# The Keskintos Astronomical Inscription Text and Interpretations

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High on the crest of the hill which overlooks the plain of Keskinto, Nikias could see the ancient observatory. Masses of pink and white oleander clustered over the crumbling walls. Green crested lizards scuttled between the stones. The flat roofed building was encircled by silver-grey olive trees, heavy with fruit.

A stone stele stood in one corner of the courtyard. Nikias ran his fingers over the columns of letters and numbers which had been chiselled into the face of the stone. The names of the planets could be read at the head of the neat columns. The numbers appeared to Nikias to indicate various positional data, phases and rotations far beyond his understanding. At the bottom of the stone stele, partially obscured by the long grasses which had begun to climb its smooth face, were larger size letters. Nikias cleared aside the grass in order to be able to read these last words... 'A thanksgiving to the Gods.'

Victor Kean<sup>1</sup>

# Introduction.

The medium of inscription on stone was employed by ancient Greek astronomers for two distinct purposes. *Parapegmata*, of which the best known and best preserved examples come from Hellenistic Miletus, correlated a schematized annual cycle of weather changes with the first and last risings and settings of constellations; these were essentially public resources, representing an important aspect of what it meant to be an astronomer from the fifth century B.C. to the 2nd century A.D.<sup>2</sup> The second variety of astronomical inscription is exemplified by Ptolemy's *Canobic Inscription* (preserved in copies of a manuscript transcription from late antiquity) and the inscription with which the present article is concerned.<sup>3</sup> These were lists of elements of models for the motions of the heavenly bodies, chiefly comprising numerical data. Formally both the *Canobic Inscription* and the Keskintos Inscription are

<sup>&</sup>lt;sup>1</sup> Kean 1991, 26–27.

<sup>&</sup>lt;sup>2</sup> See the forthcoming monograph on Greco-Roman parapegmata, Lehoux 2007; for the Miletus fragments, see Lehoux 2005.

<sup>&</sup>lt;sup>3</sup> On the *Canobic Inscription* see Hamilton, Swerdlow, and Toomer 1987 and Jones 2005.

votive inscriptions, that is, they are presented as a "thank offering" (χαριστήριον) to a god or gods. A votive inscription accompanied and explicated a gift offered to a god or gods, and the question what constituted the offering in the case of these astronomical inscriptions deserves consideration. Beyond being expressions of piety, moreover, they may have served to establish personal credit or even priority for their contents.

We possess only the last part, perhaps less than half, of the Keskintos Inscription, and this is damaged and full of obscurities. References to it in recent scholarship on Greek astronomy are few and mostly glancing. Nevertheless the inscription is a document of great significance for the history of Greek astronomy, because it provides technical information about mathematical planetary theory from a time, contemporary with or a little later than Hipparchus, from which there are scarcely any other sources. Of the fifteen surviving lines of text, the top thirteen are part of a tabulation of planetary periodicities, from which it is possible to draw inferences about the nature of the mathematical models with which they were associated. The fourteenth line defines units of arc, and the last is the dedication. The lost beginning of the inscription likely identified the astronomer who erected it (cf. the *Canobic Inscription*), and may have presented other kinds of astronomical data in addition to the lost seven lines that gave the periodicities for the sun and moon. Further speculation about this lost portion is useless.

The present study offers the first edition of the inscription since its *editio princeps* published in 1894, together with an extensive interpretation of its contents—only the third such to appear, following those of Tannery (1895) and Neugebauer (1975). The reasons why this important and fascinating document has been so neglected, as well as the need for a new edition, will become evident as we review its modern history.

The Keskintos Inscription was first discussed publicly by the epigrapher F. Hiller von Gaertringen at the June 1894 meeting of the German Archeological Institute of Berlin. According to the report of the meeting in the *Archäologischer Anzeiger* (at that time published as a supplement to the Institute's *Jahrbuch*), Hiller stated that the inscription was found 8 kilometres west of Lindos on Rhodes "an dem heutigen Orte Keskindos."<sup>4</sup> The following year Hiller published the inscription as Text No. 913 in volume 12.1 of the series *Inscriptiones Graecae* (commonly abbreviated *IG*).<sup>5</sup> The circumstances of its discovery are briefly described a bit more fully in Hiller's Latin introduction to the text, which I translate here:

Block of darkish marble, width 0.78 metres, height 0.30 metres, depth 0.29 (?) metres, damaged on the top and especially on the left side of the bottom. Found in the locality that is today called Kéσκιντος, where ancient tombs indicate that there was a district belonging to the citizens of Lindos [*Lindiorum pagum*]. The place Kéσκιντος is located about half an hour, i.e. about two kilometres from the citadel of Lartos towards the west

<sup>&</sup>lt;sup>4</sup> [Anonymous] 1894, 125.

<sup>&</sup>lt;sup>5</sup> IG 12.1, 148–149 with addenda at 207.

on the northern slope of the mountain  $O\rho\theta\eta$ . The letters, decorated with large serifs, exhibit that ornamental elegance that numerous signatures of sculptors [*tot statuariorum tituli*] show was in favour in the first century B.C. The height of the letters is about 0.008 metres, but in the final line 0.014 metres. And the whole character of the writing convinces me that we should believe it to be neither much older nor much more recent than 100 B.C. Diaco(nu)s Adelphiu of Lindos brought a squeeze of the stone to Athens, from where by the kindness of the scholar Paul Wolters it was sent to me in June of 1893. I discussed it briefly at the Archeological Institute of Berlin in June, 1894; see *Jahrb. des arch. Inst.* IX 1894 *Anzeiger* fasc. III.

Diakos Adelphiu was a resident of Lindos who supplied Hiller, and previously E. Loewy, with several squeezes of inscriptions; Paul Wolters was the Second Secretary of the German Archeological Institute at Athens from 1887–1900.

Nearly half a century later, in a book review, Hiller provided some further details of the discovery and fate of the inscription:<sup>6</sup>

Er haftet am Berge Όρθη, über dem Küstenpunkte 'ς τὰ γράμματα, der auf seine Berichtigung zu diesem Namen noch einmal zu untersuchen wäre, nahe am Dorfe Λάρδος (oder Λάρτος). Der Demenname ist noch unbekannt. Dort fand mein lindischer Gastfreund Diakos Adelphiu einen Stein, brachte den Abklatsch nach Athen ins Institut, und schickte das Original später auf meine Veranlassung nach Berlin, wo es im Museum sein dürfte.

Keskintos (Kέσκιντος) is the name of a farm estate surrounded by pine forest, at 36° 5' 20" N, 27° 59' 20" E on the northwest slope of an approximately 300 metre high hill (Hiller's 'Opθη, now designated *Stafilia* on maps) situated southwest of Lardos. At present day one reaches Keskintos by travelling about 2.3 km northwest from Lardos by the road to Moni Ypsenis and then about 1 km south along an unpaved road. I have not found Keskintos marked on any modern map; on the map of Rhodes by H. Kiepert in *IG* 12.1 it is incorrectly marked on the northeast of the hill and almost directly south of Lardos.<sup>7</sup> Hiller consistently reported the place name as bearing the masculine ending  $-o_{\zeta}$ , and I have been able to confirm that this is correct by local inquiry. Hiller further suggested on etymological grounds that the original spelling should be Kέσκινθος, and it is by "Keskinthos" that the inscription is catalogued by the Antikensammlung of the Staatliche Museen zu Berlin, where it is

<sup>&</sup>lt;sup>6</sup> Hiller von Gaertringen 1942, 165–168.

<sup>&</sup>lt;sup>7</sup> A hand-drawn map of part of Rhodes from the early 20th century Danish excavations on Lindos, reproduced in *Lindos IV*, 2, 97, shows Κέσκιντος, but is evidently just an adaptation of the *IG* map. A "castle" is marked at the corresponding spot (i.e. the false location of Keskintos) on the map of Rhodes serving as frontispiece of the first volume of Newton 1865; perhaps this refers to the "antique structure built of large, cut, blue-grey limestone blocks" seen and photographed by K. F. Kinch in 1903, for which see *Lindos IV*, 2, 87. Kinch speculated that it was the remains of a Hellenistic dam.

currently kept (inventory number SK 1472). However, for some reason Tannery always used a neuter (or accusative?) form of the name, "Keskinto," and through his authority this spelling has become prevalent in the modern scholarship.

It was presumably also through his Lindian associate that Hiller heard of the presence of ancient tombs in the vicinity, on the basis of which he hypothesized that Keskintos was the site of a Hellenistic community. Until recently no further archeological work appears to have been done on the spot, but in 1994 a brief notice was published reporting surface finds at Keskintos including an "early Christian" (presumably mid first millennium A.D.) church, some other buildings, a counterweight from an olive press, fragments of tiles and pottery, and tombs.<sup>8</sup> Unless evidence of Hellenistic habitation turns up, it seems more probable that our inscription was transported to Keskintos, perhaps from as far as Lindos, during the first millennium A.D. to be reused as building material than that it should have originally been erected in such an out-of-the-way place.

Hiller's accounts imply that he did not see the inscription when it was still in Rhodes (though he was there at work on *IG* roughly at that time), and notwithstanding the later arrival of the stone in Berlin all modern research on the text seems to have depended on Diakos Adelphiu's squeeze.<sup>9</sup> In preparing his edition, Hiller consulted several philologists (including Hermann Diels and J. L. Heiberg) and astronomers (including Martin Brendel and Norbert Herz) to little profit, but his correspondence with Paul Tannery was considerably more felicitous for the explication of the meaning of the inscription.<sup>10</sup> Hiller first approached Tannery by a letter of September 18, 1894. In a letter to Hiller dated October 1 Tannery set out the first of his important insights, that the numbers in the right set of columns are consistently ten times the corresponding numbers in the left set—not a trivial observation since there were many misreadings in Hiller's transcription. In the same letter Tannery gave an interpretation of the meaning of the four distinct kinds of planetary period that was fundamentally sound. On December 12 he reported to Hiller his third insight, that all the numbers reflected a common "great period" of 29140 or 291400 years. Tannery published

<sup>&</sup>lt;sup>8</sup> Volanakis 1994.

<sup>&</sup>lt;sup>9</sup> According to Tannery 1895a (= Tannery 1912, 488) the squeeze was sent to Hiller in Berlin in June of 1893. A photograph of the squeeze was published in Herz 1894 (cf. p. 1136 note 1); another photograph of the squeeze in Tannery 1939, facing p. 119, is compromised by pencilled outlining of the letters. Neugebauer 1975, 698 note 2 states that D. Price obtained a new squeeze after the Second World War, but apparently this gave rise to no published scholarship. A squeeze is also preserved in the archive of the *Inscriptiones Graecae* at the Berlin-Brandenburg Academy of Sciences, Berlin.

<sup>&</sup>lt;sup>10</sup> For the correspondence between Hiller and Tannery see Tannery 1939, 142–187. Tannery's correspondence with Herz, on pp. 119–141, is also dedicated to the inscription. Tannery published four articles concerning the inscription in short succession. Tannery 1895a was his principal detailed discussion; Tannery 1895b is a brief summary; Tannery 1895c is a response to Herz 1894 containing some expansions of Tannery's interpretations; and Tannery 1895d concerns the metrology of line 14 in relation to ancient units of time. All were reprinted in Tannery 1912.

his interpretation of the inscription in 1895; since then the only significant contribution to the subject is Neugebauer's 1975 commentary, which addresses the problem of the nature of the planetary theory underlying the numbers.<sup>11</sup>

In August, 2005, having prepared a provisional transcription with the aid of an excellent photograph provided by the Berlin Antikensammlung, I had the opportunity to directly inspect and collate the inscription where it is currently stored in the catacombal basement of the Pergamonmuseum. Because of the weathered condition of the stone, the writing proved to be most legible when viewed with light cast from various directions almost parallel to the stone's surface; by this method it proved possible to see several traces of lettering that had been invisible to Hiller.

Ideally one would prefer to base one's interpretation of the inscription on a text read unambiguously from the stone without any assumptions about what should or can be written there. Since, however, the surface of the stone is damaged to the point where many letters cannot be securely read merely from the extant traces, the tasks of transcription and interpretation cannot proceed independently beyond the first stages. Hiller's *IG* text preceded Tannery's researches, and it reports several readings, not marked as uncertain, that Hiller replaced in the *corrigenda* after Tannery had convinced him that the numbers in the right columns should be ten times those in the left columns. Even this corrected text will appear doubtful to a careful reader of Tannery's papers and letters and Neugebauer's commentary. I believe that my new transcription maintains an appropriate middle course between paleographical agnosticism and wilfulness. I have accepted the following general assumptions as sufficiently well established on the basis of independently secure readings to be usable in limiting the admissible readings elsewhere:

1. The text in cols. i–iii of lines 1–13 is identical to that in cols. v–vii. When the inscription was complete, four consecutive lines were associated with each planet, named in cols. i and v using their non-theophoric Greek names. Cols. ii–iii and vi–vii cycle through the same four texts for each planet. I omit detailed textual notes for the readings in these columns.

2. The numbers in col. viii are ten times the numbers in col. iv.

3. Cols. iv and viii contain numbers of various kinds of periods, identified by the text in cols. ii–iii and vi–vii, associated with the planet named in cols. i and v. Col. iv gives the number of periods contained in 29140 solar years, while col. viii gives the number of periods contained in 291400 solar years. (It is not important at this stage to determine just what kind of solar year, e.g. tropical or sidereal, is intended.)

4. The periods  $\kappa \alpha \tau \dot{\alpha} \mu \eta \kappa \sigma \zeta$  in lines 2, 6, and 10 are cycles of the planet's mean or true motion in longitude, i.e. circuits of the ecliptic relative to a reference longitude (again

<sup>&</sup>lt;sup>11</sup> Neugebauer 1975 v. 2, 698–705.

this could be either sidereal or tropical). The periods  $\kappa \alpha \tau \dot{\alpha} \sigma \chi \hat{\eta} \mu \alpha$  in lines 1, 5, 9, and 13 are synodic cycles of the planet. Hence the sum of these two numbers for Mars, Jupiter, or Saturn is the number of longitudinal revolutions of the sun during the same time, i.e. exactly 29140 or 291400. The ratio of longitudinal to synodic periods for each planet is reasonably accurate.

5. The periods  $\kappa \alpha \tau \dot{\alpha} \pi \lambda \dot{\alpha} \tau \circ \zeta$  in lines 3, 7, and 11 are latitudinal periods, i.e. circuits of the ecliptic relative to the nodal line. These numbers differ only slightly from the numbers of periods  $\kappa \alpha \tau \dot{\alpha} \mu \eta \kappa \circ \zeta$ , so that the underlying theory assumes a slowly shifting nodal line.

As an aid to reading doubtful traces of the higher order digits, one can estimate the theoretically expected numbers of periods  $\kappa\alpha\tau\dot{\alpha}\,\mu\eta\kappa\sigma\varsigma$  and  $\kappa\alpha\tau\dot{\alpha}\,\sigma\chi\eta\mu\alpha$  in 29140 sidereal or tropical years from the actual durations of the planets' longitudinal and synodic periods according to modern theory (Table 1). The difference between the two kinds of year turns out to have a negligible effect on the numbers.

Anticipating the line-by-line commentary below, I would express complete confidence in my reading of all the numerals in lines 6–15, that is, the sections concerning Jupiter and Saturn and the units of arc. I am equally confident of the numerals for Mars' periods in longitude and relative position in lines 2 and 5, almost as sure of its periods in latitude in line 3, but doubtful of the correct readings for its periods in depth in line 4, where the traces in cols. iv and viii are very difficult to discern and appear to be inconsistent with respect to one of the digits. Too little of Mercury's periods in relative position in line 1 can be read to be of much use.

## **Description.**

(Cf. Fig. 1.) The stone's dimensions are 77 cm width, 31.5 cm height, 14 cm depth. The two sides and the bottom are dressed faces, whereas the top face is broken; we can infer from the contents of the inscription that more than seven lines of text, that is, 12 cm or more, are missing at the top. A large piece is also broken off the lower left corner. The inscribed front surface is weathered and chipped.

Planet	Sidereal Longitudinal periods	Synodic periods	Tropical Longitudinal periods	Synodic periods
Mercury	29140	91852	29140	91848
Mars	15494	13646	15494	13646
Jupiter	2455	26685	2456	26684
Saturn	989	28151	990	28150

Table 1. Numbers of longitudinal and synodic periods in 29140 solar years estimated from modern theory.



Fig. 1. The Keskintos Astronomical Inscription (IG 12.1, 913, = SK 14472) Staatliche Museen zu Berlin, Preußischer Kulturbesitz, Antikensammlung, Photo Johannes Laurentius.

Concerning the general style and probable date of the script I have nothing to add to Hiller's statement in his *IG* preface. Writing to Tannery on January 7, 1895, Hiller explained that in his opinion the writing was too "*maniriert*" to belong to the beginning of the second century B.C., while the second half of the first century B.C. was also to be ruled out, in part because of the use of iota adscript.<sup>12</sup> Hiller had made a special study of the dating of Rhodian inscriptions involving the tracing of families of the sculptors named in them.<sup>13</sup>

A single abbreviation is attested in line 14, where  $\mu \circ \iota \rho \widehat{\omega} v$  is represented by a small  $\circ \iota$  suspended above M, i.e.  $\mathcal{K}$ , very much as in astronomical papyri of the first century A.D. and later.<sup>14</sup> The numeral form for 6 ( $\varsigma$ , the so-called "stigma") is  $\Box$ , and that for 900 ( $\lambda$ , "sampi") is T. There is no clearly legible instance of 90 ( $\circ$ , "qoppa"), though I would guess from the visible traces, especially line 9 col. iv, that it took the form  $\Im$ .

## Line-by-line paleographical commentary.

1. The sole surviving line belonging to Mercury runs along the broken upper edge of the stone. In col. iv there appear to be remains of letters, but none can be identified with the slightest confidence. In col. viii it may be presumed that the first sign was a number of myriads, of which only the right bottom serif of the M can be seen, followed by a clear H and parts of two strokes of the next letter: a vertical stroke at the upper left and a vertical stroke at the bottom and a bit further right. Since the expected number of synodic periods of Mercury in this column is close to 918500, the most likely values for this damaged digit are 4 (Y) or 5 ( $\Phi$ ), of which only Y is consistent with the traces.

2. In col. iv the M is unclear (left half visible only) but certain from context. The traces of the next letter are suggestive of E, while those of the following letters are visible but unidentifiable. Following the numerals in most lines of col. iv is a horizontal stroke. No certain instances of this mark can be seen in col. viii. I suspect that this is an early form of the zero sign  $\overline{\phantom{a}}$  frequently attested in astronomical papyri of the Roman period, here indicating that the number of periods is exactly a whole number.<sup>15</sup> It is noteworthy that the mark does not appear in line 11 (*pace* Hiller in the *IG* transcription) where a fraction does follow the whole number.

3. The traces of the letter read as T are also compatible with  $\lambda$ .

4. In col. iv a pit has obliterated much of the M. Only upper strokes of the number of myriads, read here as  $\Delta$ , are visible, and would also be consistent with reading A. The traces of the next letter are strongly suggestive of  $\lambda$  (T is also admissible). Only the bottom of the final digit is visible, and consistent with  $\Delta$  or E, the counterparts of the two possible readings

<sup>&</sup>lt;sup>12</sup> Tannery 1939, 176–177.

<sup>&</sup>lt;sup>13</sup> Hiller von Gaertringen 1894.

<sup>&</sup>lt;sup>14</sup> See for example the first century A.D. procedure text *P. Oxy. astron.* 4136 lines 9 and 19, Jones 1999 v. 2, 14 and plate II.

<sup>&</sup>lt;sup>15</sup> For forms of the zero sign in papyri see Jones 1999 v. 1, 61–62.

of the corresponding digit in col. viii. In col. viii the letter following the number of myriads looks like A (alternatively,  $\Delta$ ) in particular one can see a notch that seems to be the vertex of this letter. I am unable to reconcile these traces with the traces of the corresponding digit in col. iv, in which I have slightly more confidence. Traces of what appear to be all four "spokes" of X are visible, though there is sufficient damage to the surface to leave the reading open to doubt. The last letter's right part is missing; what remains is suggestive of N, though M is also possible.

5. The traces of the letters following the initial M are illegible.

6. The traces of the first two letters are suggestive of BY; the remainder is illegible.

7. In col. iv the first letter cannot be read.

9. A pit has obliterated the letters following M in col. viii.

10. Only the vertical stroke of the 9 in col. iv is visible. In col. viii only the bottom half of K is visible.

11. In col. viii only what appears to be a bit of the loop of 9 is visible.

12. In col. viii surface damage has rendered unclear the number of myriads.

13. In col. iv only the top right part of B survives. In col. viii the Y is unclear.

14. The numerals in this line were marked as such by bars over the letters, a common convention for indicating that letters are not to be construed as Greek (hence not required for numbers in a table). A bar is visible over  $T\Xi$ , over the  $\Theta$  and K of  $\Theta\Psi K$  (thus confirming against Tannery's disbelief that  $\Theta$  is indeed part of the numeral), over the last visible letter of the line, and perhaps over the illegible isolated letter, a trace of which is visible along the broken edge of the stone to the left of the beginning of the legible part of the line. A pit has obliterated the "thousands" stroke preceding  $\Theta\Psi K$ . Of the last letter, besides the stroke marking it as a numeral, there remains a small vertical stroke, apparently a serif, at the upper left, which is compatible with K but not with B (the reading proposed by Tannery).

## Text and Translation.

In the text, obliterated letters are bracketed, while a dot under a letter indicates that its reading is not certain on purely paleographical grounds. A dot with no letter above indicates unidentifiable traces of a letter. In the translation, brackets have the same meaning as in the text, but no brackets are used in partially legible words. Digits in serious doubt are underlined, and "x" designates digits that are too uncertain even to guess at.

viii	[9A] M [′]HY[ ]	ΙΕ Μ 'Δ <b>λ</b> Κ	IE M 'ATE	NXY, M	іг М 'СҮП	в М 'ΔФ	Β Μ ኅΔΦΞ	K∆ M ′BX	Kر M ['رکا]	[′]0àK	ებიმ,	КZ Й ́́́́АΨЕ	кн М ҮАҮП	
vii	διέξοδο[ι]	ζωιδια[κοὶ]	τρ[0]πικοί	μεριδρομαί	<b>້</b> ວາຂ໌ຮຸ້ <b>ວ</b> ້ວວາ	ζωιδιακοί	τροπικοί	μεριδρομαί	διέξοδοι	ζωιδιακοί	τροπικοί	μεριδρομαί	διέξοδοι	
vi	[κατὰ] σχημα	κατὰ μηκος	κατὰ πλάτος	κατὰ βάθος	κατὰ σχημα	κατὰ μῆκος	κατὰ πλάτος	κατὰ βάθος	κατὰ σχημα	κατὰ μῆκος	κατὰ πλάτ[ος]	κα[τ]ὰ βάθος	κατὰ σχημα	
٨	Στίλβ[οντος]	Πυρόεντος	Πυρόεντος	Πυρόεντος	Πυρόεντος	Φαέθοντος	φαέθοντος	Φαέθοντος	Φαέθοντος	Φαίνοντος	Φαίνοντος	Φα[ίνο]ντος	Φαίνοντος	
iv		A M 'EY QB	A M 'EYAÇ[]		Å M (T[XM]H	– NYB[]	− SNYG	B Μ 'ΔΣΞ —	в М 'ÇХŶ—	<b>^</b> фВ —	λΠΘ ΣΙζ	Β Μ ′ZPOς —	B MMH' HPMH	
iii	[διέξοδοι]	[ζωι]διακοί	[τρο]πικοί	[περι]δρομαὶ	[διέ]ξοδοι	[ζ]οιδιακοί	τροπικοί	περιδρομαὶ	διέξοδοι	ζωιδιακοὶ	τροπικοί	περιδρομαὶ	ວ່າຂໍຮຸດວັດເ	
ii	[κατὰ σχημα]	<u>κατ</u> ά μηκ[o]ς	κατὰ πλάτ[ος]	κατὰ βά[θος]	κατὰ σχ[ημα]	κατὰ [μι]κος	κατ[ὰ πλ]ἀτος	κατὰ βάθος	κατὰ σχημα	κατὰ μῆκος	κατὰ πλάτος	κατὰ βάθος	[κατὰ] ἀΧῆμα	
i	Σ[τίλβοντος]	Πυ[ρόεντ]ος	Πυρόεντος	Πυρόεντος	Πυρόεντος	Φαέθοντος	φάέθοντος	[Φαέ]θοντος	[Φαέθ]οντος	[Φαίνο]ντος	[Φαίνοντος]	[Φαίνοντος]	[Φαίνοντος]	
					5					10				

].[...]...ό κύκλος μο(ιρῶν) Τ<del>Ξ, σ</del>τιγμῶν [']Θ<del>ΨΚ. ἡ</del> μοῦρα στιγμῶν Κ̈́[<del>Ζ.]</del> ]...αις χαριστήριον

15

viii	[91]84xx	154920	154360	$40\underline{1}6\underline{5}0$	136480	24500	24560	242600	266900	9920	9896	271760	281480
vii	passages	zodiacals	tropicals	revolutions	passages	zodiacals	tropicals	revolutions	passages	zodiacals	tropicals	revolutions	passages
vi	[In] relative position	In longitude	In latitude	In depth	In relative position	In longitude	In latitude	In depth	In relative position	In longitude	In latitude	In depth	In relative position
Λ	Mercury	Mars	Mars	Mars	Mars	Jupiter	Jupiter	Jupiter	Jupiter	Saturn	Saturn	Saturn	Saturn
iv	XXXX	15492	15436	40 <u>9</u> 6x	13648	2450	2456	24260	26690	992	989 216	27176	28148
iii	[passages]	zodiacals	tropicals	revolutions	passages	zodiacals	tropicals	revolutions	passages	zodiacals	tropicals	revolutions	passages
ii	[In relative position]	In longitude	In latitude	In depth	In relative position	In longitude	In latitude	In depth	In relative position	In longitude	In latitude	In depth	[In] relative position
.1	Mercury	Mars	Mars	Mars	Mars	Jupiter	Jupiter	Jupiter	Jupiter	Saturn	[Saturn]	[Saturn]	[Saturn]
					2					10			

]... A circle comprises 360 degrees or 9720 stigmai. A degree comprises 2[7] points.

15 ] to ... a thank-offering.

## The dedication in line 15 and the votive offering.

It will be convenient to discuss the inscription from the bottom up, beginning with the dedication and ending with the planetary table. A formula in the dative case, and ending with a feminine plural, is required in line 15, and the space preceding the first legible letter would correspond to approximately eighteen letters. The most likely restoration is  $\theta \epsilon o i \varsigma \pi \hat{\alpha} \sigma \iota \kappa \alpha i \pi \dot{\alpha} \sigma \alpha \iota \varsigma$ , "to all the gods and goddesses," or some variant of this common formula.

Votive offerings in ancient Greek society were occasioned by many circumstances, for example success in war or in athletic competitions, coming of age, recovery from illness or survival in calamity, receipt of public office or honours.<sup>16</sup> A great range of articles could be donated to the gods, often related to the reason for the offering (e.g. spoils of war, the discus of an athletic victor, or a model of a healed body part), but often something else such as a statue. Ptolemy's *Canobic Inscription* begins with the words "To the Saviour God Claudius Ptolemy [scil. dedicates] the first principles and models of astronomy," and thus appears to be an instance of the comparatively rare category in which the textual or factual content of the inscription is itself the votive offering.<sup>17</sup> Perhaps the Keskintos Inscription was like this too, but it is possible, even probable, that the astronomical data recorded in the inscription were meant as a kind of caption explaining or supplementing a visual display of the planetary system.

## The units of arc in line 14.

Roughly the first third of the penultimate line of the inscription, probably one or more phrases comprising about thirty letters, is lost; like what follows, it may have consisted of relations among units, perhaps of time. The surviving text defines two units of arc, the familiar  $\mu o \hat{i} \rho \alpha$  or "degree" equivalent to  $\frac{1}{360}$  of a circle's circumference, and a unit called a  $\sigma \tau i \gamma \mu \eta$  or "point," which does not seem to be attested, at least by that name, elsewhere in our sources for Greek astronomy and mathematics; for clarity we will refer to these units as *stigmai* (singular *stigme*). The inscription gives as the number of *stigmai* in a circle's circumference  $\Theta \Psi K$ , which would straightforwardly be read as 9720. In accordance with the way that numerals are written elsewhere in the inscription, one would expect the theta to be preceded by a small elevated arc signalling "thousands." As it happens, the surface of the stone has a large pit where this sign would be, though this is not apparent in Herz's photograph of the squeeze. From the equation of 9720 *stigmai* and 360 degrees Hiller restored the numeral for the numeral was visible on his squeeze.

Tannery objected strenuously against reading 9720. His primary reason, so far as one can tell from his published statements and correspondence, was that he thought that a unit

<sup>&</sup>lt;sup>16</sup> The most comprehensive treatment of the topic remains Rouse 1902.

<sup>&</sup>lt;sup>17</sup> For other instances of inscription as votive object see McLean 2002, 252. Chaniotis 1988, 278–283 catalogues attested votive offerings connected with literature and science, but many of these accompanied a separate article.

15

of arc defined as such a curious fraction of a degree was incredible.<sup>18</sup> He therefore chose to interpret the theta of the numeral as a symbol for  $\kappa \acute{o} \kappa \acute{o} \lambda o \upsilon$ , making the phrase read "(a circle comprises) 720 points of a circle," from which one derives the equivalence of 1 degree to 2 *stigmai*. Tannery alleged that the sign in question could not possibly be a true theta, first because he thought it was slightly smaller than the thetas in the previous lines, and secondly because the "thousands" sign is missing; Hiller dismissed both arguments, pointing out that the sizes of letters in the inscription are not rigidly constant and that the surface where the thousands sign should be is damaged.<sup>19</sup> Against Tannery's hypothesis stands also the fact that the circle-and-central-dot symbol for  $\kappa \acute{o} \kappa \lambda o \varsigma$  is unattested in any ancient inscription and is not known to have been used in manuscripts before the Middle Ages. Nevertheless subsequent scholarship has adopted Tannery s reading unquestioningly. Direct examination of the stone now provides two decisive proofs that 9720 is the correct reading: first, the raised horizontal bar that Hiller saw above TE in the same line, marking it as a numeral, can also be seen above the  $\Theta$  and K of  $\Theta \Psi K$ ; secondly, part of a stroke of the first letter of the numeral at the end of the line turns out to be visible, and is consistent with K but not B.

There are no known Greek parallels for a metrology based on dividing the circle into 9720 units. As we shall see, however, the planetary table almost certainly embodies the assumption that almost all planetary periodicities are contained an exact whole number of times in 29160 Egyptian calendar years (and absolutely all in 291600 Egyptian years), and 9720 is precisely one third of 29160. This is surely no accident, though just what it means is not immediately obvious since on the face of it the *stigme* is a unit of arc, not of time. I would suggest that the inscription's author was intentionally relating spatial "circles" to temporal "cycles." When we come to discuss the latitudinal periods in the planetary table, we will see that individual periods of a planet are indeed thought of on analogy with circles and divided into 360 units.

This connection with the long common periods of the planetary table may be sufficient to account for the definition of the *stigme*; but there is good reason to believe that it simultaneously served another function more directly associated with the measurement of arcs. Among several attested ancient values for the apparent size of the sun's and moon's disk, we have Ptolemy's statement in the *Canobic Inscription* that "at the mean distances of the Sun and Moon at syzygies, the diameter of either luminary subtends at the sight <sup>1</sup>/<sub>162</sub> of a right angle," which would make the disks <sup>1</sup>/<sub>168</sub> of a circle.<sup>20</sup> The *stigme* of the Keskintos Inscription would be precisely <sup>1</sup>/<sub>15</sub> of this arc. We know, moreover, from Ptolemy, *Planetary Hypotheses* 1 part 2.5, that Hipparchus had estimated the apparent breadths of the stars and planets in terms of fractions of the sun's disk, among which he adduces as examples that Hipparchus thought that the breadth of Venus was <sup>1</sup>/<sub>10</sub> the breadth of the sun, and that of the smallest fixed star (presumably meaning the smallest deserving notice)

<sup>&</sup>lt;sup>18</sup> Tannery 1895a, 51–52 (= Tannery 1912, 489–490), and 1939, 140–187 passim.

<sup>&</sup>lt;sup>19</sup> Tannery 1939, 147 and 151–152.

<sup>&</sup>lt;sup>20</sup> Section 13 of the text in Jones 2005, 74–75.

#### SCIAMVS 7

planet	name	meaning
Mercury	Στίλβων	twinkling
Venus	Φωσφόρος	light-bringer
Mars	Πυρόεις	fiery
Jupiter	Φαέθων	gleaming
Saturn	Φαίνων	shining

Alexander Jones

Table 2. The non-theophoric names of the planets.

was 1/30 the sun's breadth.<sup>21</sup> (Ptolemy gives his own similarly expressed estimates of the disks of the remaining planets and the brightest stars.) Thus investigations of cosmic sizes and distances could give rise to a metrology based on small units of arc independent of degrees. Interestingly, two passages of the Sanskrit *Pañcasiddhāntikā* of Varāhamihira, likely derived ultimately from Greek sources, specify a division of the moon's disk into 15 units.<sup>22</sup> One might guess that for the author of our inscription 1/15 of the sun's or moon's breadth constituted the smallest observable interval.

The *Canobic Inscription* does not state where Ptolemy obtained his estimate that the apparent diameters of the sun and moon at mean distance are <sup>1</sup>/<sub>162</sub> of a right angle. In *Almagest* 4.9, however, in a passage that is clearly referring to the parts of his lunar theory that were recorded in the *Canobic Inscription* but subsequently revised, Ptolemy writes that he took over his original figures for the apparent diameters of the moon and the earth's shadow from Hipparchus. Confusingly, he here gives this Hipparchian lunar diameter as "approximately" <sup>1</sup>/<sub>650</sub> of a circle, which is not in exact agreement with the *Canobic Inscription*. There is no way to determine with certainty which number is Hipparchus' and which is a rounding; it would certainly be noteworthy if the influence of Hipparchus' work could be found in the Keskintos Inscription, not least because Hipparchus himself is known to have worked on Rhodes from the late 140s to the early 120s B.C. (On the other hand we will see that Hipparchus' influence is conspicuously absent in the inscription's choice of year length.)

## The planetary table: terminology and sequence.

Two sets of names for the five planets were in use in Greek astronomy during the Hellenistic period.<sup>23</sup> The more familiar theophoric names of the pattern "the star of Hermes" ( $\delta \tau o \hat{\nu}$  'Epµo $\hat{\nu} \, d\sigma \tau \eta \rho$ ) became established during the fourth century B.C. The Keskintos Inscription, however, employs a scheme of descriptive names that are first attested in some anonymous planetary observation reports from the third century B.C. preserved in Ptolemy's *Almagest*, and that remained in use into Roman times in astronomical and, subsequently, astrological

<sup>&</sup>lt;sup>21</sup> Goldstein 1967, 8; Morelon 1993, 74–75.

<sup>&</sup>lt;sup>22</sup> Pañcasiddhāntikā 5.4 and 14.38, for which see Neugebauer and Pingree 1971 v. 2, 49 and 92. I am grateful

to Dennis Duke for drawing my attention to these passages.

<sup>&</sup>lt;sup>23</sup> Cumont 1935.

17

texts, sometimes in combination with the theophoric names. The descriptive names and their meanings are listed in Table 2. The sequence in which the planets were listed (Venus, Mercury, Mars, Jupiter, Saturn) is one of the common Greek orderings, based on the principle that the duration of the longitudinal period is correlated with the planet's distance from the earth; thus in the inscription the planets are in ascending order of presumed distance. This principle left indeterminate the relative distances of Mercury, Venus, and the sun, which all have one year for their mean longitudinal period. Of the six possible permutations of the three bodies, five are attested in ancient sources.<sup>24</sup> We have no reliable grounds for tracing traditions within Greek astronomy through these orderings, so no inferences should be drawn about the Keskintos Inscription's place in such traditions.

Each of the four kinds of period associated with each planet is specified doubly, by a prepositional phrase such as κατὰ μῆκος, "in longitude," and by an adjective or noun such as ζφδιακοί, "zodiacals." The adjectives imply an understood masculine noun, perhaps κύκλοι, "cycles," or χρόνοι, "time-intervals." Neither part of the specification is self-explanatory, so we are dealing with technical nomenclature, not definitions.

The first three prepositional phrases would translate literally (if the context was not celestial) as "in length," "in breadth," and "in depth." This three-dimensional terminology was common in late Hellenistic and Roman period astronomy in the sense of "with respect to motion parallel to the ecliptic," "with respect to motion north and south of the ecliptic," and "with respect to motion towards and away from the earth." The terms "length" and "breadth" are normally translated as "longitude" and "latitude," while "depth" is sometimes less satisfactorily rendered as "anomaly." An unidentified third century A.D. commentator on Ptolemy s lunar tables quotes a passage from an astronomical writer named Apollinarius (date uncertain, but not later than the second half of the second century A.D.) defining periods or "restitutions" (ἀποκαταστάσεις) of latitude, longitude, and depth as follows:<sup>25</sup>

"Restitution of latitude" is the name given to the time interval in which the centre of the moon is placed on the ecliptic and then has revolved around the turning-points of latitude and is restored to the plane of the ecliptic. "Restitution of depth" is the name given to the time interval in which the part of the surface of the star's sphere that is furthest from the earth, having gone from the part of its own motion that is furthest from the earth, is restored again to its furthest position from the earth. "Restitution of longitude" is the name given whenever the centre of any star whatsoever, having set out from the plane of one of the circles described through the poles of the zodiac circle and having progressed around the zodiacal circle, is restored again to the same plane, the one from which it began travelling.

Apollinarius' definitions of the periods of longitude and latitude are fairly straightforward and obviously applicable to a planet as well as to the moon. The period of depth is more

<sup>&</sup>lt;sup>24</sup> Neugebauer 1975 v. 2, 690–693.

<sup>&</sup>lt;sup>25</sup> Jones 1990, 38–41.

obscure—is the "star's sphere" meant to be the visible object, or an epicycle?—though it is clear enough that to speak of such a period implies that one is assuming a model of motion in which the planet's distance from the earth is not constant, thus ruling out homocentric spheres.

The secondary epithets of these three periods in the Keskintos Inscription add little information. "Zodiacal" is an obviously appropriate adjective to apply to a period in longitude, whereas "tropical" is less apposite for a latitudinal period, implying as it does a misleading analogy between latitudinal motion in an ecliptic frame of reference and the sun's north-south oscillation between the solstices ( $\tau p \delta \pi \sigma \iota$ ) in an equatorial frame of reference. The application of "revolutions" ( $\pi \epsilon \rho \iota \delta \rho o \mu \alpha \iota$ ) to the periods in depth reinforces our sense that these periods are to be thought of in terms of kinematic models involving combinations of circular motions.

The fourth planetary period in the inscription, "in relative position, passages" ( $\kappa \alpha \tau \dot{\alpha} \sigma \chi \hat{\eta} \mu \alpha \delta \iota \dot{\xi} \delta \delta \sigma \iota$ ), is absent from Apollinarius' list, and in fact there seems to be no other extant text that groups a period with a name resembling this together with periods in longitude, latitude, and depth. In this instance it is the secondary epithet for which we have an informative parallel.<sup>26</sup> In his *Commentary on Aristotle's De Caelo* 2.12 (ed. Heiberg 496) Simplicius explains that the period of revolution of the third and fourth spheres in Eudoxus' homocentric models for a planet is "the time interval in which it goes from appearance [ $\dot{\alpha}\pi \dot{\alpha} \phi \phi \dot{\alpha} \varepsilon \omega \varsigma$ ] to the next appearance having passed through [ $\delta \iota \varepsilon \xi \iota \dot{\omega} v$ ] all the relative positions [ $\sigma \chi \dot{\varepsilon} \sigma \varepsilon \iota \varsigma$ ] with respect to the sun, which is the time interval that the astronomers call a time interval of passage [ $\delta \iota \varepsilon \xi \dot{\omega} \delta \upsilon \chi \rho \dot{\omega} v v$ ]." This is obviously the planet's synodic period, thought of in terms of the planet s cycle of varying elongation from the sun. Simplicius'  $\sigma \chi \dot{\varepsilon} \sigma \iota \varsigma$  would be synonymous with the inscription's  $\sigma \chi \hat{\eta} \mu \alpha$ , a term that also occurs in astrological texts in the related senses of "aspect" (astrologically significant elongation between any two heavenly bodies) and "lunar phase."

The terminology for the periods tells us that the inscription's author was thinking in terms of a three-dimensional conception of planetary motion, not merely a phenomenological description of apparent positions relative to the zodiacal belt as in Babylonian astronomy. At the same time the fact that he assumed a degree of regularity in planetary motion that would justify speaking of periods covering tens and hundreds of thousands of years, involving slow revolutions of some components completed on a commensurate time scale, tells us that the conception was probably grounded in mathematics rather than natural philosophy. Hence we are led to interpret the inscription as a summary of parameters of a system of kinematic geometrical models, comparable to Ptolemy's *Canobic Inscription*. Ptolemy's inscription gave the complete set of numerical parameters required to determine all the spatial proportions of his planetary system, for example the ratios of radii of the various circles composing the models, as well as a set of epoch orientations for every element. If the Keskintos inscription was similarly comprehensive, it would have had to be many times longer than the extant fragment. But I rather doubt that it gave much more than the

<sup>&</sup>lt;sup>26</sup> Tannery 1895a, 54 note 1 (= Tannery 1912, 494 note 1).

periodicities; for although these periodicities imply some kind of modelling, there is no need to presume that the models were as completely worked out in relation to empirical data as Ptolemy's are.

For convenient reference we will use the notations listed in Table 3 for the numbers of periods in the inscription; these are adapted with slight modification from Neugebauer's discussion.

## The common periods or Great Years.

The planetary table in the Keskintos Inscription, as explained by Tannery, records for each planet and kind of period the number of such periods encompassed respectively in 29140 solar years (col. iv) and 291400 solar years (col. viii). The numbers 29140 and 291400 do not appear explicitly in the preserved part of the table but can be obtained by adding the numbers of longitudinal and synodic periods for any of the superior planets, since the number of times that the sun is in conjunction with a superior planet must be the difference between the number of the sun's revolutions in longitude and the number of the planet's own slower revolutions in longitude:

$$A = Y - L \tag{1}$$

In principle we should be able to obtain the totals 29140 or 291400 from any of the six pairs of numbers L + A or  $L_{10} + A_{10}$  in the extant part of the table. Unfortunately because of damaged and lost letters only the pair L and A for Saturn is entirely and unambiguously preserved. For Mars and Jupiter we have securely read numbers for  $L_{10}$  as well as numbers for A or  $A_{10}$  that are secure in the highest-order digits, which is enough to satisfy us that the totals are always the same (it would make no sense to have totals only *nearly* the same for different planets).

The fact that the planetary periodicities in the inscription are expressed in terms of how many are contained in long periods of 29140 and 291400 years could mean two things: either the author of the inscription believed that the numbers in question were exact, i.e. that all periodicities that are listed as whole numbers are exactly completed in the long intervals, or the author used these long periods merely as a convenient and uniform way using as few fractions as possible. Here the fractional period assigned to Jupiter's latitudinal

	col. iv	col. viii
Longitudinal periods ("length")	L	I
Latitudinal periods ("breadth")	B	$egin{array}{c} L_{10} \ B_{10} \end{array}$
Periods in depth	G	$G_{10}^{-10}$
Synodic periods ("relative position")	Α	$A_{10}$
Solar years	Y	$Y_{10}$

Table 3. Notations for the numbers of periods according to the inscription.

motion in line 11, to be discussed below, is the "exception that proves the rule": there would be no reason to include a fraction unless all the other numbers lacking fractions were to be understood as exact. The long periods of the planetary table are thus instances of "Great Years," cycles of presumed cosmic repetition—the clearest example so far known in a Greco-Roman source of a Great Year applied to technical mathematical astronomy rather than philosophical or astrological cosmology.<sup>27</sup>

In Indian astronomy long combined periods (*yugas*) are used in a similar way to express periodicities, and are interpreted as signifying a universal return of the cosmic system to an initial state, and especially because of this the Keskintos Inscription has been remarked on as a witness to a variety of Greek astronomy scarcely attested in classical sources but that fed into the development of mathematical astronomy in India.<sup>28</sup> For my part, I am fully confident that sooner or later we will find proof in papyri of the Roman period that as late as Ptolemy's time common periodicities were employed in certain varieties of Greek mathematical astronomy. The Indian planetary theories that employ kinematic modelling represent a considerably more advanced understanding of planetary motion than underlies the Keskintos Inscription, closely resembling Ptolemy's models though with generally poorer numerical parameters.<sup>29</sup>

Specific durations of Great Years are attested in the fringes of Greco-Roman astronomical and astrological literature, although seldom in connection with precise astronomical data.<sup>30</sup> Some of these attested periods were constructed by multiplying together valid periods for the individual heavenly bodies. For example, a "cosmic restitution" ( $\kappa o \sigma \mu \kappa \dot{n} \dot{\alpha} \pi \sigma \kappa \alpha \tau \dot{\alpha} \sigma \tau \alpha \sigma \tau \zeta$ ) of 1753200 years, reported in several texts, was generated by multiplying together short periods of years for each planet that roughly encompass a whole number of longitudinal and synodic periods (30 for Saturn, 12 for Jupiter, 8 for Venus, and so on), a period of 25 Egyptian calendar years (of 365 days) that contains nearly a whole number of lunar months, and a "Sothic" period of 1461 Egyptian years that contains a whole number of  $365^{1/4}$  day solar years. Another frequently attested rationale was to choose numbers that were numerologically attractive, especially numbers rich in small prime factors such as 2, 3, and 5, or indeed to combine these factors with 365 or 1461.<sup>31</sup> It is often not made explicit

<sup>&</sup>lt;sup>27</sup> For an excellent discussion of Great Years in philosophical and astrological contexts see de Callataÿ 1996 (the Keskintos Inscription is not mentioned).

<sup>&</sup>lt;sup>28</sup> Neugebauer 1975 v. 2, 704–705; Toomer 1984, 422 note 12. *Yugas* bearing an astronomical meaning connected with planetary conjunctions first show up in works of the early fifth century A.D. that reflect extensive Greek influence. Note, however, that the fundamental concept of the *yuga* appears to antedate the transmission of Greek astronomy into India. See de Callataÿ 1996, 30–31.

<sup>&</sup>lt;sup>29</sup> Duke 2005.

<sup>&</sup>lt;sup>30</sup> Neugebauer 1975 v. 2, 604–607 and 618, and de Callataÿ 1996, 253–258.

<sup>&</sup>lt;sup>31</sup> In calling such derivations "numerological," I do not mean that there was necessarily a mystical element in the underlying thought, only that relations involving small whole numbers were believed to have a role in celestial structures.

whether the assigned lengths of these Great Years are to be understood as numbers of solar or Egyptian years, though Great Years divisible by the 1461-year Sothic period or the 25-year lunisolar cycle only make astronomical sense in terms of Egyptian years.

What sort of Great Year were 29140 and 291400 years? 29140 can be divided into factors 2 (twice), 5, 31, and 47. While 47 is the number of years in the shorter and less accurate of the two Babylonian Goal Year period relations for Mars (cf. the next section), the remaining factors have no apparent astronomical significance, nor do they form a numerologically appealing set.<sup>32</sup> The search for interesting properties of 29140 and 291400 is, in fact, a blind alley. The significant number, almost certainly, was not 291400 solar years, but 291600 Egyptian years. If one assumes that the solar year is exactly  $365^{1/4}$  days, the number of solar years contained in 291600 Egyptian years would be:

$$291600 \times \frac{365}{365^{1/4}} \approx 291400.41 \text{ solar years}$$
(2)

which one might reasonably round off at the expense of a negligible increase in the implied length of the solar year (approximately 365.25051 days). Unlike 291400, 291600 is a composite of the smallest possible factors,  $2^4 \times 3^6 \times 5^2$ . I have already drawn attention to the corroborating circumstance that 29160 is also exactly thirty times 9720, the number of *stigmai* in a circle according to line 14 of the inscription.<sup>33</sup>

The choice of these particular products of 2, 3, and 5 likely arose out of the realization that the Sothic period, 1461 Egyptian years, which is the shortest period containing whole numbers of both Egyptian years and years of exactly  $365^{1/4}$  days, is very close to 1458, i.e.  $2 \times 3^6$ . A period of 1458 Egyptian years might have seemed too short to allow for a sufficiently accurate represention of the planet's longitudinal and synodic periods by whole numbers. Multiplication of 1458 by 20 or by 200 (numbers that might intentionally contain as factors the 25-year lunisolar period and the 8-year periodicity of Venus) would have resulted in the Great Years of the inscription.

For the fact that the inscription lists the numbers of each period in both 29140 and 291400 years (or rather, following my suggestion, 29160 and 291600 Egyptian years) Tannery's rationale seems adequate: while most of the periods were supposed to be completed an integer number of times in the shorter interval, a few, including the attested case of Saturn's periods in latitude, had fractional residues which could be resolved by multiplying

<sup>&</sup>lt;sup>32</sup> Tannery 1895c, 328 (=Tannery 1912, 515–516) noted that Saturn's periods in longitude and relative position in the inscription have a common factor 124, so that they reduce to a period relation comprising 235 years, 8 longitudinal periods, and 227 synodic periods. Since the periods for Mars and Jupiter are not exactly reducible to reasonably short period relations, I do not think we should conclude that the 29140-year period was constructed from a supposed 235-year relation for Saturn.

<sup>&</sup>lt;sup>33</sup> It is also tempting to connect the difference of 20 between 29140 and 29160 with the discrepancies of 20 noted in the relations among the planetary periods, but I do not see any satisfactory way to rationalize one in terms of the other.

everything by 10.<sup>34</sup> It is less obvious why the author did not simply use the larger numbers, although we may be grateful that he did not since the redundancy has proved so helpful in reading the numerals. The most plausible explanation is that the system assumed that the very long Great Year of 291600 Egyptian years could be broken up into shorter but imperfect Great Years in which *almost* all the periodicities are exactly completed, a concept analogous to the hierarchy of *yugas* (*Kalpa, Mahâyuga*, etc.) of Indian texts.

Incidentally, the author of the inscription would have obtained different results if he had begun with a more accurate value for the tropical or sidereal year; for example, dividing 291600 by Hipparchus' tropical year of  $365^{1/4} - \frac{1}{300}$  days would yield a quotient of approximately 291403.07. The fact that the author assumed a solar year exactly equal to (or at most negligibly different from)  $365^{1/4}$  days does not by any means imply that the inscription was earlier than Hipparchus' writings on the length of the year and precession (which date from the 120s B.C.). Theon of Smyrna (ed. E. Hiller, 172–173) and a papyrus table of solar mean motions, *P. Oxy. astron.* 4174a, are both witnesses to the currency in the second century A.D. of a model of the sun's motion involving separate solar periodicities for longitude, latitude, and "depth" (thus paralleling the Keskintos Inscription's treatment of planetary periodicities), according to which the sun's period of longitude is  $365^{1/4}$  days, with no distinction drawn between a tropical and sidereal period.<sup>35</sup> (In subsequent discussions we will refer to this as "Theon's solar model.")

### The longitudinal and synodic periods.

All but one of the numbers for the periods in the preserved part of the inscription are expressed as integers. These numbers must have originated as roundings, that is, the astronomer who devised the table began with some value for the mean duration of each period, perhaps expressed as a period relation *not* based on the 29160 year interval, and determined to the nearest whole number how many such periods would be contained in 29160 years. (The numbers of periods in col. viii were of course simply obtained by multiplying these rounded quotients by ten.) A reasonable hypothesis to begin with is that for each planet a period relation derived from observations or from earlier tradition was scaled to fit a total interval of 29140 solar years, and then the numbers were rounded to the nearest whole number. We may thus provisionally consider each number in col. iv to express the original assumed ratios of the periodicities to a precision of  $\pm^{1}/_{2}$  period in 29140 solar years, so that the period relations can be written as follows:

*Mars:* 29140 years =  $13648 \pm \frac{1}{2}$  synodic periods =  $15492 \pm \frac{1}{2}$  longitudinal periods *Jupiter:* 29140 years =  $26690 \pm \frac{1}{2}$  synodic periods =  $2450 \pm \frac{1}{2}$  longitudinal periods *Saturn:* 29140 years =  $28148 \pm \frac{1}{2}$  synodic periods =  $992 \pm \frac{1}{2}$  longitudinal periods 1 year = 365.25 days (3)

<sup>&</sup>lt;sup>34</sup> Tannery 1985a, 53 and 1895c, 321.

<sup>&</sup>lt;sup>35</sup> Jones 2000.

We may further think of each period as representing a revolution of some component of a kinematic model; for example in a Ptolemy-style epicyclic model a period of longitude would represent one revolution of the epicycle around the earth relative to a longitudinally fixed direction, while one period of relative position would represent both one revolution of the epicycle around the earth relative to the sun, and one revolution of the planet around the epicycle relative to the direction from the earth to the epicycle. (It does not matter for our present purposes whether the planetary models imagined by the inscription's author were structurally the same as Ptolemy's.) By multiplying the number of periods contained in 29140 years by  $360^{\circ}$  and dividing by 29140, we translate it into a mean annual motion in longitude or anomaly. The mean daily motion can also be obtained by dividing the annual motion by the length of the year.

In this way we can compare the period relations built into the Keskintos Inscription with other planetary period relations attested in ancient astronomy, both to get a comparative sense of its accuracy and to see whether there it has any obvious relation to the other known systems. The sets of period relations that it seems most appropriate to compare with the inscription are the so-called Goal Year and ACT relations of Babylonian astronomy and the relations that Ptolemy adopted in the *Almagest*. Naturally we need only look at the relations for the superior planets.

The Goal Year relations are the basis of a category of Babylonian astronomical text of which we have numerous examples from the last three centuries B.C.<sup>36</sup> These Goal Year Texts were designed to enable the forecasting of astronomical phenomena on the basis of observations made in previous years. Thus for a year y one would collect the observed phenomena of Venus recorded for the year y - 8, but those for Saturn recorded for the year y - 57, because Venus and Saturn approximately repeat their synodic cycles at the same stages of the solar (sidereal) year respectively after 8 and 57 solar years. In the Goal Year Texts certain kinds of observations of Mars are quoted from the year y - 47 and others from y - 79; and similarly there are some observations of Jupiter from y - 71 and from y - 83. If the repetitions were exact, they would imply the following period relations:

Mars (short):	47 sidereal years = 22 synodic periods = 25 longitudinal period	ods
Mars (long):	79 sidereal years = 37 synodic periods = $42 \log$ . periods	
Jupiter (short):	71 sidereal years = $65$ synodic periods = $6$ long. periods	
Jupiter (long):	83 sidereal years = 76 synodic periods = 7 long. periods	
Saturn:	59 sidereal years = 57 synodic periods = $2 \log$ . periods	(4)

No specific length of the sidereal year in days is embedded in the scheme. It is believed that in practice the Babylonian astronomers used small corrections when applying the Goal Year periods to predictions, but the precise nature of these corrections is very imperfectly

<sup>&</sup>lt;sup>36</sup> On Goal-Year Texts see Hunger 1999 (with a translation of a specimen text); the underlying period relations are listed in Neugebauer 1975 v. 1, 554–555, and Hunger and Pingree 1999, 167–169.

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known.<sup>37</sup> According to Ptolemy (*Almagest* 9.2), Hipparchus knew of at least some of the Goal Year relations (we do not know whether he had Mars' 47-year period or Jupiter's 83-year period). It is not clear whether Hipparchus' version incorporated correction terms.

The Babylonian arithmetical models for predicting planetary phenomena, known as "ACT schemes" from the acronym of Neugebauer's edition of the pertinent texts, *Astronomical Cuneiform Texts*, are built upon period relations that are considerably more accurate than the Goal Year relations: <sup>38</sup>

Mars:284 sidereal years = 133 synodic periods = 151 long. periodsJupiter:427 sidereal years = 391 synodic periods = 36 long. periodsSaturn:265 sidereal years = 256 synodic periods = 9 long. periods(5)

The length of the sidereal year was not defined. The ACT schemes are also attested in Roman period Greek papyri.

Ptolemy's relations, the most accurate of the ancient sets, are expressed in Ptolemy's tropical frame of reference as corrections of the Goal Year relations known to Hipparchus:

Mars:	79 tropical years + 3;13 days	
	= 37 synodic periods	
	= 42 (tropical) longitudinal periods + $3;10^{\circ}$	
Jupiter:	71 tropical years – 4;54 days	
	= 65 synodic periods	
	= 6 (tropical) longitudinal periods $-4;50^{\circ}$	
Saturn:	59 tropical years + 1;45 days	
	= 57 synodic periods	
	= 2 (tropical) longitudinal periods + 1;43°	
1 tropica	11 year = 365;14,48 days	
1 siderea	al year = $365; 15, 24, 31, 22, 27, 7$ days	(6)

Translated into a sidereal frame of reference using to Ptolemy's values for the tropical and sidereal year (which is given explicitly only in the *Planetary Hypotheses*), these become:

Mars: 79 sidereal years + 4;1 days
= 37 synodic periods
= 42 longitudinal periods + 2;23°
Jupiter: 71 sidereal years - 4;11 days
= 65 synodic periods
= 6 longitudinal periods - 5;33°

<sup>&</sup>lt;sup>37</sup> Hunger and Pingree 1999, 203–205.

<sup>&</sup>lt;sup>38</sup> Neugebauer 1955. For the period relations, see Neugebauer 1975 v. 1, 423 (Table 9), and Hunger and Pingree 1999, 244.

Saturn:	59 tropical years + 2;21 days	
	= 57 synodic periods	
	= 2 longitudinal periods + 1;8 $^{\circ}$	(7)

Table 4 gives the mean motions in sidereal longitude and anomaly and the ratio of synodic to sidereal longitudinal periods for the superior planets derived from modern theory and from the ancient period relations. (We assume that the solar years of the Keskinto Inscription may be regarded as sidereal years.) The errors in the Keskintos Inscription's implied mean motions in longitude and anomaly are considerably larger than the uncertainties arising from rounding off the numbers of periods in 29140 years. In the case of Mars the ratio of synodic to longitudinal periods is very nearly the same as in the 79-year Goal Year period relation. The ratio for Saturn is comparably inaccurate, but in the opposite direction, to that of the Goal Year relation, while the ratio for Jupiter is as poor as the 71-year Goal Year relation, but again in the opposite direction. Either the inscription's numbers originated in a set of period relations different from any of the sets that we know of, or the author manipulated his data more than just by rounding the numbers of periods. The shortest period relations that would generate the inscription's numbers by mere rounding are:

Mars:	79 years = 37 synodic periods = 42 long. periods	
Jupiter:	226  years = 207  synodic periods = 19  long. periods	
Saturn	235  years = 227  synodic periods = 8  long. periods	(8)

We will see, however, that the numbers were likely modified for nonastronomical reasons.

## The periods in latitude.

The periods in latitude in the inscription presumably count revolutions of the planet with respect to a nodal line of its model, just as the moon's latitudinal periods (dracontic months) count mean or true revolutions of the moon relative to its nodes. We cannot tell whether the underlying theory attempted to deal with the synodic component in planetary latitude. In the several theories of planetary latitude that Ptolemy proposed through his career, the nodal line is always sidereally fixed, which would make latitudinal periods indistinguishable from longitudinal periods, whereas in some Indian planetary theories the nodal lines have gradual motions. The inscription's numbers of latitudinal periods are always slightly different from the numbers of longitudinal periods, so that the nodal line of each planet must be assumed to have a slow motion, direct for Mars and Saturn, but retrograde for Jupiter.

Neugebauer rightly remarks that the true nodal motions of the planets are too slow to have been detectable in antiquity, but his conclusion that the numbers "obviously... cannot be based on real observations" is a *non sequitur*.<sup>39</sup> The fact that Ptolemy situated all the planets' nodal lines at multiples of 10° from their apsidal lines shows how difficult it was to establish their accurate positions even on the basis of a sophisticated latitude theory

<sup>&</sup>lt;sup>39</sup> Neugebauer 1975 v. 2, 702.

	Modern	Goal Year	ACT	Almagest	Keskintos
Mars					
longitude (years)	191.406°	191.489° (47 y) 191.392° (79 y)	191.408°	191.407°	191.391°±0.006°
anomaly (years)	168.594°	168.511° (47 y) 168.608° (79 y)	168.592°	168.593°	168.609±0.006°
longitude (days)	0.52403°			0.52406°	0.52400°±0.00002°
anomaly (days)	0.46158°			0.46158°	0.46163°±0.00002°
synodic/long.	0.8808	0.8800 (47 y) 0.8810 (79 y)	0.8808	0.8808	0.8810±0.0001
Jupiter					
longitude (years)	30.350°	30.423° (71 y) 30.361° (83 y)	30.351°	30.351°	30.268°±0.006°
anomaly (years)	329.650°	329.577° (71 y) 329.639° (83 y)	329.649°	329.649°	329.732±0.006°
longitude (days)	0.08309°			0.08313°	$0.08287^{\circ} \pm 0.00002^{\circ}$
anomaly (days)	0.90252°			0.90251°	0.90276°±0.00002°
synodic/long.	10.8616	10.8333 (71 y) 10.8571 (83 y)	10.8611	10.8612	10.894±0.003
Saturn					
longitude (years)	12.221°	12.203°	12.226°	12.222°	12.255°±0.006°
anomaly (years)	347.779°	347.797°	347.774°	347.778°	347.745±0.006°
longitude (days)	0.03346°			0.03349°	0.03355°±0.00002°
anomaly (days)	0.95215°			0.95215°	0.95207°±0.00002°
synodic/long.	28.4575	28.5000	28.4444	28.4551	28.375±0.015

 Table 4. Comparison of mean motions in longitude and anomaly implied by the ancient period relations.

allowing separation of the zodiacal and synodic components of observed latitudes. It is certainly conceivable that an astronomer of Hipparchus' time might have thought he could measure a movement in the nodes on the order of a degree or so per longitudinal period, failing to realize that errors of observation, reduction, and analysis would overwhelm any actual motion. Similarly, empirical arguments could have contributed to Theon's solar model, according to which the sun has a motion in latitude with a period 1/2 day shorter than its period of longitude.<sup>40</sup>

A theory of moving nodal lines for the planets could also have originated in a belief that the nodes were aligned with other elements of the system at the beginning of the Great Year but were no longer so aligned at the present date. In the Indian planetary systems that have moving nodal lines, the motion of these lines, which is always extremely slow, seems to have been a consequence of the assumption that all the nodal and apsidal lines as well as the heavenly bodies themselves were in conjunction at the beginning of a great period.

Whether or not observations played some part in the derivation of the nodal motions in the inscription, it does appear that other *a priori* considerations were involved. The evidence for this is the relationship between the nodal motions ascribed to Jupiter and Saturn. Jupiter's is straightforwardly 6 complete longitudinal revolutions in the retrograde sense in 29140 years. For Saturn, we have the anomalous entry for the revolutions in latitude, comprising two numbers, in col. iv line 11 (the termination of the corresponding entry in col. viii is unfortunately unreadable). The reading, 989 followed by 216, is not in doubt, but what does the second number mean? Tannery acutely suggested that it expressed a fraction of one period of latitude, written either as degrees or as *stigmai* such that one complete period was either  $360^{\circ}$  or (since Tannery believed that a *stigme* was half a degree)  $720 \ stigmai.^{41}$  If we suppose that degrees were meant, then 989 periods plus  $216^{\circ}$  would be equivalent to  $989^{3/5}$  periods, so that the nodes would make  $2^{2/5}$  complete revolutions in the direct sense in 29140 years. The ratio of Saturn's nodal motion to Jupiter's would thus be exactly 12 : 30, the inverse of the ratio of their round longitudinal periods,

$$12 (L_{4} - B_{4}) = 30 (L_{b} - B_{b})$$
(9)

a pattern that I would doubt can be accidental even if the 12 : 30 ratio did not turn up elsewhere in the inscription's numbers, as we shall see it does. Equivalently, one can say that according to the inscription the nodes of Jupiter and Saturn are supposed to move equal amounts (approximately 0.88°) in opposite directions in one rounded longitudinal period.

A much more rapid nodal motion is assigned to Mars: 56 revolutions in longitude in the direct sense in 29140 years, amounting to approximately 1.30° per longitudinal period or 0.69° per year. This motion does not have an obvious numerical relationship to the motions for Jupiter and Saturn. It is hard to imagine that such a parameter could have withstood comparison with observational records covering more than an interval of a few years.

<sup>&</sup>lt;sup>40</sup> Jones 2000; Jones 1999 v. 1, 170–171 and v. 2, 164–167.

<sup>&</sup>lt;sup>41</sup> Tannery 1895a, 53 (= Tannery 1912, 492).

## The periods in depth and the model structures.

Naive expectation would be that a planetary period "in depth" should be identical to a period "in relative position," since the synodic anomaly is the most obvious component of planetary anomaly and so far as we know the only one that was known in Greek astronomy before Hipparchus' time. This is obviously not the case in the Keskintos Inscription. Tannery refused to speculate in print about how the periods in depth should be interpreted, contenting himself with the true remark that we cannot take it for granted that all features of planetary models of Hipparchus' time should be "correct" geocentric representations of astronomical reality.<sup>42</sup> Herz, however, noticed that for Jupiter and Saturn the numbers of periods in longitude and depth very nearly add up to the numbers of synodic periods, and he asserted that if the synodic periods were correctly interpreted by Tannery as periods of the planet's revolution around an epicycle relative to the instantaneous apogee of the epicycle (i.e. relative to the radius from the earth to the epicycle's centre), then the periods in depth should correspond to revolutions of the planet on its epicycle relative to a sidereally fixed direction.<sup>43</sup> As Neugebauer later pointed out, Herz appears to have committed an oversight, since in a normal epicyclic model with the planet revolving around its epicycle in the same sense as the epicycle revolves around the earth, the number of revolutions of the planet around the epicycle relative to a fixed direction ought to be the sum of the longitudinal and synodic periods, not the difference:44

$$G = A + L \tag{10}$$

In Neugebauer's interpretation, the underlying model had the planet revolving around its epicycle in the opposite sense to the epicycle's longitudinal revolution, so that the number of revolutions of the planet on the epicycle relative to a sidereally fixed direction would be the number of synodic periods minus the number of longitudinal periods:

$$G = A - L \tag{11}$$

hence also:

$$L = A - G \tag{12}$$

For Saturn and Jupiter, which have relatively small epicycles and synodic periods that are much shorter than their longitudinal periods, the choice of direction of motion of the planet on the epicycle has only a small effect on any phenomena (such as retrogradations) deduced from the model, so that it would not have been an easy matter to prove that a "same-sense" model fits observations best.

<sup>&</sup>lt;sup>42</sup> Tannery 1895a, 56–57 (= Tannery 1912, 496) and Tannery 1895c, 325–327 (=Tannery 1912, 511–515).

<sup>&</sup>lt;sup>43</sup> Herz 1894, 1142–1143.

<sup>&</sup>lt;sup>44</sup> Neugebauer 1975 v. 2, 702–704.

29

Neugebauer drew attention to two problems facing his interpretation of the periods in depth. The first is that the numbers recorded for Mars do not even nearly fit, since 13648 synodic periods minus 15492 longitudinal periods would yield a (negative) difference of 1844 for the periods in depth, which cannot be reconciled in any way with the traces in line 4 of the inscription.<sup>45</sup> (The circumstance that the difference for Mars has to be negative, unlike in the cases of Jupiter and Saturn, should not matter for the number in the inscription since it would just mean that the revolutions are performed in the opposite sense.) Two responses are possible: either we have to dismiss the apparent relation subsisting in the numbers for Jupiter and Saturn as accidental, or we have to conclude that the number for Mars had a different meaning entirely, reflecting a difference in the assumed model. We will take the latter course, and hence for the time being we restrict consideration to Jupiter and Saturn.

The second of Neugebauer's difficulties is that if we subtract the inscription's periods in depth from the synodic periods for Jupiter and Saturn, we do not get exactly the attested periods in longitude, but numbers 20 less, which we will designate L' and  $L'_{10}$ ; thus for Jupiter:

$$26690 - 24260 = 2430 = 2450 - 20 \tag{13}$$

while for Saturn:

$$28148 - 27176 = 972 = 992 - 20 \tag{14}$$

or, in general:

$$L' = L - 20 = A - G \tag{15}$$

Neugebauer considered these discrepancies of 20 periods to be "very disturbing," as indeed they are, and suggested that the periods in depth were based for some unexplained reason on an "auxiliary" time period twenty years shorter than the 29140 years associated with the longitudinal and synodic periods.

Another possibility not contemplated by Neugebauer is that the periods in depth are not periods of revolution of the planet on its epicycle relative to a sidereally fixed direction, but relative to a direction that has a gradual direct motion amounting to 20 longitudinal

<sup>&</sup>lt;sup>45</sup> Neugebauer treats the number in line 4 col. iv as entirely unreadable and gives the number in col. viii as 401680 (misprinted as 491680 in table (2) on p. 700), expressing doubt about the initial 40. Tannery wrote to Hiller on December 26, 1894 (Tannery 1939, 170) that he saw on the squeeze the letters IM above the M for myriads in col. viii rather than simply M, and since this could not be read as a numeral, he corrected it to IH, reading the entire number as 182680; in his published discussions, however, he gives the number as 182680 with scarcely any mention of uncertainty about the initial digits (Tannery 1895a, 55 [= Tannery 1912, 494] and 1895c, 323 [= Tannery 1912, 509]). I can see no trace of his alleged iota.

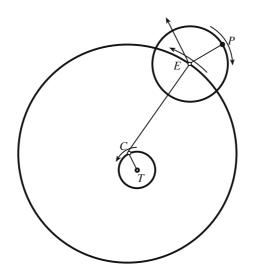


Fig. 2. Possible epicyclic model for Jupiter or Saturn in the Keskintos Inscription.

revolutions in 29140 years (or almost exactly  $^{1}/_{4^{\circ}}$  per year). This is incidentally the rate of motion of the solar apogee implied by Theon's solar model in which the sun's period in depth is  $365^{1}/_{2}$  days. It does not seem absurd to suggest that in the Keskintos Inscription it represents a shifting apsidal line for the models of Jupiter and Saturn, which would mean that the underlying models involved a second anomaly as well as the synodic anomaly. Again it is worth remarking that while Ptolemy's planetary apsidal lines are sidereally fixed, their counterparts in some Indian planetary theories have slow shifts.

Reckoning the motion of a planet on an epicycle according to a sidereal frame of reference seems unnatural in a geocentric system, although G. J. Toomer pointed out that a comparable convention exists in Indian planetary theories.<sup>46</sup> To reckon the same motion according to a frame of reference with a slow sidereal shift is still more counterintuitive. Perhaps, therefore, we should not assume that the model was epicyclic, but rather the kinematically equivalent eccentric model in which the planet performs its longitudinal revolution on the eccentre while the centre of the eccentre revolves around the earth to generate the synodic anomaly. Such a model would resemble a Tychonic model, except that the centre of the eccentre bearing the planet would not revolve around the earth with the sun's motion but with a mirror image of the sun's motion.

Fig. 2 shows the epicyclic version of the model under consideration. The centre *C* of the deferent revolves around the earth, *T*, in the direct sense (counterclockwise as seen from the north) at a rate of  $1/4^{\circ}$  per year relative to a sidereally fixed direction. The centre *E* of the epicycle revolves around the deferent in the direct sense, making one revolution relative to a sidereally fixed direction in one "period in longitude." The planet, *P*, revolves around the epicycle in the retrograde sense (clockwise) making one revolution relative to a direction parallel to *TC* in one "period in depth," but one revolution relative to the radius *CE* (or *TE*) in one "period in relative position."

<sup>&</sup>lt;sup>46</sup> Neugebauer 1975 v. 2, 704 n. 28.

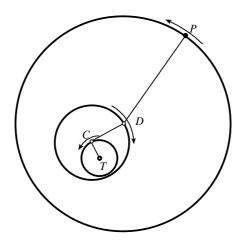


Fig. 3. Possible model for Jupiter or Saturn with revolving eccentre.

In the kinematically equivalent eccentric version (Fig. 3), the centre *C* of a small deferent revolves around the earth, *T*, in the direct sense at a rate of  $1/4^\circ$  per year relative to a sidereally fixed direction, just as in the epicyclic version. The centre *D* of the eccentre bearing the planet revolves in the retrograde sense around the deferent, making one revolution relative to the apsidal line *TC* in one "period in depth." Finally, the planet *P* revolves around the eccentre in the direct sense, making one revolution relative to a sidereally fixed direction in one "period in longitude."

A third kinematically equivalent variation (Fig. 4) is again epicyclic, but with the deferent centred on the earth, and an "eccentric epicycle," i.e. the centre of the epicycle is offset from the point *C* on the deferent that revolves with the planet's longitudinal period around the earth. The direction of the offset *CE* has the  $1/4^{\circ}$  per year motion, and the "period in depth" is that of the planet's retrograde motion around the epicycle relative to *CE*. This model structure is perhaps least like anything we know of in Greek astronomy, for what that consideration is worth, but it makes the most immediate sense of the periods as they are expressed in the inscription.

Dennis Duke has drawn my attention to one further remarkable numerical relation.<sup>47</sup> L' for Jupiter and Saturn, which we are tentatively interpreting as the planets' periods of motion around the earth relative to the reference direction that has the  $1/4^{\circ}$  per year motion (apsidal line or what have you), are respectively 2430 and 972. These numbers are respectively  $81 \times 30$  and  $81 \times 12$ , hence exactly in the the ratio between Saturn's and Jupiter's longitudinal periods rounded to whole years, 30 : 12. The fact that the common factor 81 is a numerologically appealing number and a factor (1/360) of 29160 is also surely deliberate (in fact both 2430 and 972 divide evenly into 29160).

Hence L' for this pair of planets was chosen subject to numerological constraints. If L was determined simply as the closest whole number approximating the number of the planet's longitudinal periods in 29140 years resulting from an accepted period relation, it would have to be an accident that L - L' is the same number, 20, for both planets. I think it is

<sup>&</sup>lt;sup>47</sup> Personal communication.

much more probable that L itself was adjusted (probably reduced for Jupiter and increased for Saturn) so that the differences would be equal, reinforcing the linkage between the two planets in which the author of the inscription evidently believed. Part of the astronomical inaccuracy of L can be ascribed to this adjustment.

Turning to Mars, we find that this planet's periods in depth are nearly or exactly triple the periods in relative position, thus  $3 \times 13648 = 40944$  whereas the inscription seems to give 40965 or 40964 (accepting the reading of the third digit suggested by the traces in col. iv). Thus we seem to have:

$$G = 3A + 21 \tag{16}$$

or

$$G = 3A + 20 \tag{17}$$

The discrepancy, whether 21 or 20, is difficult to interpret in terms of a model, and cautious allowance should be made for the wretched condition of legibility of the numerals in this part of the inscription—though to get rid of the discrepancy entirely one would have not only to read Y rather than X in line 4 col. viii, but also M rather than  $\Xi$  in col. iv where I judged the identification of the letter to be secure. In support of reading 40965 for *G*, with a resulting discrepancy of 21, is the circumstance that:

$$L = 81 \times 191 + 21 \tag{18}$$

which may indicate that the difference between L (which in this instance may *not* be a numerologically manipulated value) and the nearest integer multiple of 81 is again being built into G even though this time G is otherwise dependent only on A, not L.

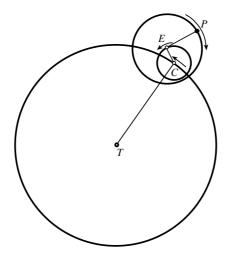


Fig. 4. "Eccentric epicycle" model for Jupiter or Saturn.

Leaving this puzzle of the small discrepancy aside, the fact that the periods in depth are approximately triple the synodic periods is remarkable in two ways. First, there is no longitudinal component in the number of periods in depth such as we found for Jupiter and Saturn. This must mean that for Mars the frame of reference is geocentric, i.e. relative more or less to the direction from the earth to the planet. There would be no obvious motivation for treating Mars differently in this respect unless the assumed model was somehow different; one possibility is that Mars was given an epicyclic model, while Jupiter and Saturn were supposed to have eccentric models along the lines of Fig. 3.

But even in a conventional geocentric frame of reference, i.e. counting revolutions of the planet on its epicycle with respect to the rotating radius from the earth to the epicycle's centre, we would expect the number of periods in depth to be equal to the number of synodic periods, not triple that number, since in any "correct" planetary theory a single cycle of variation in apparent longitudinal motion should coincide with a single cycle of phases relative to the sun, so that the retrogradations will invariably occur during the time around opposition for a superior planet, or around alternate conjunctions for an inferior planet. But here we seem to have a model in which either the motion in depth is not the primary cause of retrogradation or there will be three retrogradations in every synodic period, presumably one around opposition but the other two during stages not long after the planet's first morning visibility and not long before its last evening visibility.

Although a model involving spurious retrogradations seems hard to reconcile with any serious program of observation, we have striking confirmation from another source that there existed a tradition in early Greek astronomy that attributed to Mars three anomalistic periods for every synodic period. This is the famous passage in Simplicius' *Commentary on Aristotle's De Caelo* in which Simplicius describes Eudoxus' models of homocentric spheres. According to Simplicius (ed. Heiberg 486), the spheres in Eudoxus' planetary models that account for the anomaly have the following approximate periods:

Saturn	13 months
Jupiter	13 months
Mars	8 months and 20 days
Venus	19 months
Mercury	110 days

It has long been recognized that, alone among these periods, the one for Mars is not merely inexact but spectacularly wrong since Mars' mean synodic period is approximately 780 days, i.e. about 23 months. The usual assumption is that the transmitted text of Simplicius' commentary, or the text on which he drew, is corrupt here and attempts have been made to "emend" it.<sup>48</sup> However, 8 months of 30 days plus 20 days amounts to 260 days, i.e. exactly one third of 780 days. The Simplicius passage and the Keskintos Inscription support each other's readings as representing an authentic, if bizarre, ancient theoretical presumption

(19)

<sup>&</sup>lt;sup>48</sup> Mendell 2000, 104–105 n. 67.

about Mars' motion. I refrain from speculating whether it really goes back, as Simplicius claims, to Eudoxus and was first associated with homocentric models (which by hypothesis cannot have involved "motion in depth") or whether it was a later, Hellenistic, innovation that somehow was grafted on a retrospective account of Eudoxus' theories.

Simplicius' short anomalistic period for Mars has sometimes been defended on the grounds that, with just one anomalistic period per synodic period, Eudoxus' four-sphere model cannot be made to generate retrogradations for a planet having Mars' longitudinal and synodic periods.<sup>49</sup> The argument runs that, rather than abandon the model, Eudoxus (or a later adapter or reconstructor if we prefer) chose a parameter that would result in plausible retrogradations in the expected situations relative to the sun, even if this entailed additional retrogradations where none were wanted. Interestingly, a similar problem arises if one tries to construct a "wrong-sense" epicyclic model for Mars: the correct periodicities do not allow retrogradation to occur no matter what epicycle radius is assumed, but one can get retrogradations, including of course spurious ones, by tripling the rate of the planet's revolution on its epicycle.<sup>50</sup> This explanation perhaps leaves too many questions to be entirely satisfying. First, the retrograde arcs generated by this model are much too small (a mere 1.2° in a 70-day retrogradation if the model involves no eccentricity). Then, why not simply reverse the direction of motion on the epicycle, especially given that this would not only fix the retrogradations but also bring Mars closest to the earth around opposition when it is conspicuously brightest? And why, if this was not done, were the periods in depth of Mars' model reckoned differently from those of Saturn and Jupiter? In short, I think we are much further from understanding the theory of Mars underlying the inscription than we are with respect to the theories of Jupiter and Saturn.

## The place of the Keskintos Inscription in the history of Greek planetary theory.

To get some sense of how important the chance discovery of the Keskintos inscription is for our knowledge of how Greek planetary theory developed leading up to the well documented models of Ptolemy, it suffices to review the most important other ancient sources:

*Simplicius*, writing in the sixth century A.D., narrates the rise and fall of homocentric modelling of the sun, moon, and planets in Book 2 of his *Commentary on Aristotle's De Caelo*, of which the most famous part is his account of the models of Eudoxus and Callippus (for which we also have sketchy information in a text far closer to their time, Aristotle's *Metaphysics* 12.8). His story essentially begins with Plato and Eudoxus and ends with Autolycus, so that it spans roughly a hundred years from the early fourth to the early third century B.C., with some vague remarks about unspecified later astronomers introducing (or appropriating from the Pythagoreans) the devices of eccentres and epicycles. Simplicius appeals, directly or indirectly, to several lost authors

<sup>&</sup>lt;sup>49</sup> Heath 1913, 209–210; Aaboe 2001, 71–72.

<sup>&</sup>lt;sup>50</sup> Aaboe 1963, 5–6. Dennis Duke has pointed out to me that one would not get retrogradations if one merely doubled the number of anomalistic periods for Mars instead of tripling them.

including Eudemus and Sosigenes. The reliability of Simplicius' narrative cannot be taken for granted, and there are large divergences among modern attempts to reconstruct Eudoxus' and Callippus' models while taking Simplicius' information seriously.<sup>51</sup>

Ptolemy makes several assertions about Hipparchus' work on planetary theory in Almagest 9.2–3. He tells us that Hipparchus possessed a set of period relations for the planets' motions in longitude and anomaly, which we recognize as derived from the Babylonian Goal Year relations though possibly with correction terms that Ptolemy does not preserve for us. Moreover, he states that Hipparchus did not publish theories of his own for the planets in any work that Ptolemy knew of, but he did organize a collection of planetary observation reports and used this in a polemical work to demonstrate that the models employed by the "mathematicians" of his time disagreed with the phenomena. Ptolemy alleges that those models were defective because their authors had "made their geometrical demonstrations concerning a single, constant anomaly and retrogradation," which seems to mean that the models did not recognize the zodiacal anomaly.<sup>52</sup> Finally, Ptolemy disparages certain unnamed astronomers (*not* characterized as "mathematicians") who "chose to exhibit the uniform and circular motion [of the planets] by means of what is called the *Eternal Table-construction* [διὰ τῆς καλουμένης αἰωνίου κανονοποιίας]." According to Ptolemy, what these people produced, which seems to have been sets of tables ostensibly derived from models involving "eccentric circles or [circles] concentric with the zodiac and carrying epicycles or (by Zeus!) the combination of the two," attempted to account quantitatively for both planetary anomalies, but in a way that was both erroneous (διεψευσμένως) and nondemonstrative (ἀναποδείκτως). We have references to *Eternal Tables* (αἰώνιοι κανόνες) as sources of computed planetary positions in the Anthologiae of Ptolemy's near-contemporary Vettius Valens (6.2 ed. Pingree) and in the papyrus horoscope cast by one Titus Pitenius for a person born in A.D. 81 (*P. Lond.* 130 = GH no. 81, lines 12–13), so Ptolemy is probably speaking of developments since Hipparchus' time.<sup>53</sup> It is possible, as Toomer has suggested, that the term  $\alpha i \omega v_1 \alpha$  alludes to a use of combined periods for all the planets' periodicities as in Indian astronomy and the Keskintos Inscription.54

<sup>&</sup>lt;sup>51</sup> On the reliability of Simplicius and his relationship to his sources see the divergent views of Mendell 2000 and Bowen 2002. The classic interpretation of Eudoxus' models, Schiaparelli 1877 (= Schiaparelli 1925–1927 v. 2, 3–112), now competes with that by Mendell 1998 and the more radical reconstruction by Yavetz 1998.

 $<sup>^{52}</sup>$  It is also possible to read the phrase in question as meaning that the demonstrations in question, whatever they were, were strictly valid only on the assumption of no zodiacal anomaly although the models ostensibly involved both anomalies. But the circumstance that Hipparchus is supposed to have refuted them from observations, not

on grounds of internal consistency, supports the stronger interpretation that I put forward in the text above.

<sup>&</sup>lt;sup>53</sup> For the horoscope see Neugebauer and van Hoesen 1959, 21–28. Two allusions to αἰώνιοι κανόνες (or αἰώνια κανόνια) in Byzantine translations of works by Abu Ma'shar presumably have no connection with the ancient tables except as a verbal echo: Cumont and Boll 1904, 147 line 13 and Pingree 1968, 11 line 7. <sup>54</sup> Toomer 1984, 422 note 12.

*Indian planetary theories* are generally accepted to have been based on Greek models that either date from the period between Hipparchus and Ptolemy or from the time after Ptolemy, but with little or no influence of Ptolemy's own work. The Indian models use epicycles or eccentres to represent both anomalies, and although structurally different from Ptolemy's models as described in the texts, the motions that they generate are kinematically very close to Ptolemy's models, if one makes allowances for the different numerical parameters (which are generally inferior in the Indian texts).<sup>55</sup>

*Pliny the Elder* has by far the most to say about planetary theory among the Greco-Roman literary sources between Hipparchus' and Ptolemy's time. His discussion of astronomical matters is notoriously muddled and largely unintelligible, but there are some indisputable references to epicyclic models ("wrong sense" for the superior planets, "right sense" for the inferiors) and to the apogees of eccentric deferents (2.64–75).<sup>56</sup> It is impossible to say which among the more than forty Roman and "foreign" authors that Pliny cites for Book 2 of the *Naturalis Historia* were his maltreated sources for the section on planetary motion.

*Papyri* of the Hellenistic and Roman periods contain extremely few specifics about planetary models. The most informative document is *P. Mich.* 3.149, a second century A.D. astrological text that describes epicyclic models with specific radii (made to agree with a numerological principle) and motion of the planet in the "right sense" for the inferiors and "wrong sense" for the superiors, as in Pliny.<sup>57</sup> Inaccurately and imprecisely specified apogees imply that the models used eccentricity for the zodiacal anomaly, again as in Pliny. Numerous papyrus tables from the first century A.D. and after show that the prevailing methods of computing planetary positions among the astrologers of the time were either adaptations of the Babylonian "ACT" models or, after the second century, versions of Ptolemy's tables.<sup>58</sup> No kinematic planetary tables unrelated to Ptolemy's have yet been securely identified on papyrus.

It would hardly be an exaggeration to say that the Keskintos Inscription is the only document or artifact bearing on Greek planetary theory before Ptolemy that contains exact and (partially) intelligible information about sophisticated planetary modelling and that is actually contemporary with the science that it reports and thus not subject to any suspicion that its contents might be a reconstruction or distortion. Given the exiguous survival rate of Greco-Roman artifacts, the fact that we have such an inscription at all implies that similar documents and similar astronomical activities were reasonably common in the late Hellenistic period. Practitioners of astronomy surely ranged from a small number of people

<sup>&</sup>lt;sup>55</sup> Pingree 1978, especially 555–560; Duke 2005.

<sup>&</sup>lt;sup>56</sup> Neugebauer 1975 v. 2, 802–805.

<sup>&</sup>lt;sup>57</sup> Neugebauer 1975 v. 2, 805–808; Aaboe 1963.

<sup>&</sup>lt;sup>58</sup> Jones 1999 v. 1, 113–119.

37

(such as Hipparchus) who made important contributions down to a small number of cranks and incompetents; but most of the astronomers, like most physicians, mathematicians, philosophers, and other intellectuals of the time, would have been carrying out "normal science" in the Kuhnian sense, working the variations on an accepted set of problems and producing results that were perhaps original insofar as they were different from anyone else's but not notably better or worse. The Keskintos Inscription is thus most likely representative of typical rather than cutting-edge astronomy around 100 B.C.; but in any case we can be sure that anything we find in it or implied by it was part of the science of that time, whether or not we have parallels in our other sources.

The inscription complements Ptolemy's account of the planetary theory of Hipparchus' time, and to some extent supports its credibility. According to Ptolemy Greek planetary modelling was in flux around the second half of the second century B.C. The zodiacal anomaly was as yet not well understood, and the current mathematical models did not attempt to deal with it, or at least not in a consistent way. Moreover, the fact that Hipparchus expressed the planetary periodicities in terms of the Babylonian Goal Year period relations, with or without corrections, implies that the more accurate ACT relations were not yet known to Greek astronomers. The inscription, on the other hand, portrays a system that likely dates from a short time after Hipparchus' work. The conception of planetary motion seems to be influenced less by Babylonian mathematical astronomy than by analogies with lunar theory: thus we have shifting nodal and, it seems, shifting apsidal lines (as in Theon's solar theory), and models according to which the planet has the slowest apparent motion when furthest from the earth. The models for Jupiter and Saturn may have yielded decent agreement with the phenomena known at the time; but it was apparently understood that a model of the same structure would not work for Mars-although it is hard to believe that the model adopted instead, somehow involving a tripling of the anomalistic periods, was much better. The periodicities, which are no better than those known to Hipparchus, and perhaps worse, have been adjusted to make them conform to certain simple numerical relations, including an assumption that all periods are simultaneously completed in a long "Great Year" formed by repeatedly multiplying the smallest prime numbers together.

The role in Greek astronomy of such numerical relations, which I have characterized above as "numerology" for want of a more neutral word, deserves more consideration than it has had. They apparently have little to do with astrology (which in any case was just incipient in the Greco-Roman world around the time of our inscription), but have some affinity with harmonic theory, in which ratios of small whole numbers were commonly used to model musical intervals. Besides the Keskintos Inscription, I am aware of two instances of fairly sophisticated Greek astronomical numerology. First, in *P. Mich.* 3.149, col. i lines 9–25, values for the radii of the epicycles of the seven heavenly bodies are listed such that their sum divided by three is exactly the epicycle radius of Venus. Secondly, Ptolemy's *Canobic Inscription* gives the mean distances of the moon and sun from the earth as respectively 64 and 729 earth-radii, explicitly pointing out that these are the sixth powers of 2 and 3. This is the more interesting because we know that Ptolemy would have obtained

numbers fairly close to 64 and 729 from the empirical data on which he relied at this time, so the numerological relation is one that he discerned in the results of his calculations, not one that he imposed *a priori*. Although in the third book of his *Harmonics* (which, like the *Canobic Inscription*, seems to belong to the earlier part of his career) Ptolemy writes at length, if rather vaguely, about the applicability of harmonic science to aspects of astronomical and astrological modelling, there are scarcely any traces of interest in such numerical patterns in the *Almagest* or the other later astronomical writings. Even the pretty numerical relationship between the solar and lunar distances was discarded.

In discussing the dedication, I remarked that it could have accompanied an object visually displaying the planetary system to which the astronomical data of the inscription would then serve as a commentary. This object, if it existed, might have been a stationary pictorial representation or solid model, though the alternative possibility of a mobile or mechanical planetary device, a specimen of what was called *sphairopoiia*, is attractive. I must leave it to others having greater technological expertise to consider whether the inscription's period relations could practically have been made the basis of a gearwork device for displaying planetary motion, comparable to the well known Antikythera Device which was roughly contemporary with the inscription.<sup>59</sup> There is plenty of room here for speculative play.

Several points of resemblance between the Keskintos Inscription and Indian planetary theory have been mentioned above, in particular the long combined periods and the slowly moving nodal lines. I have no doubt that the inscription represents an early stage in the line of Greek planetary theory that evolved into the lost sources of the Indian tradition. On the other hand, the Indian models are much nearer to Ptolemy's in their structural consistency and in their representations of planetary anomaly and latitudinal motion. Between roughly 100 B.C. and A.D. 100 most of the fundamental questions concerning apparent planetary motion seem to have been settled in a way that left much less freedom for variation in modelling structures in Ptolemy's time than was possible in Hipparchus'.

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<sup>&</sup>lt;sup>59</sup> Price 1974, 62 briefly adduced the Keskintos Inscription as evidence of astronomical activity at Rhodes about the time when the Antikythera Mechanism was made (*possibly* at Rhodes). This allusion seems to have inspired an episode set in the "lush valley" of Keskinto (thick with "olive, pine, and citrus... among the oak and cypress trees"—citrus is a solecism for the Mediterranean at this date) in the fanciful but researched novella that is the first part of Kean 1991 (26–27), from which the epigraph of this paper is taken. (Cf. also Kean 1993, 71–72, a more prosaic account of the inscription for the benefit of tourists [!].)

## **References.**

- Aaboe, A. 1963. "On a Greek Qualitative Planetary Model of the Epicyclic Variety." *Centaurus* 9, 1–10.
- Aaboe, A. 2001. Episodes from the Early History of Astronomy. New York.
- [Anonymous]. 1894. "Sitzungsberichte der archäolog. Gesellschaft zu Berlin. 1894. Juni." Jahrbuch des Kaiserlich deutschen Archäologischen Instituts 9, supplement: Archäologischer Anzeiger. 122–125.
- Bowen, A. C. 2002. "Simplicius and the Early History of Greek Planetary Theory." *Perspectives on Science* 10, 155–167.
- de Callataÿ, G. 1996. *Annus Platonicus: A Study of World Cycles in Greek, Latin and Arabic Sources*. Publications de l'Institut orientaliste de Louvain 47. Louvain-la-Neuve.
- Chaniotis, A. 1988. *Historie und Historiker in den griechischen Inschriften: Epigraphische Beiträge zur griechischen Historiographie*. Heidelberger althistorische Beiträge und epigraphische Studien 4. Stuttgart.
- Cumont, F. 1935. "Les noms des planètes et l'astrolatrie chez les Grecs." *L'Antiquité Classique* 4, 5–43.
- Cumont, F., and F. Boll. 1904. *Catalogus Codicum Astrologorum Graecorum* 5.1. Brussels.
- Duke, D. 2005. "The Equant in India: The Mathematical Basis of Ancient Indian Planetary Models." Archive for History of Exact Sciences 59, 563–576.
- Goldstein, B. R. 1967. *The Arabic Version of Ptolemy's Planetary Hypotheses*. Transactions of the American Philosophical Society N.S. 57.4. Philadelphia.
- Hamilton, N. T., N. M. Swerdlow, and G. J. Toomer. 1987. "The Canobic Inscription: Ptolemy's Earliest Work." In J. L. Berggren and B. R. Goldstein, eds., *From Ancient Omens to Statistical Mechanics*. Copenhagen. 55–73.
- Heath, T. L. 1913. Aristarchus of Samos: The Ancient Copernicus. Oxford.
- Herz, N. 1894. "Über eine unter den Ausgrabungen auf Rhodos gefundene astronomische Inschrift." *Sitzungsberichte der mathematisch-naturwissenschaftlichen Classe der Kaiserlichen Akademie der Wissenschaften* (Wien) 103 IIa, 1135–1144 and plate.
- Hiller von Gaertringen, F. 1894. "Die Zeitbestimmung der rhodischen Künstlerinschriften." Jahrbuch des Kaiserlich deutschen Archäologischen Instituts 9. 23–43.
- Hiller von Gaertringen, F. 1895. Inscriptiones Insularum Maris Aegaei praeter Delum. Fasc. 1. Inscriptiones Rhodi Chalces Carpathi cum Saro Casi. Berlin.
- Hiller von Gaertringen, F. 1942. Review of C. Blinkenberg and K. F. Kinch, *Lindos. Fouilles de l'acropole 1902–1914*, vol. 2. *Göttingische Gelehrte Anzeigen* 204, 161–169.
- Hunger, H. 1999. "Non-Mathematical Astronomical Texts and their Relationships." In N. M. Swerdlow, ed., Ancient Astronomy and Celestial Divination. Cambridge, Mass. 77–96.
- Hunger, H., and D. Pingree. 1999. Astral Sciences in Mesopotamia. Handbuch der Orientalistik 1.44. Leiden.
- *IG* 12.1 = Hiller von Gaertringen 1895.

- Jones, A. 1990. Ptolemy's First Commentator. Transactions of the American Philosophical Society 80.7. Philadelphia.
- Jones, A. 1999. *Astronomical Papyri from Oxyrhynchus*. 2 vols. Memoirs of the American Philosophical Society 233. Philadelphia.
- Jones, A. 2000. "Studies in the Astronomy of the Roman Period IV. Solar Tables Based on a Non-Hipparchian Model." *Centaurus* 42, 77–88.
- Jones, A. 2005. "Ptolemy's *Canobic Inscription* and Heliodorus' Observation Reports: Text, Translation, and Notes." *SCIAMVS* 6, 53–97.
- Kean, V. 1991. The Ancient Greek Computer from Rhodes known as the Antikythera Mechanism. Anixi Attikis.
- Kean, V. 1993. Rhodes: New Light on Old Mysteries. Anixi Attikis.
- Lehoux, D. 2005. "The Parapegma Fragments from Miletus." Zeitschrift für Papyrologie und Epigraphik 152, 125–140.
- Lehoux, D. 2007. Parapegmata. Cambridge. (forthcoming)
- *Lindos IV*, 2 = Sørensen and Pentz 1992.
- McLean, B. H. 2002. An Introduction to Greek Epigraphy of the Hellenistic and Roman Periods from Alexander the Great down to the Reign of Constantine (323 B.C. – A.D. 337). Ann Arbor.
- Mendell, H. 1998. "Reflections on Eudoxus, Callippus, and Their Curves: Hippopedes and Callippopedes." *Centaurus* 40,177–275.
- Mendell, H. 2000. "The Trouble with Eudoxus." In P. Suppes, J. M. Moravcsik, and H. Mendell, eds., Ancient & Medieval Traditions in the Exact Sciences. Essays in Memory of Wilbur Knorr. Stanford. 59–138.
- Morelon, R. 1993. "La version arabe du *Livre des Hypothèses* de Ptolémée." *Mélanges de l'Institut dominicain d'Études Orientales* 21, 7–85.
- Neugebauer, O. 1955. Astronomical Cuneiform Texts. 3 vols. London.
- Neugebauer, O. 1975. A History of Ancient Mathematical Astronomy. 3 vols. Berlin.
- Neugebauer, O., and D. Pingree. 1971. *The Pañcasiddhāntikā of Varāhamihira*. 2 vols. Det Kongelige Danske Videnskabernes Selskab, Historisk-Filosofiske Skrifter 6.1. Copenhagen.
- Neugebauer, O., and H. B. van Hoesen. *Greek Horoscopes*. Memoirs of the American Philosophical Society 48. Philadelphia.
- Newton, C. T. 1865. Travels and Discoveries in the Levant. 2 vols. London.
- Pingree, D. 1968. Albumasaris de revolutionibus nativitatum. Leipzig.
- Pingree, D. 1978. "History of Mathematical Astronomy in India." *Dictionary of Scientific Biography*15. New York. 533–633.
- Price, D. 1974. Gears from the Greeks. The Antikythera Mechanism A Calendar Computer from ca. 80 B.C. Transactions of the American Philosophical Society N.S. 64.7. Philadelphia.
- Rouse, W. H. D. 1902. *Greek Votive Offerings: An Essay in the History of Greek Religion*. Cambridge.

- Schiaparelli, G. V. 1877. "Die homocentrischen Sphären des Eudoxus, des Kallippus und des Aristoteles." Abhandlungen zur Geschichte der Mathematik 1. (= Zeitschrift für Mathematik und Physik. Supplement zur historisch-literarischen Abtheilung d. 22. Jahrg.) 101–198.
- Schiaparelli, G. V. 1925–1927. Scritti sulla storia della astronomia antica. 3 vols. Bologna.
- Sørensen, L. W. and P. Pentz. 1992. Lindos IV, 2: Excavations and Surveys in Southern Rhodes: The Post-Mycenaean Periods until Roman Times and the Medieval Period. Copenhagen.
- Tannery, P. 1895a. "L'Inscription astronomique de Keskinto." *Revue des Études grecques* 8, 49–58.
- Tannery, P. 1895b. "Sur l'inscription astronomique de Keskinto." *Comptes rendus de l'Académie des Sciences* 120, 363–365.
- Tannery, P. 1895c. "Une inscription grecque astronomique." *Bulletin astronomique* 12, 317–328.
- Tannery, P. 1895d. "Sur les subdivisions de l'heure dans l'antiquité." *Revue archéologique* 26, 359–366.
- Tannery, P. 1912. Mémoires scientifiques publiés par J.-L. Heiberg et H.-G. Zeuthen. II. Sciences exactes dans l'antiquité 1883–1898. Toulouse.
- Tannery, P. 1939. *Mémoires scientifiques publiés par J.-L. Heiberg et H.-G. Zeuthen. XV. Correspondance*. Toulouse.
- Volanakis, I. [Ι. Βολανάκης] 1994. Επιφανειακές έρευνες: επισημάνσεις περισυλλογή αρχαίων. Ρόδος. Λάρδος, Κέσκιντος. Αρχαιολογικόν Δέλτιον 49 Χρονικά Β 2, 211.
- Yavetz, I. 1998. "On the Homocentric Spheres of Eudoxus." Archive for History of Exact Sciences 51, 221–278.

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