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A Manchu manuscript on arithmetic owned by Tôyô Bunko: "suwan fa yuwan ben bithe"

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I Introduction

In the Manchu-Mongol section of Tôyô Bunko in Tokyo, there is a Manchu manuscript entitled "suwan fa yuwan ben bithe" (満文算法原本)[1], which consists of 131 folios and explains elementary number theory essentially due to Euclid[2]. The purpose of this paper is to give the Romanized texts of its preface and some of its sections, and to give their English translation.

The existence of this manuscript has been known for more than forty years since the catalogue of the Manchu-Mongol section of Tôyô Bunko was published ([Poppe-Hurwitz-Okada, 1964]), but unfortunately, nobody had studied this manuscript until the author made a report in [Watanabe, 2004][3]. Recently, Tatiana A. Pang and Giovanni Stary introduced another copy[4] of this book, "suwan fa yuwan ben bithe", owned by St. Petersburg Branch of the Institute of Oriental Studies, Russian Academy of Science, in their paper [Pang-Stary, 2000]. Although their paper contains many topics of great interest, detailed description of its mathematical contents was not given there. In this paper we shall give this description by translating 29 sections from its 75 sections in the manuscript owned by Tôyô Bunko.

The importance of studying Manchu documents is now fully recognized by researchers in the political history of the Qing period, especially of the first half of that period, including the reign of the Emperor Kangxi. Most of those documents were created in and around the Court and the goverment, and contain various information which was often omitted in their Chinese versions. The handwritten copy of "suwan fa yuwan ben bithe" owned by Tôyô Bunko is not an exception. It contains many linguistic and mathematical corrections reflecting the process of creating a textbook. Let us overview this

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[1] The call number of this manuscript is Ma2-19-1.

[2] See [Poppe-Hurwitz-Okada, 1964] for bibliographical data.

[3] According to Tatsuo Nakami, a researcher in the United Kingdom took an interest in this manuscript nearly thirty years ago, but no progress was made at that time (private communication).

[4] Bibliographical data on this copy can be found in the catalogue of this Branch written by Pang [Pang, 2001].

feature of the manuscript.

The original manuscript is divided to the following three parts;

(1) the primary main part, written in a beautiful blockprint form directly on the physical body of the manuscript,

(2) the secondary main part, which consists of words written in the same beautiful blockprint form used in (1) on small pieces of paper attatched to parts of the primary main part,

(3) the correction part, which consists of letters, words and phrases written in a very rough cursive form often accompanied with deletion of letters belonging to the primary main part.

Note that the correction part is written in thin black ink, not in red ink.

Since the contents of the correction part of earlier sections were reflected in those of the primary main part of later sections (see the notes to Section 1 for details), we may conclude that the contents of the two main parts and those of the correction part were written almost simultaneously. We have to consider the possibility that the manuscript is not a copy but the original, because it would be a troublesome job to copy a manuscript by hand preserving a wide difference between the various styles of handwriting. It is possible that the correction part is the original written by the Emperor Kangxi himself. The fact that this part was written in black ink does not exclude this possibility, because "suwan fa yuwan ben bithe" is thought to be written around 1689, and this date overlapped with the period of mourning of the Empress Xiaozhuang, who was a wife of Hong Taiji and the grandmother of the Emperor Kangxi.

Now let us describe the contents of this paper. The parts of the manuscript translated in this paper are the following

LIST 1: Preface, Sections 1-7, 11, 15-20, 22, 25, 27, 28, 37, 38, 41, 42, 45, 50, 51, 60, 62, 63, 75.

Propositions and algorithms in other sections are cited in these sections. We shall also give a brief summary of them. The sections containing these propositions and algorithms are the following

LIST 2: Sections 12-14, 23, 31, 34, 35, 39, 40, 61.

So very roughly speaking, this paper covers the contents of one half of the manuscript.

For each part of the manuscript appearing in List 1, we give their Romanized Manchu texts first. Also, we give their translations into English. These translations are rather literal for the following two reasons. One is that some technical terms were created and modified in this manuscript, and as a result, they are not sufficiently fixed. The other is that the correction part contains many linguistic comments, some of which are concerned with an appropriate choice of Manchu case suffixes. To show the difference between the original texts and the corrections to them, we had to make a literal translation, often preserving the number of terms in verbs. To make amends for difficulties in understanding caused by literal translation, we give a brief summary of each section in the modern language of mathematics. We also give there the correspondence between the mathematical contents of this manuscript and those of the *Elements* and the *Shuli Jing Yun* (數理精蘊) in several cases[5]. Comments and remarks associated with particular words or phrases or sentences are collected in Notes.

There are many problems arising from this manuscript and other Manchu documents on natural sciences. We refer to [Watanabe, 2004] and [Watanabe, 2005] for some of them.

Professor Sae Okamoto gave the author an opportunity to use resourses which are often difficult to access for an outsider, introduced him to the society of active researchers in the humanities, and always encouraged him to continue this work. Professor Teruhiro Hayata and other members of the Japanese Association for Manchu and Qing Studies (Manzokushi kenkyûkai) were so kind that they always gave rapid and detailed answers to the author's questions, which were long, tedious and sometimes elementary. The author expresses his gratitude to all of them for their kindnesses, and in addition, to Professor Hideki Kawahara for his valuable comments and suggestions, and for providing an opportunity to write this paper. The author also thanks to Professor Nobuo Miura, Professor Lewis Pyenson, the referees and the editor for their giving valuable comments in the early or the final stage of this work.

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Notations

Notations in Romanized Manchu Texts

Romanization of Manchu texts in this paper almost completely follows the way of Möllendorff. The only exception is the Romanization of the Manchu genitive case suffix. When this case suffix is spelled as an independent word, we will transliterate it as '-i', inserting a space between the preceding word, and distinguish it from the ordinary isolated form of the Manchu letter 'i'. This distinction is necessary, because this manuscript uses the second sign 'yi' (乙) of the Ten Stems (十干) in the traditional Chinese system of the calender instead of a Western mathematical symbol '*B'*. This symbol appears so often that we have to distinguish it from the Manchu genitive suffix.

In the Romanized Manchu texts of the manuscript as well as in the English translation,

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[5] For the *Elements* we refer to Heath's classical translation *The Thirteen Books of Euclid*, Dover, New York, 1956. For *Shu Li Jing Yun* we refer to.the facsimile edition contained in *Zhongguo Kexue Jishu Dianji Tonghui* (中國科學技術典籍通彙).

letters belonging to the correction part are given in italics. A deleted letter or a sequence of letters accompanied with the secondary main part is enclosed with a pair of doubled square brackets ‘[[,]]’, and a deleted letter or a sequence of letters associated with the correction in the correction part is enclosed with a pair of single square brackets ‘[,]’. An added letter or a sequence of letters as in the secondary main part and the correction part is enclosed with a pair of a single round brackets ‘(,)’. Inserted notes in the original manuscript are enclosed with a pair of braces ‘{, }’. A doubled question mark ‘??’ indicates that there is an illegible letter of sequence of letters. A single slash ‘/’ indicates a line break, and a doubled slash ‘//’ indicates a half-folio break. A pair of pointed brackets ‘⟨, ⟩’ indicates the words inserted by the author of this paper.

Other Notations and Conventions

In the English translation of Manchu texts, we shall always express the title of a book or of a document by enclosing it with a pair of double quotation marks. Although this convention violates established rules in publishing, it is necessary in our case because we have to distinguish the titles of books from the English translation of the correction part.

The title of a Manchu book is expressed by enclosing the faithful transliteration of its Manchu titles with a pair of double quotation marks. No capitalization is made here, because there is no distinction between the uppercase and the lowercase letters in Manchu script.

List of Abbreviations

In this paper we shall use the following abbreviations:

SLJY	the *Shuli Jing Yun* (數理精蘊).
SfYbSLJY	the *Suanfa Yuanben* (算法原本) in the *Shuli Jing Yun*.
DGYB	“daicing gurun –i yooni bithe” or the *Da Qing Quanshu* (in Manchu and Chinese, 大清全書)
GTCL	“(dergici toktobuha) ge ti ciowan lu bithe” (in Manchu, 欽定格體全録, the *Manchu Anatomy*).
MaGHYB	“gi ho yuwan ben bithe” (in Manchu, 滿文幾何原本, *The Principles of Quantities*).
MaSFYB	“suwan fa yuwan ben bithe” (in Manchu, 滿文算法原本, *The Principles of Calculation*).

II Romanized Manchu Texts and their Translation

II.0. TITLE

II.0.1. Romanized Manchu Texts

suwan fa yuwan ben bithe /
uheri nadanju sunja meyen /

II.0.2. Translation

"Book of the Principles of Calculation"
Seventy-five sections altogether

II.0.3. Notes

(l.1) suwan fa yuwan ben bithe

In the catalogue of Tôyô Bunko the English title of this manuscript is "Fundamentals of Calculation". Judging from the contents of this manuscript, either of the two words 'principles' and 'fundamentals' is appropriate, but it seems to us that 'principles' is slightly better than 'fundamentals'. The reason is as follows.

There is another Manchu book on mathematics entitled "gi ho yuwan ben bithe" (満文幾何原本). This book was written by a Jesuit or Jesuits, and was intended to form a counterpart to this "suwan fa yuwan ben bithe", as is clear from the structure of their titles. According to [Pang-Stary, 2000], the preface to the first chapter of this book explains the meaning of a noun phrase 'gi ho yuwan ben', which is a Manchu expression of the official pronunciation of the sequence of Chinese characters '幾何原本' in Mandarin Chinese at that time, as 'ton –i sekiyen sere gisun'. This Manchu noun phrase means 'the source (i.e. the fountainhead) of numbers'. If the author of that book had wanted to mean 'the fundamentals of numbers', in other words, if he had wanted to mean the basis on which the whole of mathematics are constructed, he could have said 'ton –i da sekiyen' or 'ton –i fulehe da' instead of 'ton –i sekiyen'.

We remark that Robert Morrison (1782-1834) translated the Chinese phrase 'ji he yuan ben' (幾何原本) as 'the principles of quantities' in his famous English-Chinese dictionary[6].

(l.2) uheri nadanju sunja meyen

This manuscript consists of 131 folios, and two of them are devoted to the preface. So one section cosists of 1.7 folios on average. Note that this noun phrase refers to the number of sections, but not to the name of the chapter to which these sections belong. See the notes to the first line of Section 1.

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[6] Robert Morrison, *A Dictionary of the Chinese Language*, Vol.3, East India Company's Press, Macao, 1822.

II.1. PREFACE

II.1.1. Romanized Manchu Texts

suwan fa yuwan ben bithe sioi: /

ton -i giyan umesi narhūn. ton -i baitalan umesi amba: tuttu ambula tuwame. akūmbume kimcire de / teisulehelengge gemu tacire ursei baita wakangge akū bicibe. ton be getukelerengge. yargiyan -i / hafuka saisai [nenden ohobi] (*neneme jabuci acarange*): jaka be hafure sara be akūmbure deribun. [aici] urunakū ede / [bidere] (*bisirengge*): unenggi ton be getukeleme muteci sarangge urunakū yargiyan ombi: sarangge yargiyan / oci. baita [icihiyarangge] (*de nikeburede*) urunakū akdun bime. kenehunjerakū ombi: kenehunjerakū oci. / tumen baita yongkiyambi: aikabade ton be getukeleme muterakū oci. sarangge urunakū / . (eden) [[ekiyehun]] ombi: sarangge (eden) [[ekiyehun]] oci. jaka de tunggalaha manggi. kenehunjerengge labdu / yaya baita de farfabumbi:

tere anggala [abka na -i] (*dergi fejergi duin dere julhe ne -i*) mohon akū. inenggidari baita (f.1a)// narhūn ser serengge. ton de [holbobuhakūngge] (*horibuhakūnge*) emke akū:

terei giyan -i narhūn babe / gisureci. abkai du -i forhošome aššara. ba na -i onco amba. šun biya -i šurdeme forhošoro / usiha oron -i gurire ilire. niyalmai baita -i bodoro [kicere] (*banjibure*) tumen jaka -i mayara muturengge / toktoho giyan. toktoho ton akūngge akū. aikabade ton -i giyan be hafume. terei / baitalan -i arga be getukeleme muteci. arga be bahafi ton be forhošome. ton ci / arga be toktobume bodoro celere [be] (*de*) saci ojorakūngge akū ombi:

baitalan -i amba / babe gisureci. [gurun -i jalin ningge.] ton be (*gurun de*) getukeleci. gurun -i jalin banjibure fayabure / jeku ciyanliyang ni tucire dosire be bodome saci ombi: jiyanggiyūn ohongge. / ton be getukeleci. erin ucuri be tuwame dosire bederere. labdu komso be forhošome (f.1b)// baitalara be bahanambi: usin nimalan -i urse ton be saci. erin de acabume tarire jodoro be bahambi / hūdašara urse ton be saci. tucire dosire madabure fusembure be bahambi: jai faksisa / futa mishan be saha manggi. teni umesi lak sere hošonggo muheliyen be bahambi: weilere urse / kemun durun be bahanaci. weilere ararangge teni narhūn mangga ombi: ere jergi hacingga / baitalan -i tumen duwali tanggū hacin de. ton be waliyaci. muterakū ombi:

tuttu ere arga be / ureburakūci ojorakū bime. geli terei giyan be [getukelerakūci] (*akūmburakū oci*) ojorakū: aikabade / sibkiha kimcihangge [yongkime] (*yongkiyame*) getuken. sekiyen eyen be šuwe hafuka. giyan kooli be [yongkiyame] (*wacihiyame*) / ulhihengge waka oci. uthai duka de dosifi. boo -i dolo isinara unde gese: terei / galai arara. yasai tuwara. mujilen gūnin de tebure. umesi sure -i hafunarangge. largin (f.2a)// [hacin] (*bargin*) de [ufarabufi] (*kūthūbufi*). nike[n](*bure ba*) akū de isinarakūngge komso: tuttu suwan fa yuwan ben sere emu / bithe be arafi. tacire ursede (*neneme*) erei šošohon -i giyan be urebure [de acibuha]: amala arga / tome terei giyan be leolefi. sara be yargiyalabume. baitalan de kenehunjeburakū obuha: unenggi / urebufi badarambume muteci. [absi oci] (*eiterecibe*) terei doro be baharakūngge akū ombi: uttu ofi / sioi araha: (f.2b)//

II.1.2. Translation

"Book of the Principles of Calculation", Preface.

The principle of numbers is very subtle, and the utilization of numbers is very extensive. Thus the elucidation of numbers (*is what*) [has been the first priority for] wise men, who really understand things thoroughly, (*should first answer*,) though there is nothing wrong in the affairs of the people who learn from all that they have encountered in their wide and thorough investigation. The beginning of investigating things thoroughly and extending 〈our〉 knowledge to the utmost (*is that which*) certainly exists here. If we are really able to elucidate numbers our knowledge is certainly based on facts. If our knowledge is based on facts, then (*it is*) [the results of managing our affairs are] certainly reliable and we are free from doubts (*when we depend on this knowledge in our affairs*). If we are free from doubt ten thousand affairs will be perfect. If we are not able to elucidate numbers our knowledge is certainly (deficient.) [[insufficient 〈in quantity〉.]] If our knowledge is (deficient) [[insufficient 〈in quantity〉,]] then we have many doubts and are confused of any affairs when we encounter things unexpectedly.

Besides this, there is nothing that has not been (*restrained by*) [related with] numbers in the cases in which the endless daily affairs (*of all times 〈observed〉 in the two vertical and four 〈horizontal〉 directions 〈of the universe〉*) [in the heaven and the earth] are said to be subtle and fine.

Concerning the subtle points of the principles 〈of numbers〉, there is nothing without the definite principles and the definite numbers in the growth and the decline of ten thousands of things, such as the rotating motion of the celestial sphere, the width and the area of the 〈whole〉 earth and its parts, the revolution of the sun and the moon going around us, the 〈irregular〉 movements of planets and astronomical places, and planning and [making efforts] (*production*) in 〈all〉 human affairs.

If we are able to understand the principles of numbers thoroughly and to elucidate the methods of its utilization, then (*there is nothing*) [it is impossible] that we cannot understand(, *when*) [how] we change the numerical values after we have obtained 〈particular〉 methods, and 〈conversely〉 determine methods from the numerical values 〈which we have already obtained〉.

Concerning the extensive points of its utilization, [it is something for the state. If] (*if*) we elucidate numbers (*in 〈affairs related with〉 the state*), then we can calculate and understand the amounts of production and consumption for the state and transfers of provisions and money 〈between the imperial storehouses〉. If those who were appointed to the generalship can elucidate numbers, then they can go out and back 〈around their camps〉 by watching for opportunities, and to use larger and smaller troops by shifting their positions. If farmers understand numbers, then they can sow and weave in suitable seasons. If merchants understand numbers, then they can go in and out 〈of marketplaces〉 and increase their profits in the form of money and livestock. Next as for carpenters, they obtain very exact squares and circles after they have understood 〈the mathematical principles in〉 their ropes and inked strings. If craftsmen understand their rulers and molds, then and only then their works will be fine and expertly made. In ten thousands of types and hundreds of kinds of various ways of utilizations belonging to these classes, if we throw numbers away we will become incompetent.

Thus we should practice these methods, and in addition, we should extend their principles to the utmost. If the results of deep and detailed investigation are not (*completely*) clear, and the acts of this investigation are not the acts of making the source and the derivations ⟨of theories⟩ transparent and of getting (*an*) [a complete] intuition on the princples and the rules ⟨of numbers⟩ (*without omission*), then the situation is similar to that of the case in which we entered a gate but have not reached the inside of the house yet. It is rare that we do not lose (*the ground on which we are based*) [our support] after the acts of writing something by our hand, of observing something by our eyes, of keeping something in our mind and of passing through difficulties very wisely have been jumbled by a mess. Thus we wrote a book entitled“ The Principles of Calculation”and first made students practice the ⟨fundamental⟩ principles forming outlines ⟨of this book⟩. Later we discussed the principles of each method, verified our knowledge, and made the readers free from doubts in applying ⟨the results of this book⟩. If we can really extend these principles and methods through repeated practices, (*in any case*) we never fail to find our way. With this ⟨remark⟩ we close this preface.

II.1.3. Notes

(f.1a, l.2) ton -i giyan

A Manchu noun ‘giyan’ is used in this preface in an inconsistent and rather confused way. In many philosophical and ethical contexts, this noun ‘giyan’ is an established Manchu term for the idea of ‘li’ (理, ‘the Principle’), which plays important roles in the ontology and the ethics of Neo-Confucianism when it is coupled with the idea of ‘qi’ (氣, ‘sukdun’ in Manchu), or of ‘shi’ (事). We also recall that the word ‘li’ is often found in a triplet ‘li’, ‘xiang’ (象) and ‘shu’ (數) and that these words are used as fundamental technical terms in interpretations of the *Yi Jing* (易經, the *Book of Changes*) by Neo-Confucianists.

Now let us consider the first sentence of this preface (f.1a, l.2). Since the word ‘giyan’ is not coupled with ‘sukdun’ but with ‘baitalan’ there, and ‘baitalan’ is a Manchu equivalent for the term ‘yong’ (用), which is not coupled with the term ‘li’ in the standard doctorine of Neo-Confucianism, we cannot regard this ‘giyan’ as one of the Manchu technical term in Neo-Confucianism. The author of this manuscript refers to a contrast between theoretical sciences and practical techniques later (f.1b, l.1-f.2a, l.5), so it seems probable that he meant a Western binary opposition between a theory and an application by using the words ‘giyan’ and ‘baitalan’.

On the other hand, the noun phrase ‘mayara muturengge’ in the third line of f.1b corresponds to a Chinese phrase ‘xiao chang’ (消長), which is an important concept in the philosophy of the *Yi jing* and expresses the changes in everything, so the pair of ‘giyan’ and ‘ton’ in the phrase ‘toktoho giyan. toktoho ton’ (f.1b, l.4) corresponds to the pair of ‘li’ (理) and ‘shu’ (數), which should be explained in the contexts of Neo-Confucianism.

(f.1a, l.4) **(*acarange*)**

The standard spelling of this word is ‘acarangge’. The corrector did not have to take care of the exactness of spellings, even if he checked a carefully written manuscript.

(f.1a, l.4) **jaka be hafure sara be akūmbure**

This phrase is the traditional Manchu equivalent for a famous Chinese phrase ‘ge wu zhi zhi’ (格物致知), which are the first two steps in the so-called Eight Steps in the *Da Xue* (大学, the *Great Learning*). Here we followed the classical English translation of this phrase by James Legge.

It is probable that the source of a noun ‘nenden’ in ‘[nenden ohobi]’ in this line is also a passage from the Manchu translation of the *Da Xue*. A Chinese phrase ‘zhi suo xian hou’ (知所先後, ‘to know what is first and what is last’) is translated as follows: ‘nenden amaga be saha de. doro de hanci ombi: ’

(f.1a, l.4) **[aici]**

The use of Manchu interrogative adverb ‘aici’ (‘what sort of?’) in this sentence is not grammatically correct. Since it does not make sense, we omit the translation. It is possible that the author of this manuscript thought that ‘aici’ was the combination of an interrogative pronoun ‘ai’ and an ablative case suffix ‘ci’, which is used in a comparative construction. If this is true, the meaning of the original sentence may be as follows:

> The beginning of the first two steps in the Eight Steps may be found in our various acts, but the most important one is found in the study of numbers.

(f.1a, l.9) **(*dergi fejergi duin dere*)**

This word corresponds to a Chinese classical notion of ‘liu he’ (六合), which means the universe considered as the combination of the heaven, the earth and the basic four directions on the earth. The corrector’s choice of words is more rhetorical than the original one.

(f.1b, l.2) **abkai du**

The literal meaning of the noun phrase ‘abkai du’ in this line is ‘the hipbone of heaven’. Here the celestial sphere, which was thought to be a material being with a thin, wide and hard body, is compared with a hipbone. Since ‘du’ in this noun phrase represents a concrete thing that can rotate and move, we can exclude other possibilities.

The north celestial pole was compared with a hip joint. We can find an example of this comparison in Vol.10 of “han –i banjibuha sing li jing -i bithe”, which is the Manchu translation of the *Yuzuan Xingli Jingyi* (御纂性理精義, *The Essence of the Philosophy of Neo-Confucianism*) and was published by the government in 1717. The example is as follows:

> 問北辰曰，北辰是天之樞紐. (Chinese original.)
> be cen -i oron be fonjiha de. hendume. be cen -i oron. uthai abka -i horgikū šošohon. (Manchu translation)
> (A student of Master Zhu (i.e. Zhu Xi 朱熹, 1130-1200) asked his teacher what Bei-chen is. The Master said that the place of Bei-chen (i.e. the north celestial pole) was nothing but the center of the mechanism of the heaven and controled its movement.)

In the above passages a Chinese noun ‘shu’ (樞, ‘a pivot’) corresponds to a Manchu noun ‘horgikū’. ‘horgikū’ means ‘a pivot’ in “daicing gurun –i yooni bithe” (*Da Qing Quanshu*, 大清全書), a Manchu-Chinese dictionary published in 1683[7], but it also means ‘a hip joint’, as is seen in “han –i araha nonggime toktobuha manju gisun –i buleku bithe” (*Yuzhi Zengding Qing Wenjian*, 御製增訂清文鑑), an official Manchu-Chinese dictionary published in 1773. It was natural to compare the celestial sphere with a hipbone in case that this ‘shu’ (樞) was taken as a hip joint.

(f.1b, l.2) **forhošome**
Throughout this manuscript the intermediate form of the two Manchu letters ‘-g-’ and ‘-h-’ are used without distinction. Since the main part of this manuscript was written beautifully, this confusion was not a result of the carelessness of copyists making this manuscript. The corrector, who always checked Manchu expressions severely, made no correction in these cases. According to Teruhiro Hayata, this confusion reflects the real confusion in the pronunciation system, and was common in the Manchu literature created in the early Qing period before the goverment started to force the normative spellings through their officinal Manchu or Manchu-Chinese dictionaries[8].

(f.1b, l.3) **oron**
In general, the meaning of this Manchu noun is a place which is or will be occupied with something. In our context, this word is the Manchu translation of a Chinese noun ‘chen’ (辰), which means a point on the the celestial sphere which is not occupied by a star but plays a role in the traditional astronomy in China.

(f.2a, l.7) **[yongkiyame] (*wacihiyame*)**
The difference between two Manchu verbs ‘yongkiya-’ and ‘wacihiya-’ is as follows; the former means the state that everything exits, and the latter means the state that nothing remains.

II.1.4. Remarks

There is a blockprint copy of *MaGHYB* in the St. Petersburg Branch of the Institute of

[7] For this dictionary we refer to [Hayata-Teramura, 2004].
[8] private communication, 2003.

Oriental Studies. Tatiana A. Pang and Giovanni Stary gave the whole text of the preface to the first chapter of this blockprints in their paper ([Pang-Stary, 2000]). The preface of that book gives a short explanation of its title and its editorial policy.

On the other hand, the preface to this manuscript is long and substantial, as we saw in the above translation. The fact that this preface spent more than twenty lines in order to explain the usefulness of mathematics suggests that this book was not intended to be personal lecture notes only for the emperor, but to be a textbook for the public.

Another thing which should be noted about this preface is a strong influence of the terminology of Neo-Confucianism on its expressions. A good example of this influence is the use of the phrase 'jaka be hafure sara be akūmbure' in the fourth line of f.1a. It seems probable that this influence made the whole of this manuscript acceptable to well-educated people in China, and that this acceptability yielded a sequence of translation of this manuscript into Chinese, the last of which is now known as the *Suanfa Yuanben* (算法原本) in the *Shuli Jing Yun*. In this sense, this preface can be regarded as a result of compromise with the traditional thoughts in China made by Jesuits.

Any compromise of this kind was impossible after the Chinese Rites Controversy had changed into a serious diplomatic issue between Rome and Peking in 1706. We can find an example of this fact in another Manchu manuscript on Western science entitled "ge ti ciowan lu bithe" (格體全録, the *Manchu Anatomy*)[9]. This book was written by a French Jesuits Dominique Parrenin (1665-1741) in the 1710s in compliance with the order of the Emperor Kangxi and deals with anatomy and clinical medicine developed in early-modern Europe. It has not been translated into Chinese yet. The contents of its first three sections, which play the role of the preface to the whole book, have nothing to do with Neo-Confucianism in any sense. They are entirely based on the theology of Thomas Aquinas, the philosophy of Aristotle, and a moderate form of the theory of the human machine essentially due to Descartes. In addition, in the opening paragraph of its first section, Parrenin made a strict and explicit distinction between God and the material heaven, which was a philosophical and theological point of issue in the Rites Controversy. It seems probable that these points prevented the *Manchu Anatomy* from being translated and published. (See [Watanabe 2005] for details.)

Political conditions strongly affected the development of natural science in China.

II.2. SECTION 1

II.2.1. Romanized Manchu Texts

suwan fa yuwan ben bithe. ujui fiyelen: /
uju /
emke serengge. ton -i fulehe: geren emu -i ishunde acahangge be ton sembi: ton -i labdu

9 Our arguments are based on the three copies of this book possesed in Tôyô Bunko. Their call numbers are Ma2-16-1, Ma2-16-2 and Ma2-16-3.

komso / amba ajigen -i teksin akū be. akū obume muterakū: neihen obume teksileki seci. urunakū emu / toktoho kooli bihede. teni terei lak sere be bahaci ombi:
tuttu ajige ton be jibsime / nonggiha manggi. amba ton de tehereci. ere ajige ton be. amba ton be kemne[re](*me wacihiyara*) lak serengge sembi: /
duibuleci. amba ton jakūn. ajige ton juwe bifi. juwe be ilan ubu nonggici. jakūn de urunakū / teherere be dahame. ere juwe sere ajige ton. uthai jakūn sere amba ton be kemne[re](*me wacihiyara*) lak serengge / ombi:
ajige ton de udu ubu be nonggiha seme. amba ton de tehererakū oci. ere ajige ton be (f.3a)// amba ton be kemne[re](*me wacihiyara*) lak serengge waka sembi:
duibuleci. amba ton jakūn. ajige ton ilan bifi / ilan be emu ubu nonggici. ninggun ofi. jakūn ci ajige ombi: ilan be juwe ubu nonggici. uyun / ofi. geli jakūn ci amba ombi: ere gesengge. uthai kemne[re](*me wacihiyara*) lak serengge waka ombi:
jai ajige / ton -i amba ton be [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*) muterengge oci. ere ajige ton be amba ton -i emu neihen ubu / sembi: aikabade ajige ton amba ton be [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*) muterakūngge oci. ere ajige ton be / amba ton -i emu neihen akū ubu sembi: tuttu juwe serengge. jakūn be kemneme muterengge ofi. / juwe uthai jakūn -i emu neihen ubu ombi: ilan serengge. jakūn be kemneme muterakūngge ofi. ilan / uthai jakūn -i emu neihen akū ubu ombi:
komso be jafafi. labdu be kemneme. ajige be jafafi / amba be kemneme. terei [lak seme] acanara be [(*wacihiyame*)] bairede. majige hono tašaraburakūngge. inu damu (f.3b)// terei ubu be neihen obure arga be bahara de bi: (f.4a)//

II.2.2. Translation

"The Principles of Calculation" Book, the first chapter.
The first,
A unit is the root of numbers. That which is produced from many units combined with each other is called a number. We are not able to remove the irregularity of numbers in ⟨their various⟩ sizes and multiplicities. If we want to make them uniform and regular, we will be able to do this exactly only in the case in which we have a definite rule.

Thus we say that a small number is that which measures (*and exhausts*) a large number [(*exhaustively*)] [exactly] when ⟨the side of⟩ the small number turns equal to ⟨that of⟩ the large number after we have added the small number to itself repeatedly.

For example, ⟨suppose that⟩ the large number is 8 and ⟨that⟩ the small number is 2, and if we add 2 to itself three times the result is certainly equal to 8, so it follows that this small number 2 is that which measures (*and exhausts*) the large number 8 [(*exhaustively*)] [exactly].

We say that a small number is not that which measures (*and exhausts*) a large number [(*exhaustively*)] [exactly] when ⟨the side of⟩ the small number does not become equal to ⟨that of⟩ the large number, no matter how many times we add the small number ⟨to the former side⟩.

For example, if the large number is 8, the small number is 3, and we add 3 to itself once, then the result will be 6 and it is smaller than 8. If we add 3 to itself twice, then the result will be 9 and it is larger than 8. Things which are similar to this is just numbers on which measuring (*and exhausting*) is not exact.

We next say that a small number is a uniform part of a large number when this small number is that which can measure (*and exhaust*) the large number [(*exhaustively*)] [exactly]. If a small number is that which cannot measure (*and exhaust*) a large number [(*exhaustively*)] [exactly], then we say that this small number is a non-uniform part of the large number. So 2 is just a uniform part of 8 because 2 is that which can measure 8. 3 is just a non-uniform part of 8 because 3 is that which cannot measure 8.

When we seek the exactness of measurement in which we measure a quantity with large multiplicity after taking another one with small multiplicity, or we measure a quantity of large size after taking another one of small size, only the acquisition of the ⟨above⟩ method of making those parts uniform enables us not to make the slightest error.

II.2.3. Notes

(f.3a, l.1) ujui fiyelen

Throughout this manuscript each of its sections is referred as 'ere fiyelen –i *M*-ci meyen', which means '*M*-th section of this chapter', so all of its sections belong to only one chapter. A noun phrase 'ujui fiyelen' in the first line of this section shows that this unique chapter is the first chapter of this book. From a commonsense point of view, the adjective 'first' is necessary only when this book contains two or more chapters. It seems probable that the original plan made by the author of this manuscript contained more definitions and theorems from Euclid's *Elements*.

(f.3a, l.3) ton -i fulehe

In general, a Manchu noun 'ton' means a number or numbers. Throughout this manuscript, this noun, as a technical term of mathematics, means a natural number or natural numbers. The unit 1 is often excluded from the category of 'ton'.

A Manchu noun 'fulehe' means a root or roots of plants. This noun, as a technical term of mathematics, has two different meanings in this manuscript. One is a generator or generators of an algebraic system and its examples are found here, and in Section 11 and Section 36. The other meaning is a square or cubic root (plural is also possible as in other nouns in Manchu) and its examples are found in Section 6 , Section 7, Section 55, Section 56 and Section 58. To reflect this confusion in the usage of the Manchu word 'fulehe', we always take a common English word 'a root' or 'roots' as an English equivalent for it.

(f.3b, l.4) [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*)

The author of this manuscript suggested a Manchu phrase 'lak seme kenme-' ('to measure exactly') as an equivalent for 'measure' in the sense of Euclid. The corrector replaced this Manchu phrase with 'wacihiyame kemne-' ('to measure exhaustively') at first, and again with 'kemneme wacihiya-' ('to measure and exhaust'). The corrector is so industrious that he corrected the original phrase ‘lak seme kenme-’ in this way at more than 300 places. This type of correction is found only in the pages from Section 1 to Section 41. In the pages from Section 42 to the end of this manuscript, the expression ‘kemneme wacihiya-’ is used instead of ‘lak seme kemne-’ from the beginning. This observation yields the following facts:

(a) The primary and secondly main parts of the preface and Sections 1-41 were written first.
(b) Second, the correction part of the preface and these sections was written.
(c) Third, the primary and secondly main parts of Sections 42-75 were written.
(d) Last, the correction part of the preface and these sections was written.

Note that a Chinese verb phrase ‘du jin’ (度盡), which means ‘to measure’ in the sense of Euclid in *SLJY*, is a word-for-word translation of this Manchu verb phrase ‘kemneme wacihiya-’.

(f.3a, l.5) ajige ton
The distinction between the positive degree and the comparative degree of a Manchu adjective is always made by adding a certain adverbial phrase to this adjective in the latter case. The author of this manuscript could made this distinction explicitly by doing so. For example, he could say

> (yaya juwe ton bifi.) ere juwe ton –i dorgide ajige ton
> (Suppose that we have two arbitrary numbers.) The smaller number of these two numbers

instead of a simple phrase ‘ajige ton’, but he did not. It is doubtful that readers of this manuscript were aware of the fact that in European languages, for example, in Latin, the adjective corresponding to ‘ajige’ in this line took the comparative form. From this reason we used the positive form ‘small’ instead of the comparative form ‘smaller’.

(f.3b, l.8-f.4a, l.1) komso … bi:
We can find a word-for-word translation of this Manchu sentence into Chinese in the last sentence of the Section 1 of the first volume of *SfYbSLJY*. The Chinese sentence is as follows:

> 以小度大，以寡御多，求其恰符而毫無舛者，惟在得其平分之法而已.

(f.3b, ll.3-4) ajige / ton -i amba ton be [(wacihiyame)] [lak seme] kemneme (*wacihiyame*) muterengge oci.
There is no difference between the meaning of the next two sentences:

(a) ajige ton. amba ton be kemneme wacihiyame muterengge:
(a’) ajige ton. amba ton be kemneme wacihiyame mutembi:

(a) is a nominal sentence consisting of two noun phrases ‘ajige ton’ and ‘amba ton be kemneme wacihiyame muterengge’. (a’) contains a verb phrase ‘kemneme wacihiyame mutembi’. Motoki Nakajima pointed out that this type of Manchu sentence was created under the strong influence of classical Chinese and it was considered as an academic style of writing by Manchus in the Qing period. Nominal sentences of type (a) are prefered in this manuscript, too.

II.2.4. Remarks

This section contains two fundamental definitions. One is the definition of a natural number as a composite of units, and the other is the definiton of measurability and immeasurability of a natural number by another natural number in the sense of Euclid. The first definition corresponds to Def.2 in Book VII of the *Elements* and Sec.1 of Vol.1 of *SfYbSLJY*. The second definition corresponds to Def.3-5 in Book VII of the *Elements* and Sec.1 of Vol.1 of *SfYbSLJY*.

II.3. SECTION 2

II.3.1. Romanized Manchu Texts

jai. /

ton -i hacin udu labdu bicibe. eiterecibe juru sonihon ci tucirakū: aibe juru seci. / juwe gulhun neihen ubu -i ton inu: aibe sonihon seci. juwe gulhun neihen ubu banjinarakū ton inu /

juwe. duin. ninggun. jakūn. juwan. -i jergi ton -i gesengge be. neihen juwe ubu obume dendeci. gemu gulhun / ton banjiname ofi. erebe juru ton sembi: ilan. sunja. nadan. uyun. juwan emu -i jergi ton -i / gesengge be. neihen juwe ubu obume dendeci. gemu gulhun ton banjinarakū ofi. erebe sonihon ton / sembi:

geli ajige juru ton be jafafi. amba juru ton be dendere de. juru ubu banjinaci / ere jergi amba ton be. juru ubui juru ton sembi:

duibuleci. duin -i jergi ajige / juru ton be baitalafi. gūsin juwe -i jergi amba juru ton be. neihen jakūn ubu (f.5a)// obume dendeci. jakūn serengge. juru. ubui ton be dahame. ere gūsin juwe. uthai / juru ubui juru ton ombi:

geli ajige juru ton be jafafi. amba juru ton be neihen / dendere de. sonihon ubu banjinaci. ere jergi ton be. sonihon ubui juru ton sembi: /

duibuleci. ninggun -i jergi ajige ton be baitalafi. gūsin -i jergi amba ton be neihen / sunja ubu obume dendeci. sunja serengge. sonihon ubui ton be dahame. ere gūsin uthai / sonihon ubui juru ton ombi:

geli ajige sonihon ton be. jafafi. amba sonihon ton be. / dendere de. sonihon ubu banjinaci. ere jergi ton be sonihon ubui sonihon ton sembi:

duibuleci. / sunja -i jergi ajige ton be. baitalafi. tofohon -i jergi amba ton be. neihen ilan ubu obume dendeci. ilan / serengge. sonihon ton bime. tofohon inu sonihon ton ofi. tuttu ere tofohon uthai sonihon ubui sonihon (f.5b)// ton ombi: (f.6a)//

II.3.2. Translation

The second,

There exist a lot of kinds of numbers, but each of them is either even or odd in any case. Concerning what numbers we call even, they are numbers with two integral uniform parts. Concerning what numbers we call odd, they are numbers which do not generate two integral uniform parts.

If we divide those which are similar to numbers such as 2, 4, 6, 8, 10 into two uniform parts, then in every case integral numbers are produced and hence they are called even numbers. If we divide those which are similar to numbers such as 3, 5, 7, 9, 11 into two uniform parts, then in every case integral numbers are not produced and hence they are called odd numbers.

Moreover, if parts which are of even numbers are produced when we take a small even number and divide a large even number by it, then we call a large number of this kind an even-times even number.

For example, if we divide a large even number like 32 into eight uniform parts by using a small even number like 4, 8 is even and it follows immediately that this number 32 is an even-times even number.

Moreover, if parts which are of odd numbers are produced when we take a small even number and divide a large even number by it, then we call a large number of this kind an odd-times even number.

For example, if we divide a large number like 30 into five uniform parts by using a small number like 6, then 5 is odd and it follows immediately that this number 30 is an odd-times even number.

Moreover, if parts which are of odd numbers are produced when we take a small odd number and divide a large odd number by it, then we call a large number of this kind an odd-times odd number.

For example, if we divide a large number like 15 into three uniform parts by using a small number like 5, then 3 is an odd number and 15 is also an odd number, so this 15 turns out to be an odd-times odd number immediately.

II.3.3. Remarks

This section contains two definitions. One is the definition of an even and odd number, and the other is the definiton of an even-times even, odd-times even and odd-times odd number. The first definition corresponds to Def.6-7 in Book VII of the *Elements* and Sec.2 of Vol.1 of *SfYbSLJY*. The second definition corresponds to Def.8-11 in Book VII of the *Elements* and Sec.2 of Vol.1 of *SfYbSLJY*.

II.4. SECTION 3

II.4.1. Romanized Manchu Texts

ilaci /

yaya ton be jafafi. ton be kemnere de. aikabade amba ton bifi. erebe [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro / ajige ton akū oci. ere jergi ton be. emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemnere ton akūngge / sembi:

duibuleci sunja. nadan. juwan emu -i sonihon ton -i gesengge. gemu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / ojoro ajige ton akūngge: ainu seci. aikabade juwe. eici ilan. eici duin be / baitalafi. sunja be kemnehe. nadan be kemnehe. juwan emu be kemnehe seme. urunakū acanarakū be / dahame. tuttu ere ilan ton be. emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge sehebi: /

geli emu udu amba ton bifi: ere be gemu [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ajige ton akū oci. ere / udu amba ton be. ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge sembi:

duibuleci. tofohon. (f.7a)// jakūn ere juwe ton oci. ere juwe ton de [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ajige ton akū be dahame / emu ci tulgiyen. ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge ombi: ainu seci. aikabade eici juwe. / eici duin be baitalafi. udu jakūn be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ocibe. tofohon be [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*). muterakū: / eici ilan. eici sunja be baitalafi. udu tofohon be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ocibe. jakūn be [(*wacihiyame*)] [lak] / [seme] kemneme (*wacihiyame*) muterakū: tuttu ofi tofohon. jakūn ere juwe ton -i gesengge be. emu ci tulgiyen / ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge sehebi:

jai sunja. ninggun. uyun. ere ilan ton -i gesengge. gemu [(*wacihiyame*)] [lak seme] / kemne[re](*me wacihiyara*) ajige ton akūngge: erebe inu emu ci tulgiyen. ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge sembi: adarame seci / ilan be baitalafi. udu ninggun. uyun. be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ocibe. sunja be [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*) muterakū: juwe be / baitalafi. udu ninggun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ocibe. sunja. uyun be [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*) muterakū: (f.7b)// tuttu ofi sunja. ninggun. uyun ere ilan ton be. inu emu ci tulgiyen. ede tede / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge sehebi: (f.8a)//

II.4.2. Translation

The third,

When we take an arbitrary number and measure another number with it, if we have a large number and but do not have any small ⟨nontrivial⟩ number which can measure (*and exhaust*) it [(*exhaustively*)] [exactly], then a number of this kind is called what no numbers other than 1 measure [(*exhaustively*)] [exactly].

For example, those which are similar to odd numbers such as 5, 7 and 11, are all what no numbers other than 1 can measure (*and exhaust*) [(*exhaustively*)] [exactly]. Concerning how it occurs, even if we tried to meaure 5, to measure 7 and to measure 11 by using 2 or 3 or 4, this ⟨measurement⟩ would never be exact and it follows that we called these three numbers what no numbers other than 1 can measure (*and exhaust*) [(*exhaustively*)] [exactly].

Moreover, if we have a series of large numbers but do not have any small ⟨nontrivial⟩ number which measures (*and exhausts*) all of them [(*exhaustively*)] [exactly], then we call these large numbers what no numbers can measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there.

For example, as for two numbers 15 and 8, we do not have a small number which can measure (*and exhaust*) these two numbers [(*exhaustively*)] [exactly], and it follows that these two numbers are what no numbers other than 1 measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there. Concerning how it occurs, we are not able to measure (*and exhaust*) 15 [(*exhaustively*)] [exactly], though we can measure (*and exhaust*) 15 [(*exhaustively*)] [exactly] by using 3 or 5. We therefore said that those which are similar to these two numbers 8 and 15 were what no numbers other than 1 measured (*and exhausted*) [(*exhaustively*)] [exactly] here and there.

Next as for 5 and 6 and 9, and any triple of numbers which are similar to them, we do not have a small number which can measure (*and exhaust*) all of them [(*exhaustively*)] [exactly]. We also call them what no numbers can measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there. Concerning how it occurs, although we can measure (*and exhaust*) 6 and 9 [(*exhaustively*)] [exactly] by using 3, we are not able to measure (*and exhaust*) 5 with it [(*exhaustively*)] [exactly]. Although we can measure (*and exhaust*) 6 [(*exhaustively*)] [exactly] by using 2, we are not able to measure (*and exhaust*) 5 and 9 with it [(*exhaustively*)] [exactly]. Hence we also said that these three numbers, 5 and 6 and 9, were what no numbers could measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there.

II.4.3. Notes

(f.7a, l.3) emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemnere ton akūngge

This noun phrase means ‘a prime number’ or ‘prime numbers’. Throughout this manuscript an adjective corresponding to ‘prime’ is not given. A strong tendency to adopt a long descriptive phrase as a technical term on science is common to both of this manuscript and “ge ti ciowan lu bithe” (“Manchu anatomy”)

(f.7b, ll.5-6) emu ci tulgiyen / ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge

This noun phrase means ‘relatively prime numbers’.

II.4.4. Remarks

This section contains two definitions. One is the definition of a prime number, and the other is the definiton of two or more natural numbers which are relatively prime. The first definition corresponds to Def.12 in Book VII of the *Elements* and a part of Sec.16 of Vol.1 of *SfYbSLJY*. The second definition corresponds to Def.13 in Book VII of the *Elements* and a part of Sec.16 of Vol.1 of *SfYbSLJY*.

II.5. SECTION 4

II.5.1. Romanized Manchu Texts

duici /

yaya ton be jafafi ton be kemnere de. aikabade amba ton bifi. ajige ton be baitalafi. [lak] / [(*wacihiyame*)] [seme] kemne[ci](*me wacihiyaci*). oci. ere jergi ton be. emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton bisirengge sembi: /

duibuleci. amba ton juwan juwe bifi. ajige ton -i duin be baitalafi. aikabade duin be / juwe ubu nonggici. uthai juwan juwe be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: tuttu ere jergi ton be / emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton bisirengge sehebi:

geli yaya juwe amba ton bifi. emu / ajige ton be baitalafi [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. ere jergi juwe amba ton be. emu ci tulgiyen / ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton bisirengge sembi:

duibuleci. juwan juwe. juwan ninggun ere juwe / amba ton bifi. ajige ton -i duin be baitalafi. duin be juwe ubu nonggici. uthai juwan (f.9a)// juwe be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): duin be ilan ubu nonggici. uthai juwan ninggun be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): / tuttu juwan juwe. juwan ninggun. ere juwe amba ton. uthai emu ci tulgiyen. ede tede [lak] / [(*wacihiyame*)] [seme kemnere] (*kemneme wacihiyara*) ton bisirengge ombi:

geli juwan. tofohon. orin -i jergi ilan ton [be] (*oci*). [inu emu ci] / [tulgiyen. ede tede lak seme kemnere ton bisirengge sembi] (*aikabade*) [: duibuleci]. ajige ton sunja be baitalafi. / sunja be emu ubu nonggi[ci.](*ra de*) uthai juwan be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): sunja be juwe ubu nonggi[ci.](*ra de*) / tofohon be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): sunja be ilan ubu nonggi[ci](*ra de*). uthai orin be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): / tuttu ere juwan. tofohon. orin ere ilan ton. inu emu ci tulgiyen ede tede [(*wacihiyame*)] [lak seme] / kemne[re](*me wacihiyara*) ton bisirengge ombi:

jai aikabade juwe ton. ilan ton. eici geren amba ton -i dorgi / emu ton. geren amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. ere geren amba ton be. inu emu ci (f.9b)// tulgiyen ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton bisirengge sembi:

duibuleci. nadan. orin emu. gūsin / sunja -i jergi ilan ton oci. nadan be baitalafi. uthai ere nadan be kemneci ombime. / gūwa be inu kemneci ombi: aikabade. nadan be juwe ubu nonggici. uthai orin emu be [lak] [(*wacihiyame*)] / [seme] kemne[mbi](*me wacihiyambi*): nadan be duin ubu nonggici. uthai gūsin sunja be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): tuttu / nadan. orin emu. gūsin sunja ere ilan ton. inu emu ci tulgiyen. [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton / bisirengge ombi: (f.10a)//

II.5.2. Translation

The fourth,

When we take an arbitrary number and measure another number ⟨with it⟩, if we have a large number and we can measure (*and exhaust*) it [(*exhaustively*)] [exactly] by using a small number, a ⟨large⟩ number of this kind[10] is called that which has a number other than 1 which measures (*and exhausts*) it [(*exhaustively*)] [exactly].

For example, ⟨suppose that⟩ the large number is 12, and if we use a small number 4 and add 4 to itself two times, then we can measure (*and exhaust*) 12 [(*exhaustively*)] [exactly]. Thus we said that a number of this kind was that which has a number other than 1 which measure (*and exhaust*) it [(*exhaustively*)] [exactly].

Moreover, ⟨suppose that⟩ we have two arbitrary large numbers, and if we can measure (*and exhaust*) them [(*exhaustively*)] [exactly] by using a small number, two large numbers of this kind are called those which have a number other than 1 which measures (*and exhausts*) them [(*exhaustively*)] [exactly] here and there.

For example, ⟨suppose that⟩ 12 and 16 are these two large numbers, and if we use a number 4 and add 4 to itself two times, then we measure (*and exhaust*) 12 [(*exhaustively*)] [exactly]. If we add 4 to itself three times, then we measure (*and exhaust*) 16 [(*exhaustively*)] [exactly]. Thus these two large numbers 12 and 16 are just those which have a number other than 1 which measures (*and exhausts*) them [(*exhaustively*)] [exactly] here and there.

Moreover, (*as for three numbers such as 10 and 15 and 20,*) we also call (*them*) [three numbers such as 10 and 15 and 20] those which have a number other than 1 which can measure (*and exhaust*) them [(*exhaustively*)] [exactly]. (*If*) [When] we use a small number 5 and add 5 to itself once, (*then*) we measure (*and exhaust*) 10 [(*exhaustively*)] [exactly]. (*If*) [When] we add 5 to itself two times, (*then*) we measure (*and exhaust*) 15 [(*exhaustively*)] [exactly]. If we add 5 to itself three times, then we measure (*and exhaust*) 20 [(*exhaustively*)] [exactly]. Thus these three numbers 10 and 15 and 20 are also those which have a number other than 1 which measures (*and exhausts*) them [(*exhaustively*)] [exactly] here and there.

Next if a number in ⟨the set of⟩ two numbers, or three numbers, or many numbers can measure (*and exhaust*) the many numbers [(*exhaustively*)] [exactly], then these numbers

[10] One may want to replace the phrase ‘a ⟨large⟩ number of this kind’ with ‘this ⟨large⟩ number’, but the original Manchu text is ‘ere jergi ton’, not ‘ere ⟨amba⟩ ton’.

are also called those which have a number other than 1 which measures (*and exhausts*) them [(*exhaustively*)] [exactly] here and there.

For example, as for three numbers such as 7 and 21 and 35, by using 7 we can immediately measure this number 7 and, at the same time, we can also measure the other ones. If we add 7 to itself two times, then we measure (*and exhaust*) 21 [(*exhaustively*)] [exactly]. If we add 7 to itself four times, then we measure (*and exhaust*) 35 [(*exhaustively*)] [exactly]. Thus these three numbers 7 and 21 and 35 are also those which have a number other than 1 which measures (*and exhausts*) them [(*exhaustively*)] [exactly] here and there.

II.5.3. Notes

(f.7a, l.3) **emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton bisirengge**

This noun phrase means ‘a composite number’ or ‘composite numbers’.

(f.9a, ll.2-3) **emu ci tulgiyen. ede tede [lak] / [(*wacihiyame*)] [seme kemnere] (*kemneme wacihiyara*) ton bisirengge**

This noun phrase means ‘numbers with a nontrivial common divisor’.

II.5.4. Remarks

This section contains two definitions. The first one is the definition of a composite number as a natural number which cannot be measured with any nontrivial natural number. The second one is the definition of two or more natural numbers which are not relatively prime. The first definition corresponds to Def.14 in Book VII of the *Elements* and a part of Sec.15 of Vol.1 of *SfYbSLJY*. The second definition corresponds to Def.15 in Book VII of the *Elements* and a part of Sec.15 of Vol.1 of *SfYbSLJY*.

II.6. SECTION 5

II.6.1. Romanized Manchu Texts

sunjaci /

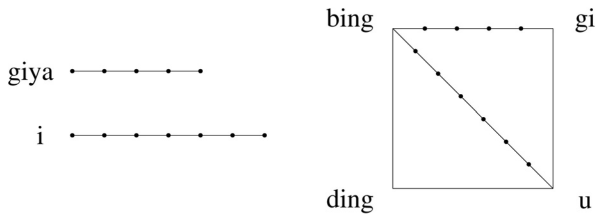
yaya juwe jijun. juwe dere. juwe beye bifi. aikabade ton be baitalafi. / terei c'y ts'un be toktobuci ojorongge oci. ere jergi jijun. / dere. beye be. ede tede [(*wacihiyame*)] [lak seme] kemneme (*wacihiyaburengge*) [bisirengge] sembi:

duibuleci. / giya. i sere juwe jijun bifi. tere giya sere jijun golmin sunja / c'y. i sere jijun golmin nadan c'y oci. ere juwe jijun i c'y / t'sun be toktobure de. sunja. nadan -i ton be baitalaci ojoro be / dahame. uthai ede tede [(*wacihiyame*)] [lak seme] kemneme (*wacihiyaburengge*) [bisirengge] ombi: adarame / seci. yaya ton eici sonihon. eici juru. udu labdu bicibe. (f.11a)// aikabade terei ton be toktobure de. emke be baitalaci. [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorakūngge / akū ombi: te emu c'y be baitalafi. ere giya. i sere juwe jijun be kemnere de. emu / c'y be duin ubu nonggici.

uthai giya sere jijun be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): emu c'y be / ninggun ubu nonggici. uthai i sere jijun be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*):

geli aikabade / eici juwe jijun. eici juwe dere. eici juwe beye. bifi. ton be baitalafi. c'y / t'sun be toktobume muterakū oci. erebe ede tede [(*wacihiyara*)] [lak sere] kemneme (*wacihiyaburakūngge*) [akūngge] sembi: /

duibuleci. bing ding u gi sere emu durbejengge dere -i. bing gi sere emu ergi. / dere sunja c'y ojoro. ere durbejengge arbun -i bing u sere hošotoloho jijun / nadan c'y funceme ojoro oci. ere funcehe (gargata ton wacihiyaci ojorakū) [[??]] ofi. tuttu ton be (f.11b)// baitalafi. [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*) muterakū: uttu ofi ere durbejengge arbun -i emu dere. / jai erei hošotoloho jijun. uthai ede tede [(*wacihiyara*)] [lak sere] kemneme (*wacihiyaburakū*) [akū] juwe jijun / ombi: ere jijun -i turgun be amala encu narhūšame suhebi: (f.12a)//



The figures in f.11a

II.6.2. Translation

The fifth,

⟨Suppose that⟩ we have two arbitrary lines or two arbitrary surfaces or two arbitrary solids, and if they are objects the numerical size of which we can determine by using numbers, we call lines or surfaces or solids of this kind those which are measured (*and exhausted*) [(*exhaustively*)] [exactly] here and there.

For example, ⟨suppose that⟩ we have two line segments Jiya and Yi, and if the length of the line segment Jiya is 5 chi and the length of the line segment Yi is 7 chi, we can use numbers 5 and 7 when we determine the numerical size of these two line segments, so it follows immediately that they are those which are measured (*and exhausted*) [(*exhaustively*)] [exactly] here and there. Concerning how it occurs, arbitrary numbers, some of which may be odd numbers, and others of which may be even numbers, are numerous, however, there is nothing which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] by using the unit when we make those numbers definite. Now when we use 1 chi and measure these two line segments Jiya and Yi, if we add 1 chi to itself four times, then it measures (*and exhausts*) the line segment Jiya [(*exhaustively*)] [exactly]. If we add 1 chi to itself six times, then it measures (*and exhausts*) the line segment Yi [(*exhaustively*)] [exactly].

Moreover, if we have two lines or two surfaces or two solids, and if they are objects the numerical size of which we are not be able to determine by using numbers, we call them those which are not measured (*and exhausted*) [(*exhaustively*)] [exactly] here and there.

For example, when the side Bing-Ji of a square Bing-Ding-Wu-Ji is 5 chi long and a diagonal line Bing-Wu of this square is 7 chi long and more, (we cannot exhaust) [[??]] this extra (fragment of a number,) [[??]] and thus we are not be able to measure (*and exhaust*) them [(*exhaustively*)] [exactly] by using numbers. Such being the case, ⟨firstly⟩ a side of this square, and secondly, ⟨one of⟩ its diagonals, are just two line segments which are not measured (*and exhausted*) [(*exhaustively*)] [exactly] here and there. Later we will explain the details of the situation of these line segments in a different place.

II.6.3. Notes

(f.11a, l.3) c'y ts'un

These Manchu words came from Chinese words 'chi' (尺) and 'cun' (寸), and they mean the concrete values of length or area or volume in this context.

(f.11a, l.5) giya. i

Two words 'giya' and 'i' came from Chinese words 'jia' (甲) and 'yi' (乙), which were the first and the second of the so-called Ten Stems (十干) in the traditional Chinese system of the calender. The notation 'giya' in Manchu alphabet represents the official pronunciation of the character 甲 in Mandarin Chinese at that time, because the process of palatization had not been completed yet, as we see in the *Yinyun Chanwei* (音韻闡微, the *Bringing the Subtleties in the Pronunciation System to Light*), an official guide to the system of normative pronunciation in Mandarin Chinese published in 1726.

Throught this manuscript the Chinese names of the elements of the Ten Stems and the Twelve Branches (十二支) transliterated in Manchu alphabet are used as symbols expressing mathematical objects, such as numbers or points. These transliterated names are mere symbols and have no ordinary meaning as is used in the calendar sytem, because each of the Ten Stems and the Twelve Branches has its own name in Manchu, for example, 'niowanggiyan' for 'jia' and 'niohon' for 'yi' respectively, and in every Manchu document these Manchu names are used to express the dates in the Chinese calender system.

II.6.4. Remarks

This section explains the correspondence between various numbers and the values of the length or the area or the volume of intervals. This explanation corresponds to the one given in Sec.26 of Vol.2 of *SfYbSLJY*.

II.7. SECTION 6

II.7.1. Romanized Manchu Texts

ningguci. /

ton be ishunde teherebu[mbi](*me kamcimbi*) sere giyan adarame seci. juwe ton bifi. ere ton -i dorgi de / (*emu serengge udu*) [udu emu] bisire songkoi. tere ton de udu ubu nonggifi teherebure be henduhebi: /

duibuleci. ninggun. juwan ere juwe ton be ishunde teherebure de. ere ninggun -i dorgi de. / daci (*emu serengge*) ninggun [emu] bifi. tuttu ninggun be juwan ubu obuci. bahara ninju. ishunde / teherebuhe ton ombi: uttu ofi emu be. ninggun de duibulere duibulen. uthai / juwan be ninju de duibulere duibulen -i adali ombi:

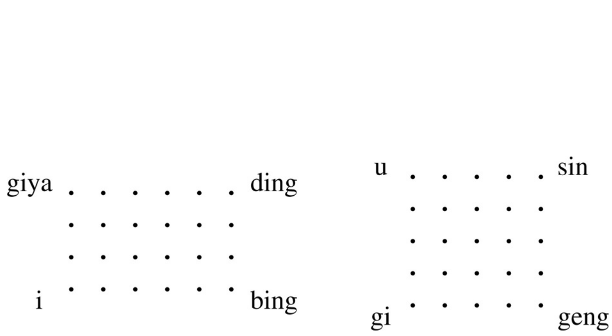
yaya amba ajige juwe / ton be ishunde teherebu[re](*me kamcire*) de baha ton be. necin durbejengge ton sembi:

duibuleci. / duin. ninggun ere juwe ton be ishunde teherebuci orin duin bahara adali: (f.13a)// adarame seci. duin. ninggun ere juwe ton be. tongki arafi / neihen faidame. undu duin tongki ilibufi giya i obure. hetu / ninggun tongki ilibufi giya ding obure. ere ninggun be. duin mudan / faidara ohode. uthai giya i bing ding sere necin durbejengge / ton banjinambi: tuttu amba ajige juwe ton be ishunde teherebu[re](*me kamcire*) de / baha ton be. necin durbejengge ton sehebi:

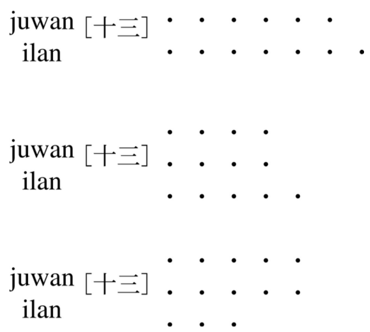
aikabade ishunde / teherehe juwe ton be ishunde teherebu[ci](*me kamcici*). baha / ton be tob (*necin*) durbejengge [necin] ton sembi:

duibuleci. / sunja be sunja de (*kamcici*) [teherebuci] orin sunja bahara (*jergingge*) [adalingge] (f.13b)// inu: adarame seci. unenggi ere orin sunja -i ton be. inu tongki obufi neihen / faidame. jecen tome sunja obuci. uthai u gi geng sin sere tob necin durbejengge / banjinambi: tuttu ishunde teherehe juwe ton be teherebu[re](*me kamcire*) de baha ton be. tob / necin durbejengge ton sehebi: terei emu jecen -i (*ton sunja oci*) [sunja ton]. uthai tob durbejengge -i / fulehe ohobi:

geli yaya ton [udu] emu (*serengge udu*) bisirengge be. tongki be inu udu obume arafi / (*aikabade*) ton [be] durbejengge (*oci*) [obure turgunde]. tongki be teni durbejengge obume faidaci ombi: / aikabade ton durbejengge ome banjinarakū oci. tongki be inu durbejengge obume / faidame (*banjinarak*ū) [muterakū]: uthai juwan ilan -i durbejengge banjinarakūngge be. udu tongki arafi / gūnin cihai faidacibe. ainaha seme durbejengge banjinarakū ojoro adali: nirugan be (f.14a)// tuwaha de. uthai saci ombi: (f.14b)//



The figures in f.13b



The figures in f.14b

II.7.2. Translation

The sixth,

Concerning what the principle of (*placing numbers close together to bring balance between them*) [bringing balance between numbers] is, we mean that we have two numbers and bring balance ⟨between them⟩, by adding one number to itself a certain number of times, in accordance with the existence of a certain number of units in the other number.

For example, when we bring balance between the two numbers 6 and 10, we have six units in 6 from the beginning and hence a number 60, which is obtained by making 6 tenfold, is identical to the number which was produced by bringing balance ⟨between them⟩. Such being the case, the compared pair given by comparing 1 with 6 is just equivalent to the compared pair given by comparing 10 with 60.

The number which is obtained when we (*place*) [bring balance between] two arbitrary numbers, a large one and a small one, (*close together to bring balance between them*) is called a number of the rectangular type.

For example, it is similar to the fact that we obtain 24 when we bring balance between two numbers 4 and 6. Concerning how it occurs, if we make the two numbers 4 and 6 into points and arrange them in lines uniformly, and if we set six points horizontally and denote them by Jiya-Yi, and if we set four points vertically and denote them by Jiya-Ding, and if we arrange ⟨the set of⟩ these six ⟨points⟩ in a line four times, then a rectangle Jiya-Yi-Bing-Ding is naturally produced. We therefore called the number which was obtained when we (*placed*) [brought balance between] two unequal numbers (*close together while bringing balance between them*) a number of the rectangular type.

The number which we obtain when we (*place*) [bring balance between] two equal numbers (*close together while bringing balance between them*) is called a number of the (*plain-*)square type. For example, it is (*of the same kind as*) [similar to] the fact that we obtain 25 if we (*place*) [balance] 5 (*close to*) [with] 5. Concerning how it occurs, if we really make this number 25 into points and arrange them in lines uniformly, too, and set

⟨the number of points⟩ on each border to 5, then a plane square Wu-Ji-Geng-Xin is naturally produced. We therefore called the number which was obtained when we (*placed*) [brought balance between] two equal numbers (*close together while bringing balance between them*) a number of the (*plain*-)square type.

Moreover, ⟨suppose that⟩ we make a certain number of points for an arbitrary number containing a certain number of units, then we can arrange the points in a rectangle, only if the number is of the rectangular type. If a number is not generated as ⟨a number⟩ of the rectangular type, then we also cannot arrange points in any rectangle. ⟨The situation is⟩ just similar to the fact that even if we make a number 13, which is not produced as ⟨a number⟩ of the rectangular type, into points and arrange them as we like, it does not produce any rectangle. We can understand ⟨the details of these situations⟩ when we see the figures ⟨in f.14b⟩.

II.7.3. Notes

(f.13a, l.2) ton be ishunde teherebu[mbi](*me kamcimbi*)

The author of this manuscript suggested an expression ‘ton be ishunde teherebumbi’ as a Manchu equivalent for an English phrase ‘to multiply numbers together’, but the corrector modified this expression as ‘ton be ishunde teherebume kamcimbi’.

The meaning of a Manchu verb ‘kamci-’ is similar to that of another Manchu verb ‘acabu-’, which is often used to express addition in this manuscript. Both verbs mean ‘to put two or more things together’. The meaning of ‘acabu-’ is more general than that of ‘kamci-’ and this verb is regarded as an unmarked expression. ‘kamci-’ is regarded as a marked expression, and it has an additional meaning that the identities of the two or more things which are put together are preserved in the process expressed by this verb.

It seems probable that the choice of technical terms in Manchu done in this manuscript reflects this difference between the meanings of the two verbs. In this manuscript, multiplication is regarded as construction of a rectangle and the result of this operation is identified with the value of the area of this rectangle. In this case the two arguments of multiplication can be reconstructed from the length of the two adjacent sides of this rectangle, so in a certain sense the identities of the two arguments are preserved in the process of multiplication. On the other hand an addition is regarded as construction of a union of two line segments, both of which correspond to the two arguments of addition. It is impossible to recover the two line segments from their union, and this implies that the identities of the two arguments are not preserved in the process of addition. So the verb ‘acabu-’ is not appropriate for the technical term for multiplication.

The author of this manuscript was not sensitive to the above difference on the meaning of these two verbs. In Section 27, he used an expression ‘teherebume acabu-’ instead of ‘teherebume kamci-’ four times, and in all cases this expression was replaced with ‘teherebume kamci-’ or ‘teherebu-’ by the corrector.

(f.13a, l.6) duibulere duibulen

The definition of the notion of 'duibulere duibulen' is not found in this manuscript. It seems that it was given in *MaGHYB*, because the sixth chapter of that manuscript is often cited when theorems or algorithms about 'duibulere duibulen' are needed in this manuscript.

We may consider that this noun phrase is a Manchu technical term for 'proportion', and probably it was constructed from the Chinese equivalent 'bi li' (比例) for 'proportion'. 'duibulere' is the participle form of a Manchu verb 'duibule-', the meaning of which is 'to compare'. Since the noun 'duibulen' is derived from the same verb, the phrase 'duibulere duibulen' can be considered to be a redundant expression. Probably, this expression is based on the explanation on the character 'li' (例) given in Vol. 8 of the *Shuo Wen Jie Zi* (説文解字, the *Analysis of Simple Graphs as an Explanation of Complex Characters*). It says 'the character 'li' (例) means comparison (or a comparison)' ('例, 比也').

Note that in general the notion of 'duibulen' does not always imply that the number of the compared objects is always equal to 2. If we restrict ourselves to the examples in this manuscript, we cannot exclude the possibility that 'duibulen', as a technical term of mathematics, means not only 'a compared pair' but 'a compared set'.

(f.13a, l.8) necin durbejengge ton

This Manchu noun phrase means 'a plane number' or 'plane numbers' in the sense of Euclid. The adjective 'necin' as a technical term of mathematics means 'plane' or 'two-dimensional'. In many Manchu-Chinese dictionaries written in the Qing period, a Chinese equivalent for 'necin' is 'ping' (平). A word-by-word translation of 'necin durbejengge ton' is 'ping fang shu' (平方数).

(f.13b, l.8) tob (*necin*) durbejengge [necin] ton

This noun phrase means 'a square number' or 'square numbers'.

(f.14a, ll.4-5) tob durbejengge -i fulehe

This noun phrase means 'a square root' or 'square roots'.

II.7.4. Remarks

This section contains three definitions. The first one is the definition of multiplication. The second one is the definition of a compsite number as a product of two nontrivial natural numbers. The third one is the definition of a square number and its square root. The first definition corresponds to Def.16 in Book VII of the *Elements* and a part of Sec.3 of Vol.1 of *SfYbSLJY*. The second definition corresponds to Def.17 in Book VII of the *Elements* and Sec.4 of Vol.1 of *SfYbSLJY*. The third definition corresponds to Def.19 in Book VII of the *Elements* and Sec.4 of Vol.1 of *SfYbSLJY*.

II.8. SECTION 7

II.8.1. Romanized Manchu Texts

nadaci. /

yaya ilan ton bifi ishunde (*teherebume kamcire de*) [tehereburede] baha ton be. iliha / durbejengge ton sembi: terei da bihe ton be. iliha durbejengge / jecen -i ton sembi:

duibuleci. giya i bing sere ilan ton bifi. / giya sere juwe. i sere ilan be. ishunde teherebuci. ninggun / bahambi: aikabade ere ninggun be tongki arafi neihen faidaci. / ding u sere necin durbejengge ton ombi: geli bing sere ton -i / duin. baha ton -i ninggun be ishunde teherebufi. bahara ton orin / duin be. orin duin tongki arafi. ninggun ton be duin mudan (f.15a)// neihen faidaci. gi ding u geng sere iliha durbejengge ton be / bahambi: tuttu ere jergi yaya ton be. aikabade tongki arafi. / jibsime nonggime neihen faidara de. iliha durbejengge banjinaci ojoro ton be. / uthai iliha durbejengge ton sembi:

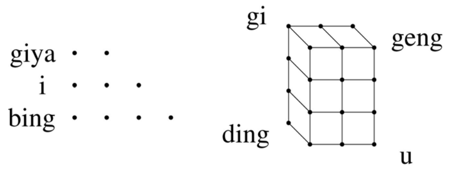
jibsime nonggime neihen faidacibe. iliha / durbejengge banjinarakūngge be. uthai iliha durbejengge. waka. ton sembi:

geli / aikabade ishunde teherere ilan ton bifi. (ere be ishunde) [[ere tere be ishunde]] teherebu[rede](*me kamcirede*) baha / ton be. duin durbejengge ton sembi:

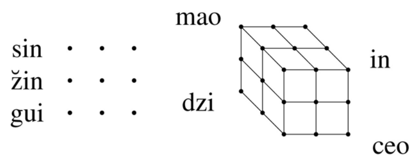
duibuleci. sin žin gui sere ilan ton bifi. / ton tome gemu ilata oci. sin sere ilan ton. žin sere / ilan ton be ishunde teherebuci. uyun be bahafi durbejengge (f.15b)// ton ombi: geli ere uyun. gui sere ton -i ilan be ishunde teherebuci. orin nadan be bahafi. / uthai [(*iliha*)] duin durbejengge ton ombi: adarame seci. ilan be ilan de ishunde teherebure de. / uyun be bahafi. dze ceo sere durbejengge ton ojoro be dahame. ere uyun be. ilan de geli / teherebure de orin nadan be bahafi. tongki arafi. jibsime nonggime neihen faidaci. uthai / mao dze ceo in sere duin durbejengge ton banjinambi:

jai ere jergi yaya ton be. aikabade / tongki arafi. jibsime nonggime neihen faidara de. duin durbejengge banjinaci ojoro ton be / uthai duin durbejengge ton sembi: erei emu jecen -i ton be. uthai duin durbejengge -i / fulehe sembi: aikabade jibsime nonggime neihen faidaha seme duin durbejengge banjinarakū[ngge be] oci / [uthai] (*duin*) durbejengge [(*ton*)] waka (*ton*) [ton] sembi:

uthai juwan juwe. duin durbejengge (*waka*) [ojorakū] ton ojoro (f.16a)// (*adali*) [gesengge inu]: adarame seci. juwan juwe be tongki arafi. juwe be baitalafi. emu jecen / obume faidafi. da ton -i songkoi teherebuci duin be bahafi. durbejengge ton ombi: / dahime teherebuci jakūn bahara be dahame. udu iliha durbejengge ton bicibe. juwan / juwe ci ajigen ofi. duin ekiyehun ombi: geli aikabade ilan be baitalafi. emu jecen / obume faidafi. da ton -i songkoi teherebuci. uyun be bahafi durbejengge ton ombi: / dahime teherebuci orin nadan be bahafi. juwan juwe ci amba ofi. tofohon fulu / ombi: tuttu ere juwan juwe tongki be gūnin cihai faidaha seme duin durbejengge / banjinarakū ofi. erebe duin durbejengge waka ton sehebi: (f.16b)//



The figures in f.15a



The figures in f.15b

II.8.2. Translation

The seventh,

⟨Suppose that⟩ we have three arbitrary numbers, and when we (*place them close together to*) bring balance among them, the number which is obtained ⟨in this process⟩ is called a number of the rectangular-parallelopipedic type. The numbers which we had at the begininng are called numbers corresponding to the sides of rectangular parallelopipeds.

For example, ⟨suppose that⟩ we have three numbers Jiya and Yi and Bing, and if we bring balance between Jiya and Yi, which are equal to 2 and 3 respectively, then we obtain 6. If we make this number 6 into points and arrange them in lines uniformly, then we have Ding-Wu, which is a number of the rectangular-parallelopipedic type. Moreover, if we make the number 24, which is obtained by bringing balance between the number Bing and the number 6, the former of which is equal to 4 and the latter of which was obtained ⟨in the above process⟩, into twenty-four points and arrange the six numbers[11] uniformly four times, then we obtain a number of the rectangular-parallelopipedic type Ji-Ding-Wu-Geng. We therefore say that arbitrary numbers of this kind, ⟨in other words,⟩ numbers which can produce rectangular parallelopipeds if we make them into points and arrange these points uniformly by adding them repeatedly, are just numbers of the rectangular-parallelopipedic type. Those which do not produce rectangular parallelopipeds even if we arrange them uniformly by repeated addition are called numbers which are not of the rectangular-parallelopipedic type.

Moreover, ⟨suppose that⟩ we have three equal numbers, then the number obtained by (*placing them close together and*) bringing balance among them is called a number of the cubic type.

For example, ⟨suppose that⟩ we have three numbers Xin and Ren and Gui, and if each of number is equal to 3 in all cases, then when we bring balance between Xin and Ren, both of which are three numbers respectively[12], we obtain 9 and it is just a number of the rectangular type. Moreover, if we bring balance between this number 9 and the number Gui, which is equal to 3, then we obtain 27 and it is just a number of the [(*rectangular-parallelopipedic*)] cubic type. Concerning how it occurs, when we balance 3 with 3 we obtain 9 and the result is equal to Zi-Chou, ⟨see the figure in f.15b,⟩ which is a num-

[11] It seems that 'the six points' is correct. See II.8.3.
[12] 'both of which are three numbers respectively' should be replaced with 'both of which are equal to the number 3 respectively'. See II.8.3.

ber of the square types, and following this, we obtain 27 when we balance this 9 with 3, and if we make it into points and arrange them uniformly by repeated addition, then a number of the cubic type, which is equal to Mao-Zi-Chou-Yin, is generated.

We next say that arbitrary numbers of this kind, ⟨in other words,⟩ numbers which can produce cubes if we make them into points and arrange these points uniformly by repeated addition, are just numbers of the cubic type. We say that the number which corresponds to one of their borders is just the root of a cube. If any cube is not generated, no matter how freely we arrange the points uniformly by repeated addition, then we say that it is [not] a (*number*) [number] which is (*not*) of the (*cubic*) [square] type. ⟨The situation is⟩ similar to that of the case in which 12 is a number which (*is not*) [will not be] of the cubic type. Concerning how it occurs, ⟨suppose that⟩ we make ⟨the number⟩ 12 into points, and if we use 2 ⟨which is identified with two points⟩ and arrange ⟨these two points⟩ in a line which is regarded to be an border ⟨of a square⟩, and if we balance it with the original number ⟨2⟩, then we obtain 4 and ⟨this 4⟩ is a number of the square type. If we balance ⟨this 4 with the original number 2⟩ again, we obtain 8, so it follows that ⟨this 8⟩ is smaller than 12 and falls below ⟨12⟩ by 4, though it is a number of the rectangular -parallelopipedic type.

Moreover, if we use 3 ⟨which is identified with three points⟩ and arrange ⟨these three points⟩ in a line which is regarded to be an border ⟨of a square⟩, and if we balance it with the original number ⟨3⟩, then we obtain 9 and ⟨this 9⟩ is a number of the square type. If we balance ⟨this 9 with the original number 3⟩ again, we obtain 27, so it follows that the result is larger than 12 and exceeds ⟨the latter⟩ by 15. Therefore any cube will not be produced, no matter how freely we arrange these twelve points, and hence we said that this ⟨12⟩ was a number which was not of the cubic type.

II.8.3. Notes

(f.15a, ll.2-3) iliha durbejengge ton

This Manchu noun phrase means ‘a solid number’ or ‘solid numbers’ in the sense of Euclid. A word-by-word translation of ‘iliha durbejengge ton’ is ‘li fang shu’ (立方数).

(f.15a, ll.8-9) sin sere ilan ton. žin sere ilan ton

The literal difference between two Manchu noun phrases ‘ilan –i ton’ and ‘ilan ton’ is whether the Manchu genitive case suffix ‘-i’ exists or not, however, the meanings of these two phrases are far different. The former one means ‘the number 3’, and the latter one means ‘three numbers’. From the mathematical point of view, the expression ‘ilan ton’ in the primary main part of the text is wrong, but the corrector made no correction in this case. Here we gave a literal translation.

(f.15a, l.9) ninggun ton

Probably, ‘ninggun tonki’ (‘six points’) is correct.

(f.15b, l.7) duin durbejengge

Throughgout this manuscript this Manchu phrase means ‘of a cube’ or ‘of cubes’.

The notion of ‘duin durbejengge’ is rather ambiguous in *GTCL*. In the section dedicated to the bones of feet, cuboid bones are described as being ‘duin durbejengge’, and we can consider that this adjective means ‘cubic’, as is the case with the examples in this manuscript. On the other hand in the section dedicated to adominal muscles, bellies of rectus abdominis muscles are described as being ‘duin durbejengge’. Nobody can have a cubic belly of this muscle, however fat he or she is.

(f.16a, ll.7-8) duin durbejengge ton –i felehe
The meaning of this noun phrase is ‘a cubic root’ or ‘cubic roots’.

II.8.4. Remarks

This section contains two definitions. The first one is the definition of a natural number which is given by the product of three nontrival natural numbers. The second one is the definition of a cubic number and its cubic root. The first definition corresponds to Def.18 in Book VII of the *Elements* and Sec.7 of Vol.1 of *SfYbSLJY*. The second definition corresponds to Def.20 in Book VII of the *Elements* and Sec.4 of Vol.1 of *SfYbSLJY*.

II.9. SECTION 11

II.9.1. Romanized Manchu Texts

juwan emuci. /
yaya amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro geren ajige ton be acabufi. ede ton -i fulehe -i / emu (*serengge*) be nonggifi. banjinaha uheri ton. kemnebure da amba ton de lak seme tehereci. ere da / amba ton be. yongkiyaha ufihi -i ton sembi:
duibuleci. da amba ton ninggun oci. / geren ajige ton juwe ocibe. ilan ocibe. gemu ninggun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ome ofi. / ere [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) juwe sere ilan serengge de. ton -i fulehe -i emu be nonggici. / bahara ninggun. da amba ton de tehereme ofi. ere ninggun be. uthai yongkiyaha / ufihi -i ton sembi:
geli duibuleci. da amba ton orin jakūn oci. geren ajige ton / juwe. duin. nadan. juwan duin be baitalafi. gemu orin jakūn be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ome (f.22a)// ofi. ere [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) juwe. duin. nadan. juwan duin de. ton -i fulehe -i emu be / nonggici. bahara orin jakūn da amba ton de tehereme ofi. ere orin jakūn be. / uthai yongkiyaha ufihi -i ton sembi: (f.22b)//

II.9.2. Translation

The eleventh,
⟨Suppose that⟩ we combine many small numbers which can measure (*and exhaust*) an arbitrary large number [(*exhaustively*)] [exactly] and ⟨that⟩ we add the unit, which is the

root of numbers, to the result, and if the sum which is produced ⟨in this process⟩ is exactly equal to the original large number which was measured ⟨in the same process⟩, then this original number is called a number with a complete quantity.

For example, if the original number is 6, then the many small numbers, one of which is either 2 or 3, can measure (*and exhaust*) 6 [(*exhaustively*)] [exactly], so a number 6, which is obtained by adding 1, the root of numbers, to ⟨the sum of⟩ these numbers 2 and 3, both of which measure (*and exhaust*) the original large number [(*exhaustively*)] [exactly], is equal to the original large number, and we therefore say that this number 6 is just a number with a complete quantity.

Moreover, if the original large number is 28, then we will use many small numbers such as 2 and 4 and 7 and 14, and be able to measure (*and exhaust*) 28 [(*exhaustively*)] [exactly] with all of them, so a number 28, which is obtained by adding the unit, the root of numbers, to ⟨the sum of⟩ these numbers 2 and 4 and 7 and 14, all of which measure (*and exhaust*) the original large number [(*exhaustively*)] [exactly], is equal to the original large number, and we therefore say that this number 28 is just a number with a complete quantity.

II.9.3. Notes

(f.22a, l.4) ufihi

In many Manchu dictionaries, a Manchu noun 'ufihi' is explained as 'a part' or 'a portion' or 'a share' and it is difficult to distinguish it from another Manchu noun 'ubu'.

On the other hand, in *GTCL*, there is a distinct difference between the meanings of these two words. The noun 'ubu' means 'a component' or 'components', as is seen from one of its examples 'tuwa –i ubu', which means 'a component consisting of the element of fire (in human blood)' in the sections dedicated to feverish diseases. The noun 'ufihi' means 'the quantity of a concrete thing or of its part', as is seen in the same sections.

In this manuscript the noun 'ufihi' appears six times. The three of them are found in this section and form a component of a noun phrase 'yongkiyaha ufihi –i ton', which means 'a perfect number'. Other three examples are found in Section 55, and all of them mean the values of the area of certain regular squares.

It seems probable that the concrete size of parts of natural numbers, which is the quantitative side of the notion of a part, was treated in this section and hence the noun 'ufihi' was selected here.

II.9.4. Remarks

This section contains the definitions of a perfect number. It corresponds to Def.23 in Book VII of the *Elements*. In *SLJY*, this definition was omitted.

II.10. SECTION 15

II.10.1. Romanized Manchu Texts

tofohoci. /

teherehekū juwe ton bifi. ere juwe ton be. [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) ojoro ton bisire akū be bairengge:

juwe da ton -i / dorgi ajige ton -i songkoi. amba ton be wacihiyame ekiyembufi. / funcehe ton be ajige ton obufi. da ajige ton be geli / wacihiyame ekiyembume. ere songkoi forgošome emdubei ishunde / wacihiyame ekiyembuhei emu de isitala. nenehe ton be [(*wacihiyame*)] [lak seme] / kemne[ci](*me wacihiyaci*) ojoro ton gemu akū oci. tere da amba / ajige juwe ton be. emu (*sere*) ci tulgiyen ede tede [lak] [(*wacihiyame*)] (f.26a)// [seme] kemne[ci](*me wacihiyaci*) ojoro ton akūngge sembi:

duibuleci. giya i sere juwan emu. bing / ding sere nadan. ere teherehekū juwe ton bifi. ere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / ojoro ton bisire akū be baiki seci. bing ding sere nadan -i songkoi. giya i / sere juwan emu be ekiyembuci. ding i sere duin funcembi: ere duin -i songkoi. / geli bing ding sere nadan be ekiyembuci. u ding sere ilan funcembi: ere ilan -i / songkoi. geli i ding sere duin be ekiyembuci. gi i sere emke funcembi: uttu / emdubei forgošome. ishunde ekiyembuhei. geren ton -i dorgi emu de isinaha manggi. nenehe ton be / [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton akū ofi. / {ai turgunde [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorakū seci. ilan serengge. / duin be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorakū. duin serengge. nadan be [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*) ojorakū. nadan serengge juwan / emu be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorakū ofi kai:} giya i. bing ding sere juwe teherehekū ton be. (f.26b)// uthai emu ci tulgiyen [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton akūngge sembi: aikabade emu ci / tulgiyen. giya i. bing ding sere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton bi seci. / cendeme geng sere ton be arafi kemneme tuwa: geng serengge aikabade giya i. bing / ding sere juwe ton be kemneme muteci. bing ding daci giya ding ni gese be dahame. / geng serengge inu urunakū giya ding sere ton be [(*wacihiyame*)] [lak seme] kemneme (*wacihiyame*) mutembi: giya ding be / kemneme muteci. geli da giya i be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ome ofi. julergi meyen -i songkoi. / ding i be inu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) acambi: ding i be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*). urunakū / ding i de teherere bing u be. [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: bing u be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / geli da bing ding be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ome ofi. julergi meyen -i songkoi inu u ding be (f.27a)// [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) acambi: u ding be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. inu urunakū / u ding de teherere ding gi be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: ding gi be [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*) oci. geli neneme ding i be

[(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ohobi: ding i. / ding gi be kemneci oci. julergi meyen -i songkoi gi i be inu [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) ombi: ere gi i emu (*serengge*) [ofi kai]: emu oci. tere [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) ojoro geng serengge. inu emu: ton (*de dosirak*ū) [waka]: ton (*de dosirak*ū) [waka] oci. / emke ci tulgiyen. ede tede [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akū be saci ombi: /
geli duibuleci. giya i sere juwan jakūn. bing ding sere sunja. ere teherehekū / juwe ton bifi. ere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton bisire akū be (f.27b)// baiki seci. bing ding sere sunja -i songkoi. giya i sere / juwan jakūn be wacihiyame ekiyembufi. funcehe ilan ding / i ombi: {ainu ilan funcembi seci. juwan jakūn be dahime ilan mudan ekiyembuci. / urunakū tofohon ekiyere be dahame. tuttu ilan funcembi:} / ere ilan -i songkoi. da bing ding sere sunja be geli / ekiyembuci. ding gi sere juwe funcembi. ere juwe -i / songkoi ding i sere ilan be geli ekiyembuci. gi i / sere emke funcembi: ere songkoi emdubei forgošome geren / ton -i dorgi be ekiyembume genehei. emke de isitala. / julergi ton be. yooni [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton akū (f.28a)// ofi. ere giya i. bing ding sere juwe da ton be. uthai emke ci tulgiyen. ede tede / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akūngge sembi: [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akū turgun. gemu julergi leolen -i / adali: (f.28b)//

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The figure in f.26a

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        gi
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The figure in f.28a

II.10.2. Translation

The fifteenth,

⟨Suppose that⟩ we have two unequal numbers. To seek whether a ⟨nontrivial⟩ number which can measure (*and exhaust*) these two numbers [(*exhaustively*)] [exactly] exists or not ⟨is as follows⟩.

Having decreased the number which is larger between the two original numbers by the ⟨other⟩ smaller number ⟨repeatedly and⟩ exhaustively, we set a ⟨new⟩ small number to the number remained ⟨in this process⟩, and ⟨having finished this resetting⟩ we decrease the original smaller number ⟨by this new small number repeatedly and⟩ exhaustively again, and following this ⟨procedure⟩, we continue changing the roles of numbers and decreasing them one by another ⟨repeatedly and⟩ exhaustively at a steady pace, and during this process, if we have no ⟨nontrivial⟩ number which can measure (*and exhaust*) the former numbers [(*exhaustively*)] [exactly] at all until the number 1 is attained, then those original two numbers, the larger one and the smaller one, are called what no numbers other than (*the unit*) [1] can measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there.

For example, if we have two unequal numbers Jiya-Yi and Bing-Ding, which are equal to 11 and 7 respectively, and want to seek whether a number which can measure (*and exhaust*) these two numbers [(*exhaustively*)] [exactly] exists or not, ⟨the procedure is as follows;⟩ if we decrease Jiya-Yi, which is equal to 11, by Bing-Ding, which is equal to 7, then Ding-Yi, which is equal to 4, remains. Moreover, if we decrease Bing-Ding, which is equal to 7, by this number 4, then Wu-Ding, which is equal to 3, remains. Moreover, if we decrease Yi-Ding, which is equal to 4, by this number 3, then Ji-Yi, which is equal to 1, remains. After the number 1 was attained as one of the many numbers while we were changing the roles of numbers and decreasing them one by another at a steady pace in this manner, we have no ⟨nontrivial⟩ number which can measure (*and exhaust*) the former numbers [(*exhaustively*)] [exactly], {Concerning the details of why we cannot measure (*and exhaust*) them [(*exhaustively*)] [exactly], ⟨the measurements is imposible⟩ because the number 3 cannot measure (*and exhaust*) 4 [(*exhaustively*)] [exactly], and the number 4 cannot measure (*and exhaust*) 7 [(*exhaustively*)] [exactly], and the number 7 cannot measure (*and exhaust*) 11 [(*exhaustively*)] [exactly].} hence we say that the two unequal numbers Jiya-Yi and Bing-Ding are just what no numbers other than 1 can measure (*and exhaust*) [(*exhaustively*)] [exactly]. If we assume that there is a number other than 1 which can measure (*and exhaust*) the two numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly], denote this number by Geng and try to measure the two number with it. If Geng can measure the two numbers Jiya-Yi and Bing-Ding, then Geng can also [(*exhaustively*)] [exactly] measure (*and exhaust*) the number Jiya-Ding certainly, because Bing-Ding is equal to Jiya-Ding from the beginning. If Geng can measure Jiya-Ding, then ⟨Geng⟩ can [(*exhaustively*)] [exactly] measure (*and exhaust*) the original number Jiya-Yi as well, and hence ⟨Geng⟩ should also measure (*and exhaust*) Ding-Yi [(*exhaustively*)] [exactly], according to the previous section. If ⟨Geng⟩ measures (*and exhausts*) Ding-Yi, then ⟨Geng⟩ can [(*exhaustively*)] [exactly] measure (*and exhaust*) Bing-Wu, which is equal to Ding-Yi, certainly. If ⟨Geng⟩ measures (*and exhausts*) Bing-Wu [(*exhaustively*)] [exactly], then ⟨Geng⟩ can [(*exhaustively*)] [exactly] measure (*and exhaust*) the original Bing-Ding as well, and hence ⟨Geng⟩ should also measure (*and exhaust*) Wu-Ding [(*exhaustively*)] [exactly], according to the previous section. If ⟨Geng⟩ can measure (*and exhaust*) Wu-Ding [(*exhaustively*)] [exactly], then ⟨Geng⟩ can also [(*exhaustively*)] [exactly] measure (*and exhaust*) Ding-Ji, which is equal to Wu-Ding, certainly. If ⟨Geng⟩ can measure (*and exhaust*) Ding-Ji [(*exhaustively*)] [exactly], then from the beginning ⟨Geng⟩ can [(*exhaustively*)] [exactly] measure (*and exhaust*) Ding-Yi as well. If ⟨Geng⟩ measures Ding-Yi and Ding-Ji, ⟨Geng⟩ can also measure (*and exhaust*) Ji-Yi [(*exhaustively*)] [exactly], according to the previous section. [It is the case because this] (*This*) Ji-Yi is equal to (*the unit*) [1]. If ⟨Ji-Yi⟩ is equal to 1, the number Geng, which can measure (*and exhaust*) ⟨Ji-Yi⟩ [(*exhaustively*)] [exactly], is also equal to 1. ⟨Geng⟩ (*does not belong to the class of numbers*) [is not a number]. If ⟨Geng⟩ (*does not belong to the class of numbers*) [is not a number], we can see that the original two numbers are what no numbers other than 1 can measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there.

Moreover, for example, ⟨suppose that⟩ we have two unequal numbers Jia-Yi and Bing-Ding, which are equal to 18 and 5 respectively, and if we want to seek whether a number which can measure (*and exhaust*) these two numbers [(*exhaustively*)] [exactly] exists or not, ⟨the proceedure is as follows;⟩ if we decrease Jiya-Yi, which is equal to 18, by Bing-Ding, which is equal to 5, ⟨repeatedly and⟩ exhaustively, then the number 3, which remained ⟨in this process⟩ is the value of Ding-Yi. {Concerning why the number 3 remains, if we decrease 18 ⟨by 5⟩} three times over, then we certainly subtract 15 ⟨from 18⟩, so it follows that 3 remains.} If we decrease the original Bing-Ding, which is equal to 5, by this number 3, then Ding-Ji, which is equal to 2, remains. If we decrease Ding-Yi, which is equal to 3, by this number 2, then Ji-Yi, which is equal to 1, remains. Since there is no ⟨nontrivial⟩ number which can measure (*and exhaust*) all of the former numbers [(*exhaustively*)] [exactly] until the number 1 is attained, while we are changing the roles of numbers in the many numbers and decreasing them at a steady pace by following this ⟨procedure⟩, we say that these two original numbers Jiya-Yi and Bing-Ding are just what no numbers other than 1 can measure (*and exhaust*) [(*exhaustively*)] [exactly] here and there. The details of the lack of numbers which measure (*and exhaust*) them [(*exhaustively*)] [exactly] are the same as in the previous arguments.

II.10.3. Notes

(f.26a, ll.2-3) teherehekū … bairengge:

This section explains an algorithm due to Euclid, which gives a way to determine whether two given integers have a nontrivial common divisor or not. The first paragraph of this section plays a role of the heading of this section, and in the original Manchu text it contains only one sentence consisting of a noun phrase, which is constructed from the following complex sentence

(a) teherehekū juwe ton bifi. ere juwe ton be. [lak seme] [(*wacihiyame*)] kemne[ci](*me wacihiyaci*) ojoro ton bisire akū be baimbi:
We have two unequal numbers and seek whether a ⟨nontrivial⟩ number which can *measure and exhaust* these two numbers exists or not.

by replacing the non-perfective finite suffix '-mbi' in the Manchu verb 'baimbi' with the combination of an imperfect participle suffix '-re' and a nominalizer suffix '-ngge'. The first half of the sentence (a), 'teherehekū … bifi', gives the assumption of the algorithm in this section, and its second half 'ere … baimbi' gives the purpose of the algorithm. It is difficult to translate this single Manchu noun phrase into a single English noun phrase because of the difference between the syntax of the two languages. So in the above translation we divide this Manchu noun phrase into two English sentences, the first one is an imperative sentence, which gives the assumption of the algorithm, and the second one is an infinitive phrase, which gives the purpose of the algorithm. Throughout this manuscript, all of the first paragraphs in the sections dedicated to algorithms have the same structure as described above, and they will be translated into English in the same way.

(f.26a, l.4) wacihiyame ekiyembufi
In this manuscript, subtraction is usually expressed by the following phrases containing a Manchu verb 'ekiyembu-':

(a) 'Q_1 -i dorgici Q_2 be ekiyembumbi' ('to subtract Q_2 from Q_1'),
(b) 'Q_1 –i songkoi Q_2 be ekiyembumbi' ('to decrease Q_2 by Q_1').

The result of (a) is given by $Q_1 - Q_2$, and that of (b) is given by $Q_2 - Q_1$. Note that the role of the noun accompanied by the Manchu case suffix 'be' is different.

(f.27a, l.6) julergi meyen
Section 14 of this manuscript deals with the following

PROPOSITION. Let a, b, c be natural numbers satisfying $b \neq c$. Assume that the number a measures two numbers b and $b+c$. Then the number a also measures the number c.

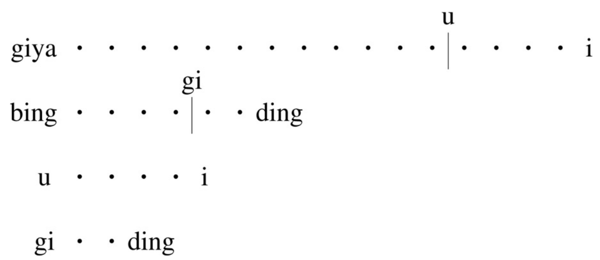
II.10.4. Remarks

This section deals with an algorithm for deciding whether two unequal natural numbers are relatively prime or not. The Euclidean algorithm is given here. The contents of this section are found in Prop.1 in Book VII of the *Elements* and Sec.18 of Vol.1 of *SfYbSLJY*.

II.11. SECTION 16

II.11.1. Romanized Manchu Texts

juwan ningguci. /
ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro teherehekū juwe amba / ton bifi. ere ajige ton be bairengge:
duibuleci. ajige / ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro. giya i sere juwan ninggun. / bing ding sere ninggun -i teherehekū juwe amba ton bifi. ere be / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ajige ton be baiki seci. bing ding sere ninggun -i / songkoi. giya i sere juwan ninggun be wacihiyame ekiyembu. duin / funcefi u i ombi: {ainu duin funcembi seci. giya i sere juwan ninggun -i / dorgide. bing ding sere ninggun -i juwe mudan bisire / turgunde. tuttu juwan juwe be / ekiyembuci uthai duin funcembi:} ere u i sere duin -i songkoi. bing (f.29a)// ding sere ninggun be ekiyembu. juwe funcefi. gi ding ombi: ere gi ding sere juwe -i songkoi. / u i sere duin be geli ekiyembu. uthai funcerengge akū ofi. ere gi ding serengge. giya i. / bing ding sere da amba ton be. [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton ombi: adarame seci. gi ding ni songkoi. / u i be wacihiyame ekiyembufi. funcehengge akū be dahame. gi ding serengge. u i be urunakū / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) be kenehunjere be akū: [(*wacihiyame*)] [lak seme] kemne[rengge](*me wacihiyarangge*) oci. u i daci bing gi -i / gese be dahame. gi ding inu bing gi be urunakū [(*wacihiyame*)] [lak seme] kemne[mbime](*me wacihiyambime*). geli beyebe [lak seme] [(*wacihiyame*)] / kemne[mbi](*me wacihiyambi*): uttu bing gi. gi ding sere juwe meyen be [(*wacihiyame*)] [lak seme] kemne[rengge](*me wacihiyarangge*) ofi. ere fiyelen -i / juwan juweci meyen -i songkoi. gi ding urunakū bing ding ni uheri ton be [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) ombi: julergi de bing ding ni songkoi. giya u be juwe mudan ekiyembuhe be dahame. (f.29b)// ere bing ding urunakū giya u be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): giya u be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*). gi ding. / geli bing ding be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) be dahame. [inu] ere fiyelen -i juwan ilaci meyen -i songkoi. / gi ding. inu urunakū giya u be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): giya u. u i sere juwe meyen be [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*). geli ere fiyelen -i juwan juweci meyen -i songkoi. tere gi ding. inu urunakū / giya i -i uheri ton be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): ere songkoi oci. gi ding. uthai giya i / bing ding sere juwe da amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro be saci ombi: (f.30a)//



The figure in f.29a

II.11.2. Translation

The sixteenth,

⟨Suppose that⟩ we have two unequal large numbers which can be measured (*and exhausted*) with a small number [(*exhaustively*)] [exactly]. To seek this small number ⟨is as follows⟩.

For example, ⟨suppose that⟩ we have two unequal large numbers Jiya-Yi and Bing-Ding, which are equal to 16 and 6 respectively and can be measured (*and exhausted*) with a small number [(*exhaustively*)] [exactly], and if we want to seek this small number, decrease Jiya-Yi, which is equal to 16, by Bing-Ding, which is equal to 6, ⟨repeatedly and⟩ exhaustively. Then 4 remains and ⟨this 4⟩ is equal to Wu-Yi. ⟨See the figure.⟩ {Concerning why 4 remains, since we have twice Bing-Ding, which is twice 6, in Jiya-Yi, which is equal to 16, ⟨the number which we have to subtract from 16 is 12, and⟩ if we subtract 12 from Jiya-Yi, then 4 remains.} Decrease Bing-Ding, which is equal to 6, by this Wu-Ding, which is equal to 4, then 2 remains and ⟨this 2⟩ is equal to Ji-Ding. ⟨See the figure.⟩ Moreover, decrease Wu-Yi, which is equal to 4, by this Ji-Ding, which is equal to 2. Then there is no remainder, and it follows that this Ji-Ding is a number which can measure (*and exhaust*) the original large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly]. Concerning how it occurs, since nothing remains after we have decreased Wu-Yi by Ji-Ding ⟨repeatedly and⟩ exhaustively, we have no doubt that Ji-Ding certainly measures (*and exhausts*) Wu-Yi [(*exhaustively*)] [exactly]. If Ji-Ding is that which measures (*and exhausts*) Wu-Yi [(*exhaustively*)] [exactly], Ji-Ding also [(*exhaustively*)] [exactly] measures (*and exhausts*) Bing-Ji certainly and, at the same time, ⟨Ji-Ding⟩ measures (*and exhausts*) itself [(*exhaustively*)] [exactly], because from the beginning Wu-Yi is equal to the same number as Bing-Ji is equal to. So ⟨Ji-Ding⟩ is that which measures (*and exhausts*) the two line segments Bing-Ji and Ji-Ding [(*exhaustively*)] [exactly], and according to the twelfth section of this chapter, Ji-Ding can [(*exhaustively*)] [exactly] measure (*and exhaust*) the sum Bing-Ding certainly. Since we decreased Jiya-Wu by Bing-Ding twice before, this Bing-Ding [(*exhaustively*)] [exactly] measures (*and exhausts*) Jiya-Wu certainly. If ⟨Bing-Ding⟩ measures (*and exhausts*) Jiya-Wu [(*exhaustively*)] [exactly], Ji-Ding [(*exhaustively*)] [exactly] measures (*and exhausts*) Bing-Ding as well, therefore Ji-Ding also [(*exhaustively*)] [exactly] measures (*and exhausts*) Jiya-Wu certainly, according to the thirteenth section of this chapter. If it measures the two line segments Jiya-Wu and Wu-Yi, then according to the twelfth section of this chapter again, that number Ji-Ding also [(*exhaustively*)] [exactly] measures (*and exhausts*) the sum Jiya-Yi certainly. In this way we can see that Ji-Ding can measure (*and exhaust*) the two original large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly].

II.11.3. Notes

(f.29b, ll.7-8) ere fiyelen -i juwan juweci meyen

Section 12 of this manuscript deals with the following

PROPOSITION. Let $a, b, c, \cdots$ be arbitrary natural numbers. Then every common divisor of the numbers $a, b, c, \cdots$ is a divisor of the sum $a+b+c+\cdots$.

This proposition corresponds to a part of Sec.15 of Vol.1 of *SfYbSLJY*.

(f.30a, l.2) ere fiyelen -i juwan ilaci meyen

Section 13 of this manuscript deals with the following

PROPOSITION. Let a, b, c be arbitrary natural numbers. Assume that the number a measures the numbers b and the numbers b measures the number c. Then the number a also measures the number c.

II.11.4. Remarks

This section deals with an algorithm for finding a nontrivial common divisor of two unequal natural numbers when these two numbers are not relatively prime. The Euclidean algorithm is used here. The contents of this section found in Prop.2 in Book VII of the *Elements* and Sec.17 of Vol.1 of *SfYbSLJY*.

II.12. SECTION 17

II.12.1. Romanized Manchu Texts

juwan nadaci. /

geren ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro teherehekū juwe / amba ton bifi. ere geren ajige ton -i dorgi amba ton be / bairengge:

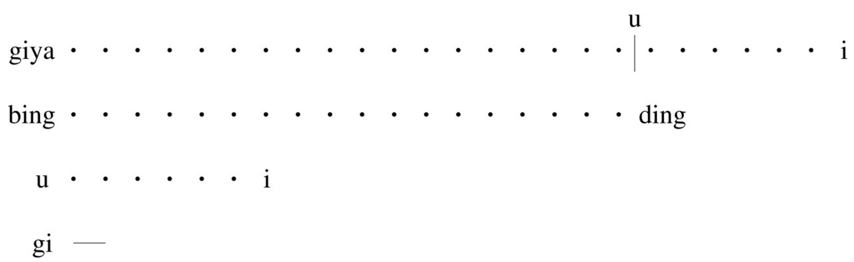
julergi meyen -i songkoi emdubei wacihiyame ekiyembu: / julergi ton be bahafi ekiyembume wacihiyara ton. uthai / geren ajige ton -i dorgi amba ton ombi:

duibuleci. / ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro giya i sere / orin duin. bing ding sere juwan jakūn -i teherehekū juwe / amba ton bifi. ere be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i (f.31a)// dorgi amba ton be baiki seci. giya i sere orin duin be bing ding sere juwan / jakūn -i songkoi wacihiyame ekiyembu. ninggun funcefi. u i ombi: ere ninggun -i songkoi. bing ding / sere juwan jakūn be geli wacihiyame ekiyembu. uthai ekiyembume wajifi funcerengge akū ombi: / funcerengge akū ofi. tere u i sere ninggun. uthai juwe da amba ton be [(*wacihiyame*)] [lak seme] / kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba ton ombi: adarame seci. geren ajige ton -i dorgi de. / ere giya i. bing ding sere juwe amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ninggun ci amba / ton akū ofi: aikabade ere giya i bing ding sere juwe amba ton be [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) ojoro u i sere ninggun ci amba ton bi seci. taka gi sere be arafi / cendeme kemne. aikabade gi serengge. bing ding be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. urunakū (f.31b)// bing ding de teherere giya u be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: giya u be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) / geli giya i sere ton be daci [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) oci. ere fiyelen -i juwan duici meyen -i / songkoi. inu urunakū u i be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: u i be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / oci. gi serengge. urunakū u i ci amba akū be saci ombi:

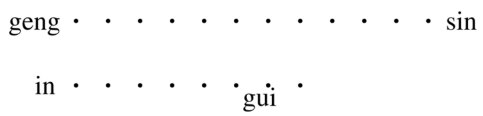
aikabade da / teherehekū juwe ton -i dorgi. amba ton be ajige ton -i songkoi wacihiyame ekiyembure de. / ekiyembume wacihiyafi funcerengge akū oci. ere ajige ton. uthai da teherehekū juwe / ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba ton ombi:

duibuleci. geng / sin sere juwan jakūn. žin gui sere ninggun. teherehekū juwe da ton bifi. ere / ninggun -i songkoi. tere juwan jakūn be wacihiyame ekiyembure de. ekiyembume (f.32a)// wacihiyafi. funcerengge akū be dahame. ere ninggun / uthai geng sin. žin gui sere juwe da ton be / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba ton / ombi:

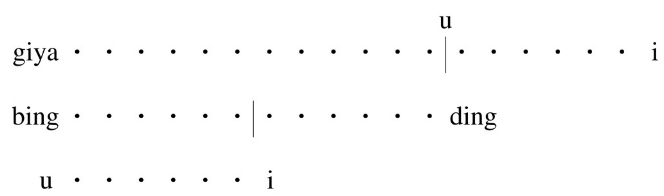
ere meyen -i leolen. julergi meyen -i turgun -i / emu adali: (f.32b)//



The figure in 31a



The figure in f.32a



The figure in f.33a

II.12.2. Translation

The seventeenth,

⟨Suppose that⟩ we have two unequal large numbers which can be measured (*and exhausted*) with many small numbers [(*exhaustively*)] [exactly]. To seek the largest number among these many small numbers ⟨is as follows⟩.

According to the previous section, decrease the numbers ⟨repeatedly and⟩ exhaustively at a steady pace. Then the number with which we can exhaust the former numbers by subtraction is just the largest number of these many small numbers.

For example, ⟨suppose that⟩ we have two unequal large numbers Jiya-Yi and Bing-Ding, which are equal to 24 and 18 respectively and can be measured (*and exhausted*) with a small number, and if we want to seek the largest number among the many small numbers which measure (*and exhaust*) these ⟨two numbers⟩ [(*exhaustively*)] [exactly], decrease Jiya-Yi, which is equal to 24, by Bing-Ding, which is equal to 18, ⟨repeatedly and⟩ exhaustively. Then 6 remains and ⟨this 6⟩ is equal to Wu-Yi. ⟨See the figure.⟩ Decrease Bing-Ding, which is equal to 18, by this number 6 in addition, then there is no remainder after we have finished decreasing. Since there is no remainder, that number Wu-Yi, which is equal to 6, is just the the largest number among these many small numbers which measure (*and exhaust*) the two original numbers ⟨Jiya-Yi and Bing-Ding⟩ [(*exhaustively*)] [exactly]. Concerning how it occurs, ⟨it is the case⟩ because among the many small numbers there is no ⟨nontrivial⟩ number which is larger than 6 and can measure and exhaust these two numbers Jiya-Yi and Bing-Ding. If we assume that there is a number which can measure (*and exhaust*) these two large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly] and is larger than Wu-Yi, which is equal to 6, then denote this number by Ji temporarily and try to measure ⟨Jiya-Yi and Bing-Ding with it⟩. If Ji can measure (*and exhaust*) Bing-Ding [(*exhaustively*)] [exactly], then it can [(*exhaustively*)] [exactly] measure (*and exhaust*) Jiya-Wu, which is equal to Bing-Ding, certainly. If Ji can measure (*and exhaust*) Jiya-Wu [(*exhaustively*)] [exactly] and, in additon, if Ji can measure (*and exhaust*) the number Jiya-Yi [(*exhaustively*)] [exactly] at the beginning, according to the fourth section of this chapter, it can also [(*exhaustively*)] [exactly] measure (*and exhaust*) Wu-Yi certainly. If ⟨Ji⟩ can measure (*and exhaust*)

Wu-Yi [(*exhaustively*)] [exactly], we can see that the number Ji is certainly not larger than Wu-Yi.

When we decrease the number which is larger of the two original unequal numbers by the other smaller number ⟨repeatedly and⟩ exhaustively, if there is no remainder after we have finished decreasing and exhausting, this small number is just the largest number of these many small numbers which measure (*and exhaust*) the two original unequal numbers [(*exhaustively*)] [exactly].

For example, ⟨suppose that⟩ we have two unequal original numbers Geng-Xin and Ren-Gui, which are equal to 18 and 6 respectively, and when we decrease that number 18 by this number 6 ⟨repeatedly and⟩ exhaustively, there is no remainder after we have finished decreasing and exhausting it, so it follows that this number 6 is just the largest number of many small numbers which measure (*and exhaust*) the two original numbers Geng-Xin and Ren-Gui [(*exhaustively*)] [exactly].

The arguments in this section are the same as those on the situations in the previous section.

II.12.3. Remarks

This section deals with an algorithm for finding the greatest common divisor of two unequal natural numbers when these two numbers are not relatively prime. As in the previous section, the Euclidean algorithm is used here. The contents of this section are found in Prop.2 in Book VII of the *Elements*.

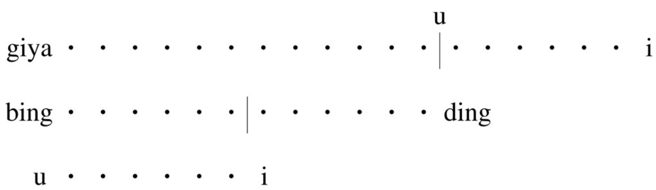
II.13. SECTION 18

II.13.1. Romanized Manchu Texts

juwan jakūci. /

yaya tehereheku̅ juwe amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro geren ajige / ton bici. ere geren ajige ton -i dorgi yaya emke. inu urunakū ere / jergi ajige ton -i dorgi amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: /

duibuleci. giya i sere juwan jakūn. bing ding sere juwan juwe -i / tehereheku̅ juwe amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro. geren ajige / ton -i dorgi juwe bici. ere ajige ton -i juwe. inu urunakū ere / jergi geren ajige ton -i dorgi amba ton be kemneci ombi: adarame / seci. giya i sere juwan jakūn -i dorgici. bing ding sere juwan (f.33a)// juwe be ekiyembuci. funcehe ninggun u i ombi: geli u i sere ninggun -i songkoi. bing ding sere / juwan juwe be wacihiyame ekiyembuci. uthai ekiyembume wacihiyafi. funcerengge akū ombi: funcerengge / akū oci. julergi meyen -i songkoi. [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ajige ton -i dorgi ninggun sere amba ton be / bahambi. te julergi de ajige ton -i juwe. giya i. bing ding sere juwe amba ton be kemneci / ombi sehebi: juwe amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me *wacihiyaci*) oci. aikabade giya i dorgici bing ding be / ekiyembuci funcerengge u i ofi. tere juwe serengge. ere fiyelen -i juwan duici meyen -i songkoi. / inu urunakū u i -i ninggun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: ere songkoi oci. juwe amba ton be / gemu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro yaya ajige ton bici. ere ajige ton. inu urunakū ere jergi geren ajige / ton -i dorgi amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro be saci ombi: (f.33b)//



The figure in f.33a

II.13.2. Translation

The eighteenth,

If there are many small numbers which can measure (*and exhaust*) two arbitrary unequal numbers [(*exhaustively*)] [exactly], then an arbitrary one among these many small numbers can also [(*exhaustively*)] [exactly] measure (*and exhaust*) the largest number among the small numbers of this kind certainly.

For example, if we have a number 2 among the many small numbers which can [(*exhaustively*)] [exactly] measure (*and exhaust*) two unequal large numbers Jiya-Yi and Bing-Ding, which are equal to 18 and 12 respectively, then this small number 2 can also [(*exhaustively*)] [exactly] measure (*and exhaust*) the largest number among the small numbers of this kind certainly.

Concerning how it occurs, if we subtract Bing-Ding, which is equal to 12, from Jiya-Yi, which is equal to 18, we have Wu-Yi, which is equal to 6. ⟨See the figure.⟩ If we decrease Bing-Ding, which is equal to 12, by Wu-Yi, which is equal to 6 ⟨repeatedly and⟩ exhaustively, then we have no remainder after we have decreased it exhaustively. If there is no remainder, then according to the previous section we obtain a large number 6 among the many numbers which measure (*and exhaust*) them [(*exhaustively*)] [exactly]. Now ⟨let us recall that⟩ we said before that the small number 6 could measure (*and exhaust*) two large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly]. When ⟨this 6⟩ can measure (*and exhaust*) the two large numbers [(*exhaustively*)] [exactly], if we subtract Bing-Ding from Wu-Yi, then the remainder is Wu-Yi, and the number 2 can also [(*exhaustively*)] [exactly] measure (*and exhaust*) Wu-Yi, which is equal to 6, certainly, according to the fourteenth section of this chapter. In this way we can see that if we have arbitrary small numbers which can measure (*and exhaust*) both of the two large numbers [(*exhaustively*)] [exactly] then these small numbers can also [(*exhaustively*)] [exactly] measure (*and exhaust*) the largest number of the small numbers of this kind certainly.

II.13.3. Remarks

This section deals with the following

PROPOSITION. Suppose that two unequal natural numbers are given. Then every common divisor of them measures the greatest common divisor of them.

II.14. SECTION 19

II.14.1. Romanized Manchu Texts

juwan uyuci. /

ilan ton be bifi. ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro ojorakū be / bairengge:

ere fiyelen -i tofohoci meyen -i songkoi. ere ilan ton -i / dorgi juwe ton be. [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ajige ton bisire akū be / baica. aikabade [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorongge akū oci. uthai ere / fiyelen -i ilaci meyen -i songkoi. ere ilan ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) / ton akū ombi: aikabade ere juwe ton be. ajige ton -i [(*wacihiyame*)] [lak seme] / kemne[ci](*me wacihiyaci*) oci. ere fiyelen -i juwan nadaci meyen -i songkoi. ere jergi / geren ajige ton -i dorgi amba ton be baica: geli ere ton. (f.34a)// jai da ilaci ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ajige ton bisire akū be baica: aikabade [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*) ojoro ton akū oci. ere ilan ton ajige ton de kemnebuci ojorakū ton ombi: / aikabade ere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ajige ton bici. ere da ilan ton. ajige ton de / [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro ton ombi:

duibuleci. giya i sere juwan juwe. bing ding sere orin. / u gi sere orin sunja -i jergi teherehekū ilan ton bifi. ere ilan ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro / ajige ton bisire akū be baiki seci. ere fiyelen -i tofohoci meyen -i songkoi. giya i sere / juwan juwe. bing ding sere orin. ere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ajige ton bisire / akū be baica: baha duin uthai [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba ton ombi: / geli duin sere. orin sunja sere juwe ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ajige ton bisire akū be baicara de. (f.34b)// duin -i songkoi orin sunja be wacihiyame ekiyembuci emke funcembi: emke funceci. uthai da ilan / ton be. emke ci tulgiyen. [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akū ombi: adarame seci. aikabade giya i sere / juwan juwe , bing ding sere orin. ere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ajige ton bici. julergi / meyen -i songkoi. inu urunakū geren ajige ton -i dorgi amba ton -i duin be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombihe: / te duin be. jai u gi -i orin sunja be

[(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ajige ton akū be dahame. uthai / giya i. bing ding. u gi sere ilan ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ajige ton akū ombi:
ere arga be / baitalaci. teherehekū duin ton. sunja ton. jai hacingga teherehekū ton. ajige ton de [(*wacihiyame*)] [lak seme] / kemne[buci](*me wacihiyabuci*) ojoro ojorakū be inu saci ombi:
duibuleci. orin duin. gūsin. gūsin uyun. dehi / uyun sere duin ton bifi. erebe [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton bisire akū be baiki seci. (f.35a)// ere fiyelen -i tofohoci meyen -i songkoi. orin duin. gūsin sere juwe ton be. ede tede ishunde / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton be baicaci. ninggun bahambi: geli ere meyen -i songkoi. ninggun sere (*gūsin*) [orin] uyun / sere be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton be baicaci. ilan bahambi. geli ilan sere. dehi uyun sere be [lak] [(*wacihiyame*)] / [seme] kemne[re](*me wacihiyara*) ton bisire akū be baicame. ilan -i songkoi dehi uyun be wacihiyame ekiyembuci emke funcembi. / emke funceci. ere duin ton uthai ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton ombi: erebe / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton adarame akū sere turgun. julergi leolen -i emu adali: (f.35b)//

giya · · · · · · · · · · · · · i

bing · ding

u · gi

The figure in f.34a

II.14.2. Translation

The ninteenth,

⟨Suppose that⟩ we have three numbers. To seek whether they can be [(*exhaustively*)] [exactly] measured (*and exhausted*) with a small number ⟨is as follows⟩.

According to the fifteenth section of this chapter, examine whether a small ⟨nontrival⟩ number which can measure (*and exhaust*) two numbers of these three numbers [(*exhaustively*)] [exactly] exists or not. If we do not have that which can measure (*and exhaust*) them [(*exhaustively*)] [exactly], then according to the third section of this chapter there is no number which can measure (*and exhaust*) these three numbers [(*exhaustively*)] [exactly]. If we can measure (*and exhaust*) these two numbers with a small number [(*exhaustively*)] [exactly], then according to the seventeenth section of this chapter, examine the large⟨st⟩ number among the many small numbers of this kind. Moreover, examine whether a small ⟨nontrivial⟩ number which can [(*exhaustively*)] [exactly] measure (*and *exhaust*) ⟨firstly⟩ this number, and secondly the original third number, exists or not. If there is no small ⟨nontrivial⟩ number which can measure (*and exhaust*) them [(*exhaustively*)] [exactly], then these three number are numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any small ⟨nontrivial⟩ number. If there is a small ⟨nontrivial⟩ number which can measure (*and exhaust*) these two numbers [(*exhaustively*)] [exactly], then these three original numbers are numbers which can be measured (*and exhausted*) with a small ⟨nontrivial⟩ number [(*exhaustively*)] [exactly].

For example, ⟨suppose that⟩ we have three unequal numbers such as Jiya-Yi and Bing-Ding and Wu-Ji, which are equal to 12 and 20 and 25 respectively, and if we want to seek whether a small ⟨nontrival⟩ number which can [(*exhaustively*)] [exactly] measure (*and exhaust*) these three numbers exists or not, then according to the fifteenth section of this chapter, examine whether a small ⟨nontrival⟩ number which can [(*exhaustively*)] [exactly] measure (*and exhaust*) these two numbers Jiya-Yi and Bing-Ding, which are equal to 12 and 20 respectively, exists or not. The number 4 which we have obtained ⟨in this process⟩ is just the large⟨st⟩ number among the many small numbers which measure (*and exhaust*) them [(*exhaustively*)] [exactly]. Moreover, when we examine whether a small ⟨nontrival⟩ number which measures and (*exhausts*) the two numbers 4 and 25 [(*exhaustively*)] [exactly] exists or not, if we decrease 25 by 4 ⟨repeatedly and⟩ exhaustively, 1 remains. If 1 remains, then there is no number other than 1 which measures (*and exhausts*) the three original numbers [(*exhaustively*)] [exactly]. Concerning how it occurs, if we had a small number which could [(*exhaustively*)] [exactly] measure (*and exhaust*) these two numbers Jiya-Yi and Bing-Ding, which were equal to 12 and 20 respectively, then according to the previous section, we could also [(*exhaustively*)] [exactly] measure (*and exhaust*) the number 4, which was the large⟨st⟩ number among the many small numbers, certainly. Now there is no small number which [(*exhaustively*)] [exactly] measures (*and exhausts*) ⟨firstly⟩ the number 4, and secondly Wu-Ji, which is equal to 25, so it follows immediately that there is no small number which can measure (*and exhaust*) the three numbers Jiya-Yi and Bing-Ding and Wu-Ji [(*exhaustively*)] [exactly].

If we use this method, then we can also see whether four or five unequal numbers ⟨firstly⟩, or various unequal numbers secondly, can be measured (*and exhausted*) with a small number [(*exhaustively*)] [exactly] or not.

For example, ⟨suppose that⟩ we have four numbers 24 and 30 and 39 and 49, and if we want to seek whether a number which can measure (*and exhaust*) them [(*exhaustively*)] [exactly] exists or not, then according to the fifteenth section of this chapter, we obtain 6 when we examine numbers which can measure (*and exhaust*) the two numbers 24 and 30 [(*exhaustively*)] [exactly] in the same way here and there. Moreover, according to this section, if we examine numbers which measure (*and exhaust*) 6 and (*39*) [29] [(*exhaustively*)] [exactly], we obtain 3. Moreover, if we decrease 49 by 3 ⟨repeatedly and⟩ exhaustively while examining whether a number which measures (*and exhausts*) 3 and 49 [(*exhaustively*)] [exactly] exists or not, 1 remains. If 1 remains, these four numbers are just numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any small ⟨nontrivial⟩ number. The details of the lack of numbers which measure (*and exhaust*) them [(*exhaustively*)] [exactly] are the same as in the previous arguments.

II.14.3. Notes

(f.35b, l.1) ishunde

Many English-speaking authors explain the meaning of this Manchu adverb as 'mutually'. *DGYB* explains it as '互相' or '彼此相同'. Here we followed the second explanation.

II.14.4. Remarks

This section deals with an algorithm for deciding whether three unequal natural numbers are relatively prime or not. The Euclidean algorithm is used here. The contents of this section are found in Prop.3 in Book VII of the *Elements*.

II.15. SECTION 20

II.15.1. Romanized Manchu Texts

orici. /

geren ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro teherehekū yaya ilan ton / bifi. ere geren ajige ton -i dorgi amba ton be bairengge: /

duibuleci. geren ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro. / giya i sere juwan ninggun. bing ding sere juwan juwe. u gi / sere jakūn -i teherehekū ilan ton bifi. ere ajige ton -i / dorgi amba ton be baiki seci. ere fiyelen -i juwan nadaci meyen -i / songkoi. neneme giya i. bing ding sere juwe ton be [lak] [(*wacihiyame*)] / [seme] kemne[re](*me wacihiyara*) ajige ton -i dorgi amba ton be baici. geng sin sere (f.36a)// duin -i ton be bahambi: ere geng sin sere duin. u gi be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro / ojorakū be baica. aikabade nirugan -i songkoi u gi sere jakūn be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / oci. ere geng sin sere duin. uthai baire ton ombi: adarame seci. geng sin / serengge. u gi be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro. geli julergi de gisurehe songkoi. giya i. / bing ding be inu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro be dahame. urunakū da ilan ton be [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*) ojoro be saci ombi: tere baha duin. ai turgunde da ilan ton be / [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro geren ajige ton -i dorgi amba ton ombi seci. geren / ajige ton -i dorgi de. ere ilan ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro. duin / sere ton ci amba ningge akū turgun: aikabade ere ilan ton be. [lak] [(*wacihiyame*)] (f.36b)// [seme] kemne[ci](*me wacihiyaci*) ojoro geng sin sere duin -i ton ci amba ningge be / seci. žin obume arafi. cendeme kemne. unenggi žin sere ton. giya / i. bing ding. u gi sere ilan amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. / ere fiyelen -i juwan jakūci meyen -i songkoi. giya i. bing ding / sere juwe ton be [(*wacihiyame*)] [lak seme] kemne[he](*me wacihiyaha*) geren ajige ton -i dorgi geng / sin sere amba ton be inu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: geng / sin be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*). urunakū geng sin ci amba akū be / saci ombi: geli aikabade jai nirugan -i songkoi ilaci / ton -i u gi be ninggun obuci. ere baha geng sin sere duin -i (f.37a)// ton uthai ninggun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorakū ombi. / udu ninggun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorakū bicibe. / ere geng sin sere duin. u gi sere ninggun urunakū / ishunde gūwa ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) / ojoro ton ombi: ai turgun seci. geng sin / serengge. giya i. bing ding sere juwe ton be / [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojorongge ofi. ere / fiyelen -i duici meyen -i songkoi. ere geng / sin. jai giya i. bing ding sere. (f.37b)// ilan ton gemu ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro ton ombi: tere u gi sere ton inu timu de gisurehe / songkoi. giya i. bing ding ni emgi uheri ilan ton gemu ajige ton de ishunde [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) / ojoro ton ombi: uttu oci. geng sin sere. u gi sere juwe ton. inu ajige ton de ishunde / [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro ton ombi: ajige ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojoro ton oci. jai ere / fiyelen -i juwan nadaci meyen -i arga be baitalafi. ere [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i dorgide amba / ton be baisu: baha juwe sere ton be. dze ceo obu. ere ton giya i. bing ding. u gi sere da ilan ton be / [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) amba ton ombi: dze ceo ai turgunde. giya i. bing ding. u gi sere ilan ton be [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) ombi seci. dze ceo serengge. geng sin be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ome ofi. ere fiyelen -i juwan ilaci meyen -i / songkoi. inu urunakū giya i. bing ding sere juwe amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: giya i. bing ding (f.38a)// sere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombime. geli da u gi sere ilaci ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / ojoro be dahame. ere dze ceo urunakū giya i. bing ding. u gi sere ilan da ton be gemu [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*) ombi: dze ceo sere juwe ton. adarame da ilan ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro geren / ajige ton -i dorgi amba ton ombi seci. ere juwe sere ton ci tulgiyen. da ilan ton be [(*wacihiyame*)] [lak seme] / kemne[ci](*me wacihiyaci*) ojoro amba ton akū turgun: aikabade da ilan ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro dze ceo / sere juwe ton ci amba ton bi seci. erebe in obume arafi cendeme kemne. in serengge. aikabade ere / giya i. bing ding sere juwe amba ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. ere fiyelen -i juwan jakūci meyen -i / songkoi inu urunakū geng sin be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi. geli daci u gi be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / ombi sehe be dahame. geng sin. u gi be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. inu juwan jakūci meyen -i songkoi. (f.38b)// urunakū dze ceo be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: dze ceo be [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) oci. araha in serengge. urunakū dze ceo

ci amba / akū be saci ombi:
aikabade da ilan amba ton -i jergi. duin / ton. sunja ton. jai hacingga ton bici. ere arga be baitalafi. inu / ere ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba ton be / bahaci ombi:
duibuleci. geren ajige ton -i [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro / giya i sere orin duin. bing ding sere juwan ninggun. u gi sere juwan / juwe. geng sin sere jakūn. ere duin ton bifi. ere be [(*wacihiyame*)] [lak seme] / kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba ton be baiki seci. julergi (f.39a)// songkoi. giya i sere orin duin. bing ding sere juwan ninggun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro / geren ajige ton -i dorgi amba ton be bairede. jakūn bahambi. geli jakūn. juwan juwe be [lak] [(*wacihiyame*)] / [seme] kemne[ci](*me wacihiyaci*) ojoro geren ajige ton -i dorgi amba ton be bairede. duin bahambi: jai ere / baha duin. geng sin sere duici ton -i jakūn be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ojorakū be baica. / baicaci jakūn be urunakū [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: jakūn be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro be / dahame. ere duin uthai da duin ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) geren ajige ton -i dorgi amba / ton ombi: ere leolehe turgun julergi songko: (f.39b)//

giya · · · · · · · · · · · · · · · · i

bing · · · · · · · · · · · · ding

u · · · · · · · · gi

geng · · · · sin

in —

The figure in f.36a

giya · · · · · · · · · · · · · · · · i

bing · · · · · · · · · · · · ding

u · · · · · · gi

geng · · · · sin

dzi · · ceo

in —

The figure in f.37b

II.15.2. Translation

The twentieth,

⟨Suppose that⟩ we have three arbitrary unequal numbers which can be measured and (*exhausted*) with many small ⟨nontrivial⟩ numbers [(*exhaustively*)] [exactly]. To seek the large⟨st⟩ number among these many small ⟨nontrivial⟩ numbers ⟨is as follows⟩.

For example, ⟨suppose that⟩ we have three unequal numbers Jiya-Yi and Bing-Ding and Wu-Ji, which are equal to 16 and 12 and 8 respectively and can be measured (*and exhausted*) with many small ⟨nontrivial⟩ numbers [(*exhaustively*)] [exactly], and if we want to seek the large⟨st⟩ number among these small ⟨nontrivial⟩ numbers, then according to the seventeenth section of this chapter, we first seek the large⟨st⟩ number among these many small ⟨nontrivial⟩ numbers which can measure (*and exhaust*) the two numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly], and if this is done we obtain Geng-Xin, which is equal to 4. Examine whether this Geng-Xin, which is equal to 4, can measure (*and exhaust*) Wu-Ji [(*exhaustively*)] [exactly] or not. If we can measure (*and exhaust*) Wu-Ji, which is equal to 8, with ⟨Geng-Xin⟩ [(*exhaustively*)] [exactly] as ⟨we see⟩ in the figure ⟨in f.36a⟩, then this Geng-Xin is just the number which we are seeking. Concerning how it occurs, since Geng-Xin can measure (*and exhaust*) Wu-Ji [(*exhaustively*)] [exactly] and moreover, according to what we said before, ⟨Geng-Xin⟩ can also measure (*and exhaust*) Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly], we can see that it can measure (*and exhaust*) the three original numbers ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩ [(*exhaustively*)] [exactly]. Concerning the details of the fact that which the number 4, which we have obtained ⟨in that process⟩, is the large⟨st⟩ number among the many small numbers which can measure (*and exhaust*) the three original numbers ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩ [(*exhaustively*)] [exactly], ⟨it is the result of⟩ the fact that there is nothing larger than the number 4, which can measure (*and exhaust*) the three original numbers [(*exhaustively*)] [exactly], among the many small ⟨nontrivial⟩ numbers. If we talk about a thing which is larger than Geng-Xin, which is equal to 4, and can measure (*and exhaust*) these three numbers ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩ [(*exhaustively*)] [exactly], denote it by Ren and try to measure ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩. If the number Ren can really measure (*and exhaust*) the three large numbers Jiya-Yi and Bing-Ding and Wu-Ji [(*exhaustively*)] [exactly], then according to the eighteenth section of this chapter, it can [(*exhaustively*)] [exactly] measure (*and exhaust*) Geng-Xin, which is the large⟨st⟩ number among the many small ⟨nontrivial⟩ numbers which can [(*exhaustively*)] [exactly] measure (*and exhaust*) the two numbers Jiya-Yi and Bing-Ding. We can see that if ⟨Ren⟩ can measure (*and exhaust*) Geng-Xin [(*exhaustively*)] [exactly] then ⟨Ren⟩ is certainly not larger than Geng-Xin. Moreover, if we set the third number Wu-Ji to 6 in accordance with the second figure ⟨in this section⟩, then the number Geng-Xin, which we have obtained and is equal to 4, cannot measure (*and exhaust*) 6 [(*exhaustively*)] [exactly]. Although it cannot measure (*and exhaust*) 6 [(*exhaustively*)] [exactly], these ⟨numbers⟩ Geng-Xin and Wu-Ji, which are equal to 4 and 6 respectively, are certainly numbers which can be [(*exhaustively*)] [exactly] measured (*and exhausted*) with another small number in the same way. Concerning the details of this fact, Geng-Xin is that which can measure (*and exhaust*) the two numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly], and hence all of these three numbers, ⟨firstly⟩ Geng-Xin, and secondly Jiya-Yi and Bing-Ding, are numbers which can be measured (*and exhausted*) with a small ⟨nontrivial⟩ number [(*exhaustively*)] [exactly]. As for the number Wu-Ji, together with Jiya-Yi and Bing-Ding, all of these three numbers are numbers which can be [(*exhaustively*)] [exactly] measured (*and exhausted*) with a small ⟨nontrivial⟩ number in the same way, according to the statement given in the theme. If so, the two numbers Geng-Xin and Wu-Ji are also numbers which can be [(*exhaustively*)] [exactly] measured (*and exhausted*) with a small ⟨nontrivial⟩ number in the same way. If they are numbers which can be measured (*and exhausted*) with a small ⟨nontrivial⟩ number [(*exhaustively*)] [exactly], next seek the large⟨st⟩ number among these many small ⟨nontrivial⟩ numbers which measure (*and exhaust*) them [(*exhaustively*)] [exactly], by using the method of the seventeenth section of this chapter. Denote the number 2, which we have obtained ⟨in this process⟩, by Zi-Chou. This number is the large⟨st⟩ number which measure (*and exhaust*) the three numbers Jiya-Yi and Bing-Ding and Wu-Ji [(*exhaustively*)] [exactly]. Concerning the details of the fact that Zi-Chou can measure (*and exhaust*) three numbers Jiya-Yi and Bing-Ding and Wu-Ji [(*exhaustively*)] [exactly], Zi-Chou can measure (*and exhaust*) Geng-Xin [(*exhaustively*)] [exactly], and hence it can also measure (*and exhaust*) the two large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly], according to the thirteenth section of this chapter. ⟨Zi-Chou⟩ can measure (*and exhaust*) the two large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly] and, at the same time, ⟨Zi-Chou⟩ can [(*exhaustively*)] [exactly] measure (*and exhaust*) the third original number Wu-Ji in addition, so it follows that this Zi-Chou can measure (*and exhaust*) all of the three original numbers Jiya-Yi and Bing-Ding and Wu-Ji [(*exhaustively*)] [exactly]. Concerning the details of the fact that the two numbers Zi and Chou[13] is the large⟨st⟩ number among the many small numbers which measure (*and exhaust*) the three original numbers ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩ [(*exhaustively*)] [exactly], ⟨it is the result of⟩ the fact that except this number 2 there is no large number which can measure (*and exhaust*) the three original numbers ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩ [(*exhaustively*)] [exactly]. If we assume that there is a number which is larger than the two numbers Zi and Chou[14] and ⟨this larger number⟩ can measure (*and exhaust*) the three original numbers ⟨Jiya-Yi and Bing-Ding and Wu-Ji⟩ [(*exhaustively*)] [exactly], denote this ⟨larger number⟩ by Yin and try to measure ⟨the two large numbers with it⟩. As for Yin, if it can measure (*and exhaust*) the two large numbers Jiya-Yi and Bing-Ding [(*exhaustively*)] [exactly], then according to the eighteenth section of this chapter, it can also [(*exhaustively*)] [exactly]

[13] Of course, ‘the two numbers Zi and Chou’ should be replaced with ‘the number 2, which is called Zi-Chou’. See II.15.3.

[14] The phrase ‘the two numbers Zi and Chou’ should be replaced with ‘the number 2, which is called Zi-Chou’ again. See II.15.3.

measure (*and exhaust*) Geng-Xin certainly. Moreover, we said that ⟨Yin⟩ could measure (*and exhaust*) Wu-Ji [(*exhaustively*)] [exactly] from the beginning, so it follows that if ⟨Yin⟩ can measure (*and exhaust*) Geng-Xin and Wu-Ji [(*exhaustively*)] [exactly] then ⟨Yin⟩ can also [(*exhaustively*)] [exactly] measure (*and exhaust*) Zi-Chou certainly, according to the eighteenth section again. If it can measure (*and exhaust*) Zi-Chou [(*exhaustively*)] [exactly], we can see that Yin, which we made ⟨in the above process⟩, is certainly not larger than Zi-Chou.

If we have four numbers or five numbers like the original three numbers ⟨discussed here⟩, and next to them, various numbers ⟨of this kind⟩, then we can also obtain the large⟨st⟩ number among the many small numbers which measure (*and exhaust*) these numbers by using this method.

For example, ⟨suppose that⟩ we have four numbers Jiya-Yi and Bing-Ding and Wu-Ji and Geng-Xin, which are equal to 24 and 16 and 12 and 8 respectively and can be measured (*and exhaustd*) with many small numbers [(*exhaustively*)] [exactly], and if we want to seek the large⟨st⟩ number among the many small numbers which measure (*and exhaust*) them [(*exhaustively*)] [exactly], we obtain 4 when we seek the large⟨st⟩ number among the many small numbers which [(*exhaustively*)] [exactly] measure (*and exhaust*) ⟨the two numbers⟩ Jiya-Yi and Bing-Ding, which are equal to 24 and 16 respectively, we obtain 8. Moreover, when we seek the large⟨st⟩ number among the many small numbers which measure (*and exhaust*) 8 and 12 [(*exhaustively*)] [exactly], we obtain 4. Next examine whether we can [(*exhaustively*)] [exactly] measure (*and exhaust*) the number 4 and the fourth number Geng-Xin, the former of which was obtained ⟨in this process⟩ and the latter of which is equal to 8. If we examine it, then we can [(*exhaustively*)] [exactly] measure (*and exhaust*) 8 ⟨with 4⟩ certainly. Since we can measure (*and exhaust*) 8 ⟨with this 4⟩ [(*exhaustively*)] [exactly], this number 4 is just the large⟨st⟩ number among the many small numbers which measure (*and exhaust*) the four original numbers [(*exhaustively*)] [exactly]. The details which we have discussed ⟨here⟩ are the same as ⟨we gave⟩ before.

II.15.3. Notes

(f.38b, l.3) dze ceo sere juwe ton

The meaning of this noun phrase is ‘the two numbers Zi and Chou’. The correct Manchu phrase should be ‘dze ceo sere juwe –i ton’ (‘the number 2, which is called Zi-Chou’), but no correction was made by the corrector. The noun phrase ‘juwe ton’ in the sixth line of f.38b has the same problem.

II.15.4. Remarks

This section deals with an algorithm for finding the greatest common divisor of three unequal natural numbers when these two numbers are not relatively prime. As in the previous section, the Euclidean algorithm is used. The contents of this section are found in Prop.3 in Book VII of the *Elements*.

II.16. SECTION 22

II.16.1. Romanized Manchu Texts

orin juweci. /
yaya juwe ton bifi. aikabade erebe tere be ishunde teherebu[ci](*me kamcici*) / meni meni baha ton urunakū ishunde gese ombi:
duibuleci. / giya sere ton ilan. i sere ton duin bifi. giya be. / i de teherebuci. bing sere ton be bahambi: i be. / giya de teherebuci. ding sere ton be bahambi: ere baha bing / ding sere juwe ton. urunakū ishunde gese ombi: adarame / seci. emu sere be. u obufi. ere fiyelen -i ningguci meyen -i / songkoi. u sere emu be. giya sere ton -i ilan de (f.43a)// duibulerengge. i sere ton -i duin be. bing sere ton -i juwan juwe de duibulere duibulen -i / adali ombi: uttu ofi gi ho yuwan ben -i ningguci fiyelen -i uyuci meyen -i songkoi. giyan be / forgošome duibulere duibulen -i arga be baitalaci. u sere emu be. i sere ton -i / duin de duibulerengge. giya sere ton -i ilan be. bing sere ton -i juwan juwe de / duibulere duibulen -i adali ombi: geli ere fiyelen -i ningguci meyen -i songkoi. u sere / emu be. i sere ton -i duin de duibulerengge. giya sere ton -i ilan be. ding sere / ton -i juwan juwe de duibulere duibulen -i adali ombi: uttu ofi ere bing. ding / sere juwe ton -i meni meni ton be. giya sere ton de duibulerengge. gemu u sere / emu be. i sere ton -i duin de duibulere duibulen -i adali ombi: duibulere (f.43b)// duibulen adali oci. ere bing. ding sere juwe ton. urunakū ishunde gese ton ombi: (f.44a)//

giya · · · i · · · · u ·

bing · · · · · · · · · · · ·

ding · · · · · · · · · · · ·

The figure in f.43a

II.16.2. Translation

The twenty-second,
⟨Suppose that⟩ we have two arbitrary numbers, and if we (*place them closely together to*) balance one with another, then each of the numbers which we obtain ⟨in this process⟩ is equal to the other.

For example, ⟨suppose that⟩ we have a number Jiya and a number Yi, which are equal to 3 and 4 respectively, and if we balance Jiya with Yi we obtain a number Bing. If we balance Yi with Jiya we obtain a number Ding. These two numbers Bing and Ding, which we have obtained ⟨in this process⟩, are certainly equal. Concerning how it occurs, after we have denoted 1 by Wu, we find that the result of comparing Wu, which is equal to 1, with Jiya, which is equal to 3, is equivalent to the compared pair given by comparing Yi, which is equal to 4, with Bing, which is equal to 12, according to the sixth section of of this chapter. Such being the case, according to the ninth section of the sixth chapter of "The Principles of Quantities", if we use the method of compared pairs given by exchanging ratios in the process of comparison, the result of comparing Wu, which is equal to 1, with Yi, which is equal to 4, is equivalent to the compared pair given by comparing Jiya, which is equal to 3, with Bing, which is equal to 12. Moreover, according to the sixth section of this chapter, the result of comparing Wu, which is equal to 1, with Yi, which is equal to 4, is equivalent to the compared pair given by comparing Yi, which is equal to 4, with Ding, which is equal to 12. Such being the case, the result of comparing each of these two numbers Bing and Ding with the number Jiya is equivalent to the compared pair given by comparing Wu, which is equal to 1, with Yi, which is equal to 4, in all cases. If the compared pairs are equivalent, these two numbers Bing and Ding are certainly equal numbers.

II.16.3. Remarks

This section deals with the following

PROPOSITION. Multiplication in the set of natural numbers is commutative.

This proposition corresponds to Prop.16 in Book VII of the *Elements*. In *SLJY* this theorem is omitted.

II.17. SECTION 25

II.17.1. Romanized Manchu Texts

orin sunjaci. /
yaya emu ton be. geren ton de teherebufi. baha geren ton be. ishunde duibulerengge. / da bihe geren ton (*be*) [de] ishunde duibulere duibulen -i adali ombi:
duibuleci. ninggun sere / emu ton bifi. jakūn sere. juwan sere ton de teherebuci. ninggun be. jakūn de teherebure de. / dehi jakūn be bahambi: ninggun be. juwan de teherebure de. ninju be bahambi: ere jakūn be / juwan de duibulerengge. dehi jakūn be ninju de duibulere duibulen -i adali ombi: / adarame seci. ninggun be. jakūn de teherebure de dehi jakūn bahara be dahame. ere fiyelen -i / ningguci meyen -i songkoi. emu be. ninggun de duibulerengge. jakūn be. dehi jakūn de / duibulere duibulen -i adali ombi: geli ninggun be. juwan de teherebure de. ninju bahara be (f.47a)// dahame. ineku ningguci meyen -i songkoi. emu be. ninggun de duibulerengge. juwan be. ninju de / duibulere duibulen -i adali ombi: uttu oci. jakūn be. dehi jakūn de duibulerengge. juwan be. / ninju de duibulere duibulen -i adali ofi. gi ho yuwan ben -i ningguci fiyelen -i uyuci meyen -i / songkoi. forgošome duibuleci. jakūn be juwan de duibulerengge. dehi jakūn be. ninju de / duibulere duibulen -i adali ombi: uttu ofi. da bihe juwe ton -i jakūn sere. jai juwan / serengge be. ninggun de meni meni teherebufi. bahara juwe ton -i dehi jakūn sere. ninju serengge be. / ishunde

duibulere duibulen adali ojoro be saci ombi:
jai aikabade emu ton be baitalafi / geren ton be teherebure de. da ton inu teherebuhe geren ton -i dorgide bici. terei giyan / ere adali:
duibuleci. emu ninggun sere ton be baitalafi. ninggun sere. jakūn sere juwe (f.47b)// ton de teherebuci. gūsin ninggun. dehi jakūn be bahambi: ninggun inu terei dorgide / bisire be dahame. uthai ninggun be jakūn de duibulerengge. gūsin ninggun be dehi / jakūn de duibulere duibulen -i adali ombi: (f.48a)//

II.17.2. Translation

The twenty-fifth,
The result of comparing ⟨a pair chosen from⟩ the many numbers obtained by balancing an arbitrary number with many numbers is equivalent to the compared pair given by comparing ⟨*the corresponding pair of*⟩ [them with] the many numbers which we had at the beginning.

For example, ⟨suppose that⟩ we have a number 6 and if we balance it with ⟨each of⟩ numbers 8 and 10, then we obtain 48 when we balance 6 with 8. We obtain 60 when we balance 6 with 10. The result of comparing this number 8 with 10 is equivalent to the compared pair given by comparing 48 with 60. Concerning how it occurs, since we obtain 48 when we balance 6 with 8, according to the sixth section of this chapter, the result of comparing 1 with 6 is equivalent to the compared pair given by comparing 8 with 48. Moreover, since we obtain 60 when we balance 6 with 10, according to this sixth section, the result of comparing 1 with 60 is equivalent to the compared pair given by comparing 10 with 60. If so, the result of comparing 8 with 48 is equivalent to the compared pair given by comparing 10 with 60, and if we change their paring in the process of comparison as we did in the ninth section of the sixth chapter of "The Principles of Quantities" Book, the result of comparing 8 with 10 is equivalent to the compared pair given by comparing 48 with 60. Such being the case, we can see that the compared pair given by comparing the two numbers 8 and 10, which we had at the beginning, and the compared pair given by comparing the two numbers 48 and 60, both of which were obtained by balancing each of the original two numbers ⟨8 and 10⟩ with 6, are equivalent.

Next if we balance many numbers by using another number and the ⟨latter⟩ original number also belongs to ⟨the set⟩ the many numbers which will be balanced, the principles in that ⟨situation⟩ is similar to this ⟨case⟩.

For example, if we use a number 6 and balance each of two numbers 6 and 8 with it, we obtain 36 and 48. Since 6 also belongs to them, the result of comparing 6 with 8 is just equivalent to the compared pair given by comparing 36 with 48.

II.17.3. Remarks

This section contains two propositions. The first one is the following

PROPOSITION A. Let a,b,c be arbitrary natural numbers. Then we have $ab:ac$

$= b : c$.

The second one is the following

PROPOSITION B. Let a, b be arbitrary natural numbers. Then we have $a^2 : ab = a : b$.

The first proposition corresponds to Prop.18 in Book VII of the *Elements* and a part of Sec.3 of Vol.2 of *SfYbSLJY*. In the latter case we inferred this correspondence from the examples of this proposition used here. It is doubtful that the authors of *SLJY* were conscious that the commutativity of multiplication is not trivial.

II.18. SECTION 27

II.18.1. Romanized Manchu Texts

orin nadaci. /

yaya geren ton bifi. emu ton de teherebume (*kamcici*) [acabuci]. bahara geren ton be ishunde / duibulerengge. geren da ton be ishunde duibulere duibulen adali ombi:

duibuleci. / ninggun. jakūn sere juwe ton bifi. sunja de (*teherebuci*) [teherebume] [(*kamcici*)] [acabuci]. gūsin. dehi sere / juwe ton be bahafi. uthai ninggun be jakūn de duibulerengge. gūsin be dehi de / duibulere duibulen -i adali ombi: adarame seci. ere fiyelen -i orin juweci meyen -i / songkoi. aikabade sunja be. ninggun de. jakūn de (*teherebuci*) [teherebume acabuci]. bahara juwe / ton. ninggun be. sunja de. jakūn be. sunja de teherebuci bahara ton -i gese ombi: / te ere fiyelen -i orin sunjaci meyen -i songkoi. sunja be. ninggun de teherebure. jakūn de (f.51a)// teherebure de. bahara ton be. ishunde duibulerengge. ninggun be jakūn de duibulere / duibulen -i adali ombi: uttu ofi. ninggun. jakūn sere juwe ton be. sunja de / (*teherebuci*) [teherebume acabuci]. bahara juwe ton be ishunde duibulerengge. ninggun be jakūn de / duibulere duibulen -i adali ombi: (f.51b)//

II.18.2. Translation

The twenty-seventh,

⟨Suppose that⟩ we have many arbitrary numbers and if we (*place*) [combine] each of them and another number (*close together*) to balance the former one with the latter one, then the results of comparing pairs chosen from the many numbers which are obtained ⟨in this process⟩ are equivalent to the ⟨corresponding⟩ compared pairs given by comparing pairs of the many original numbers ⟨respectively⟩.

For example, ⟨suppose that⟩ we have two numbers, 6 and 8, and if we (*balance each of them with 5,*) [combine each of them with 5] [(*put each of them close to 5*)] [to bring balance between them,] then we obtain two numbers 30 and 40, and then the result of comparing 6 with 8 is equivalent to the compared pair given by comparing 30 with 40. Concerning how it occurs, according the twenty-second section of this chapter, two numbers which are obtained by balancing 5 with 6, and with 8, are equal to numbers obtained (*by balancing 6 with 5, and 8 with 5,*) ⟨*respectively*⟩. [by combining 6 with 5, and 8 with 5, and bringing balance between them ⟨respectively⟩.] Now according to the twenty-fifth section of this chapter, the result of comparing the numbers which are obtained by balancing 5 with 6 and 5 with 8 is equivalent to the compared pair given by comparing 6 with 8. Such being the case, the result of comparing the two numbers which are obtained when we balance each of two numbers 6 and 8, with 5 is equivalent to the compared pair given by comparing 6 with 8.

II.18.3. Remarks

This section deals with the following

PROPOSITION. Let *a*, *b*, *c* be arbitrary natural numbers. Then we have $ac : bc = a : b$.

The only difference between the propositions in Section 25 and this section is the order of multiplication.

II.19. SECTION 28

II.19.1. Romanized Manchu Texts

orin jakūci. /

emu duibulere ton bifi. ere duibulen de teherere ishunde sirandume / duibulere ton be bairengge:

duibuleci. (*ilan be sunja de*) [[??]] duibulere / ton bifi. ere duibulen de teherere. ishunde sirandume duibulere udu / ton be baiki seci. ilan be. da ton -i songkoi. (*teherebuci*) [teherebume acabuci]. / uyun bahambi: geli ilan be. sunja de (*teherebuci*) [teherebume acabuci]. tofohon / bahambi: geli sunja be. da ton -i songkoi (*teherebuci*) [teherebume acabuci]. orin / sunja bahambi: ere baha uyun. tofohon. jai orin sunja sere ilan / ton. uthai ilan be. sunja de. duibulere duibulen de teherere. (f.52a)// ishunde sirandume duibulere ilan ton ombi: adarame seci. ere fiyelen -i orin sunjaci meyen -i / songkoi. ilan be. da ton -i songkoi (*teherebure*) [teherebume acabure]. jai sunja de (*teherebure*) [teherebume acabure] oci. / bahara uyun. tofohon sere juwe ton be ishunde duibulerengge. ilan be. sunja de duibulere duibulen -i / adali ombi: geli sunja be ilan de (*teherebure*) [teherebume acabure]. jai da ton -i songkoi (*teherebure*) [teherebume] / [acabure] oci. bahara tofohon. orin sunja sere juwe ton be duibulerengge. ere fiyelen -i / orin sunjaci meyen -i songkoi. ilan be. sunja de duibulere duibulen -i adali ombi: tuttu / ilan be. sunja de duibulerengge. uyun be. tofohon de duibulere duibulen -i adali ombi: geli / uyun be. tofohon de duibulerengge. tofohon be. orin sunja de duibulere duibulen -i adali / ofi. ere uyun. tofohon. jai orin sunja sere ilan ton ishunde sirandume duibulere duibulen -i (f.52b)// ton ombime. ilan be sunja de duibulere duibulen -i adali ojoro be saci ombi:

aikabade / geli ishunde sirandume duibulere adali duin ton be baiki seci. neneme ilan be. uyun de. / tofohon de. orin sunja de (*teherebuci*) [teherebume acabuci]. orin nadan. dehi sunja. nadanju sunja be bahambi: / geli sunja be. orin sunja de (*teherebuci*) [teherebume acabuci]. emu tanggū orin sunja be bahambi: ere bahara. / orin nadan. dehi sunja. nadanju sunja. emu tanggū orin sunja sere duin ton. ilan be. sunja de / duibulere duibulen de teherere. ishunde sirandume duibulere adali duin ton ombi: adarame seci. / ilan be. uyun de. tofohon de. orin sunja de (*teherebuci*) [teherebume acabuci]. ere fiyelen -i orin sunjaci / meyen -i songkoi. bahara orin nadan. dehi sunja. nadanju sunja -i ton be ishunde duibulerengge. uyun. / tofohon. orin sunja be ishunde duibulere duibulen -i adali ombi: orin nadan be. dehi sunja de (f.53a)// duibulerengge. ilan be. sunja de duibulere duibulen -i adali ombi: geli dehi sunja be. nadanju / sunja de duibulerengge. ilan be. sunja de duibulere duibulen -i adali ombi: te ere orin sunja be. / ilan de (*teherebure*) [teherebume acabure]. jai sunja de (*teherebure*) [teherebume acabure] oci. nadanju sunja. emu tanggū orin / sunja sere juwe ton be bahambi: erebe julergi meyen -i songkoi duibuleci. ilan be. sunja de / duibulere duibulen -i adali ombi: uttu oci. ere orin nadan. dehi sunja. nadanju sunja. / emu tanggū orin sunja sere duin ton. ishunde sirandume duibulere adali duin ton ombime. geli / ilan be sunja de duibulere duibulen -i adali ton ombi: (f.53b)//

ilan uyun	sunja tofohon		orin [juwan] sunja
orin nadan	dehi sunja	nadanju sunja	emu tanggū orin sunja

The figure in f.52a

II.19.2. Translation

The twenty-eighth,

⟨Suppose that⟩ we have a compared ⟨pair of⟩ numbers. To seek numbers which are successively compared so that ⟨all of⟩ those compared pairs are equivalent to this compared pair ⟨is as follows⟩.

For example, ⟨suppose that⟩ we have a compared pair given by comparing (*3 with 5*) [[??]], and if we want to seek several numbers which are successively compared so that ⟨all of⟩ those compared pairs are equivalent to this compared pair, then when we (*balance*) [combine] 3 with the original number ⟨3⟩ [to bring balance between them] we obtain 9. Moreover, if we (*balance 3 with 5*) [combine 3 with 5 to bring balance between them] we obtain 15. Moreover, if we (*balance 5*) [combine 5] with the original number ⟨5⟩ [to bring balance between them] we obtain 25. The three numbers 9 and 15 and 25, which were obtained ⟨in this process⟩, are just numbers which are successively compared so that ⟨all of⟩ those compared pairs are equivalent to the compared pair given by comparing 3 with 5. Concerning how it occurs, according to the twenty-fifth section of this chapter, the result of comparing two numbers 9 and 15, which are obtained by (*balancing*) [combining] 3 with the original number ⟨3 firstly⟩ and with 5 [to bring balance between them] secondly, is equivalent to the compared pair given by comparing 3 with 5. Moreover, according to the twenty-fifth section of this chapter, the result of comparing two numbers 15 and 25, which are obtained by (*balancing*) [combining] 5 with 3 ⟨firstly⟩ and with the original number [to bring balance between them] secondly, is equivalent to the compared pair given by comparing 3 with 5. Thus the result of comparing 3 with 5 is equivalent to the compared pair given by comparing 9 with 15. Moreover, the result of comparing 9 with 15 is equivalent to the compared pair given by comparing 15 with 25, so we can see that ⟨firstly⟩ these three numbers 9 and 15, and secondly 25 are compared pairs of numbers which are successively compared and, at the same time, their compared pairs are equivalent to the compared pair given by comparing 3 with 5.

If, in addition, we want to seek four numbers which are successively compared so that ⟨all of⟩ those compared pairs are equivalent, we first obtain 27 and 45 and 75 ⟨respectively⟩, when we (*balance*) [combine] 3 with ⟨each of three numbers⟩ 9 and 15 and 25 [to bring balance between them]. Moreover, if we (*balance*) [combine] 5 with 25 [to bring balance between them] we obtain 125. The four numbers 27 and 45 and 75 and 125, which are obtained ⟨in this process⟩, are just numbers which are successively compared so that ⟨all of⟩ those compared pairs are equivalent to the compared pair given by comparing 3 with 5. Concerning how it occurs, if we (*balance*) [combine] 3 with ⟨each of three numbers⟩ 9 and 15 and 25 [to bring balance between them], then according to the twenty-fifth section of this chapter, the results of comparing ⟨pairs from⟩ the ⟨each of three⟩ numbers 27 and 45 and 75, which are obtained ⟨in this process⟩, is equivalent to the compared pairs given by comparing 9 and 15 and 25. The result of comparing 27 with 45 is equivalent to the compared pair given by comparing 3 and 5. Moreover, the result of comparing 45 with 75 is equivalent to the compared pair given by comparing 3 and 5. Now if we (*balance*) [combine] this 25 with 3 ⟨firstly⟩ and with 5 secondly [to bring balance between them], we obtain two numbers 75 and 125. If we follow the previous section and compare them, the result is equivalent to the compared pair given by comparing 3 and 5. If so, the four numbers 27 and 45 and 75 and 125 are four numbers which are successively compared so that ⟨all of⟩ those compared pairs are equivalent and, at the same time, equivalent to the compared pair given by comparing 3 with 5.

II.19.3. Notes

(f.52a, l.2) **ishunde sirandume duibulere ton**

This noun phrase is a Manchu term for the notion of 'numbers in continued proportion' in the *Elements*. There are several variants of this phrase as follows:

(a) 'ishunde sirandume duibulere duibulen adali geren ton' (in Section 52, 54)
(b) 'ishunde sirandume duibulere duibulen -i ton' (in Section 55-57)

The Chinese noun phrase 'xiang lian bi li zhi shu' (相連比例之數), which is used as an equivalent for 'numbers in continued proportion' in *SfYbSLJY*, coincides with a word-by-word translation of the last expression in Manchu.

II.19.4. Remarks

This section gives an algorithm for finding a finite geometric sequence the common ratio of which is equal to a given rational number.

II.20. SECTION 37

II.20.1. Romanized Manchu Texts

gūsin nadaci. /
ishunde [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū juwe ton bifi. aikabade / emu ajige ton. ere juwe ton -i dorgi. emu ton be [(*wacihiyame*)] [lak seme] / kemne[ci](*me wacihiyaci*) oci. jai emu ton. ere ajige ton. ishunde [(*wacihiyame*)] [lak seme] / kemne[buci](*me wacihiyabuci*) ojorakū ton ombi:
duibuleci. ishunde [(*wacihiyame*)] [lak seme] / kemne[buci](*me wacihiyabuci*) ojorakū duin sere uyun sere juwe ton be. giya i / obufi geli emu ajige ton -i ilan be bing obufi. ere juwe ton -i / dorgi i sere ton -i uyun be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. tere / emu giya sere ton -i duin. bing sere ajige ton -i ilan. ishunde (f.66a)// [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton ombi: adarame seci. aikabade bing giya sere juwe ton be / [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton bi seci. taka cendeme ere ton be ding obufi kemne. unenggi / ding serengge. bing be kemneci oci. bing daci i be [(*wacihiyame*)] [lak seme] kemne[rengge](*me wacihiyarangge*) be dahame. ere / fiyelen -i juwan ilaci meyen -i songkoi. ding inu urunakū i be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi: i be / [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombime. julergi de geli giya be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*) sehe be dahame. giya i sere / juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) oci. ere juwe ton. ishunde [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton / waka ombi: unenggi ishunde [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton waka oci. timu de gisurehe / giya i sere ishunde [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton waka sehengge de acanarakū ombi: timu de / acanarakū turgunde. giya bing sere juwe ton. urunakū ishunde [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton ombi: (f.66b)//

II.20.2. Translation

The thirty-seventh,
⟨Suppose that⟩ we have two numbers each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly], and if a small number can measure (*and exhaust*) one of these two numbers [(*exhaustively*)] [exactly], then the other number and this small number are numbers each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly].

For example, we ⟨first⟩ denote the two numbers 4 and 9, each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly], by Jiya and Yi ⟨respectively⟩, and in addition, denote the small number, which is equal to 3, by Bing, and if ⟨Bing⟩ can [(*exhaustively*)] [exactly] measure (*and exhaust*) the number Yi, which is one of the two numbers and is equal to 9, then that number Jiya and the small number Bing, which are equal to 4 and 3 respectively, are numbers each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly]. Concerning how it occurs, if we assume that there is a ⟨nontrivial⟩ number which can measure (*and exhaust*) the two numbers Bing and Jiya, then try to denote this number by Ding temporarily and measure ⟨Bing and Jiya with Ding⟩. If Ding can really measure Bing, then according to the thirteenth section of this chapter, Ding can also [(*exhaustively*)] [exactly] measure (*and exhaust*) Yi certainly, because from the beginning Bing is that which measures (*and exhausts*) Yi. ⟨Ding⟩ can measure (*and exhaust*) Yi, and we said before that it measures (*and exhausts*) Jiya [(*exhaustively*)] [exactly] as well, so it follows that the two numbers Jiya and Yi are not numbers each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly] if Ding can measure (*and exhaust*) these two numbers [(*exhaustively*)] [exactly]. If they are really not numbers each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly], it contradicts the statement in the theme ⟨of this section⟩ that Jiya and Yi are not numbers each of which cannot be measured (*and exhausted*) with the other one [(*exhaustively*)] [exactly]. Since it contradicts the theme, the two numbers Jiya and Bing are not numbers each of which cannot be [(*exhaustively*)] [exactly] measured (*and exhausted*) with the other one certainly.

II.20.3. Notes

(f.66b, ll.7-8) timu de … acanarakū ombi:

'waka' in this clause is redundant, but was not deleted by the corrector. Here we followed the original text.

II.20.4. Remarks

This section asserts the following

PROPOSITION. Let a, b be natural numbers such that each of them cannot measure the other one. If another natural number c can measure one of them, say a, then b cannot measure c, and vice versa.

Of course, this statement is false. A counterexample is given by putting $a = 4$, $b = 6$ and $c = 2$. The correct form of this proposition is as follows:

PROPOSITION'. Let a, b be relatively prime. If one of them, say a, and another natural number c are relatively prime, then b and c are relatively prime.

The example of this section also satisfies the latter statement.

Note that the same type of confusion can be found in Section 36 of this manuscript.

The proposition in Section 36 is similar to that of Prop.22 in Book VII of the *Elements*, but the condition of being relatively prime, which is found in the conclusion of the latter proposition, was replaced with the condition of being mutually immeasurable in the conclusion of the former proposition. The corrector made no correction on both propositions.

The correct form of the proposition in this section corresponds to Prop.23 in Book VII of the *Elements*. On the other hand, any form of this proposition was omitted in *SLJY*.

II.21. SECTION 38

II.21.1. Romanized Manchu Texts

gūsin jakūci. /

juwe ton bifi. emke emken -i encu emu ton de emgilere de. ere emgilehe / juwe ton be. gemu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro emu ajige ton akū oci. / ere juwe da ton be ishunde kamcime teherebure de bahara emu ton. / jai julergi encu emu ton be. gemu [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton / akū ombi: duibuleci. giya sere nadan. i sere ilan -i juwe ton bifi. / ere juwe ton -i dorgi giya sere nadan ocibe. i sere ilan ocibe. yaya emke. encu / emu bing sere ton -i jakūn de emgilere de. yaya ton de gemu [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) / ojorakū oci. nadan be ilan de kamcifi bahara orin emu be ding obuha (f.67a)// manggi. ere ding sere ton. bing sere ton -i emgi. inu yaya ton de gemu [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton / ombi: adarame seci. aikabade ding sere. bing sere juwe ton de [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ojoro ton bi / seci. taka cendeme u arafi kemne. aikabade u be jafafi. ding sere ton be [lak seme] [(*wacihiyame*)] / kemne[ci](*me wacihiyaci*) oci. ere u urunakū emu ton be songkolofi teni kemneci ohobi: ere ton be / gi obufi cendeme tuwa. unenggi u serengge. gi sere ton -i songkoi. ding be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) / oci. gi sere ton -i dorgide emu serengge udu bici. ding sere ton -i dorgide u serengge / inu udu ubu bisire be dahame. uthai ere fiyelen -i ningguci meyen -i songkoi. araha u. / gi sere juwe ton be ishunde teherebuci. ding sere ton ombi: julergi de giya be i de / kamcici. ding sere ton be bahambi sehe be dahame. uthai ere fiyelen -i gūsin emuci (f.67b)// meyen -i songkoi. u be giya de duibulerengge. i be gi de duibulere duibulen -i adali ombi: / duibulen adali oci. daci giya sere. bing sere juwe ton. yaya ton de gemu [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) / ojorakū ton sehe bime. geli u serengge. bing be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*) ombi sehe be dahame. julergi / meyen de gisurehe songkoi. urunakū u sere. giya serengge. inu yaya ton de gemu [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) / ojorakū ton ombi: [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū turgunde. ere fiyelen -i gūsin sunjaci meyen -i / songkoi. ere u. giya sere juwe ton. ishunde duibulere duibulen adali geren ton -i dorgi umesi / ajige ton ombi: umesi ajige ton ojoro be dahame. ere fiyelen -i gūsin duici meyen -i songkoi. / u. giya sere juwe ton. ishunde duibulen adali i. gi sere juwe ton be [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) de. / u serengge. i be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*). giya serengge. gi be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*). uttu oci. julergi de (f.68a)// u serengge. bing be [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*) sehe be dahame. u urunakū. bing. i sere juwe ton be / [(*wacihiyame*)] [lak seme] kemne[mbi](*me wacihiyambi*): bing. i sere juwe ton be [(*wacihiyame*)] [lak seme] kemne[ci](*me wacihiyaci*). ere bing. i sere juwe ton. / yaya ton de gemu [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ton waka ombi: [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū / ton waka oci. timu -i gisun de acanarakū ombi: timu de acanarakū oci. giya i sere juwe / ton. bing sere gūwa ton be. aikabade [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akū oci. i. ding serengge. bing / serengge be. inu urunakū [(*wacihiyame*)] [lak seme] kemne[re](*me wacihiyara*) ton akū be saci ombi: (f.68b)//

giya nadan   i ilan   bing jakūn

ding orin emu

u emu   gi emu

The figure in f.67a

II.21.2. Translation

The thirty-eighth,

⟨Suppose that⟩ we have two numbers, and when we couple each of them with a number which is different ⟨from them⟩, if there is no small ⟨nontrivial⟩ number which can measure (*and exhaust*) these coupled two numbers [(*exhaustively*)] [exactly] in each case, then there is no ⟨nontrivial⟩ number which can [(*exhaustively*)] [exactly] measure (*and exhaust*) both of the ⟨two⟩ numbers, ⟨firstly⟩ the number which is obtained when we place the two original numbers close together and bring balance between them, and secondly, the other different number which was given before.

For example, ⟨suppose that⟩ we have two numbers Jiya and Yi, which are equal to 7 and 3 respectively, and when we couple an arbitrary one from these two numbers, each of which may be equal to 7 or 3, with the other different number Bing, which is equal to 8, and if ⟨neither of these pairs⟩ cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, then after we have denoted a number 21, which was obtained by placing 7 close to 3, by Ding, ⟨we see that⟩ this number Ding, together with the number Bing, also constitute ⟨a pair of⟩ numbers which cannot be measured (*and* *exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number. Concerning how it occurs, if we assume that for the two numbers Ding and Bing there exists a ⟨nontrivial⟩ number which can measure (*and exhaust*) both of them [(*exhaustively*)] [exactly], try to denote this number by Wu temporarily and measure ⟨Ding and Bing with it⟩. If we take Wu and ⟨Wu⟩ can measure (*and exhaust*) the number Ding [(*exhaustively*)] [exactly], then this Wu can certainly measure (*and exhaust*) ⟨Ding⟩ [(*exhaustively*)] [exactly] when it follows a ⟨nontrivial⟩ number. Denote this number by Ji and try ⟨to measure numbers⟩. If the number Ji can really measure (*and exhaust*) Ding [(*exhaustively*)] [exactly] by following the number Ji, we have a certain number of times Wu in the number Ding when we have a certain number of units in the number Ding, so it follows immediately that if we bring balance between the two numbers Wu and Ji, both of which we constructed ⟨in the above process⟩, we have the number Ding, according to the sixth section of this chapter. We said before that if we placed Jia close to Yi we obtained the number Ding, so it follows immediately that the result of comparing Wu with Jiya is equivalent to the compared pair given by comparing Yi with Ji, according to the thirty-first section of this chapter. If the compared pairs are equivalent, since we said at the beginning that the two numbers Jiya and Bing are ⟨a pair of⟩ numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number and, at the same time, we said that the number Wu can measure (*and exhaust*) Bing [(*exhaustively*)] [exactly], both of the numbers Wu and Jia are also ⟨a pair of⟩ numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, according to what we stated in the previous section. Since ⟨the pair Wu and Jia⟩ cannnot be measured (*and exhausted*) [(*exhaustively*)] [exactly] ⟨with any nontrivial number⟩, these two numbers Wu and Jia are the smallest ⟨pair of⟩ numbers among the many ⟨pairs of⟩ numbers the compared pairs of which are equivalent, according to the thirty-fifth section of this chapter. Since they are the smallest ⟨pair of⟩ numbers, according to the thirty-fourth section of this chapter, the number Wu measures (*and exhausts*) Yi [(*exhaustively*)] [exactly] and the number Jiya measures (*and exhausts*) Ji [(*exhaustively*)] [exactly] when the two numbers Wu and Jiya [(*exhaustively*)] [exactly] measure (*and exhaust*) the two numbers Yi and Ji, the compared pair of which is equivalent ⟨to the pair of the former numbers⟩. If so, since we said before that the number Wu measured (*and exhausted*) Bing [(*exhaustively*)] [exactly], Wu [(*exhaustively*)] [exactly] measures (*and exhausts*) the two numbers Bing and Yi certainly. If ⟨Wu⟩ measures (*and exhausts*) the two numbers Bing and Yi [(*exhaustively*)] [exactly], then these two numbers Bing and Yi are not ⟨a pair of⟩ numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number. If they are not ⟨a pair of⟩ numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] ⟨with any nontrivial number⟩, then it contradicts the statement of the theme. If it contradicts the theme, then we can see that if there is no number which measures (*and exhausts*) the two numbers Jiya and Yi and the other number Bing [(*exhaustively*)] [exactly] then certainly there is no number which in fact measures (*and exhausts*) the numbers Yi and Ding[15] and the number Bing [(*exhaustively*)] [exactly].

II.21.3. Notes

(f.67b, l.9-f.68a, l.1) ere fiyelen -i gūsin emuci meyen

Section 31 of this manuscript deals with the following

PROPOSITION. If two plane numbers $a = cd$ and $b = ef$ (c, d, e, f are natural numbers) satisfies $a = b$, then we have $c : e = f : d$.

This proposition corresponds to the second half of Prop.19 in Book VII of the *Elements*. The first half of that proposition corresponds to the proposition in Section 30 of this manuscript.

(f.68a, l.5) ere fiyelen -i gūsin sunjaci meyen

Section 35 of this manuscript deals with the following

PROPOSITION. Take two natural numbers a, b and fix them. Define a set M of pairs of natural numbers by $M := \{(x, y) | x : y = a : b\}$. If a and b are relatively prime, then the pair (a, b) is the smallest in M.

This proposition corresponds to Prop.21 in Book VII of the *Elements*.

(f.68a, ll.6-7) umesi ajige ton

Throughout this manuscript, a Manchu adverb ‘umesi’ (in English, ‘very’) expresses the superlative of adjectives such as ‘amba’ (in English, ‘large’) or ‘ajige’ (in English, ‘small’) in mathematical contexts.

(f.68a, l.7) ere fiyelen -i gūsin duici meyen

Section 34 of this manuscript deals with the following

PROPOSITION. Take two natural numbers a,b and fix them. Define a set M of pairs of natural numbers by $M := \{(x, y) | x : y = a : b\}$. If the pair (a, b) is the smallest in M, then for any element (c, d) of M the number a measures c and the number b measures d.

This proposition corresponds to Prop.20 in Book VII of the *Elements*.

(f.68b, ll.5) i. ding serengge.

‘i’ in this phrase is redundant, but was not deleted by the corrector. Here we followed the original text.

[15] The phrase ‘the numbers Yi and Ding’ should be replaced with ‘the number Ding’ again. See II.21.3.

II.21.4. Remarks

This section deals with the following

PROPOSITION. Let *a*, *b*, *c* be natutral numbers. If *a* and *b* are relatively prime and, at the same time, *b* and *c* are relatively prime, then *ab* and *c* are also relatively prime.

This proposition corresponds to Prop.24 in Book VII of the *Elements*.

II.22. SECTION 41

II.22.1. Romanized Manchu Texts

dehi emuci. /
emu ton de [(*wacihiyame*)] [lak seme] kemne[burakū](*me wacihiyaburak*ū) juwe ton bifi. ere juwe / ton -i meni meni (*tob*) [duin] durbejengge. iliha durbejengge ton / araha manggi. ere araha juwete (*tob*) [duin] durbejengge ton. / jai juwete iliha durbejengge ton. juwe juwe -i gemu emu / ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ombi:
duibuleci. giya / sere juwe. i sere ilan -i juwe ton. emu ton de [lak] [(*wacihiyame*)] / [seme] kemne[buci](*me wacihiyabuci*) ojorakū oci. ere juwe sere. ilan serengge -i / (*tob*) [duin] durbejengge ton ara: duin sere. uyun serengge be. (f.73a)// bing. ding obufi. juwe iliha durbejengge ton -i jakūn. orin nadan be araha manggi. / u gi obu: ere araha meni meni juwete durbejengge ton. gemu emu ton de [lak] [(*wacihiyame*)] / [seme] kemne[buci](*me wacihiyabuci*) ojorakū ombi: adarame seci. giya i sere juwe ton. emu ton de / [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū turgunde. ere fiyelen -i gūsin uyuci meyen -i songkoi. / giya -i araha (*tob*) [duin] durbejengge bing sere ton. jai i sere ton. urunakū emu / ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ombi: ere bing i sere juwe ton. emu ton de / [(*wacihiyame*)] [lak seme] kemne[burakū](*me wacihiyaburak*ū) be dahame. inu ere fiyelen -i gūsin uyuci meyen -i songkoi. i -i / araha (*tob*) [duin] durbejengge ding sere ton. jai bing sere ton. inu urunakū emu ton de / [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ombi: ere ding bing sere juwe ton. da bihe giya i (f.73b)// sere juwe ton -i juwe (*tob*) [duin] durbejengge ofi. tuttu da bihe juwe ton -i juwe (*tob*) [duin] durbejengge. / emu ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū be saci ombi: geli julergi de gisurehe / songkoi. giya i sere juwe ton. emu ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū oci. geli / ere fiyelen -i gūsin uyuci meyen -i songkoi. giya sere ding sere juwe ton. emu ton de / [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ombi: bing i sere juwe ton. inu emu ton de [lak] [(*wacihiyame*)] / [seme] kemne[buci](*me wacihiyabuci*) ojorakū ojoro. geli bing ding sere juwe ton. inu emu ton de [lak] [(*wacihiyame*)] / [seme] kemne[buci](*me wacihiyabuci*) ojorakū oci. giya bing sere juwe ton emke emken -i. i ding sere juwe ton -i / emke emken de emgileci. ere juwete. gemu emu ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me

wacihiyabuci) ojorakū be / dahame. julergi meyen -i songkoi. giya be bing de (teherebuci) [[??]]. u sere ton be bahara. i be (f.74a)// ding de (teherebuci) [[??cici]] gi sere ton be bahara turgunde. ere bahara u gi sere juwe ton. / urunakū emu ton de [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū ombime. ere u gi sere juwe ton. / da bihe giya i sere juwe ton de araha iliha durbejengge ton ombi:
aikabade / geli giya i be. u gi de (teherebume kamcifi) [[??mcime teherebufi.]] bahara juwe ton. jai ere jergi / geren ton -i juwete durbejengge ton be. inu ere songkoi leoleci. gemu emu ton de / [(*wacihiyame*)] [lak seme] kemne[buci](*me wacihiyabuci*) ojorakū be saci ombi: (f.74b)//

II.22.2. Translation

The forty-first,
⟨Suppose that⟩ we have two numbers which cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, and after we have constructed numbers of the (*square*) [cubic] type and numbers of the rectangular-parallelopipedic type from each of these two numbers, ⟨firstly⟩ the pair of numbers of the (*square*) [cubic] type constructed ⟨in this process⟩, and secondly, the pair of numbers of the rectangular-parallelopipedic type, cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number in each case.

For example, if two numbers Jiya and Yi, which are equal to 2 and 3 respectively, cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, then construct numbers of the (*square*) [cubic] type. After we have denoted the number 4 or 9 by Bing or Ding ⟨respectively⟩ and constructed 8 and 27, which are two numbers of the cubic type, denote them by Wu and Ji ⟨respectively⟩. All of these constructed pairs, which consist of numbers of the square types, cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number. Concerning how it occurs, since the two numbers Jiya and Yi cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, ⟨firstly⟩ the number Bing, which is of the (*square*) [cubic] type and was constructed from Jiya, and secondly, the number Yi, certainly cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, according to the thirty-ninth section of this chapter. These two numbers Bing and Yi cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, so it follows that ⟨firstly⟩ the number Ding, which is of the (*square*) [cubic] type and was constructed from Yi, and secondly, the number Bing, also cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number certainly, according to the thirty-ninth section of this chapter again. These two numbers Ding and Bing are the two squares of the two numbers Jiya and Yi, which we had at the begininng, ⟨respectively,⟩ therefore we can see that the two squares of the two numbers which we had at the begininng cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number.

Moreover, according to what was stated before, if the two numbers Jiya and Yi cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any number, then according to the thirty-ninth section of this chapter again, the two numbers Jiya and Ding cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number. If the two numbers Bing and Yi also cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, and in addition, if the two numbers Bing and Ding also cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, then when we couple each of the two numbers Jiya and Bing with each of the two numbers Yi and Ding, all of these pairs cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number, and since we obtain a number Wu when we balance Jia with Bing and since we obtain a number Ji when we balance Yi with Ding, according to the previous section, it follows from this ⟨incommensurability⟩ that these two numbers Wu and Ji, which are obtained ⟨in this process⟩, certainly cannnot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number and, at the same time, these two numbers Wu and Ji are numbers of the rectangular-parallelopipedic type which were constructed from the two numbers Jiya and Yi, which we had at the beginning.

Moreover, if we ⟨firstly⟩ discuss the two numbers which are obtained by placing Jiya and Yi with Wu and Ji close together ⟨respectively⟩ and bringing balance between them, and secondly discuss pairs of the numbers of the square types, which are many numbers of this kind, in this way, then we can see that all of them cannot be measured (*and exhausted*) [(*exhaustively*)] [exactly] with any ⟨nontrivial⟩ number.

II.22.3. Notes

(f.73a, l.3) **(*tob*) [duin] durbejengge**

As we explained in the comments on Section 6 and 7, in this manuscript the meaning of a phrase 'tob durbejengge' is 'of a regular square' or 'of regular squares', and the meaning of a phrase 'duin durbejengge' is 'of a cube' or 'of cubes'. Since the theorem of this section is the incommensurability of the squares or the cubes of two relatively prime numbers, 'tob durbejengge' is correct in this case.

The explanations for a Manchu adjective 'durbejengge' given in the dictionaries written in the Qing period are rather obscure. It seems to us that this adjective means 'having corners' in some cases, and it is possible that the author of this manuscript misunderstood that the adjective phrase 'duin durbejengge' means 'having four corners' when he wrote this part of the manuscript.

(f.73a, l.3) **iliha durbejengge**

A Manchu adjective phrase 'iliha durbejengge' appears twenty times in this manuscript. It means 'of a rectangular parallelopiped' or 'of rectangular parallelopipeds' in the wider sense in 80 % of its examples, and in the rest of them it means 'of a cube' or 'of cubes'. The latter cases, which contain the example in this line, are found only in this section.

Note that in most of this manuscript Manchu equivalents for an adjective pharse ‘of a cube’ or ‘of cubes’ are ‘duin durbejengge’ (there are 13 examples in total) and ‘durbejengge beyei’ (there are 6 examples in total and all of them are found in Section 51 and Section 56), so the use of ‘iliha durbejengge’ as an equivalent for ‘of a cube’ or ‘of cubes’ is irregular. Serious errors in the terminology of geometry occurred in this section.

(f.73a, l.4) ere fiyelen –i gūsin uyuci meyen

Section 39 of this book deals with the following

PROPOSITION. If two natural numbers a and b are relatively prime, then the square a^2 of a and the number b are relatively prime.

This proposition corresponds to Def.25 in Book VII of the *Elements*.

(f.73b, l.9) ere ding bing sere juwe ton

Although the correct phrase is ‘ere bing ding sere juwe ton’, the corrector made no correction here.

(f.74a, l.9) julergi meyen

Section 40 of this book deals with the following

PROPOSITION. Suppose that four natural numbers a_1, a_2, b_1, b_2 are given. If a_i and b_j are relatively prime for each i and j, then the products a_1a_2 and b_1b_2 are relatively prime.

This proposition corresponds to Def.26 in Book VII of the *Elements*.

II.22.4. Remarks

This section deals with the following

PROPOSITION. Let a, b be natutral numbers. If a and b are relatively prime, then their squares a^2 and b^2 are relatively prime. Their cubes a^3 and b^3 are also relatively prime.

The last remark in this section says that a^4 and b^4 are also relatively prime. This proposition corresponds to Prop.27 in Book VII of the *Elements*.

II.23. SECTION 42

II.23.1. Romanized Manchu Texts

dehi juweci. /

yaya ton de kemneme wacihiyaci ojorakū emu ton de / gūwa emu ton be kemneme wacihiyaci ojorakū oci. ere / juwe ton be inu emu ton de kemneme wacihiyaci ojorakū / ombi:

duibuleci. ton de kemneme wacihiyaci ojorakū giya / sere ton -i sunja de. gūwa bing sere ninggun -i ton be / kemneme wacihiyaci ojorakū oci. ere giya. bing sere sunja. / ninggun

-i juwe ton be urunakū yaya emu ton de kemneme / wacihiyaci ojorakū ombi: adarame seci. aikabade giya (f.75a)// bing sere juwe ton be. kemneme wacihiyaci ojoro emu ton bi seci. ere giya sere ton. ton de / kemneme wacihiyaci ojorakū ton waka ombi: unenggi ton de kemneme wacihiyaci ojorakū ton / waka oci. timu -i gisun de acanarakū ombi: timu de acanarakū ofi. ere giya sere / bing sere juwe ton be urunakū emu ton de kemneme wacihiyaci ojorakū be saci ombi: (f.75b)//

giya i bing
nadan ilan jakūn

ding
orin emu

u gi
emu emu

The figure in f.75a

II.23.2. Translation

The forty-second,

If we cannot measure and exhaust a number with another number which we cannot measure and exhaust with any ⟨nontrivial⟩ number, then we cannot to measure and exhaust these two numbers with any ⟨nontrivial⟩ number, too.

For example, when we cannot measure and exhaust a number Bing, which is equal to 6, with a number Jiya, which is equal to 5 and cannot be measured and exhausted with any ⟨nontrivial⟩ number, we cannot measure and exhaust these two numbers Jiya and Bing, which are equal to 5 and 6 respectively, with any ⟨nontrivial⟩ number certainly. Concerning how it occurs, if we assume that there is a ⟨nontrivial⟩ number which can measure and exhaust these two numbers Jiya and Bing, this number Jiya is not a number which we cannot measure and exhaust with any ⟨nontrivial⟩ number. If it is really not a number which we cannot measure and exhaust with any ⟨nontrivial⟩ number, then it contradicts the statement of the theme. Since it contradicts the theme, we can see that we certainly cannot measure and exhaust these two numbers Jiya and Bing with any ⟨nontrivial⟩ number.

II.23.3. Remarks

This section deals with the following

PROPOSITION. Let p be an arbitrary prime number and q be a natural number which cannot be measured by p. Then p and q are relatively prime.

This proposition corresponds to Prop.29 in Book VII of the *Elements*.

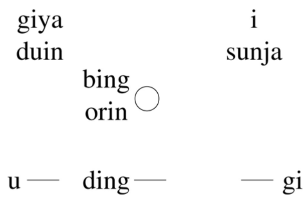
II.24. SECTION 45

II.24.1. Romanized Manchu Texts

dehi sunjaci. /

yaya ton de kemneme wacihiyaci ojorakū juwe ton bifi. ere juwe ton be / emke emken -i kemneme wacihiyaci ojoro geren ton -i dorgi umesi ajige ton be / bairengge:

duibuleci. ton de kemneme wacihiyaci ojorakū giya sere duin. i / sere sunja. juwe ton bifi. ere juwe ton be. emke emken -i kemneme wacihiyaci / ojoro geren ton -i dorgi umesi ajige ton be baiki seci. ere giya. i / sere duin. sunja -i juwe ton be. ishunde teherebufi bahara orin be bing / obuha de. uthai bahara ton inu ombi: adarame seci. giya i sere / juwe ton be ishunde teherebufi bing sere ton be bahaci. ere fiyelen -i (f.80a)// orin ilaci meyen -i songkoi. tere giya i sere juwe ton. emke emken -i bing sere emu ton be kemneme / wacihiyaci ombime. geli giya i sere emke emken -i kemneme wacihiyara geren ton -i dorgide. bing ci ajigen ningge / akū ombi: aikabade bing ci ajigen ningge bi seci. cendeme ding [obu[me](*ci leoleci*)] (*leoleki*) [arafi kemne]. unenggi ding ni / ton de. giya i -i ton be emke emken -i kemneme wacihiyaci oci. urunakū meni meni emu ton be / songkolofi kemnerengge: aikabade giya -i ton be jafafi. u -i ton -i songkoi. ding ni ton be kemneme / wacihiyaci oci. i -i ton de. inu gi -i songkoi ding ni ton be kemneme wacihiyaci ojoro be / dahame. ere fiyelen -i gūsin [(*emuci*)] [sunjaci] (*ilaci*) meyen -i songkoi giya be i de duibulerengge. gi be u de / duibulere (*duibulen -i*) adali ombi: julergi de giya i sere juwe ton be. ton de kemneme wacihiyaci ojorakū / ton sehe be dahame. ere fiyelen -i gūsin sunjaci meyen -i songkoi giya i sere juwe ton. ishunde (f.80b)// (*duibulere*) duibulen adali geren ton -i dorgi umesi ajige ton ombi: umesi ajige ton ofi. ere fiyelen -i / [(*gūsin emuci*)] [gūsin duici] (*gūsin duici*) meyen -i songkoi urunakū ishunde (*duibulere*) duibulen adali (*u*) gi [u] sere juwe ton be kemneme / wacihiyaci ombi: giya de. gi be kemneme wacihiyara. i de u be kemneme wacihiyara be dahame. / giya. i be ishunde teherebuci. bing sere ton be bahambime. geli giya be u de teherebuci. / ding sere ton be bahambi: uttu ofi ere fiyelen -i orin sunjaci meyen -i songkoi. i be (*gi*) [u] de / duibulerengge. bing be ding de duibulere (*duibulen -i*) adali ombi: geli julergi de. i de (*gi*) [u] be / kemneme wacihiyaci ombi sehe be dahame. bing de urunakū ding be kemneme wacihiyaci ombi: / bing de. ding be kemneme wacihiyaci oci. bing serengge. urunakū ding ci ajige ojoro. / ding serengge. bing ci amba ojoro be saci ombi: (f.81a)//



The figure in f.80a

II.24.2. Translation

The forty-fifth,

⟨Suppose that⟩ we have two numbers which we cannot measure and exhaust with any number. To seek the smallest number among many numbers which can measure and exhaust these two numbers one by one[16] ⟨is as follows⟩.

For example, ⟨suppose that⟩ we have two numbers Jiya and Yi, which are equal to 4 and 5 respectively and cannot be measured and exhausted with any ⟨nontrivial⟩ number, and if we want to seek the smallest number of many numbers which can measure and exhaust these two numbers one by one, ⟨the desired number⟩ is in fact the number which is immediately obtained when we bring balance between Jiya and Yi, which are equal to 4 and 5 respectively, and denote the number 20, which is obtained ⟨in this process⟩, by Bing. Concerning how it occurs, if we place two numbers Jiya and Yi close together and obtain the number Bing, then according to the twenty-third section of this chapter those two numbers Jiya and Yi can measure and exhaust the number Bing one by one and, at the same time, there is nothing smaller than Bing among many numbers which measure and exhaust Jiya and Yi one by one. If we assume that there is something smaller than Bing, [denote it by Ding and measure. If] [(*If we denote it by Ding and discuss it, and if*)] (*let us* ⟨*denote it by Ding and*⟩ *try to discuss* ⟨*this*⟩ *Ding. If*) we can really measure and exhaust the numbers Jiya and Yi with the number Ding one by one, then ⟨the two numbers Jiya and Yi are⟩ certainly what we measure by following a number in each case. If we take the number Jiya and can measure and exhaust the number Ding with it by following a number Wu, then we can measure and exhaust the number Ding with the number Yi by following a number Ji, so it follows that the result of comparing Jiya with Yi is equivalent (*to the compared pair*) given by comparing Ji with Wu, according to the thirty[(*-first*)][-fifth](*-third*) section of this chapter. We said before that the two numbers Jiya and Yi are numbers which cannnot be measured and exhausted with with any ⟨nontrivial⟩ number, so it follows that the two numbers Jiya and Yi give the smallest ⟨sequence of⟩ numbers among ⟨sequences of⟩ many numbers the compared pairs of which are equivalent, according to the thirty-fifth section of this chapter. Since they are the smallest numbers, they can certainly measure and exhaust the two numbers (*Wu and*) Ji [and Wu], the compared pair of which is equivalent, according to the [(*thirty-first*)] [thirty-fourth] (*thirty-fourth*) section of this chapter. Since we ⟨can⟩ measure Ji with Jiya, and (*Wu*) [Ji] with Yi, we obtain the number Bing if we bring balance between Jiya and Yi and, at the same time, we also obtain the number Ding if we balance Jiya with Wu. Such being the case, according to the twenty-fifth section of this chapter, the result of comparing Yi with Ji is equivalent to the compared pair given by comparing Bing with Ding. Moreover, we said before that we could measure and exhaust Ji with Wu, so it follows that we can certainly measure and exhaust Ding with Bing. If we can measure and exhaust Ding with Bing, then we can see that Bing is certainly smaller than Ding and

[16] This statement is wrong. See II.24.3.

that Ding is larger than Bing.

II.24.3. Notes

(f.80a, l.2-3) ere juwe … geren ton

This section deals with an algorithm which gives the least common multiple of two relatively prime numbers, but the summary of this section given as the first sentence is wrong. The accusative case suffix 'be' in this noun phrase is a grammatical error. It should be replaced with the dative-locative suffix 'de'. Moreover, the conditional converb 'wacihiyabuci' should be repalaced with the the conditional converb of the causative form 'wacihiyabu-' of the verb 'wacihiya-'. The correct phrase and its meaning are as follows:

> ere juwe ton be / emke emken -i kemneme wacihibuyaci ojoro geren ton
> (many numbers which we can be measured and exhausted with these two numbers one by one)

The Manchu accusative case suffix 'be' and the converb 'wacihiyaci' in 'ere juwe … geren ton' (f.80a, ll.5-6) have the same problem. The corrector made no correction in both cases. Here we followed the original text.

(f.80a, l.9-f.80b. l.1) ere fiyelen -i orin ilaci meyen

Section 23 of this manuscript deals with the following

PROPOSITION. Let a, b be arbitrary natural numbers. Their product ab can be measured by a and is equal to the result of repeated summation $a+\cdots+a$ (b times).

(f.80b, l.4-5) urunakū meni meni … songkolofi kemnerengge:

If a number is a multiple of another number, then there exists the third number which gives the multiplicity. This sentence is a remark on the existence of the third number.

(f.80b, l.5-6) aikabade giya … wacihiyaci ojoro

This part tells the following facts; the number Ding is a multiple of the number Jiya and its multiplicity is equal to Wu. The number Ding is a multiple of the number Yi and its multiplicity is equal to Ji.

(f.80b, l.7) ere fiyelen -i gūsin [(*emuci*)] [sunjaci] (*ilaci*) meyen -i songkoi

From the mathematical point of view, 'gūsin sunjaci meyen' is wrong and 'gūsin *emuci* meyen' is correct. Section 33 of this manuscript deals with the following

PROPOSITION. Let a be an arbitrary natural number and b, c be its divisors. Then we have $(a/b):(a/c)=c:b$.

Hence 'gūsin *ilaci* meyen' is possible when we have a proposition giving the relation between measurement and division.

(f.81a, ll.1-2) **ere fiyelen -i [(*gūsin emuci*)] [gūsin duici] (*gūsin duici*) meyen -i songkoi**

From the mathematical point of view, 'gūsin duici meyen' is correct and 'gūsin *emuci* meyen' is wrong.

II.24.4. Remarks

This section deals with an algorithm for finding the least common multiple of two relatively prime numbers. The contents of this section are found in Prop.34 and, possibly, Prop.39 in Book VII of the *Elements* and a part of Sec.21 of Vol.1 of *SfYbSLJY*. Note that the statement in *SLJY* does not refer to the minimality of common multiple found by this algorithm. It also does not refer to the condition on the relative primeness of the two natural numbers.

II.25. SECTION 50

II.25.1. Romanized Manchu Texts

susaici. /

yaya duibulere duibulen -i geren ton bifi. aikabade ujui ton. / wajima ton. gūwa ton de kemneme wacihiyaci ojorakū ton oci. / ere geren ton. ishunde duibulere duibulen adali geren ton -i / dorgi umesi ajige ton ombi:

duibuleci giya -i duin. i -i ninggun. bing ni / uyun -i duibulen adali ton bifi. aikabade ujui ton -i giya -i duin. / wajima ton -i bing ni uyun be. gūwa emu ton de kemneme wacihiyaci / ojorakū oci. ere geren ton. uthai ishunde duibulen adali geren ton / dorgi umesi ajige ton ombi: adarame seci. aikabade ishunde (f.90a)// duibulen adali geren ton -i dorgi de. duin. ninggun. uyun ci ajige ton bi seci. / cendeme ding u gi sere be arafi leoleki. giya i bing ni ishunde duibulerengge. / ding u gi de adali oci. gi ho yuwan ben -i ningguci fiyelen -i juwan ilaci meyen -i / songkoi. giya be bing de duibulere duibulen. ding be gi de duibulere duibulen -i / adali ombi: giya. bing ni ton be kemneme wacihiyara ton akū turgunde. ere fiyelen -i / gūsin sunjaci meyen -i songkoi ishunde duibulen adali geren ton -i dorgi umesi / ajige ton ombi: geli ere fiyelen -i gūsin duici meyen -i songkoi. giya bing sere / ton. ishunde duibulen adali geren ton be kemneme wacihiyaci ojoro be dahame. uthai / ding gi be kemneme wacihiyaci ombi: ding gi be kemneme wacihiyaci. ding gi. giya bing ci (f.90b)// amba be dahame. giya bing ci ajige ton akū be saci ombi: {ere gese ton be. eici sirandume / duibulere duibulen ocibe. eici / sirandume duibulere duibulen akū ocibe. / gemu duwali songkoi bodombi:} (f.91a)//

giya i bing
duin ninggun uyun

ding — u — — gi

The figure in f.90a

II.25.2. Translation

The fiftieth,

⟨Suppose that⟩ we have many arbitrary numbers of compared pairs, and if we cannot measure and exhaust the first number and the last number with another ⟨nontrivial⟩ number, then these many numbers give the smallest ⟨sequence of⟩ numbers among ⟨sequences of⟩ many numbers the compared pairs of which are equivalent ⟨to the given compared pairs⟩.

For example, ⟨suppose that⟩ we have ⟨three⟩ numbers Jiya and Yi and Bing, which are equal to 4 and 6 and 9 respectively and give equivalent compared pairs, and if we cannot measure and exhaust the first number Jiya and the last number Bing, which are equal to 4 and 9, with another different number, then these many numbers ⟨Jiya and Yi and Bing⟩ just give the smallest ⟨sequence of⟩ numbers among ⟨sequences of⟩ many numbers the compared pairs of which are equivalent ⟨to the given compared pairs⟩. Concerning how it occurs, if we assume that there is ⟨a sequence of⟩ numbers which is smaller than ⟨the sequence given by⟩ 4 and 6 and 9 among ⟨sequences of⟩ many numbers the compared pairs of which are all equivalent, then let us try to ⟨take this smaller sequence and⟩ write down numbers Ding and Wu and Ji and discuss them. If the results of comparing Jiya and Yi and Bing are equivalent to the results on Ding and Wu and Ji, then according to the thirteenth section of the sixth chapter of “The Principles of Quantities”, the compared pair given by comparing Jiya with Bing is equivalent to the compared pair given by comparing Ding with Ji. Since there is no number which measures and exhausts the numbers Jiya and Bing, according to the thirty-fifth section of this chapter, they give the smallest ⟨sequence of⟩ numbers among ⟨sequences of⟩ many numbers the compared pairs of which are equivalent. Moreover, according to the thirty-fourth section of this chapter, the numbers Jiya and Bing can measure and exhaust many numbers the compared pairs of which are equivalent ⟨to them⟩, so it follows immediately that they can measure and exhaust Ding and Ji. If they measure and exhaust Ding and Ji, Ding and Ji are larger than Jiya and Bing, so it follows that we can see that there is no pair of numbers smaller than Jiya and Bing. {In the same way we compute all of the numbers which are similar to these ⟨numbers⟩, whether they are compared pairs given by successive comparison, or not.}

II.25.3. Remarks

The terminology in this section is rather ambiguous. If we assume that a noun phrase ‘ishunde duibulere duibulen adali geren ton’ in the fourth line of f.90a means a finite arithmetic sequence consisting of natural numbers, in other words, if we assume that an adjective ‘sirandume’ was dropped in this noun phrase, we can say that this section gives the following

PROPOSITION. Consider the set of all finite geometric sequences consisting of natural numbers and having a given length and a given common ratio. If the first term and the last term of a geometric sequence of this kind are relatively prime, then this sequence is the smallest in the set defined above.

Our assumption can be justified from two reasons. First, the technical term in Manchu for the notion of a geometric sequence is not sufficiently fixed. Second, the example given in this section is a triple of natural numbers 4, 6 and 9 and it is a finite geometric sequence with a common ratio 3/2.

If the above explation is correct, this algorithm corresponds to Prop.1 in Book VIII of the *Elements*. Combined with the previous section, it also corresponds to a part of Sec.19 of Vol.2 of *SfYbSLJY*. Note that in *SLJY* the proposition in the previous section and the algorithm in this section are not clearly distinguished.

II.26. SECTION 51

II.26.1. Romanized Manchu Texts

susai emuci. /

emu duibulen -i ton bifi. ere duibulen -i adali. ishunde / sirandume duibulere emu udu umesi ajige ton be bairengge: /

duibuleci. giya -i jakūn be. i -i juwan juwe de duibulere / ton bifi. ere duibulen -i adali. ishunde sirandume duibulere / emu udu umesi ajige ton be baiki seci. ere fiyelen -i / dehi duici meyen -i songkoi. jakūn. juwan juwe -i ishunde / duibulere duibulen -i adali geren ton -i dorgi umesi ajige / ton be baifi. juwe. ilan be baha manggi. bing ding obu. (f.92a)// geli ere fiyelen -i orin jakūci meyen -i songkoi. juwe be ilan de duibulere / duibulen -i adali ishunde sirandume duibulere ton be baime. juwe be da ton de kamcifi. / bahara duin be. u obu: geli juwe. ilan be ishunde teherebufi. bahara ninggun be / gi obu: geli ilan be da ton de kamcifi. bahara uyun be geng obu: ere bahara / duin. ninggun. uyun -i ilan ton. uthai jakūn. juwan juwe -i duibulere duibulen -i / adali ojoro. ishunde sirandume duibulere umesi ajige ton ombi: adarame seci. / ere fiyelen -i orin jakūci meyen -i songkoi. ere u gi geng sere ilan ton. bing be / ding de. eici giya be i de duibulere adali ishunde sirandume duibulere / ton ombime. geli bing ni juwe. ding ni ilan -i juwe ton. ishunde duibulere duibulen (f.92b)// adali ton -i dorgi umesi ajige ton ofi. ere fiyelen -i dehi emuci meyen -i / songkoi. ere juwe ton de araha meni meni tob durbejengge ton. jai u -i duin. / bing ni juwe de araha tob durbejengge ton. geng ni uyun. ding ni ilan de / araha tob durbejengge ton be. gemu emu ton de kemneme wacihiyaci ojorakū ombi: emu / ton de kemneme wacihiyaci ojorakū ofi. julergi meyen -i songkoi. ere u -i duin. gi -i ninggun. / geng ni uyun. ishunde duibulen adali geren ton -i dorgi umesi ajige ton ojoro be saci ombi: /

aikabade duin ton. ishunde sirandume duibulere duibulen adali umesi ajige ton be baiki seci. / ere fiyelen -i orin jakūci meyen -i songkoi. bing ni juwe be. u -i duin de. gi -i / ninggun de. geng ni uyun de meni meni ishunde teherebuci. jakūn. juwan juwe. (f.93a)// juwan jakūn be bahambi: geli ding ni ilan. geng ni uyun be ishunde teherebuci. orin /

nadan be bahambi: ere bahara jakūn. juwan juwe. juwan jakūn. orin nadan -i duin ton. / uthai baire ishunde duibulere duibulen adali sirandume duibulere duin umesi ajige / ton inu ombi: adarame seci. ere duin ton. ere fiyelen -i orin jakūci meyen -i / songkoi. terei juwe. ilan. jai jakūn. juwan juwe -i ton. gemu adali ishunde / sirandume duibulere duibulen -i ton ombi: geli ere fiyelen -i dehi / emuci meyen -i songkoi. jakūn. orin nadan -i uju wajima -i juwe / ton. emu ton de kemneme wacihiyaci ojorakū ton ombi: emu ton de / kemneme wacihiyaci ojorakū ofi. ere fiyelen -i susaici meyen -i songkoi. ere ishunde (f.93b)// sirandume duibulere duin ton. duibulen adali geren ton -i dorgi umesi ajige ton / ojoro be saci ombi: / ere songkoi bodome leoleci. geli juwe hacin be saci ombi:
emu hacin aikabade / ishunde adali sirandume duibulere duibulen -i ilan ton -i dorgi umesi ajige ton bici. / ere ilan ton -i ujui ton. wajima ton. urunakū gemu tob durbejengge ton ombi: /
duibuleci. u -i duin. gi -i ninggun. geng ni uyun -i ilan ton oci. erei ujui / ton -i duin. wajima ton -i uyun. gemu tob durbejengge ton ojoro be saci ombi: /
aikabade ere jergi duin ton bici. ere duin ton -i ujui ton. wajima ton. urunakū / durbejengge beyei ton ombi:
duibuleci. jakūn. juwan juwe. juwan jakūn. orin nadan. ere (f.94a)// duin ton -i ujui ton -i jakūn. wajima ton -i orin nadan. gemu durbejengge beyei ton / ojoro be saci ombi: /
jai emu hacin. aikabade ere u -i duin. gi -i ninggun. geng ni uyun -i jergi ishunde / adali sirandume duibulere duibulen -i geren ton -i dorgi umesi ajige ton bici. / ere ton -i ujui ton. wajima ton. ere fiyelen -i dehi emuci meyen de gisurehe / songkoi. urunakū emu ton de kemneme wacihiyaci ojorakū ton ombi: (f.94b)//

giya	i
jakūn	juwan juwe
bing	ding
juwe	ilan

u	gi	geng
duin	ninggun	uyun

The figures in f.92a

II.26.2. Translation

The fifty-first,

⟨Suppose that⟩ we have numbers of a compared pair. To seek the smallest sequence of numbers successively compared so that ⟨all of⟩ those compared pairs are equivalent to this compared pair ⟨is as follows⟩.

For example, ⟨suppose that⟩ we have numbers given by comparing Jiya, which is equal to 8, with Yi, which is equal to 12, and if we want to seek the smallest sequence of numbers successively compared so that ⟨all of⟩ those compared pairs are equivalent to this compared pair, then according to the forty-fourth section of this chapter, we seek the smallest ⟨pair of⟩ numbers of many ⟨pairs of⟩ numbers which are equivalent to the compared pairs given by comparing 8 and 12, and after we have obtained 2 and 3, denote them by Bing and Ding respectively. Moreover, according to the twenty-eighth section of this chapter, seek numbers which are successively compared so that ⟨all of⟩ their compared pairs are equivalent to the compared pair given by comparing 2 with 3, and denote the number 4, which is obtained by placing 2 close to the original number ⟨2⟩, by Wu. Moreover, denote the number 6, which is obtained by bringing balance between 2 and 3, by Ji. Moreover, denote the number 9, which is obtained by placing 3 close to the original number ⟨3⟩, by Geng. ⟨The sequence of⟩ these three numbers 4 and 6 and 9, which is obtained ⟨in this process⟩, is just the ⟨smallest sequence of⟩ numbers successively compared so that ⟨all of⟩ their compared pairs are equivalent to the compared pair given by comparing 8 with 12. Concerning how it occurs, according to the twenty-eighth section of this chapter, these three numbers Wu and Ji and Geng are numbers which are successively compared so that ⟨all of⟩ their compared pairs are equivalent to the compared pair given by comparing Bing with Ding, or Jiya with Yi and, at the same time, the two numbers Bing and Ding, which are equal to 2 and 3 respectively, are the smallest ⟨pair of⟩ numbers among ⟨pairs of⟩ numbers the compared pair of which are equivalent, so according to the forty-first section of this chapter, ⟨firstly a pair of⟩ numbers of the square type which is constructed from these two numbers ⟨2 and 3⟩, and secondly, ⟨a pair of⟩ numbers of the square[17] type each of which is constructed from ⟨the pair⟩ Wu and Bing or ⟨the pair Geng and Ding⟩, which is equal to a pair 4 and 2 or a pair 9 and 3 respectively, cannot be measured and exhausted with any ⟨nontrivial⟩ number. Since we cannnot measure and exhaust them with any ⟨nontrivial⟩ number, according to the previous section we can see that ⟨the sequence of⟩ these numbers Wu, which is equal to 4, and Ji, which is equal to 6, and Geng, which is equal to 9, give the ⟨smallest sequence of⟩ numbers the compared pairs which are equivalent.

If we want to seek four numbers ⟨constituting⟩ the smallest ⟨sequence of⟩ numbers the successively-compared pairs of which are equivalent, then according to the twenty-eighth section of this chapter, if we balance Bing, which is equal to 2, with Wu or Ji or Geng, which is equal to 4 or 6 or 9 respectively, then we obtain 8 and 12 and 18. Moreover, if we bring balance between Ding, which is equal to 3, and Geng, which is equal to 9, then we obtain 27. In fact, these four numbers 8 and 12 and 18 and 27, which are obtained ⟨in this process⟩, just give the desired smallest ⟨sequence of⟩ four numbers successively compared so that ⟨all of⟩ their compared pairs are equivalent. Concerning how it occurs, as for these four numbers, according to the twenty-eighth section of this chapter, ⟨firstly a

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[17] There is confusion about terminology in this part of the Manchu text. '⟨a pair of⟩ numbers of the square type each of which is constructed from ⟨the pair⟩ Wu and Bing or ⟨the pair Geng and Ding⟩, which is equal to a pair 4 and 2 or a pair 9 and 3 respectively' should be replaced with '⟨a pair of⟩ numbers of the cubic type constructed from Bing and Ding, which are equal to 2 and 3 respectively'

pair of⟩ numbers 2 and 3 from them, and secondly, ⟨a pair of⟩ numbers 8 and 12, are numbers of compared pairs which are given by successive comparison and are all equivalent. Moreover, according to the forty-first section of this chapter, the two numbers 8 and 27, which are the first one and the last one ⟨respectively⟩, are numbers which we cannot measure and exhaust with any ⟨nontrivial⟩ number. Since we cannot measure and exhaust them with any ⟨nontrivial⟩ number, according to the fifty-first section of this chapter we can see that ⟨the sequence of⟩ these four numbers successively compared is the smallest ⟨sequence of⟩ numbers among the many numbers the compared pairs of which are equivalent.

If we compute and discuss ⟨these objects⟩ in this way, we can understand ⟨the following⟩ two things.

The first thing ⟨is as follows;⟩ if we have the smallest ⟨sequence of⟩ numbers among ⟨the sequences of⟩ three numbers of compared pairs successively compared so that the compared pairs are equivalent to one another, both of the first number and the last number of these three numbers are certainly numbers of the plane-square type.

For example, as for Wu and Ji and Geng, which are equal to 4 and 6 and 9 respectively, we can see that both of their first number and their last number, which are equal to 4 and 9 respectively, are numbers of the plane-square type.

If we have four numbers of this kind, the first number and the last number of these four numbers are certainly numbers of the solid-square type.

For example, as for 8 and 12 and 18 and 27, we can see that both of the first number and the last number of these four numbers are numbers of the solid-square type.

The second thing ⟨is as follows;⟩ if we have the smallest ⟨sequence of⟩ numbers such as these Wu and Ji and Geng, which are equal to 4 and 6 and 9 respectively, among the many numbers of compared pairs which are given by successive comparison and are equivalent, then the first number and the second number of these numbers are certainly numbers which we cannot measure and exhaust with a ⟨common⟩ number, according to what was stated in the forty-first section of this chapter.

II.26.3. Remarks

(f.93a, ll.2-3) u -i duin. / bing ni juwe de araha tob durbejengge ton

This noun phrase is not correct. The correct phrase is 'u -i duin. bing ni juwe de araha durbejengge ton' or 'bing ni juwe de araha durbejengge beyei ton'. The adjective phrase 'tob durbejengge' in the next line has the same problem. The corrector made no corection in both cases.

II.26.4. Remarks

This section gives an algorithm for findng the smallest geometric sequence consisting of natural numbers and satisfying the condition that its common ratio is equal to a given rational number. This algorithm corresponds to Prop.2 in Book VIII of the *Elements*.

Combined with the previous section, it also corresponds to a part of Sec.19 of Vol.2 of *SfYbSLJY*.

II.27. SECTION 60

II.27.1. Romanized Manchu Texts

ninjuci. /

yaya geren ton bifi. aikabade ujui ton ci. wajima ton de / isitala gemu ilhi aname neigen nonggihangge oci. erebe / neigen nonggiha duibulere duibulen -i ton sembi:

duibuleci. / juwe. duin. ninggun. jakūn. juwan. juwan juwe -i ninggun ton / bici. erei ujui ton ci. wajima ton de isitala gemu / ilhi aname ton tome neigen juwete nonggibuha be dahame. / ere geren ton. uthai neigen nonggiha duibulere duibulen -i / ton ombi:

aikabade geren ton bifi. ujui ton ci. (f.109a)// wajima ton de isitala gemu ilhi aname neigen ubu nonggihangge oci. ere geren / ton be. ubu nonggiha duibulere duibulen -i ton sembi:

duibuleci. juwe. duin. / jakūn. juwan ninggun. gūsin juwe. ninju duin -i ninggun ton bici. ere ton. ujui / ton ci wajima ton de isitala gemu neigen emte ubu nonggibuhangge ofi. tuttu / ere ton be ubu nonggiha duibulere duibulen -i ton sembi: (f.109b)//

juwe	duin	ninggun	jakūn	juwan◯	juwan juwe
juwe	duin	jakūn	juwan ninggun	gūsin juwe	ninju duin

The figure in f.109a

II.27.2. Translation

The sixtieth,

⟨Suppose that⟩ we have many arbitray numbers, and if all of these numbers are those which were produced by adding a number uniformly one after another from the first number to the last number, then we call them numbers of compared pairs which were uniformly increased.

For example, if we have six numbers such as 2 and 4 and 6 and 8 and 10 and 12, then all of them are produced by adding 2 to each number uniformly one after another from the first number to the last number, so it follows that these many numbers are just numbers of compared pairs which were uniformly increased.

⟨Suppose that⟩ we have many arbitrary numbers, and if all of these numbers are those which were produced by adding a multiple ⟨of the preceding number⟩ with a uniform multiplicity one after another from the first number to the last number, then we call them numbers of compared pairs to which multiples were added.

For example, if we have six numbers such as 2 and 4 and 8 and 16 and 32 and 64, then all of them are produced by increasing each number by itself uniformly once one after the other from the first number to the last number, therefore these many numbers are called numbers of compared pairs to which multiples were added.

II.27.3. Notes

(f.109a, l.3) neigen nonggiha duibulere duibulen -i ton

This Manchu noun phrase means 'a finite arithmetic sequence which is strictly increasing'. The object of the Manchu verb 'nonggi-' is the common difference (see Section 75). Each term of the sequence are the object of the Manchu verb 'nonggibu-', which is the passive/causative form of the verb 'nonggi-'.

II.27.4. Remarks

This section contains two definitions. One is the definition of a finite arithmetic sequence, and the other is the definiton of a geometric sequence with the common ratio 2. This section corresponds to a part of Sec.31 of Vol.2 of *SfYbSLJY*.

II.28. SECTION 62

II.28.1. Romanized Manchu Texts

ninju juweci. /

yaya neigen nonggibuha geren ton bifi. aikabade ere ton -i / dorgi juwe ton be acabuci. bahara ton. juwe dalbai / ishunde adali oron giyalabuha juwe ton be acabuha ton de / teherembi:

duibuleci giya -i sunja. i -i jakūn. bing ni / juwan emu. ding ni juwan duin. u -i juwan nadan. gi -i / orin. geng ni orin ilan. sin -i orin ninggun. žin -i orin / uyun -i uyun ton bici. ere uyun ton de neigen nonggibuha / ilan -i ton be gui obu: ere uyun ton -i dorgi. yaya (f.112a)// juwe ton be ishunde acabure de. aikabade bing geng sere juwe ton be acabuci. / gūsin duin -i ton be bahambi: ere bing geng ni juwe dalbade bisire ishunde adali / oron giyalabuha giya. žin -i juwe ton be acabuci. bahara ton. inu gūsin duin / ofi teherembi: adarame seci. ere geren ton neigen nonggibuha ton be dahame. / ere fiyelen -i ninjuci meyen -i songkoi žin sere ton. sin sere ton ci fulu / ojorongge. gui -i ton ombi: i sere ton. giya sere ton ci fulu ojorongge. / inu gui -i ton ombi: fulu ojorongge gemu gui -i ton be dahame. aikabade žin / sere ton -i sin sere ton ci fulu oho gui -i ilan be gaifi. giya -i / sunja de acabuci. jakūn ombi: uttu oci. giya -i ton. i de teherembi: (f.112b)// sin -i ton. žin de teherembi: gemu ishunde teherere be dahame. aikabade giya be / žin de. i be sin de meni meni juwete ton be ishunde acabuci. bahara juwe / ton. urunakū meni meni ishunde teherembi: ere songkoi leoleci. i be sin de. / bing be geng de meni meni juwete ton be ishunde acabuci. bahara juwe ton. inu / urunakū meni meni ishunde teherembi: ere uttu teherere turgunde. giya be žin de. / bing be geng de meni meni juwete ton be ishunde acabuci. bahara juwe ton. gemu / meni meni ishunde teherere be saci ombi: (f.113a)//

II.28.2. Translation

The sixty-second,

⟨Suppose that⟩ we have many arbitrary numbers which are uniformly increased, and if we combine two numbers among these numbers, then the number which we obtain ⟨in this process⟩ is equal to the number given by combining the two numbers which lie on the both sides and are separated by the same number of places.

For example, if we have nine numbers such as Jiya and Yi and Bing and Ding and Wu and Ji and Geng and Xin and Ren, which are equal to 5 and 8 and 11 and 14 and 17 and 20 and 23 and 26 and 29 respectively, then denote the number 3, which was uniformly added ⟨to them⟩, by Gui. When we combine two arbitrary numbers among these nine numbers, if we combine the two numbers Bing and Geng, then we obtain a number 34. If we combine the two numbers Jiya and Ren, which lie on the both sides of these Bing and Geng and are separated by the same number of places, then the number which is obtained ⟨in this process⟩ also turns out to be 34 and is equal ⟨to the sum of Bing and Geng⟩. Concerning how it occurs, since these many numbers are the numbers which were uniformly increased, according to the sixtieth section of this chapter, the number by which the number Ren exceed the number Xin is equal to the number Gui. The number by which the number Yi exceed the number Jiya is also equal to the number Gui. The numbers by which one exceed another are all equal to Gui, so it follows that if we get Gui, which is equal to 3 and by which the number Ren exceed the number Xin, and if we combine ⟨Gui⟩ with Jiya, which is equal to 5, then we have 8. If so, the number Jiya will be equal to Yi. The number Xin will be equal to Ren. Since they will be equal in all cases, if we combine each pair, ⟨in other words⟩, Jiya with Ren and Yi with Xin respectively, then the two numbers which are obtained ⟨in this process⟩ are certaily equal. If we follow this way and discuss ⟨this phenomena, we see that⟩ if we combine each pair, ⟨in other words⟩, Yi with Xin and Bing with Geng respectively, then the two numbers which are obtained ⟨in this process⟩ are also certainly equal. Since these ⟨numbers⟩ are equal in this manner, we can see that if we combine each pair, ⟨in other words,⟩ Jiya with Ren and Bing with Ding respectively then all of the two numbers which are obtained ⟨in this process⟩ are equal.

II.28.3. Remarks

This section deals with the following

PROPOSITION. Let $a_1, a_2, \cdots, a_n$ be a finite arithmetic sequence which is strictly increasing. Then for any natural numbers *i*, *j*, *d* satisfying the inequalities $d < i < j < n - d$ we have $a_i + a_j = a_{i-j} + a_{j+d}$.

A special case of this propositions found in Sec.11 of Vol.2 of *SfYbSLJY*.

II.29. SECTION 63

II.29.1. Romanized Manchu Texts

ninju ilaci. /
yaya neigen nonggibuha duibulere duibulen -i ton bifi. erei / yaya emu ton -i juwe dalbai ton be acabufi bahara ton be / dulin bukdaci. uthai dulimbai ton de teherembi: /
duibuleci. juwe. duin. ninggun. jakūn. juwan. juwan juwe -i / neigen nonggibuha duibulen -i ninggun ton bifi. ere ton -i / dorgi jakūn sere ton -i juwe dalbai ninggun. juwan be / acabufi bahara juwan ninggun be dulin bukdaci. bahara jakūn -i / ton. dulimbai ton -i jakūn de teherembi: adarame (f.114a)// seci. erei ton tome gemu neigen juwete nonggibuhangge ofi. ninggun -i ton. jakūn ci / juwe ekiyehun. juwan -i ton. jakūn ci juwe fulu: te fulu oho juwe be. / juwan ci ekiyembuci jakūn funcembi: ere ekiyembuhe juwe be ninggun de nonggici / jakūn ombi: uttu ofi ninggun. juwan sere juwe ton be acabuci bahara ton. / dulimbai ton -i jakūn ci emu ubu fulu ombi: erebe dulin bukdaci. ini / cisui jakūn de teherere be saci ombi:
geli yaya ere jergi juwe dalbai / adali oron giyalabuha juwe ton be acabufi dulin bukdaha ton. inu gemu dulimbai / ton de teherembi:
duibuleci. jakūn sere ton -i juwe dalbai emte oron giyalabuha / duin. juwan juwe sere juwe ton be acabufi bahara juwan ninggun -i ton be dulin (f.114b)// bukdaci. jakūn bahafi dulimbai ton -i jakūn de teherembi: erei teherere / turgun. leolen. gemu julergi emu songko: (f.115a)//

juwe	duin	ninggun	jakūn	juwan	juwan juwe

The figure in f.114a

II.29.2. Translation

The sixty-third,
If we have arbitrary numbers of compared pairs which were uniformly increased and halve the number which was obtained by combining numbers lying on the both sides of an arbitrary number, then the result is equal to the middle number.

For example, ⟨suppose that⟩ we have six numbers of compared pairs which were uniformly increased, 2 and 4 and 6 and 8 and 10 and 12, and if we halve the number 16 which is obtained by combining 6 and 10, which lie on the both sides of a number 8 in these numbers, then a number 8, which is obtained ⟨in this process⟩, is equal to the middle number 8.

Concerning how it occurs, since ⟨this sequence⟩ is that which is produced by increasing each number uniformly by 2, the number 6 falls below 8 by 2, and the number 10 exceeds 8 by 2. Now if we subtract the number 2, which is equal to the excess, from 10, then 8 remains. If we add this subtracted number 2 to 6, then the result is equal to 8. Such being the case, the number which is obtained by combining two numbers 6 and 10 exceeds the middle number 8 by once 8. We can see that if we halve ⟨this obtained number⟩ the result naturally coincides with ⟨the middle number⟩ 8.

Moreover, the number given by combining two numbers of this kind, which lie on the both sides ⟨of a number⟩ and are separated ⟨from the middle number⟩ by the same number of places and by halving ⟨the result of this combination⟩ is also equal to the middle number.

For example, if we halve a number 16, which is obtained by combining two numbers 4 and 12, which lie on the both side of the number 8 and are separated from it by one place, then we obtain 8 and it is equal to the middle number 8. As for the details of these equalities, all arguments keeps the same track as before.

II.29.3. Remarks

This section deals with the following

PROPOSITION. Let $a_1, a_2, \cdots, a_n$ be a finite arithmetic sequence which is strictly increasing. Then for any natural numbers i satisfying the inequalities $0 < i < n$ we have $\left(a_{i-1} + a_{i+1}\right)/2 = a_i$.

A special case of this propositions found in Sec.12 of Vol.2 of *SfYbSLJY*.

II.30. SECTION 75

II.30.1. Romanized Manchu Texts

nadanju sunjaci. /

yaya neigen nonggibuha duibulere duibulen -i oron -i ton / udu be sambime. neigen nonggiha ton. geren ton -i uheri / ton be inu safi. ere geren ton -i dorgi umesi amba. / umesi ajige juwe ton be bairengge:

duibuleci. giya. i. / bing. ding. u. gi sere neigen nonggibuha duibulere / duibulen -i oron -i ton -i ninggun be sambime. neigen nonggiha / ton -i duin. geren ton -i uheri ton -i nadanju jakūn be / inu safi. ere geren ton -i dorgi umesi amba. umesi (f.130a)// ajige juwe ton be baiki seci. uheri ton -i nadanju jakūn be oron -i ton -i / ninggun de nikebume faksalaci. juwan ilan -i ton be bahafi. uju wajima juwe / ton -i emu dulin -i ton ombi: geli ere juwan ilan de emu ubu nonggiha de / bahara orin ninggun. uju wajima -i juwe ton -i uheri ton ofi. ere fiyelen -i ninju / emuci meyen -i songkoi. oron -i ton -i ninggun be emke ekiyembufi. funcere sunja be. / neigen nonggiha ton -i duin de teherebuci. bahara orin. umesi amba ton ci / umesi ajige ton be ekiyembufi funcehe ton ombi: geli neneme baha orin ninggun. ere orin be / ishunde ekiyembuci. funcere ninggun umesi ajige ton de emu ubu

nonggibuha ton ombi: / ere ton be dulin bukdaci. bahara ilan. uthai baire umesi ajige ton inu: (f.130b)// ere ilan be. neneme baha orin de acabuci. bahara orin ilan. uthai baire / umesi amba ton inu:
ere arga -i fulehe. ninju emuci. ninju duici / meyen -i adali: (f.131a)//

giya	i	bing	ding	u	gi
ilan	nadan	juwan emu	tofohon	juwan uyun	orin ilan

The figure in 130a

II.30.2. Translation

The seventy-fifth,

⟨Suppose that⟩ we know the number of places of arbitrary compared pairs which were uniformly increased and, at the same time, we also know the uniformly added number and the sum of many numbers. To seek the two numbers, the largest one and the smallest one of these many numbers ⟨is as follows⟩.

For example, ⟨suppose that⟩ we know the number of places of compared pairs which were uniformly increased, Jiya and Yi and Bing and Ding and Wu and Ji, to be 6 and ⟨that⟩ at the same time we also know the uniformly added number to be 4 and the sum of many numbers to be 78, if we want to seek the two numbers, the smallest one and the largest one of the many numbers, we obtain a number 13 when we scatter the sum 78 while depending on the number 6 of places[18], and it is equal to the half of the ⟨sum of the⟩ two numbers, the first one and the last one. Moreover, the number 26, which is obtained by increasing 13 by itself, is equal to the sum of the two numbers, the first one and the second one, and according to the sixty-first section of this chapter, if we balance a number 5, which remains after we have decreased the number 6 of places by 1, with 4, which is the uniformly added number, then the number 20 which is obtained ⟨in this process⟩ is equal to the number which remained after we had subtracted the smallest number from the largest number. Moreover, a number 6, which remains when we decrease each of the number 26 which was obtained before and this number 20 by the other, is equal to the number given by increasing the smallest number by itself. The number 3, which is obtained by halving this number, is in fact just the smallest number. A number 23, which is obtained by combining this number 3 with the number 20, which was obtained before, is in fact just the largest number which we are seeking.

The roots of this method is similar to those of the sixtieth section and the sixty-first section.

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[18] The meaning of this part is ‘we obtain a number 13 when we divide the sum 78 by 6, the number of places’, but the author of this manuscript used an unusual expression here. See II.30.3.

II.30.3. Notes

(f.130b, ll.1-2) nadanju jakūn … nikebume faksalaci

In this manuscript, division is usually expressed by the following phrases containing a Manchu verb 'dende-':

(a) 'Q_1 be baitalafi Q_2 be dendembi' ('to divide Q_2 by Q_1'),

(b) 'Q_1 –i songkoi Q_2 be dendembi' ('to divide Q_2 by Q_1').

The use of a verb 'faksala-' ('to scatter' or 'to separate' in English) in this section is an unusual one.

(f.130b, ll.4-5) ere fiyelen -i ninju / emuci meyen -i songkoi

Scetion 61 of this manuscript deals with the following

PROPOSITION. Let $a_1, a_2, \cdots, a_n$ be a finite arithmetic sequence which is strictly increasing and d be its common difference. Then for each $i \in \{1, 2, \cdots, n\}$ we have $a_i = a_1 + (i-1)d$.

II.30.4. Remarks

This section gives an algorithm for finding the values of the first term and the last term of a finite arithmetic sequence which is strictly increasing from the values of its common difference, its length and its sum.

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