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A preliminary study in the critical assessment of diagrams in Greek mathematical works

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I Introduction: critical text vs. uncritical diagrams in modern editions

We read Greek mathematical works in critical editions, or in modern translations of these. The editor provides a text reconstructed from the primary manuscripts through established procedures of textual criticism, and we can examine the editor's individual decisions by consulting the critical apparatus.

The text of a mathematical work is almost always accompanied by diagrams. It is quite certain that these existed in the original, because it is often impossible to understand the text without them. However, the figures accompanying the text in the editions we read, have not been critically assessed using procedures similar to those applied to the text. Indeed, the figures in modern editions are often different from any found in the extant manuscripts, and sometimes the editors seem to have invented new figures on the basis of ideas of mathematical consistency and generality.[1]

Critical examination of the diagrams is therefore necessary. Such an undertaking requires a comparison of the diagrams in a number of manuscripts, but this is not an easy task. Some diagrams are hard to see because of the poor condition of the manuscript; lines and labels are not always readable. Hence, one should first transcribe the figures, just as one transcribes the text when producing an edition.

This article reports a preliminary study of the diagrams using a simple computer program. We discuss the general characteristics of the figures, and then make a case study of the particularly complicated situation found for proposition III.25 of the

[1]The diagrams have not been altogether ignored by historians. Ver Eecke, in his translation of the *Spherics* of Theodosius (1927), sometimes refers to the drawings in the manuscripts or in previous editions (propositions II.10, 19, 20). However, his aim was to give more general, mathematically correct figures.

Jones, in his edition of Book VII of Pappus (1986), gave an apparatus to the diagrams (2:620–627), an attempt which should be noted as a forerunner to the more recent studies of diagrams (such as [De Young 2005] and [Keller 2005]), for which the influence of [Netz 1999] has been decisive.

Elements. Appendix 1 describes the program we used to transcribe the figures (the program is distributed gratis), and Appendix 2 gives example transcriptions: the figures of all 48 propositions of Book I of the *Elements*, in six principal manuscripts (four in Greek, two in Latin).

II General characteristics of the diagrams in manuscripts

II.1 Overspecification

When we look at the diagrams in manuscripts, the first thing we realize is that they are much less general than those printed in modern editions. Where modern editors shows us parallelograms, in the manuscripts we find rectangles, even squares (e.g., Appendix 2, prop. I.35, 36, 42–45). For a proposition treating any triangle, modern editions give a scalene, general triangle, while we often encounter an isosceles or right triangle in manuscripts (e.g., Appendix 2, prop. I.4, 8, 17–21, 25, 26, 47).

Let us call this phenomena in the manuscript diagrams *overspecification*. A question that immediately comes to mind is whether the overspecification originates with the ancient authors, or is the result of simplification and modification on the part of medieval scribes. Although it is difficult to give a definitive answer to this question, we are inclined to think that overspecification had its origin in antiquity. There are two principal reasons for this position. Firstly, this phenomenon can be found in almost every proposition and it is unlikely that independent modification of the diagrams occurred on such a large scale. Secondly, as we will see below, some figures in the manuscripts are simply incorrect as an accurate metrical representation of the geometric objects, although they are nonetheless capable of representing the geometric situation at issue. These “incorrect” drawings suggest that diagrams are not meant to be a strict reproduction of the spatial relationships of geometric objects along the lines of a photograph, but are rather meant to be a schematic representation. Overspecification can also be understood as a feature of schematic representation.

II.2 Incorrect diagrams

Sometimes the diagrams in the manuscripts are simply wrong and yet, nevertheless, serve quite well to represent the configuration of the geometric objects treated in the propositions that they accompany.

A conspicuous example was pointed out by Reviel Netz: in Archimedes’ *Sphere and Cylinder*, the figure of a dodecagon inscribed in a circle appears several times. The manuscript diagrams represent the sides of the dodecagon not by straight lines but by concave curves, so that they can easily be distinguished from the circumference of the circle.[2] In a similar vein, in proposition 14 of Archimedes' *Method*, where a semicircle and parabola almost coincide, the arc of the parabola is represented by two straight lines.[3] Although these particular "incorrect" diagrams may result from some misunderstanding on the part of the scribes, there are other kinds of "incorrect" figures that are difficult to attribute to simple scribal error or alteration.

In the works on spherics, in which circles are generated as sections of a sphere, we find drastically simplified representations. Theodosius' *Spherics* proposition I.6 (Fig. 1) shows the circle ΓΔ passing through the center, H, of a sphere, and two other smaller circles, AB and EZ, in the same sphere.[4] So the three circles in the figure are sections of one sphere with center H, where ΘH and HK are not necessarily in a line. The figure is incorrect if we expect a visual representation in linear perspective, but it still serves its purpose as a schematic representation of the objects at issue.

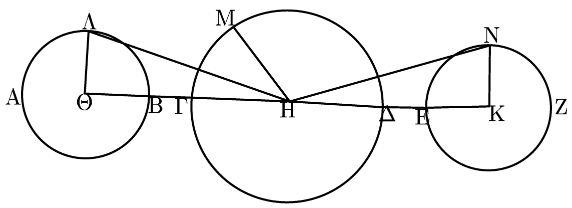
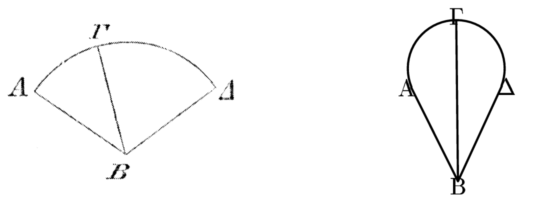


Fig. 1: Theodosius' *Spherics* I.6 (Vat. Grec. 204)



Heiberg **Vat. Grec. 204**

Fig. 2: Euclid's *Catoptrics*, figure of prop. 5

We find "incorrect" figures even where the lines are not so complicated as to make it difficult to represent them all correctly. Proposition 5 of Euclid's *Catoptrics* treats a concave spherical mirror. In one manuscript diagram, the center of the sphere (B in Fig. 2) is obviously outside of the sphere.[5]

[2]See [Netz 2004], proposition I. 26, 28, 30, 32, 33 etc. In this edition, the figures are accompanied by a critical apparatus. Similar diagrams can be seen in some of the manuscripts of Euclid's *Elements* IV.16, where a fifteen-sided polygon is inscribed in a circle.

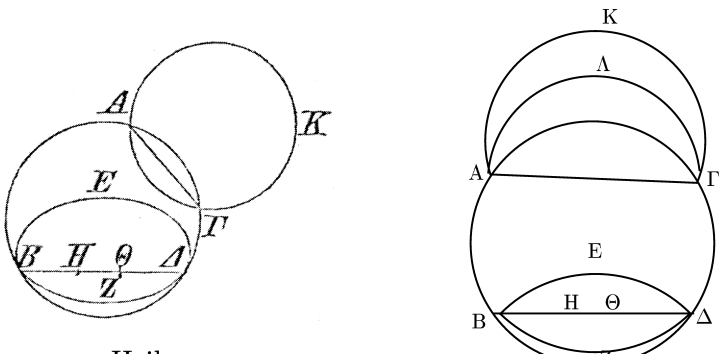
[3]See [Netz, Saito and Tchernetska 2001–2002, Pt.1,13; Pt.2,125].

[4]The proposition proves that if the circle ΓΔ is a great circle and (1) if AB and EZ are at equal distance from the center of the sphere, they are equal to each other, while (2) if AB is farther from the center than EZ, then AB is smaller than EZ.

[5]I would like to thank Michela Malpangotto and Riccardo Bellé for drawing my attention to the examples cited from *Spherics* and *Catoptrics* respectively.

Sometimes a mathematically incorrect diagram is drawn in order to present a situation assumed in a proof by *reductio ad absurdum.* In proposition 13 of Book III, Euclid shows that two circles cannot touch each other at more than one point. One of the circles is ΑΒΔΓ while the second circle is either ΒΔΕΖ (consisting of two arcs) for the case of touching internally, or ΑΛΓΚ (a lune, not a circle!) for the case of touching externally.[6]

Heiberg replaces the second circle touching externaly by a circle *intersecting* the first circle at two points, Α and Γ, which fails to represent the hypothesis of touching at two points. Since the hypothesis is impossible, the "incorrect" figures in the manuscripts are, in fact, a better representation of the situation. What is most problematic, of course, is that Heiberg does not mention his decision to deviate from the manuscripts.[7]



Heiberg   **codex P**

Fig. 3: Proposition III.13

III Proposition III.25: A case study

III.1 III.25 in Heiberg's edition

The text of a mathematical work has variants: it differs from one manuscript to another. The same holds true for the diagrams. Let us take a rather complicated example of the variants of a diagram: proposition 25 of Book III of the *Elements* in the Greek tradition.

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[6]We show here the figure redrawn from codex **P** (for the siglum, see Appendix 2). All the manuscripts that I have been able to examine, including some belonging to the Arabic and Latin traditions, agree in representing the second circle by a lune.

[7]Heiberg also made the decision to delete point Λ, which is present in the diagrams of all principal manuscripts (except **p**, which has something like an iota). Strangely enough, however, only **B** contains this label in the text (and **P** and **V** in a more recent hand). This discrepancy between text and diagram may have been Heiberg's justification for eliminating point Λ from his edition.

This proposition is notorious for its Arabo-Latin variants and our study, although initially restricted to the Greek tradition, will reaffirm the complexity of its transmission. The proposition is the problem of drawing a complete circle from a given segment, ABΓ.

Let us sketch the argument following the Greek text. Euclid joins the extremities of the given segment, A and Γ, bisects this line at Δ, and then drops a perpendicular BΔ to AΓ.[8] Next, he joins A and B, and constructs an angle BAE at point A, equal to angle ABΔ. Thus the triangle ABE is isosceles, and it is easy to show that E is the center of the segment, and hence will be the center the completed circle.

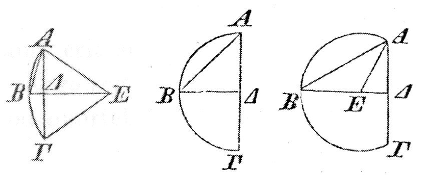

Fig. 4: Three diagrams of prop. III.25 in Heiberg's edition

What seems somewhat strange is that the text introduces a division into three cases before constructing the angle BAE, according to whether the angle ABΔ is greater than, equal to, or less than the angle BAΔ. Accordingly, the point E falls on the prolongation of BΔ, on Δ (in which case it is no longer necessary to introduce the point E), or between BΔ. In terms of the given arc ABΓ, these three cases correspond to the case of ABΓ being respectively less than, equal to, or greater than semicircle.[9] In Heiberg's edition, there are three figures according to these three cases. There seems to be no problem if one looks only at this edition or translations based on it.

III.2 The diagrams of III.25 in the Greek manuscripts

The situation of the manuscript diagrams, however, is quite complicated. Let us begin with the Bodleian manuscript **B** (Table 1).[10] In this manuscript, the three diagrams that we see in Heiberg's edition only appear in the margin.[11] In the proper

[8]We see at once that the center of the circle lies on BΔ, or its prolongation. This is stated as a corollary to the first proposition of Book III, and we expect to draw another chord AB, then draw a perpendicular to this chord at its midpoint. The intersection of the perpendiculars to the two chords is the center of the circle. But this is not the way the proof is structured, which casts some doubt on the authenticity of the corollary to prop. III.1.

[9]This correspondence is, in fact, stated at the end of each case.

[10]For the sigla of the manuscripts, see Appendix 2.

[11]Moreover, Heiberg has rotated the diagrams for some unknown reason.

place for the diagram,[12] there appears only one figure, which we have redrawn in Table 1.[13] The *single diagram* in codex **B** strikes us as quite strange. The arc ΑΒΓ is no doubt a semicricle. Then, what is the point Ε? It must be the center of the segment, so that it should coincide with point Δ. This is an impossible diagram. The only solution is to recall other "incorrect" diagrams in manuscripts that we have seen. The reader is expected to understand from this figure that Ε is the center and ΕΑ, ΕΒ, ΕΓ are equal to each other. The straight line from Α to an unnamed point between Β and Δ suggests that this figure was intended for all three cases. This unnamed point is probably the center of the circle in the third case, in which the arc ΑΒΓ is greater than semicircle, while the center falls on the point Ε in the first case, in which the arc ΑΒΓ is less than semicircle. In the second case (semicircle), the lower part of the figure is ignored and point Δ is the center.

What is striking is that the *single diagram* appears in most of the principal Greek manuscripts except **p**, one of the Parisian manuscripts. The other Parisian codex, **q**, has the same drawings, except that the unnamed point between Β and Δ has the name Θ, which never appears in the text.[14] Codex **P**, the Vatican manuscript predilected by Heiberg because it does not have traces of Theon's intervention, also shows one official figure, with point Θ named. There are also the three diagrams in margin. The only difference is that all the circle segments are prolonged to form complete circles.[15]

If we had only these manuscripts, we would conclude that the *single diagram* served for all the three cases. However, there are other manuscripts which lack the line corresponding to ΒΘ.

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[12]In manuscript **B**, as in many others, the text is narrowed at the end of each proposition to make space for the diagram. Hence, we may call the figure in the negative space of the text the *official* diagram. Figures in the margin often appear at the beginning of a long proposition, for which the official diagram is not visible until a later page opening. This is not, however, the case for proposition III.25 in codex **B**. All the figures appear on the same page.

[13]Hereafter, we use the terms *single diagram* and *three diagrams* to distinguish these different figures both appearing in Greek manuscripts.

[14]It is hard to decide which is the official diagram in manuscript **q**. The diagrams in this manuscript are usually drawn in the negative space of the text, but in proposition III.25, all the figures appear in margin. In the right margin at the beginning of the proposition, we have the first of the *three diagrams*, anticipating the figure on the next page (we have not redrawn this anticipating figure in Table 1). Then on the next opening, we have all of the *three diagrams* in the top margin. But in the left margin of the same page, we have larger drawings of two figures: the *single diagram* (as in **P**, that is, with the line for the third case, and the point between **B** and Δ named Θ), and the one of the *three diagrams* for the third case again (called *another diagram* in Table 1).

[15]The point Θ also appears in each of the three diagrams, with no apparent purpose. This is probably due to an incompetent scribe.

Table 1: prop. III.25 in Greek manuscripts

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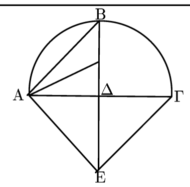
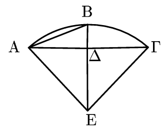
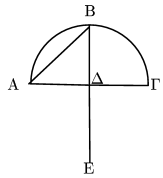
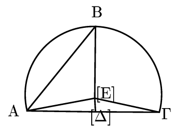
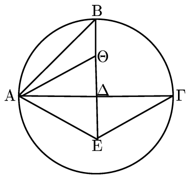
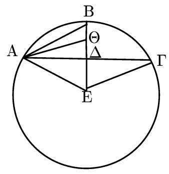
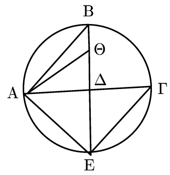
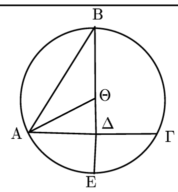
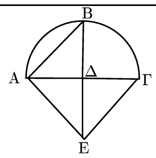
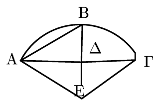
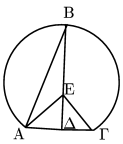
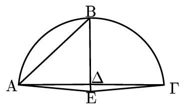
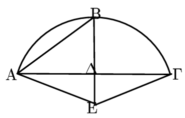
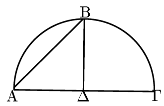
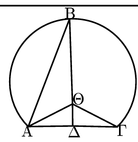
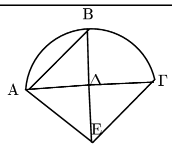
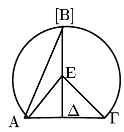
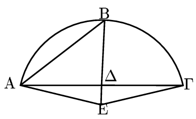
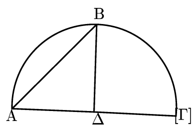
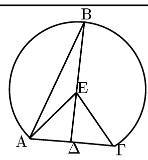
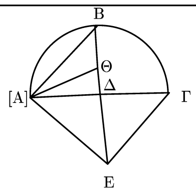
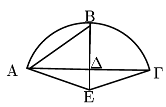
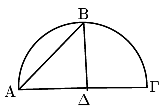
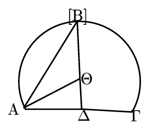
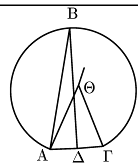
B				
P				
b				
V				
F				
p				
q				
				 **another figure in codex q**

The sigla are those given by Heiberg: **F** = Laurentianus XXVIII, 3, 10th century. **p** = Paris. Gr. 2466, 12th century. **q** = Paris. Gr. 2344. 12th centruy. See Appendix 2 for other sigla.

The Bologna manuscript, **b**, has one official diagram and two (not three) diagrams in the margin, omitting that for the second case (semicircle). Perhaps the scribe considered this case covered by the official figure. In the official, *single diagram*, the line AΘ is omitted. The Florence manuscript, **F**, has only two figures, both in the margin, so that it is difficult to decide which one is official. The figures are much like those in the Bologna manuscript, although lacking two of the three diagrams. The scribe seems to have thought that the single diagram stands for the first two cases. The Vienna manuscript **V**, has both the single diagram and the three diagrams, however, both are given space where the text is narrowed (so both were official and probably present in the antegraph from which the text was copied). The *single diagram* lacks line AΘ. What is intriguing in this manuscript is that point E is very near point Δ in the *single diagram*, probably showing the scribe's recognition of, and bewilderment at, the fact that point E must be the center, although it does not appear to be so. It is likely that the antegraph of **V** had a figure like that in **BFbq**.

In **b**, **F** and **V**, the diagrams without the line AΘ allow two interpretations concerning the *single diagram*. Either line AΘ was suppressed, because the scribe thought that this case was covered by the third of the *three diagrams*, or the *single diagram* originally did not include the line AΘ, representing only the first case, and the line AΘ was added later to make it applicable to the third case.[16]

III.3 Which is older, the single diagram or the three diagrams?

It is tempting to assume that the *single diagram* is the original and that the text has undergone some alteration (multiplication of cases), since it is otherwise difficult to explain the existence of a figure which does not correspond exactly to the text. That this is in a sense an "incorrect" diagram adds to the credibility of this thesis, because a medieval scribe would not have dared to create such a drawing (we may say *difficilius diagramma potius*, applying the principle *difficilior lectio potior*). Moreover, we have already seen a similar "incorrect" figure in *Catoptrics*.

This thesis also leads to the assumption that the unessential division into three cases, was a result of later elaboration. Indeed, the text contains locutions rare in the *Elements*. At the end of the first case, the author impatiently says "it is clear" (δῆλον), and to conclude each of the second and third case, the word "clearly" (δηλαδή) is used: both are rare in the *Elements*.[17]

Moreover when we examine the Arabo-Latin tradition of this proposition, the problem becomes even more complicated.

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[16] It was standard practice for ancient authors to treat only one representative case of possible cases [Vitrac 2004, 16]. For example, in proposition I.7, Euclid does not argue the case in which the point Δ falls within the triangle ABΓ.

[17] The former is used only in X.9, 44, 111 (and δηλονότι in IV.8; VI.7; X.47), while the latter in X.4, 10; XI.3, XII.4, 17; XIII.15.

Table 2:
prop. III.24 (=Gk. 25) in some Latin manuscripts[18]

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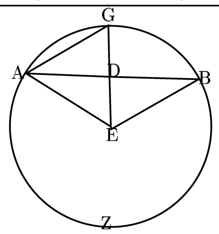
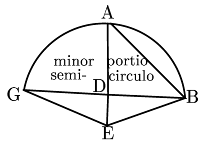
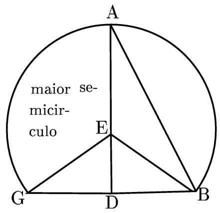
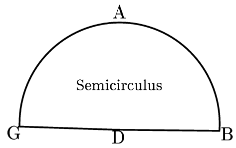
<tr><td>GB</td><td>
</td><td>
</td><td>
</td><td>
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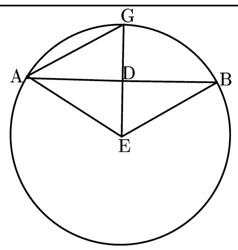
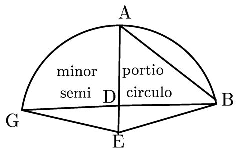
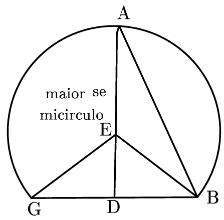
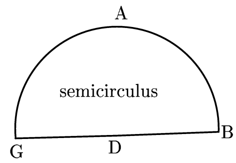
<tr><td>GR</td><td>
</td><td>
</td><td>
</td><td>
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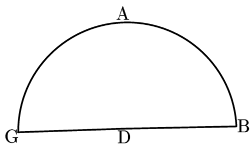
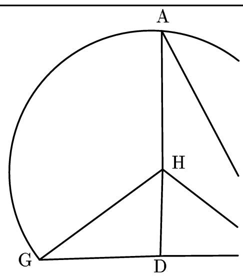
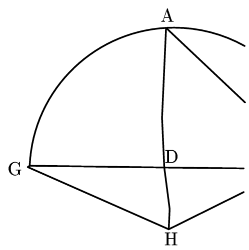
<tr><td>AB</td><td></td><td>
</td><td>
</td><td>
</td></tr>
</table>

GB = Bruges 521 (Gerard). **GR** = Vat. Rossiano 579 (Gerard).
AB = Bruges 529 (Adelard).

[18] In this table, we have tentatively redrawn the figures from some of the manuscripts of the Latin translations, without being exhaustive.

The Gerard manuscripts are two of the three that contain the best text [Busard 1984, XXIV], and the last of the three, Vat. lat. 7299, also has quite similar figures. The order of the three diagrams differs from the order of argument in the text, but is the same in these three manuscripts (from top to bottom in Rossiano, from left to right in other two).

The only Adelard manuscript that we have reproduced is Bruges 529 (the so-called Adelard I), which is one of the most important witnesses to this tradition [Busard 1983, 20ff]. The figures are in the margin and have been partially cut off.

III.4 Arabo-Latin tradition of the proposition III.24 (Greek III.25)

The situation of this proposition in Arabo-Latin tradition (III.24 in this tradition) is skillfully summarized in [Vitrac 2004, 40]. Following Vitrac, we will give an overview of the Arabo-Latin tradition.

Both the Latin translations of Gerard of Cremona and Adelard of Bath have three cases with three diagrams, as the Greek text (Table 2). The solution is essentially the same. However, the distinction of cases is based not on the comparison of angles but on the magnitude of the given segment. The first Latin case is that of the semicircle (second case in Greek), then the case of the segment greater than semicircle (third case in Greek), and finally the segment less than semicircle.

The Arabic version of an-Nayrīzī presents essentially the same argument in the same order as Gerard and Adelard, although the text is amplified by references to earlier propositions.[19]

Thus there are two three-case versions of the proposition, one in Greek and another in the Arabo-Latin tradition. What makes the situation even more complicated is the existence of alternative proofs with only one diagram, in an-Nayrīzī (after III.31) and in Gerard (after III.30). As Vitrac [2004, 31n109] points out, these two are not the same; an-Nayrīzī, nonetheless, speaks of the three cases using a diagram very much similar to that of the *single diagram* in Greek codex **B** (the proof is attributed to Heron), while Gerard, does not mention the division of cases and uses a diagram that resembles the first of the *three diagrams* in codex **P** without the line AΘ (no attribution to Heron). It should be added that some Arabic versions have only one figure.

It seems, nevertheless, probable that the common diagram in codex **B** and in the alternative proof in an-Nayrīzī derives from the same Greek source. Anyway, this complicated and intriguing situation deserves a thorough examination of all the available sources in all three traditions (Greek, Arabic and Latin), both the text and diagrams, which is beyond the scope of the present study.

We hope to have shown that there is much to be learned from a study of the manuscript diagrams, and that transcribed (or redrawn) figures like those in Table 1 are useful, even indispensable, for research.

In Appendix 1, we briefly describe the program we used to make all the figures in this article (except the reproduction from Heiberg's editions). In Appendix 2, we redraw the diagrams of the 48 propositions of Book I of the *Elements* as found in six manuscripts.

[19] See the bibliography for the editions of these versions.

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I would like to thank the referees, for their valuable comments, and Satoshi Tojo, specialist in information science, for suggesting the word "oversimplification" to designate the particular features of the manuscript diagrams.

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Appendix 1: Redrawing diagrams by computer program

Deriving metrical data from the diagram

We have developed computer programs to register the metrical data of the diagrams and redraw them. The function of such a program is very simple.[20] It displays an image file on the screen and registers the coordinates of chosen points under an arbitrary label name. If, for example, the first registered point has coordinates (577, 510) and is accompanied in the manuscript by the label A, one selects this point and types "A". The program lists the data as follows:

```
1,577,510,A
```

Inputting other points, one obtains a list of points:

```
1,577,510,A
2,808,511,B
3,697,311,G
4,357,511,D
5,1046,508,E
```

One then inputs the lines. If there is a triangle ABG, i.e., the points A–B–G–A are connected by straight line segments, this is input in the following manner:

[20] There are two programs both in beta version. One is Diagram, a very simple program whose functions are explained here. The other is called Sctriptorium, a program for making critical editions of text, to which a function has been added to handle figures. Both can be downloaded from my site (`http://www.greekmath.org/diagram/`).

They are distributed gratis, under the conditions of the GNU GPL.

```
line,1,2,3,1
```

Circles are input in the same manner. If there are two circles, passing through A, G, E and B, G, D respectively, the program lists this as follows:

```
circle,1,3,5
circle,2,3,4
```

In this way, one has five points, a triangle and two cirlces. The example data are those representing the figure of the first proposition of *Elements* Book I, in codex **P**. Then, from this data alone, with no reference to the original image file, a redrawn image can be made.

One could use any number of methods to draw the new figure from the data. The drawings in this article are made by LaTeXand converted into EPS files.

What is preserved and what is lost in the redrawn diagrams

As is clear from the process of redrawing, the relative positions of labeled points are reproduced accurately. The straight lines, however, always appear as perfectly straight, which is not actually always the case, especially if they are drawn freehand. This can partly be adjusted by entering some intermediate points and redrawing the line as a sort of zigzag (see prop. I.13 in codex **B**). Moreover, the width of lines is completely ignored. Some lines in the redrawn figures appear thicker; this is a (provisory) convention for designating later additions and corrections to the original drawing.

The position of labels can be registered as additional information, which is used to place the labels in the redrawn diagrams in Appendix 2.[21] The size and style of labels are ignored and we always use capital letters for labels. As usually happens in the text, some labels are difficult to read, some are unreadable, and some have disappeared. We use a question mark for labels whose reading is not certain, square brackets for those that are unreadable.

When one cannot find a label corresponding the letter name used in the text and/or existing in the figures of other manuscripts, it is difficult to decide whether it was originally missing or has disappeared over time. It may simply be so faint that it is not visible in the microfilm. Sometimes labels seem to have disappeared because the margin of the paper has been cut off in the process of binding (for example, see the figures in Table 2 above). In this article, we used square brackets both for unreadable labels (whose faint image we see on the film), and for completely invisible labels (when we are sure of its existence – the position of the label is decided

[21] For example, the position of point Θ in codex **B**, proposition I.2, although quite distant from the point it indicates, reproduces the situation in the manuscript.

somewhat arbitrarily). Some more precise convention should be established in the future.

We have not distinguished those figures neatly drawn by ruler and compass from those casually drawn by freehand, although they give a very different visual impression.[22]

Finally, in this article we have not attempted to reproduce the size of each drawing. Instead, we have redrawn each figure at approximately the same size, in order to be able to print as many figures as possible in the given space.

Appendix 2: figures in Book I of the *Elements*

This appendix contains the figures of all 48 propositions of *Elements* Book I, redrawn from six manuscripts, along with reproductions of Heiberg's figures (1883). The notes are provisory and are not exhaustive. The limitations and conventions discussed in Appendix 1 apply to all the redrawn diagrams.

We have chosen four of the six principal Greek manuscript that Heiberg used in his 1883 edition of the *Elements*. This selection is partly made for logistical reasons; the diagrams are in poorer condition in the two Greek manuscripts (**Fp**) that we have excluded (or postponed) at this time. Instead of these, we have included two Latin manuscripts of Gerard's translation (**GB, GR**),[23] in order to show some of the general characteristics of the Arabo-Latin tradition, of which the most conspicuous is the horizontal reflection of the diagrams.[24]

In most cases, it is easy to identify the diagram that was drawn when the text was copied, or shortly thereafter. We call this the *official* diagram and we have only redrawn official diagrams, except in some rare cases. There are many other diagrams, usually in the margin, which we call *alternative* diagrams. Some appear on an opening preceding or following that of the official diagram such that the diagram corresponding to the text is always visible (thus they are additional rather than alternative). Some diagrams have been copied later from other manuscripts. It is relatively rare that someone has attempted to emend a diagram that is in error, either by correcting the diagram itself (e.g., I.33 **B** and I.45 **P**), or by making new drawings (e.g., I.27 **B**).

The following are general descriptions of the manuscripts whose diagrams we have redrawn. The sigla are those of [Heiberg 1883] for Greek manuscripts; for Gerard manuscripts those of [Busard 1984] with **"G"** added.

[22] For a conspicuous case, see the description of codex **B** at the beginning of Appendix 2 below.

[23] For a discussion of the choice of these two manuscripts, see note 18.

[24] For example, if there is a horizontal line AB, with A on the left and B on the right in the Greek manuscripts, in the Arabo-Latin tradition, A and B are reversed, probably because Arabic reads from right to left.

- **P:** Vat. Grec. 190. 12th century.
  - Text: Written in two colums. This is the only manuscript with no trace of Theonian intervention, and was therefore much used by Heiberg in his edition.
  - Diagrams: Drawn at the end of each proposition, occupying the whole width of one column. Drawn neatly with ruler and compass; however, right angles are not exact (see I.33, 47).
  - Alternative diagrams: There are many, in different hands. Most of them anticipate the official diagram.
- **B**. Bodleianus Dorvillianus 301 (X,1 inf.2,30 in Heiberg). Written in 888.
  - Text (initial part, to the middle of proposition I.14): Written in a single column. The initial part is written extremely casually – or "carelessly," to quote Heath (1925, 1:47). Scholars agree that the leaves of this part have been lost and replaced.
  - Text (subsequent part, from the middle of proposition I.14): Written in a single column, in an elegant hand.
  - Diagrams (initial part): The text is narrowed, usually at the end of each proposition, and the figures are drawn in the negative space of the text. This space, however, is too small in some cases and here the diagrams are drawn in the margin. The drawings are drawn freehand and are far from accurate.
  - Diagrams (subsequent part): Drawn in the negative space of the text as in the first part (the space is much wider), and with much more precision in fine lines, using ruler and compass. However, sometimes the limited space seems to have affected the diagrams (see I.47, where the height of the space seems to have been too small).

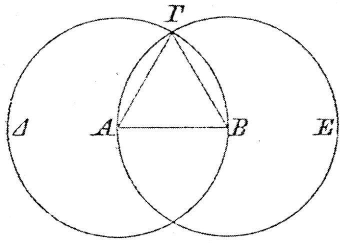
    The space for the diagram is not always at the end of a proposition. It is often made at the end of a page, while the text goes on to the next page.

    Quite a few pages of this manuscript are in poor condition and it is sometimes hard to read the labels, or even to see the diagram itself.
  - Alternative diagrams: Many in various hands, mostly in the same opening as the official diagrams.
- **b**: Bologna, biblioteca comunale, 18–19. 11th century.
  - Text: Written in a single column.
  - Diagrams: Drawn at the end of each proposition, where the text space is narrowed to make space for diagrams. Drawn in fairly thick lines, not very meticulously, with ruler and compass, except some simple diagrams.

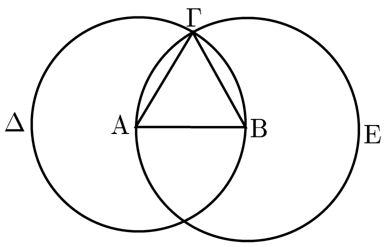
- Alternative diagrams: Few, and all in more recent freehand (I.7, 16, 22, 27, 43). In the official space for the diagram of I.48, we find two figures for I.47 (see I.48 in this appendix).

- **V**: Vindobonensis phil. gr. 31 (103 in another catalogue).
  - Text: Written in a single column.
  - Diagrams: Drawn in the negative space of the text, as in codex **B** but not at the end of a page. If there is a page turn within a proposition, the space for the diagram is usually found just after the page turn. There is no space for propositions I.6, 42 whose diagrams appear in the margin.
  - Alternative diagrams: Many in various hands, mostly on the same opening, some before or after the page turn.
- **GB**: Bruges 521. 14th century.
  - Text: Written in a single column.
  - Diagrams: Drawn exclusively in the outer, side margin. For a wide diagram (e.g., I.22), the text recedes according to the shape of the diagram to give more space. Hence, it is possible that the text and the diagrams were copied by the same scribe. As the diagrams are drawn in the side margins whose width is limited, multiple figures tend to be arranged vertically (e.g., I.24, 42, 45; see also I.44). The leaves are often warped causing the diagrams to appear deformed in the microfilm images (corrected approximately in the redrawn figures).
  - Alternative Diagrams: None. Unlike the Greek manuscripts, this manuscript seems to have had few, in any, readers.
- **GR**: Vat. Rossiano 579. 14th century.
  - Text: Written in two columns.
  - Diagrams: Drawn exclusively in the side margins, both inner and outer. The text leaves space for the diagrams more often than in **GB**, probably because the margin space is more limited with the text in two columns.
  - Alternative Diagrams: None (as in **GB**).

Proposition I.1



Heiberg



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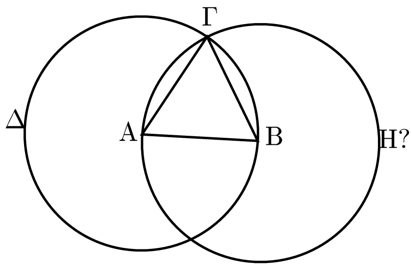

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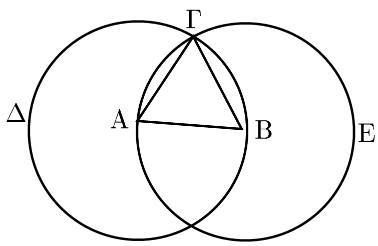

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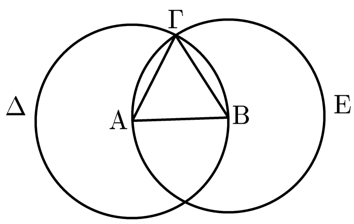
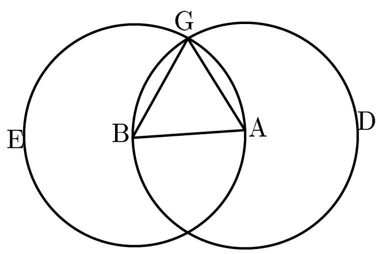
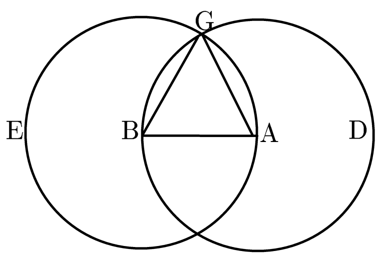


Codex V

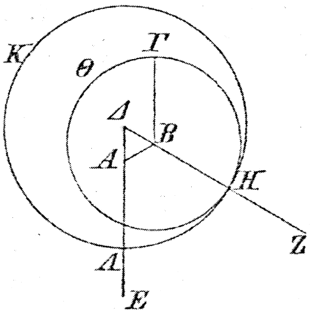


Codex GB

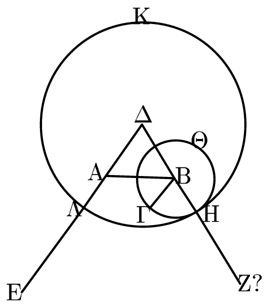


Codex GR

Proposition I.2



Heiberg



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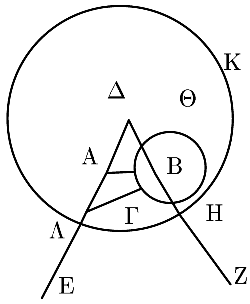

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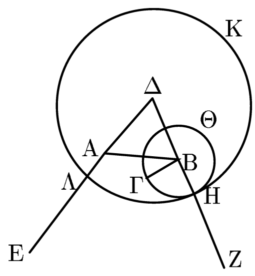

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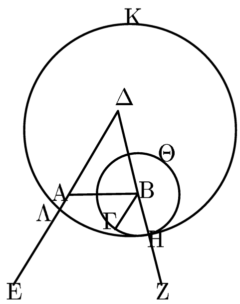

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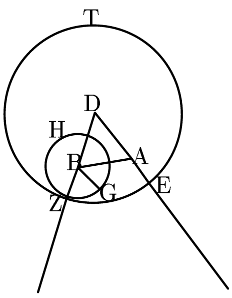

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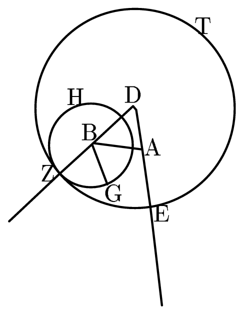
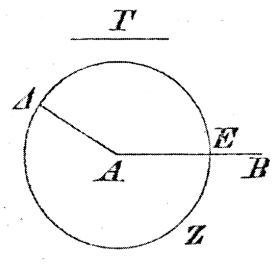


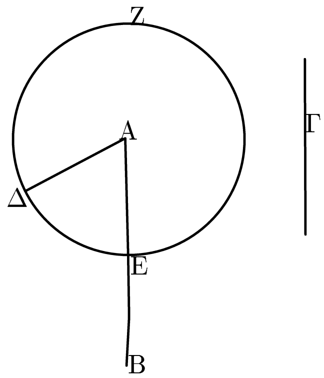
Codex GR

Proposition I.3



Heiberg

GR: Line G may be hidden by the binding.



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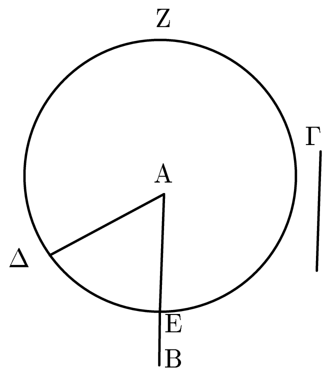

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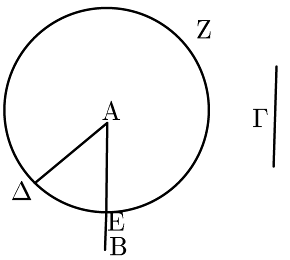

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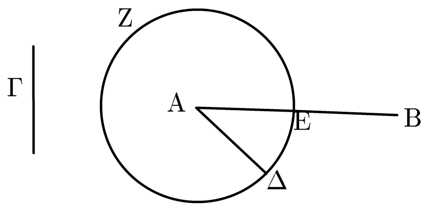
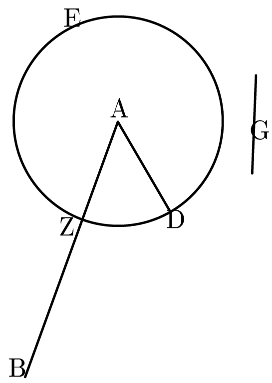
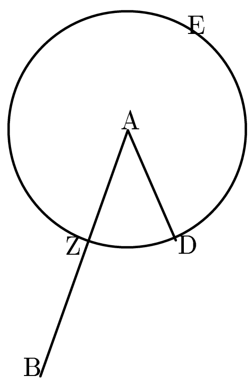


Codex V

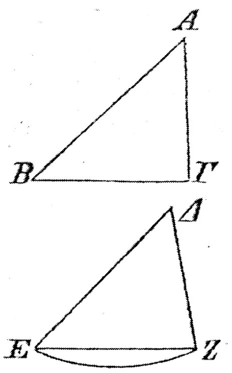


Codex GB

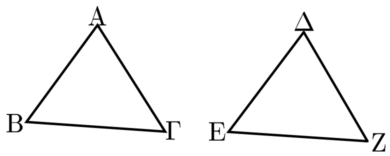


Codex GR

Proposition I.4



Heiberg



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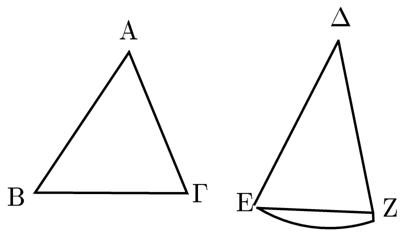

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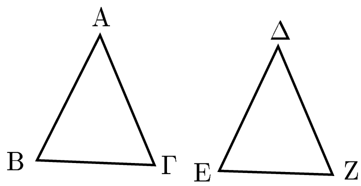

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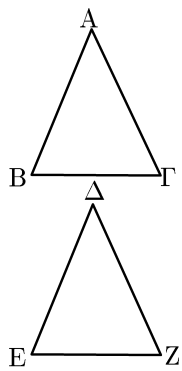

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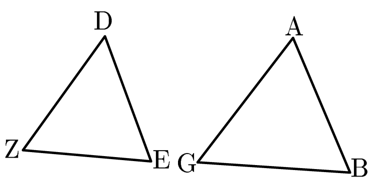
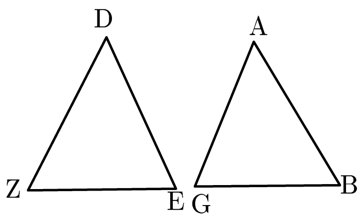
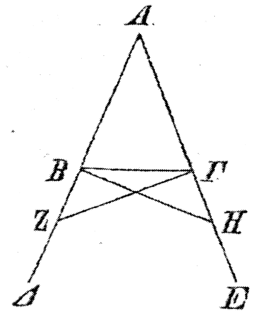


Codex GB

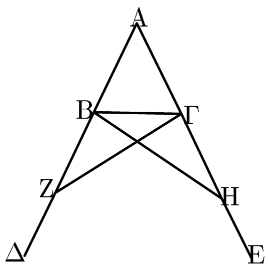


Codex GR

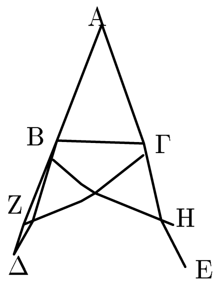
Proposition I.5



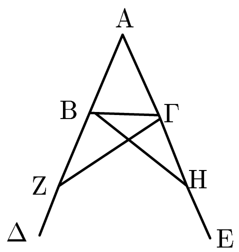
Heiberg



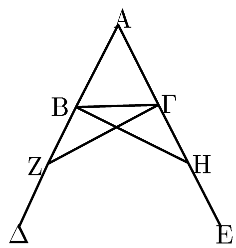
Codex P



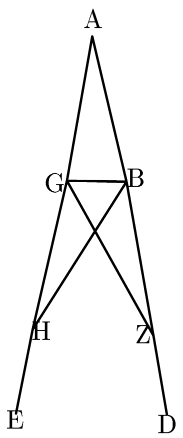
Codex B



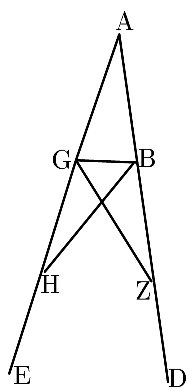
Codex b



Codex V

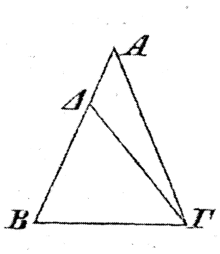


Codex GB

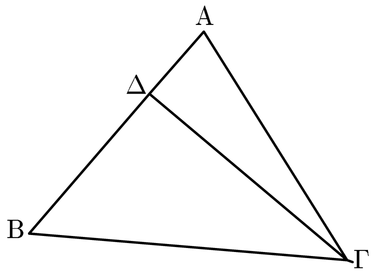


Codex GR

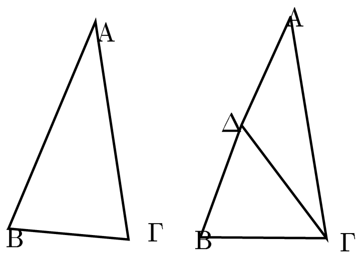
Proposition I.6



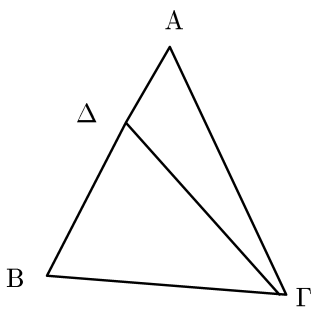
Heiberg



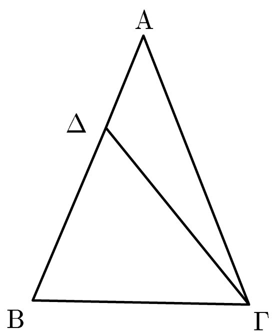
Codex P



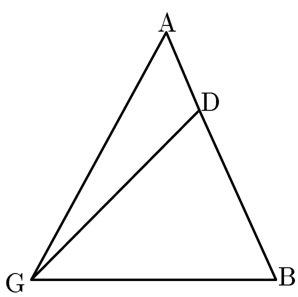
Codex B



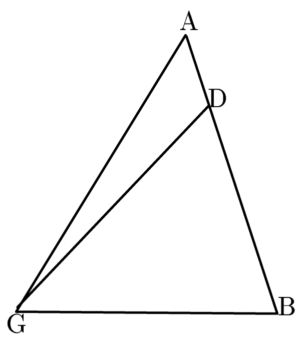
Codex b



Codex V

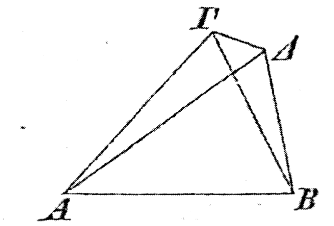


Codex GB

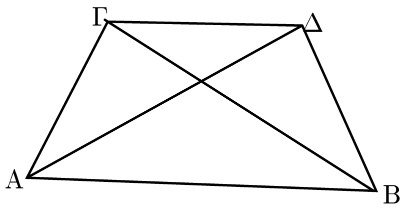


Codex GR

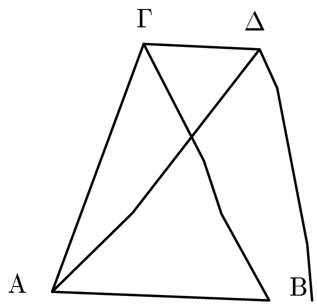
Proposition I.7



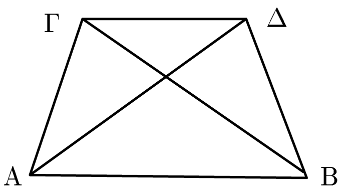
Heiberg



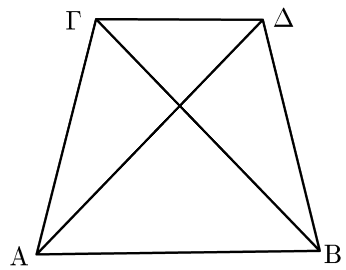
Codex P



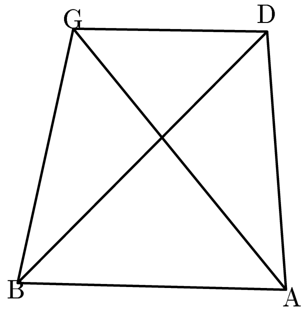
Codex B



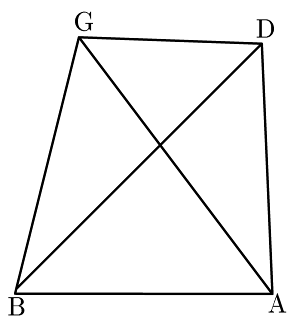
Codex b



Codex V

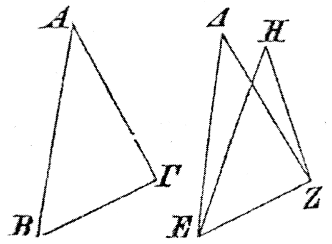


Codex GB

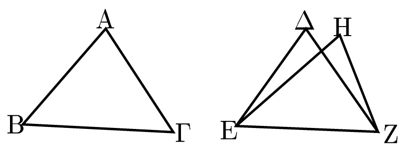


Codex GR

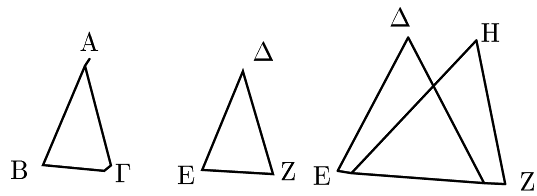
Proposition I.8



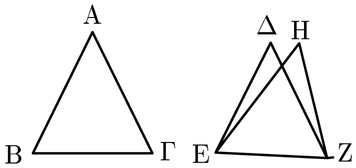
Heiberg



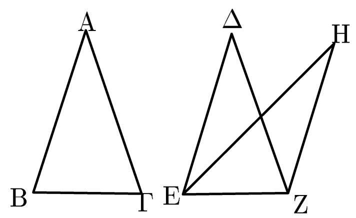
Codex P



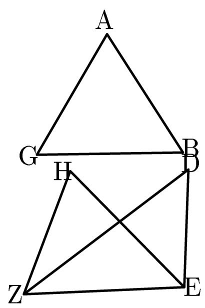
Codex B



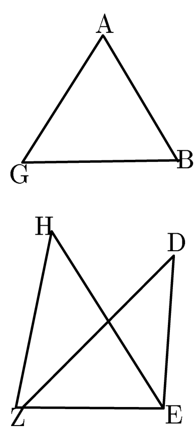
Codex b



Codex V

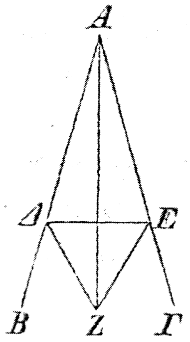


Codex GB

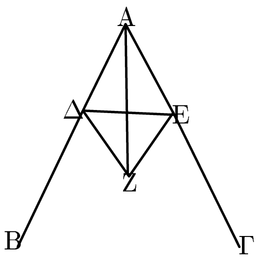


Codex GR

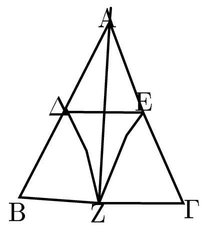
Proposition I.9



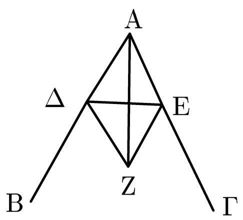
Heiberg



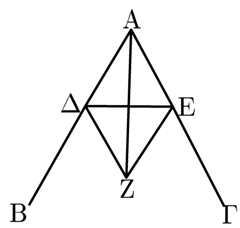
Codex P



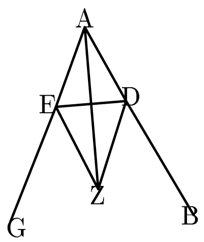
Codex B



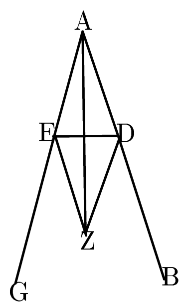
Codex b



Codex V

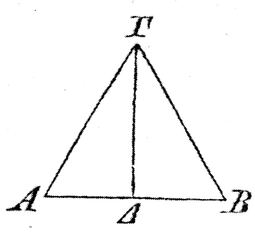


Codex GB

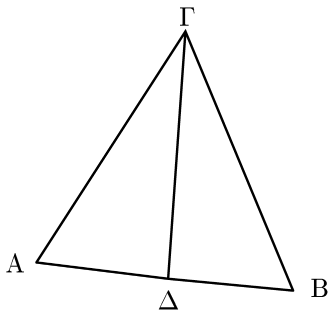


Codex GR

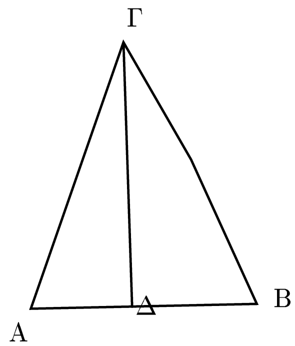
Proposition I.10



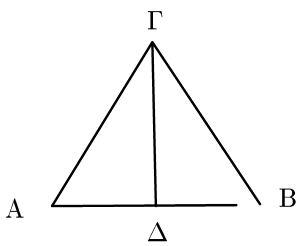
Heiberg



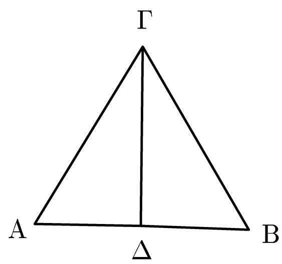
Codex P



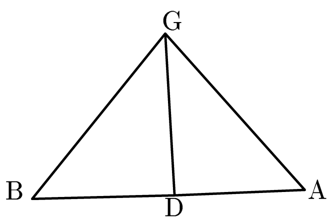
Codex B



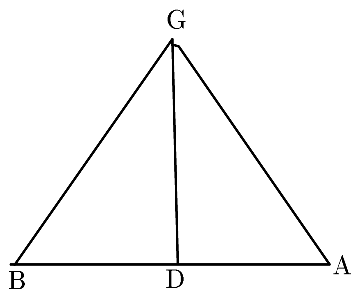
Codex b



Codex V

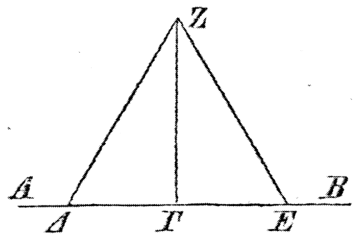


Codex GB

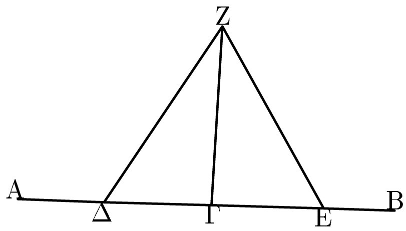


Codex GR

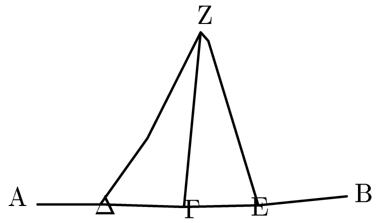
Proposition I.11



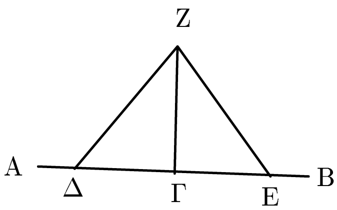
Heiberg



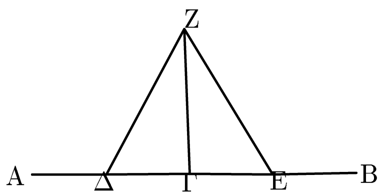
Codex P



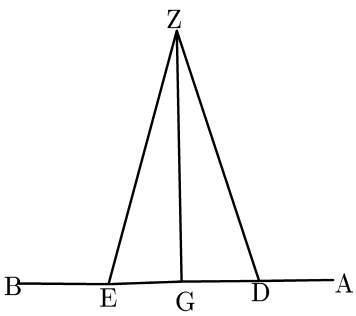
Codex B



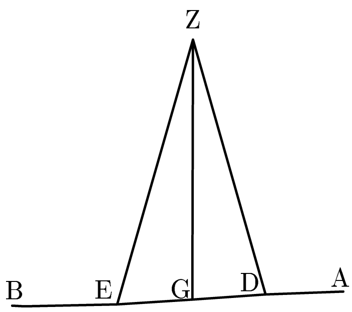
Codex b



Codex V

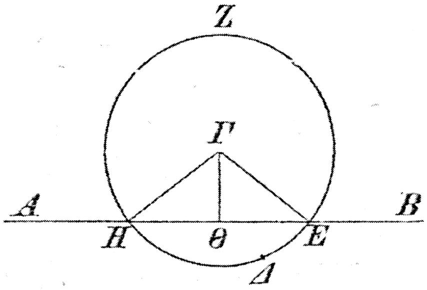


Codex GB



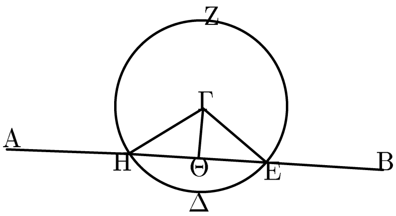
Codex GR

Proposition I.12

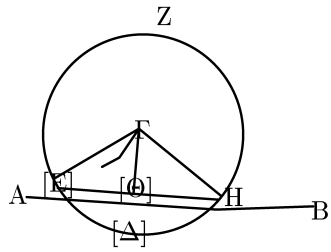


Heiberg

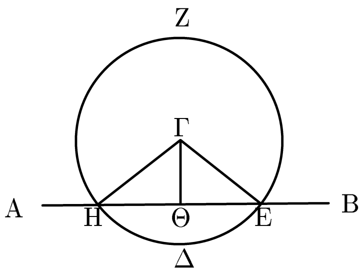
GR: Label A is probably hidden by the binding.



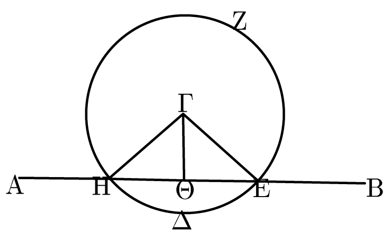
Codex P



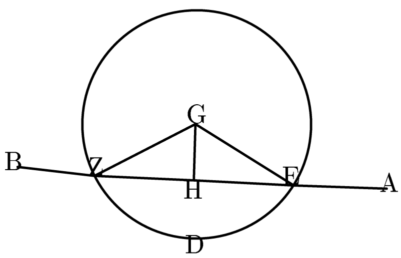
Codex B



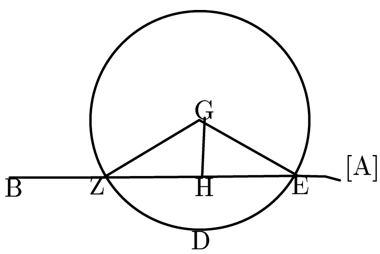
Codex b



Codex V

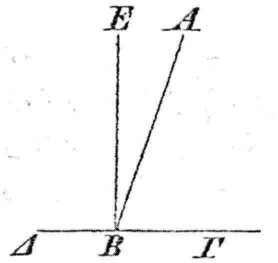


Codex GB



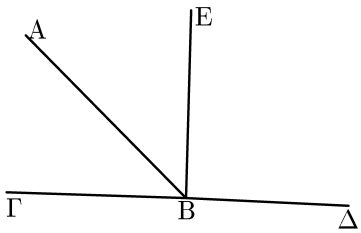
Codex GR

Proposition I.13

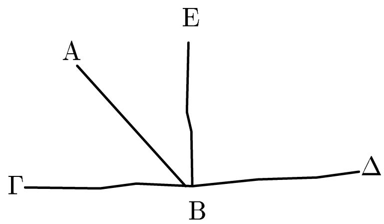


Heiberg

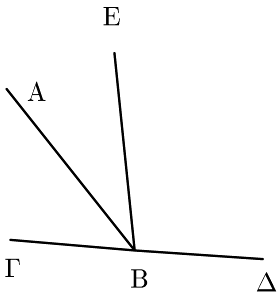
Heiberg's diagram is horizontally reflected.



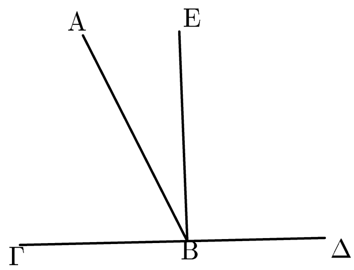
Codex P



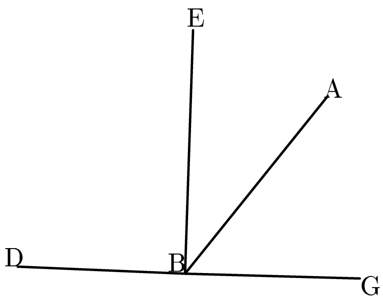
Codex B



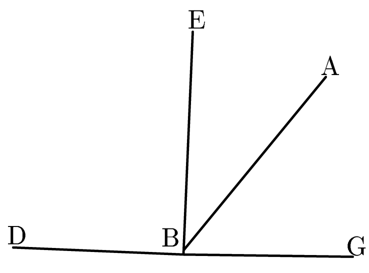
Codex b



Codex V

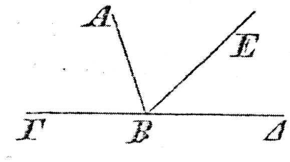


Codex GB



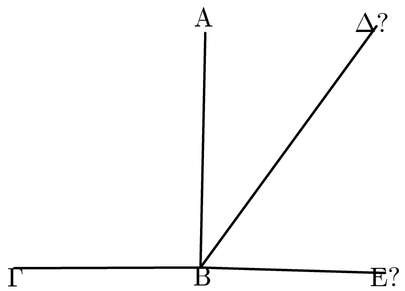
Codex GR

Proposition I.14

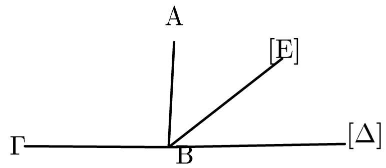


Heiberg

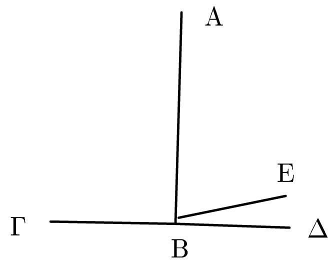
P: Labels Δ, E are vary faint, but seem to be exchanged. **GB GR:** The diagrams are not horizontally reflected in Gerard's translation.



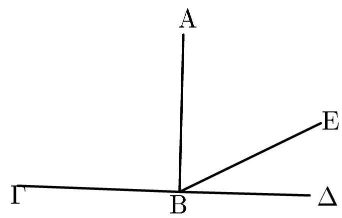
Codex P



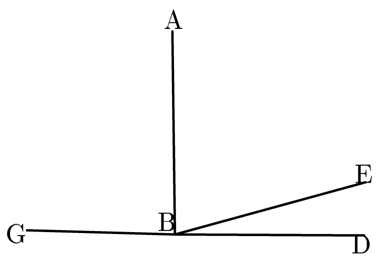
Codex B



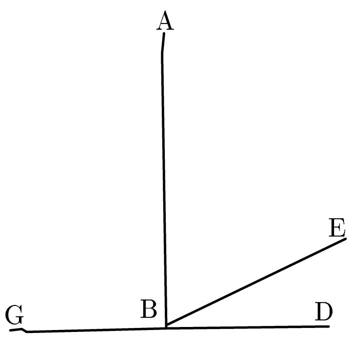
Codex b



Codex V

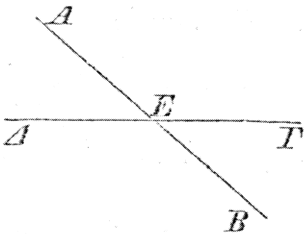


Codex GB



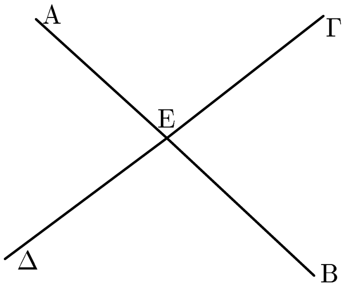
Codex GR

Proposition I.15

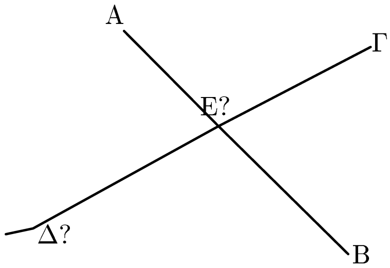


Heiberg

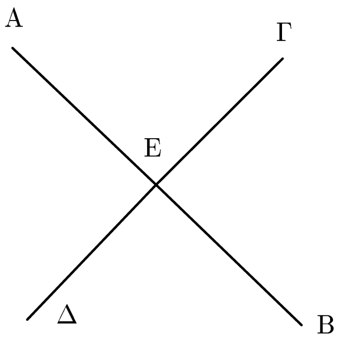
b: Label E is faded and rewritten on the intersection of AB and ΓΔ. **GB GR:** The diagrams are rotated, not horizontally reflected. **GR:** Label B is hidden by the binding.



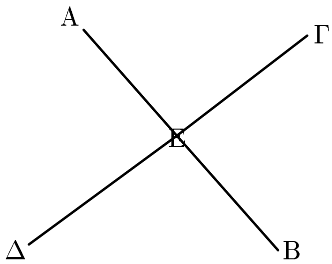
Codex P



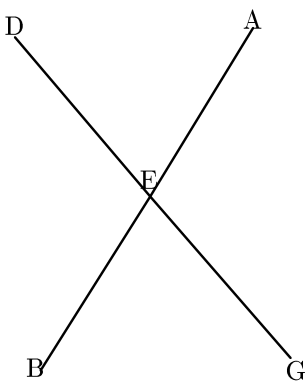
Codex B



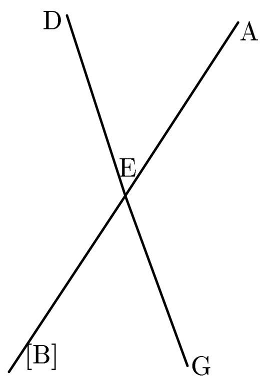
Codex b



Codex V

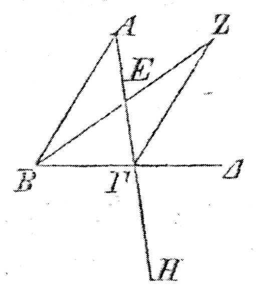


Codex GB



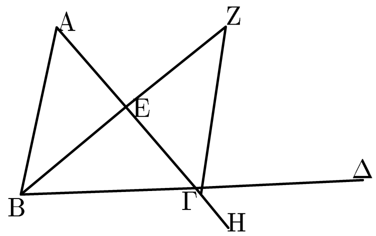
Codex GR

Proposition I.16

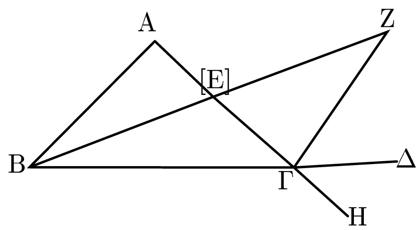


Heiberg

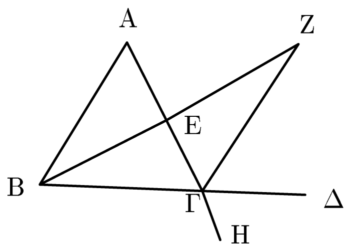
B: The lines have completely faded out around the point of intersection, where the label E should be. **PbV:** Another diagram, in the margin, by a more recent hand, for another case briefly mentioned at the end of the text, in which BΓ is bisected and the angle BΓH is proved to be greater than the angle ABΓ.



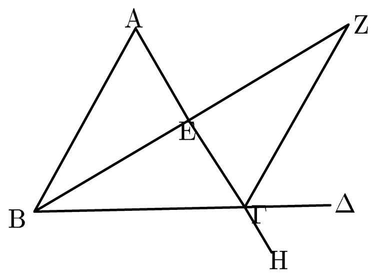
Codex P



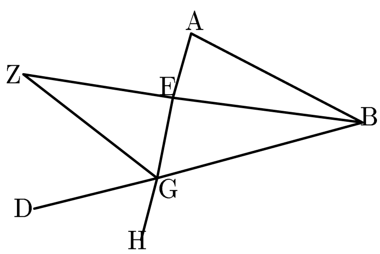
Codex B



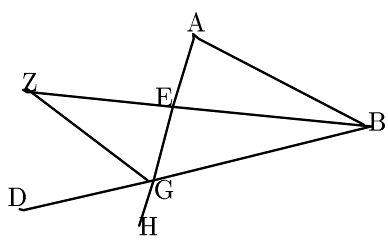
Codex b



Codex V

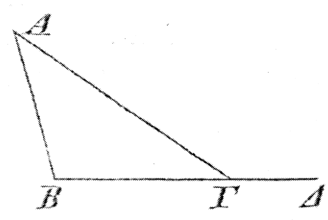


Codex GB

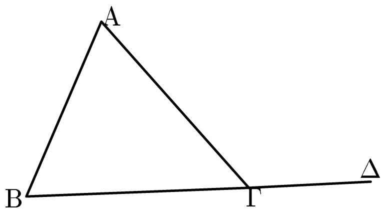


Codex GR

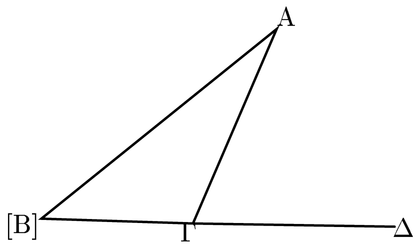
Proposition I.17



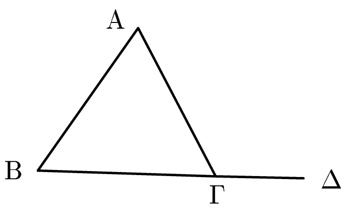
Heiberg



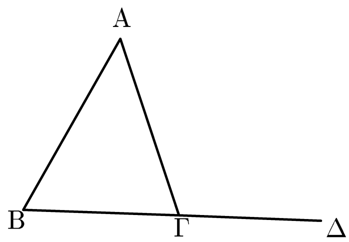
Codex P



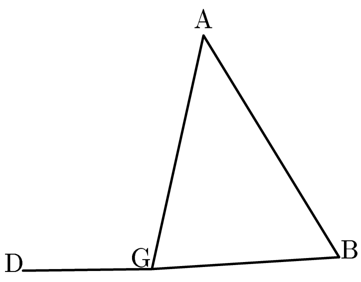
Codex B



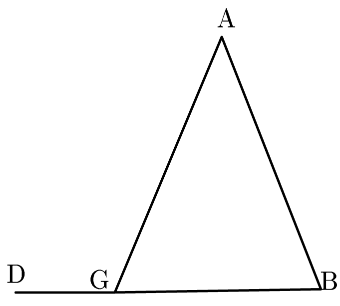
Codex b



Codex V

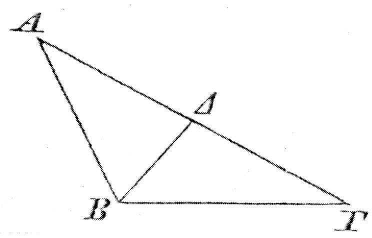


Codex GB

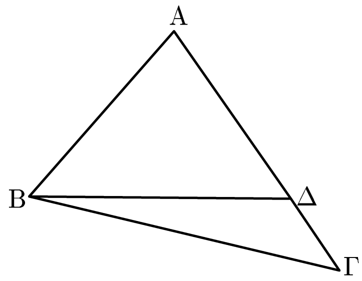


Codex GR

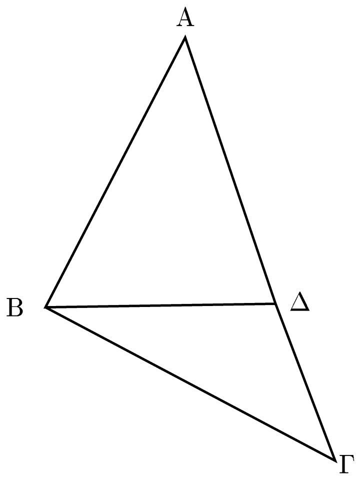
Proposition I.18



Heiberg



Codex P



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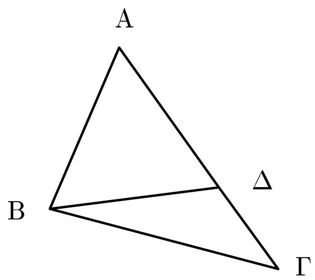

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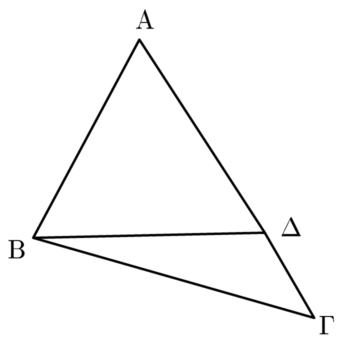
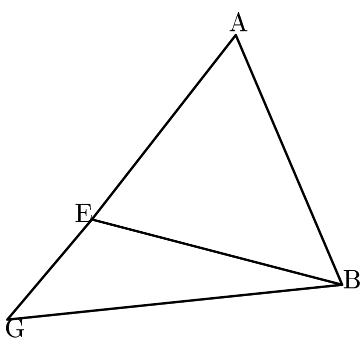
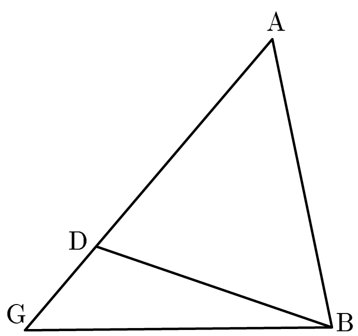


Codex V

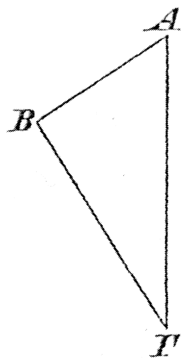


Codex GB



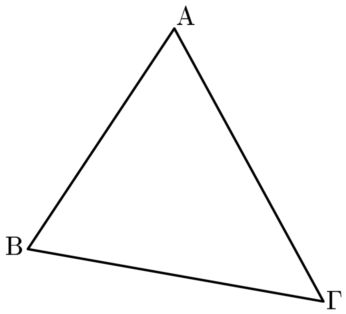
Codex GR

Proposition I.19

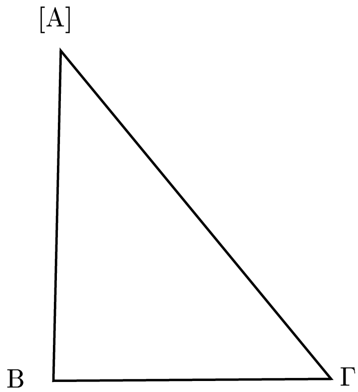


Heiberg

V: Label A cannot be seen, although the lines are clear.



Codex P



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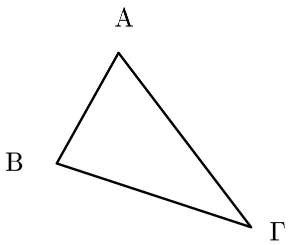

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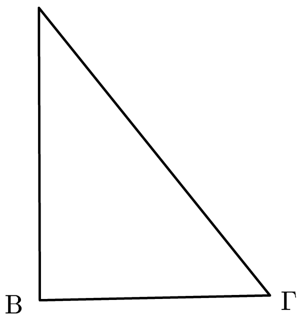
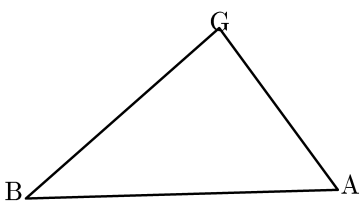
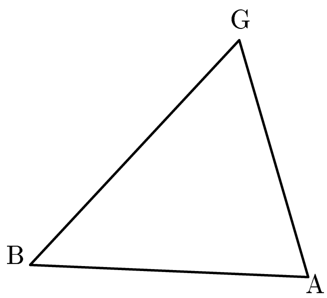


Codex V

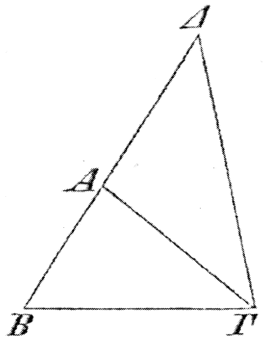


Codex GB

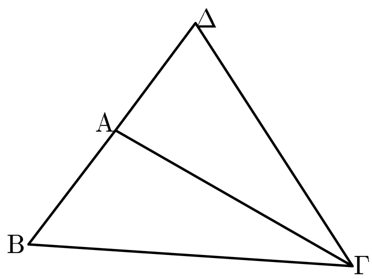


Codex GR

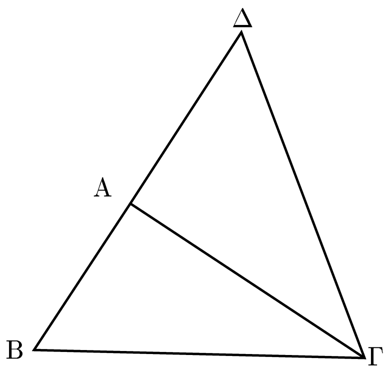
Proposition I.20



Heiberg



Codex P



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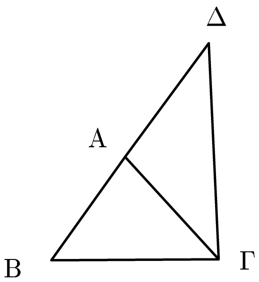
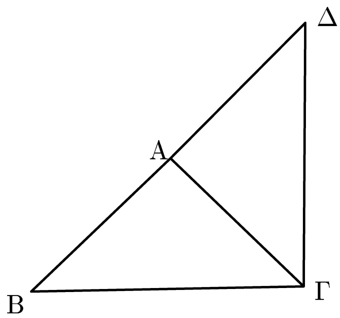
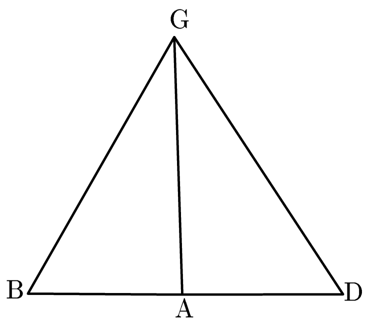


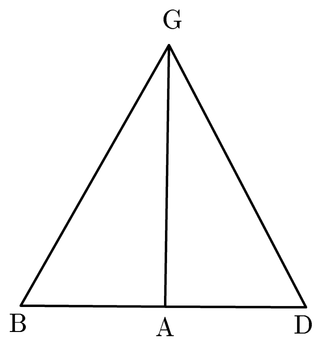
Codex b



Codex V

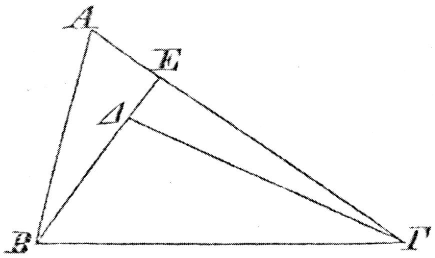


Codex GB

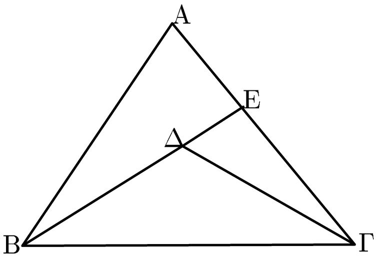


Codex GR

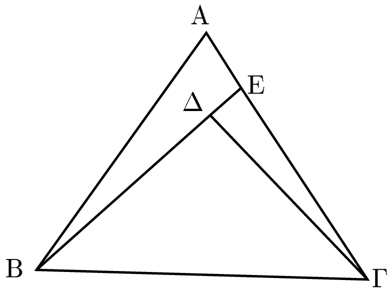
Proposition I.21



Heiberg



Codex P



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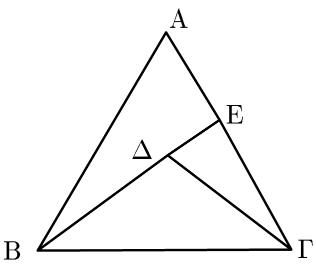
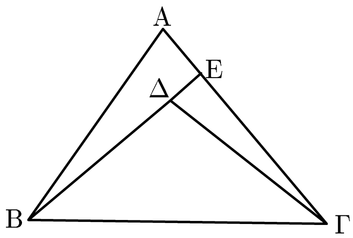
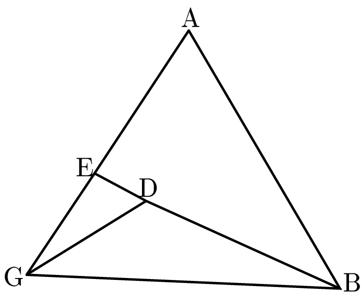


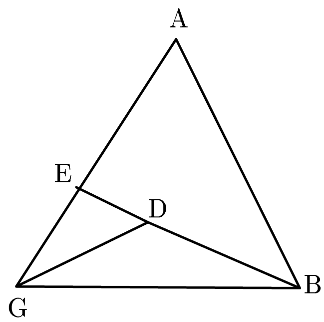
Codex b



Codex V

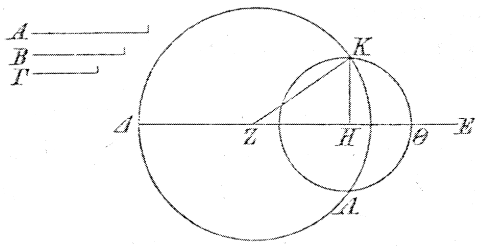


Codex GB

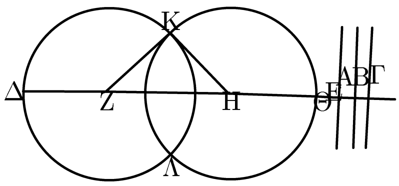


Codex GR

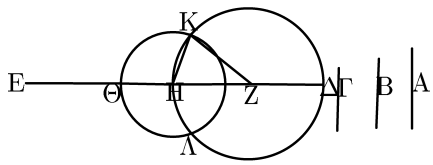
Proposition I.22



Heiberg



Codex P



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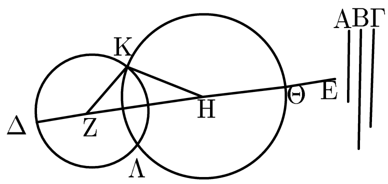
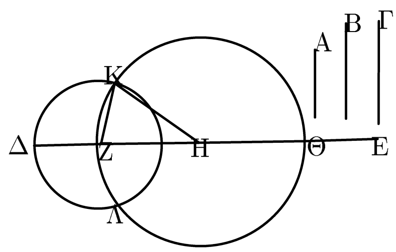
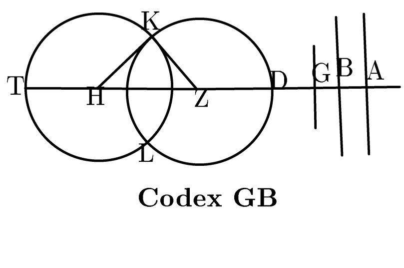


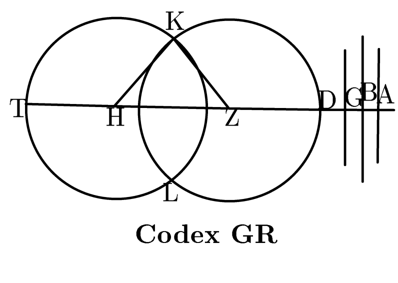
Codex b



Codex V

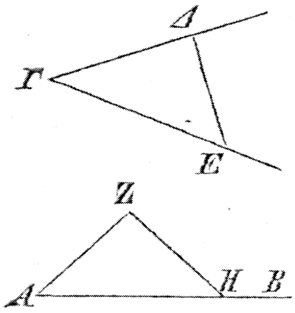


Codex GB

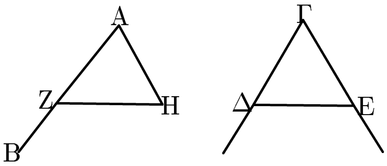


Codex GR

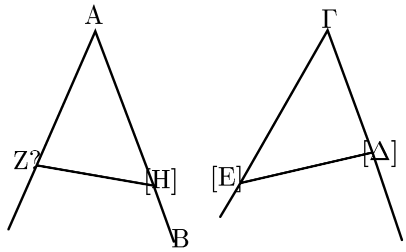
Proposition I.23



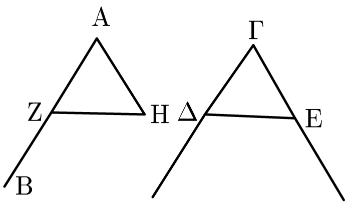
Heiberg



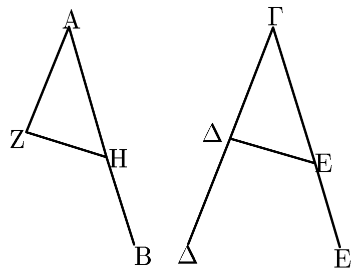
Codex P



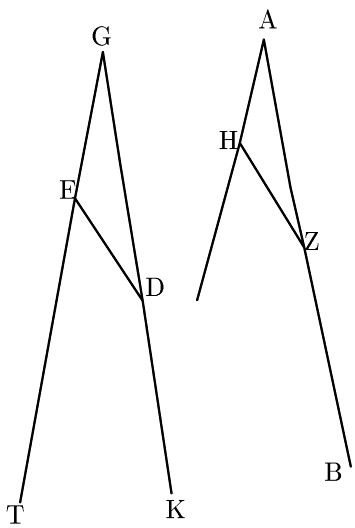
Codex B



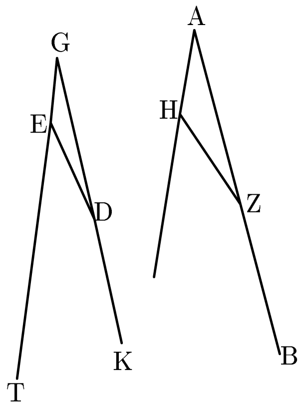
Codex b



Codex V

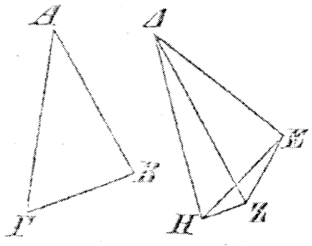


Gerard B

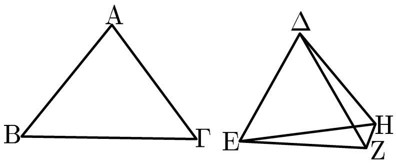


Codex GR

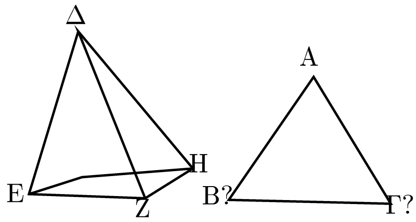
Proposition I.24



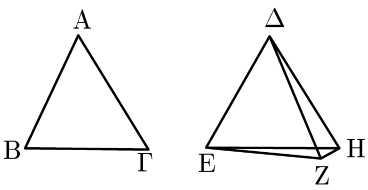
Heiberg



Codex P



Codex B



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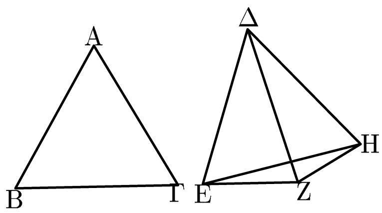
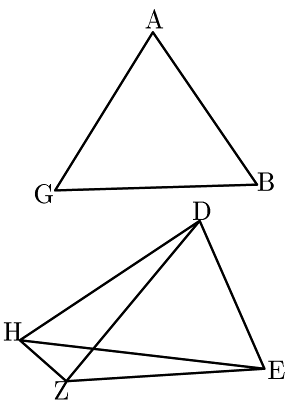
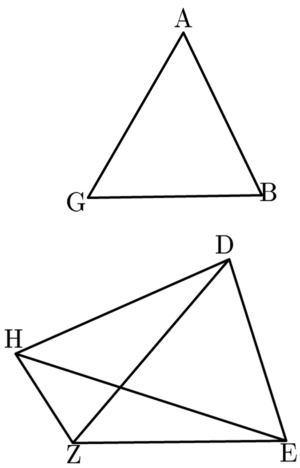


Codex V

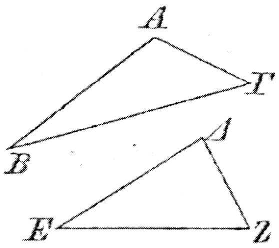


Codex GB

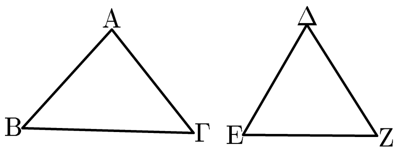


Codex GR

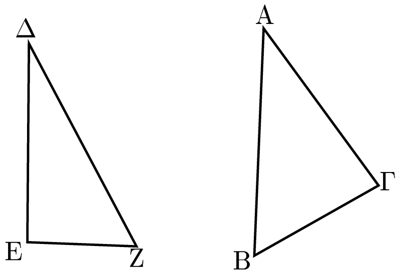
Proposition I.25



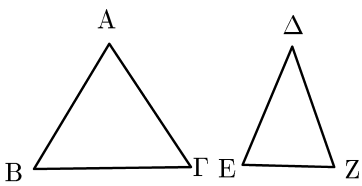
Heiberg



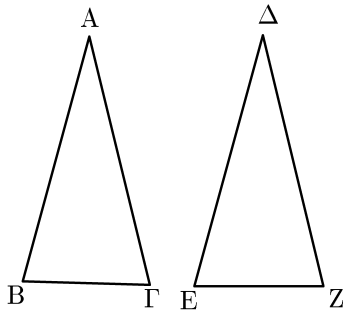
Codex P



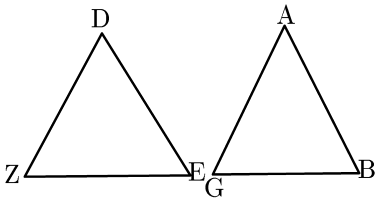
Codex B



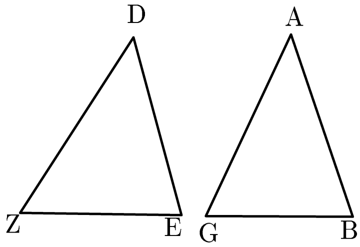
Codex b



Codex V

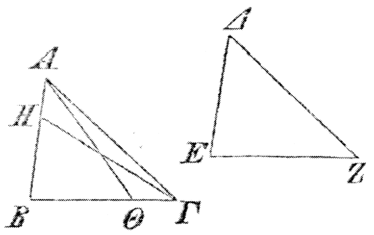


Codex GB

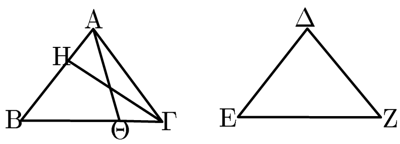


Codex GR

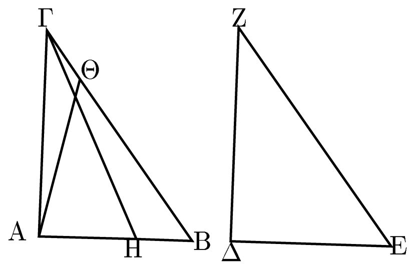
Proposition I.26



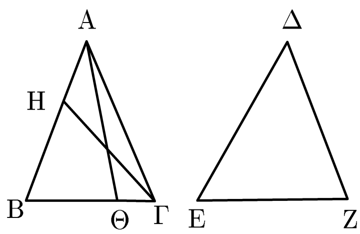
Heiberg



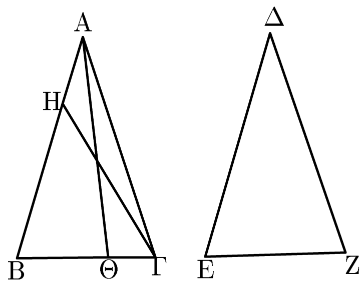
Codex P



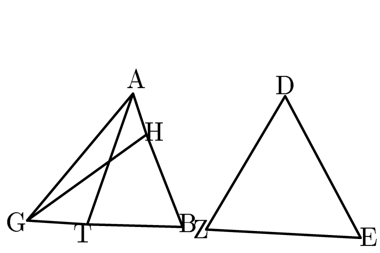
Codex B



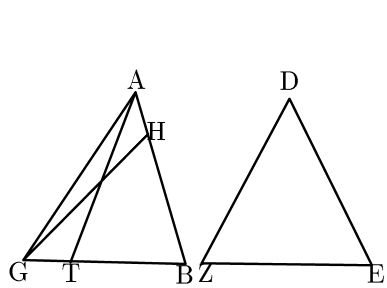
Codex b



Codex V

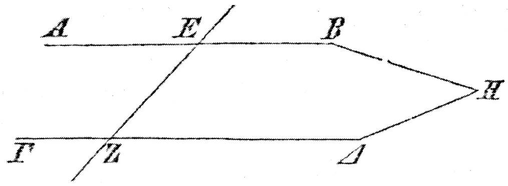


Codex GB



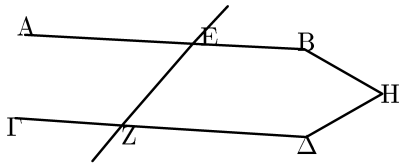
Codex GR

Proposition I.27

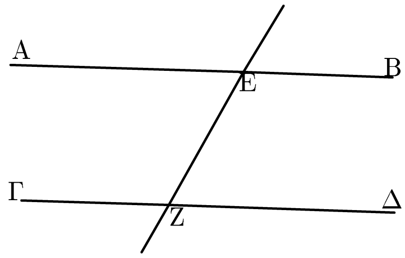


Heiberg

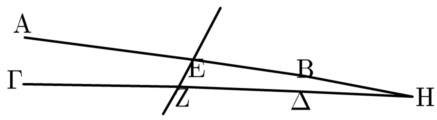
B: Point H is missing. Another diagram in the side margin of **B** by a recent freehand resembles that of codex **P**.



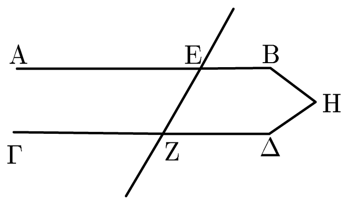
Codex P



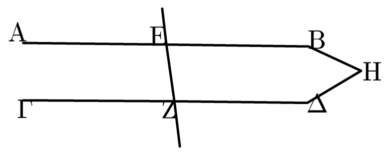
Codex B



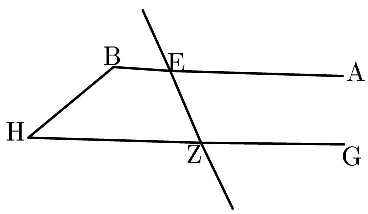
codex B (in the bottom margin)



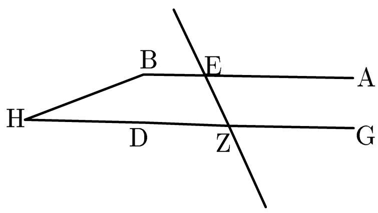
Codex b



Codex V

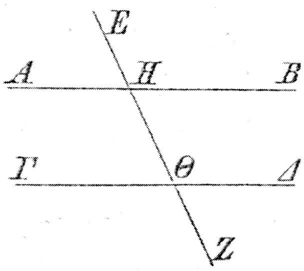


Codex GB

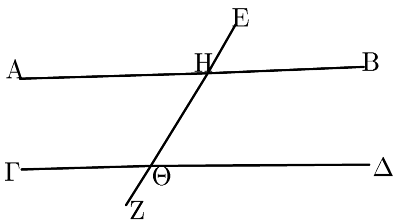


Codex GR

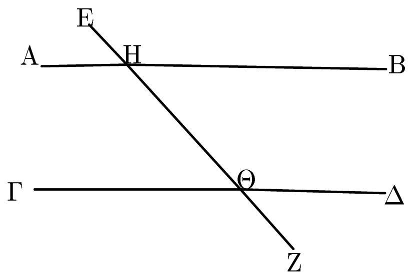
Proposition I.28



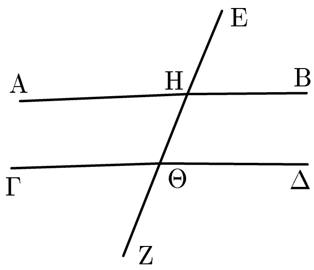
Heiberg



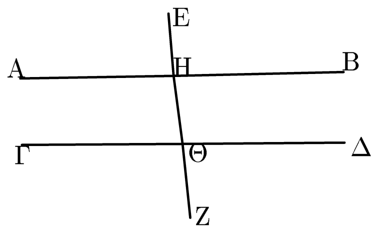
Codex P



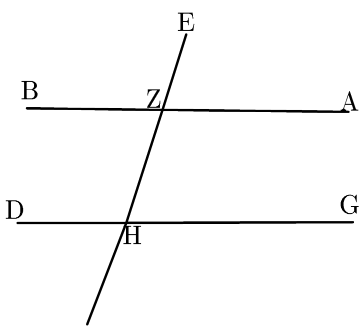
Codex B



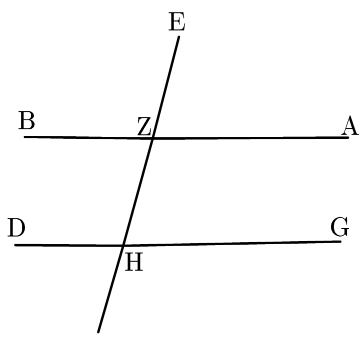
Codex b



Codex V

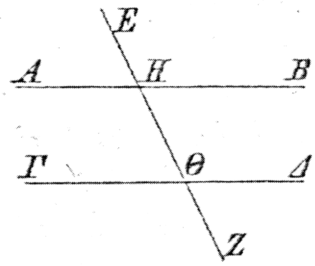


Codex GB



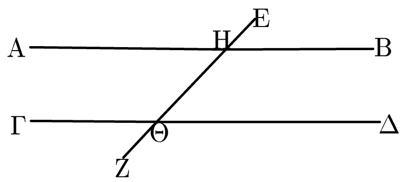
Codex GR

Proposition I.29

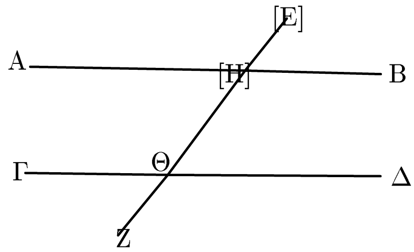


Heiberg

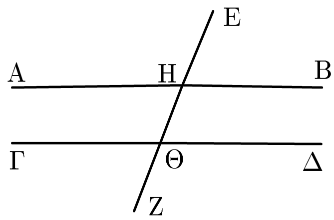
B: the whole figure is very faint.



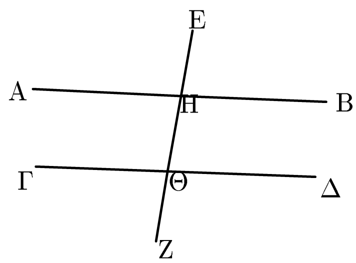
Codex P



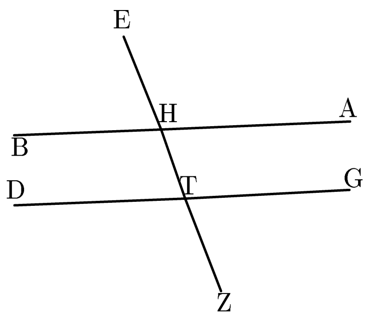
Codex B



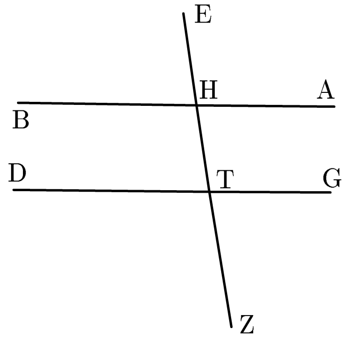
Codex b



Codex V

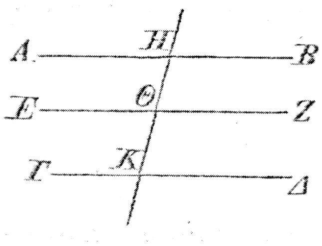


Codex GB

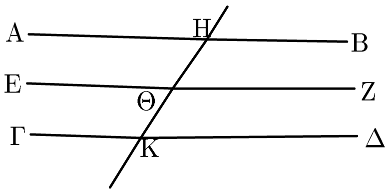


Codex GR

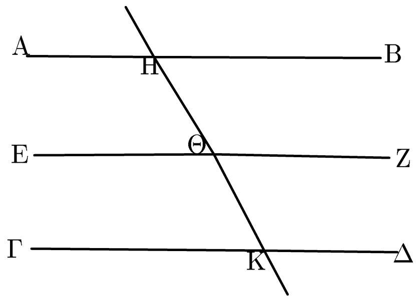
Proposition I.30



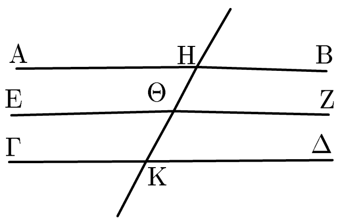
Heiberg



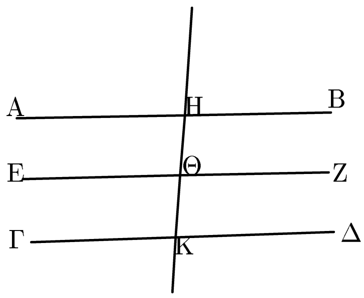
Codex P



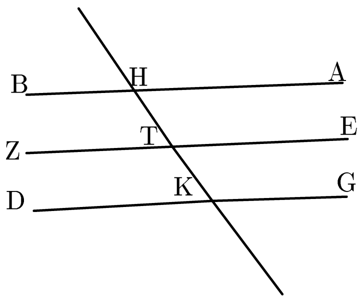
Codex B



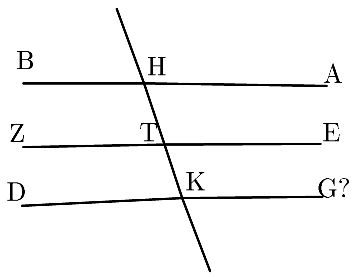
Codex b



Codex V

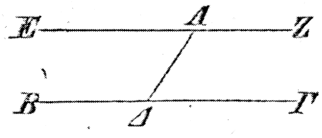


Codex GB

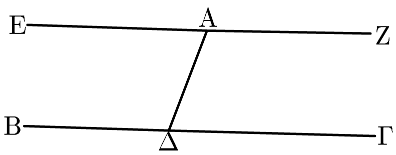


Codex GR

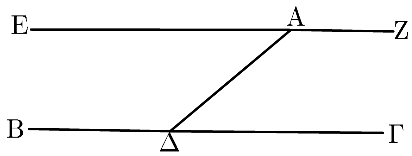
Proposition I.31



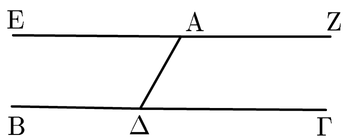
Heiberg



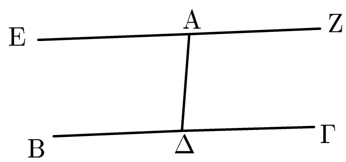
Codex P



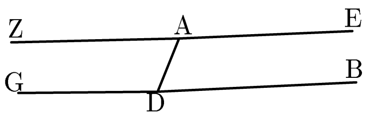
Codex B



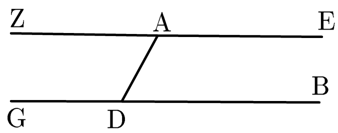
Codex b



Codex V

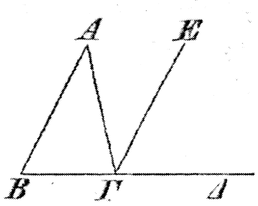


Codex GB

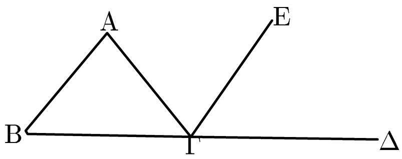


Codex GR

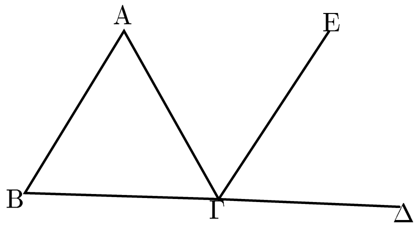
Proposition I.32



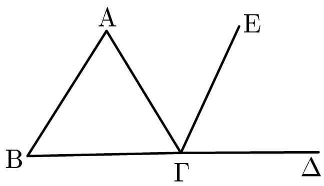
Heiberg



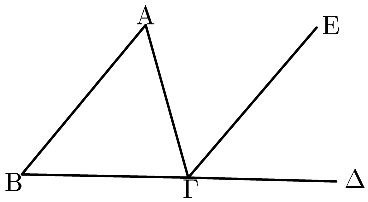
Codex P



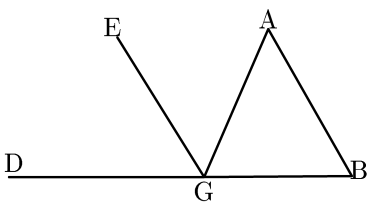
Codex B



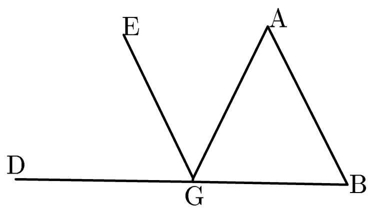
Codex b



Codex V

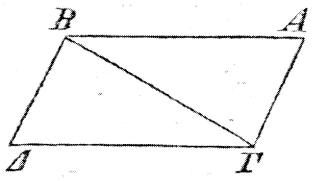


Codex GB



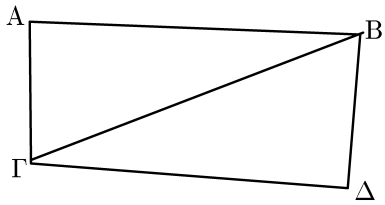
Codex GR

Proposition I.33

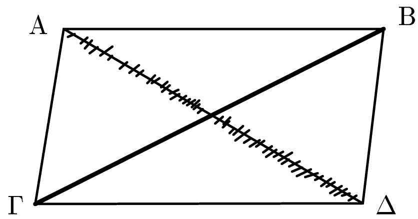


Heiberg

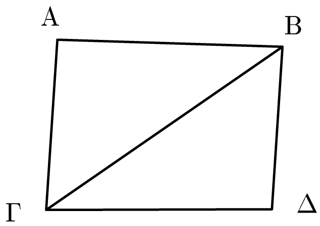
B: Line AΔ deleted and BΓ added by another hand.



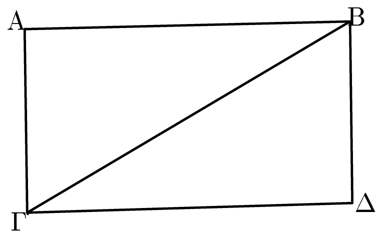
Codex P



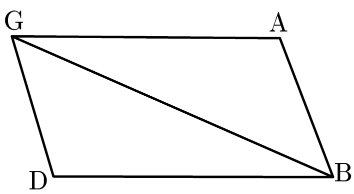
Codex B



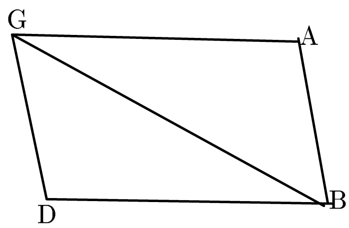
Codex b



Codex V

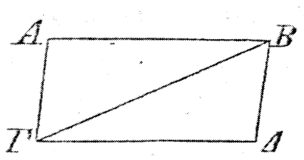


Codex GB



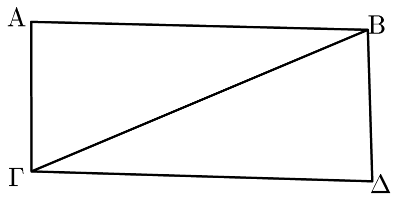
Codex GR

Proposition I.34

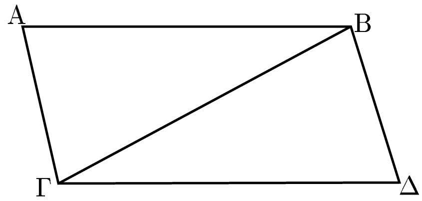


Heiberg

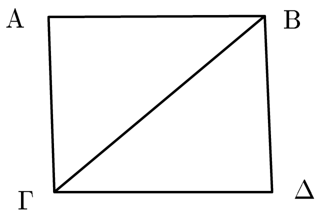
GB GR: The diagrams are rotated, not horizontally reflected.



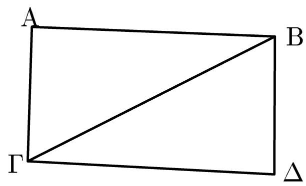
Codex P



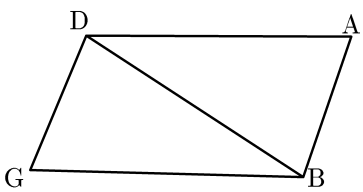
Codex B



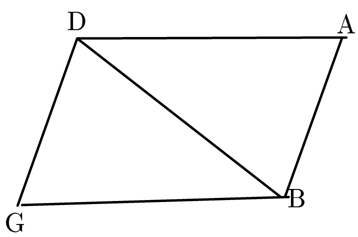
Codex b



Codex V

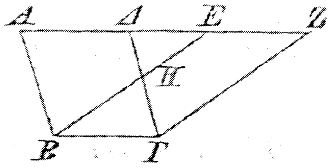


Codex GB

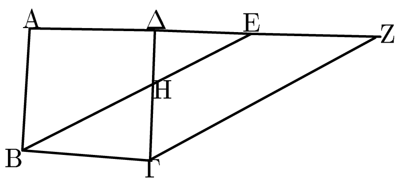


Codex GR

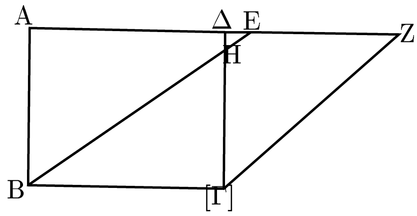
Proposition I.35



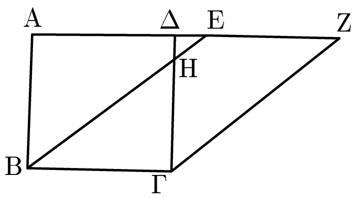
Heiberg



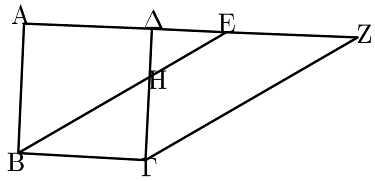
Codex P



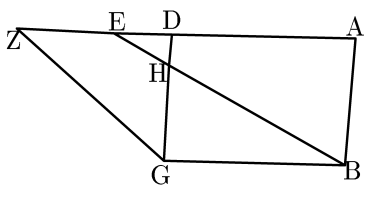
Codex B



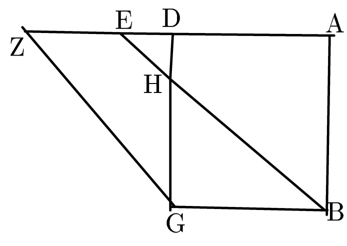
Codex b



Codex V

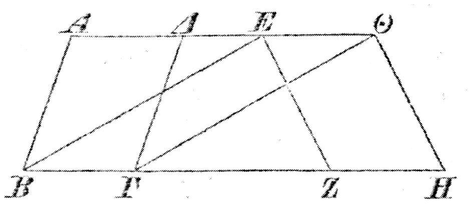


Codex GB

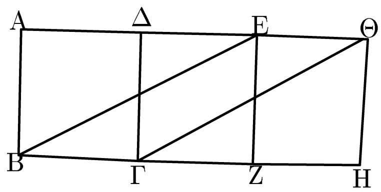


Codex GR

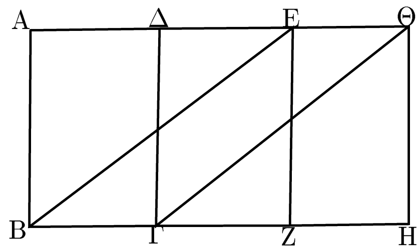
Proposition I.36



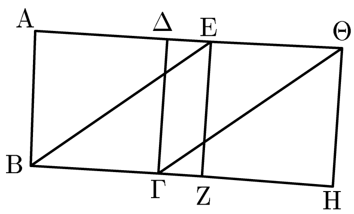
Heiberg



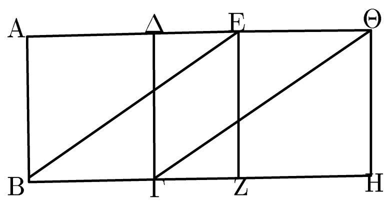
Codex P



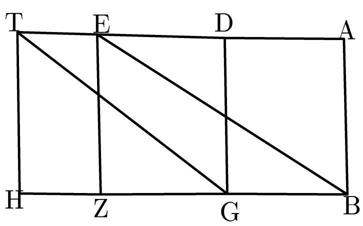
Codex B



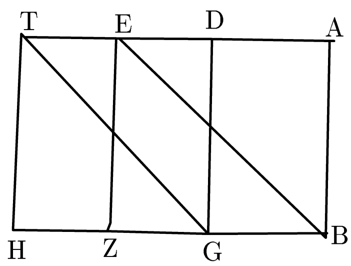
Codex b



Codex V

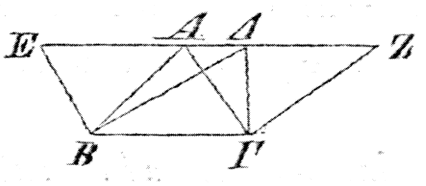


Codex GB

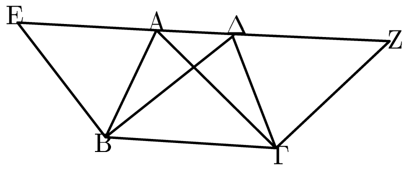


Codex GR

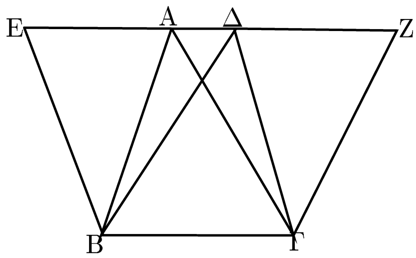
Proposition I.37



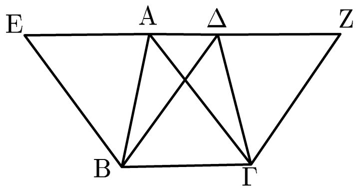
Heiberg



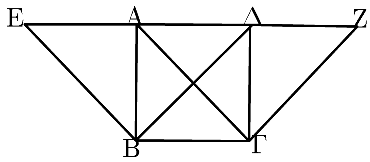
Codex P



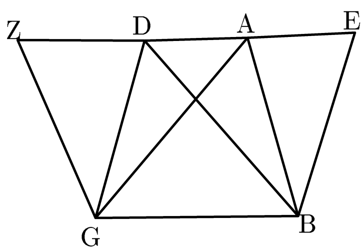
Codex B



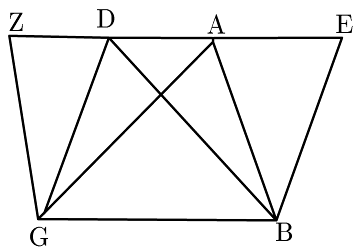
Codex b



Codex V

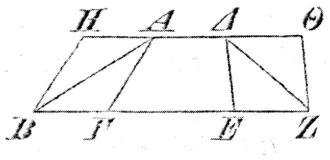


Codex GB

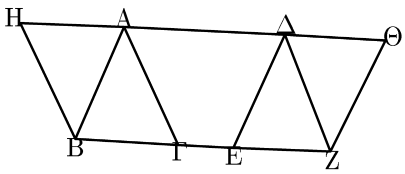


Codex GR

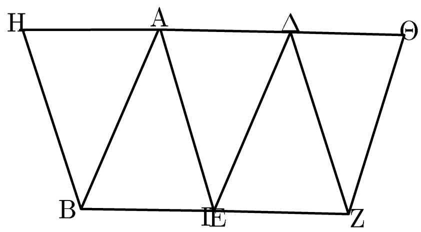
Proposition I.38



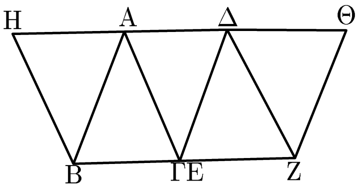
Heiberg



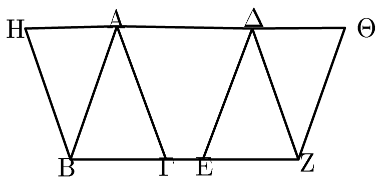
Codex P



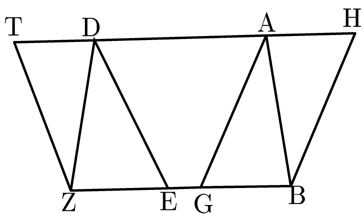
Codex B



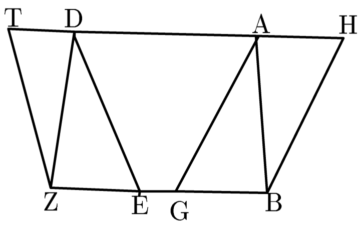
Codex b



Codex V

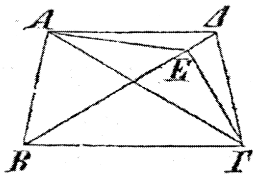


Codex GB

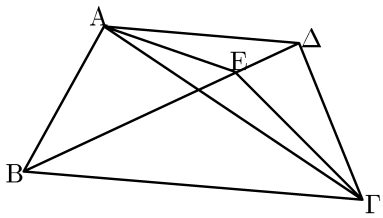


Codex GR

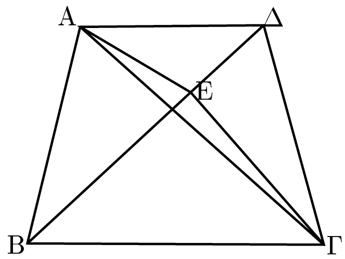
Proposition I.39



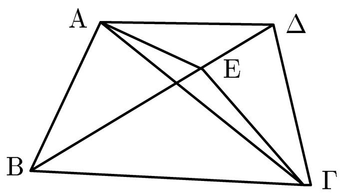
Heiberg



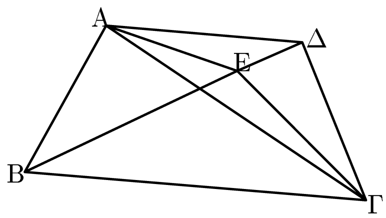
Codex P



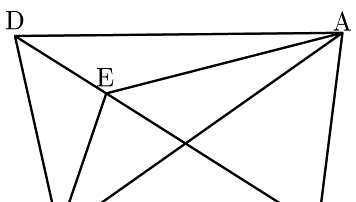
Codex B



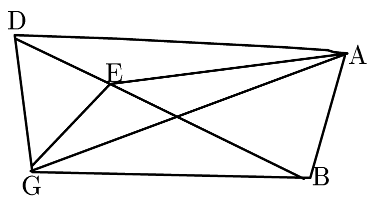
Codex b



Codex V

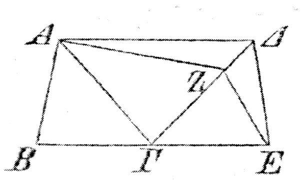


Codex GB



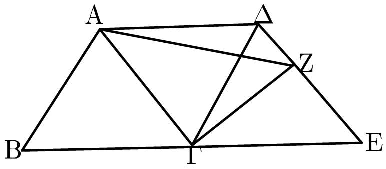
Codex GR

Proposition I.40

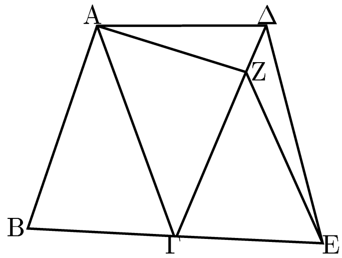


Heiberg

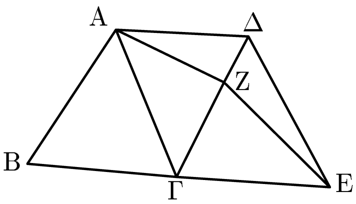
P: Point Z is erroneously taken on the line EΔ.



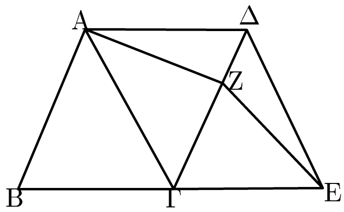
Codex P



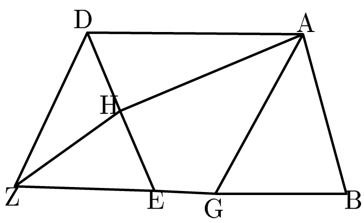
Codex B



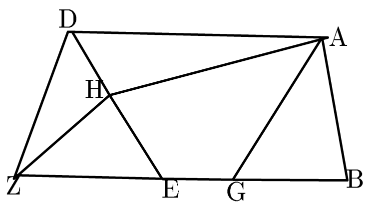
Codex b



Codex V

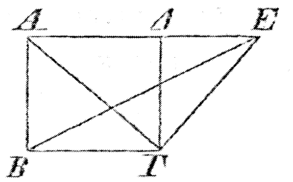


Codex GB



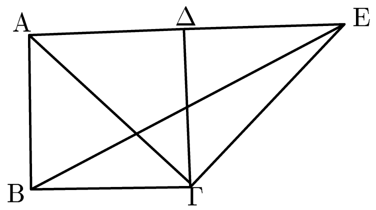
Codex GR

Proposition I.41

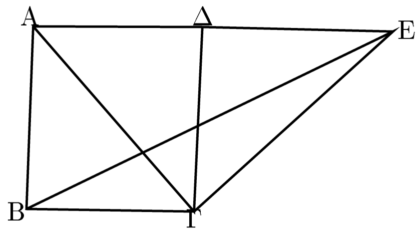


Heiberg

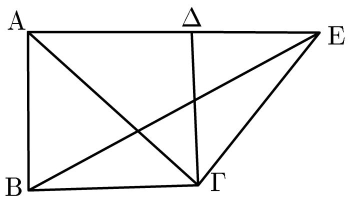
GB: Line GE is missing.



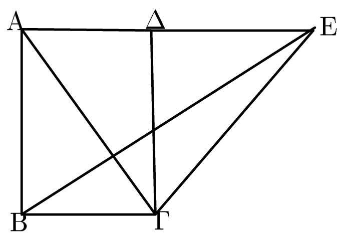
Codex P



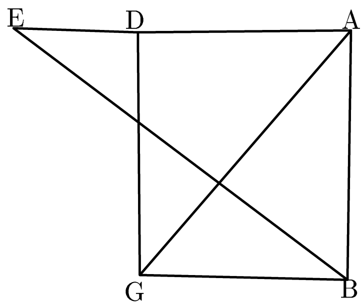
Codex B



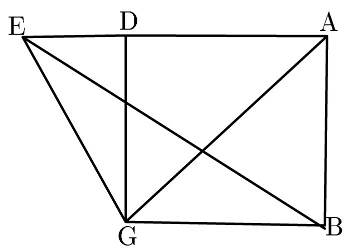
Codex b



Codex V

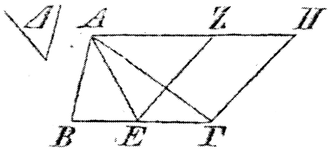


Codex GB

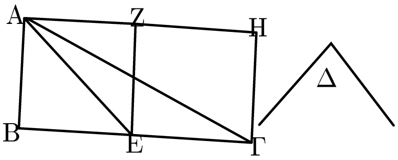


Codex GR

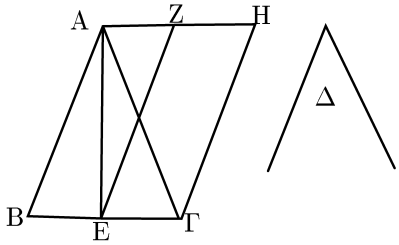
Proposition I.42



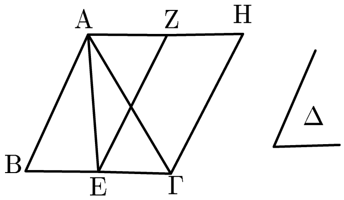
Heiberg



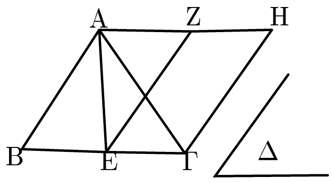
Codex P



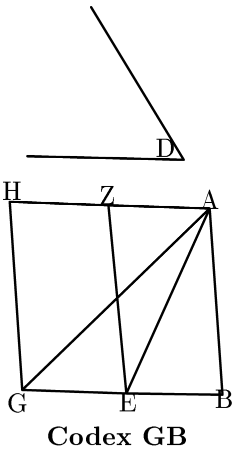
Codex B



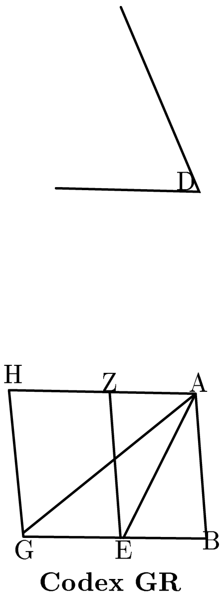
Codex b



Codex V

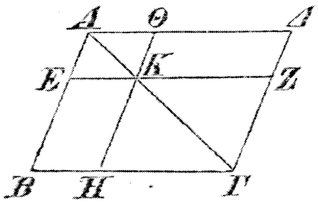


Codex GB

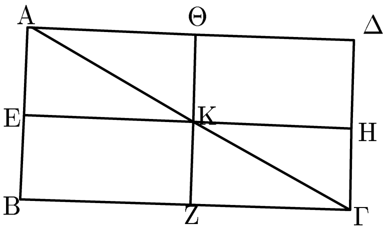


Codex GR

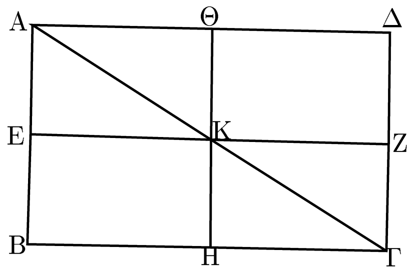
Proposition I.43



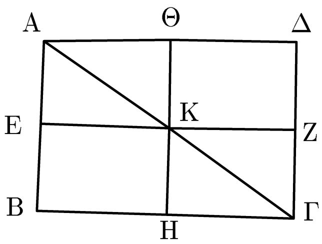
Heiberg



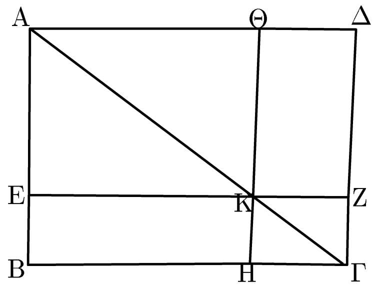
Codex P



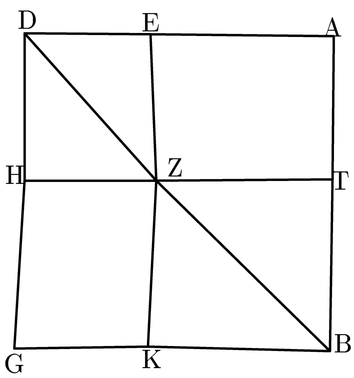
Codex B



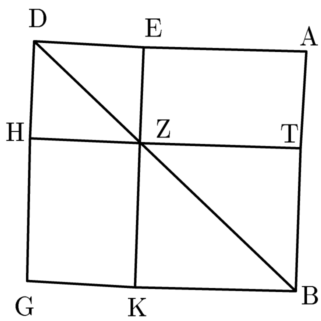
Codex b



Codex V

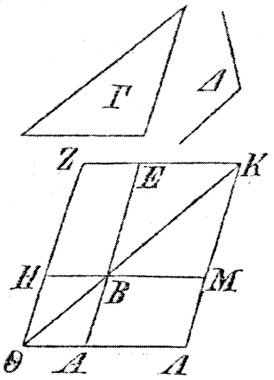


Codex GB



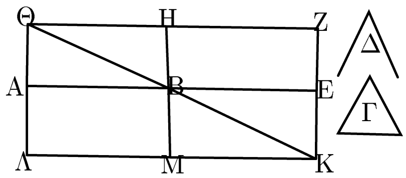
Codex GR

Proposition I.44

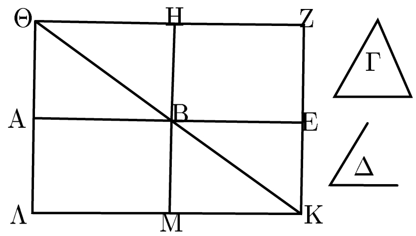


Heiberg

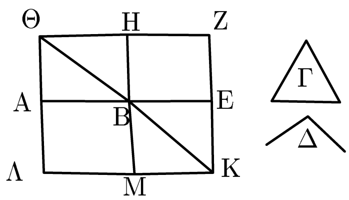
GR: The two parts of the figure appear in the right and left margin separately.



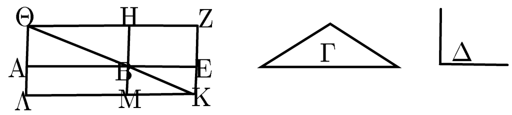
Codex P



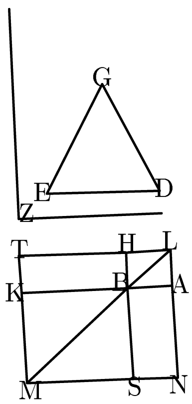
Codex B



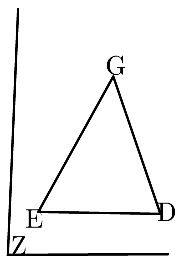
Codex b



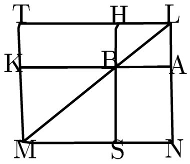
Codex V



Codex GB

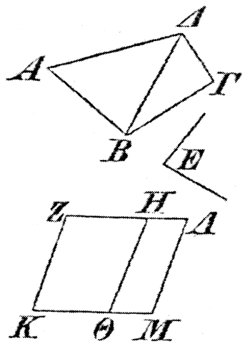


Codex GR (right margin)



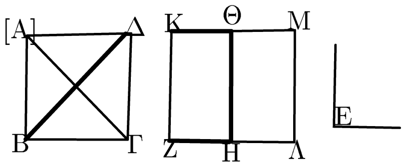
Codex GR (left margin)

Proposition I.45

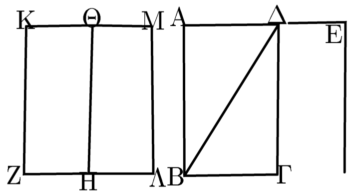


Heiberg

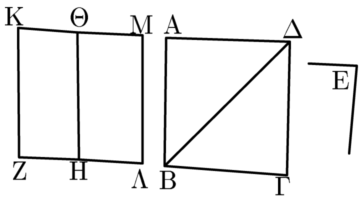
P: BΔ, ZH, HΘ, ΘK drawn by freehand (probably by a more recent hand, here shown by thicker lines).



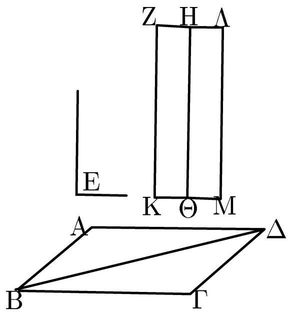
Codex P



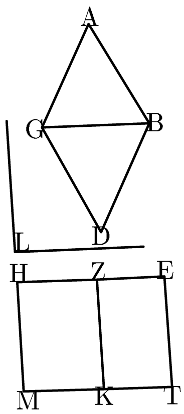
Codex B



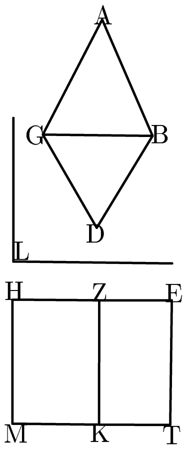
Codex b



Codex V

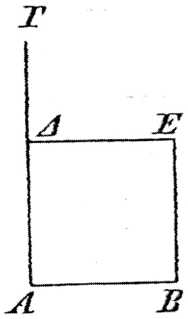


Codex GB



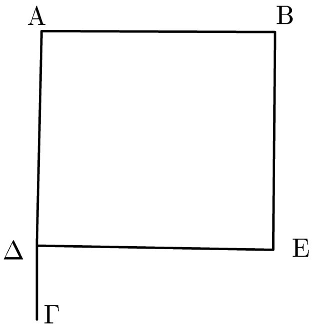
Codex GR

Proposition I.46

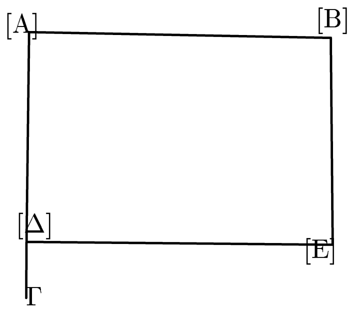


Heiberg

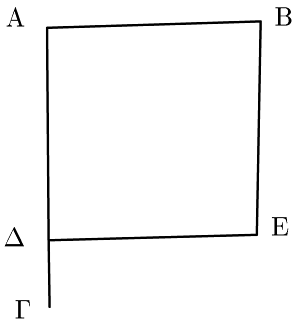
B: The whole figure is very faint.



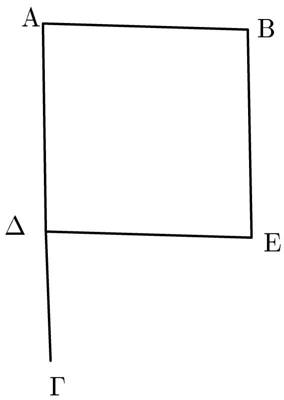
Codex P



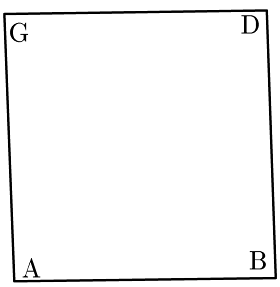
Codex B



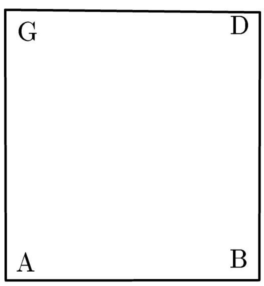
Codex b



Codex V

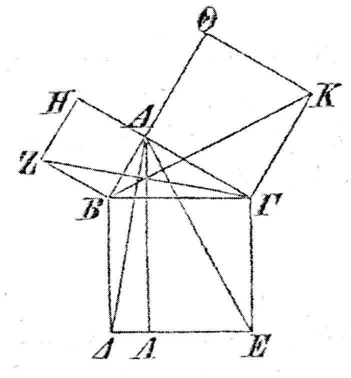


Codex GB



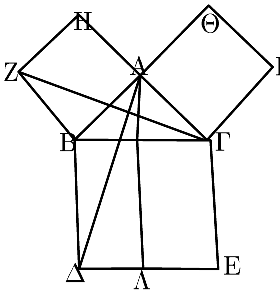
Codex GR

Proposition I.47

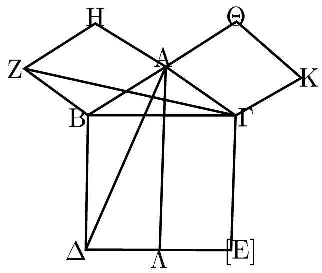


Heiberg

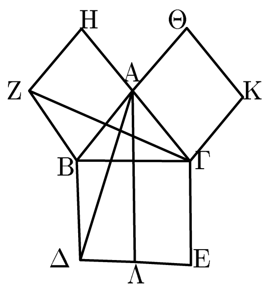
V: Lines AE and BK are drawn freehand. Numbers are written by more recent hands. **PV:** For three sides 5, 5, 7;4,15,50 (**V:** 7;4,15), and for three squares 25, 25, 50, all in Arabic numerals. **b:** 5, 5 for sides of upper squares and 25 for their areas in Greek numerals. **B:** 3, 4, 5 for sides and 9, 16, 25 for squares in Greek numerals; the same set of numbers appear in diagrams in the margin of **P**(Greek), **V**(one diagram in Greek, another in Arabic). These mss. contain other writings that I have not been able to decipher.



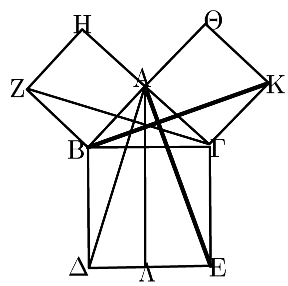
Codex P



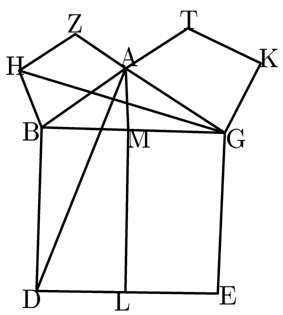
Codex B



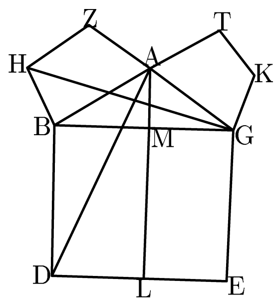
Codex b



Codex V

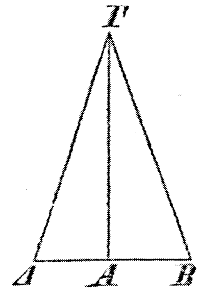


Codex GB



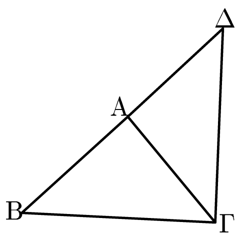
Codex GR

Proposition I.48

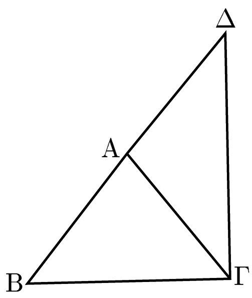


Heiberg

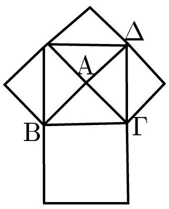
b: Two diagrams for I.47 (not I.48) appear in the space left for diagram in the text of I.48. One is similar to that in **b** I.47 (not shown here; see I.47), and the other is shown below (a figure similar to this figure appears also in **P** in the lower margin of I.47). The diagram for I.48 appears in the lower margin.



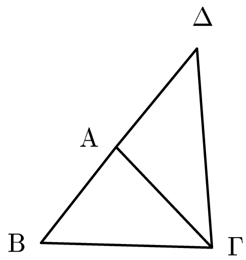
Codex P



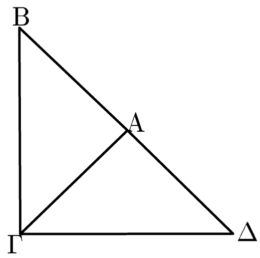
Codex B



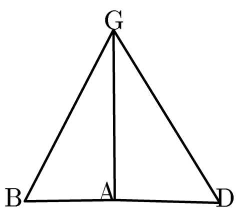
Codex b



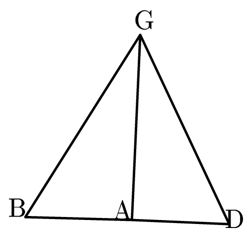
Codex b (in margin)



Codex V



Codex GB



Codex GR

(Received: December 4, 2006)