inscription on the Astronomical Bureau)* by Yang Huan. This important text is presented in full in the Appendix, as translated by Niu Weixing 鈕衛星 (Shanghai Jiaotong University) and Kim Taylor (Needham Research Institute); the text is among many preserved in a vast anthology.<sup>28</sup> Here we note first of all that, according to this, the dimensions of the observatory are 200 × 150 paces, where the pace is 5 feet. If the foot is taken as approximately 30 cm, this makes the base 300 × 225 m.<sup>29</sup> An examination of detailed maps of Beijing shows that this is indeed just the dimension of the Examinations Hall (Gong Yuan 貢院) situated immediately north of the Ming Observatory; at least in the North-South direction it is 300 m long, and the distance from its western edge to the wall is some 200 m. These sites are marked in the map shown in Fig. 6. It is therefore very plausible indeed that the Yuan observatory was originally on this very site. Certainly the present Observatory is far smaller than its Yuan predecessor. There is a detailed drawing of the terrace of the Ming Observatory and its various instruments, and this shows clearly that it measures only roughly 40 feet square, far smaller than the Yuan

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<sup>27</sup> *History of Chinese Astronomy*, Beijing, 1981, p 214. This contrasts with the simpler view that the Ming observatory was constructed by the Jia-jing Emperor of the Ming Dynasty, in his second year (1523), as remarked, for example, by Gaubil (1809), p. 399; cf. Arlington (1935), 155.

<sup>28</sup> The Complete Library in Four Branches of Literature (四庫全書), Vol. 1367, pp.209-211. The contents of the *Ming* are summarised in the *History of Chinese Astronomy*, Beijing, 1981, p 214.

<sup>29</sup> In one Chinese commentary these dimensions are interpreted as 123 m × 92 m., but without any supporting argument; *Achievements of Ancient Chinese Astronomy*, p. 134.

Observatory.<sup>30</sup> From a recent map of Beijing we can fix the coordinates of the Ming observatory as 116;25,33 East, 39;54,19 North.

The central part of Beijing in the Ming period is defined by two contiguous rectangles, framing the Tatar city in the north and the Chinese city in the south; see Fig. 4. The northern rectangle replaces one created under the preceding Mongol period by Kubilai Khan *ca* 1280, when it was known as Dadu (Great Court). In this transformation the southern wall of the Tatar city was moved south by some 700m. As to the situation of the Observatory in relation to the wall, we note that the wall marking the Southern boundary of the Tatar city in the Yuan period is known to have been replaced by the road Zhangan that extends from the Tian Anmen gate eastwards towards the wall, but without quite reaching it. The present-day Zhangan street is continued in a more or less straight line as far as the wall, but with a change of name to Jianguomen Nei before it reaches the wall; see Fig. 6. In any event, this road marks the line of the ancient wall of Dadu at a point lying between the extant Ming observatory and the Examinations Hall. In this way, quite apart from its size, one can understand how the Ming observatory cannot have been built precisely on the site of the Yuan observatory, because the latter lay just within the corner of the East and South walls of the Yuan city. In the *Taishiyuan Ming* the observatory is described as lying ‘under the East wall’.

The Yuan observatory consisted of three stories together with a terrace, the Lingtai. It was constructed in year 16 (1279), more than a year later than the earliest observation made by Guo. Further, there is a list of the instruments placed in the observatory,

On the top of the Lingtai:

Simplified Equatorial Torquetum (jianyi 簡儀),

Upward-looking Instrument (yangyi 仰儀).

Beneath the Simplified Equatorial Torquetum:

Direction-determining Board (zhengfangan 正方案).

On a separate small platform to the left (east) side of the Lingtai:

Exquisite Armillary Sphere (linglonghunyi 玲瓏渾儀).

To the right of the Lingtai:

High Gnomon (gaobiao 高表).

There has been a most interesting attempt at an illustration of the observatory on the basis of the description given in the *Taishiyuan Ming*. This is reproduced in Fig. 3, taken

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<sup>30</sup> For the drawing, Verbiest (1687), facing p.1; Lecomte (1696), Vol.1, facing p. 92; reproduced in Needham (1959), Plates LXVI. Photographs are given by Needham (1959) Plate LXVII, and Yi Shitong (1989), p. 15.

from *Achievements of Ancient Chinese Astronomy*, edited by the Astronomy Bureau of Beijing.<sup>31</sup>

These and many more instruments are listed in the Biography, referred to several times already, an account that was the basis of Wylie's valuable summary of his activity.<sup>32</sup> As remarked above, from this account, we learn only that the instruments were to be established on elevated ground, but no reference is made there to an observatory by the city wall.

Now we see among the instruments on the Lingtai that there was a High Gnomon, 'Gao Biao', the same name that was used by Guo Shoujing for his 40 foot structure. Here no dimensions are given, and in fact there is so little detail that we cannot even be altogether sure that the shadow definer was used. However, the gnomon established on the Ling Tai cannot be the same as that used by Guo Shoujing. For the first observation, in the list of 98 that we are studying here, was made at the end of year 14, 1277 December 10, whereas the Ling Tai was built well over a year later, in the Spring of year 16, around 1279 March 14. Its High Gnomon is described as being on top of the Ling Tai, so that this particular gnomon only dates from year 16. Moreover in the Biography there are noted even earlier observations which served to fix the Winter Solstice of 1276, given as Dec 14, year 13, day (5,11)35 8½ ke.<sup>33</sup> Thus at least two and a half years separate the observations with the 40 feet gnomon from the construction of the Astronomical Bureau, its Lingtai, and its superimposed gaobiao.

Indeed we have already found that the latitude of the site of the observations places it not at the site of the observatory built within the corner of the Tatar city walls, but some 4 minutes of latitude to the south.

### **The site of the observations**

The latitude determined by the analysis of the 98 shadow lengths is 39;49,48 ±0;0,50, or effectively 39;50±0;1. This places the site some four minutes south of the observatories of the Yuan and Ming periods.

That the gnomon was placed some way to the south of Beijing was already inferred by Gaubil, who provides some surprising material in addition to arguments about the latitude.<sup>34</sup> He worked from only 6 of the shadow lengths (nos. 17, 53, 62, 63, 65, 71 in our list) and yet the latitude of the site that he derived, 39;50, agrees with that found here.

In remarks not published until 1809, Gaubil writes

Le tatou de Cocheou-king, est sans contredit la cour de Cobilay à la Chine, et c'est au moins en partie la ville tartare de Pékin. La tour des mathématiques et la ville

<sup>31</sup> *Achievements of Ancient Chinese Astronomy*, Beijing, 1987, p.134.

<sup>32</sup> Peking Edition, Vol.13, p. 3847; Wylie (1897), p.5-7.

<sup>33</sup> Yuan Shi chap 164, Peking Edition, p. 3849.

<sup>34</sup> Gaubil (1809), p. 399-400.

chinoise furent bâties sous la règne de Kiatsing de la dynastie Ming. Notre maison française est plus boréale que l'extrémité australe de la ville chinoise de plus de 9 li: or à 5 ou 6 li au sud de l'extrémité de la ville chinoise, on voit des restes du lieu où étaient les instruments de Cocheou-king, et l'observatoire n'en était pas loin; il paraît certain qu'il était au sud de la ville chinoise d'aujourd'hui.

The Dadu of Guo Shoujing is assuredly the court of Kubilai in China, and is at least in part the Tatar city of Peking. The mathematical tower and the Chinese city were constructed during the reign-period of Jia-jing<sup>35</sup> of the Ming dynasty. Our French House is to the north of the southern limit of the Chinese city by more than 9 li; now 5 or 6 li south of the limit of the Chinese city one sees the remains of the place where were the instruments of Guo Shoujing, and his observatory was not far away. It appears to be certain that it was to the south of the present-day Chinese city.<sup>36</sup>

and further, 'Our French house in Peking is at the latitude 39;54 or ;55, the observatory of Guo Shoujing being some 0;4 to the south...'.<sup>37</sup>

Gaubil's positions are readily confirmed. The position of the Jesuit house is shown on the sketch map by Arlington<sup>38</sup> from which one can determine that the distance 9 li to the southern boundary of the Chinese City is some 6.2 km (making the li in Gaubil's time about 0.69 km). From a modern map one would make the latitude of the house nearer to 39;55,30. The Jesuit House in the western suburbs is shown in Fig. 5, which also shows the position of the Jin capital due south of it. A point 5 or 6 li, about 3.8 km, south of the southern boundary of the Chinese City is then about 39;50, as Gaubil said. Therefore a place 5 or 6 li, or 4 minutes of latitude, to the south of the southern limit of the Chinese city agrees perfectly well with the latitude found in the detailed analysis given here.

Here Gaubil makes a valuable distinction between the offices of the observatories and the location of Guo's gnomon. Moreover, it is especially intriguing to read of 'the remains of the place where were the instruments of Guo Shoujing'. Perhaps Gaubil had in fact been shown remains of instruments, or at least the site, established in the Jin period.

Dong Zuobin, in the course of his study of the great tower at Yangcheng, considered the observations recorded by Guo Shoujing. In order to illustrate the procedure of interpolation he went through three examples, using the three triples 13, 14 and 17 of Table 2.<sup>39</sup> Further, he explained how to discover the latitude of the site from the height of the tower and the length of the shadow, and arrived at the latitude 39;51.5, but without

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<sup>35</sup> See Note 27.

<sup>36</sup> Gaubil (1809), p. 400.

<sup>37</sup> Gaubil (1809), p. 401.

<sup>38</sup> Arlington (1935), p. 234.

<sup>39</sup> Dong Zuo-bin (1939), p. 62-5.

giving any detail of his calculation<sup>40</sup>. Nor did he discuss the implications for the precise site of the observations, assuming only that they were made at Dadu. He provided a naturalistic drawing to show how the gnomon and its scale might have appeared as situated in an open field.<sup>41</sup> He assumed, that is to say, not that the gnomon was situated at an observatory, but that it was in the open air, rather in accordance with the account in the *Biography*.<sup>42</sup> Although we arrive at a slightly different latitude, and with real statistical control, Dong Zuobin's figure is nevertheless quite close to it, and consistent with the assumption of a site in the Jin capital Zhongdu.

Needham, treating of the great gnomons from a larger perspective writes<sup>43</sup>

According to the *Yuan Shi*, three other places [apart from Yang Cheng] (Tatu (now Peking), Shangtu (the Xanadu of the English poet, and the Mongol imperial summer capital in Chahar province), and Nanhai in Kuangtung) were destined to receive gnomons of 40 ft., but only at Yang-chhêng and Peking was one actually set up, and Yang-chhêng alone (the 'centre of the earth') ranked so high as to have the tower as well.<sup>44</sup>

I cannot find in the *Yuan Shi* or elsewhere any supporting text for this remark. Moreover, in the list of latitudes of various towns, Guo Shoujing gives the solstitial shadow lengths for some of these, but it is only for Dadu that the shadow is one that would be for a 40 foot gnomon, the others being for an 8 foot gnomon.<sup>45</sup> In any case, for Yang Cheng, only the latitude is given. This indicates that there was a 40 foot gnomon only at Dadu. Needham, to be sure, does not commit himself to the view that the shadow observations in the *Yuan Shi* were made at Yang Cheng rather than at Beijing, but his great emphasis on the former has encouraged people to believe that this was the case. He reproduces the photograph of the tower published by Dong Zuobin, with caption, 'In its present form the structure is a Ming renovation of the building erected by Kuo Shou-Ching about 1276 for use with the 40 ft. gnomon'.<sup>46</sup> This suggests at least Needham's belief that this was the site of the observations listed in the *Yuan Shi*. In fact, according to the *Ming Shi*, in the Jiajing year 7 (1528), a 40 foot wooden gnomon was erected, and one must question whether there was previously any such gnomon on the site.<sup>47</sup>

The mistaken belief regarding the site of the shadow measurements arose not only as a result of Needham's work, but is apparently encouraged among visitors to the tower itself,

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<sup>40</sup> Dong Zuo-bin (1939), p. 113-5.

<sup>41</sup> Dong Zuo-bin (1939), p. 58, Fig. 15.

<sup>42</sup> *Yuan Shi*, Chapter 164, p. 3847.

<sup>43</sup> Needham (1957), pp. 296-7, shown in figs 115-7.

<sup>44</sup> Needham (1957), pp. 297.

<sup>45</sup> *Yuan Shi*, Chapter 48, p. 1000 seq.

<sup>46</sup> Dong Zuo-bin (1939); Needham (1959), p. 296 seq.

<sup>47</sup> *Ming Shi*, Chapter 25, p. 359.

where there is, I believe, a display to show how the shadows were measured with the yingfu. The problem is that Needham did not quote from either Gaubil or Dong Zuobin to show that they calculated the latitude of the site.

### **The latitude of Dadu according to Guo Shoujing**

Guo had made his own determination of the latitude, or rather, in his own terms, the altitude of the pole star, of Dadu (Beijing).<sup>48</sup> In Chapter 48 he gives a table of pole star altitudes for a wide range of Chinese sites. Among these Dadu is assigned the altitude  $40\frac{3}{4}\frac{1}{12}$ , that is 40.8333.<sup>49</sup> Translated into Western degrees this is 40;14,47, which is well to the north of Beijing, in fact it is about 38 km north of the northern wall of the Tatar city. There is therefore a great disagreement not only with the figure calculated from the shadow lengths, but also with that of the Yuan observatory.

Another difficulty emerges when one analyses the tables of day-length provided by Guo. In Chapter 55 he gives tables of the declination, of the distance from the pole star to points on the ecliptic, and of the length of day as a function of the degree of the ecliptic. These values of the length of the day were calculated according to Guo's version of spherical trigonometry, and depend on the obliquity of the ecliptic and the latitude. It has to be said that in these matters the Chinese procedure falls well short of that available in Greek-Islamic sources, as has been well explained by Martzloff and others<sup>50</sup>. However, once the peculiar nature of the methods is followed one can calculate the latitude which had been assumed for this table. The calculations of day length do not agree absolutely with the figures in the table, and when one calculates the latitude implied by the various values, over the whole range of the ecliptic, one finds that the latitude falls in the range 40.65 to 40.8, with an average around 40.75. If not so far from the value that Guo has stated,  $40\frac{3}{4}\frac{1}{12}$ , it is not in particularly close agreement.

These difficulties with the latitude are in some contrast to the rather precise value of the obliquity given by Guo, 23.9030, equivalent to 23;33,32 in a circle of 360 degrees. This compares favourably with the value 23;32,6 calculated for 1280, including nutation.

## **Conclusion**

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<sup>48</sup> It is not to be forgotten that the spherical shape of the earth was not employed by Chinese astronomers, so that it would be wrong to read into these figures the concept of latitude.

<sup>49</sup> Expressed as 40 tai qiang; see the remarks above on this way of expressing fractions.

<sup>50</sup> Martzloff (1997), p. 328 seq. One must disregard the otiose speculation by Needham in Beer (1961) and Needham (1962), p. 48-50, who argued for the adoption of Greek-Islamic trigonometry as early as the time of Yi-xing in the 8<sup>th</sup> century.

From the observations of meridian shadow lengths recorded as having been made by Guo Shoujing beginning in 1276 the latitude of the site may be inferred, and is found to be  $39;49,48 \pm 0;0,50$ . This measure of the latitude not only agrees essentially with results found by Gaubil and by Dong Zuobin, but gives to them greater substance. This latitude is situated slightly to the south of the 'Chinese City', the southern part of Dadu (Beijing). Indeed it is at the edge of the area of the Jin capital Zhongdu. Such a site agrees well with the account in the Biography of Guo Shoujing. It quite rules out, moreover, the assumption found in some quarters that the observations had been made at the great tower at Yangcheng in Henan province.

There is included also a translation of the Taishiyuan Ming, a most important contemporary description of the observatory erected in Beijing in 1279. This observatory was situated immediately to the north of the present Observatory. It was certainly not the actual site of the shadow measurements, which were begun before 1279, and at a point some four minutes of arc to the south.

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<sup>51</sup> As confirmed by Laplace's remark in the issue for 1811, p.429.

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**Table 1 Shadow measurements of Guo Shoujing**

The observations are grouped around the Winter Solstice (W.S.) and Summer Solstice (S.S.) in the years 14-16.

Intercalary months are marked \*.

No.	Year	Month, day	Day-cycle	Shadow	Date	JD
			W.S. year 14			
1	14	11,14	ji-hai (6,12)36	79.4855	1277 December 10	2187826
2	14	11,21	bing-wu (3,7)43	79.5410	1277 December 17	2187833
3	14	11,22	ding-wei (4,8)44	79.4550	1277 December 18	2187834
4	14	11,9	jia-wu (1,7)31	78.6355	1277 December 5	2187821
5	14	11,26	xin-hai (8,12)48	78.7935	1277 December 22	2187838
6	14	11,27	ren-zi (9,1)49	78.5500	1277 December 23	2187839
7	14	11,28	gui-chou (10,2)50	78.3045	1277 December 24	2187840
8	14	11,1	bing-xu (3,11)23	75.9865	1277 November 27	2187813
9	14	11,2	ding-hai (4,12)24	76.3770	1277 November 28	2187814
10	14	12,6	geng-shen (7,9)57	75.8501	1277 December 31	2187847
11	14	10,21	bing-zi (3,1)13	70.9710	1277 November 17	2187803
12	14	12,16	geng-wu (7,7)7	70.7600	1278 January 10	2187857
13	14	12,17	xin-wei (8,8)8	70.1465	1278 January 11	2187858
14	14	6,5	gui-hai (10,12)60	13.0800	1277 July 7	2187670
15	15	5,1	gui-wei (10,8)20	13.0385	1278 May 23	2187990
16	15	5,2	jia-shen (1,9)21	12.9205	1278 May 24	2187991
			S.S. year 15			
17	15	5,19	xin-chou (8,2)38	11.7775	1278 June 10	2188008
18	15	5,28	Geng-xu (7,11)47	11.7800	1278 June 19	2188017
19	15	5,29	xin-hai (8,12)48	11.8055	1278 June 20	2188018
20	14	12,15	ji-si (6,6)6	71.3430	1278 January 9	2187856
21	15	11,2	xin-si (8,6)18	70.7595	1278 November 17	2188168
22	15	11,3	ren-wu (9,7)19	71.4060	1278 November 18	2188169
23	14	12,12	Bing-yin (3,3)3	72.9725	1278 January 6	2187853
24	14	12,13	Ding-mao (4,4)4	72.4545	1278 January 7	2187854
25	14	12,14	mou-chen (5,5)5	71.9090	1278 January 8	2187855
26	15	11,4	gui-wei (10,8)20	71.9575	1278 November 19	2188170
27	15	11,5	jia-shen (1,9)21	72.5050	1278 November 20	2188171
28	15	11,6	yi-you (2,10)22	73.0335	1278 November 21	2188172
29	14	12,7	xin-you (8,10)58	75.4170	1278 January 1	2187848

30	14	12,8	ren-xu (9,11)59	74.9595	1278 January 2	2187849
31	14	12,9	gui-hai (10,12)60	74.4860	1278 January 3	2187850
32	15	11,9	mou-zi (5,1)25	74.5205	1278 November 24	2188175
33	15	11,10	ji-chou (6,2)26	75.0035	1278 November 25	2188176
34	15	11,11	geng-yin (7,3)27	75.4495	1278 November 26	2188177
			W.S. year 15			
35	15	11,19	mou-xu (5,11)35	78.3185	1278 December 4	2188185
36	15	*11,9	mou-wu (5,7)55	78.3635	1278 December 24	2188205
37	15	*11,10	ji-wei (6,8)56	78.0825	1278 December 25	2188206
38	15	11,12	xin-mao (8,4)28	75.8815	1278 November 27	2188178
39	15	11,13	ren-chen (9,5)29	76.3015	1278 November 28	2188179
40	15	*11,15	jia-zi (1,1)1	76.3665	1278 December 30	2188211
41	15	*11,16	Yi-chou (2,2)2	75.9530	1278 December 31	2188212
42	15	*11,17	bing-yin (3,3)3	75.5045	1279 January 1	2188213
43	15	11,8	ding-hai (4,12)24	74.0375	1278 November 23	2188174
44	15	*11,20	Ji-si (6,6)6	74.1200	1279 January 4	2188216
45	15	*11,21	geng-wu (7,7)7	73.6145	1279 January 5	2188217
46	15	6,26	mou-yin (5,3)15	14.4525	1278 July 17	2188045
47	15	6,27	ji-mao (6,4)16	14.6380	1278 July 18	2188046
48	16	4,2	mou-yin (5,3)15	14.4810	1279 May 13	2188345
18	15	5,28	geng-xu (7,11)47	11.7800	1278 June 19	2188017
49	16	4,29	Yi-si (2,6)42	11.8630	1279 June 9	2188372
50	16	4,30	bing-wu (3,7)43	11.7830	1279 June 10	2188373
			S.S. year 16			
51	16	4,19	Yi-wei (2,8)32	12.3695	1279 May 30	2188362
52	16	4,20	bing-shen (3,9)33	12.2935	1279 May 31	2188363
53	16	5,19	yi-chou (2,2)2	12.2640	1279 June 29	2188392
54	16	3,21	mou-chen (5,5)5	16.3950	1279 May 3	2188335
55	16	6,16	ren-chen (9,5)29	16.0995	1279 July 26	2188419
56	16	6,17	gui-si (10,6)30	16.3110	1279 July 27	2188420
57	16	3,2	ji-you (6,10)46	21.3050	1279 April 14	2188316
58	16	7,7	ren-zi (9,1)49	21.1955	1279 August 15	2188439
[59	16	7,8	gui-chou (10,2)50	21.4865	1279 August 16	2188440]
60	16	3,1	mou-shen (5,9)45	21.6110	1279 April 13	2188315
59	16	7,8	gui-chou (10,2)50	21.4865	1279 August 16	2188440
61	16	3,1	jia-yin (1,3)51	21.9155	1279 August 17	2188441
62	16	2,18	yi-wei (2,8)32	26.0345	1279 March 31	2188302
63	16	7,21	bing-yin (3,3)3	25.8990	1279 August 29	2188453

64	16	7,22	ding-mao (4,4)4	26.2590	1279 August 30	2188454
65	16	7,23	geng-chen (7,5)17	32.1955	1279 March 16	2188287
66	16	8,5	geng-chen (7,5)17	31.5965	1279 September 12	2188467
67	16	8,6	xin-si (8,6)18	32.0265	1279 September 13	2188468
68	16	1,19	ding-mao (4,4)4	38.5015	1279 March 3	2188274
69	16	8,18	gui-si (10,6)30	37.8230	1279 September 25	2188480
70	16	8,19	jia-wu (1,7)31	38.3105	1279 September 26	2188481
			W.S. year 16			
71	16	10,24	mou-xu (5,11)35	76.7400	1279 November 29	2188545
72	16	11,25	ji-si (6,6)6	76.5600	1279 December 30	2188576
73	16	11,26	geng-wu (7,7)7	76.1425	1279 December 31	2188577
74	16	10,18	ren-chen (9,5)29	74.0525	1279 November 23	2188539
75	16	10,19	gui-si (10,6)30	74.5450	1279 November 24	2188540
76	16	10,20	jia-wu (1,7)31	75.0250	1279 November 25	2188541
77	16	11,28	ren-shen (9,9)9	75.3200	1280 January 2	2188579
78	16	11,29	gui-you (10,10)10	74.8525	1280 January 3	2188580
79	16	12,1	jia-xu (1,11)11	74.3650	1280 January 4	2188581
80	16	12,2	yi-hai (2,12)12	73.8715	1280 January 5	2188582
81	16	10,16	geng-yin (7,3)27	73.0150	1279 November 21	2188537
82	16	12,3	bing-zi (3,1)13	73.3200	1280 January 6	2188583
83	16	12,4	ding-chou (4,2)14	72.8425	1280 January 7	2188584
84	16	10,14	mou-zi (5,1)25	71.9225	1279 November 19	2188535
85	16	10,15	ji-chou (6,2)26	72.4690	1279 November 20	2188536
86	16	12,5	mou-yin (5,3)15	72.2725	1280 January 8	2188585
87	16	10,7	xin-si (8,6)18	67.7450	1279 November 12	2188528
88	16	10,8	ren-wu (9,7)19	68.3725	1279 November 13	2188529
89	16	10,9	gui-wei (10,8)20	68.9775	1279 November 14	2188530
90	16	12,12	jia-shen (1,9)21	68.1450	1280 January 15	2188592
91	16	10,1	yi-hai (2,12)12	63.8700	1279 November 6	2188522
92	16	12,18	xin-mao (8,4)28	64.2975	1280 January 21	2188598
93	16	12,9	ren-chen (9,5)29	63.6250	1280 January 22	2188599
94	16	9,22	bing-yin (3,3)3	57.8250	1279 October 28	2188513
95	16	12,28	xin-chou (8,2)38	57.5800	1280 January 31	2188608
96	16	12,29	ren-yin (9,3)39	56.9150	1280 February 1	2188609
97	16	9,20	jia-zi (1,1)1	56.4925	1279 October 26	2188511
96	16	12,29	ren-yin (9,3)39	56.9150	1280 February 1	2188609
98	17	1,1	gui-mao (10,4)40	56.2500	1280 February 2	2188610

**Table 2 List of triples considered by Guo**

The three observations are indicated by the notation (1,2,3}, referring to the numbered list of observations in Table 1.

1 (1,2,3)	20 (51,52,53)
2 (4,5,6)	21 (54,55,56)
3 (6,7,4)	22 (57,58,59)
4 (8,9,10)	23 (60,59,61)
5 (11,12,13)	24 (62,63,64)
6 (14,15,16)	25 (65,66,67)
7 (17,18,19)	26 (68,69,70)
8 (20,21,22)	27 (71,72,73)
9 (24,25,26)	28 (75,76,78)
10 (23,24,27)	29 (77,78,76)
11 (30,33,29)	30 (74,75,79)
12 (30,31,32)	31 (79,80,74)
13 (35,36,37)	32 (81,82,83)
14 (38,39,40)	33 (84,85,86)
15 (39,40,41)	34 (81,85,86)
16 (38,41,42)	35 (87,88,90)
17 (43,44,45)	36 (91,92,93)
18 (46,47,48)	37 (94,95,96)
19 (18,49,50)	38 (97,96,98)

## Appendix

Translation by Niu Weixing 鈕衛星 (Shanghai Jiaotong University) and Kim Taylor (Needham Research Institute) of

### INSCRIPTION ON THE ASTRONOMICAL BUREAU

Taishiyuan Ming (太史院銘)

Authored by Yang Huan 楊桓 (1234-1299), Gentleman of the School Books (校書郎) at the Astronomical Observatory and preserved in Compilation of Yuan Literature, Vol.17 (元文類 17 卷) (edited by Su Tianjue 蘇天爵, dated 1334), as contained in The Complete Library in Four Branches of Literature (四庫全書) Vol. 1367, pp.209-211.

### INTRODUCTION<sup>52</sup>

The inscription on the Astronomical Bureau was written to commemorate the construction of the new Astronomical Bureau for the Yuan dynasty in 1279. This astronomical bureau was built in the Yuan capital of Dadu (大都) or modern-day Beijing. It was to house the most sophisticated astronomical instruments of the day, many of which were designed and constructed by the renowned Yuan dynasty astronomer, Guo Shoujing (1231-1316). These included the equatorial torquetum, the scaphe and the 40 foot gnomon. The calendrical data collected from these instruments was used by Guo Shoujing in the compilation of the Shoushi li (授時曆) calendrical system. So accurate was this calendar that it was to remain the court calendar until the end of the Ming dynasty in 1644.

Astronomy in ancient China had a state function. The study of the heavenly bodies was highly organised and institutionalised. Each dynasty had to function according to a precise astronomical calendar in order to demonstrate that the Emperor was governing in accordance with the movements of the heavenly bodies. A deviation from such a calendar would imply that the Emperor was no longer in sync with the heavens and would thus initiate activities such as military excursions or the harvesting of crops on inauspicious days. When a new dynasty was set up, it was therefore not unusual for the new Emperor to wish to discard the calendar of his predecessor and to realign his dynasty with the stars by setting up a new calendar.

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<sup>52</sup> This translation was undertaken jointly by Niu Weixing of Shanghai Jiaotong University and Kim Taylor of the Needham Research Institute, Cambridge. We would like to thank Raymond Mercier for his assistance and encouragement in putting the translation together. We are also grateful to the Needham Research Institute for allowing us to present a preliminary version at a text-reading seminar and are indebted to all the scholars, in particular, Christopher Cullen, for their pointed and pertinent comments.

The inscription on the Astronomical Bureau describes such a situation. It most likely took the form of an inscribed plaque which hung on one of the exterior walls. Some of the more famous instruments in the Bureau also had such plaques attached to them. This particular inscription gives us invaluable information about the purpose for which the Astronomical Bureau was built in the context of the new Emperor's rule, details of its location within the capital, a description of its structural layout, the astronomical instruments with which it was equipped and their location in the Bureau, as well as the names and rank of the key members of its staff. The Inscription is written in a highly-stylised language that makes it only accessible to specialists of the history of Chinese astronomy. This translation aspires to render the message of the Inscription accessible to all.

### TRANSLATION<sup>53</sup>

Heaven tired of the disorder on Earth, and looked for learned men who could act as leaders of the people. It was according to this decree that the Emperor of Divine Military Might Genghis Khan (taizu shengwu huangdi 太祖聖武皇帝) grew up with regal bearing, so that with his spiritual and mighty [powers] he might suppress and remove the misfortunes and difficulties [caused by man], and thus accordingly, the precious fate of the Yuan dynasty was set. There came a succession of sages, each as virtuous as the previous one. Some of their achievements were accumulative and some were a legacy [of the previous sages]. This [continued] until the present day when there came the filial Emperor of great glory, with the highest authority to pass down the Way, benevolent and cultured, of righteous ferocity, and endowed with sagacity and spiritualness. From a remote princely mansion, he personally led the army. There was an advance of the military, but no war [because the enemy had, in fright, already fled]. The will of heaven was [the same as] the will of man and all pledged allegiance [to this new Emperor]. [His Empire] reached China and was set up. [There began] major construction in the cities and towns. Sages were appointed and those with ability were put to use. Numerous divisions of government were established. Once the government structure and state ideology were in place, rituals were determined and music ceremonies made [in order to] perform ceremonies to the family ancestors in the temples. Generals continued to be sent to all directions [where there were] those who did not submit [to the new emperor's rule]. And then finally [the Emperor] could start to unite the 6 sides of the box [ie. the Universe - that which is above, below, behind, in front, to the right, to the left] until the boundaries [of his Empire] met the limits of Heaven. To live in peace, with no [need for] action.

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<sup>53</sup> The translation has been kept as close to the original text as possible. This means that some of the wording and cadences of the English sentences might appear a bit stilted, but this should hopefully reveal the mannerisms of a highly-stylised text of this kind. All names and titles have been included on their first appearance in romanised Chinese (pinyin) and in Chinese characters. All words in square brackets are our own and are intended to fill in the blanks where the meaning has not been expressed in the Chinese but are necessary for a coherent English sentence.

Birds and fish and animals and plants: all are steeped in benevolence and mercy. Among the common people, there are no undesired events. The men only have to till the land and the women only have to weave. The workers manufacture and the merchants trade. Each person labours over his own clothes and his own food. The Emperor carefully considered that of those who were involved in the lives of the common people, there were none who were not primarily engaged [in these activities]. For agricultural matters are the root of all that with which the Four Peoples [the Scholar, the Farmer, the Worker, the Merchant] feed and clothe themselves. And so specialised departments were set up to supervise their work. And it was thought furthermore [by the Emperor] that in order to assist and promote their industry, the Astronomical Bureau should be set up, in order to make clear the Way of the Heavens, and thus respectfully provide the common people with a seasonal timetable.

In the Zhiyuan year 13 [1276], the Emperor was continuing to follow the Calendar of Great Brightness (Damingli 大明曆).<sup>54</sup> But it had become old and had lost its precision. Therefore [the Emperor] desired to create his own system. Wang Xun 王恂 of the rank Advisor to the Crown Prince (Taizi zanshan 太子贊善),<sup>55</sup> whose profession it was to be proficient at the art of calculating, was given all [the following areas of] the advancing and lagging behind of the moon and sun, the acceleration and retardation of the five planets [those that can be seen with the naked eye - Mercury, Venus, Mars, Jupiter and Saturn], the progression and retrogression, emergence and disappearance [of the planets], the position of the Centre Star<sup>56</sup> at dusk and at dawn - all these issues were given to him to deduce and extrapolate from, in order to determine the four seasons. Not long afterwards, he was promoted to the rank of Director of the Astronomical Bureau (Taishiling 太史令). Then there was Guo Shoujing 郭守敬 of the rank Master of the Waterways (Dushuijian 都水監), who, being intelligent and understanding the movements of the Heavens, was a genius at designing systems. All the instruments of armillary sphere, celestial globe, gnomon and clepsydra (yi 儀, xiang 象, biao 表, lou 漏) [which served to] tell the time of day and to measure the movements of the celestial bodies, were given to him to calibrate and fine-tune. Not long afterwards, he was awarded the position of Associate Director of the Astronomical Bureau (Tongzhi Taishishi 同知太史事) [ie. privy to the same information as the Director of the Astronomical Bureau]. Once the calendar was complete, he was promoted to Director of the Astronomical Bureau. Then there was Xu Heng 許衡, formerly the Left Aide of the Palace Secretary

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<sup>54</sup> The Calendar of Great Brightness was originally created by Yang Jishao 楊紱紹 in the year 1127 and served as the calendar of the Jin Dynasty from 1137-1181. It was then revised by Zhao Zhiwei 趙知微 in 1181 and continued to be used by the Jin from 1182-1234. It also served as the calendar for the early Yuan dynasty from 1206-1280.

<sup>55</sup> All translations of official titles have been taken from Hucker (1985).

<sup>56</sup> The Centre Star (zhongxing 中星) refers to the star located exactly at the meridian of the southern sky at dusk and at dawn - a process also known as meridian transit? For more information see Cullen (1996), p. 19.

(Zhongshu Zuocheng 中書左丞), who was famous among the sages of the day. All [that involved] a thorough inquiry into the Paths of Heaven and careful consideration of the increases and decreases [of the planetary movements] was given to him to lead and direct. He was assisted by Yang Gongyi 楊恭懿 of the rank Academician of Scholarly Worthies (Jixian Xueshi 集賢學士). The one who gave overall support and who assisted from the beginning to the end until the matter was complete, was in fact Zhang Wenqian 張文謙, formerly of the rank Left Aide of the Palace Director (Zhongshu Zuocheng 中書左丞) turned Great Supervisor of Agriculture (Dasinong 大司農). After a short time he came to balance the roles of Great Academician of the Institute for the Glorification of Literature (Zhaowenguan Daxueshi 昭文館大學士) and Director of the Astronomical Bureau (Taishiyuanshi 太史院事). As for all [matters to do with] construction labour and [building materials of] earth, wood, metal and stone, these were given to Jia Zhen 假貞 of the rank Minister of Works (Xinggongbu shangshu 行工部尚書) and Head of the Imperial Manufactories (Shaofujian 少府監) to manage and regulate. All to do with the beautifying of literary embellishments and craftsmen's decorations on the instruments of the armillary sphere, celestial globe, gnomon and clepsydra, should be referred to Ah Naga 阿納噶 of the rank Grand Minister of Education (Dasi tu 大司徒).

In the spring of the year 16 [1279], a fortuitous spot was chosen beneath the eastern wall of the capital city, and there began supervision of the building of the walls [around the worksite]. Its north-south length was 200 bu (布), its breadth less a quarter of this [ie. 150 bu].<sup>57</sup> In the middle a Spiritual Platform (Lingtai 靈臺)<sup>58</sup> was erected, and in the remaining seven zhang [presumably the height] three layers of building were built. The middle and lower layers were all surrounded by a corridor of rooms (wu 廡). On the lower level, the central room on the south-facing side was the main office (guanfu 官府), which served as a general management room for the administration of the Observatory. Its leader had the title of Director (ling 令), his direct subordinate was titled Associate Director (Tongzhiyuanshi 同知院事), his direct subordinate was titled Assistant Commissioner (Qianyuanshi 僉院事), and they were meant to assist the Director. There was also a replicate [set of people] which reported to the Emperor, but their numbers were unfixed. Of those belonging to this group, there were the Managers (zhushi 主事), the Translators (lingyishi 令譯史),<sup>59</sup> the Administrative Clerks (ganshi 幹事) and those who looked after the storehouse (kuju zhi si 庫局之司).

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<sup>57</sup> It is to be assumed that bu (布) came to replace, by some accident, its homonym bu (步), meaning pace. The dimensions are discussed above in the section 'Observatories in Beijing'.

<sup>58</sup> Needham (1959), p. 189, translates lingtai directly as astronomical observatory. Literally it means a raised platform which served as a stage for astronomical observances. It was also thought to have spiritual powers capable of defending the observatory from evil.

<sup>59</sup> Presumably the translators were responsible for translating astronomical documents from Muslim nations. For more information on such a transfer of knowledge see 'The Astronomical

The rooms to the left and right [of this central room on the bottom layer] served as the rooms where the various scholars could meet and discuss. Altogether there were 70 people who calculated and observed the calendrical system. [These 70 people] were divided into two Bureaus. One was called Calculation (tuisuan 推算) and its officials included the Head of the Five Offices (wuguanzheng 五官正), the Director of Calendrical Calculations (baozhangzheng 保章正), their [respective] deputies (fu 副) and calendrical clerks (zhangli 掌曆), and these gathered separately [ie. apart from other scholars in the Observatory] in the Dawn Chamber (zhaoshi 朝室). The second was called Observational Verification (ceyan 測驗). Its officials were the Gentlemen of the Spiritual Platform (lingtailang 靈臺郎), the Astronomical Observers (jianhou 監候), and their deputies. The third [counting from the main office, guanfu] was called the Water Clock (louke 漏刻). Its officials were the Supervisor of the Water Clocks (qiehuazheng 繫壺正) and the Time Keeper (sichenlang 司辰郎), and these gathered separately from the others in the Dusk Chamber (xishi 夕室). All the instruments were taken out from and put back into the Yin Chamber.

On the middle layer there was the Fire Chamber (lishi 離室)<sup>60</sup> which was used to align the radiance of Earth-shine (lie yingyao 列景曜).<sup>61</sup> In the Wind Chamber (xunshi 巽室) the water-powered clepsydra-driven armillary sphere (shuiyun huntian hulou 水運渾天壺漏) was installed. In the Earth Chamber (kunshi 坤室) the Celestial Globe (huntianxiang 渾天象) and the Planar Chart (gaitiantu 蓋天圖) were installed. The Thunder and Swamp Chambers (zhenshi, duishi 震室, 兌室) were used to chart the visible and invisible stars on the celestial spherical and planar maps [as evidenced from] different positions in the north and south. The Water Chamber (kanshi 坎室) was used to find the position of the Master of the Year (taisui 太歲).<sup>62</sup> The Heaven Chamber (qianshi 乾室) was used to store the books of astronomical observations. And the Mountain Chamber (genshi 艮室) was used to store the worksheets of calendrical system calculations, both past and present.

On the top of the Spiritual Platform two pieces of equipment, the Simplified Equatorial Torquetum (jianyi 簡儀) and the Upward-looking Instrument (yangyi 仰儀),

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Instruments of Cha-ma-lu-ting: their Identification, their Relations to the Instruments of the Observatory of Maragha', in Hartner (1968), p. 215-259. A broad up to date review is given by Allsen (2001), especially Chapter 17 'Astronomy'.

<sup>60</sup> These rooms are all named after the Eight Trigrams (bagua 八卦).

<sup>61</sup> Earth-shine - the effect when the sunlit earth illuminates by reflection the unlighted part of the moon. The Chinese also called it dexing 德星 or jingxing 景星; Needham (1959), p. 189.

<sup>62</sup> Taisui refers to an imaginary star, also known as Suiyin 歲陰 or Taiyin 太陰. According to Needham (1959), p. 402, it was an invisible counter-Jupiter, which moved around in the opposite direction to the planet itself. Jupiter, with the other planets, appears to move eastwards or anti-clockwise through the stars, so a shadow-planet was invented to move with them, accompanying the sun.

were set up.<sup>63</sup> The Direction-determining Board (zhengfangan 正方案) was arranged beneath the Simplified Equatorial Torquetum. To the left of the Spiritual Platform there was a separate small platform. Along the walls, corridors with rooms (wu) were constructed in order to decorate its four outer sides. On its [flat] roof was installed an Exquisite Armillary Sphere (linglonghunyi 玲瓏渾儀). To the right of the Spiritual Platform was erected a Lofty Gnomon (gaobiao 高表). In front of the gnomon [ie. south] was a pavilion, and to the north of the gnomon a Stone Shadow Template (shigui 石圭) was laid out. On the face of the Stone Shadow Template was carved [a scale] to measure the shadow [calibrated into] zhang (丈), chi (尺), cun (寸) and fen (分). To the side of the Template was tacked on an additional [structure] which, when the Template was exposed to the sun's rays, could act as a shadow-measuring gauge.<sup>64</sup> To the front of the Spiritual Platform [ie. the south] in the east and west corners [of the courtyard], the bureaus concerned with the work of sealing the calendar were set up. Directly to the south [ie. between these two bureaus] were the offices of Sacrificial Foods and the School of Arithmetic (shenchu suanxue 神厨算學) whose administrative structure was set up as above.

From the beginning, the debate concerning the reformation of the calendar was underway. And officials were sent in the four directions to the places of Korea (Gaoli 高麗), Haikou (chongyai 瓊崖), Chengdu 成都, and Helin 和林 to measure the shadows and to corroborate their findings. This was a re-enactment of the [imperial] orders of the brothers of Xihe (羲和).<sup>65</sup> In addition, prefectures within 5,000 li south of the city of Shangdu 上都, such as Dongping<sup>66</sup> 東平, Yangcheng<sup>67</sup> 陽城, Eji<sup>68</sup> 鄂吉, each sent an official to measure and test so as to obtain [celestial] measurements both near and far [from Shangdu].

At the winter solstice of year 17 [1280] the new calendar was presented to the Emperor. Officials were promoted and rewarded [with monetary gifts], not all of the same value. In 1281 [the new calendar] was released to be implemented across the whole nation. In the year 18 [1282] Zhao Bingwen 趙秉溫 of the rank Vice Director of the Palace Library (mishu shaojian 秘書少監) was promoted to the ranks of the Academician of the Institute for the Glorification of Literature (zhaowenguan xueshi 昭文館學士) and

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<sup>63</sup> A detailed description of the astronomical instruments employed can be found in the Yuan Shi, Chap.48.

<sup>64</sup> This is most likely to have been the Shadow Definer yingfu (景符), referred to in the article above.

<sup>65</sup> Regarded as the Foundation charter of Chinese official astronomy, the story of the brothers Xi and He receiving an imperial commission from the legendary Emperor Yao 堯, to observe and record the movements of the Heavens, is recorded in the first chapter of the Book of Documents (Shujing 書經). Needham (1959), p. 186-88, tells their story, as does Cullen (1996), p. 3-4.

<sup>66</sup> In modern-day Shandong, directly south of Beijing.

<sup>67</sup> Formerly regarded as the centre of China near modern-day Zhengzhou in Henan province.

<sup>68</sup> Near modern-day Wuhan in Hubei province.

the Associate Director of the Astronomical Bureau (zhitaishiyuanshi 知太史院事). The following year 19 [1283] there came a decree from the Crown Prince that Li Qian 李謙 of the rank Advisor in the Establishment of the Heir Apparent (yude 諭德) should compile the discussions [of calculating] the calendar. In the year 21 [1284] Eergensali 諤爾根薩里 of the rank Left Imperial Attendant (zuosi yifengyu 左司儀奉禦) was promoted to the rank of Academician of Scholarly Worthies (jixian xueshi 集賢學士) and shortly afterwards he was promoted to the rank of Grand Academician (daxueshi 大學士) and at the same time in charge of affairs of the Astronomical Bureau (taishiyuanshi 太史院事). Then in the spring of the year 23 [1286] it [the Discussions of the Calendar (liyi 曆儀)] was presented to the Emperor together with the Canons of the Calendar (lijing 曆經). Altogether there were 21 volumes. As for the remaining matters which were still not yet complete, a memorial was presented to the Emperor suggesting that the official Huan should become involved in these discussions. [And thus it was that] the prefaces for [works] such as the Canons of the Calendar and the Format of the Calendar (lishi 曆式), and the stylised poems (ming 銘) for instruments such as the gnomon, clepsydra, armillary sphere and celestial globe came to be presumptuously written by your servant Huan. And thus with the most reverential and deepest of kowtows, the prefaces and the stylised poems which commemorate the initial phase of the original establishment of the Observatory read as follows:

天鑒下民，亂靡有定。孰能一之，聖哲受命。

Heaven inspects the people below. Within the madness there is method. Who could be capable of uniting them? It is the sages and wisemen who have received this fate.

太祖神武，始開乾坤。創業垂法，以貽後昆。

The spiritual and mighty emperor Taizu [Genghis Khan] is the one who began to open up Heaven and Earth. He created the dynasty and set up a system of laws to bequeath it to his later generations.

續緒紹功，划除妖昏。逮今聖皇，天錫勇智。

He continued the work which [the emperors before] had left behind, and removed all evil and shady elements. Today's Divine Emperor is bestowed by Heaven with bravery and wisdom.

內修法度，外遣將帥。伐罪吊民，罔越厥志。

Within the nation He [the Emperor] set up law and system. Outside the nation He sent generals to penalise crimes and to pacify the people. And He did not deviate from this purpose.

炎方歸命，赦其后至。武功告成，萬國來萃。

The southern areas received [the Emperor's] mandate, and [He] pardoned their lateness in

joining [the Empire]. The military achievements are complete and the myriad countries gather together [as one].

同軌同文，重譯奉贄。小大悉臣，師旅以寧。

[In this new Empire] the same standards and same language [are imposed]. It is through multiple translations that audiences with the emperor are requested [ie. the breadth of the Empire is such that to communicate with the Emperor one has to transmit one's message across a number of nations and thus through a variety of languages]. The small and the big all submit [to the new regime], and the army thus remains tranquil.

思与万方，永保太平。黎民定居，蕃息生生。

[The Emperor] contemplates the numerous places [and wishes them] to forever remain at peace. The ordinary people settle down, and they beget many future generations.

為衣而蠶，為食而耕。士勸其賢，工勸其能。

In order to have clothes to wear, they cultivate silkworms. In order to have food to eat, they till the fields. For scholars, encourage their wisdom. For workers, encourage their talents.

關陘夜開，商旅通行，民惟勤克，罔适天宜。

Let the passes be open at night so that the merchants might freely travel. The common people are only capable of being industrious and do not know how to adapt to Heaven's will.

匡之翼之，以煩聖思。乃立太史，法遵黎羲。

Help them and assist them, for this is disturbing the Emperor's peace of mind. Thus the Astronomical Bureau was established, as a method of following Li and Xi [ancient mythical astronomers].

欽若天道，敬授民時。教其動作，時種時獲。

Respectfully submit to the Way of the Heavens, respectfully supply the common people with a time-table. Teach them their rituals of labour, when it is auspicious to sow and when it is auspicious to reap.

教其趨向，是宜是吉。五禮之舉，選時為日。

Teach them the directions to take [in their actions], those which are appropriate and those which are propitious. For the holding of the Five Rituals,<sup>69</sup> auspicious times should be

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<sup>69</sup> The Five Rituals are, traditionally, Auspicious Ritual (jili 吉禮), the Fierce Ritual (xiongli 凶禮), the Army Ritual (junli 軍禮), the Guest Ritual (binli 賓禮), and the Honour Ritual (jiali 嘉

chosen as dates.

代卜代筮，不勞龜策。期措斯民，康壽之域。

Instead of divining [with oracle-bones] and instead of fortune-telling [with yarrow-stalks], one does not need turtles or yarrow-stalks. One hopes to put these people in a position of [determining their own] good health and long life.

民祝聖皇，眉壽万年。民祝聖皇，五福駢臻。

The people wish the divine Emperor a long and extended life. The people wish the divine Emperor a life replete with the Five Happinesses.<sup>70</sup>

民祝聖皇，億兆子孫。七政順軌。陰陽調均。

The people wish the divine Emperor innumerable sons and grandsons. The seven heavenly bodies [ie. the sun, moon and five main planets] follow their paths smoothly. Yin and Yang are in balance.

時雨時暘，化育秋春。蕩蕩巍巍，順德何言。

It rains when it is supposed to rain. And the sun comes out when it is supposed to come out. So that all can flourish and be plentiful throughout the year. How sublime!! [The Emperor's] moral excellence is such that there is no describing it.

天覆地載，太平無垠。

That which Heaven covers and Earth supports will have limitless peace and harmony.

### Chinese text

The following is the Chinese text corresponding to the section beginning at the 3<sup>rd</sup> paragraph of the translation, 'In the spring of 1279', and continuing as far as 'whose administrative structure was set up as above.' The lines are numbered according to the columns of the published text, pages 210b-211a.

- 4。十六年春擇美地，得都邑僦墉下，始治役，垣
- 5。縱二百布武，橫減四之一，中起靈臺，餘七丈，爲層三，中
- 6。下皆周以廡，其下面日中室爲官府，以總聽院政，長曰
- 7。令，次同知院事，次僉院事，以宰輔之重，領於上者，無定
- 8。員，其屬有主事，有令譯史，有榦事，有庫局之司，左右旁

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禮).

<sup>70</sup> The Five Happinesses are long-life, riches, nobility, peace and contentment, and hoards of sons and grandsons.

9. 室，以會司屬議，凡推測星歷，諸生七十人，泣以二局，一  
 10. 曰推算，其官有五官正，有保章正，有副，有掌歷，分集於  
 11. 朝室。二曰測驗，其官有靈臺郎，有監候，有副。三曰漏刻，  
 12. 其官有絜壺正，有司辰郎，分集於夕室。凡器用出納，於  
 13. 陰室中層。離室以列景曜。巽室以措水運渾天壺漏。坤  
 14. 室以措渾天象蓋天圖。震兌二室以圖南北畢方渾天  
 15. 蓋天之隱見。坎室以位太歲。乾室以貯天文測驗書。艮  
 16. 室以貯古今推算歷法。臺顛設簡仰二儀，正方案勇簡  
 1. 儀下。靈臺之左，別爲小臺，際葶周廡，以華四外，上措玲  
 2. 瓏渾儀。靈臺之右立高表，表前爲堂，表北甃石圭，圭面  
 3. 刻度景丈尺寸分，圭旁夾以連葶可圭，上露天日，爲度  
 4. 景計。靈臺之前東西隅，置印歷工作局。次南神厨算學，  
 5. 設位如上。

### References (Appendix)

Supplementary to those already listed.

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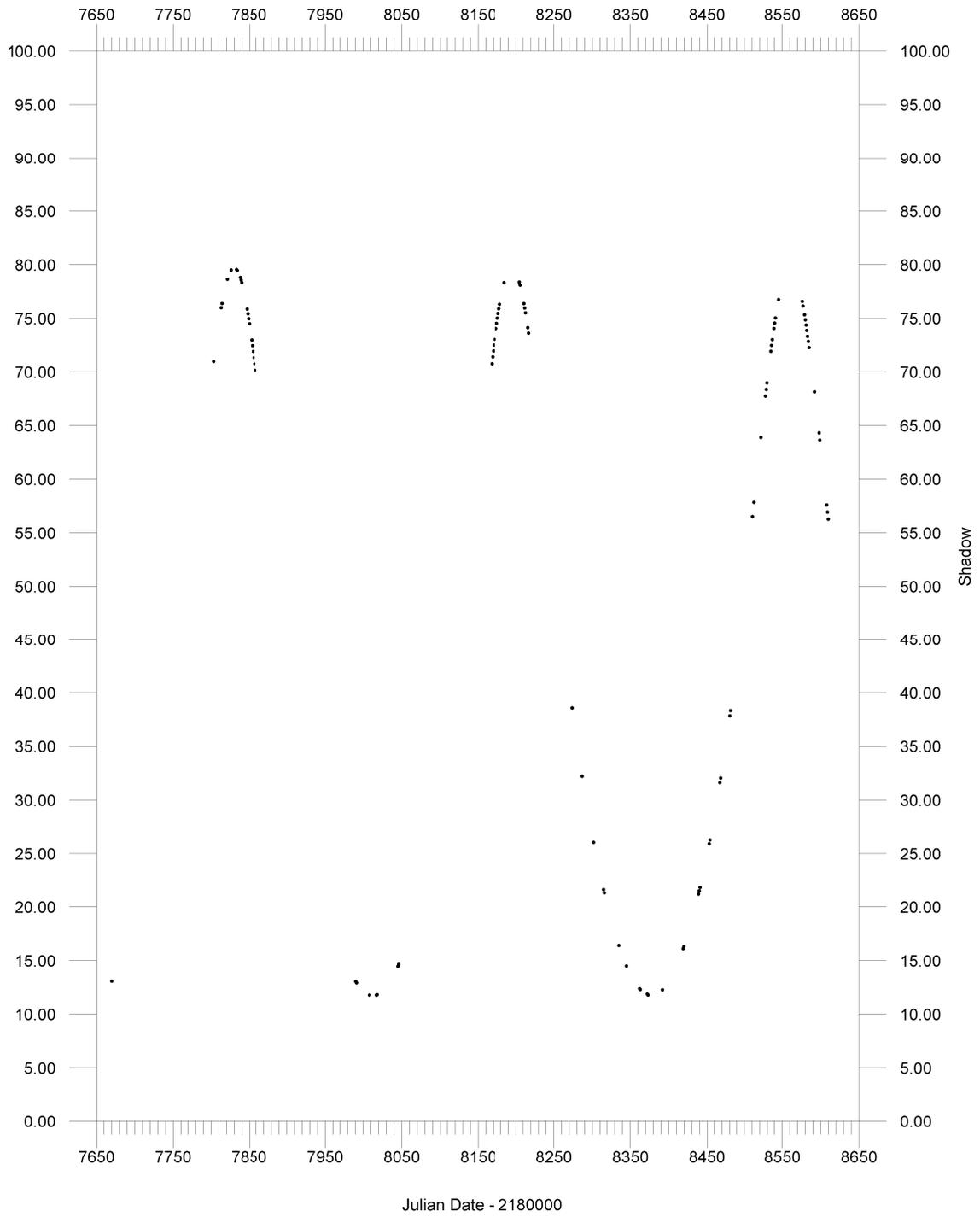


Fig.1. The 98 shadow lengths, in feet, plotted against Julian Day Number.

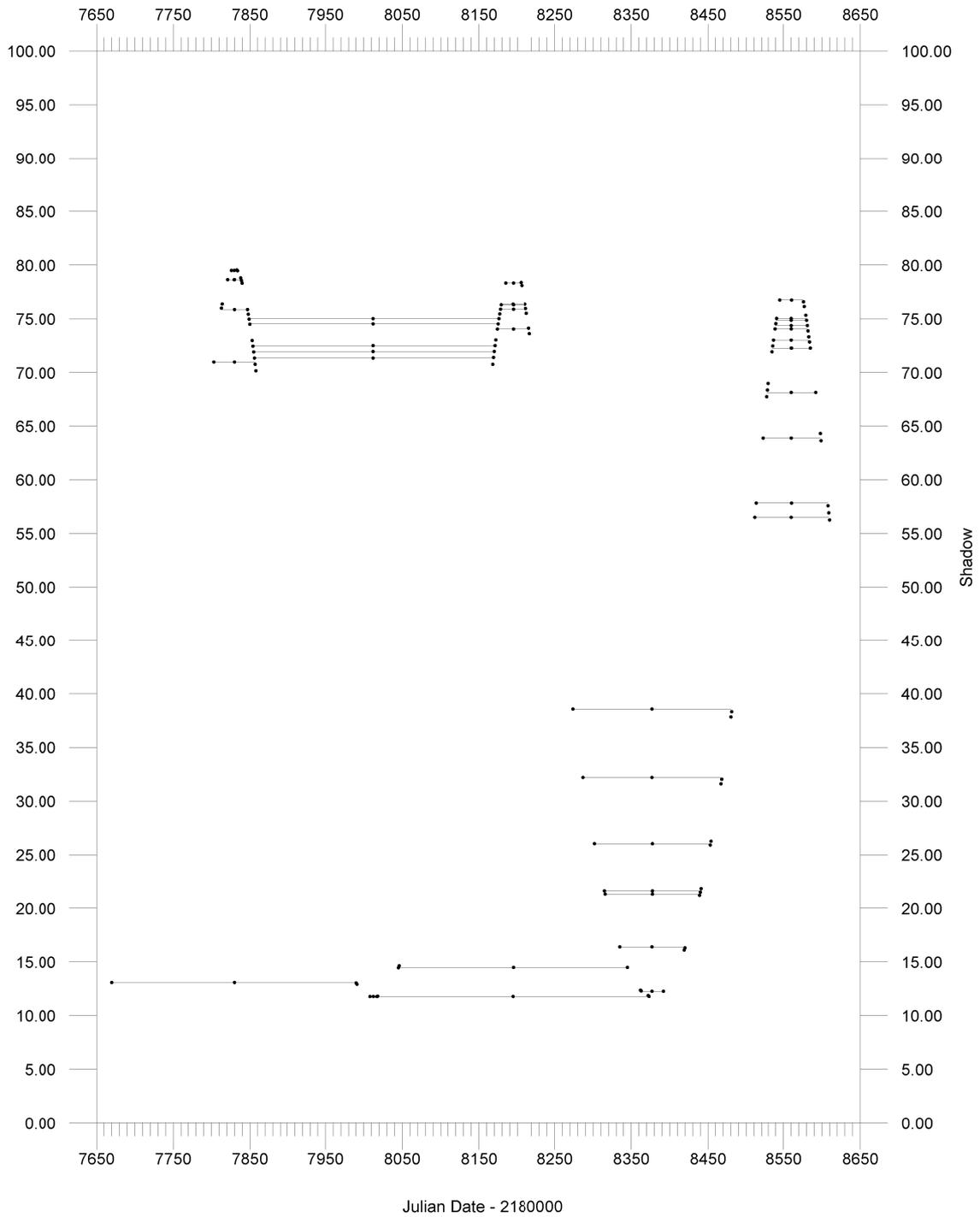


Fig.2. The 98 shadow lengths, in feet, plotted against Julian Day Number, and showing the triples and date of solstice

Fig.2. The 98 shadow lengths, in feet, plotted against Julian Day Number, and showing the triples and date of solstice.

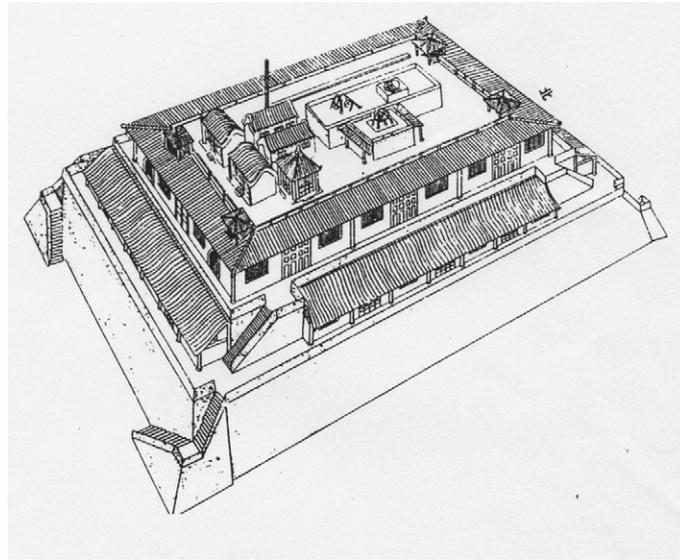


Fig.3. Conjectural reconstruction of the Yuan Observatory; *Achievements of the Ancient Astronomy of China* (1987), p.133.

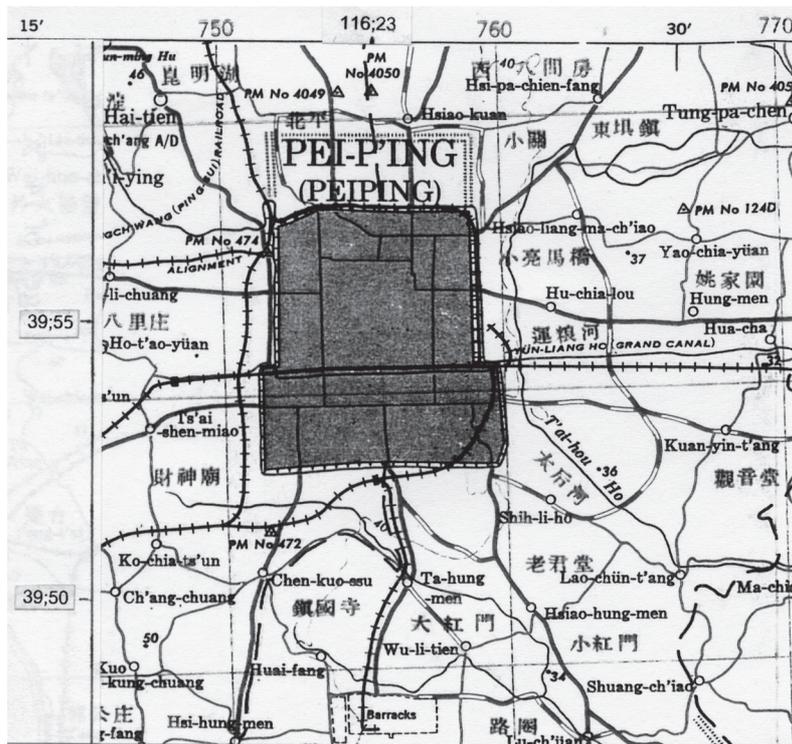


Fig. 4 Beijing as defined by the walls constructed in the Ming period, ca. 1420, together with the northern boundary (shown dotted) of Dadu, the Yuan capital.

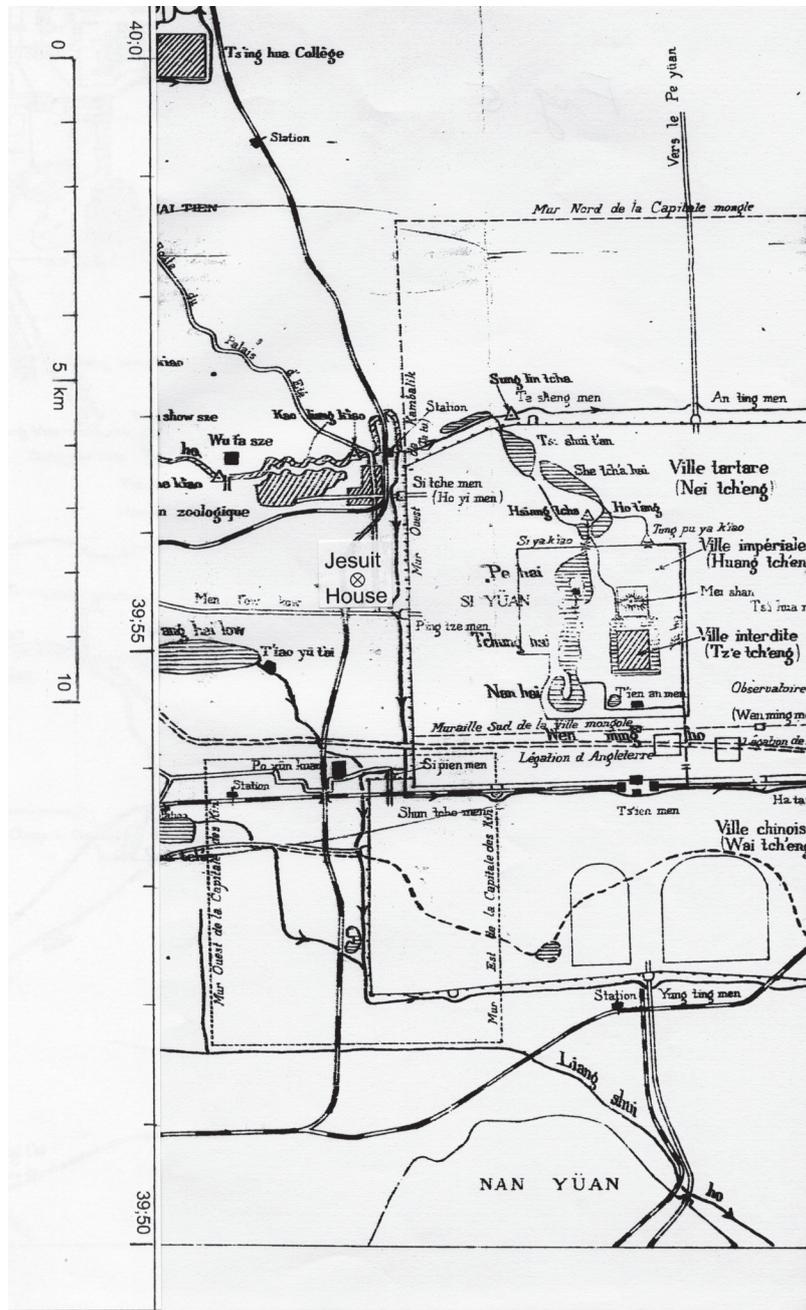


Fig.5. The Western suburbs of Beijing, showing the boundary of the Jin capital (Zhongdu), and also the situation of the Jesuit House. Taken from Bouillard (1925).

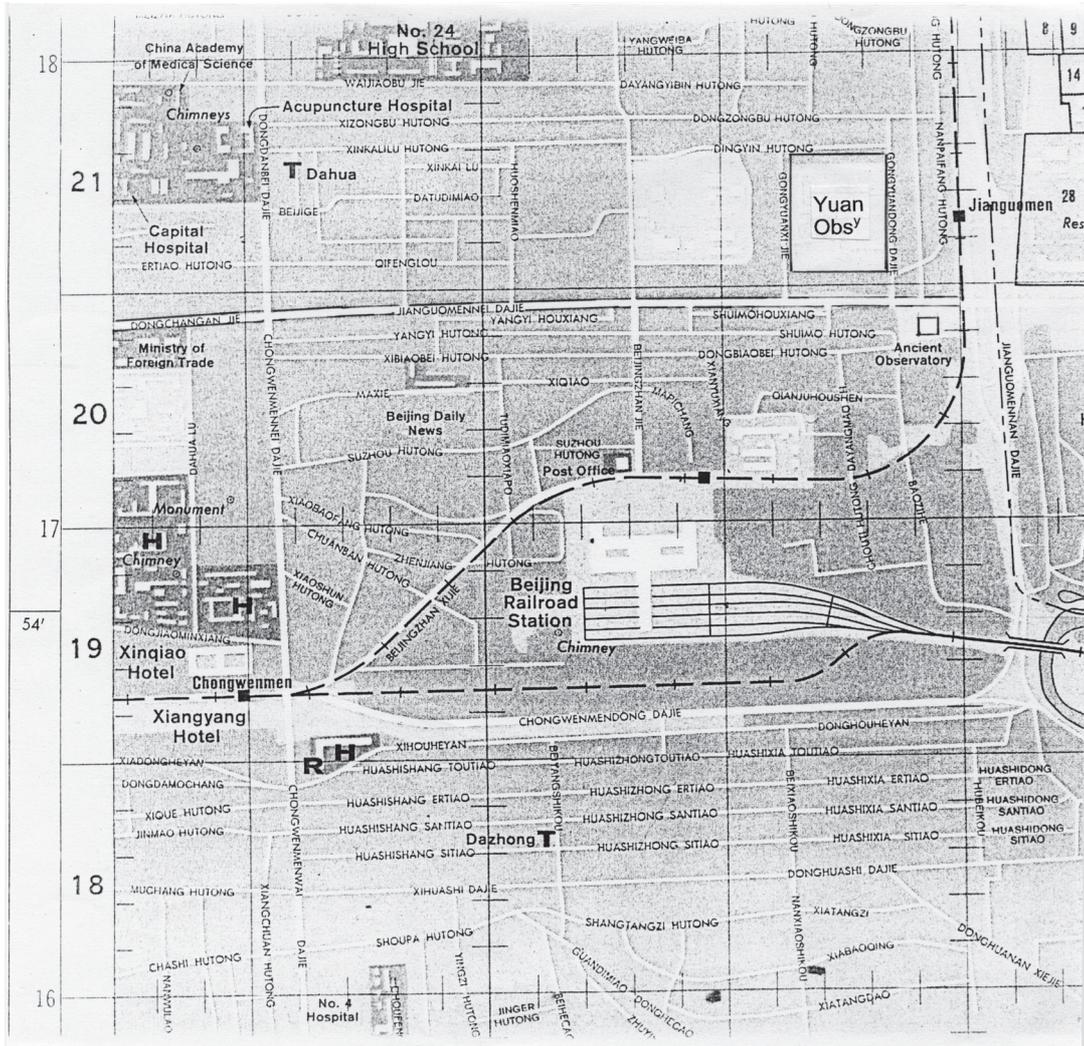


Fig.6. A street map of Beijing showing the extant Ming Observatory, and just north of it, the site of the former Yuan Observatory. These are separated by Zhanagan street, and its continuation, Janguomen Nei street, which together mark the line of the southern wall of Dadu.

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