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# The Arabic version of Ptolemy's *Planisphere* or *Flattening the Surface of the Sphere*: Text, Translation, Commentary

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There are currently no known manuscripts containing the Greek text of Ptolemy's *Planisphere*.[1] Nevertheless, there are two medieval translations from which an assessment of the original work can be made. The oldest of these is in Arabic, undertaken by an unknown scholar, presumably as part of the Baghdad translation movement [Kunitzsch 1993, 97; Kunitzsch 1995, 150–153]. There has never been any doubt that this was based on a Greek text written by Ptolemy, and those who are familiar with Ptolemy's *Almagest* will notice many similarities of style and structure between the two texts, despite the different rhetorical tendencies of Greek and Arabic prose. Moreover, Ptolemy assumes that the reader of the *Planisphere* has already read his *Almagest*, which he refers to in a number of places.

In the 12$^{\text{th}}$ century, a loose Latin translation was made by Hermann of Carinthia on the basis of a different Arabic version than that found in the two known Arabic manuscripts.[2] Hermann's version was edited by Heiberg [1907, 227–259], translated into German by Drecker [1927], and has been the source of much of the modern scholarship on the text.[3] Hermann's text served as the basis for the early-modern Latin editions, the most influential of which was by Commandino [1558], who appended a commentary that includes a study in linear perspective.[4]

Although Anagnostakis [1984] produced an English translation and study of one of the Arabic manuscripts as part of his dissertation, the text has never been formally

[1] The 10$^{\text{th}}$-century Suidas gives the Greek title as the Ἅπλωσις ἐπιφανείας σφαίρας, *Simplification of the Sphere* [Adler 1928–1938, 254]. Kaufmann corrected the fist word to ἐξάπλωσις, which he took as "unfolding" [Pauly-Wissowa 1894–1980, vol. 2, 1801], and Neugebauer [1975, 870–871] accepted this correction. It is possible to read either of these words as "unfolding" or "spreading out."

[2] An earlier Latin translation survives only in fragments [Kunitzsch 1993]. There is too little of this text to be able to say much about the Arabic source.

[3] There is also a medieval Hebrew translation, but this appears to have been based on the Latin [Lorch 1995, 276, n. 11].

[4] Commandino's text, as well as two earlier modern editions, was reprinted by Sinisgalli and Vastola [1992], who also provided an Italian translation. Commandino's commentary is reprinted, with an Italian translation, by Sinisgalli [1993].

edited. The present study supplies a critical edition of the extant Arabic text, along with our translation and commentary. The commentary provides a new reading of the text that encompasses the entire treatise and integrates it into its context as a work in the Greek mathematical tradition.

There have been relatively few historical studies of the *Planisphere* as a whole. Probably, the most useful summary of the mathematics underlying Ptolemy's approach is by Neugebauer [1975, 857–868], who read the text as principally concerned with the construction of astrolabes [Neugebauer 1949, 247–248]. The commentary of Sinisgalli and Vastola [1992] is likewise useful for understanding the text in terms of modern mathematical methods and projective geometry. In neither case, however, is there much attempt to understand Ptolemy's project in terms of ancient mathematical methods. Moreover, both of these studies are based on the Latin text. Anagnostakis [1984], in his study of the Arabic text, gave a commentary to the whole treatise, but did not attempt to situate the overall approach and goals of the treatise in the context of ancient mathematical methods. These last issues have been addressed in papers by Berggren [1991], who sought to understand Ptolemy's aim in the *Planisphere* by comparison with his *Geography*, and Lorch [1995], who compared Ptolemy's methods with those of a medieval commentator. Our reading makes use of these studies; however, we make some key differences of interpretation, which are discussed in the commentary. Moreover, ours is the first reading based on a critical edition of the oldest extant version of the text.

The *Planisphere* is the first known treatise that develops a plane diagram of the celestial sphere using methods mathematically related to stereographic projection. Although Ptolemy wrote the text, the methods contained in it probably go back at least as far as Hipparchus [Neugebauer 1975, 868–869]. Moreover, the text appears to have been written for the advanced student, or expert, in mathematical astronomy. For these reasons, the *Planisphere* should be of great interest to historians of the ancient and medieval exact sciences. By studying this text, we may learn what sort of knowledge could be assumed on the part of a mathematically competent reader in the $2^{\text{nd}}$ century, and probably for a number of centuries before this time. In this way, historians of astronomy can produce a more detailed picture of the mathematical methods of ancient astronomers, and historians of mathematics can develop a broader understanding of the range and methods of Greek mathematics.

The kinds of mathematical thought preserved in the *Planisphere* are especially important if we are interested in those cultures that inherited the Greek mathematical tradition. The Greek conception of the celestial sphere, in both its geometric and arithmetic articulations, was of great interest during the medieval period to scholars working in Sanskrit, Pahlavi, Arabic, Hebrew and Latin. Although many modern historians have a tendency to draw disciplinary divisions between astronomy and mathematics, such distinctions would hardly have been evident to ancient and medieval scholars. In particular, there would have been little of the institutional and professional segregation that we now take so much for granted. The different ways of categorizing and arranging the mathematical sciences were nearly as numerous as the practitioners, but mathematical studies of the celestial sphere were always seen as an important branch of the exact sciences. In the medieval and early modern periods, the projection of the sphere onto the plane became a fruitful area of new research and Ptolemy's text was understood as fundamental to the field. It is our hope that this study will bring new understanding to the endeavors of the ancient and medieval scholars who investigated the fundamental mathematical structures of their cosmos.

# I Editorial Procedures

We have prepared the text on the basis of images of the only two manuscripts presently known and available.

**I**: Istanbul, Aya Sofya 2671
**T**: Tehran, Khān Malik Sāsānī

Anagnostakis [1984, 226–267] printed the first of these in facsimile, while the second is described in detail by Kunitzsch [1994a], who collated the two and listed what he considered to be the superior readings of **T**. A third MS is listed by Beaurecueil [1956, 19] as having belonged to the Maktabat Riʾāsat al-Maṭbūʿāt in Kabul; however, this library has not survived the recent wars.

Our apparatus refers to three other sources which are not MSS but which have been useful in establishing the text.

**Mas**: Maslama's notes
**Her**: Hermann's Latin translation
**Ana**: Anagnostakis's English translation

The 10$^{\text{th}}$-century Andalusian astronomer Abū al-Qāsim Maslama ibn Aḥmad al-Faraḍī al-Majrīṭī studied the treatise and produced a series of notes and supplementary material of use for the construction of astrolabes [Vernet and Catalá 1965, 1998; Kunitzsch and Lorch 1994]. As well as being useful for understanding the mathematics of the treatise, Maslama's notes contain sixteen citations, all but one of which can be usefully compared to the text contained in **TI**. There are, however, differences between the text Maslama quotes and that preserved in **TI**. These are usually minor, but in places they are enough to show that the two versions could not be edited so as to produce a single text (for examples, see lines 423, 481, 498, **Mas**107).

Hermann's Latin translation was made on the basis of some version of Maslama's edition and included Maslama's notes and additional material [Kunitzsch and Lorch 1994, 34–71]. We have used this text to justify a number of corrections to the Arabic, especially in the numbers. The superior readings found in Hermann's text are probably corrections introduced by Maslama, or less likely Hermann, as opposed to evidence for a more pristine text. Nevertheless, they represent medieval readings without which the text would, in a number of cases, make no mathematical or astronomical sense. It would also be possible to correct many of the letter names of geometric objects on the basis of Hermann's text, but since he often uses different lettering, and sometimes a slightly different figure, this would be more trouble than it could be worth.

There are numerous errors in the letter names of geometric objects. In the early part of the treatise, a more recent hand has corrected many of these in **T**. Moreover, in his translation, Anagnostakis [1984, 99–101] introduces many corrections, most of which concern the letter names. Where we follow these corrections, they are attributed to one or both of these sources.

## I.1 Orthography

In the edited text, we have attempted to follow the orthography of the manuscripts, so that where **TI** agree on a particular form, we follow that even when it differs from modern conventions.[5] For example, since they both write ثلث for ثلاث, we have printed the former. Where they disagree, however, we follow modern practice. For example, since **I** has هى whereas **T** usually has هي, we print هي. On the other hand, since they also disagree, for example, on احدى and احدي, we print احدى.

Because **I** is more liberal in the use of diacritical marks, we have allowed ourselves to be guided by it and sometimes include a shadda, tanwin and short vowel signs. Often we directly follow **I** in this, but at times we silently insert them with no manuscript authority in the interest of clarity. We also neglect such marks in **I** where they are unnecessary. For example, **I** often includes the shadda of the assimilated sun letters.

There is relatively little use of hamza in the manuscripts. **I** sometimes includes an initial hamza for clarity, or seemingly at random, but neither MS makes use of the medial or final hamza. Thus in both MSS, we find جز for جزء or شي for شيء, and so they are printed in the text. Nevertheless, in a few words, we include a medial or final hamza in order to differentiate them from similar words or make explicit their grammatical form. The reader should understand that these are not found in the manuscripts. As is common in medieval MSS, when the vowel is kasra, the seat of the absent hamza is the dotted ي. Thus, we find دايرة for دائرة, قايم for قائم, and so

[5] Outside of the context of the text, however, we use modern orthography.

in every case.

## I.2 Correcting the Text

In the interest of mathematical and astronomical intelligibility, we have often found it necessary to correct the text. Nevertheless, we have attempted to balance this need with a desire to stay true to the evidence of the manuscripts. Although there are places were we have changed a grammatical form, inserted or deleted material that seems requisite, our tendency has been to follow a policy of minimal intervention.

We do not know who produced the original translation, so we cannot compare the style of this treatise with others by the same author. Passages that may strike us as strange could simply be due to idiosyncratic features of the translator's prose style. The translator may himself have faced obstacles in understanding his sources. He was, no doubt, working with a very limited number of Greek manuscripts, which would themselves have presented numerous difficulties. Hence, where we can make some possible sense of the manuscript idiom no matter how strained or unnatural it may appear to us, we have let the text stand. For example, the beginning of lines 122–125 makes little sense, but is grammatically coherent. Instead of rewriting the passage, we have preserved the text as it is and translated freely. In any case, all changes from the manuscript sources are noted in the apparatus.

As discussed above, the corrections that we have most consistently introduced are in the numbers and the letter names of geometric objects. It is clear that these are highly susceptible to the inaccuracies of manuscript transmission, and we have felt little hesitation in correcting them or following the corrections of previous readers. Nevertheless, in the case of the numbers we have always justified our corrections on the basis of other occurrences of the same value in the text, other medieval sources or simple arithmetical operations implied by the text. There is, however, one number that is clearly incorrect but which we have not changed. The number $25;30^{\mathrm{p}}$ at line 224 is certainly wrong; however, the value in Hermann's translation ($\approx 55;59^{\mathrm{p}}$) does not agree exactly with that derived by computation ($56;1,17^{\mathrm{p}}$).[6] Moreover, it is not possible to assume that the stated value for the arc subtending this length ($55;40^{\circ}$) was precisely calculated from the chord table, since recomputing with the chord table often shows minor discrepancies with the numbers in the text. Hence, we cannot know exactly what value was found in the original translation, much less in the Greek.

As mentioned above, there are a few places where we have added or deleted some words in the interest of clarity. Although there are some cases where it is clear that changes were introduced in the process of transmission, we must bear in mind that the medieval scribes were not in the habit of introducing deliberate changes to the texts [Dallal 1999, 66]. Moreover, they were fairly careful to transmit the text

[6] See page 93, note 83.

accurately as it was written, so that we must admit the possibility that some of the obscure passages are faithful reproductions of the original translation.[7]

An example of such ambiguity is found in lines 206–215. In this treatise, a square is expressed as "the square of some line times itself" (مربع خط ما في مثله), where the expression for the line can be a letter name, a few words, or an extended phrase.[8] In lines 206–215, the Pythagorean Theorem (*Elem.* I 47) is twice asserted for the squares of specific objects in the diagram and the expressions for the squared lines become somewhat involved. The result is that in three of five cases the actual expression for the square (مربع ⋯ في مثله) has dropped out (in the other case, the square is stated as a number). Since some reference to the squares is mathematically required, we have no satisfying alternative. Either the original translator assumed that it would be obvious to the reader that the squares were intended and deliberately omitted the actual phrases asserting this, or at some point in the transmission a copyist intensionally or accidentally dropped nine words in six different places. These alternatives are problematic because the full expression for the square occurs not only in the first instance but also in the third, while it is hard to imagine a conscientious scribe introducing systematic errors of this kind. There are, of course, other possibilities – the original translator may have gotten sloppy at this point, the text may have been garbled in the transmission and corrected by a scribe who did not fully understand the mathematics, the Greek source(s) may already have contained errors, and so forth. In the edited text, we have added the expression for the squares in brackets, mindful of the fact that this may not represent any medieval version of the Arabic.

## I.3 Editing the Diagrams

Although the copyist of **I** left empty boxes in the text where the diagrams should appear, no figures were ever drawn. Hence, **T** is our only evidence for the diagrams of this version of the Arabic text. In some sense, this has made the task of editing the figures easier. Nevertheless, since there are errors in the diagrams, we have chosen not to reproduce them exactly in our edition of the text, but to strike a balance between this and redrawing them to suit the mathematical requirements of the material they accompany, noting all differences between our reproduced figures and the originals. Moreover, since the text is provided with a translation, we have redrawn figures for this that we consider to be fully consistent with the mathematics involved. Hence, readers of the Arabic text may find it useful to consult the diagrams for the translation as well as those for the text.

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[7] For example, the scribe of **T** checked his work, added a number of missing words and phrases in the margins and included a brief note near the colophon stating that the copy was true.

[8] See page 46 for a discussion of this idiom.

The diagrams accompanying the text preserve the internal and relative scale, orientation, shape, and label positioning of the originals.[9] Features such as color, relative line weight and letter shape are not preserved. Since we have reproduced the shape of the lines and circles quite closely, it should be possible for the reader to see at a glance which lines have been drawn with a guide and which by freehand. It should be noted, however, that some of the circles are not true, although they all appear to have been drawn with an instrument. The curvature of the circles is reproduced accurately in our figures. Wherever our reproduction departs from the MS, it is so noted in the apparatus.

## II Translation Procedures

The exposition in this treatise is often not as clear as one would like. Having been translated from Ptolemy's highly structured Greek into Arabic, a language not perfectly suited to the kind of sentence that Ptolemy liked to write, there are a number of passages that readers may find obscure.

In an attempt to mitigate these difficulties, we have not tried to maintain literal faithfulness to the wording of the technical terminology, but to its meaning. Nevertheless, since those who do not read Arabic may also be interested in the literal expressions, we provide a discussion of these phrases.[10] This section also serves as a partial index to the technical terminology. Hence, we list the line numbers where the stated terms are found. We do not, however, list line numbers for words and phrases that occur numerous times or that we translate consistently throughout.

*Astronomical and Geographic Terms*

The expression for the equator is "the circle of the equalizer of the day" (دائرة معدل النهار), or less often, with ellipsis, "the equalizer of the day" (معدل النهار, 18, 30, 46, 133, 157, 425). We translate both expressions with *equator*.

The phrase translated as *meridian*, "the circle of midday" (دائرة نصف النهار) is also quite consistent. Ptolemy uses the term *meridian* to refer to any great circle through the celestial poles, so that it is generally independent of any local coordinates. The meridians through the equinoctial and the solstitial points respectively, known as the equinoctial and solstitial colures, sometimes play a significant geometric role in the treatise. Where their status as colures is of little importance these are simply called, and translated as, *meridians* (14, 404, 503). In the one case where the equinoctial colure is used as such it is again called, and translated as,

[9]We are grateful to Ken Saito for giving us computer programs designed by himself and Paulo Mascellani that are useful for reproducing MS diagrams.

[10]The terminology in this section should be compared with the list provided by Kunitzsch [1994b] for the Arabic versions of the *Almagest*.

the *meridian* (449). The solstitial colure, on the other hand, is called "the meridian that goes through the two poles" (دائرة نصف النهار التي تمر بالقطبين, 440, 461), and "the circle that goes through the two poles" (الدائرة التي تمر بالقطبين, 485, 514). In one case it is referred to, in the plane, as "the straight line that goes through both two poles" (الخط المستقيم الذي يمر بالقطبين جميعًا, 429). We have translated these expressions fairly literally.

For the ecliptic, on the other hand, there is more variety. In the beginning of the treatise, we generally encounter "the circle of the inclined sphere" (دائرة الفلك المائل, 6, 24, 43, 49, 51, 72, 93, 183), but this gradually gives way to "the circle of the sphere of the signs" (دائرة فلك البروج, 47, 52, 77, 80, 83, 87, 89, 90, 126, 129, 167, 240, 323), which in turn gives way to the simplified "circle of the signs" (دائرة البروج, 177, 279, 311, 429, 436, 439, 442, 449, 459, 462, 465*, 466*, 483, 488, 496, 498, 515).[11] We render these three expressions as the *ecliptic*. The most common phrase, however, is "the circle that goes in (or through) the middle of the signs" (الدائرة التي تمر في (او ب)وسط البروج, 39, 54, 74, 134, 147, 174, 180, 186, 199, 288, 306, 339(2), 399, 424, 431, 432, 434, 508, 510, 519, 523). This we have translated as *the circle through the signs*.

The horizon is called either "the circle of the horizon" (دايرة الافق, 51, 73, 78, 80, 82, 88, 89, 90, 169, 281, 289, 293, 319, 321, 342, 343, 350) or simply "the horizon" (الافق, 157(2), 166, 180, 274(2), 277, 283, 318). In some cases, the former expression clearly denotes the mathematical object that represents the horizon on either the sphere or planisphere while the later means the local horizon. In other cases, however, the situation is ambiguous or the distinction is not clearly maintained. Nevertheless, since such a distinction may have been intended, we have translated the former expression as *the horizon circle* and the later simply as *the horizon*.

The two most important classes of lesser circles, and the only ones of any relevance for the project of this treatise, are the circles parallel to the equator and the ecliptic, now generally called the parallels of declination and latitude. Because these circles come up so often, it will be useful to refer to them as $\delta$-circles and $\beta$-circles, since they are sets of points of equal declination, $\delta$, and celestial latitude, $\beta$, respectively. The $\delta$-circles are called "the circles parallel to the equator" with a number of trivial grammatical variants (الدوائر الموازية لدائرة معدل النهار, 7, 10, 11, 18, 19, 30, 46, 58, 70, 98, 133, 146, 157, 202, 246, 274, 348, 370, 405, 410, 415, 445, 448, 487, 507, 509, 517, 523). The $\beta$-circles are called "the circles parallel to the ecliptic," again with variants (الدوائر الموازية لدائرة البروج, 439, 441, 448, 459, 462, 466, 483, 487, 496, 498, 512, 514). We have translated these various expressions rather closely.

The four cardinal points of the ecliptic are defined either by the intersections

[11]Line numbers followed by an asterisk indicate instances in the edited text, but not in the manuscripts. Line numbers followed by an *e* indicate that the expression has undergone ellipsis.

of the ecliptic with the equator or by its points of tangency with the two equal $\delta$-circles known as the Tropics of Cancer and Capricorn. The expression that we translate as the *tropics* is "the two circles of the place of turning" (دائرتا المنقلبين, 36, 40, 114, 166, 528, 528), which are sometimes specified as "the circle of the summer (or winter) place of turning" (دائرة المنقلب الصيفي (او الشتوي) 38(2), 40(e), 41, 123, 124). We translate these as the *summer* or *winter tropic*. The *solstitial points* are called "the two points of the places of turning" (نقطتي المنقلبين 134, 147, 178, 282), and the *summer solstice* is "the point of the summer place of turning" (نقطة المنقلب الصيفي, 310). The *equinoctial points*, on the other hand, are called "the two points of equality" (نقطتي الاستواء, 176, 233, 264), and once "the two points of evenness" (نقطتي الاعتدال, 307). These are also occasionally specified as "the point of the spring (or fall)" (نقطة الربيع (او الخريف), 42(2), 311, 333, 334). We translate these as the *vernal* or *autumnal point*.

The subject of rising-times of arcs of the ecliptic, which in the *Almagest* is usually discussed with some form of the verb "to rise" (ἀναφέρειν, συναναφέρειν), is handled in this treatise by "ascensions" (مطالع). Because the rising-times of arcs of the ecliptic are measured by the arcs of the equator that rise with them, which are converted to times by the identity $1° = 4$ minutes, the word *maṭāliʿ* can denote either the time or the co-ascendant arc. Hence, there is some ambiguity in the text and it would be possible to translate some occurrences of the term as "co-ascension."[12] Nevertheless, we have preserved the ambiguity and always translated with *rising-times*.

In Ptolemy's studies of rising-times, the fundamental case, by which all other cases are measured, is the situation known as *sphaera recta*, in which the observer is on the equator. This is referred to numerous times as "the upright sphere" (الكرة المستقيمة). This situation is contrasted with that known as *sphaera obliqua*, in which the observer is at any other latitude. The latter case is referred to only once, as "the inclined sphere" (الكرة المائلة, 270). These two expressions are translated literally.

A geographic latitude is specified by the Arabic transliteration of the Greek "inclination" (إقليم, κλίμα, 157, 315, 322, 338), which means a geographic region at roughly the same latitude. In both Greek and Arabic, however, this can carry the technical meaning of *latitude* and we have translated it as such.[13]

[12] For example, see lines 308–314 and page 97. Naṣīr al-Dīn al-Ṭūsī, in his *Memoir on Astronomy*, defines مطالع as a "co-ascension" [Ragep 1993, 282].

[13] Ptolemy generally uses κλίμα in this way in the *Almagest* [Toomer 1984, 42, n. 32], and also once in his *Geography* [Berggren and Jones 2000, 111].

*Mathematical Terms*

The distinction in Greek mathematical texts between a radius as geometric object and as the interval with which a circle is constructed appears to be preserved in the Arabic [Fowler and Taisbak 1999; Sidoli 2004]. The former, "the [line] from the center" (ἡ ἐκ τοῦ κέντρου), is translated with "half of the diameter" (نصف القطر, 92, 93, 122, 123, 127, 128(2), 168, 184, 199(2), 202, 207, 208, 212, 214, 246, 248, 275, 276, 348, 370) while the latter, the "interval" (διάστημα), is rendered as "distance" (بعد, 22(2), 33, 46, 99, 100, 274(2), 413(2), 418, 447, 450, 491). We translate the former as *radius* and the later as *distance*.

The Arabic, on the other hand, makes a linguistic distinction between an arc and a circumference, which Ptolemy would not have done in Greek. We have translated قوس with *arc* and "the bounding line" (الخط المحيط, 453, 455, 458) as *circumference*.

The Greek phrase "the [square] upon the [line] $AB$" (τὸ ἀπὸ τῆς $AB$) has been rendered in Arabic as "the square of $AB$ [multiplied] by itself" (مربع ا ب في مثله, 205(2), 207, 208*, 208, 212*, 213*, 351(2), 456),[14] or more often simply "$AB$ times itself" (ا ب في مثله, 63, 64, 211, 212, 457, 471, 472(2), 473, 475, 476, 477(2), 478(2)). Although the Arabic expression stresses an arithmetic operation which is not stated in the Greek, it is clear that Ptolemy fully intended his readers to understand the Greek expression as both a geometric object and an arithmetic operation. Hence, we have not hesitated to translate such phrases with *AB squared*, or *the square of AB*.

We have translated a number of descriptive phrases with modern technical terms. Thus, where the Arabic has "what remains from the semicircle" (ما بقي من نصف الدائرة, 106, 109, 116), we translate with *the supplement*. In this text, "the remaining angle" (الزاوية الباقية, 193, 300) means *the complementary angle* and is translated as such. Likewise, "the opposite angle" (الزاوية المقابلة, 194) is translated as *the vertical angle*. The *hypotenuse* is expressed as "the (line) that subtends the right (angle)" with ellipsis in two cases (الخط الذي يوتر الزاوية القائمة, 211(e), 223, 255, 325, 356(e), 375).

The expression for the chord of an arc is a fairly literal translation of the usual Greek idiom for chord, "the straight line that subtends arc $AB$" (الخط المستقيم الذي يوتر قوس ا ب, 109(2), 117, 119, 135, 137, 148, 150, 159, 160). Despite the fact that in other Arabic texts there is a single word for chord, وتر, we have translated all variations of the longer expression with some form of *the chord of arc AB*.

[14] The Arabic preposition في, used to indicate multiplication commonly means "in." Like all prepositions, however, it has a range of meanings many of which are context specific. In the context of multiplication, we translate it with *by*.

Equality, and related concepts, are handled in a number of different ways in the treatise. The vast majority of cases are handled with grammatical variations on the root سوي. Most often, one object is said to be *equal to* another object using مساوٍ and the preposition ل (27(2), 66, 67(2), 101, 104, 106, 193, 194, 196, 211, 236, 291, 292, 293, 298, 299(2), 300, 301, 304, 306, 310, 312, 393*, 394, 412, 413, 420(2), 421(3), 422, 431, 454(2), 456(2), 457(2), 468, 469(2), 474, 475, 481, 494). Less often, two or more objects are said to be mutually *equal* using متساوٍ (always in the feminine متساوية, 44(2), 59, 97, 186, 302, 303, 307, 515). There is also one case in which the congruence of two triangles is asserted by stating that they are "equisided and equiangular" (مساوي الاضلاع والزوايا, 192, see also note 72). Finally, a property of two objects is said to be the *same* using سواء. This last construction is used six times to assert the equality of distances (12, 24, 68, 71, 98, 188). We have translated these expressions fairly literally. Products and squares, or the degree value of angles, are sometimes asserted as equal by stating that they are "similar" (مثل, 63, 64, 85, 86, 226, 237, 259, 328, 359, 378); however, they are more often said to be "equal" (مساوٍ). In one case, مثل is also used for equal line segments (86). We have translated this use of مثل as *equal*. There are two cases, however, where مثل is used in the sense of "similar" but not equal (411, 450), and we have translated accordingly.

As usual in medieval Arabic texts, a proportion is asserted by stating that the ratio of $A$ to $B$ is *as* (ك) the ratio of $C$ to $D$. This can be contrasted with Greek texts, where the two ratios are generally said to be "the same" (αὐτός).

We use numerals, employing the notations for sexagesimal fractions introduced by Neugebauer and others, whereas the text writes out numbers in longhand. Thus, where the text reads "one hundred parts and two parts and four minutes and forty-five seconds" (مائة جزء وجزئين واربع دقائق وخمس واربعين ثواني) we translate with $102;4,45^{\mathrm{p}}$. The other abbreviations we use are $x^{\circ}$ for degrees (درجة), $x^{\mathrm{t}}$ for time degrees (زمان, $1^{\mathrm{t}} = 4$ minutes) and $x^{\circ\circ}$ for half degrees (درجة, $2^{\circ\circ} = 1^{\circ}$). Note that $x^{\circ\circ}$ is introduced only for convenience, reflecting a mathematical, not textual, distinction.[15]

### *Representing the sphere*

The basic project of the treatise is to construct a plane diagram of a sphere, hence there are many references to "the solid sphere" (الكرة المجسمة). The majority of these are general references to the sphere as a mathematical object (13 times), however, there are also a number of specific references to the sections of the *Almagest* in which Ptolemy uses solid geometry to address the same topics as are covered in this

[15] Toomer, following B. Goldstein, uses this notation in his translation of the *Almagest* [Toomer 1984, 8].

treatise (114, 158, 173, 232, 269, 337, 399). We have translated all of these references literally.

The Arabic translator has used the verb "to imagine" (وهم, V) to translate the equivalent Greek expression (νοεῖν), which is used to discuss aspects of the geometric objects that are not fully depicted in the diagram [Netz 1999, 52–56]. In the first case, we are asked to *imagine* that certain straight lines represent meridian circles (14). In the next case, we imagine that the movement of the sphere is in a certain direction, whereas in the text the movement of the stars is effected by changing the position of the horizon, not that of the celestial sphere (280).[16] In the final three cases, we imagine that an object depicted in the plane of the figure is in fact in its proper place on the solid sphere (441, 486, 498). This is a standard idiom in Greek mathematical texts for directing the reader's attention to the solid objects which are the true subject of discussion and are only adumbrated by the diagram.

Geometric objects in the plane of the diagram are discussed prepositionally as being "in place of" objects on the sphere (مكان, 9, 14, 48, 56, 73, 81, 501, 514; بدل 403, 492). That these two prepositions indicate a relationship of representation is made clear in one place where the planar object is said to "substitute for" the solid object (تقوم مقام, 436). We have translated both of the prepositions as *representing* and the second phrase as *to stand in for*. Because it is often useful to distinguish between an object on the sphere and the object that stands in for it in the planisphere, we will introduce a special terminology for this purpose. Hence, we will speak of the *r*-ecliptic, *r*-horizon and *r*-meridian to refer to the circles and lines that represent these objects in the plane. It is, however, important to note that although Ptolemy sometimes distinguishes between an object on the sphere and the plane object that represents it, he often does not.

An object in the plane that we would call a projection is said to be a "correlate" of the solid object that it represents (نظير, 23, 49, 53, 57, 182, 432, 512). Properties that obtain on the sphere are said to obtain "in potential" on the plane (بالقوة, 52, 53, 71, 75, 91, 189, 433). The expression *bi-l-quwwa* often translates the Greek δυνάμει and occasionally κατὰ δύναμιν.[17] The former term is used by Ptolemy with a range of meanings to do with capacity, effect and function.[18] The later expression was used once in the *Almagest* to mean "in effect" [Heiberg 1898–1903, 275]. Whatever the original Greek, in this treatise the expression describes the way in which mathematical relationships between objects exist in the planisphere. For example, a circle in the planisphere is said to *functionally* bisect another when the line joining

[16] This is made explicit in *Planis.* 10.

[17] For κατὰ δύναμιν, see, for example, Thābit's translation of Nicomachus' *Introduction to Arithmetic* [Kutsch 1959, 41]

[18] Δυνάμει is also used twice in the *Almagest* with the specialized mathematical meaning of "equal in square" [Heiberg 1898–1903, 35]. This usage is clearly not that intended in the present text.

their intersections passes through the point on the planisphere that represents the center of the bisected circle, whereas in the plane this line is not actually a diameter nor the point a center. The Greek term *dynamei* was used by Aristotle, and many following him, to mean "potentially," and Hermann translated بالقوة with *potentia* [Heiberg 1907, 230 ff.]. Nevertheless, Ptolemy rarely uses the term in this way, and certainly it was not so intended in this text. In two cases, objects are asserted to be *functionally correlates* (53, 433). It is not clear from the context that there is any conceptual difference between these correlates and the others.

Since the treatise concerns the construction of a plane diagram of the sphere, one of its primary goals is to show that this representation is "consistent" with the sphere (موافق, 8, 46, 94, 269, 316, 399). The first time this term is used it seems to indicate a general congruence between the sphere and the planisphere, but in subsequent usage it becomes clear that the planisphere is said to be *consistent* when it can be used to generate the same numerical results as are found using solid geometry.

### *Non-technical terminology*

We have been less systematic in the translation of non-technical idioms. Because our aim was to render the work into good English while remaining faithful to our understanding of the meaning of the Arabic text, in many places our translation is not precisely literal.

The translation of the Arabic dual may be taken as an example of our general practice. Because Arabic has a dual, Arabic authors naturally, and necessarily, use it whenever two objects are discussed. English authors, however, only point out that there are two objects when this is somehow significant. Hence, when the reader can be assumed to know that we are discussing two objects, we render the dual with the simple plural. Consistently translating the dual as *two* adds an emphasis that we believe was not intended by the Arabic author.

Since in Arabic, as in any other language, most common words have multiple meanings whose difference significations are not always well expressed by a single English word, we have used different words to try to convey these different meanings. For example, both the verb وضع and the noun formed on the same root are used in related, but different, ways. The verb sometimes means to logically *assume* and other times to geometrically *set out*. The noun sometimes means something more concrete like *place* and sometimes something more abstract like *situation*. We have, hence, translated these words according to our understanding of the Arabic author's intent.

There is one interesting non-technical construction that warrants further comment. The Arabic author often refers to previous passages of this work with the expression قد تقدّمنا followed by ف and another verb, again in the perfect, secondperson plural (92, 93, 98, 111, 166, 216, 244, 270, 275, 320, 342).[19] The second verb expresses whatever we previously did, such as *proved* (بيّنّا), *set out* (وضعنا), *explained* (أوضحنا), and so forth. This construction almost certainly translates the Greek genitive absolute. Ptolemy was rather fond of the genitive absolute and we find it used in a number of different ways in his works that survive in Greek. One of these uses, however, is certainly that conveyed by the above expression. That is, he uses it to refer to material previously treated in the same work. We always translate this Arabic expression with some use of the word *previously*.

Brackets are used as follows. Square brackets, [ ], enclose explanatory additions not found in the Arabic text but which we believe are necessary to the argument. Square brackets are also used to enclose text that is not found in the MSS but which we have added to our edition of the Arabic. Parentheses, ( ), are added merely for clarity and enclose phrases that are found in the Arabic but which may be read as parenthetical.

## III The Structure of the Treatise

The Arabic treatise is presented as undivided, continuous prose and it is unlikely that the Greek original contained any formal divisions.[20] Nevertheless, certain clear shifts of topic are apparent in both the subject matter and Ptolemy's exposition. Accordingly, readers and editors of the text have introduced various divisions. There have been two significant proposals for dividing the text: (1) that found in Maslama's notes and (2) that in Heiberg's edition of Hermann's translation.[21] Although, in one case, Maslama's sectioning is preferable to Heiberg's, we have followed the latter, since this is the version of the treatise that is most likely to be compared with the present text.

In fact, however, neither of these divisions is entirely satisfactory. For example, Heiberg separates *Planis.* 4–7, whereas they obviously belong together, while Maslama joins *Planis.* 19 & 20, although they treat quite different subjects. Moreover, they both take *Planis.* 2 & 3 as separate sections, whereas *Planis.* 2 is clearly a lemma to *Planis.* 3 and it begins with a statement of what is demonstrated in *Planis.* 3. In order to help the reader navigate the text, and to provide a more detailed system of references for our commentary, we propose a new division of the text, while adhering to Heiberg's section numbers.

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[19] The قد is sometimes absent, but the structure of the rest of the expression is always the same.

[20] Although the medieval full stop, ⨀, is found at the end of some sections, it is missing at the end of others. Furthermore, it is found at the end of sentences that do not conclude sections and there is generally little agreement between the MSS. Indeed, the scribe of **I** used a mark very similar to this to fill space at the end of a line, with no stop intended.

[21] See Kunitzsch and Lorch [1994, 97] for a concordance of the numbering schemes.

A clear division of subject matter, as well as Ptolemy's remarks, allows us to separate the treatise into two parts. The first part introduces the role of the equator and the $r$-meridians, the construction of the $r$-ecliptic, the $r$-horizons, and the $r$-$\delta$-circles, and provides a computational treatment of rising-time phenomena. The second part introduces the construction of $r$-$\beta$-circles and addresses special topics in the practical implementation of the planisphere. We group sections together when they address a single, coherent topic and we treat them in a single section of our commentary. Finally, some sections contain significant, internal changes of subject. In order to refer specifically to these subtopics, we introduce subsection numbers. In the following list, we give the line numbers of the Arabic text for the sections and subsections. The divisions are also noted in the margins of the translation.

**Part I**

1.1 (Discussion, 4–8): General introduction to the project of the treatise.

1.2 (Description, 9–22): Description of the basic features of the planisphere, and the procedure for drawing the circle representing a great circle at a given inclination to the equator. The claim that such circles bisect the equator.

1.3 (Theorem, 23–29): Proof of this claim, using the $r$-ecliptic as an example.

1.4 (Description, 30–43): General claim that the planisphere preserves key mathematical features of the sphere. Description of how this works in the case of the relationship between the ecliptic and the equator.

1.5 (Description, 43–50): Description of how the features of the planisphere are used to divide the $r$-ecliptic into quadrants and signs.

2.1 (Discussion, 51–53): Enunciation for *Planis.* 2 & 3. (Repeated at the beginning of *Planis.* 3.)

2.2 (Theorem, 54–72): Lemma for *Planis.* 3. Proof that an $r$-meridian intersects the $r$-ecliptic at points corresponding to diametrically opposite points on the sphere.

3 (Theorem, 73–91): Proof that an $r$-horizon, drawn so as to bisect the equator, also *functionally* bisects the $r$-ecliptic. That is, their intersections correspond to points that are diametrically opposite on the sphere.

4.1 (Discussion, 92–94): Introduction to the next few sections and generally to *Planis.* 4–13.

4.2 (Metrical Analysis, 95–110): Analysis showing that if the absolute declination of a pair of equal $\delta$-circles is given, the radii of the corresponding $r$-$\delta$-circles have given ratios to the radius of the equator.

4.3 (Calculation, 111–131): Calculation of the radii of the $r$-tropics given the obliquity of the ecliptic, $\varepsilon$, and the radius of the equator. Calculation of the radius of the $r$-ecliptic and the distance of the center of the $r$-ecliptic from the center of the equator.

5 (Calculation, 132–143): Calculation of the radii of the $r$-$\delta$-circles that are 30° in celestial longitude from the solstitial points.

6 (Calculation, 144–155): Calculation of the radii of the $r$-$\delta$-circles that are 60° in celestial longitude from the solstitial points.

7 (Calculation, 156–171): Calculation of the radii of the $r$-$\delta$-circles tangent to the great circle of the horizon at 36° in terrestrial latitude.

8.1 (Description, 172–197): Description of how the features of the planisphere are applied to rising-time phenomena at the latitude of the equator.

8.2 (Calculation, 198–234): Calculation of the rising-times of the signs about the equinoxes (Pisces, Aries, Virgo, Libra) at the latitude of the equator. (The procedure in this calculation will also be used in *Planis.* 9, 12 & 13.)

8.3 (Metrical Analysis, 235–239): A metrical analysis giving a simpler way to calculate the rising-times of the signs at the latitude of the equator.

9 (Calculation, 240–269): Calculation of the rising-times of the remaining signs at the latitude of the equator.

10.1 (Description, 270–287): Description of how the features of the planisphere are applied to rising-time phenomena at the paradigm latitude of Rhodes, 36°.

10.2 (Theorem, 288–303): Proof that when the solstices are on the horizon, the $r$-horizon intersects the equator at points equidistant from the equinoxes.

10.3 (Description, 303–307): Description showing how the geometry of the planisphere makes it clear that the rising-times of equal arcs of the ecliptic about one and the same equinox are equal.

10.4 (Discussion, 308–314): Introduction of the arc of ascensional difference and the relationship between this arc and the length of daylight.

11 (Calculation, 315–337): Calculation, using the ascensional difference at the paradigm latitude of 36°, of the rising-times of the quadrants about the equinoxes and the time difference between the longest or shortest daylight and equinoctial daylight.

12 (Calculation, 338–367): Calculation, using the ascensional difference, of the rising-times of the signs on either sides of the equinoxes (Pisces, Aries, Virgo, Libra) at 36° latitude.

13 (Calculation, 368–397): Calculation, using the ascensional difference, of the rising-times of the of the remaining signs at 36° latitude.

**Part II**

14.1 (Discussion, 398–399): Summary of the results so far.

14.2 (Problem, 400–423): To construct the $r$-$\delta$-circles on an arbitrary plate with a given southernmost bounding circle.

15.1 (Discussion, 424–427): General introduction to *Planis.* 15–19.
15.2 (Problem, 428–433): Construction of the point representing the pole of the ecliptic.
15.3 (Description, 434–438): Description of circles representing great circles through the poles of the ecliptic.
16 (Problem, 439–458): To construct the circle that represents a given $\beta$-circle. Construction of the $r$-$\beta$-circle along with the $r$-$\delta$-circle that intersects it at the equinoctial colure. Proof that the $r$-$\beta$-circle intersects the $r$-$\delta$-circle.
17 (Theorem, 459–482): Proof that $r$-$\beta$-circles are non-concentric.

18 (Problem, 483–497): To construct an $r$-$\beta$-circle that extends beyond a southernmost bounding circle.

19 (Problem, 498–505): To construct the line representing the $\beta$-circle passing through the hidden pole.

20.1 (Discussion, 506–515): Introductory remarks on drawing on the plate a system of equatorial and ecliptic circles and lines. Summary of the relevant results established above.
20.2 (Discussion, 516–530): Practical methods for drawing the grid of circles and lines representing both the ecliptic and equatorial coordinate systems.

*Logical Structure*

Insofar as it develops theorems, computations and problems that are employed constructively as the work progresses, the *Planisphere* is a treatise of deductive mathematics. There are, however, no explicit, prefatory statements of the mathematical or astronomical assumptions, as is found in a number of the preserved works of Greek mathematical astronomy.

We exhibit the internal structure of the treatise in Table 1. The table shows that certain sections are used repeatedly (as *Planis.* 1, 4, 7 & 8), while others are isolated results (as *Planis.* 13 & 18). There are no series of theorems that lead successively to a final result, as we find in many Greek mathematical works. This means that there are no theorems that should be read as mere lemmas to the following theorems. Moreover, the division into two sections is also reflected in the structure. Whereas the sections of the first part show a strong dependence on the forgoing theorems, those in the second part are much more independent of the results of this treatise, relying instead on elementary geometry.

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
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| *Alm* | | | | • | • | • | • | | | | | | | | | | | | | |
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Table 1: The logical structure of the treatise. A section in the column headings is supported by each unit marked with a bullet in the row headings. The column denoted *Alm* refers specifically to *Alm.* I 14–16, while *CT* refers to the chord table methods set out in *Alm.* I 13.

At the same time that he produces the internal deductive framework exhibited in Table 1, Ptolemy assumes that the reader can provide justifications for steps in the geometric argument or computation on the basis of background knowledge in Euclidean geometry and the trigonometric methods of the chord table. In many cases, he seems to expect the reader to know that he is referring to specific propositions in the *Elements* or *Alm.* I. For example, he assumes the following toolbox: *Elem.* I, 10, 32, 47, III 3, 4, 21 (and its conv.), 26, 27, 28, 31 (and its conv.), 35 (and its conv.) & 36, IV 5, V 16, VI 1–4, 6, 17, XI 3, 19, and *Alm.* I 13-16.[22] As can be seen, this is a fair range of propositions from the *Elements* including theorems in plane geometry, ratio theory and its application to geometry, and solid geometry. Furthermore, Ptolemy assumes his reader is knowledgeable in chord-table trigonometry as set out in *Alm.* I 13 and has access to the right ascensions and declinations of the degrees of the ecliptic derived in *Alm.* I 14–16. There are also, however, a number of places where the steps in Ptolemy's argument cannot be justified by reference to a single theorem in the *Elements* but require the general knowledge of geometry that a Greek mathematical reader of the 2$^{\text{nd}}$ century could be assumed to posses. In these cases, in the notes to our translation we have supplied arguments along the lines of the ancient methods.

[22]The concept of the toolbox was introduced by Saito [1985], however, a good general overview is given by Netz [1999, 216–239].

# IV Text

بسم الله الرحمن الرحيم وصلّى الله على محمّد
كتاب بطلميوس من اهل قلاوذية
في تسطيح بسيط الكرة قال

[١]

إنّه لما كان من الممكن، يا سورا، وممّا ينتفع به في ابواب كثيرة أن يوجد في بسيط مسطح الدواير التي تقع في الكرة المجسمة كأنّها مبسوطة، رايت أنّ ممّا يجب في حق العالم أن اكتب لمن اراد معرفة ذلك كتابًا بايجاز ابيّن فيه على ايّ وجه يمكن أن ترسم دايرة الفلك المايل، والدواير الموازية لدايرة معدل النهار، والدواير المعروفة بدواير نصف النهار، على ما يصير به جميع ما يعرض في ذلك موافقًا لما يظهر في الكرة المجسمة.

وقد يتهيّأ لنا هذا الغرض الذي قصدنا له متى استعملنا مكان دواير نصف النهار خطوطًا مستقيمة، ورتّبنا الدواير الموازية لدايرة معدل النهار ترتيبًا يتهيّأ به اولا أن تكون الدواير العظيمة المرسومة من الدواير المايلة الماسة للدواير الموازية لدايرة معدل النهار، التي بعدها عن جنبتيها بعد سوا، تقطع ابدًا دايرة معدل النهار بنصفين. ويلتأم لنا ذلك على هذه الصفة.

نضع دايرة معدل النهار دايرة ا ب ج د، وأنّها حول مركز ه. ونخط فيها قطرين، يتقاطعان على زوايا قايمة، وهما خط ا ج وخط ب د. ونتوهّم هذين الخطين مكان دواير نصف النهار. وأنّ نقطة ه القطب الشمالي، لأنّ القطب الاخر لا يمكن وضعه في بسيط مسطح، اذ كان بسيطه يمتدّ الى ما لا نهاية له، كما سنبيّن ذلك فيما بعد. واذا كان القطب الشمالي هو الظاهر في بلداننا دايمًا، فالأولا أن نستعمله خاصّة فيما نريد رسمه.

ومن البيّن أنّ الدواير الموازية لمعدل النهار، التي هي اميل الى الشمال من دايرة معدل النهار، ينبغي أن ترسم داخل دايرة ا ب ج د، والدواير الموازية، التى هي اميل الى الجنوب، يجب أن ترسم خارجًا عنها. فنخرج خطي ا ج ب د، ونفصل من الدايرة عن جنبتي نقطة ج قوسين متساويتين، وهما ج ز ج ح. ونصل خط د ط ز وخط د ح ك. ونجعل نقطة ه مركزًا، ونرسم ببعد خط ه ط وببعد خط ه ك دايرة ط ل ودايرة ك م.

فاقول إنّ هاتين الدايرتين هما نظيرتا الدايرتين من الدوير التي في الكرة المجسمة عن جنبتي دايرة معدل النهار، بعدهما منها بعد سوا، وأنّ دايرة الفلك المايل، التي ترسم على مركز يقطع خط ط م بنصفين حتى تماس هاتين الدايرتين على نقطة ط وعلى نقطة م، تقسم دايرة

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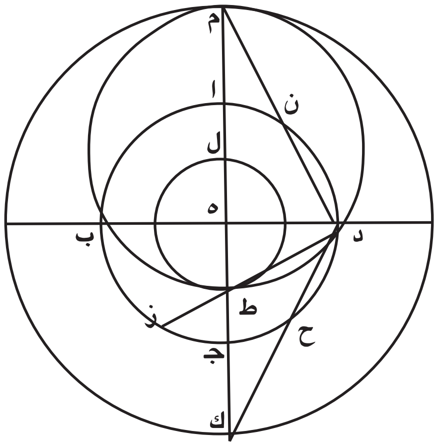
2 بطلميوس] بطليموس **T**. 12 ويلتأم] ونلتأم **T**. 13 نضع] نضع ان **T**. ه] و **I**. 14 مكان] فكان **I**. 20 ج] د **TI**, *corr.* **Ana**. 21 متساويتين] متساويين **TI**. 22 ك م] طم **TI**, *corr.* **T Ana**. 24 بعد] بعدًا **I**.

ا ب ج د بنصفين، اعني أنّها تمر بنقطة ب ونقطة د. برهان ذلك أن نصل خط د ن م، فلأنّ قوس ا ن مساوية لقوس ج ح، التي هي مساوية لقوس ج ز، تكون قوس ن د ز نصف دايرة. فزاوية م د ط اذًا قايمة، والدايرة التي ترسم على قطر ط م من مثلث م د ط القايم الزاوية تمر بنقطة د. فهي اذًا تقسم دايرة معدل النهار بنصفين.

فقد تبيّن من ذلك أنّا في جميع الدواير الموازية لمعدل النهار اذا فصلنا عن جنبتي نقطة ج قسيًا يكون مقدارها بحسب بعد كل واحد من هذه الدواير من دايرة معدل النهار، ووصلنا اطراف القسي بنقطة د بخطوط مستقيمة، وجعلنا ما تفصله الخطوط المستقيمة من خط ه ك ابعادًا، وجعلنا نقطة ه مركزًا، وادرنا دواير، كان القياس في ذلك على هذا المثال الذي وضعناه.




ومن البيّن أنّا وإن وضعنا كل واحدة من قوسي ز ج ج ح ثلثًا وعشرين درجة واحدى وخمسين دقيقة بالتقريب، بالدرج التي بها دايرة معدل النهار وهو دايرة ا ب ج د ثلثماية وستين درجة، وهي البعد فيما بين دايرة معدل النهار وبين كل واحدة من دايرتي المنقلبين في الدايرة المرسومة على قطبي دايرة معدل النهار، كانت دايرة ط ل، من الدايرتين المرسومتين على نقطة ط وعلى نقطة م، دايرة المنقلب الصيفي ودايرة ك م دايرة المنقلب الشتوي. وعلى هذا المثال تكون الدايرة المرسومة على نقطة م ونقطة ب ونقطة ط ونقطة د، وهي الدايرة التي تمر في وسط البروج، تماس دايرتي المنقلبين على نقطة ط، وهي المنقلب الصيفي، وعلى نقطة م، وهي المنقلب الشتوي، وتقسم دايرة معدل النهار بنصفين على نقطتي ب د. فتكون نقطة ب نقطة الربيع، ونقطة د نقطة الخريف، لأنّ حركة الكل إنّما هي كأنّها من نقطة ب نحو نقطة ا ثم الى نقطة د. الّا أنّ قسمة دايرة الفلك المايل الى البروج لا يمكن أن تقع على

27 ا ن] ار I. 28 م د ط[1]] مكط TI, *corr.* T Ana. ط م] طر TI, *corr.* Ana. م د ط[2]] مكط TI, *corr.* T Ana. 29 تمر] تم ثم I. د] ب TI, *corr.* Ana. 36 المنقلبين] المنقلبتين I. 38 م] ل TI. 40 المنقلبين] المنقلبتين I. 43 قسمة] القسمة I.

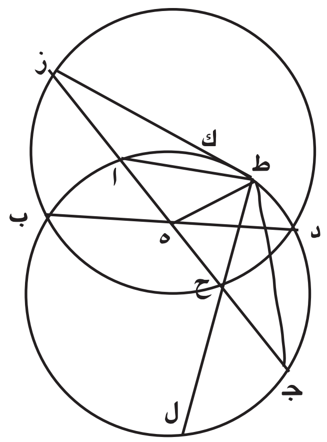
قسي متساوية ولا قسمته الى اربعة اجزا ايضًا تقع على قسي متساوية كلها. لكن إنّما قسمته على ما ينبغي على هذا المثال فقط، اعني أن تجعل أوايل البروج على النقط التي عليها تقسم الدواير الموازية لمعدل النهار، التي ترسم بالطريق الذي قد أوضحنا على البعد الموافق لبعد كل واحد من البروج من دايرة معدل النهار في الكرة المجسمة، دايرة فلك البروج. فأنّ بهذه الدرجة وحده تكون جميع الخطوط المستقيمة التى تجاز على قطب ه مكان دواير نصف النهار تمر من دايرة الفلك المايل على الاجزا التى هي نظاير الاجزا المتقابلة على القطر في الكرة المجسمة.

[٢]

وتكون جميع دواير الافق، التى ترسم على مثل ما رسمت دايرة الفلك المايل، ليس إنّما تقسم دايرة معدل النهار فقط بنصفين، لكنّها تقسم ايضًا دايرة فلك البروج بنصفين بالقوة، اعني أنّها ايضًا ترسم على الاجزا التي هي بالقوة نظاير الاجزا المتقابلة على القطر في الكرة المجسمة.

فلتكن دايرة معدل النهار دايرة ا ب ج د حول مركز ه. والدايرة التي تمر في وسط البروج دايرة ز ب ح د، وتقطع دايرة معدل النهار بنصفين على نقطة ب ونقطة د. ونجيز على قطب ه مكان دايرة نصف النهار خطًا مستقيمًا، كيف ما وقع. وليكن خط ز ا ه ح ج.

فاقول إنّ نقطتي ز ح نظيرتان للنقطتين اللتين هما متقابلتان على القطر في الكرة المجسمة، اعني أنّ الدواير الموازية لدايرة معدل النهار التي ترسم على هاتين النقطتين تفصل قوسين متساويتين عن جنبتي دايرة معدل النهار على المثال الذي وصفناه، كما أنّ ذلك شي يعرض في الكرة المجسمة ايضًا.




45 اعني أن تجعل ... دايرة فلك البروج] *cited* **Mas**, قد *omit.* **Mas**. 51 دواير] دواير دواير **I**.
55 ز ب ح د] رحك **I**. د] ك **I**. 56 ز ا ه ح ج] ز ا ح ج **I**, ز ا ح ج *with* ه *in marg.* **T**.
59 متساويتين] متساوين **I**.

برهان ذلك أن نخرج من نقطة ه خطًا مستقيمًا على زوايا قايمة على خط ا ج، وهو خط ه ط. ونصل خط ا ط وخط جـ ط وخط ز ك ط وخط ط ح ل. فمن البيّن أنّ زاوية ا ط جـ قايمة، وذلك أنّ قوس ا ط جـ نصف دايرة. ولأنّ ضرب خط ز ه في خط ه ح مثل ه د في مثله، اعني أنّه مثل ه ط في مثله، تكون نسبة خط ز ه الى خط ه ط كنسبة خط ه ط الى خط ه ح. [فـ]يكون مثلث ز ط ح ايضًا قايم الزاوية، وزاوية ز ط ح هي القايمة. فزاوية ز ط ل اذًا مساوية لزاوية ا ط جـ. فاذا سقطنا زاوية ا ط ح مشتركة، صارت زاوية ك ط ا الباقية مساوية لزاوية ح ط جـ الباقية. فقوس ك ا ايضًا مساوية لقوس جـ ل. فقد بيّنّا أنّ خطي ط ك ز ط ل لما وصلا بطرفي القوسين اللتين بعدهما من دايرة معدل النهار بعد سوا، وكان مخرجهما من النقطة التي بعدها من نقطة ا ونقطة جـ الربع، وهي نقطة ط، اخذنا في خط ز جـ نقطة ز ونقطة ح، وهي النقط التي عليها ترسم الدايرتين الموازيتين لدايرة معدل النهار اللتين بعدهما عنها بعد سوا. ولذلك، يكون خط ز ه ح قد مرّ بالنقط التي هي بالقوة على قطر دايرة الفلك المايل.

[٣]

واقول إنّا وإن رسمنا دايرة اخرى مايلة عن دايرة معدل النهار مكان دايرة الافق حتى تكون هذه الدايرة تقسم دايرة معدل النهار وحدها بنصفين، كان موضعا تقاطع هذه الدايرة والدايرة التي تمر في وسط البروج متقابلتين على القطر بالقوة، اعني أنّ الخط الذي يصل بينهما يمر بمركز دايرة معدل النهار.

فلتكن ايضًا دايرة معدل النهار دايرة ا ب جـ د حول مركز ه، ودايرة فلك البروج دايرة ح ب ط د، ولتقسم دايرة معدل النهار بنصفين على قطر ب ه د. ودايرة الافق دايرة ح ا ط جـ، وتقسم هذه الدايرة ايضًا دايرة معدل النهار بنصفين على قطر ا ه جـ. وليكن التقاطع المشترك لدايرة فلك البروج ودايرة الافق نقطة ح ونقطة ط. فاقول إنّا إن وصلنا نقطة ح بمركز ه بخط مستقيم مكان دايرة نصف النهار، وأخرجنا ذلك الخط على الاستقامة، صار الى نقطة ط.

برهان ذلك أنّا نصل خط ح ه، ونخرجه على الاستقامة حتى يقطع دايرة الافق، وهي دايرة ح ا جـ، على نقطة ط. فاقول أنّ نقطة ط مشتركة لدايرة فلك البروج ايضًا، وهي دايرة

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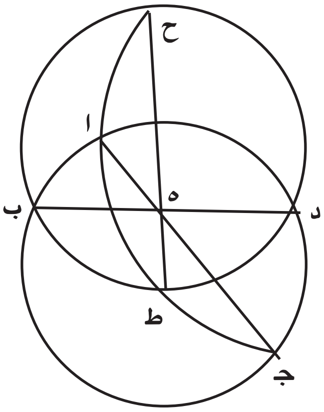
62 ط ح ل] د ط ح *in marg.* **T**. 63 ه ح] ه جـ **I**. 65 فيكون] يكون **TI**. 66 اذًا] *omit.* **I**. 67 ح ط جـ] جـ ط ل **I**, صح ح ط ج *in marg.* **T**. 70 النقط] النقطة **I**. 71 بالنقط] بالنقطة **TI**. 74 موضعا] موضعي *corr. to* موضعا **T**. 75 يصل] يتصل **I**. 77 مركز ه] مركزه **TI**. دايرة ح ب ط د] دايرة *omit.* **I**, حيطك **TI**, *corr.* **T Ana**. 78 ب ه د] بهك **TI**, *corr.* **Ana**. 80 وصلنا] فصلنا **I**. 82 أنّا نصل ... على نقطة ط] *cited* **Mas**, خط ح ه: خط ه ح **Mas**, الاستقامة: استقامة **Mas**, وهي دايرة ح ا جـ *omit.* **Mas**. 82 ح ه] جـ ه **TI**, *corr.* **Ana**.

ح ب ط د. فلأنّه قد اخرج في دايرة ح ا ط ج خطا ح ط ا ج متقاطعان على نقطة ه، يكون خط ح ه في خط ه ط مثل خط ا ه في خط ه ج، وكذلك خط ا ه في خط ه ج مثل خط ب ه في خط ه د. فخط ب ه اذًا في خط ه د مثل خط ح ه في خط ه ط. فخطا ب د ط ح اذًا في دايرة واحدة. فيجب من ذلك أن تكون نقطة ط على دايرة فلك البروج، وهي دايرة ح ب ط د، وقد كنّا وصفنا أنّها على دايرة الافق، وهي دايرة ح ا ط ج. فالخط الذي يصل بين موضعي تقاطع دايرة فلك البروج ودايرة الافق هو خط قد يمر بمركز دايرة معدل النهار، وهو نقطة ه. فقد تبيّن من ذلك أنّ دايرة الافق ودايرة فلك البروج يتقاطعان على نقطتين متقابلتين على القطر بالقوة. وذلك ما اردنا أن نبيّن.




[٤]

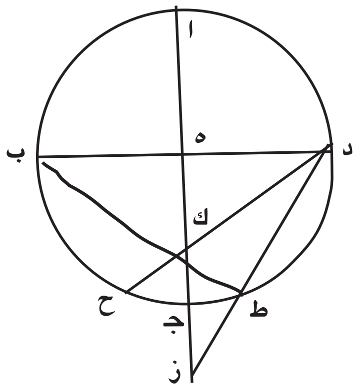
فاذ قد تقدّمنا فبيّنّا ذلك، فلننظر الآن الى ما نسبة انصاف اقطار الدواير المتوازية، التي ترسم على بروج دايرة الفلك المايل، الى نصف قطر دايرة معدل النهار، التي تقدّمنا فوضعناها حتى نعلم أن مطالعها توجد بالعدد ايضًا موافقة لما يظهر في الكرة المجسمة.

فلتكن ايضًا دايرة معدل النهار دايرة ا ب ج د حول مركز ه. ونخرج منها قطرين يتقاطعان على زوايا قايمة، وهما ا ج ب د. ونخرج خط ا ج على الاستقامة الى نقطة ز. ونفصل عن جنبتي نقطة ج قوسين متساويتين، وهما ج ح ج ط. ونصل خط د ك ح وخط د ط ز. وقد تقدّمنا فأوضحنا أنّ الدواير الموازية لدايرة معدل النهار، التي بعدها عنها بعد سوا، ما كان منها

84 ح ب ط د] حطك **T**, جبطك **I**, *corr.* **Ana**. خطا] *omit.* **I**. متقاطعان] فيتقاطعان **T**. 85 ح ه] ج ه **TI**, *corr.* **Ana**. وكذلك] ولذلك **TI**. 88 ح ب ط د] هطك **TI**, *corr.* **T Ana**. وهي] وهذه **I**. 89 هو] وهو **TI**. 90 يتقاطعان] سقاضعتان **I**. 92 نسبة] نشبة **TI**, proportio **Her**. 94 المجسمة] المستقيمة **TI**, in spera corpora applanes et decliui (applanos, a planis *in other* MSS) **Her**. 95 منها] منهما **I**. 96 زوايا] زوا **I**. ا ج ب د] اح يد **I**. ونفصل ز.] ونفصل **I**. 97 متساويتين] متساويين **TI**. 98 بعد سوا] بعد ما سوا **I**.

اميل الى الشمال فإنّما رسم على مركز ه وببعد ه ك، وما كان منها اميل الى الجنوب فعلى مركز ه وببعد ه ز.

ويتبيّن لنا أنّ نسبة خط ه ز الى خط ه ك على هذا المثال. لأنّ قوس ج ح مساوية لقوس ج ط فقوس ب ح، وقوس ب ج ط، مجموعين هما نصف دايرة. فالزوايا المقابلة لها، اعني زاوية ه د ك وزاوية ه د ز، مجموعين مساويتين لزاوية قايمة. وزاوية ه د ك مع زاوية ه ك د ايضًا قايمة، فزاوية ه د ز مساوية لزاوية ه ك د. فمثلث ز ه د اذًا القايم الزاوية شبيه بمثلث د ه ك القايم الزاوية، فنسبة خط ز ه الى خط ه د كنسبة خط د ه الى خط ه ك. لكن نسبة قوس ب ط الى ما بقي من نصف الدايرة، اعني القوس المساوية لقوس ب ح، كنسبة زاوية ه د ز الى زاوية ه ز د، وكنسبة القوس التي على خط ه ز من الدايرة المرسومة على مثلث د ه ز القايم الزاوية الى القوس التي على خط ه د من هذه الدايرة بعينها. فنسبة الخط المستقيم الذي يوتر قوس ب ط الى الخط الذي يوتر ما بقي من نصف الدايرة، اعني قوس ب ح، كنسبة خط ز ه الى خط ه د وكنسبة خط د ه الى خط ه ك.




فاذ قد تقدّمنا فحصّلنا ذلك، فنضع اولا في مثل هذه الصورة كل واحد من قوسي ج ح ج ط ثلثًا وعشرين درجة واحدى وخمسين دقيقة وعشرين ثانية، بالدرج التي بها دايرة ا ب ج د ثلثماية وستين درجة، وهي الدرج التي وضعنا أنّها البعد فيما بين دايرة معدل النهار وبين كل واحدة من دايرتي المنقلبين في كلامنا في الكرة المجسمة ايضًا.

فتكون قوس ب ط ماية وثلثة عشر درجة واحدى وخمسين دقيقة وعشرين ثانية، بالدرج التي بها هذه الدايرة ثلثماية وستين درجة، وقوس ب ح ما بقي من نصف الدايرة وهو ست

99 ه ك] ه ك هر I. 105 ه د] ه ك TI, *corr.* **T Ana.** د ه] ك ه TI, *corr.* **T Ana.** 107 ه ز د] ه ز ك TI, *corr.* **T Ana.** ه ز] ه د TI, *corr.* **T Ana.** 108 ه د] ه ك TI, *corr.* **T Ana.** 110 كنسبة[1]] حا[ا]شـ[ـيـ]ـة، لانّ نسبة ب ط الى ط د كنسبة جبن زاوية ب د ط اعني خط ه ز الى جبن زاوية د ب ط اعني خط د ه *in marg.* **T**. 112 ج ط] خط **I**. دقيقة وعشرين ثانية] وعشرين ثانية *omit.* **I**, دقيقة وعشرين ثانية *above* **T**. 115 ب ط] رط **I**. 116 وقوس] وقوس وقوس **I**.

وستون درجة وثمان دقايق واربعون ثانية· والخط المستقيم الذي يوتر قوس ب ط مایة جز وثلث وثلثون دقيقة وثماني وعشرون ثانية، بالاجزا التي بها القطر مایة وعشرون جزًا فقد وضعنا ذلك في كتاب المجسطي، والخط الذي يوتر ب ح خمسة وستون جزًا من هذه الاجزا وتسع وعشرون دقيقة· نسبة خط ز ه اذًا الى خط ه د ونسبة خط ه د الى خط ه ك هي نسبة مایة جز وثلث وثلثين دقيقة وثمان وعشرين ثانية الى خمسة وستين جزًا وتسع وعشرين دقيقة· ولذلك خط ه د، الذي هو نصف قطر دايرة معدل النهار، بالاجزا التي هو بها ستين جزًا بتلك الاجزا يكون نصف قطر دايرة المنقلب الشتوي، وهو خط ه ز، اثنان وتسعون جزًا وثمان دقايق وخمس عشرة ثانية، ونصف قطر دايرة المنقلب الصيفي تسعة وثلثين جزًا واربع دقايق وتسع عشرة ثانية·

وقد وضح من ذلك أنّ قطر دايرة فلك البروج، اذا كانت تماس هاتين الدايرتين بطرفي قطرها، هو نصف قطريهما جميعًا، وهو مایة واحدى وثلثون جزًا واثنتا عشر دقيقة واربع وثلثون ثانية، بالاجزا التي يكون بها نصف قطر دايرة معدل النهار ستين جزًا، وأنّ نصف قطر دايرة فلك البروج خمسة وستون جزًا وست وثلثون دقيقة وسبع عشرة ثانية· والخط الذي بين مركزه ومركز دايرة معدل النهار يكون ستة وعشرون جزًا من هذه الاجزا واحدى وثلثين دقيقة وثمانيا وخمسين ثانية·

[٥]

ونضع ايضًا كل واحدة من قوسي ح جـ جـ ط عشرين درجة وثلثين دقيقة وتسع ثواني، وهو البعد فيما بين دايرة معدل النهار وبين الدايرتين الموازيتين لمعدل النهار اللتين تفصلان من الدايرة التي تمر بوسط البروج عن جنبتي نقطتي المنقلبين ثلثين درجة، حتى يكون قوس ب ط مایة وعشر درجات وثلثين دقيقة وتسع ثواني، والخط المستقيم الذي يوترها ثمانية وتسعين جزًا وخمسًا وثلثين دقيقة وسبعًا وخمسين ثانية، وقوس ب ح تسعًا وستين درجة وتسعًا وعشرين دقيقة واحدى وخمسين ثانية، والخط المستقيم الذي يوترها ثمانية وستين جزًا وثلث وعشرين دقيقة واحدى وخمسين ثانية· فنسبة خط ز ه اذًا الى خط ه د ونسبة خط ه د ايضًا الى خط ه ك، هي نسبة ثمانية وتسعين جزًا وخمس وثلثين دقيقة وسبع وخمسين ثانية الى ثمانية وستين جزًا وثلث وعشرين دقيقة واحدى وخمسين ثانية· فالاجزا التي بها

120 ه د[2] ] هك **TI**, *corr.* **T Ana**. 122 ولذلك] وكذلك **I**. ه د] هك **TI**, *corr.* **T Ana**. بالاجزا] فالاجزا **I**. 123 ه ز] هو **TI**, *corr.* **T Ana**. وتسعون] وسبعون **I**. 127 قطرها] قطر بها **TI**, *corr.* **T**. 128 التي يكون] التي بها نكون **I**. 130 ستة وعشرون] سبعة وعشرون **TI**, XXVI **Her**. 132 جـ ط] خط **I**. وثلثين ... ثواني] وتسعًا وثلثين دقيقة **TI**, puncta XXX secundas IX **Her**. 133 تفصلان] يفصلان **T**. 135 وثلثين ... ثواني] وتسعًا وثلثين دقيقة **TI**, puncta XXX secundas IX **Her**. يوترها] توترها **I**. 137 وتسعًا وعشرين] واحدى وعشرين **TI**, XXIX **Her**. 138 وثلث] وستا **I**, وثنا **T**. ه د] ه ك **TI**, *corr.* **T Ana**. 139 ه د] ه ك **TI**, *corr.* **T Ana**.

يكون خط ه د ستين جزًا، يكون بها خط ه ز ستة وثمانين جزًا وتسعًا وعشرين دقيقة واثنين واربعين ثانية، وخط ه ك يكون واحدًا واربعين جزًا من هذه الاجزا وسبعًا وثلثين دقيقة وخمس عشرة ثانية.

[٦]

وعلى هذا المثال، نضع كل واحدة من قوسي ح جـ و جـ ط احدى عشرة درجة وتسعًا وثلثين دقيقة وتسعًا وخمسين ثانية، وهي البعد في الدايرة العظمى التي ترسم على قطبي دايرة معدل النهار فيما بين دايرة معدل النهار وبين الدايرتين الموازيتين لها اللتين يفصلان ستين درجة من الدايرة التي تمر بوسط البروج عن جنبتي نقطتي المنقلبين. فتكون جملة قوس ب ط مائة درجة ودرجة وتسعًا وثلثين دقيقة وتسعًا وخمسين ثانية، والخط المستقيم الذي يوترها ثلثة تسعين جزًا ودقيقتين واربع عشرة ثانية، وتكون قوس ب ح ثمانيًا وسبعين درجة وعشرين دقيقة، والخط المستقيم الذي يوترها خمسة وسبعين جزًا وسبعًا واربعين دقيقة وثلثًا وعشرين ثانية. فنسبة خط ز ه الى خط ه د ونسبة خط د ه الى خط ه ك هي نسبة ثلثة وتسعين جزًا ودقيقتين واربع عشرة ثانية الى خمس وسبعين جزًا وسبع واربعين دقيقة وثلث وعشرين ثانية، والاجزا التي يكون بها خط ه د ستين جزًا، بها يكون خط ه ز ثلثة وسبعين جزًا وتسعًا وثلثين دقيقة وسبع ثواني، وخط ه ك ثمانية واربعين جزًا من هذه الاجزا واثنين وخمسين دقيقة.

[٧]

وكذلك إن جعلنا كل واحدة من قوسي ح جـ و جـ ط اربعًا وخمسين درجة، وهي بعد الدايرتين الموازيتين لمعدل النهار اللتين يماسها الافق الذي في إقليم رودس، وهو الافق الذي مثّلنا به في الكرة المجسمة، عن جنبتي دايرة معدل النهار، كان في هذا ايضًا قوس ب ط مائة واربعًا واربعين درجة، والخط الذي يوترها مائة واربعة عشر جزًا وسبع دقايق وسبعًا وثلثين ثانية، وقوس ب ح ستًا وثلثين درجة، والخط الذي يوترها سبعة وثلثين جزًا واربع دقايق وخمسًا وخمسين ثانية. ونسبة خط ز ه الى خط ه د وخط د ه الى خط ه ك هي نسبة مائة واربعة عشر جزًا وسبع دقايق وسبع وثلثين ثانية الى سبعة وثلثين جزًا واربع دقايق وخمس وخمسين ثانية. فالاجزا التي بها يكون خط ه د ستين جزًا، بها يجتمع خط ه ز ايضًا مائة واربعة وثمانين جزًا وتسعًا وثلثين دقيقة وثمانيًا واربعين ثانية، ويكون خط ه ك تسعة عشر جزًا من هذه الاجزا وتسعًا وعشرين دقيقة واثنين واربعين ثانية. ومن البيّن أنّه لما كان هذين

141 ه د] ه ك **TI**, *corr.* **T Ana**. 144 جـ ط] خط جـ ط **I**. 146 دايرة] *in marg.* **I**. 149 درجة] دقيقة, *above* درجة **I**. 153 خط ه د] يكون خط ه د **TI**. 156 وكذلك] ولذلك **TI**. جـ ط] خط جـ ط **I**. اربعًا وخمسين] خمسًا واربعين **TI**, LIIII **Her**. 159 وسبع] وتسع **TI**, VII **Her**. 163 ه ز] ه د **TI**, *corr.* **T Ana**.

الخطين اذا جمعا كانا قطر الافق الذي تقدّمنا فوضعناه، كما يكون من قطري دايرتي المنقلبين قطر دايرة فلك البروج، صار هذا القطر مايتين واربعة اجزا وتسع دقايق وثلثين ثانية، بالاجزا التي بها يكون قطر دايرة معدل النهار ماية وعشرين جزًا. ويجب من ذلك أن يكون نصف قطر دايرة الافق ماية جز وجزين واربع دقايق وخمس واربعين ثانية، ويكون الخط الذي بين مركز هذه الدايرة ودايرة معدل النهار اثنين وثمانين جزًا من هذه الاجزا وخمسًا ثلثين دقيقة وثلث ثواني. وذلك ما اردنا أن نبيّن.

[٨]

واذ قد وضعنا ذلك فلنبيّن أن في مثل هذه الصورة ايضًا يرى مقادير المطالع وجميع ما يعرض فيها على مثال ما بيّنّا في الكرة المجسمة.

فلتكن دايرة معدل النهار دايرة ا ب جـ د حول مركز ه، والدايرة التي تمر بوسط البروج دايرة ز ب ح د حول نقطة ط. ونخرج قطرين يمران بنقطة ه التي هي مركز دايرة معدل النهار ومكان دايرة نصف النهار. احدهما يمر بالتقاطع الذي على نقطتي ب و د، وهما نقطتي الاستوا، وهو خط ب ه د. والاخر يمر بمركز دايرة البروج، وهو خط ز ط ه ح، فيحدث نقطتي المنقلبين، وهما ز و ح.

وليكن قصدنا اولا أن نبيّن ما يطلع في الكرة المستقيمة من دايرة معدل النهار مع اجزا الدايرة التي تمر في وسط البروج. فلأنّ الافق في الكرة المستقيمة وضعه وضع دايرة نصف النهار، والخطوط المستقيمة في هذه الصورة التي تجاز على قطب دايرة معدل النهار، وهي نقطة ه، هي نظاير دواير نصف النهار، فمن البيّن أنّ كل واحدة من قوسي ز ب ح د، وهما ربعا دايرة الفلك المايل، يطلع مع كل واحدة من قوسي ا ب جـ د، وهما ربعا دايرة معدل النهار، ويتوسّطان السما معها ويغربان معها، لأنّ خط ب د في دايرة ز ب ح د يقسمه نصف القطر، وهو خط ط ح، بنصفين وعلى زوايا قايمة على نقطة ه.

فنفصل من الدايرة التى تمر بوسط البروج قوسين متساويتين، وهما قوس ب ك وقوس د ل، ونجيز خط ك م ه ن وخط ل س ه ع. فاذ كنّا قد بيّنّا أن نقطتي ك ل ونقطتي ع ن تمر بها الدواير المتوازية، التي بعدها عن جنبتي دايرة معدل النهار بعد سوا، حتى أنّ نقطة ك هي مقابلة لنقطة ن بالقوة، ونقطة ل مقابلة لنقطة ع.

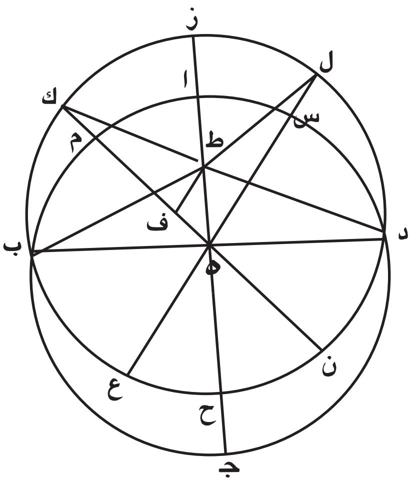
168 بها يكون] يكون بها I. 169 وخمس واربعين ثانية] وخمس ثواني TI, secundas XLV Her. 172 واذ ... في الكرة المجسمة] *cited* **Mas**, وضعنا: وضفنا **Mas**, أن *omit.* **Mas**, ايضًا: إنّه **Mas**, مثال *omit.* **Mas**. 174 فلتكن] فليكن **I**. 176 ومكان] وفكان **I**. ب و د] ب و د **TI**, *corr.* **Ana**. 178 ز و ح] ب و د **T**. 182 ز ب ح د] رب جـ د **I**. 183 ا ب جـ د] ا ب ح د **I**. 186 متساويتين] متساويين **T**. وهما] وهما وهما **I**. قوس] قوسا **TI**. 187 ل س ه ع] سهع **TI**. ع ن] ع ز **TI**, *corr.* **Ana**. 188 تمر] يمر **I**. 189 ن] ب **TI**, *corr.* **Ana**.

إن نحن وضعنا اولا أنّ قوس ب ك برج الحوت، فمن البيّن أنّ قوس ل د تكون برج الميزان، وعلى هذا المثال قوس ب ع تحتوى على برج الحمل وقوس ن د تحتوى على برج السنبلة. لكنّا إن وصلنا خطوط ك ط ل ط، كان مثلث ك ه ط مساوي الاضلاع والزوايا لمثلث ل ه ط، فزاوية ك ه ط مساوية لزاوية ل ه ط، والزوايا الباقية، اعني زاوية ك ه ب وزاوية ل ه د، مساوية بعضها لبعض وللزوايا المقابلة لها. فاذ كانت هذه الزاوية عند مركز دايرة معدل النهار، فأنّ القسي ايضًا التي تطلع من دايرة معدل النهار مع كل واحد من البروج التي وضعناها مساوية بعضها لبعض. فإن نحن وجدنا مقدار واحدة من هذه القسي، كأنّا وجدنا مقدار قوس م ب، فقد حصّلنا مع ذلك ما كنّا في طلبه من المطالع.

فنخرج من نقطة ط عمودًا على خط ك ه، وليكن خط ط ف. فاذ قد بيّنّا أنّ الاجزا التي بها نصف قطر دايرة معدل النهار ستين جزًا يكون خط ط ك، وهو نصف قطر الدايرة التي تمر في وسط البروج، خمسة وستين جزًا وست وثلثين دقيقة وسبع عشرة ثانية، وخط ه ط، وهو الخط الذي بين مركز هذه الدايرة ومركز دايرة معدل النهار، ستة وعشرين جزًا واحدى وثلثين دقيقة وثمان وخمسين ثانية، وخط ه ك، وهو نصف قطر الدايرة الموازية لدايرة معدل النهار التي ترسم على راس الحوت وراس العقرب، اعني التي تمر بنقطتي ك ل، يكون من هذه الاجزا ثلثة وسبعين جزًا وتسع وثلثين دقيقة وسبع ثواني، فمثلث ه ط ك معلوم.

فإن اضفنا الى خط ك ه مربع ك ط في مثله منقوص منه مربع ط ه في مثله حدث فضل خط ك ف على خط ف ه. الّا أنّ كل دايرتين يتقاطعان، وتقسم الدايرة العظمى منهما الدايرة الصغرى بنصفين، كم كان مقدار الدايرتين، فأنّ مربع نصف قطر الدايرة العظمى في مثله منقوص منه [مربع] الخط الذي بين مركزيهما [في مثله] يحدث عنه مربع نصف قطر الدايرة الصغرى في مثله. وذلك أنّا على مثال ما فعلنا في هذه الدواير إن وصلنا خط ب د، كان الخط الذي يوصل بين موضعي التقاطع يمر بمركز ه في الدايرة الصغرى. فاذ كانت زاوية د ه ط قايمة، فأنّ خط ط د في مثله، وهو الخط الذي يوتر الزاوية، مساوٍ لما يجتمع من خطي ط ه و ه د كل واحد في مثله. فيجب من ذلك أن يكون فضل [مربع] نصف قطر الدايرة التي تقسم دايرة معدل النهار بنصفين [في مثله]، كم كان مقدارها، عند [مربع] الخط الذي بين مركزيهما [في مثله] ثلثة الف وستماية جز من الاجزا التي بها يكون نصف قطر دايرة معدل النهار ستين جزًا.

190 ب ك] نك هر **TI**, *corr.* **Ana**. 196 مساوية] مساويًا **I**, مساو **T**. 200 خمسة وستين] خمسة وخمسين **TI**, LXV **Her**. 203 الحوت] الجوزا **TI**, piscium **Her**. 205 حدث] حدت **I**. 208 منقوص ... في مثله] si tetragonus distantie centrorum subtrahatur **Her**; مربع, في مثله *omit.* **TI**. 209 ب د] طد **TI**, *corr.* **Ana**. 210 فاذ] فاذا **I**. 211 مساوٍ] مساويًا **TI**. 212 مربع] *omit.* **TI**. 213 في مثله] *omit.* **TI**. مربع] *omit.* **TI**. 214 في مثله] *omit.* **TI**.




**T** has ز and ن transposed. There are also two extraneous objects neither mentioned in the text nor reproduced here. A point ق is placed between point م and the intersection of lines ب ط and ك ه, and a line joins it to point ط.

ومن قبل أنّ خط ه ك ايضًا يحسب ما تقدّمنا فوضعناه، هو ثلثة وسبعون جزًا من هذه الاجزا وتسع وثلثون دقيقة وسبع ثواني، إن اضفنا الى ذلك الفضل، وهو ثلثة الف وستماية جز، حصل لنا فضل خط ك ف عند خط ف ه، وهو ثمانية واربعون جزًا من هذه الاجزا واثنتان وخمسون دقيقة واثنتان واربعون ثانية. فاذ انقصنا ذلك من ثلثة وسبعين جزًا وتسع وثلثين دقيقة وسبع ثواني واخذنا نصف ما بقي، وهو اربعة وعشرون جزًا وست واربعون دقيقة وخمس وعشرون ثانية، كان خط ه ف اثنا عشر جزًا وثلث وعشرون دقيقة واثنا عشرة ثانية، بالاجزا التي وضعنا أنّ خط ه ط بها ستة وعشرون جزًا واحدى وثلثون دقيقة وثماني وخمسون ثانية. فالاجزا التي بها خط ه ط، وهو الخط الذي يوتر زاوية ه ف ط القايمة، ماية وعشرين جزًا بها يكون خط ه ف ايضًا خمسة وعشرين جزًا وثلثين دقيقة بالتقريب، والقوس التي يوترها خمسًا وخمسين درجة واربعين دقيقة، بالدرج التي بها تكون الدايرة التي حول مثلث ه ط ف القايم الزاوية ثلثماية وستين درجة. فزاوية ه ط ف، التي هي مثل زاوية ف ه ب، اذ كانت زاوية ط ه ب ايضًا قايمة، بالدرج التي بها تكون زاويتين قايمتين ثلثماية وستين درجة، بها تكون خمسًا وخمسين درجة واربعين دقيقة، وبالدرج التي بها يكون اربع زوايا قايمة ثلثماية وستين درجة، بها تكون هذه الزاوية سبعًا وعشرين درجة وخمسين دقيقة. ولما كانت هذه الزاوية عند مركز دايرة معدل النهار، صارت قوس ب م ايضًا سبعًا وعشرين درجة [وخمسين دقيقة]، بالدرج التي بها دايرة معدل النهار ثلثماية وستين درجة.

216 ه ك] هط **TI**, *corr.* **Ana**. 217 الفضل] المفضل **I**. 224 خمسة … بالتقريب] partes LV cum punctis fere LIX **Her**. 225 يوترها] توترها **I**. 226 ثلثماية] وثلثماية **I**. 227 اذ] اذا **I**. 228 تكون] يكون **I**. 229 ثلثماية] وثلثماية **I**. هذه] هذه *in marg.* **T**. 231 وخمسين دقيقة] *omit.* **TI**, cum puncta L **Her**.

فقد تبيّن لنا من ذلك على مثل ما بيّنّا في الكرة المجسمة أنّ كل واحد من البروج التي عند نقطة الاستوا، اعني الحوت والحمل والسنبلة والميزان، في وضع الكرة المستقيمة يطلع مع هذه السبع وعشرين درجة وخمسين دقيقة من دايرة معدل النهار.

وقد كان يمكننا أن نبيّن ذلك بقول اسهل من هذا على هذا المثال، خط ك ه في ه ن مساوٍ لخط ب ه في ه د وخط ب ه في ه د ثلثة الف وستماية جز. فاذا نحن قسمنا ذلك على خط ك ه، كان خط ه ن معلومًا. لكنّ خط ك ه يفضل على خط ه ن بمثل خط ه ف مرتين، فخط ه ف ايضًا معلوم. وخط ط ه معلوم، والزاوية القايمة التي عند نقطة ف معلومة، فزاوية ه ط ف اذًا معلومة.

[٩]

فنضع ايضًا في مثل هذه الصورة قوس ب ك من دايرة فلك البروج قوسي برجين حتى تكون نقطة ك راس الدلو ونقطة ل راس القوس، وقبالة هذه النقط على القطر تكون نقطة ن راس الاسد ونقطة ع راس الجوزا. فمن البيّن أنّا إن بيّنّا كم مقدار قوس ب م من دايرة معدل النهار، فأنّا قد عرفنا الازمان التي فيها يطلع في الكرة المستقيمة كل واحد من البروج التي تقدّمنا فوضعناها.

ومن البيّن ايضًا أنّ خطي ك ط ط ه يبقي مقدارهما على ما كان، وخط ك ه يزداد مقداره. من قبل أنّا قد بيّنّا أنّ نصف قطر الدايرة الموازية لدايرة معدل النهار، التي ترسم على راس القوس وراس الدلو، هو [ستة] وثمانون جزًا وتسع وعشرون دقيقة واثنتان واربعون ثانية، بالاجزا التي بها يكون نصف قطر دايرة معدل النهار ستين جزًا، فإن قسمنا اجزا الفضل، وهي ثلثة الف ستماية جز، على خط ك ه، اعني إن قسمناها على ستة وثمانين جزًا وتسع وعشرين دقيقة واثنين واربعين ثانية، خرج لنا فضل خط ك ف على خط ف ه واحدًا واربعين جزًا من هذه الاجزا وسبعًا وثلثين دقيقة وخمس عشرة ثانية. واذا نقصنا ذلك من ستة وثمانين جزًا وتسع وعشرين دقيقة واثنين واربعين ثانية، واخذنا نصف ما يبقي، وهو اربعة واربعون جزًا واثنتان وخمسون دقيقة وسبع وعشون ثانية، خرج لنا خط ف ه اثنين وعشرين جزًا وست وعشرين دقيقة وثلثة عشر ثانية بالتقريب، بالاجزا التي كان بها خط ط ه ستة وعشرين جزًا واحدى وثلثين دقيقة وثمان وخمسين ثابية. فالاجزا التي يكون بها خط ط ه الذي يوتر الزاوية القايمة ماية وعشرين جزًا، بها يكون خط ه ف ماية جز وجز وثمان وعشرين دقيقة، والقوس التي يوترها ماية وخمسة عشر درجة وثمان وعشرين دقيقة، بالدرج

235 يمكننا] يمكنا I. 239 فزاوية … معلومة] cited Mas, ودلك ما اردنا أن نبيّن add. Mas. 240 ب ك] ك ل TI, $\overline{bk}$ Her, corr. T. 241 تكون[2]] يكون I. 247 هو] وهو TI. ستة وثمانون] ثمانون TI, LXXXVI (36 in one MS) Her. واثنتان] واسان I. 249 جز] جزوٍ I. 250 وعشرين] وىلىں T, وىلثں I. واحدًا] واحدى TI. 252 وهو] وهو وهو I.

التي تكون بها الدايرة التي حول مثلث ه ط ف القايم الزاوية ثلثماية وستين درجة. فتكون زاوية ه ط ف، التي هي مثل زاوية ف ه ب، ماية وخمسة عشر درجة وثمان وعشرين دقيقة، بالدرج التي بها تكون زاويتين قايمتين ثلثماية وستين درجة. فامّا الدرج التي يكون بها اربع زوايا قايمة ثلثماية وستين درجة، فتكون سبع وخمسين درجة واربع واربعين دقيقة. ولما كانت هذه الزاوية عند مركز دايرة معدل النهار، صارت قوس ب م ايضًا سبعًا وخمسين درجة واربع واربعين دقيقة.

فإن نقصنا من ذلك مطالع البروج التي عند نقطتي الاستوا، التي بيّنّا أنّها سبع وعشرين درجة وخمسين دقيقة، خرج لنا الازمان الباقية، تسع وعشرين زمانًا واربع وخمسين دقيقة، وهي التي [فيها] يطلع في الكرة المستقيمة كل واحد من هذه البروج، اعني الدلو والثور والاسد والعقرب. ومن البيّن أنّ كل واحد من هذه الاربعة البروج الباقية، اعني القوس والجدي والجوزا والسرطان، يطلع في الازمان التي تبقي من قوس ربع واحد، وهو تسعون زمانًا، وهي اثنان وثلثون زمانًا وستة عشر دقيقة. وذلك موافق لما بيّنّاه في الكرة المجسمة.

[١٠]

ويتبع ذلك أن ننظر هل يتهيّأ في الكرة المايلة ايضًا تلك المطالع باعيانها التي تقدّمنا فذكرناها من مطالع البروج على ما في هذه الصورة.

ونستعمل ايضًا على طريق المثال الدايرة الموازية لمعدل النهار التي استعملناها في كتاب المجسطي، اعني الدايرة التي تمر بجزيرة رودس. وارتفاع القطب الشمالي في هذه الدايرة عن الافق ست وثلثون درجة. وامّا الافق الذي يرسم بالدواير الموازية لمعدل النهار، التي بعدها البعد الذي تقدّمنا فبيّنّاه، فأنّ نصف قطره ماية جز وجزان واربع دقايق وخمس واربعون ثانية، بالاجزا التي بها يكون نصف قطر دايرة معدل النهار ستين جز، ويكون الخط الذي بين مركز دايرة هذا الافق ودايرة معدل النهار اثنين وثمانين جزًا بهذه الاجزا وخمسًا وثلثين دقيقة وثلث ثواني.

فنجعل دايرة معدل النهار ا ب جـ د حول مركز ه، ودايرة البروج ز ب ح د حول مركز ط. ونتوهّم حركة الكرة، اذ كانت نقطة ه قد وضعت القطب الشمالي، كأنّها من نقطة د نحو نقطة جـ ثم الى نقطة ب ثم الى نقطة ا. ونرسم اولا من هذه الدايرة من دواير الافق قوسين تمران بنقطتي المنقلبين، وهما نقطتي ز و ح. ولتكونا قوسي ز ك ح ل ز م ح ن. فمن البيّن أنّه متى كان الافق وضعه وضع قوس ز ك ح ل، كان ما على نقطة ز وعلى نقطة ك

258 فتكون] فيكون I. 259 درجة] *omit.* I. 261 فتكون] فيكون I. 266 فيها يطلع] فيها *omit.* TI, تطلع I. 275 وخمس واربعون ثانية] واربعون ثانية I, واربع واربعون ثانية T, secunde XLV **Her.** 280 القطب] القطبا I. 282 تمران] يمران T. ز و ح] جـ روح **TI**, *corr.* **Ana.** ولتكونا] وليكونا T. ز ك ح ل] زطحل **TI**, *corr.* **Ana.** 283 ز ك ح ل] رطح ل **TI**, *corr.* **Ana.**

طالعًا وما على نقطة ح وعلى نقطة ل غاربًا، ومتى كان وضعه وضع قوس ز م ح ن، كان الامر بخلاف ذلك، اعني أنّ ما كان على نقطتي ن و ح فهو طالع وما كان على نقطتي م و ز فهو غارب، اذ كانت حركة الكرة إنّما هي كأنّها من نقطة د نحو نقطة جـ، وكان قطب ه قد وضع انّه ظاهر ابدًا.

فاذ كنّا قد بيّنّا أنّ الدايرة التي تمر بوسط البروج ليس هي وحدها تقسم دايرة معدل النهار بنصفين لكن تقسمه معًا بنصفين دواير الافق التي ترسم على هذا المثال، فيجب من ذلك أن يكون الخطان المستقيمان اللذان فصلا فيما بين مواضع التقاطع، اعني خط ك ل وخط م ن، يمران بمركز ه. فمن البيّن أنّ قوس ك ا ايضًا من دايرة معدل النهار مساوية لقوس جـ ل، وقوس ا م مساوية لقوس جـ ن.

ولكن قوس م ا ايضًا مساوية لقوس ك ا. وذلك أنّا إن جعلنا مركزي دايرة الافق على هذين الوضعين، نقطة س ونقطة ع، ووصلنا خطوط س ه ع ه س ط ع، فكان خط س ط ع مستقيمًا وعلى زوايا قايمة على خط ز ح، وكان خط س ه على زوايا قايمة على خط ك ل، وخط ع ه على زوايا قايمة على خط م ن، وكانت الخطوط التي تخرج اليها اعمدة من المراكز تقسّمها الاعمدة بنصفين، صارت اضلاع كل واحد من مثلثي ه ط ع ه ط س مساوية لاضلاع الاخر، وصارا قايمتين الزاويتين، وزاوية ط ه ع من مثلث ه ط ع هي المساوية لزاوية ط ه س من مثلث ه ط س. ولكن زاوية م ه ع مساوية لزاوية ك ه س، وذلك أنّ كل واحدة منهما قايمة. فزاوية م ه ا اذًا الباقية مساوية لزاوية ك ه ا الباقية. ولذلك صارت قوس م ا مساوية لقوس ك ا.

فالقوي اذًا التي تبتدىء من نقط ك م ل ن وتنتهي الى نقطتي ا جـ متساوية. والقسي ايضًا التي تبتدىء من حيث ذكرنا وتنتهي الى نقطتي ب د متساوية. ولما كانت قوس ب ح تطلع مع قوس ب ن وقوس ز ب مع قوس ك ب، وهذه القوس مساوية لقوس ب ن، ولذلك قوس د ز ايضًا تطلع مع قوس د ك وقوس ح د مع قوس د ن، وهذه القوس مساوية لقوس د ك، فقد تبيّن من ذلك ايضًا أنّ قسي الدايرة التي تمر في وسط البروج التي بعدها من نقطة واحدة بعينها من نقطتي الاعتدال بعد سوا تطلع في ازمان متساوية.

وايضًا لما كانت قوس ز ب ناقصة عن مطالع الكرة المستقيمة بقوس ك ا، وقوس ح د، وهي القوس المقابلة لهذه القوس على القطر، تفضل على مطالع الكرة المستقيمة بقوس جـ ن، وهذه

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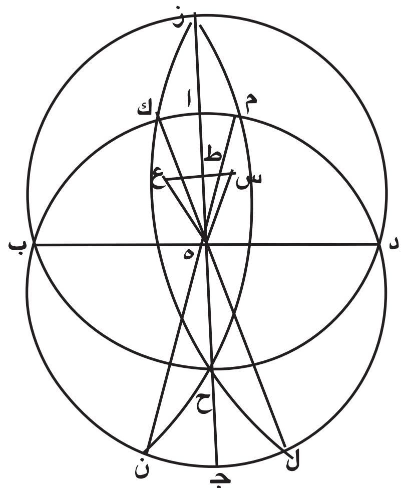
285 ن و ح] نوح **TI**, *corr.* **Ana.** 286 م و ز] م ر ن **TI.** 287 ظاهر] ظاهرًا **I.** 291 جـ ل] جـ ا **TI**, *corr.* **Ana.** 295 ز ح، وكان] ز ح ن كان **TI.** 296 م ن] كن **TI**, *corr.* **Ana.** 297 بنصفين] بنصفين نصفين **TI**, *corr.* **Ana.** 302 فالقوي اذًا التي] فالشي اذًا الذي **TI.** متساوية] مساويت **I.** 303 التي] الذي **I.** 305 د ز] د ن **TI**, *corr.* **Ana.** ح د] جـ د **TI**, *corr.* **Ana.** د ن] در **I.** 307 بعينها] بعينها *in marg.* **T**, نفسها **I.** تطلع] يطلع **I.**




القوس مساوية لقوس كا، وكانت نقطة ح نقطة المنقلب الصيفي، فمن البيّن أنّ في هذه الصورة ايضًا يكون مقدار نقصان قسي دايرة البروج التي عند نقطة الربيع عن المطالع التي في الكرة المستقيمة بمقدار زيادة القسي المساوية لها المقابلة لها على القطر على هذه المطالع باعيانها. وممّا يسهّل معرفته مع بيان ذلك أنّ اقصر ما يكون [من] النهار ينقص عن نهار الاستوا بقوسي كا و نج، واطول ما يكون من النهار يزيد على نهار الاستوا بهاتين القوسين.

[١١]

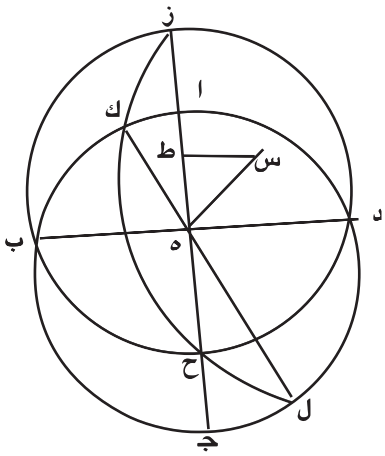
فاذ قد عرفنا ذلك فلننظر في هذا الإقليم الذي وضعناه هل يوجد ولا فضل ما بين اطول ما يكون من النهار او اقصر ما يكون منه وبين نهار الاستوا موافقًا لما يعرض في الكرة المجسمة.

فنضع صورة مثل هذه الصورة فيها الافق الذي يمر بنقط ز ك ح ل وحده. وليكن غرضنا أن نجد مقدار قوس كا. فنجعل ايضًا مركز دايرة الافق الذي هذه حاله نقطة س. ونصل خطي س ط س ه، فيكونان عمودين على خطي زح كل لما تقدّمنا فبيّنّاه. فمن اجل أنّا قد بيّنّا أنّ خط ه س، وهو الخط الذي بين مركز دايرة معدل النهار وبين مركز دايرة افق هذا الإقليم الذي وضعناه، هو اثنان وثمانون جزًا وخمس وثلثون دقيقة وثلث ثواني، بالاجزا التي وضع أنّ خط ه ط، وهو الخط الذي بين مركز هذه الدايرة وبين مركز دايرة فلك البروج، بها ستة وعشرين جزًا واحدى وثلثون دقيقة وثمان وخمسون ثانية، فالاجزا اذًا التي يكون بها خط ه س، وهو الخط الذي يوتر الزاوية القايمة،ماية وعشرين جزًا، بها يكون خط ه ط ايضًا

313 من] *omit.* **TI**. 314 على نهار الاستوا] على الاستوا **I**. 315 ولا] اولا **TI**. 316 او اقصر ما] او اقصر **I**.
318 ز ك ح ل] ز ك ل **I**. 319 هذه] *in marg.* **T**. 321 مركز دايرة افق] دايرة الافق **I**.

ثمانية وثلثين جزًا وثلثًا وثلثين دقيقة بالتقريب، والقوس التي عليه سبعًا وثلثين درجة [وثلثين دقيقة]، بالدرج التي بها تكون الدايرة التي حول مثلث ه س ط القايم الزاوية ثلثماية وستين درجة. فزاوية ط س ه اذًا، التي هي مثل زاوية ا ه ك، تكون سبعًا وثلثين درجة وثلثين دقيقة، بالدرج التي بها تكون زاويتين قايمتين ثلثماية وستين درجة، وتكون ثمان عشرة درجة وخمسًا واربعين دقيقة، بالدرج التي بها يكون اربع زوايا قايمة ثلثماية وستين درجة. ولما كانت هذه الزاوية عند مركز دايرة معدل النهار، صارت قوس ا ك ايضًا ثمان عشرة درجة وخمسًا واربعين دقيقة.

فكل واحد اذًا من الربعين اللذين عند نقطة الربيع يكون واحدًا وسبعين زمانًا وخمس عشرة دقيقة، وكل واحد من الربعين اللذين عند نقطة الخريف يكون ماية وثمانية ازمان وخمسًا واربعين دقيقة. ولذلك يكون فضل ما بين اطول ما يكون من النهار واقصر ما يكون منه وبين نهار الاستوا سبعة وثلثين زمانًا وثلثين دقيقة، ويكون من ساعات الاستوا ساعتين ونصف على مثل ما بيّنّا في الكرة المجسمة.




[١٢]

ولكن نجد مطالع البروج ايضًا في هذا الإقليم الذي وضعناه. نضع على هذا المثال دايرة معدل النهار والدايرة التي تمر في وسط البروج حول قطري ب د و ز ح، ونفصل من الدايرة التي تمر في وسط البروج قوس ب ط.

ولتكن قوس برج واحد اولا، ومن البيّن أنّه برج الحوت. ونصل خط ط ه ل، ونذير دايرة الافق الذي تقدّمنا فوضعناه مارة بنقطتي ط ل. ولتقطع دايرة معدل النهار علي نقطتي م ن.

326 [2] وثلثين دقيقة] *omit.* **TI**, cum punctis XXX **Her**. 327 تكون] يكون **I**. ثلثماية] وثلثماية **I**. 328 ط س ه] طهس **TI**, *corr.* **Ana**. 333 اللذين] الّذين **I**. 339 ب د] ب ك **TI**, *corr.* **Ana**. 341 ولتكن] وليكن **I**. ومن] من **I**.

ونصل خط م ه ن، ونخرج ايضًا من نقطة س، وهي مركز دايرة الافق، خطين مستقيمين، وهما س ه و س ط، ونخرج منها عمودًا على خط ط ل، وهو خط س ع.

وقد بيّنّا قبيل أنّ قوس ك م هي التي بها ينقص برج الحوت وبرج الحمل من الجانبين عن مطالعهما في الكرة المستقيمة، ويفضل بها برج السنبلة وبرج الميزان على مطالعهما في الكرة المستقيمة.

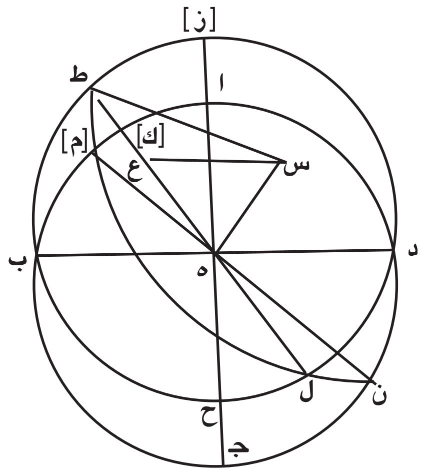
وقد بيّنّا أنّ خط ه ط، وهو نصف قطر الدواير الموازية لدايرة معدل النهار التي ترسم على راس الحوت، هو ثلثة وسبعون جزًا تسع وثلثون دقيقة وسبع ثواني، بالاجزا التي وضع بها خط ه س، وهو الخط الذي بين مركز دايرة معدل النهار ومركز دايرة الافق، اثنين وثمانين جزًا وخمسًا وثلثين دقيقة وثلث ثواني، وأنّ فضل مربع خط ط س في مثله على مربع خط ه س في مثله ثلثة الف وستماية جز من هذه الاجزا. واذا قسمنا ذلك على مثال ما فعلنا قبل على ثلثة وسبعين جزًا وتسع وثلثين دقيقة وسبع ثواني، وفعلنا ما يتلوا ذلك كما فعلنا في الكرة المستقيمة، خرج لنا خط ه ع اثني عشر جزًا وثلثًا وعشرين دقيقة واثنتي عشرة ثانية، بالاجزا التي بها يكون خط ه س اثنين وثمانين جزًا وخمس وثلثين دقيقة وثلث ثواني. فالاجزا التي يكون بها خط ه س، وهو الذي يوتر الزاوية القايمة،ماية وعشرين جزًا، بها يكون خط ه ع ثمانية عشر جزًا ودقيقة بالتقريب، والقوس التي عليه سبع عشرة درجة وست عشرة دقيقة، بالاجزا التي بها تكون الدايرة التي حول مثلث ه س ع ثلثماية وستين درجة. فزاوية ه س ع ايضًا، التي هي مثل زاوية ك ه م، تكون سبعة عشر درجة وست عشر دقيقة، بالدرج التي بها تكون زاويتين قايمتين ثلثماية وستين درجة، وتكون ثمان درج وثمان وثلثين دقيقة، بالدرج التي بها يكون اربع زوايا قايمة ثلثماية وستين درجة. فقوس ك م ايضًا هي ثمان درج وثمان وثلثين دقيقة، بالدرج التي بها تكون دايرة معدل النهار ثلثماية وستين درجة.

وقد كانت مطالع كل واحد من الاربعة البروج التي وضعناها في الكرة المستقيمة سبعة وعشرين زمانًا وخمسين دقيقة. فاذا نقصنا منها هذه الثمانية الازمان والثمان والثلثين الدقيقة، خرج لنا مطالع كل واحد من برجي الحوت والحمل تسعة عشر زمانًا واثنتي عشرة دقيقة. فاذا زدنا على هذا ذلك بعينه، خرج لنا مطالع كل واحد من برجي السنبلة والميزان ستة وثلثين زمانًا وثمان وعشرين دقيقة.

[١٣]

فنضع ايضًا في مثل هذه الصورة قوس ب ط قوس برجين، اعني الحوت والدلو، حتى يقرّ ساير ما ذكرناه على حاله.

344 منها] منهما **TI**, *corr.* **Ana**. 349 هو] وهو **TI**. 354 اثني] اثنا **TI**. جزًا وثلثًا] جزًا من هذه الاجزا وثلثًا **TI**. واثنتي] واثنتا **TI**. 357 عليه] عليها **I**. 360 تكون] يكون **I**. وتكون] ويكون **I**. 361 بها يكون] بها يكون بها **I**. درجة] جزًا **TI**, *corr.* **Ana**.




Three labels mentioned in the text are missing in **T**.

فيكون ه ط، وهو نصف قطر الدايرة الموازية لدايرة معدل النهار التي ترسم على راس الدلو، ستة وثمانين جزًا من هذه الاجزا وتسعًا وعشرين دقيقة واثنين واربعين ثانية. واذا قسم ذلك على ثلثة الف وستماية جز على مثال ما ذكرنا فيما تقدّم، خرج لنا خط ه ع اثنين وعشرين جزًا وستًا وعشرين دقيقة وثلث عشرة ثانية، بالاجزا التي بها يكون خط ه س اثنين وثمانين جزًا وخمسًا وثلثين دقيقة وثلث ثواني. والاجزا ايضًا التي بها يكون خط ه س، وهو الخط الذي يوتر الزاوية القايمة، ماية وعشرين جزًا، بها يكون خط ه ع اثنين وثلثين جزًا وستًا وثلثين دقيقة بالتقريب، والقوس التي عليه احدى وثلثين درجة واثنين وثلثين دقيقة، بالدرج التي بها تكون الدايرة التي ترسم حول مثلث ه س ع القايم الزاوية ثلثماية وستين درجة. فزاوية ه س ع اذًا، التي هي مثل زاوية ك ه م، هي احدى وثلثون درجة واثنتان وثلثون دقيقة، بالدرج التي بها تكون زاويتان قايمتان ثلثماية وستين درجة، وهو خمس عشرة درجة وستة واربعون دقيقة، بالدرج التي بها يكون اربع زوايا قايمة ثلثماية وستين درجة. فقوس ك م اذًا، وهي الفضل المشترك بين مطالع البرجين اللذين وضعناهما وبين المطالع في الكرة المستقيمة، تكون خمس عشرة درجة وستًا واربعين دقيقة، بالدرج التي بها تكون دايرة معدل النهار ثلثماية وستين درجة.

وقد كانت مطالعها في الكرة المستقيمة سبعًا وخمسين زمانًا واربع واربعين دقيقة، فإن نقصنا هذه الخمس عشرة درجة والست والاربعين دقيقة من سبع وخمسين درجة واربع واربعين دقيقة، خرج لنا مطالع الحوت ومطالع الدلو مجموعين واحدًا واربعين زمانًا وثمان

370 فيكون] ويكون **TI**. 371 ستة] لستة **TI**. 372 جز] وتسع واربعين دقيقة *add.* **TI**, *corr.* **Ana**. 378 اذًا] *omit.* **I**. 379 تكون] يكون **TI**. 380 بالدرج] بالاجزا **I**. 381 ك م] ه ك م **I**. 382 تكون[1]] يكون **I**. 383 درجة] جزًا **TI**, *corr.* **Ana**. 385 دقيقة] الدقيقة **T**. سبع وخمسين] خمس واربعين **I**. 386 خرج] خرجت **TI**. واحدًا] واحدى **I**.

وخمسين دقيقة. وامّا مطالع الدلو وحده، فيكون اثنين وعشرين زمانًا وستًّا واربعين دقيقة، من قبل أنّ الحوت كان يطلع في تسعة عشر زمانًا واثنتي عشرة دقيقة. وإن زدنا الخمس عشرة الدرجة والست والاربعين الدقيقة على سبع وخمسين درجة واربع واربعين دقيقة، خرج لنا مطالع الاسد ومطالع السنبلة، ويكون مطالعهما اذا جمعت ثلثة وسبعين زمانًا وثلثين دقيقة. فامّا مطالع الاسد وحده، فتكون سبعة وثلثين زمانًا ودقيقتين، من قبل أنّ السنبلة ايضًا كانت تطلع في ستة وثلثين زمانًا وثمان وعشرين دقيقة. ومن البيّن أنّ الثور ايضًا يطلع في ازمان مساوية لازمان طلوع الدلو، وهي اثنان وعشرون زمانًا وست واربعون دقيقة، والعقرب يطلع في ازمان مساوية لازمان طلوع الاسد، وهي سبع وثلثون درجة ودقيقان، وأنّ طلوع كل واحد من الجدي والجوزا يكون في الازمان الباقية في هذا الربع، وهي تسعة وعشرون زمانًا وسبع عشرة دقيقة، وأنّ طلوع كل واحد من السرطان والقوس يكون في الازمان الباقية من هذا الربع ايضًا، وهي خمسة وثلثون زمانًا وخمس عشرة دقيقة، على ما يليق بغرضنا المتقدّم.

[١٤]

فقد بيّنّا أنّ في هذه الصورة ايضًا التي في بسيط مسطح يكون الامر في مطالع بروج الدايرة التي تمر في وسط البروج وجميع ما يتبع ذلك موافقًا لما بيّنّاه في الكرة المجسمة.

ولكن نجعل الصورة على حسب مقدار الموضع المعلوم الذي نريد أن نرسم فيه ما ذكرناه، وحتى يتهيّأ لنا فيه أن نرسم وضع الكواكب الثابتة، إن اردنا ذلك.

وإن اردنا أن نضع فيه الشي الذي يسمّي خاصّة في آلات الساعات العنكبوت، فأنّا نضع الدايرة التي تكون خارج الدواير كلها واعظمها دايرة ا ب ج د حول مركز ه. ونخط بدل دواير نصف النهار قطرين يتقاطعان على زوايا قايمة. وليكونا خطي ا ج ب د. ونفصل من عند نقطة د قوس د ز. وليكن مقدارها بمقدار بعد الدايرة الموازية لدايرة معدل النهار التي وضعناها عن القطب الجنوبي في الكرة المجسمة. ونخرج من نقطة ج خطًا موازيًا لخط ه د، وليكن خط ج ح. ونصل خط د ز ح، ونخرج من نقطة ح عمودًا على خط ه د، وهو خط ح ط.

فاقول إنّا إن فعلنا مثل ما فعلنا فيما تقدّم حتى نفصل من عند نقطة ج قوسًا، هي بعد كل واحدة من الدواير الباقية الموازية لدايرة معدل النهار الى جانب الموافق، ووصلنا خطوطًا مستقيمة من نقطة د وبين اطراف القسي التي نفصل مثل خط د ك ج مثلًا إن كان غرضنا

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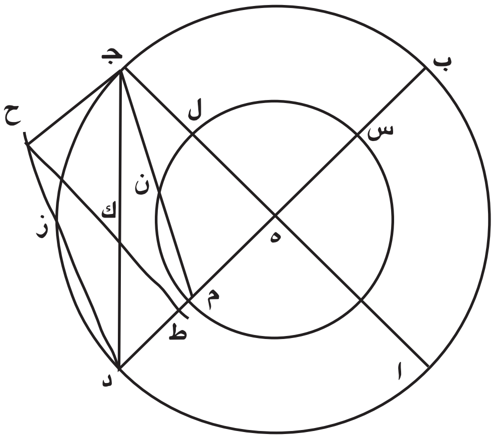
388 تسعة] اثنتي **TI**, *corr.* **Ana**. 393 مساوية] متساوية **TI**. واربعون] وثلثون **TI**, *corr.* **Ana**. 397 زمانًا] *omit.* **I**. 399 بيّنّاه] ذكرناه **I**. 400 ولكن ... إن اردنا ذلك] *cited* **Mas**, فيه *omit.* **Mas**, وضع: مواضع **Mas**. 400 ذكرناه] ذكرنا **T**. 401 لنا فيه أن] لنا ان **I**. 403 بدل] *omit.* **I**. 407 وليكن] ولتكون **T**. 408 ح ط] خط **I**.

أن نرسم دايرة معدل النهار، وجعلنا خط ه ل مساويًا لخط ط ك، وجعلنا نقطة ه مركزًا، وادرنا ببعد مساوٍ لبعد ه ل دايرة ل م س، كان وضع هذه الدايرة وضع دايرة معدل النهار.

وكل واحدة من الدواير الباقية على هذا المثال، وذلك أنّا إن فعلنا عكس ذلك على ما بيّنّا فيما تقدّم حتى نضع دايرة معدل النهار دايرة ل م س ونرسم في سطحها الدايرة الموازية لها التي بعدها عنها الى الجنوب بقدر قوس شبيهة بقوس ج ز، كانت دايرة ا ب ج د هي الدايرة التي نرسم. فنصل خط م ج وليقطع دايرة ل م، وهي دايرة معدل النهار، على نقطة ن حتى تكون دايرة ا ب ج د مرسومة على مركز ه وببعد ه ج، كما فعلنا اخيرًا.

فاقول إنّ قوس م ن شبيهة بقوس د ز. برهان ذلك أنّ نسبة خط د ه الى خط ه ج كنسبة خط د ط الى خط ط ك، وخط د ه مساوٍ لخط ه ج، فخط د ط ايضًا مساوٍ لخط ط ك. ولكن خط ط ك مساوٍ لخط ه م، فخط د ط ايضًا مساوٍ لخط ه م. وخط ط ح ايضًا مساوٍ لخط ه ج ومواز له، فخط د ز اذًا موازٍ لخط م ن. فزاوية ه م ن مساوية لزاوية ه د ز، فقوس س ل ن اذًا شبيهة بقوس ب ج ز، فقوس م ن الباقية شبيهة بقوس د ز الباقية.




There are corrections to the diagram in **T**. The figure was originally labeled, and probably drawn, by a copyist at an orientation about 135° clockwise from the copyist of the text. The configuration of triangle ج ح د in the original drawing was incorrect. The original, ruled lines were then crossed out; these are omitted from the reproduction. Lines ح د and ح ط were added by freehand. The labels ك and ز were probably added at this time, since they are oriented about 20° counterclockwise from the text. The orientation of the MS labels is not reproduced.

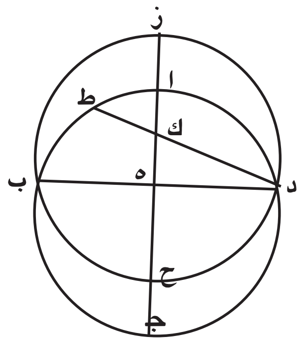
412 ط ك] هك **TI**, *corr.* **Ana**. 412 نقطة] نقطة نقطة **I**. 417 ن] ل **TI**, *corr.* **Ana**. 420 كنسبة خط] لخط **TI**. 422 د ز] د ن **TI**, *corr.* **Ana**. ه د ز] ه د ن **TI**, *corr.* **Ana**. 423 س ل ن] سكن **I**. شبيهة ... د ز الباقية] *cited as* شبيهة بقوس ج ز، وذلك ما اردنا أن نبيّن **Mas**.

[١٥]

وينبغي ايضًا أن نتمم غرضنا بأن نبيّن كيف نرسم الدواير التي حالها عند الدايرة التي تمر في وسط البروج كحال الدواير التي تقدّم ذكرها عند معدل النهار، حتى يمكنا أن نضع الكواكب التي رصدت وعرفت مواضعها بقياسها الى هذا الفلك من غير أن نستعمل اولا الاضلاع التي لها بقياسها الى دايرة معدل النهار.

فلتكن اولا دايرة معدل النهار من الدواير التي توضع في الصفيحة دايرة ا ب جـ د حول مركز ه، ودايرة البروج دايرة ز ب د، والخط المستقيم الذي يمر بالقطبين جميعًا خط ز ا ه ح جـ، وخط الذي يمر بموضع التقاطع من دايرة معدل النهار خط ب ه د. فإن نحن فصلنا قوس ب ط وجعلناها مساوية للقوس التي بين قطبي دايرة معدل النهار والدايرة التي تمر في وسط البروج، ووصلنا خط د ك ط، كانت نقطة ك نظيرة قطب الدايرة التي تمر في وسط البروج بالقوة. ومن البيّن أنّ ذلك يكون بحسب ما اوضحنا.

وكانت الدواير التي تمر بهذه النقطة وبالنقط التي هي متقابلة على القطر في الدايرة التي تمر في وسط البروج قايمة لدايرة معدل النهار ايضًا بنصفين. وتكون هذه الدواير المرسومة هي التي تقوم مقام الدواير العظام القايمة على دايرة البروج على زوايا قايمة، لأنّا قد بيّنّا أنّ جميع الدواير بالجملة التي تقطع احدى هاتين الدايرتين اللتين وضعناهما على القطر، فأنّها تقطع الدايرة الاخرى الباقية على القطر ايضًا.




**T** has ن where we print ز.

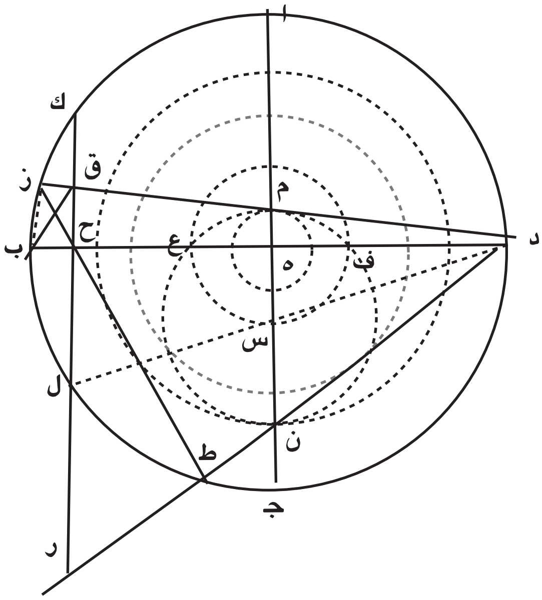
[١٦]

وقد يمكننا أن نضع في الصفيحة الدواير الموازية لدايرة البروج ايضًا على هذا المثال.

424 وينبغي ... معدل النهار] *cited* **Mas**, وينبغي ايضًا: وايضًا ينبغي **Mas**, نرسم: ترسم **Mas**, في وسط: بوست **I**, 429 ز ب د] ز جـ د **TI**, *corr.* **Ana**. بالقطبين] بالنقطتين **I**. 430 ز ا ه جـ ح] زاهحجن **I**, هحجن *add. in marg.* زا **T**, *corr.* **Ana**. 431 فصلنا قوس] قوس *in marg.* **T**. 437 فأنّها ... على القطر ايضًا] *cited* **Mas**. 439 وقد يمكننا ... الى هذا المثال] *cited* **Mas**, لدايرة البروج: لدايرة فلك البروج **Mas**.

[فـ]نجعل دايرة نصف النهار التي تمر بالقطبين دايرة ا ب جـ د حول مركز ه. وليكن المحور ب ه د، ونتوهّم نقطة د القطب الخفي، وقطر دايرة معدل النهار ا ه جـ، وقطر احدى الدواير الموازية لدايرة البروج خط ز ح ط. وليكن غرضنا أن نضع الدايرة التي هذا الخط قطر لها في الصفيحة. فنجيز على نقطة ح خطًا موازيًا لخط ا ه جـ، وليكن خط ل ح ك، ونصل خطوط د م ز د ن ط د س ل.

فمن البيّن أنّ الدايرة التي قطرها خط ز ط ترسم حول قطر م ن. وذلك أنّها تماس الدايرتين الموازيتين لدايرة معدل النهار اللتين بعدهما عنها بقدر قوسين ا ز جـ ط، ولذلك ترسم هاتان الدايرتان ببعدين ه م ه ن.




Three circles and two lines that are mentioned in the text are missing in **T**. The only internal circle in **T**, represented in gray, is incorrect. Missing or corrected objects are represented with dotted lines. A number of labels have been moved. In **T**, س marks the intersection of the gray circle and line ا جـ, ف marks the intersection of the gray circle and line ب د, and ع seems to mark the same point as ح. **T** has ط and ز transposed. In fact, another ط has been written opposite the original ز, marking the same point, presumably added by a later reader who noticed the problem with the location of ط.

ولكن لما كانت الدايرة الموازية لدايرة معدل النهار التي قطرها ل ك تقسمها الدايرة الموازية لدايرة البروج التي قطرها خط ز ط بنصفين على دايرة نصف النهار التي قطرها خط ب د، وهذه الدايرة ايضًا ترسم ببعد ه س مثل دايرة س ع ف، فيجب أن نبيّن أنّ الدايرة التي ترسم

440 فنجعل] نجعل **TI**. 442 ز ح ط] خط **I**. 444 د م ز] دمن **TI**, *corr.* **Ana**. د ن ط] ديط **I**. 446 ا ز جـ ط] بقوس ازحط **TI**. 449 التي[2]] الى **TI**. 450 الدايرة[1]] الدواير **TI**.

حول قطر م ن تمر بنقطتي ع ف.

ونصل خطي ب ز ب ق، ونخرج خطي ك ل د ط حتى يلتقيا على نقطة ر. فمن قبل أنّ زاويتي ب ز ق ب ح ق قايمتان، تكون نقط ب ح ق ز على خط محيط بداية واحدة. فزاوية ب ق ر مساوية لزاوية ب ز ط التي هي مساوية لزاوية ب د ر، فنقط ب ر د ق ايضًا على خط محيط بداية واحدة. والذي يكون من ضرب خط ق ح في خط ح ر مساوٍ للذي يكون من ضرب خط ب ح في خط ح د. واذا كان كذلك فهو مساوٍ لمربع خط ح ل في مثله. فخط م ه ايضًا في خط ه ن مساوٍ لخط ه س في مثله الذي هو مساوٍ لخط ف ه في خط ه ع، فنقط م ع ن ف اذًا ايضًا على خط محيط بداية واحدة.

[١٧]

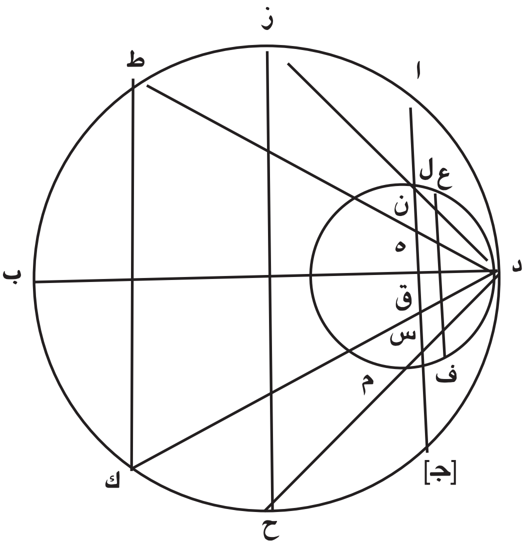
وينبغي أن نبيّن أنّ مراكز الدواير الموازية لداية البروج ايضًا التي ترسم على هذا المثال تكون مختلفة ابدًا.

فلتكن داية نصف النهار التي تمر بالقطبين داية ا ب ج د حول مركز ه، والمحور خط ب ه د، وقطر داية معدل النهار خط ا ج، وقطري دايرتين موازيتين لداية البروج خطي ز ح ط ك. ونصل خطوط د ل ز د م ح د ن ط د س ك، ونرسم حول مثلث د ن س داية ع س ف. ونصل خط ع ف، ونقسم خط ل م بنصفين على نقطة ق. فمن البيّن أنّ الداية التي على قطرها ز ح الموازية لداية البروج ترسم على قطر ل م، وأنّ الداية التي قطرها ط ك الموازية لداية البروج ترسم حول قطر ن س.

واقول إنّ هاتين الدايرتين ليس مركزهما واحدًا بعينه، اعني أنّ نقطة ق لا تقسم خط ن س ايضًا بنصفين. برهان ذلك أنّ قوس ز ط مساوية لقوس ك ح، ولذلك تكون زاوية ز د ط مساوية لزاوية ح د ك، وقوس ن ع مساوية لقوس س ف، فخطا ل م ع ف متوازيان. فنسبة خط د ل الى خط ل ع كنسبة خط د م الى خط م ف، ولكن نسبة خط د ل الى خط ل ع كنسبة خط د ل في مثله الى خط د ل في خط ل ع، ونسبة خط د م الى خط م ف كنسبة خط د م في مثله الى خط د م في خط م ف. فنسبة خط د ل في

451 قطر] قطن I. 453 ز] ف TI, *corr.* **Ana**. خط محيط] محيط I. 455 ق ح] فخ I. 456 ح ر] جد TI, *corr.* **Ana**. 457 ه ن] ه ر TI, *corr.* **Ana**. 458 فنقط … محيط بداية واحدة] *cited* **Mas**; فنقط: فنقطة **IT**, فنقط **Mas**, puncta **Her**; م ع ن ف: ن ع م ف **Mas**; اذًا, ايضًا, خط, واحدة *omit.* **Mas**. 459 وينبغي … تكون مختلفة ابدًا] *cited* **Mas**, ايضًا *omit.* **Mas**. 461 فلتكن] فليكن **I**. حول] حول حول **I**. 463 د ن س] دلس **TI**, *corr.* **Ana**. 465 البروج] معدل النهار **TI**, *corr.* **Ana**. ترسم] نرسم **I**. 466 البروج] معدل النهار **TI**, *corr.* **Ana**. 467 ق] ف **TI**, *corr.* **Ana**. 468 تكون] يكون **I**. 469 ح د ك] ح د ل **TI**, *corr.* **Ana**. 472 فنسبة] ونسبة **TI**.

مثله اذًا الى خط د ل في خط ل ع كنسبة خط د م في مثله الى خط د م في خط م ف. ولكون الدايرة، يكون خط د ل في خط ل ع مساويًا لخط س ل في خط ل ن، ويكون خط د م في خط م ف مساويًا لخط ن م في خط م س. فنسبة خط د ل في مثله الى خط س ل في خط ل ن كنسبة خط د م في مثله الى خط ن م في خط م س، واذا بدّلنا، كانت نسبة خط د ل في مثله الى خط د م في مثله كنسبة خط س ل في خط ل ن الى خط ن م في خط م س. ولكن خط د م في مثله اعظم من خط د ل في مثله، اذ كان خط د م اطول من خط د ل، فخط ن م في خط م س اعظم من خط س ل في خط ل ن. وخط ن س مشترك مع خط ل ن ومع خط م س، فخط م س اذًا اطول من خط ل ن. ولكن خط ل ق مساوٍ لخط م ق، فخط ن ق اذًا اطول من خط ق س. فليس نقطة ق مركز الدايرة التي يكون خط ن س قطرًا لها.

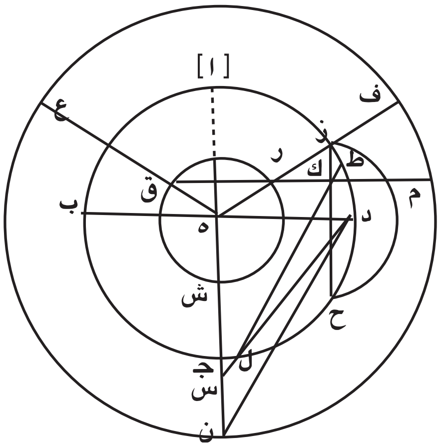



As stated in the text, line ا ج should be the diameter of the equator and point ه the center of circle ا ب ج د. The lines ط ك, ز ح should not be perpendicular to axis د ه ب. Line د ح should meet line ا ج and circle د ع ف in a single point marked with م. The label ج is missing although mentioned in the text. Labels ن and س have been moved. In **T**, they mark the intersections of lines د ط, د ك with circle د ع ف.

474 ولكون] ولكان **TI**. 476 ل ن] لر **TI**, *corr.* **Ana**. بدّلنا] خالفنا **TI**, permutatim (alternatim *in two* MSS) **Her**. 477 د م] دل **I**. ن م] بم **I**. 479 ن س] بس **I**. 481 ن ق] بق **TI**, *corr.* **Ana**. فليس ··· قطرًا لها] *cited* **Mas**, ق: و **Mas**, مركز ··· قطرًا لها: ن س لمركز الدايرة تمر بنقطتي **Mas**, وذلك ما اردنا أن نبيّن *add.* **Mas**. ق] ف **TI**, *corr.* **Ana**. 482 ن س] يس **I**.

[١٨]

ويجب الآن، لمكان الدواير الموازية لدايرة البروج التي ليس هي محصورة في الصفيحة لكن يقع بعضها في القطعة التي لا تظهر وهي غير مرسومة من الكرة، اعني الدواير التي تقطع الدايرة الخفية ابدًا، أن نضع ايضًا الدايرة التي تمر بالقطبين دايرة ا ب ج د حول مركز ه. وليكن المحور خط ب د، ونتوهّم نقطة د القطب الخفي، وقطر دايرة معدل النهار خط ا ج، وقطر الدايرة الموازية لها الخفية دايمًا خط ز ح، وقطر الدايرة التي يقاطع هذه الدايرة، الموازية لدايرة البروج خط ط ك ل.




Line د ن should pass through point ح. If it did, however, the outer circle would extend outside the space left for the diagram. In **T**, ا and the dotted part of line ا ج are missing, although mentioned in the text.

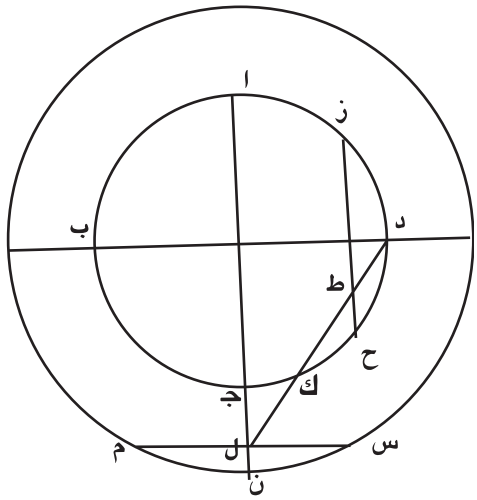
ونرسم على خط ز ح نصف دايرة، ولتكن ز م ح. ونخرج خطًا موازيًا لخط ه د، وليكن خط ك م. فمن قبل أنّا أن اخرجنا خط ا ج ن وخطي د ح ن د ل س، كانت الدايرة التي ترسم ببعد ه ن، كأنّها دايرة ع ن، هي الدايرة الخفية دايمًا في الصفيحة، وكانت الدايرة التي ترسم بدل الدايرة التي على خط ط ك ل تمر هي ايضًا بنقطة س، وتقسم الدايرة الخفية دايمًا بقسي شبيهة بقوس ح م، اذ كان خط ك م هو فصل مشترك لسطحيهما. وذلك أنّا أن رسمنا حول مركز ه دايرة مساوية لدايرة ز م ح، كأنّا رسمنا دايرة ق ر ش، واخرجنا خط م ك ر ق، واخرجنا خطي ه ق ع ه ر ف، جعلنا قوسي ن ع ن ف شبيهتين بقوسي ش ق ش ر. فتكونان شبيهتين بقوس ح م. وتكون الدايرة الموازية لدايرة البروج التي ترسم على خط ط ل

483 ويجب ... من الكرة] *cited* **Mas**, لدايرة البروج: لدايرة فلك البروج **Mas**, ليس: ليست **Mas**, هي *omit.* **Mas**, من الكرة: في الكرة **Mas**. 490 ا ج ن] أخر **I**. د ح ن] دحر **I**. 491 ه ن] من **TI**, *corr.* **Ana**. 492 وتقسم] وتقيم **TI**. 494 ق ر ش] قرس **I**. 496 فتكونان] فيكونان **I**. شبيهتين] شبيهين **TI**. وتكون] ويكون **I**.

تمر في الصفيحة بنقط ع س ف.

[١٩]

ومن البيّن أنّا وإن توهّمنا في مثل هذه الصورة الدايرة الموازية لدايرة البروج التي ترسم على نقطة د، كأنّا جعلناها الدايرة التي ترسم على خط د ك، واخرجنا خط د ك للعرض الذي ذكرناه، واجزنا على نقطة ل خط م ل س على زوايا قايمة على خط ا ج ن، كان هذا الخط في الصفيحة مكان الدايرة التي قطرها خط د ط ك. وذلك أنّ جميع الخطوط المستقيمة التي تخرج من نقطة د مارة بهذه الدايرة هي في سطح واحد، وهو سطح الدايرة، والفصل المشترك لهذا السطح ولسطح دايرة معدل النهار، هو خط م ل س، وذلك أنّ سطح دايرة نصف النهار ايضًا التي على خط ا ج هي على زوايا قايمة على كل واحد من هذين السطحين اللذين ذكرناهما.




[٢٠]

فعلى هذا المثال الذي بيّنّا يجب أن يرسم في الصفيحة قياس لما في الكرة المجسمة الدواير التي توجد بسمت دايرة معدل النهار، ما كان منها من دواير نصف النهار وما كان منها موازيًا لمعدل النهار، والدواير التي توجد بسبب الدايرة التي تمر في وسط البروج.

ومن قبل أنّ قطب دايرة معدل النهار يكون ايضًا مركزًا لهذه الدايرة ولجميع الدواير

497 ع س ف] س ف I. 498 ومن البيّن ... على نقطة د] cited Mas, لدايرة فلك البروج: لدايرة البروج Mas, على قطرها دل: ترسم على نقطة د Mas. 499 جعلناها] جعلناهما T. للعرض] للغرض I, ad latitudinem Her. 503 وذلك أنّ ... اللذين ذكرناهما] cited Mas, بد: ا ج Mas, هي: هو Mas, هذين omit. Mas. 504 هي على زوايا] هي زوا I. 509 ولجميع] و بجميع I.

الموازية لها، فأنّ جميع دواير نصف النهار تكون مع ذلك خطوطًا مستقيمة. وأنّ قطب الدايرة التي تمر في وسط البروج لا يكون مركزًا لهذه الدايرة ولا لواحدة من الدواير الاخر التي هي نظاير الدواير الموازية لها. وأنّ واحدة من هذه الدواير تكون بلا مركز، اعني تكون خطًا مستقيمًا. وأنّ الدواير العظيمة التي ترسم وتمر بهذا القطب تكون خلاف ذلك، وذلك أنّ الدايرة التي هي مكان الدايرة التي تمر بالقطبين تكون خطًا مستقيمًا، تقع على مراكز الدواير الموازية لدايرة البروج، وتكون الدواير الباقية دواير غير أنّها تكون غير متساوية.

وجب من ذلك أن يكون ممكنًا في الاوضاع التي توجد بقياس الى دايرة معدل النهار أن نرسم الكواكب من غير أن نرسم جميع الدواير بمسطرة تقسم على نسب الدواير الموازية لدايرة معدل النهار وبقسمة دايرة معدل النهار وحدها. وامّا في الاوضاع التي توجد بقياس الى الدايرة التي تمر في وسط البروج فليس يمكن ذلك. لكن يجب أن نرسم جميع الدواير او اكثرها لكي نستدلّ بها على المواضع التي يجب أن توضع فيها الكواكب.

ومن اصلح الامور في أن يستوفي في كل واحد من هذين الرسمين ما يستعمل في الكرة المجسمة أن نضع الدواير التي [توجد] بسبب دايرة معدل النهار، ما كان منها من دواير نصف النهار وما كان من الدواير الموازية لدايرة معدل النهار، والدواير التي توجد بسبب الدايرة التي تمر في وسط البروج على مثال ما عليه الاكر المضروبة. فإن كان لا يمكن رسم جميع ذلك في الصفيحة، فينبغي أن نرسم فيها الدواير التي تمر بدرجتين او بثلث درج او بست درج، اذ كان ذلك رسمَ متوسّطٍ، من قبل أنّ هذه الثلثة الاعداد هي عدد مشترك لثلثين درجة التي هي درج كل واحد من البروج، ولاربع وعشرين درجة، وهي البعد فيما بين دايرة معدل النهار وبين كل واحدة من دايرتي المنقلبين بالتقريب، حتى يقع معما يرسم من الدواير دايرتا المنقلبين ودواير نصف النهار التي تمر بالبروج، ولا يكون في الابعاد التي توجد على غير هذا المثال اختلاف.

تم كتاب بطلميوس من اهل
قلاوذية في تسطيح بسيط الكرة
والحمد لله وصلواته على محمد نبيه آله صحبه وسلم

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513 تكون] يكون I. 514 تكون] يكون I. على] عليه TI. 518 وبقسمة ... النهار] *in marg.* T, دواير: دايرة I. 521 يستوفي] تستوفي I. 522 توجد] *omit.* TI. 527 دايرة معدل النهار] معدل *add.* I. 528 معما] معها I. 529 ولا] فلا I. 531 بطلميوس] بطليموس I. 533 نبيه] *omit.* I. 533 صحبه وسلم] *illegible* T.

# V Translation

In the name of God, the Merciful, the Compassionate,
and may God bless Mohammad

The treatise of Ptolemy, of the people of Claudia

*On Flattening the Surface of the Sphere* He said

**[1]**

Since it is possible, Oh Syrus, and useful in many subjects that there be, in a flat surface, the circles that occur on the solid sphere, as though spread out, I considered it necessary with regards to the expert, that I write a treatise for whoever desired knowledge of this, in which I briefly show how it is possible to draw the ecliptic, the circles parallel to the equator, and the circles known as the meridians, so that all of what occurs in this will be consistent with what is apparent in the solid sphere.

[1.2] This aim we intend may be prepared for us when we use straight lines representing the meridians and arrange the circles parallel to the equator as a configuration, in which it is, firstly, prepared that the drawn great circles, of the inclined circles tangent to circles parallel to the equator, which are the same distance from it in both directions, always bisect the equator. This is congruous for us in the following manner.

[See Fig. 1] We assume the equator is circle $ABGD$, and that it is around center $E$. We draw in it two diameters intersecting at right angles, which are line $AG$ and line $BD$. We imagine these lines representing meridians, and point $E$ as the north pole, because it is not possible to place the other pole on a plane surface, since its plane extends without limit, as we shall show in what follows.[23] Since the north pole is always visible in our countries, it is more appropriate that we specifically use it, in that we want a drawing of it.

Clearly, the circles parallel to the equator that are north of the equator should be drawn inside circle $ABGD$, while the parallel circles that are to the south must be drawn outside of it. We produce lines $AG$, $BD$ and cut off two equal arcs of the circle on either side of point $G$, which are $GZ$ and $GH$. We join line $DTZ$, and line $DHK$. We make point $E$ a center, and we draw circle $TL$ with a distance of line $ET$, and circle $KM$ with a distance of line $EK$.

[1.3] Then, I say that these circles are the correlates of two of the circles on the solid sphere that are the same distance from the equator on either side, and that the

[23]In fact, this is never explicitly demonstrated in the extant treatise but it becomes increasingly obvious as the work progresses. Maslama provides an argument for this claim in his note accompanying *Planis.* 4–7 [Kunitzsch and Lorch 1994, 14–16].

ecliptic, drawn about a center bisecting line $TM$ such that it touches these circles at point $T$ and at point $M$, bisects circle $ABGD$, that is, it passes through point $B$ and point $D$. The proof of this is that we join line $DNM$. So, because arc $AN$ is equal to arc $GH$, which is equal to arc $GZ$, arc $NDZ$ is a semicircle. Hence, angle $MDT$ is right[24] and the circle drawn about diameter $TM$, of right triangle $MDT$, passes through point $D$. Hence, it bisects the equator.

So, it is clear from this that, for all circles parallel to the equator, if we cut off arcs on both sides of point $G$, whose magnitude depends on the distance of each of these circles from the equator, and we join the endpoints of the arcs with straight lines to point $D$, and we make what the straight lines cut off from line $EK$ distances, and we make point $E$ a center, and we describe circles, then the analog in that is in this way that we set out.[25] [1.4]

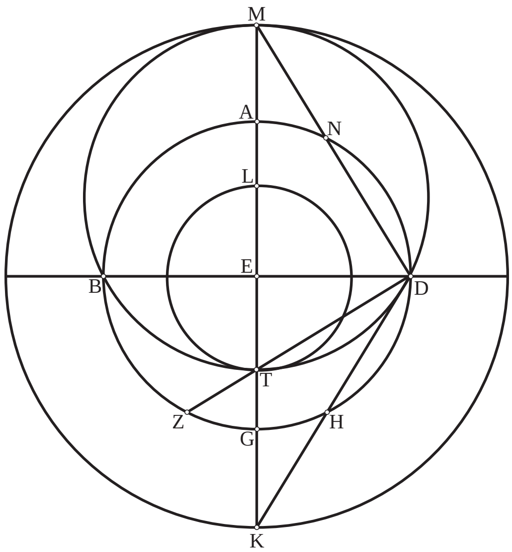


**Figure 1**: *Planisphere* 1.

Clearly, if we assume both of arcs $ZG$ and $GH$ to be approximately $23;51°$ (in the degrees in which the equator, circle $ABGD$, is $360°$), which is the distance between the equator and both of the tropics along the circle drawn through the poles of the equator,[26] then, of the two circles drawn through point $T$ and point $M$, circle $TL$ is

[24] Since it is the angle in a semicircle (*Elem.* III 31).

[25] The word that we have translated as "analog" (قياس) plays an important role in the text. A *qiyās* is a sort of reference, analogy or measure, although not generally in a numeric sense. In two cases in this text, it is used to discribe the relationship that the planisphere bears to the solid sphere (lines 33 & 506, see also page 108), while in four cases it refers to the use of coordinate systems to reference star positions (lines 426, 427, 516, 518, see also pages 103 and 109).

[26] This circle could be the solstitial colure, or indeed any meridian circle.

the summer tropic and circle $KM$ is the winter tropic. In this way, the circle drawn through point $M$, point $B$, point $T$ and point $D$ (the circle through the signs) is tangent to the tropics at point $T$ (the summer tropic) and at point $M$ (the winter tropic); and it bisects the equator at points $B$ and $D$.[27] So, point $B$ is the vernal point and point $D$ the autumnal point, because the motion of the cosmos is indeed
[1.5] as though from point $B$ toward point $A$ and then to point $D$. It is neither possible for a division of the ecliptic into signs to take place through equal arcs, nor again for its division into four parts to take place through equal arcs. Rather, its division into what is required is strictly in this way: that is, the beginnings of the signs are put at the points at which circles parallel to the equator divide the ecliptic, which are drawn according to the explained method, with the distance consistent with the distance of each of the signs from the equator in the solid sphere. For, at this degree alone, all of the straight lines passing through pole $E$, representing the meridians, cross the ecliptic at parts that are the correlates of parts diametrically opposite on the solid sphere.[28]

## [2]

Every horizon circle, drawn in the same way as the ecliptic, not only bisects the equator but also functionally bisects the ecliptic. That is, it is also drawn through parts that are functionally the correlates of parts diametrically opposite on the solid sphere.[29]

[2.2] [See Fig. 2] Let the equator be circle $ABGD$ around center $E$. The circle through the signs is circle $ZBHD$ and it bisects the equator at point $B$ and point $D$. We pass an arbitrary straight line through the pole $E$ representing a meridian. Let it be line $ZAEHG$.

I say that points $Z$ and $H$ are the correlates to diametrically opposite points on the solid sphere. That is, circles parallel to the equator that are drawn through these points will cut off equal arcs on both sides of the equator, in the way we described, just as occurs on the solid sphere as well.[30]

The proof of this is that we produce a straight line, line $ET$, from point $E$ at right angles to line $AG$. We join line $AT$, line $GT$, line $ZKT$ and line $THL$. So,

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[27] *Planis.* 1.3.

[28] In other words, only when the signs are constructed in the manner described, will the degrees determined as the beginnings of opposite signs be joined by straight lines that pass through the center of the equator.

[29] It is odd that these points should be said to be "functionally correlates" (بالقوة نظائر), since they are, in fact, the correlates. This expression is found again in *Planis.* 15 (see page 103).

[30] Ptolemy simply assumes this as an obvious fact of the relationship between the ecliptic and the meridian circles. His justification for this assumption probably comes from considerations of solid geometry (see the commentary, especially page 117).

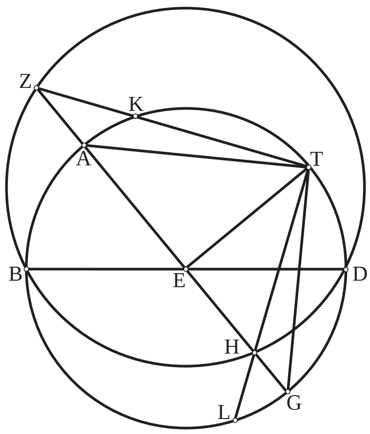


**Figure 2**: *Planisphere* 2.

clearly, angle $ATG$ is right, for arc $ATG$ is a semicircle.[31] Since the product of line $ZE$ by line $EH$ is equal to $ED$ squared, that is, equal to $ET$ squared,[32] the ratio of line $ZE$ to line $ET$ is as the ratio of line $ET$ to line $EH$.[33] [So,] triangle $ZTH$ is also right angled, and angle $ZTH$ is right.[34] Hence, angle $ZTL$ is equal to angle $ATG$. Then, if we omit the common angle $ATH$, the remaining angle $KTA$ will equal the remaining angle $HTG$. So, arc $KA$ is also equal to arc $GL$. Now, we have shown that since lines $TKZ$ and $TL$ join the endpoints of arcs that are the same distance from the equator, and their origin[35] is from the point, the distance of which from point $A$ and point $G$ is a quadrant, which is point $T$, [then] on line $ZG$ we get point $Z$ and point $H$, which are the points through which are drawn two circles parallel to the equator the same distance from it.[36] Therefore, line $ZEH$ has passed through points that are functionally on the diameter of the ecliptic.

## [3]

I say that even if we draw another circle, inclined to the equator, representing the horizon circle, so that this circle bisects the equator alone, then the two places of the intersection of this circle and the circle through the signs are functionally

[31] *Elem.* III 31.

[32] By *Elem.* III 35 in circle $ZBHD$, and since $ED$ and $ET$ are radii of circle $ABGD$.

[33] *Elem.* VI 17.

[34] *Elem.* VI 6.

[35] Literally, "the place from which they are drawn" (مخرجهما). This probably carries some notion of the *point of projection.*

[36] There does not appear to be a specific proof of this claim in *Planis.* 1. Nevertheless, it does follow from *Planis.* 1.3 & 1.4.

diametrically opposite.[37] That is, the line joining them passes through the center of the equator.

[See Fig. 3] Again, let the equator be circle $ABGD$ around center $E$, and the ecliptic circle $HBTD$, and let it bisect the equator along diameter $BED$.[38] The horizon circle is circle $HATG$, and this circle also bisects the equator along diameter $AEG$.[39] Let the intersection common to the ecliptic and the horizon circle be point $H$ and point $T$. Then, I say that if we join point $H$ with center $E$ by a straight line, representing a meridian, and we extend that line rectilinearly, it will arrive at point $T$.

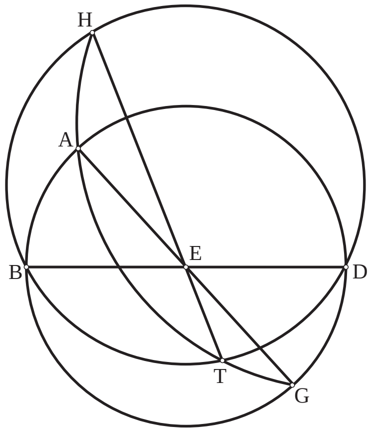


**Figure 3**: *Planisphere* 3.

The proof of this is that we join line $HE$ and produce it rectilinearly until it intersects the horizon circle, circle $HAG$, at point $T$.[40] Then, I say that point $T$ is also common to the ecliptic, circle $HBTD$.[41] So, because lines $HT$ and $AG$ have been produced in circle $HATG$ intersecting at point $E$, line $HE$ by line $ET$ is equal to line $AE$ by line $EG$, and likewise, line $AE$ by line $EG$ is equal to line $BE$ by $ED$.[42] Hence line $BE$ by $ED$ is equal to $HE$ by $ET$. Hence, lines $BD$, $TH$ are in

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[37]The claim is that if an $r$-horizon is drawn with the only condition being that it bisect the equator, then its intersections with the $r$-ecliptic can also be shown to represent diametrically opposite points. Compare this statement with *Planis.* 2.1.

[38]*Planis.* 1.3.

[39]Although the proof in *Planis.* 1.3 is stated in terms of the ecliptic, it applies to any great circle inclined to the equator.

[40]Since, by *Planis.* 2, $HET$ is a diameter of the horizon.

[41]It would have been clearer if Ptolemy had initially differentiated between $T$ as the intersection of $HE$ and circle $HAG$ and $T$ as the intersection of circle $HAG$ and circle $HBD$ and then proceeded to show that they were one and the same (see the commentary, especially page 118).

[42]By *Elem.* III 35 in circle $HATG$ and again in circle $ABGD$.

a single circle.[43] From this it follows that point $T$ is on the ecliptic, circle $HBTD$, and we had stated that it is on the horizon circle, circle $HATG$. So, the line joining the two places of the intersection of the ecliptic and the horizon is a line that passes through the center of the equator, point $E$. So, it is clear from this that the horizon circle and the ecliptic intersect at functionally diametrically opposite points. QED.

## [4]

Then, having previously demonstrated this, let us next consider the ratio of the radii of the parallel circles drawn according to the signs of the ecliptic[44] to the radius of the equator, which we previously set out, so that we come to know that their rising-times are also found numerically[45] to be consistent with what is manifest with respect to the solid sphere.

Again, let the equator be circle $ABGD$ around center $E$. We produce two of its diameters, intersecting at right angles, $AG$ and $BD$. We produce line $AG$ rectilinearly to point $Z$. We cut off two equal arcs, $GH$ and $GT$, on either side of point $G$. We join line $DKH$ and line $DTZ$. We have previously explained, of circles parallel to the equator that are the same distance from it, the one of them to the north is indeed drawn about center $E$ with distance $EK$, while that to the south [is drawn] about center $E$ with distance $EZ$.[46] [4.2] [See Fig. 4]

The ratio of line $EZ$ to line $EK$ is evident to us in this way.[47] Because arc $GH$ is equal to arc $GT$, arc $BH$ and arc $BGT$ together are a semicircle.[48] So, the angles opposite them, that is angle $EDK$ and angle $EDZ$, are together equal to a right angle.[49] Also, angle $EDK$ with angle $EKD$ is right,[50] so angle $EDZ$ is equal to angle $EKD$. Hence, right triangle $ZED$ is similar to right triangle $DEK$, so the

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[43] Converse of *Elem.* III 35.

[44] These circles are drawn through the beginnings of the signs, parallel to the equator (see *Planis.* 1.5).

[45] The expression is literally "by number" (بالعدد), and probably translates something like διὰ τῶν ἀριθμῶν, which occurs twice in the *Almagest*, where it denotes the process of producing results through computation [Heiberg 1898–1903, p. 1, 239 & 339; Toomer 1984, 157 & 211].

[46] *Planis.* 1.2.

[47] The following passage is the first piece of metrical analysis in the text. Metrical analysis was a type of Greek mathematics that was used to show generally that certain quantities could be computationally derived when other quantities were assumed as given. Although it is clear from the context that this is metrical analysis, the passage is unusual in making no mention of given magnitudes.

[48] $\widehat{BH} = 90° - \widehat{GH}$ and $\widehat{BT} = 90° + \widehat{GH}$, so $\widehat{BH} + \widehat{BT} = 180°$.

[49] *Elem.* III 31.

[50] *Elem.* I 32.

ratio of line $ZE$ to line $ED$ is as the ratio of line $DE$ to line $EK$.[51] The ratio of arc $BT$ to the supplement – that is, the arc equal to arc $BH$ – is, however, as the ratio of angle $EDZ$ to angle $EZD$, and as the ratio of the arc on line $EZ$, in the circle drawn around right triangle $DEZ$, to the arc on line $ED$, in this same circle.[52] So, the ratio of the chord of arc $BT$ to the chord of the supplement, that is arc $BH$, is as the ratio of line $ZE$ to line $ED$,[53] and as the ratio of line $DE$ to line $EK$.[54]

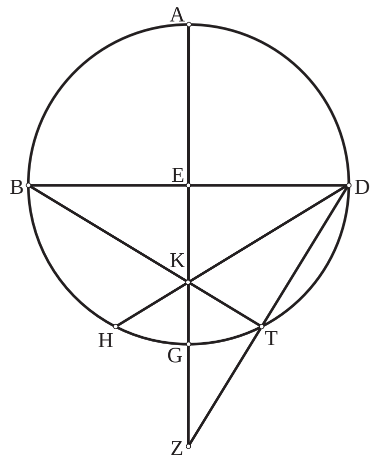


**Figure 4**: *Planisphere* 4.

[4.3] Then having previously deduced that, in a similar diagram we first assume that both of the arcs $GH$ and $GT$ are $23;51,20°$ (in the degrees in which circle $ABGD$ is $360°$), which are the degrees that we assumed for the distance between the equator and both of the tropics in our discussion with respect to the solid sphere as well.[55]

Then arc $BT$ is $113;51,20°$ (in the degrees in which this circle is $360°$), and arc $BH$, the supplement, is $66;8,40°$. The chord of arc $BT$ is $100;33,28^{\mathrm{p}}$ (in the parts in which the diameter is $120^{\mathrm{p}}$, for we have assumed this in the *Almagest*), and chord $BH$ is $65;29^{\mathrm{p}}$ (of these parts). [So,] the ratio of line $ZE$ to line $ED$, and the ratio of line $ED$ to line $EK$ is the ratio of $100;33,28^{\mathrm{p}}$ to $65;29^{\mathrm{p}}$. Therefore, line $EZ$, the radius of the winter tropic, is $92;8,15^{\mathrm{p}}$ (in those parts in which the radius of the

[51] *Elem.* VI 4.

[52] Both of these statements follow from the fact that equal arcs subtend equal angles (*Elem.* III 26).

[53] This follows from the fact that equal chords subtend equal arcs (*Elem.* III 28). In **T**, a marginal gloss, perhaps in the original hand, reads, "Note: Because the ratio of $BT$ to $TD$ is as the ratio of the side of angle $BDT$, that is line $EZ$, to the side of angle $DBT$, that is line $DE$."

[54] Since $\triangle WDZ$ is similar to $\triangle EDK$ (*Elem.* VI 4).

[55] In *Alm.* I 12, Ptolemy claims to have measured this angle using a special instrument. In fact, however, he simply assumes a traditional value of $^{11}/_{83}$ of the circumference of a circle [Heiberg 1898–1903, p. 1, 67–68; Toomer 1984, 63, n. 75].

equator, line $ED$, is $60^{\mathrm{p}}$), and the radius of the summer tropic is $39;4,19^{\mathrm{p}}$.[56]

From this it is evident that the diameter of the ecliptic (since it is tangent to these two circles at the endpoints of its diameter) is the sum of their radii, $131;12;34^{\mathrm{p}}$ (in the parts in which radius of the equator is $60^{\mathrm{p}}$), and that the radius of the ecliptic is $65;36,17^{\mathrm{p}}$. The line between its center and the center of the equator is $26;31,58^{\mathrm{p}}$ (of these parts).[57]

**[5]**

Again, we assume both arcs $HG$ and $GT$ to be $20;30,9^{\circ}$ – which is the distance between the equator and [each of] the two circles parallel to the equator that cut off $30^{\circ}$ of the circle through the signs on both sides of the solstitial points – so that arc $BT$ is $110;30,9^{\circ}$ and its chord is $98;35,57^{\mathrm{p}}$, and arc $BH$ is $69,29,51^{\circ}$ and its chord is $68;23,51^{\mathrm{p}}$.[58] Hence, the ratio of line $ZE$ to line $ED$, and also the ratio of line $ED$ to line $EK$, is the ratio of $98;35,57^{\mathrm{p}}$ to $68;23,51^{\mathrm{p}}$.[59] So, of the parts in which line $ED$ is $60^{\mathrm{p}}$, line $EZ$ is $86;29,42^{\mathrm{p}}$, and line $EK$ is $41;37,15^{\mathrm{p}}$ (of these parts).[60] [See Fig. 4]

**[6]**

In this way, we assume both arcs $HG$ and $GT$ to be $11;39,59^{\circ}$, which is the distance, along the great circle drawn through the poles of the equator, between the equator and [each of] the two circles parallel to it that cut off $60^{\circ}$ from the [See Fig. 4]

[56]The beginning of this sentence is somewhat garbled in the text. It literally reads, "Therefore line $ED$, which is the radius of the equator, in the parts of which it is $60^{\mathrm{p}}$, in these parts the radius of the winter tropic, line EZ, is $92;8,15^{\mathrm{p}}$, and the radius of the summer tropic is $39;4,19^{\mathrm{p}}$." Hermann has

> quoniam igitur *ed* semidiametros circuli recti absolute LX partium est, metiuntur quidem ex eis partibus XCII puncta VIII secunde XV linaem *ez* semidiametrum hyemalis tropici, semidiametrum autem estiui partes XXXIX puncta IIII secunde XVIIII

that is, "Since therefore, $ED$, the radius of the right circle, is simply $60^{\mathrm{p}}$, line $EZ$, the radius of the winter tropic, in fact measures $92;8,15^{\mathrm{p}}$ (of these parts), while the radius of the summer is $39;4,19^{\mathrm{p}}$" [Heiberg 1907, 234].

[57]Since the diameter of the ecliptic is the sum of the radii of the tropics, the segment between the center of the ecliptic and the center of the equator is the radii of the difference between the radii of the ecliptic and that of the summer tropic, that is, $65;36,17^{\mathrm{p}} - 39;4,19^{\mathrm{p}} = 26;31,58^{\mathrm{p}}$.

[58]The value $20;30,9^{\circ}$ is derived in *Alm.* I 14 and tabulated in *Alm.* I 15 [Heiberg 1898–1903, p. 1, 76–88; Toomer 1984, 69–72].

[59]The argument for this is given in *Planis.* 4.2 (see page 88).

[60]Calculation gives $(98;35,57^{\mathrm{p}}/68;23,51^{\mathrm{p}})60^{\mathrm{p}} = 86;29,37^{\mathrm{p}}$ and $(68;23,51^{\mathrm{p}}/98;35,57^{\mathrm{p}})60^{\mathrm{p}} = 41;37,18^{\mathrm{p}}$ respectively.

circle through the signs on both sides of the solstitial points.[61] So, the whole arc $BT$ is $101;39,59^\circ$ and its chord is $93;2,14^\mathrm{p}$, and arc $BH$ is $78;20^\circ$ and its chord is $75;47,23^\mathrm{p}$.[62] So, the ratio of line $ZE$ to line $ED$ and the ratio of line $DE$ to line $EK$ is the ratio of $93;2,14^\mathrm{p}$ to $75;47,23^\mathrm{p}$,[63] and, of the parts in which line $DE$ is $60^\mathrm{p}$, line $EZ$ is $73;39,7^\mathrm{p}$ and line $EK$ is $48;52^\mathrm{p}$ (of these parts).[64]

**[7]**

[See Fig. 4] Likewise, if we make both arcs $HG$ and $GT$ $54^\circ$, which is the distance, on either side of the equator, of [each of] the circles parallel to the equator that are tangent to the horizon at the latitude of Rhodes, which is the horizon we used as an example on the solid sphere – then, in this case as well, arc $BT$ is $144^\circ$ and its chord is $114;7,37^\mathrm{p}$, and arc $BH$ is $36^\circ$ and its chord is $37;4,55^\mathrm{p}$. The ratio of line $ZE$ to line $ED$, and line $DE$ to line $EK$, is the ratio of $114;7,37^\mathrm{p}$ to $37;4,55^\mathrm{p}$.[65] So, of the parts in which line $ED$ is $60^\mathrm{p}$, line $EZ$ again sums to $184;39,48^\mathrm{p}$ and line $EK$ is $19;29,42^\mathrm{p}$ (of these parts).[66] Clearly, since it is these lines, when summed, that are the diameter of the horizon we previously assumed – just as the diameter of the ecliptic is the diameters of the tropics – this diameter will be $204;9,30^\mathrm{p}$ (in the parts in which the diameter of the equator is $120^\mathrm{p}$). It follows from this that the radius of the horizon circle is $102;4,45^\mathrm{p}$, and the line between the center of this circle and the equator is $82;35,3^\mathrm{p}$ (of these parts).[67] QED.

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[61] This arc is derived and approximated by $11;40^\circ$ in *Alm.* I 14. The value $11;39,59^\circ$ is taken from *Alm.* I 15 (see note 58).

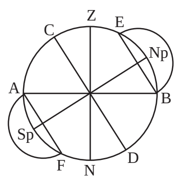
[62] Computing with the chord table gives $Crd(101;39,59^\circ) = 93;2,14^\mathrm{p}$. $78;20^\circ$ is rounded from $78;20,1^\circ$. Computing with the chord table from the value in the text gives $Crd(78;20^\circ) = 75;47,22^\mathrm{p}$, but using the slightly more precise value gives $Crd(78;20,1^\circ) = 75;47,23^\mathrm{p}$.

[63] Again, the argument for this is found in *Planis.* 4.2 (see page 88).

[64] Calculation gives $(93;2,14^\mathrm{p}/75;47,23^\mathrm{p})60^\mathrm{p} = 73;39,15^\mathrm{p}$ and $(75;47,23^\mathrm{p}/93;2,14^\mathrm{p})60^\mathrm{p} = 48;52,37^\mathrm{p}$ respectively.

[65] The traditional value for the latitude of Rhodes is $36^\circ$, which Ptolemy also uses in the *Almagest* [Heiberg 1898–1903, p. 1, 89–90 ff.; Toomer 1984, 76 ff.]. In the ancient context, the justification for using the complementary angle is probably best shown on the analemma (see page 111, note 170).

Where $AB$ is the diameter of the horizon and $CD$ the diameter of the equator, the terrestrial latitude, $36^\circ$, is the height of the north pole, $Np$, above the horizon. Hence, the $\delta$-circles tangent to the horizon, $AF$ and $BE$, are $54^\circ$ from the equator.




[66] Calculation gives $(114;7,37^\mathrm{p}/37;4,55^\mathrm{p})60^\mathrm{p} = 184;39,42^\mathrm{p}$.

[67] See note 57.

**[8]**

Since we have set that out, then let us show that, in a similar diagram, one also sees that the magnitudes of the rising-times, and all that pertains to them, are just as we showed with respect to the solid sphere.[68]

For let the equator be circle $ABGD$ around center $E$ and the circle through the signs circle $ZBHD$ around center $T$. We produce two diameters passing through point $E$, the center of the equator, representing the meridian. One of them, line $BED$, passes through the intersections at points $B$ and $D$, which are the equinoctial points. The other, line $ZTEH$, passes through the center of the ecliptic, so producing the solstitial points, $Z$ and $H$. [See Fig. 5]

First, let us proceed to show, on the upright sphere, what [parts] of the equator rise with the parts of the circle through the signs. Now, because the position of the horizon on the upright sphere is that of the meridian, and the straight lines in this diagram that pass through the pole of the equator, point $E$, are the correlates to the meridians, it is clear that both arcs $ZB$ and $HD$, which are quadrants of the ecliptic,[69] rise with both arcs $AB$ and $GD$, which are quadrants of the equator, and they culminate with them, and set with them, because, in circle $ZBHD$, the radius, line $TH$, bisects line $BD$ at right angles at point $E$.[70]

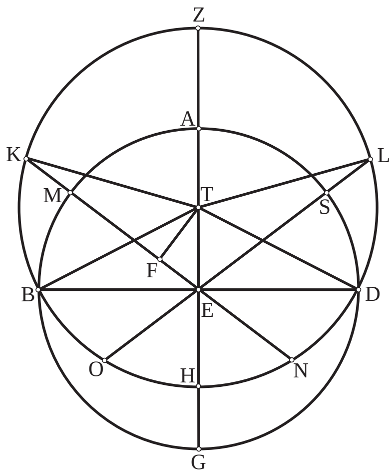



**Figure 5**: *Planisphere* 8.

So, we cut off equal arcs, arc $BK$ and arc $DL$, from the circle through the signs, and we cross line $KMEN$ and line $LSEO$. Now since, we have shown that parallel

[68]The rising-times of arcs of the ecliptic are tabulated in *Alm.* II 8 [Heiberg 1898–1903, p. 1, 134–141; Toomer 1984, 100–103].

[69]This sectioning of the ecliptic is discussed in *Planis.* 1.5.

[70]*Elem.* III 3.

circles that are the same distance from the equator, on both sides, pass through points $K$ and $L$ and points $O$ and $N$, it results that point $K$ is functionally opposite point $N$ and point $L$ is opposite point $O$.[71]

First, if we assume that arc $BK$ is the sign of Pisces, then clearly arc $LD$ is the sign of Libra, and in this way arc $BO$ makes up the sign of Aries and arc $ND$ makes up the sign of Virgo. If, however, we join lines $KT$ and $LT$, triangle $KET$ is equisided and equiangular with triangle $LET$.[72] So, angle $KET$ is equal to angle $LET$, and their complementary angles, angle $KEB$ and angle $LED$, are equal respectively and to their vertical angles. So, since this angle is at the center of the equator, then the arcs of the equator that rise with each of the signs we assumed are also equal respectively.[73] So, if we find the magnitude of one of these arcs – such as if we found the magnitude of $MB$ – then we have obtained with this what we want of the rising-times.

[8.2] Then, we produce a perpendicular from point $T$ to line $KE$. Let it be line $TF$. So, since we have shown that, of the parts of which the radius of the equator is $60^{\text{p}}$, line $TK$, which is the radius of the circle through the signs, is $65;36,17^{\text{p}}$; and line $ET$, the line between the center of this circle and the center of the equator, is $26;31,58^{\text{p}}$; and line $EK$, the radius of the circle parallel to the equator that is drawn through the beginning of Pisces and the beginning of Scorpio, passing through points $K$ and $L$, is $73;39,7^{\text{p}}$ (of these parts); then, triangle $ETK$ is given.[74]

So, if we relate[75] $KT$ squared diminished by $TE$ squared to line $KE$, there results the excess of line $KF$ over line $FE$.[76] [If,] however, any two circles, be they of any magnitude, intersect one another, and the greater circle bisects the lesser circle, then

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[71] *Planis.* 1 & 2.

[72] The technical phrase "equisided and equiangular" (مساوي الاضلاع والزوايا) probably translates a phrase such as ἰσογώνιον καὶ ἰσόπλευρον found twice in the *Almagest* [Heiberg 1898–1903, 163 & 281]. Congruence follows from the fact that arc $KZ$ equals arc $ZL$, so that $\angle KTE = \angle ETL$.

[73] This use of the singular "angle" (الزاوية) to refer to any one of a set of angles is repeated three times in the text (see pages 94, 95 and 98, below).

[74] The numbers stated in this sentence are derived in *Planis.* 4 & 6. The Arabic term معلوم, "known," probably translates the Greek δεδομένος, "given." This statement can be related to *Data* 39 [Taisbak 2003, 119–120; Menge 1896, 66–68], however, we should note that Ptolemy's use of *given* is computational, whereas Euclid's is geometric. See Berggren and Van Brummelen [2000], for a discussion of the relationship between معلوم and δεδομένος.

[75] The Arabic verb أضف (ضيف, IV) can mean to join or bring into relationship. We have used the more abstract translation to stress the peculiar nature of this operation. This is a rare case in a Greek mathematical text in which a square value is put into relation with a linear value to produce a linear result.

[76] $(KT^2 - TE^2)/KE = KF - FE$. This claim can be justified from the theory of the application of areas (*Elem.* II & VI 27–30).

the square of the radius of the greater circle diminished by the [square of] the line between their centers produces the square of the radius of the lesser circle.[77] That is to say, just as we did in these circles, if we join line $BD$, the line that joins the two places of intersection passes through the center $E$ in the lesser circle. So, since angle $DET$ is right, then line $TD$ squared, which is the hypotenuse, is equal to the sum of the squares of $TE$ and $ED$.[78] From this it follows that, whatever their magnitude, the excess of the [square of] the radius of the circle that bisects the equator over the [square of] the line between their centers is $3600^{\mathrm{p}}$ (of the parts in which the radius of the equator is $60^{\mathrm{p}}$).[79]

Because line $EK$ was also calculated to be what we previously assumed, which is $73;39,7^{\mathrm{p}}$ (of these parts),[80] if we relate to this the excess, which is $3600^{\mathrm{p}}$, we obtain the excess of line $KF$ over $FE$, which is $48;52,42^{\mathrm{p}}$ (of these parts).[81] So, when we subtract that from $73;39,7^{\mathrm{p}}$ and take half of the remainder, which is $24;46,25^{\mathrm{p}}$, line $EF$ is $12;23,12^{\mathrm{p}}$ (in the parts in which we assumed that line $ET$ is $26;31,58^{\mathrm{p}}$).[82] So, of the parts of which line $ET$, the hypotenuse to right angle $EFT$, is $120^{\mathrm{p}}$, line $EF$ is also approximately $25;30^{\mathrm{p}}$,[83] and the arc that it subtends is $55;40°$ (in the

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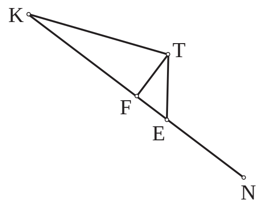
By *Elem.* I 47, $KT^2 - TE^2 = (KF^2 + TF^2) - (TF^2 + FE^2)$
$= KF^2 - FE^2$.




Since $KN$ is cut by $F$ and $E$ into equal and unequal segments, by *Elem.* II 5, $KF^2 - FE^2 = KE \times EN$.
But, $EN = FN - FE$
$= KF - FE$.
Hence, $KF^2 - FE^2 = (KE \times (KF - FE))$.
Therefore, $KT^2 - TE^2 = (KE \times (KF - FE))$.

[77]This claim follows from the configuration of the circles such that *Elem.* I 47 obtains. Considering Fig. 5, $TB^2 = EB^2 + ET^2$.

[78]The text literally reads, "equal to what is summed of the lines $TE$ and $ED$ each times (في) itself." The statement results from *Elem.* I 47.

[79]The simplification realized by pointing out that the first term of this equation is always 3600 is utilized below in *Planis.* 8.3, 9, 12 & 13.

[80]*Planis.* 6.

[81]That is $3600/73;39,7^{\mathrm{p}} = 48;52,43^{\mathrm{p}}$. Although the 3600 is, in fact, a square value, there is no indication that Ptolemy thought of it as having different units than the lengths. Nevertheless, we could be more exact and write $3600^{\mathrm{p}\times\mathrm{p}}/73;39,7^{\mathrm{p}} = 48;52,43^{\mathrm{p}}$.

[82]Note, in Figure 5, that $EF = (EK - (KF - FE))/2$.

[83]There is an error here. Calculation gives $EF = (120^{\mathrm{p}}/26;31,58^{\mathrm{p}})12;23,12^{\mathrm{p}} = 56;1,17^{\mathrm{p}}$. Hermann's text has approximately $55;59^{\mathrm{p}}$ (partes LV cum punctis fere LIX), which is better [Heiberg 1907, 239].

degrees in which the circle around right triangle $ETF$ is $360^\circ$).[84] So, angle $ETF$, which is equal to angle $FEB$ (since angle $TEB$ is also right) is $55;40^{\circ\circ}$ (in the degrees in which two right angles are $360^{\circ\circ}$); and this angle is $27;50^\circ$ (in the degrees in which four right angles are $360^\circ$). Since this angle is at the center of the equator, arc $BM$ will also be $27[;50]^\circ$ (in the degrees in which the equator is $360^\circ$).

So, it is clear to us from this, that, just as we showed with respect to the solid sphere, each of the signs at the equinoctial point – that is Pisces, Aries, Virgo and Libra – rise, in the case of the upright sphere, with these $27;50^\circ$ of the equator.[85]

[8.3] We could have shown this by a simpler argument in this way.[86] Line $KE$ by $EN$ is equal to line $BE$ by $ED$,[87] and line $BE$ by $ED$ is $3600^{\mathrm{p}}$. So, when we divide that by line $KE$, line $EN$ is given.[88] Line $KE$, however, exceeds line $EN$ by the equal of twice line $EF$.[89] So, line $EF$ is also given.[90] Line $TE$ is given, and the right angle at point $F$ is given. Hence, angle $ETF$ is given.[91]

**[9]**

[See Fig. 5] Again in the same diagram,[92] we assume arc $BK$ of the ecliptic to be the arcs of two signs, such that point $K$ is the beginning of Aquarius and point $L$ the beginning of Sagittarius; and diametrically opposite these points, point $N$ is the beginning of Leo and point $O$ the beginning of Gemini. Clearly, if we show the quantity of the magnitude of arc $BM$ of the equator, then we have determined the time degrees in which each of the signs we previously assumed rises on the upright sphere.

Clearly also, the magnitude of the lines $KT$ and $TE$ remains as it was, and the magnitude of line $KE$ increases. Because of the fact that we showed that the radius of the circle parallel to the equator, which is drawn through the beginning of Sagittarius and the beginning of Aquarius, is $86;29,42^{\mathrm{p}}$ (in the parts in which the

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[84] Computing with the chord table gives $Arc(56;1,17^{\mathrm{p}}) = 55;39,33^\circ$.

[85] *Alm.* I 16.

[86] The following passage is the second metrical analysis in the text.

[87] *Elem.* III 35.

[88] This could be justified by *Data* 55, but Ptolemy is referring to the arithmetic operation of division, as the Arabic makes clear [Taisbak 2003, 142; Menge 1896, 98–100].

[89] Note, in Figure 5, that $F$ is the midpoint of $KN$.

[90] By computation, since $KE$ and $EN$ are given. Geometrically, however, one could also justify this by an appeal to *Data* 4 & 2 [Taisbak 2003, 43 & 39; Menge 1896, 8–10 & 6].

[91] This could be justified by the theorem Taisbak calls *Data* 88*, but Ptolemy is referring to the use of the chord table [Taisbak 2003, 226].

[92] The text uses the same expression that we generally translate as "in a similar diagram" (في مثل هذه الصورة). In fact, the other occurrences of the phrase may also translate a Greek expression meaning the same diagram, since Greek mathematicians often refer to different but related figures in this way [Netz 1999, 38–40].

radius of the equator is $60^{\mathrm{p}}$),[93] if we divide the parts of the excess, $3600^{\mathrm{p}}$, by line $KE$ – that is, if we divide them by $86;29,42^{\mathrm{p}}$ – we obtain the excess of line $KF$ over line $FE$, $41;37,15^{\mathrm{p}}$ (of these parts). When we subtract that from $86;29,42^{\mathrm{p}}$ and take half of the remainder, which is $44;52,27^{\mathrm{p}}$, we obtain line $FE$, approximately $22;26,13^{\mathrm{p}}$ (in the parts in which line $TE$ is $26;31,58^{\mathrm{p}}$).[94] So, of the parts in which line $TE$, the hypotenuse, is $120^{\mathrm{p}}$, line $FE$ is $101;28^{\mathrm{p}}$,[95] and the arc that it subtends is $115;28^{\circ}$ (in the degrees in which the circle around right triangle $ETF$ is $360^{\circ}$).[96] So, angle $ETF$, which is equal to angle $FEB$, is $115;28^{\circ\circ}$ (in the degrees in which two right angles are $360^{\circ\circ}$); so, as for the degrees in which four right angles are $360^{\circ}$, it is $57;44^{\circ}$. Since this angle is at the center of the equator, arc $BM$ will also be $57;44^{\circ}$.

So, if we subtract from this the rising-times of the signs that are at the equinoctial points, which we showed are $27;50^{\circ}$,[97] we obtain the remaining time degrees, $29;54^{\mathrm{t}}$, in which each of these signs – that is Aquarius, Taurus, Leo and Scorpio – rise on the upright sphere.[98] Clearly, each of those[99] four remaining signs – that is Sagittarius, Capricorn, Gemini and Cancer – rises in the time degrees that remain from a quadrant, $90^{\mathrm{t}}$, which is $32;16^{\mathrm{t}}$. This is consistent with what we showed with respect to the solid sphere.[100]

## [10]

Following that, we consider whether on the inclined sphere these same rising-times of the signs, which we previously mentioned, are also attainable according to what is in this diagram.[101]

By way of example, we again use the circle parallel to the equator that we used in the *Almagest*, namely the circle that passes through the island of Rhodes. In this circle, the height of the north pole above the horizon is $36^{\circ}$. As for the horizon drawn by means of the circles parallel to the equator – whose distance is the distance we previously showed – its radius is $102;4,45^{\mathrm{p}}$ (in the parts in which the radius of

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93 *Planis.* 5.

94 $44;52,27^{\mathrm{p}}/2 = 22;26,13,30^{\mathrm{p}}$. These computations follow the metrical analysis in *Planis.* 8.3.

95 Calculation gives $(22;26,13^{\mathrm{p}}/26;31,58^{\mathrm{p}})120^{\mathrm{p}} = 101;28,33^{\mathrm{p}}$.

96 $Arc(101;28^{\mathrm{p}}) = 115;27,46^{\circ}$.

97 *Planis.* 8.2.

98 There are 360 time degrees in a sidereal day. Hence, $1^{\mathrm{t}} = 4$ min.

99 Literally, "these" (هذه) again, but the intention must be to distinguish between these new signs and those in the previous sentence.

100 *Alm.* I 16 [Heiberg 1898–1903, p. 1, 84; Toomer 1984, 73].

101 Here "this diagram" (هذه الصورة) refers to the planisphere generally, not to one of the individual diagrams in the treatise.

the equator is $60^{\text{p}}$), and the line which is between the center of this horizon and the equator is $82;35,3^{\text{p}}$ (of these parts).[102]

[See Fig. 6] Then we make the equator $ABGD$ about center $E$, and the ecliptic $ZBHD$ about center $T$. Since point $E$ had been assumed to be the north pole, we imagine the motion of the sphere as though it is from point $D$ toward point $G$, then to point $B$, then to point $A$. First, we draw two arcs of this one of the horizon circles passing through the solstitial points, points $Z$ and $H$. Let them be arcs $ZKHL$ and $ZMHN$.[103] So, clearly, when the position of the horizon is that of arc $ZKHL$, what is at point $Z$ and at point $K$ is rising and what is at point $H$ and at point $L$ is setting. When its position is that of arc $ZMHN$ the situation is opposite. That is, what is at points $N$ and $H$ is rising and what is at points $M$ and $Z$ is setting, since the motion of the sphere is indeed as though it is from point $D$ toward point $G$, and pole $E$ had been assumed as always visible.

[10.2] Since we have shown that not only does the circle through the signs bisect the equator but at the same time the horizon circle drawn in this way also bisects it, it follows from this that the straight lines that cut off the places of intersection, namely line $KL$ and line $MN$, pass through center $E$.[104] So, clearly, arc $KA$ of the equator is again equal to arc $GL$, and arc $AM$ is equal to arc $GN$.

Arc $MA$, however, is also equal to arc $KA$.[105] That is to say, if we put the two centers of the horizon circle at these places, point $S$ and point $O$, and we join lines $SE$, $OE$ and $STO$, then line $STO$ is straight and at right angles to line $ZH$, line $SE$ is at right angles to line $KL$, and line $OE$ is at right angles to line $MN$, [since] lines on which perpendiculars from the center fall, are bisected by the perpendiculars.[106] [So,] the sides of each of the triangles $ETO$ and $ETS$ will be equal to the sides of the other, and they will be right, and angle $TEO$, of triangle $ETO$, is the equal to angle $TES$ of triangle $ETS$.[107] Angle $MEO$, however, is equal to angle $KES$, for both of them are right. Hence, the complementary angle $MEA$ is equal to the complementary angle $KEA$. Therefore, arc $MA$ will be equal to arc $KA$. Hence, the arcs that begin at points $K$, $M$, $L$ and $N$ and end at points $A$ and $G$ are equal, and also arcs that begin where we said and end at points $B$ and $D$ are equal.

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[102] These values are derived in *Planis.* 7.

[103] Ptolemy imagines the motion of the sphere by changing the position of the $r$-horizon (see page 122). Here we have two positions of a single horizon, as usual neglecting any consideration of the southern hemisphere.

[104] *Planis.* 3.

[105] The logical connection of the statements that follow seems to have been lost in the text.

[106] The forgoing statements follow from the fact that lines $ST$, $OT$, $SE$ and $OE$ are perpendicular bisectors joining centers to chords in circles $ZKHL$ and $ZMHN$ (*Elem.* III 3).

[107] The argument seems to proceed by an implicit appeal to symmetry, since the perpendicular bisectors of equal circles fall on equal chords.

Now since arc $BH$ rises with arc $BN$ and arc $ZB$ with arc $KB$, and this arc is equal to arc $BN$, therefore arc $DZ$ also rises with arc $DK$ and arc $HD$ with arc $DN$, and this arc is equal to arc $DK$. So, it is also clear from this that arcs of the circle through the signs that are the same distance from one and the same equinoctial point rise in equal times. [10.3]

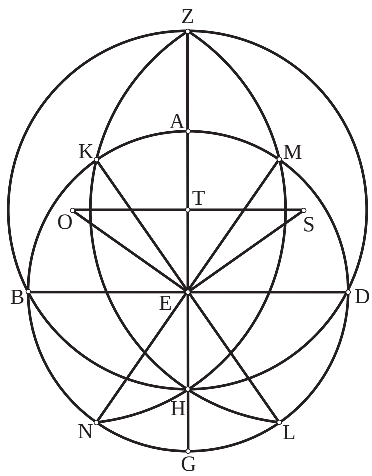


**Figure 6**: *Planisphere* 10.

Furthermore, since arc $ZB$ is less than the rising-time of the upright sphere by arc $KA$, and arc $HD$, the diametrically opposite arc, exceeds the rising-time of the upright sphere by arc $GN$,[108] and this arc is equal to arc $KA$, and point $H$ is the summer solstice, clearly in this diagram as well, the magnitude of the decreases of the arcs of the ecliptic that are at the vernal point from the rising-times on the upright sphere is the magnitude of the increase of the arcs equal to them and diametrically opposite them above these same rising-times.[109] Knowledge of this fact makes it obvious that the shortest period of daylight is less than the equinoctial daylight by arcs $KA$ and $NG$, and the longest period of daylight is greater than the equinoctial daylight by these two arcs.[110] [10.4]

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[108]Here *maṭāliʿ* (مطالع), translated as "rising-time," clearly means the co-ascendant arc of the equator from which the rising-time is determined (see page 45).

[109]This passage introduces the arc of *ascensional difference*, which is the key to Ptolemy's "easier and more methodical" (εὐχρηστότερον καὶ μεθοδικώτερον) technique for computing the rising-times on the inclined sphere developed in *Alm.* II 7 [Heiberg 1898–1903, p. 1, 125; Toomer 1984, 94–95]. The ascensional difference of an arc of the ecliptic, which is a characteristic of geographic latitude, is the difference between the time it takes that arc to rise at any given latitude and the time it takes the same arc to rise at the equator. It will form the only basis for computing oblique rising-times offered in this text. Ascensional difference is discussed by Neugebauer [1975, 36–37].

[110]The term translated here and in the following as "daylight" (النهار) is simply the word for "the

**[11]**

Now that we know this, let us next consider whether, at this assumed latitude, the excess between the longest, or shortest, period of daylight and the equinoctial daylight is found to be consistent with what occurs with respect to the solid sphere.

[See Fig. 7] So, we assume a diagram similar to this diagram in which there is only the horizon that passes through points $Z$, $K$, $H$ and $L$. Let our aim be to find the magnitude of arc $KA$. So, we again make the center of the horizon circle in this configuration point $S$. We join lines $ST$ and $SE$, so they are perpendiculars to lines $ZH$ and $KL$, from what we previously proved. So, because we have shown that line $ES$, which is the line between the center of the equator and the center of the horizon circle of this assumed latitude, is $82;35,3^{\mathrm{p}}$ (in the parts in which it was assumed that line $ET$, the line between the center of this circle and the center of the ecliptic, is $26;31,58^{\mathrm{p}}$),[111] hence, of the parts in which line $ES$, the hypotenuse, is $120^{\mathrm{p}}$, line $ET$ is also approximately $38;33^{\mathrm{p}}$, and the arc on it is $37;30°$ (in the degrees in which the circle about right triangle $EST$ is $360°$).[112] Hence, angle $TSE$, which is equal to angle $AEK$, is $37;30^{\circ\circ}$ (in the degrees in which two right angles are $360^{\circ\circ}$) and it is $18;45°$ (in the degrees in which four right angles are $360°$). Since this angle is at the center of the equator, arc $AK$ will also be $18;45°$.

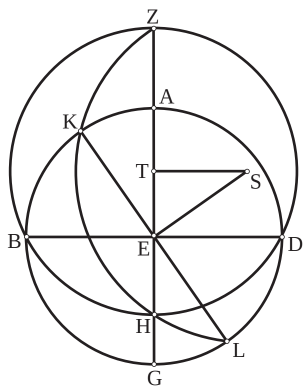


**Figure 7**: *Planisphere* 11.

Hence, both of the quadrants at the vernal point are $71;15^{\mathrm{t}}$, and both of the quadrants at the autumnal point are $108;45^{\mathrm{t}}$.[113] Therefore, the excess between the longest, or shortest, period of daylight and the equinoctial daylight is $37;30^{\mathrm{t}}$, and

day." This word, however, can mean either the period of a full day or the period of that in which the sun is above the horizon.

[111] These values are derived in *Planis.* 7 & 4.3.

[112] Calculation gives $(26;31,58^{\mathrm{p}}/82;35,3^{\mathrm{p}})120^{\mathrm{p}} = 38;33,14^{\mathrm{p}}$ and $Arc(38;33^{\mathrm{p}}) = 37;28,37°$.

[113] By *Planis.* 10.4, $90° - 18;45° = 71;15°$ and $90° + 18;45° = 108;45°$.

in equinoctial hours $2^1/_2{}^{h}$, just as we showed with respect to the solid sphere.[114]

**[12]**

We will, however, also find the rising-times of the signs at this assumed latitude. We set out, in this way, the equator and the circle through the signs around the diameters $BD$ and $ZH$, and we cut off arc $BT$ from the circle through the signs. [See Fig. 8]

First, let it be the arc of one sign, obviously the sign of Pisces. We join line $TEL$, and we describe the circle of the previously assumed horizon circle passing through points $T$ and $L$. Let it cut the equator at points $M$ and $N$. We join line $MEN$ and we again produce two straight lines, $SE$ and $ST$, from point $S$, the center of the horizon circle, and we produce from it a perpendicular to line $TL$, which is line $SO$.

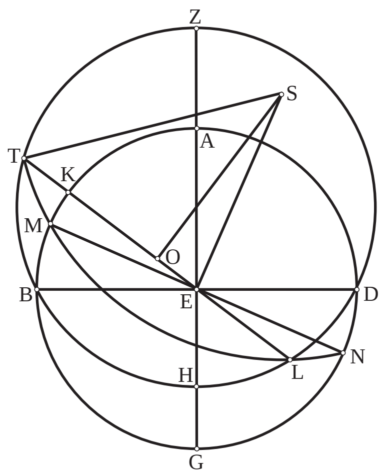


**Figure 8**: *Planisphere* 12.

We have shown, just above, that arc $KM$ is that by which the sign of Pisces and the sign of Aries fall short, on either side, of their rising-times on the upright sphere, and by which the sign of Virgo and the sign of Libra exceed their rising-times on the upright sphere.[115]

We have shown that line $ET$, the radius of the circle parallel to the equator drawn at the beginning of Pisces, is $73;39,7^{p}$ (in the parts in which line $ES$, the line between the center of the equator and the center of the horizon circle, is assumed to be $82;35,3^{p}$), and that the excess of line $TS$ squared over the line $ES$ squared is $3600^{p}$ (of these parts).[116] If, in the same way as we did above, we divide that

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[114] $37;30^\circ \times 0;04^{h/\circ} = 2;30^{h}$. The conversions between various characteristics of latitude are discussed in *Alm.* II 2–5. These specific values are derived from the latitude of Rhodes in *Alm.* II 3 [Heiberg 1898–1903, p. 1, 93–95; Toomer 1984, 781].

[115] That is, $KM$ is the ascensional difference discussed in *Planis.* 10.4.

[116] *Planis.* 6, 7 & 8.2, respectively.

by $73;39,7^{\mathrm{p}}$ and we do what follows that, just as we did for the upright sphere, we obtain line $EO$ as $12;23,12^{\mathrm{p}}$ (in the parts in which line $ES$ is $82;35,3^{\mathrm{p}}$).[117] So, of the parts in which line $ES$, the hypotenuse, is $120^{\mathrm{p}}$, $EO$ is approximately $18;1^{\mathrm{p}}$, and the arc on it is $17;16^{\circ}$ (in the degrees in which the circle around triangle $ESO$ is $360^{\circ}$).[118] So, angle $ESO$, which is equal to angle $KEM$, is also $17,16^{\circ\circ}$ (in the degrees in which two right angles are $360^{\circ\circ}$), and it is $8;38^{\circ}$ (in the degrees in which four right angles are $360^{\circ}$). So, arc $KM$ is also $8;38^{\circ}$ (in the degrees in which the equator is $360^{\circ}$).

The rising-times of each of the four assumed signs was $27;50^{\mathrm{t}}$ on the upright sphere.[119] So, when we subtract from them these $8;38^{\mathrm{t}}$, we obtain the rising-time of each of the signs Pisces and Aries as $19;12^{\mathrm{t}}$. Then, when we add those same to this, we obtain the rising-time of each of the signs Virgo and Libra, $36;28^{\mathrm{t}}$.

## [13]

[See Fig. 8] Then, again in a similar diagram we assume arc $BT$ to be the arc of two signs, that is Pisces and Aquarius, so that the rest of what we mentioned remains in the same situation.

So, $ET$, the radius of the circle parallel to the equator drawn at the beginning of Aquarius, is $86;29,42^{\mathrm{p}}$ (of these parts).[120] If that is divided according to $3600^{\mathrm{p}}$, just as we mentioned above, we obtain line $EO$ as $22;26,13^{\mathrm{p}}$ (in the parts in which line $ES$ is $82;35,3^{\mathrm{p}}$).[121] Of the parts in which line $ES$, the hypotenuse, is $120^{\mathrm{p}}$, line $EO$ is also approximately $32;36^{\mathrm{p}}$, and the arc on it is $31;32^{\circ}$ (in the degrees in which the circle drawn around right triangle $ESO$ is $360^{\circ}$). Hence, angle $ESO$, which is equal to angle $KEM$, is $31;32^{\circ\circ}$ (in the degrees in which two right angles are $360^{\circ\circ}$), and it is $15;46^{\circ}$ (in the degrees in which four right angles are $360^{\circ}$). Hence, arc $KM$, the combined difference between the rising-times of the assumed signs and the rising-times on the upright sphere, is $15;46^{\circ}$ (in the degrees in which

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[117] The method of computation is given in *Planis.* 8.2, $EO = (ET - 3600/ET)/2$. That is, $(73;39,7^{\mathrm{p}} - 3600/73;39,7^{\mathrm{p}})/2 = 12;23,12^{\mathrm{p}}$.

[118] Calculation gives $(12;23,12^{\mathrm{p}}/82;35,3^{\mathrm{p}})120^{\mathrm{p}} = 17;59,55^{\mathrm{p}}$.

[119] *Planis.* 8.2.

[120] *Planis.* 5.

[121] The Arabic reads "If that is divided by" using the standard preposition for arithmetic division (على). Since, however the operation intended is not what we mean when we say "divided by," we have translated with "divided according to" in order to distinguish this algorithm from standard division. The computation follows *Planis.* 8.2: $EO = (ET - 3600/ET)/2$, so that $(86;29,42^{\mathrm{p}} - 3600/86;29,42^{\mathrm{p}})/2 = 22;26,13^{\mathrm{p}}$.

the equator is $360°$).[122]

Their rising-times on the upright sphere were $57;44^{t}$,[123] so if we subtract these $15;46°$ from $57;44°$, we obtain the rising-time of Pisces and the rising-time of Aquarius together as $41;58^{t}$. As for the rising-time of Aquarius alone, it is $22;46^{t}$, because Pisces rises in $19;12^{t}$.[124] If we add the $15;46°$ to $57;44°$, we obtain the rising-time of Leo and the rising-time of Virgo. Their rising-times, when summed, are $73;30^{t}$. So, as for the rising-time of Leo alone, it is $37;2^{t}$, again from the fact that Virgo rises in $36;28^{t}$.[125] Clearly, Taurus also rises in times equal to the times of the rising of Aquarius, $22;46^{t}$, and Scorpio rises in times equal to the times of the rising of Leo, $37;2^{t}$, the rising of both Capricorn and Gemini is in the remaining times in this quadrant, which is $29;17^{t}$, and the rising of both Cancer and Sagittarius is in the remaining times from that quadrant,[126] which is $35;15^{t}$, as befits our original aim.[127]

**[14]**

So, we have also shown in this diagram, which is with respect to a flat surface, that the matter of the rising-times of the signs of the circle through the signs, and everything which follows that, is consistent with what we showed with respect to the solid sphere.[128]

[Now,] however, we make the diagram of a size appropriate to the given situation,[129] in which we want to draw what we mentioned, and such that it is prepared for us to draw the configuration of the fixed stars on it, if we want that. [14.2]

If we want to set out on it the thing that, particularly in horary instruments, is called the *spider*, then we set out the circle that is outside of all the circles,

[122] That is, $KM$ is the sum of the ascensional differences of the two signs. The word translated as "the combined" (المشترك) usually simply means "common." Here, however, it clearly means the sum.

[123] *Planis.* 9.

[124] *Planis.* 12.

[125] *Planis.* 12.

[126] Literally, "this quadrant" as well, but the intention must be to distinguish between them.

[127] The rising times of the quadrants are calculated in *Planis.* 11. Hence, $75;15^{t} - (19;12^{t} + 22;46^{t}) = 29;17^{t}$ and $108;45^{t} - (36;28^{t} + 37,2^{t}) = 35;15^{t}$.

[128] *Planis.* 8–13.

[129] The expression for the size of the diagram is literally "commensurate with the size" (على حسب مقدار), which probably translates something like σύμμετρον τῷ μεγέθει, used with variants three times in the *Almagest* [Heiberg 1898–1903, p. 1, 64, 351 & 403]. In a similar vein, in the Latin translation of the *Optics*, the reader is instructed to set up a bronze disk "of moderate size" (*moderate quantitatis*) [Lejeune 1989, 91]. The word translated as "situation" (موضع) can also simply mean "place." In this context, however, it probably carries the more abstract meaning.

[See Fig. 9] and the greatest of them, circle $ABGD$ around center $E$. We draw two diameters intersecting at right angles representing meridians. Let them be lines $AG$ and $BD$. We cut off arc $DZ$, beginning from point $D$. Let its magnitude be the magnitude of the distance of the assumed circle parallel to the equator from the south pole on the solid sphere. We produce a line parallel to line $ED$ from point $G$. Let it be line $GH$. We join line $DZH$ and we produce a perpendicular to line $DE$ from point $H$, which is line $HT$.

Then, I say that if we do as we did in the preceding, so that, beginning from point $G$, we cut off an arc that is the distance of each of the remaining circles parallel to the equator on the corresponding side [of $G$], and we join straight lines from point $D$ and between the endpoints of the arcs that we cut off, as line $DKG$ – for example, if our aim is to draw the equator, and we make line $EL$ equal to line $TK$, and point $E$ a center and describe circle $LMS$ with a distance equal to distance $EL$, then the position of this circle is that of the equator.

Each of the remaining circles is [drawn] in this way. That is, if we do the opposite of that, as we showed in the preceding, so that we assume the equator is circle $LMS$ and we draw in its plane a circle parallel to it, whose distance from it to the south is in the size of an arc similar to arc $GZ$, then the circle that we will draw is circle $ABGD$. So, we join line $MG$ and let it intersect circle $LM$, the equator, at point $N$, so that the circle $ABGD$ is drawn around center $E$ with distance $EG$, just as we did above.

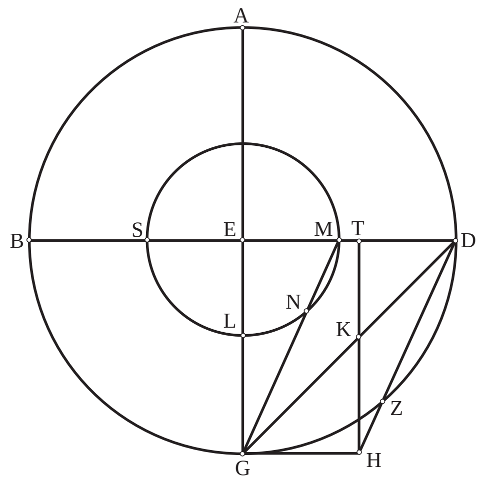


**Figure 9**: *Planisphere* 14.

I say that arc $MN$ is similar to arc $DZ$. The proof of this is that the ratio of line $DE$ to line $EG$ is as line $DT$ to line $TK$,[130] and line $DE$ is equal to line $EG$, so line $DT$ is also equal to line $TK$. Line $TK$, however, is equal to line $EM$, so line $DT$

[130] *Elem.* VI 3.

is also equal to line $EM$. Line $TH$ is also equal and parallel to line $EG$, hence line $DZ$ is parallel to line $MN$.[131] So, angle $EMN$ is equal to angle $EDZ$, hence arc $SLN$ is similar to arc $BGZ$, so the remaining arc $MN$ is similar to the remaining arc $DZ$.

**[15]**

We should also achieve our aim by showing how we draw the circles whose situation relative to the circle through the signs is as that of the circles previously mentioned relative to the equator, so that we can set out the stars whose positions are observed and determined in their measure according to this sphere without, first, using their sides in their measure according to the equator.[132]

So, first, let the equator, one of the circles set out on the plate, be circle $ABGD$ [15.2]
around center $E$, and the ecliptic circle $ZBD$, and the straight line that goes through [See Fig. 10]
both two poles line $ZAEHG$, and the line passing through the place of intersection with the equator line $BED$. If we cut off arc $BT$ and we make it equal to the arc between the poles of the equator and the circle through the signs, and we join line $DKT$, then point $K$ is functionally a correlate of the pole of the circle through the signs.[133] Clearly, this is in accordance with what we explained.

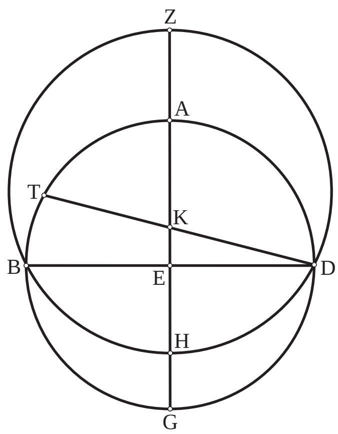


**Figure 10**: *Planisphere* 15.

The circles passing through this point and diametrically opposite points on the [15.3]
circle through the signs are bisectors of the equator as well. These drawn circles are those that stand in for the great circles perpendicular to the ecliptic, because

[131]Since $\triangle DTH$ is congruent with $\triangle EMG$.

[132]The term "sides" (اضلاع) indicates that the coordinates of a star were regarded as the sides of a spherical quadrilateral.

[133]Again, it seems strange to claim that point $K$ is the "correlate ... functionally" (نظيرة ··· بالقوة), since it is indeed the correlate (see page 84).

we have shown, in general, that all circles that diametrically intersects one of these assumed circles, diametrically intersects the other, remaining circle as well.[134]

## [16]

We can also set out, on the plate, the circles parallel to the ecliptic in this way.

[See Fig. 11] [So,] we make the meridian that passes through the two poles circle $ABGD$ around center $E$. Let the axis be $BED$, and we imagine point $D$ as the hidden pole, the diameter of the equator as $AEG$, and the diameter of one of the circles parallel to the ecliptic as line $ZHT$. Let our aim be to set out, on the plate, the circle that has this line as its diameter. So, we pass a line through point $H$ parallel to line $AEG$. Let it be line $LHK$. We join lines $DMZ$, $DNT$ and $DSL$.

Clearly, the circle whose diameter is line $ZT$ is drawn around diameter $MN$. That is to say, it touches the two circles parallel to the equator, whose distance from it are in [the size of] the arcs $AZ$ and $GT$. Therefore, these circles are drawn with distances $EM$ and $EN$.[135]

When, however, the circle parallel to the ecliptic, whose diameter is line $ZT$, bisects the circle parallel to the equator, whose diameter is $LK$, at the meridian, whose diameter is line $BD$,[136] and this circle, also, is drawn with distance $ES$, as circle $SOF$,[137] then, we must show that the circle drawn around diameter $MN$ passes through points $O$ and $F$.

We join lines $BZ$ and $BQ$, and produce lines $KL$ and $DT$ until they meet at point $R$. So, because angles $BZQ$ and $BHQ$ are right,[138] points $B$, $H$, $Q$ and $Z$ are on the circumference of a single circle.[139] So, angle $BQR$ is equal to angle $BZT$, which is equal to angle $BDR$,[140] so points $B$, $R$, $D$ and $Q$ are also on the circumference of a single circle,[141] and that which is the product of line $QH$ by line $HR$ is equal to that which is the product of line $BH$ by line $HD$.[142] Since it is like that, it is equal to line $HL$ squared.[143] So, line $ME$ by line $EN$ is also equal to line $ES$ squared,[144] which is equal to line $FE$ by line $EO$, hence points $M$, $O$, $N$ and $F$ are again on

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[134] *Planis.* 3.

[135] *Planis.* 1.

[136] This meridian is, in fact, the equinoctial colure.

[137] *Planis.* 1.

[138] $\angle BHQ$ is right by construction, while $\angle BZQ$ is the angle in a semicircle (*Elem.* III 31).

[139] Converse of *Elem.* III 31.

[140] *Elem.* III 21.

[141] Converse of *Elem.* III 21.

[142] *Elem.* III 35.

[143] Since $\triangle$s $BHL$, $HLD$ and $BLD$ are similar.

[144] Since $\triangle MED$ is similar to $\triangle QHD$ and $\triangle MND$ is similar to $\triangle QRD$.

the circumference of a single circle.[145]

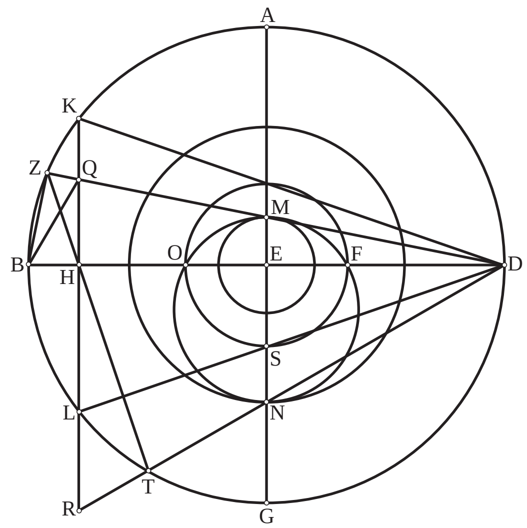


**Figure 11**: *Planisphere* 16.

## [17]

We should also show that the centers of the circles parallel to the ecliptic that are drawn in this way are always different.

Let the meridian passing through both poles be circle $ABGD$ around center $E$, and the axis line $BED$, and the diameter of the equator line $AG$, and the diameters of two circles parallel to the ecliptic lines $ZH$ and $TK$. We join lines $DLZ$, $DMH$, $DNT$ and $DSK$, and we draw circle $OSF$ around triangle $DNS$.[146] We join line $OF$ and bisect line $LM$ at point $Q$.[147] So, clearly, the circle parallel to the ecliptic, which is on its diameter $ZH$, is drawn on diameter $LM$ and the circle parallel to the ecliptic, whose diameter is $TK$, is drawn around diameter $NS$.[148] [See Fig. 12]

I say that the center of these circles is not one and the same. That is, point $Q$ does not also bisect line $NS$. The proof of this is that arc $ZT$ is equal to arc $KH$, therefore angle $ZDT$ is equal to angle $HDK$ and arc $NO$ is equal to arc $SF$.[149] So, lines $LM$ and $OF$ are parallel. Then, the ratio of line $DL$ to $LO$ is as the ratio of line $DM$ to $MF$,[150] but the ratio of line $DL$ to $LO$ is as the ratio of line $DL$ squared

[145] Converse of *Elem.* III 35.

[146] *Elem.* IV 5.

[147] *Elem.* I 10.

[148] *Planis.* 16.

[149] *Elem.* III 26 & 27.

[150] *Elem.* VI 2.

to line $DL$ by line $LO$, and the ratio of line $DM$ to $MF$ is as the ratio of line $DM$ squared to line $DM$ by line $MF$.[151] Hence, the ratio of line $DL$ squared to line $DL$ by line $LO$ is as the ratio of line $DM$ squared to line $DM$ by line $MF$. Because of the circle, line $DL$ by line $LO$ is equal to line $SL$ by line $LN$, and line $DM$ by line $MF$ is equal to line $NM$ by line $MS$.[152] So, the ratio of line $DL$ squared to line $SL$ by $LN$ is as the ratio of line $DM$ squared to line $NM$ by line $MS$, and if we alternate, the ratio of line $DL$ squared to line $DM$ squared is as the ratio of line $SL$ by line $LN$ to line $NM$ by line $MS$.[153] Line $DM$ squared, however, is greater than line $DL$ squared, since line $DM$ is longer than line $DL$,[154] so line $NM$ by line $MS$ is greater than line $SL$ by line $LN$. Line $NS$ is common with line $LN$ and with line $MS$, hence, line $MS$ is longer than line $LN$.[155] Line $LQ$, however, is equal to line $MQ$, hence, line $NQ$ is longer than line $QS$. So, point $Q$ is not the center of the circle whose diameter is line $NS$.

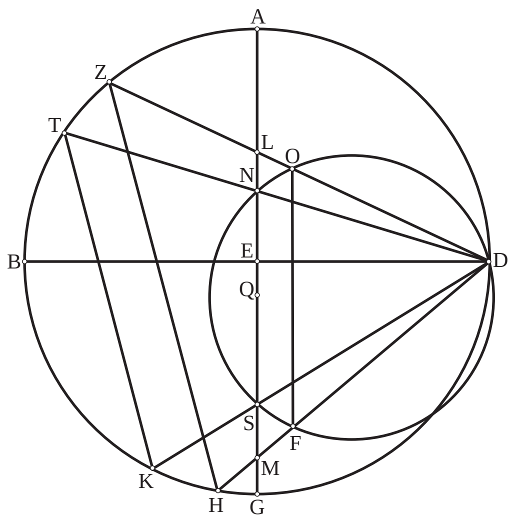


**Figure 12**: *Planisphere* 17.

## [18]

Next, for the situation of circles parallel to the ecliptic not confined to the plate,

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[151] *Elem.* VI 1.

[152] *Elem.* III 36.

[153] *Elem.* V 16.

[154] Since arc $DH <$ arc $DZ$, line $DM$ meets diameter $AG$ farther from diameter $DB$, than line $DL$. Hence, $DM > DL$.

[155] The argument appears to run as follows. Since $MS \times NM > SL \times LN$, we have $MS \times (NS + MS) > LN \times (NS + LN)$. $NS$ being common, $MS > LN$. Maslama gives a geometric argument for this claim involving auxiliary lines [Kunitzsch and Lorch 1994, 22].

part of which falls, rather, in the section of the sphere that is not visible and which is not drawn – that is, circles that intersect the always hidden circle – we must again set out the circle through the two poles as circle $ABGD$ around center $E$.[156] Let the axis be line $BD$. We imagine point $D$ as the hidden pole, line $AG$ as the diameter of the equator, the diameter of the always hidden circle parallel to it as line $ZH$, and the diameter of the circle parallel to the ecliptic that intersects this as $TKL$. [See Fig. 13]

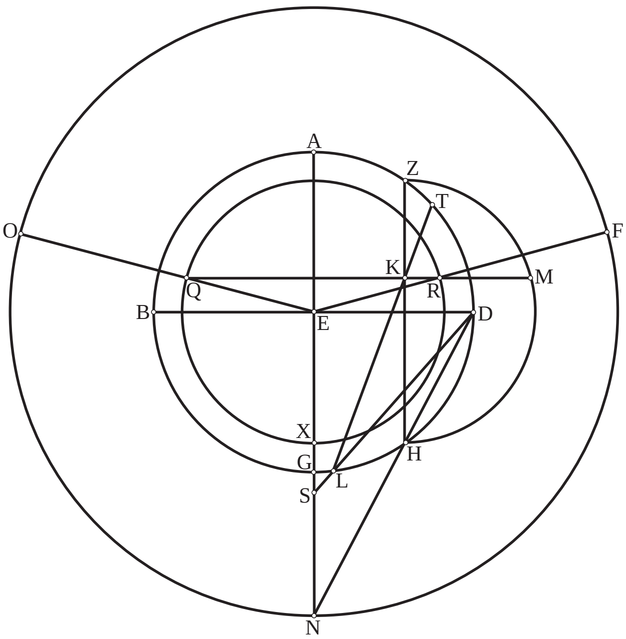


**Figure 13**: *Planisphere* 18.

We draw a semicircle on line $ZH$. Let it be $ZMH$. We produce a line parallel to $ED$. Let it be line $KM$. So, because of the fact that we extend line $AGN$ and the two lines $DHN$ and $DLS$, the circle drawn with distance $EN$, such as circle $ON[F]$, is the always hidden circle on the plate,[157] and the circle that is drawn representing the circle on line $TKL$ again passes through point $S$, and it cuts the always hidden circle in arcs similar to arc $HM$, since line $KM$ is the section common to their planes. Because, if we draw a circle about center $E$ equal to circle $ZMH$ – as if we draw circle $QRX$ – and we produce line $MRKQ$,[158] and we produce lines $EQO$ and $ERF$, then we make arcs $NO$ and $NF$ similar to arcs $XQ$ and $XR$. So, they are

[156] As the first part of this sentence makes clear, this always hidden circle (الدائرة الخفية ابدًا) is the southernmost bounding circle of a given plate. In some cases, this may be the same as the bounding circle of the region of the celestial sphere that never rises for any horizon not on the equator.

[157] *Planis.* 14.

[158] The text had $MKRQ$, following the order of the points in the manuscript diagram. We have changed the order of the letters to reflect the change in the order of the points in the diagram.

similar to arc $HM$, and the circle parallel to the ecliptic drawn on line $TL$ passes, on the plate, through points $O$, $S$ and $F$.[159]

**[19]**

[See Fig. 14] Clearly, in a similar diagram, even if we imagine the circle parallel to the ecliptic drawn through point $D$ – such as if we construct the circle drawn on line $DK$ – and we extend line $DK$ to the mentioned breadth and pass line $MLS$ through point $L$ perpendicular to line $AGN$, then this is the line on the plate representing the circle whose diameter is line $DTK$. For all straight lines produced from point $D$ passing through this circle are in a single plane, the plane of the circle, and the section common to this plane and to the plane of the equator is line $MLS$,[160] for the plane of the meridian through line $AG$ is also at right angles to both of these planes we mentioned.[161]

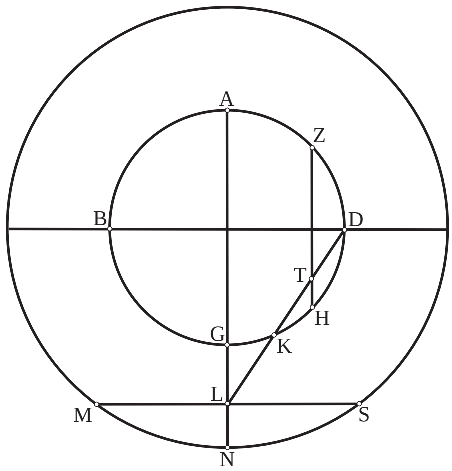


**Figure 14**: *Planisphere* 19.

**[20]**

So, in this way that we showed, an analog to what is on the solid sphere must be drawn on the plate – the circles found by way of the equator (those that are meridians, and those that are parallel to the equator), and the circles found by means of the circle through the signs.

Because, the pole of the equator is, again, a center for this circle and for all circles parallel to it, so [1] all meridians are, indeed, straight lines.[162] [2] The pole of the

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[159] *Elem.* IV 5 demonstrates the construction of a circle through three given points.

[160] *Elem.* XI 3.

[161] *Elem.* XI 19.

[162] *Planis.* 1.

circle through the signs is not a center for this circle nor for even one of the other circles that are correlates to the circles parallel to it.[163] [3] One of these circles is without a center; that is, it is a straight line.[164] [4] Great circles that are drawn and pass through this pole are different from that, for the circle representing the circle through the two poles is a straight line, on which fall the centers of the circles parallel to the ecliptic, and the remaining circles are circles, but they are unequal.[165]

It follows from this that we can draw the stars in the locations[166] found in the [20.2]
measure with respect to the equator, without drawing all circles, with only a division of the equator and a ruler divided according to the ratios of the circles parallel to the equator. As for the locations found in the measure with respect to the circle through the signs, this is not possible.[167] We must, rather, draw every circle, or most of them, in order to be guided by them regarding the positions in which the stars must be set out.

It would be best insofar as it is complete with respect to both of these drawings used on the solid sphere that we set out the circles [found] by means of the equator (those that are meridians, and those that are parallel to the equator), and the circles found by means of the circle through the signs, just as on the inscribed spheres.[168] So, if it is not possible to draw all of that on the plate, we should draw on it the circles that pass through 2°, 3°, or 6° (since this is an intermediary drawing) because these three numbers are factors[169] of 30° (the degrees of each of the signs) and 24°

[163] *Planis.* 1, 15 & 17.

[164] *Planis.* 19.

[165] These circles are introduced in the last paragraph in *Planis.* 15, although neither of the specific features mentioned here are discussed.

[166] The plural of *waḍʿ* (اوضاع), translated as "locations" here and in next sentence, can also mean "conventions" and may carry the sense of coordinates, $S(\delta, \alpha)$ or $S(\lambda, \beta)$. Hipparchus, apparently, recorded the fixed stars in equatorial coordinates [Duke 2002].

[167] Ptolemy records the stars in ecliptic coordinates in his star catalog (see *Alm.* VII 5–VIII 1).

[168] The word translated as "inscribed" (مضروب) literally means "struck" and probably refers to the method of producing the inscribed image. A similar usage of this root is found in a description of two astrolabes by a certain Ibrāhīm ibn Mamdūd al-Jallād al-Mawasilī, who admired the way the two instruments were "cast and inscribed" (سبك وضرب) [King 2005, vol. 2, 643-644]. Ptolemy is presumably referring to a well-known type of ancient star globe that included coordinate circles as guides, some set of the fixed stars and perhaps images of the constellations. Geminus, in his *Introduction to the Phenomena* (V 62–65), makes some offhand references to such inscribed spheres [Aujac 1975, 31–32; Evans and Berggren 2006, 159]. In particular, he notes that the horizon and the local meridian are not generally included among the inscribed lines. The construction of a more sophisticated star globe is described in *Alm.* VIII 3 [Heiberg 1898–1903, p. 2, 179–186; Toomer 1984, 404–407].

[169] Literally, "common numbers" (عدد مشترك).

(the approximate distance between the equator and each of the tropics) so that it happens that the two tropics and the meridians through [the beginnings of] the signs are among the circles that are drawn, and there is no difference with respect to radii that are not found in this way.

The end of the treatise of Ptolemy of the people
of Claudia *On Flattening the Surface of the Sphere.*

Praise to God, and his blessings on his prophet Mohammad,
his family and companions, and may he grant peace.

# VI Commentary

In the notes accompanying our translation, we have addressed specific textual issues and provided references for following the details of Ptolemy's arguments in the context of ancient mathematics. While these should be sufficient for understanding the steps of the proofs, a reader will, nevertheless, often be left with questions about Ptolemy's overall approach. Most of these questions arise because Ptolemy assumes a fairly advanced level of background knowledge on the part of his reader. By reading the entire treatise with an eye to what is demonstrated and what is assumed, we can, at once, develop a better understanding of Ptolemy's methods and a better sense of the readership Ptolemy saw himself as addressing.

Ptolemy's reader is assumed to have a good grasp of the principles of ancient spherical astronomy and specifically to have already mastered books I and II of the *Almagest*. There are many references to the spherical astronomy of the *Almagest* and the reader is expected to know the subjects covered, the methods developed and the specific results obtained. Ptolemy often contrasts the solid geometric methods of the *Almagest* with the planar approach of this treatise.

Most significantly, however, the reader is also assumed to already have some familiarity with the ancient geometric methods used for producing a plane diagram of the sphere that is mathematically equivalent to that produced by stereographic projection. Ptolemy, however, often proceeds in a way that is unexpected from the perspective of projective geometry [Berggren 1991, 138–142]. Hence, in reading this text, it is often more useful to situate his methods in the context of ancient solid geometry than in that of projective geometry as it was developed by medieval and early modern mathematicians. Hence in our commentary, we generally describe these aspects of Ptolemy's procedures in terms of conic theory, solid geometry and the methods of the ancient analemma.[170]

As *Planis.* 15–19 make explicit, the reader is assumed from the beginning to know that the geometric objects under discussion are constructed in various ways on a cutting plane by joining straight lines between key points on the sphere and the south pole. This construction produces a plane diagram that is mathematically equivalent to that produced by stereographic projection with the south pole as the point of projection. In medieval and, especially, early modern texts, discussions of stereographic projection are developed on the basis of two fundamental theorems, at least one of which is demonstrated at the outset.

The first of these, which we call the *circle preservation* theorem, states that the projection of any circle not passing through the point of projection is also a

[170]For overviews of the ancient and medieval analemma see Evans [1998, 132–141], Berggren [1980] and Neugebauer [1975, 839–856]. See, also, Sidoli [2005] for a discussion of the use of the analemma as a method for solving computational problems in spherical astronomy.

circle, while the projection of a circle passing through the point of projection is a straight line [Neugebauer 1975, 858–859]. The second of these, which we call the *conformality* theorem, states that the angle of intersection between any two circles, defined as the angle of the tangents at the intersection, is preserved in the projection [Neugebauer 1975, 859–860]. The earliest explicit proof of circle preservation that has survived is that of Aḥmad ibn Muḥammad ibn Kathīr al-Farghānī [Thomson 1978, 212–215, trans. of the Russian by N. D. Sergeyeva and L. M. Karpova], while the first published proof of conformality is due to Edmond Halley [1695, 204–205].[171]

Any reading of the present text must confront the ways in which Ptolemy handles these two fundamental theorems. Our reading is based on the hypothesis that Ptolemy knew a simple proof of circle preservation and assumed his readers would be familiar with this, but that he did not know any general proof of conformality and, hence, demonstrated individual cases of properties of the planisphere that are mathematically related to conformality.

As will be shown below, the proof of circle preservation is straightforward and very likely within the scope and level of background knowledge Ptolemy assumed on the part of his reader. A proof of conformality, however, is not quite so simple and, as Halley [1695, 204] says, "this not being vulgarly known, must not be assumed without a *Demonstration*." Moreover, if Ptolemy had known a general proof of conformality, many of the theorems he does give could have been stated as trivial corollaries.

Generally, what Ptolemy shows is that the angular distance between points on the equator, or on a $\delta$-circle, is preserved in the planisphere, which we will call *orthogonal angle preservation*. This could be shown from conformality by an indirect argument, however, Ptolemy will prove it for individual cases (*Planis.* 1, 8–13 & 16), presumably because, like Halley, he considered these things not generally known and hence worthy of proof. One other case of conformality is demonstrated in *Planis.* 3, in which Ptolemy shows that the intersections of circles that represent two great circles oblique to the equator correspond to diametrically opposite points. It is worth noting that the topics that the *Planisphere* addresses can all be successfully handled using the individual cases that Ptolemy demonstrates.[172]

Throughout the treatise Ptolemy often frames a proposition in terms of specific

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[171] An earlier proof of a property equivalent to conformality is preserved in the unpublished notes of Thomas Harriot [Pepper 1968, 411–412]. We will address the claim by Commandino [1558, f. 25v; Sinisgalli 1993, 146–147] that *Planis.* 16 is a proof of circle preservation in our commentary, as well as the claim by more recent readers, such as Heath [1921, 292] and Lorch [1995], that a number of the proofs concern circle preservation.

[172] Although Neugebauer [1975, 858] claims that only circle preservation was "recognized" by Greek mathematicians, the whole first part of the treatise is an argument for the conformality of the circles relevant to rising-time phenomena.

objects, such as the equator and the ecliptic, and then later asserts a more general theorem, such as concerning any oblique great circles. In general, in order to understand Ptolemy's methods, it is necessary to pay as much attention to how he carries out his proofs as to what he has to say about them. Ptolemy will often assume that the reader can supply an argument for generality by realizing that the methods of a proof can be successfully applied to similar configurations.[173] In our commentary, we will point out how proofs that Ptolemy asserts about specific objects contain more general claims and where his arguments are actually sketches of more broadly applicable methods.

The diagrams in the commentary are meant to be viewed in conjunction with those accompanying the text. Where the same object appears in both diagrams, it is given the same letter name, and where the objects are not the same but are closely related they are differentiated by primes (for example, $A$ and $A'$). Lines that are found in the original diagram are drawn in the same weight in the diagrams in the commentary, although not all original lines are included. Auxiliary lines, which are added to the diagrams in the commentary, are drawn in half weight. Where both the planisphere and the solid sphere appear in the same diagram, objects in the planisphere are highlighted by being drawn in grey.

## *Planisphere* **1**

The first section introduces the reader to the construction of objects in the plane that will stand in for objects on the sphere. As Ptolemy states, the fundamental objects are the equator, the $r$-$\delta$-circles and the $r$-meridians.

The key to understanding the treatise is the realization that Ptolemy is thinking of the planisphere as formed on a cutting plane intersecting the solid sphere. In order to distinguish Ptolemy's approach from that of pointwise projection, we will call the plane of the diagram the *cutting plane*. The underlying solid geometry is only implicit in *Planis.* 1–7, but it becomes explicit in the second part of the treatise and the style of argument that we develop here can be found in *Planis.* 15, 16, 18 & 19.

Following a common practice in Greek solid geometry, most of the diagrams in the treatise represent two, or more, different planes folded into the plane of the diagram.[174] In *Planis.* 1, the first of these is the plane of the equator, the second that of the solstitial colure.[175] Hence, in Figure 1, Ptolemy's point $D$ represents both the autumnal equinox and the south pole ($D$ and $D'$ in Figure 15).

[173] See Netz [1999, 240–270], for a more general discussion of the problem of producing generality in Greek mathematics.

[174] The exceptions are the diagrams to *Planis.* 3, 8, 10–12, which all represent a single plane.

[175] Lorch [1995, 271–273] discusses this section with two separate figures to make explicit which objects are in which plane.

In the solid configuration, the plane of the equator is the cutting plane and the south pole is the point of projection. In *Planis.* 1.2, Ptolemy simply assumes that the $\delta$-circles are represented by circles and the meridians by straight lines. Figure 15 shows why Ptolemy is justified in making these assumptions.

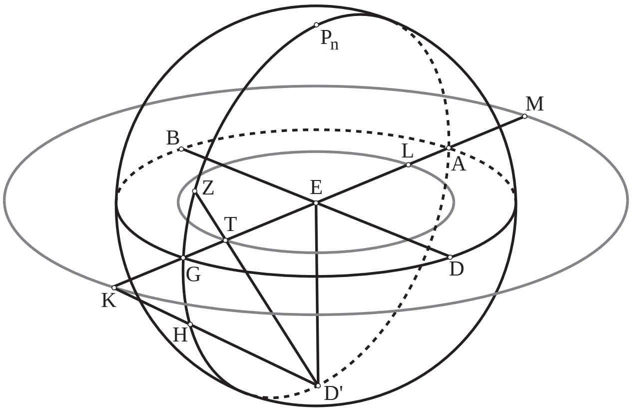


**Figure 15**: Perspective diagram of *Planis.* 1.

The cutting plane is that of the equator, $ABGD$, and two equal $\delta$-circles are imagined through points $Z$ and $H$. On the one hand, the $\delta$-circles are joined to the south pole, $D'$, by right cones whose axis, $ED'$, is perpendicular to the cutting plane. Hence, as Ptolemy states, the $\delta$-circles to the north of the equator are represented by circles inside the equator and those to south by circles outside of it. The points of the meridians, on the other hand, are joined to the south pole by lines all of which lie in planes that are perpendicular to the cutting plane and which pass through both the center of the equator and the north pole. Hence, the meridians are represented by straight lines through the center, $E$, which obviously represents the north pole.

As becomes clear in *Planis.* 1.3, however, Ptolemy also assumes, without proof, that every circle in the sphere is represented by a circle in the cutting plane. This means that there was probably a simple proof of this fact that Ptolemy could assume his readers knew.

Such a proof would be straightforward within the context of Greek geometry. Since all circles on the sphere are joined to the point of projection by cones, we can provide a simple proof based in conic theory.[176]

In *Conics* I 4 & 5, Apollonius establishes the conditions under which a cutting plane will intersect a cone in a circle. This occurs when the cutting plane is perpendicular to the axial triangle and cuts the latter in a similar triangle. The new triangle formed by the cutting plane can be similar either because (1) the base of

[176] There are medieval proofs based in similar considerations from conic theory by al-Farghānī and Jordanus of Nemore [Thomson 1978, 86–98 & 212–215]. See also Heath [1921, v. 1, 292–293] and Neugebauer [1975, 858–859].

the new triangle is parallel to the base of the axial triangle or (2) it is arranged in the position Apollonius calls subcontrary (ὑπεναντία) [Heiberg 1891, vol. 1, 18]. This serves as the basis for the following proof.

If, in Figure 16, point $P$ is the point of projection, the object that represents an arbitrary circle $AB$ is also a circle. Let the two circles parallel to the cutting plane and tangent to circle $AB$ be drawn such that $A$ and $B$ are the points of tangency. Let $C$ be the intersection of the great circle through $A$, $B$ and $P$ with the tangent circle through $B$. Join lines $AP$, $BP$ and $BC$.

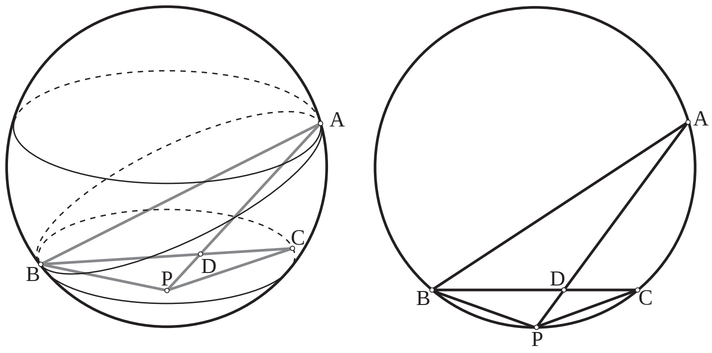


**Figure 16**: Circles on the sphere are represented by circles in the planisphere.

Then $\triangle BPA$ is an axial triangle of the cone with base $AB$ and is perpendicular to the cutting plane. It remains to show that the plane of the circle about $BC$ cuts this cone in a circle, because the cutting plane, being parallel, will cut the cone in the same kind of conic section as this plane (*Conics* I 4). Since $\widehat{BP} = \widehat{PC}$, $\angle BAP = \angle CBP$. So, since $\angle BPA$ is common, $\triangle BDP$ is similar and subcontrary to $\triangle ABP$. Therefore, the cone about $\triangle ABP$ is cut by the plane of the circle about $BC$ in a circle about diameter $BD$ (*Conics* I 5). Therefore, all circles on the sphere are represented in the plane by circles.

Ptolemy uses the fact that circles are represented by circles to construct the $r$-ecliptic as a circle tangent to the circles representing two equal $\delta$-circles. In *Planis.* 1.3, he shows that the $r$-ecliptic bisects the equator. Although he states this theorem as concerning the $r$-ecliptic, the proof itself does not depend in any way on the obliquity of the ecliptic, so that it is valid for the circle representing any great circle oblique to the equator. In fact, in *Planis.* 3, Ptolemy will apply this more general claim to the case of horizon circles. The argument that the correlates of oblique great circles bisect the equator is the first proof of a case of orthogonal angle preservation.

In *Planis.* 1.4, Ptolemy describes how this method may be used to lay out the $r$-ecliptic and the $r$-tropics, and orients the reader to the cardinal points of the $r$-ecliptic and the direction of the motion of the cosmos.

The final paragraph, *Planis.* 1.5, explains that the division of the $r$-ecliptic into quadrants and signs is not effected by constructing equal arcs, but by constructing the appropriate $r$-$\delta$-circles.

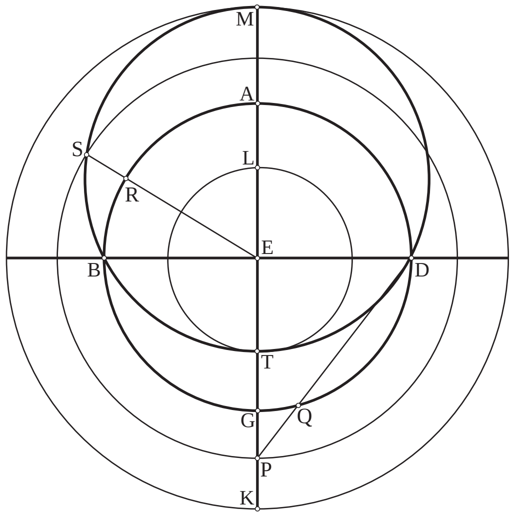


**Figure 17**: The division of the ecliptic by $\delta$-circles.

This is obvious in the case of the cardinal points of the $r$-ecliptic, but it may be useful to see an example construction of a zodiacal sign. In Figure 17, throughout the year the sun moves counterclockwise around circle $BTDM$, starting from the vernal equinox at $B$. In order to construct point $S$ as the beginning of Pisces, $\lambda = 330°$, we cut off $\overset{\frown}{GQ}$ equal to the declination of $S$, $\delta = 11;39,59°$ S (given in *Alm.* I 14 & 15), and extend $DQ$ to point $P$. If we complete a circle around $E$ through $P$ it will meet the $r$-ecliptic at $S$, the beginning of Pisces. In *Planis.* 4, Ptolemy will show how this construction can be used to compute the radius of the $r$-$\delta$-circle in terms of the radius of the equator.

Finally, Ptolemy states, without proof, that his construction will ensure that the $r$-meridians will pass through degrees of the ecliptic that correspond to diametrically opposite points. The proof of this assertion is given in *Planis.* 2. Hence, from the perspective of purely descriptive geometry, it would be simpler to find $S$ by laying off the right ascension, $\overset{\frown}{BR}$, given in *Alm.* I 16. This would, however, give us no way of computing the position of $S$ in terms of the radius of the $r$-$\delta$-circle. That is, although we could compute the right ascension, $\overset{\frown}{BR}$, we would not know the length $ES$.

## *Planisphere* **2 & 3**

The next two sections concern the relationship between the equator and $r$-horizons, which are drawn in the same way as the ecliptic. The enunciation for both sections is asserted in the beginning of *Planis.* 2, which provides a lemma, and then repeated at the beginning of *Planis.* 3.

The lemma in *Planis.* 2 shows that an $r$-meridian will intersect the $r$-ecliptic at points that correspond to diametrically opposite points.[177] What this means, and what Ptolemy will show, is that these points represent the opposite intersections of a meridian with a pair of equal $\delta$-circles.

To carry out the proof, Ptolemy again tacitly folds two different planes together to form the plane of the figure. As always, the cutting plane is the plane of the equator, but now the other plane is that of an arbitrary meridian. This is the only place in the treatise where Ptolemy folds an arbitrary meridian into the plane of the figure so that the south pole does not overlap with one of the equinoxes.

Once again Ptolemy's approach is best explained with reference to the solid configuration, as seen in Figure 18. He begins by drawing the equator, $ABGD$, and the $r$-ecliptic, $ZBHD$. He then passes an arbitrary line, an $r$-meridian, through $E$ so that it intersects both circles. He will then show that this line intersects the $r$-ecliptic at points that represent diametrically opposite points, $Z$ and $H$. In fact, what he will show is that $Z$ and $H$ represent points that are an equal distance from the equator as measured along an arc of the meridian – that is, they represent points that are joined by a diameter of the meridian.

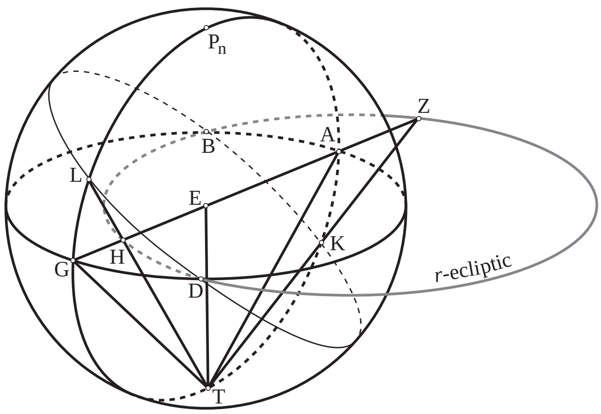


**Figure 18**: Perspective diagram of *Planis.* 2.

Ptolemy folds the plane of the meridian into that of the equator by constructing $ET$ perpendicular to $GZ$ in the plane. He then points out that $T$ functions as the point of projection and proves that $\widehat{AK} = \widehat{GL}$, so that points $Z$ and $H$ represent the points through which are drawn the $r$-$\delta$-circles of the two equal $\delta$-circles through $K$

177 Lorch [1995, 273] takes this theorem to be about the horizons for observers on the equator. The proof will, indeed, serve for such a situation; however, Ptolemy's expression of the theorem and his use of it, in *Planis.* 3, specifically refers to meridians. Moreover, when he uses this theorem for horizons at the equator in *Planis.* 8, he first reminds the reader that such horizons are geometrically equivalent to meridians.

and $L$ on the solid sphere. Hence $Z$ and $H$ represent $K$ and $L$, which in turn are joined by a diameter of the meridian.

With this as a lemma, both the construction and the proof of the next theorem can be carried out entirely within the plane. In *Planis.* 3, Ptolemy argues that an arbitrary $r$-horizon, constructed in the same way as the $r$-ecliptic, will not only bisect the equator but will also intersect the $r$-ecliptic in two points that correspond to diametrically opposite points. Using the lemma, this means that these two intersections will be joined through the center of the equator by an $r$-meridian.

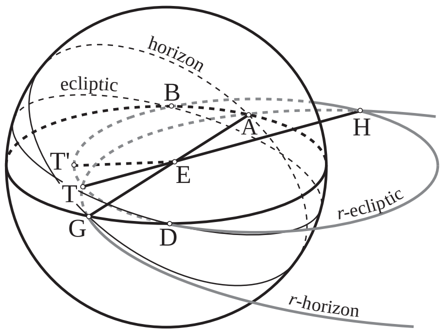


**Figure 19**: Perspective diagram of *Planis.* 3.

Although Ptolemy carries out his proof in the plane it may still be useful to consider the situation in the sphere, in order to better understand his procedure. In Figure 19, he first constructs two circles, the $r$-ecliptic and a $r$-horizon, such that they both bisect the equator. He then joins one of the intersections of these two circles, point $H$, with the center of the equator, $E$, and extends $HE$ to some point on the horizon, say $T'$. He then uses plane geometry to show that point $T'$ coincides with the other intersection of the $r$-horizon and $r$-ecliptic, point $T$.

Ptolemy's exposition is somewhat obscured by the fact that he calls both of these points $T$ in anticipation of the fact that they will be shown to be one and the same. As Maslama points out, it would have been clearer if he had proceeded by an indirect argument [Kunitzsch and Lorch 1994, 14].

It should be stated that there is never any question that $ABGD$, $HBD$ and $HAD$ are all circles. It is simply a matter of showing that point $T'$, defined as the intersection of line $HE$ and circle $HAG$, also falls on circle $HBD$. Hence, we may take *Planis.* 3 as demonstrating a case of conformality, namely that the points that represent the intersections of two great circles are joined by diametrically opposite points. In this case, the intersections of the $r$-horizon with the $r$-ecliptic are joined by $r$-meridians, and hence by a diameter of the sphere.

## *Planisphere* **4–7**

The next four sections show how the standard techniques of ancient plane trigonometry can be used to calculate the radius of an $r$-$\delta$-circle given its declination, $\delta$, and proceed to derive a number of the parameters of the planisphere using these methods.

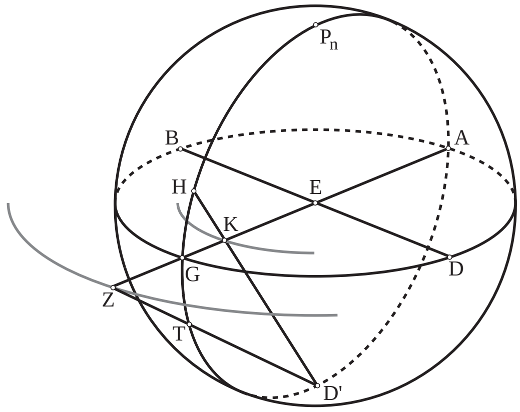


**Figure 20**: Perspective diagram of *Planis.* 4.

In Figure 20, Ptolemy sets out circle $ABGD$ as the equator and imagines a pair of equal $\delta$-circles through $H$ and $T$. Since the declination of the $\delta$-circles is given and the radius of the equator, $r_{eq}$, is always assumed to be $60^{\text{p}}$,[178] where the radius of the northern $r$-$\delta$-circle is $EK$ and the radius of the southern $r$-$\delta$-circle $EZ$, Ptolemy uses metrical analysis to show that[179]

$$\frac{Crd(90^\circ + \delta)}{Crd(90^\circ - \delta)} = \frac{EK}{r_{eq}} = \frac{r_{eq}}{EZ}.$$

Because the diameter of a circle representing any great circle tangent to a pair of $\delta$-circles is simply the sum of the radii of the two corresponding $r$-$\delta$-circles, this section also shows how to calculate the size of any great circle with a known inclination to the equator.

Ptolemy then uses these considerations to compute various parameters of the planisphere: the sizes of the $r$-tropics, the size of the $r$-ecliptic, the distance between the center of the $r$-ecliptic and the equator, the sizes of the $r$-$\delta$-circles through the beginnings of the signs, the size of an example $r$-horizon and the distance between

[178]This standard unit is stated in *Planis.* 4.3 (see page 88).

[179]In metrical analysis, given values are used to derive unknown values, which are then also said to be "given." In order to make this process explicit, we designate given numbers with numerals or variables (as $90^\circ$ and $\delta$) and the objects whose values are known on this basis with letter names (as $AB$).

the center of this $r$-horizon and the equator. All of these values will be used in sections 8–13.

For most of these computations, the declination will be given, but in the case of the $\delta$-circles through the beginnings of the signs, the declination must also be calculated from the celestial longitude. For the purposes of this treatise, Ptolemy assumes that the declination will be calculated using the sector theorem methods of the *Almagest* [Heiberg 1898–1903, p. 1, 76–78; Toomer 1984, 69–70, Sidoli 2006]. For historical reasons, however, it is worth noting that they can also be derived from the longitudes using analemma methods and plane trigonometry [Neugebauer 1975, 303–304].

*Planis.* 4–7 provide us with insight into the role of the $\delta$-circles in the mathematical development of the treatise. By showing how $\delta$-circles are used to carry out calculations, Ptolemy makes it clear that an interest in exact computation motivates his exposition. Whenever he sets out a circle that represents a circle on the sphere such as an inclined great circle or a $\beta$-circle, Ptolemy uses the two tangential $r$-$\delta$-circles. This is presumably because the diameter of the circle representing any circle is the sum of the radii of its two tangential $r$-$\delta$-circles, and the radii of these $r$-$\delta$-circles can be readily calculated.

## *Planisphere* **8 & 9**

Ptolemy now proceeds to demonstrate that the planisphere produces the same values for the rising-time phenomena as the methods of spherical geometry put forward in the *Almagest*. The constructions and demonstrations in *Planis.* 8–13 are done entirely within the plane, and Ptolemy repeatedly frames the arguments in this section as claims that the planisphere is mathematically consistent (موافق) with the sphere. The rising-times of arcs of the ecliptic was one of the major topics of ancient spherical astronomy (*Alm.* II 7–9) and the computation of their values from the geometry of the planisphere constitutes an important goal of this treatise.[180]

*Planis.* 8 & 9 compute the rising-times of the signs of the zodiac for horizons on the equator. In other words, these sections determine the right ascension of the arc of each of the signs. Ptolemy begins by orienting the reader to the diagram. In Figure 21, circle $ABGD$ is the equator about center $E$ and circle $ZBHD$ is the $r$-ecliptic about center $T$. Since every horizon at the equator coincides with the meridian of locations 90° away in terrestrial longitude, an $r$-horizon at the equator may be constructed on the planisphere in the same way as an $r$-meridian, by a straight line, as $KEN$, through the north pole, $E$.

As he makes explicit in *Planis.* 10.1, Ptolemy imagines the movement of the sphere by moving the horizon against a background of the fixed stars. Hence in Figure 21, we produce the movement of the sphere by rotating an arbitrary line

[180]See Brunet and Nadal [1981] for a discussion of rising-time phenomena in Greek astronomy.

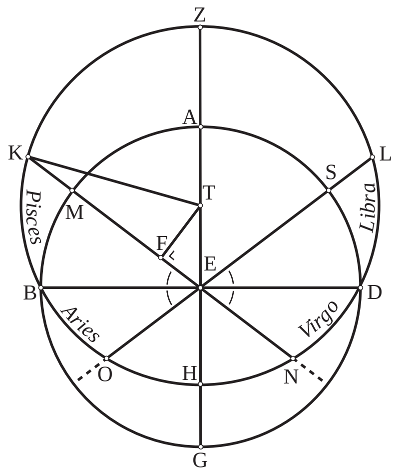


**Figure 21**: Rising-times of the signs at horizons on the equator.

passing through $E$ counterclockwise. So, where $KEN$ is the horizon, the rising of the quadrant from the winter solstice to the vernal equinox occurs as $KEN$ rotates about $E$ from the position of $ZEH$ to that of $BED$, and likewise for the other quadrants. Clearly, $\overset{\frown}{ZB}$ rises with $\overset{\frown}{AB}$, $\overset{\frown}{BH}$ with $\overset{\frown}{BG}$, $\overset{\frown}{HD}$ with $\overset{\frown}{GD}$ and $\overset{\frown}{DZ}$ with $\overset{\frown}{DA}$.

The goal of *Planis.* 8.1 is to show that if the $r$-horizon is at the beginning of Pisces, as $KEN$, the geometry of the figure can be used to calculate the rising-time of this sign. Since $\overset{\frown}{KB}$ rises with $\overset{\frown}{BM}$ as $KEN$ rotates toward $BED$, $\overset{\frown}{BM}$ is the right ascension, or time degrees, of Pisces. Hence, since 360° time degrees rise in $24^{\mathrm{h}}$, we compute the rising-time of Pisces by finding the angular value of $\overset{\frown}{BM}$. Moreover, by the symmetry of the figure, this will also be the rising-time of Virgo, Libra and Aries.

The computation, which is given in *Planis.* 8.2, is somewhat involved but can be sketched as follows. By the computations in *Planis.* 4–7, the sides of $\triangle TKE$ are given. Using two auxiliary theorems that result from the geometry of the figure, it is possible to compute the sides of $\triangle TFE$ from those of $\triangle TKE$, and using chord table methods, it is then possible to compute $\angle FTE = \angle MEB = \overset{\frown}{BM}$.

In *Planis.* 8.3, Ptolemy summarizes these results by a slightly different method, using metrical analysis. Where $KE$, the radius of the $r$-$\delta$-circle through the beginning of the sign, $r_{\delta sign}$, is given (*Planis.* 5 & 6), the first auxiliary theorem is used to show that

$$\frac{3600}{r_{\delta sign}} = EN,$$

while the second auxiliary theorem shows that

$$EK - EN = 2EF.$$

Hence,

$$\frac{r_{\delta sign} - 3600/r_{\delta sign}}{2} = EF.$$

Then in right $\triangle TFE$, $EF$ and $TE$ are given, and the angles can be computed with the chord table. This analysis forms the basis of the computations in *Planis.* 9, 12 & 13.

*Planis.* 9 uses a similar figure and the metrical analysis of *Planis.* 8.3 to find the rising-times of the remaining signs. By setting $\overset{\frown}{BK}$ equal to the two signs of Pisces and Aquarius, Ptolemy computes the right ascension of both. The difference then gives that of Aquarius alone and the complement that of Capricorn. The remaining signs are then known by symmetry. Ptolemy points out, as we have already stated, that the numbers determined in this way are the same as those derived in the *Almagest* using the methods of ancient spherical trigonometry (*Alm.* I 16).

## *Planisphere* **10–13**

The next two sections, *Planis.* 10 & 11, introduce the reader to the use of the planisphere to model rising-time phenomena at latitudes other than the equator. As in the *Almagest*, Ptolemy proceeds by using a paradigm latitude of 36°, the traditional value for Rhodes.

In *Planis.* 10, by considering the situation when the solstices are on the horizon, Ptolemy develops a basic theorem concerning the symmetry of the figure (*Planis.* 10.2), points out how the figure shows that the rising-time of equal arcs on either side of the same equinox are equal (*Planis.* 10.3), and introduces an important arc that modern scholars call ascensional difference (*Planis.* 10.4). These are the basic concepts used in *Planis.* 11–13.

Ptolemy begins, in *Planis.* 10.1, by orienting us to the figure. In Figure 22, $ABGD$ is the equator and $ZBHD$ the $r$-ecliptic. The movement of the sphere is clockwise, from $B$ toward $A$, and so on. In fact, however, this movement is once again imagined by changing the position of the horizon. The only two positions considered in *Planis.* 10 are the cases where the solstices are in the two opposite positions on the horizon, in which the horizon is $ZKL$ and $ZMN$. Since a given horizon is tangent to a pair of $\delta$-circles, the locus of the center of the $r$-horizon will be a circle about center $E$, as circle $S'XO'$. Hence, the $r$-horizon is carried as a large epicycle on the deferent $S'XO'$. When it is in the position of circle $ZKHL$, points $Z$ and $K$ are rising; in the position of $Z'K'H''L'$, points $Z'$ and $K'$ are rising; in the position of $N'H'M'Z''$, points $N'$ and $H'$ are rising; and in the position of $NHMZ$, points $N$ and $H$ are rising.

In *Planis.* 10.2, Ptolemy demonstrates that when the solstices are on the horizon, the arcs of the equator cut off by the intersections of the $r$-horizon and the equator

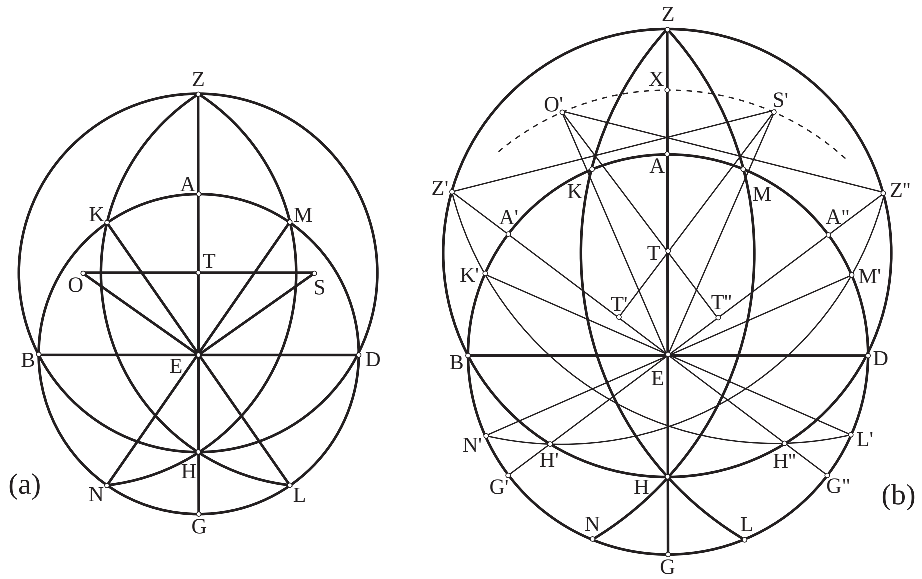


**Figure 22**: (a) Diagram for *Planis.* 10. (b) Generalization of *Planis.* 10, showing the two horizons at respectively equal times before and after their positions in (a).

are symmetrical about the cardinal points of the equator, that is $\overset{\frown}{KA} = \overset{\frown}{AM} = \overset{\frown}{GL} = \overset{\frown}{GN}$ and $\overset{\frown}{BK} = \overset{\frown}{BN} = \overset{\frown}{DL} = \overset{\frown}{DM}$.

In *Planis.* 10.3, he points out which arc of the ecliptic rise with a given arc of the equator and argues that the rising-times of equal arcs of the ecliptic are equal on either side of the same equinox. Since, however, his specific discussion of these matters is only in terms of the rising-times of quadrants, it is not immediately obvious how his discussion justifies this more general claim.

This is another example of Ptolemy giving a specific argument that provides a paradigm proof that can be reproduced as a generalization. We consider any two symmetrical positions of the horizon, say $Z'K'H''L'$ about center $S'$ and $Z''M'H'N'$ about center $Q'$, such that $\overset{\frown}{Z'Z} = \overset{\frown}{ZZ''}$. We then follow the approach of the proof in *Planis.* 10.2 to show that $\overset{\frown}{A'K'} = \overset{\frown}{A''M} = \overset{\frown}{N'G'} = \overset{\frown}{L'G''}$ as follows.

Arguments from symmetry show that $\triangle TET'$ is congruent with $\triangle TET''$, so that $\angle TET' = \angle TET''$. Both $\angle S'EK'$ and $\angle O'EM'$, however, are right, since $S'E$ and $O'E$ are perpendicular bisectors of equal chords in equal circles. Hence the differences, $\angle A'EK'$ and $A''EM'$, are equal, as are their vertical angles, $\angle N'EG'$ and $\angle L'EG''$. Therefore, $\overset{\frown}{A'K'} = \overset{\frown}{A''M} = \overset{\frown}{N'G'} = \overset{\frown}{L'G''}$.

These symmetries in the figure may be used to explain one of the fundamental facts of rising-time phenomena – namely, that rising and setting times of equal arcs of the ecliptic are equal for arcs symmetrically situated on either side of an equinox but not of a solstice.[181] $Z'B$ and $BH'$ represent two equal arcs of the ecliptic, since by *Planis.* 2, $Z'$ and $H'$ represent points on two equal $\delta$-circles. Moreover, $\overset{\frown}{Z'B}$ rises with $\overset{\frown}{K'B}$ and $\overset{\frown}{BH'}$ with $\overset{\frown}{BN'}$. Now since, $\overset{\frown}{A'K'}$ was shown to equal $\overset{\frown}{N'G'}$, $\overset{\frown}{K'B} = \overset{\frown}{BN'}$. Therefore, since $\overset{\frown}{Z'B}$ and $\overset{\frown}{BH'}$ rise with equal arcs of the equator, they rise in equal times. This argument shows the validity of the general claim made in *Planis.* 10.3. A similar argument will not hold for the solstices because the right ascension must be corrected in opposite directions on either side of a solstice.[182]

The importance of these arcs of correction, called ascensional difference, is the subject of *Planis.* 10.4. The arc of ascensional difference measures the difference between the right and oblique ascension of an arc of the ecliptic [Neugebauer 1975, 36–37]. It is defined as the arc of the equator between (a) the meridian through the intersection of the horizon with the equator and (b) the meridian through the intersection of the horizon and the ecliptic. Finding the length of this arc furnishes the simplest method of computing the oblique ascension of an arc of the ecliptic given its right ascension.

In Figure 23, let circle $ABGD$ be the equator, $ZBHD$ the $r$-ecliptic and $ZKHL$ an obliqe $r$-horizon. On the sphere, the ecliptic will be $Z'BH'D$ and the horizon $Z'KH'L$, so that $B$ is the vernal equinox, $H'$ the summer solstice, $D$ the autumnal equinox and $Z'$ the winter solstice. We imagine the motion of the sphere by rotating the equator and the ecliptic clockwise around the polar axis, $P_nP_s$, while the horizon remains stationary.[183] Then at the oblique horizon, quadrant $DH'$ sets with $\overset{\frown}{DL}$, while at an orthogonal horizon, it sets with quadrant $DG$. Hence, the ascensional difference for quadrant $DH'$ at the oblique horizon is $\overset{\frown}{LG}$.

The ascensional difference of an arc of the ecliptic between an equinox and a solstice can be used as a characteristic of latitude. At the equinoxes, as the sphere

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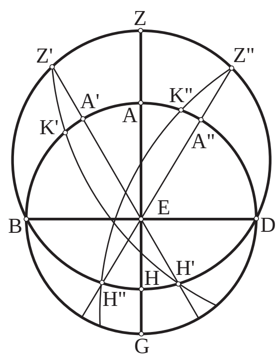
[181] The earliest mathematical treatment of this topic that has survived is Euclid's *Phaenomena* 12 & 13 [Berggren and Thomas 1996, 83–97].

[182] The planisphere makes this asymmetry in rising-times quite clear.

Let us consider the same horizon symmetrically placed on either side of the solstices, such that $Z''K''H''$ is the position of the horizon at some time before the winter solstice rises and $Z'K'H'$ its position the same amount of time after it has risen. In position $Z''K''H''$, the ascensional difference, $A''K''$, is subtracted from the right ascension, $AA''$, whereas in position $Z'K'H'$, the ascensional difference, $A'K'$, is added to the right ascension, $AA'$. Hence, the oblique ascensions, $AK''$ and $AK'$, will not be equal.




[183] This is mathematically equivalent to Ptolemy's procedure of moving the $r$-horizon against a stationary equator.

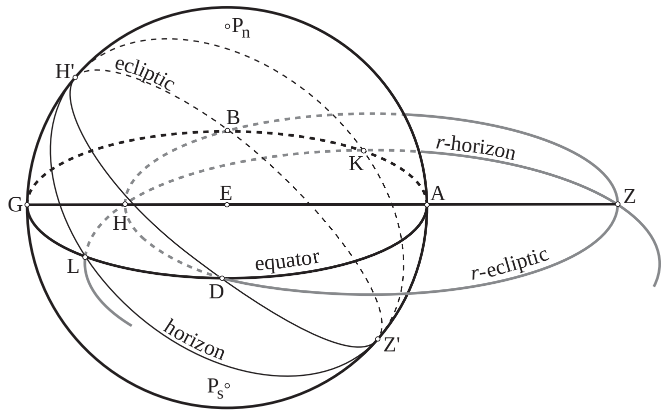



**Figure 23**: Perspective diagram showing ascensional difference.

revolves, the sun is carried on the great circle of the equator, $DGBA$, and at the solstices it is carried on $\delta$-circles imagined through $H'$ and $Z'$. Hence, at the summer solstice, when the sun is carried on the $\delta$-circle through $H'$, sunset is later than that at the equinox by the arc $\overset{\frown}{LG}$. Likewise, it rises earlier by an equal arc. Therefore, twice the ascensional difference of the principle quadrants will give the time difference between the longest or shortest daylight and the equinoctial daylight. The difference between the longest or shortest daylight and the equinoctial daylight was the most common characteristic of latitude in Greco-Roman antiquity.

In *Planis.* 11, Ptolemy uses the geometry of the planisphere and chord table methods to compute the ascensional difference for a quadrant of the ecliptic between an equinox and a solstice at the latitude of Rhodes. He then calculates the longest and shortest periods of daylight, and again points out that the values derived in this way agree with those found using the methods of spherical trigonometry set out in the *Almagest*.

*Planis.* 12 & 13 apply the corrective arc of ascensional difference developed in *Planis.* 10, the computational procedure of *Planis.* 8, and the values derived in *Planis.* 7 to compute the rising-times of the signs of the zodiac at the latitude of Rhodes, using plane trigonometry. Again, Ptolemy points out that the values for the rising-times of the signs as found in the planisphere agree with those derived in the *Almagest*. This concludes the computational sequence of *Planis.* 4–13, and indeed the whole first section.

Although historically the plane trigonometric methods of this treatise may have been used, in conjunction with the analemma, to furnish an original calculation of rising-times, this is not the function that these computations serve in Ptolemy's treatise. As they are presented in the *Planisphere*, they act as a check against an already known set of rising-time values. The methods used and the values derived constitute a strong numerical argument for the mathematical consistency between the planisphere and the solid sphere that we would attribute to the principle of conformality. That is, Ptolemy shows that if points that represent equal arcs be taken on the $r$-ecliptic, and if $r$-horizons and $r$-meridians be drawn through these, then these $r$-horizons and $r$-meridians cut off the same arcs of the equator in the planisphere as they do in the solid sphere.

## *Planisphere* **14**

In *Planis.* 14.1, Ptolemy gives a brief overview of the topics covered in the treatise so far. Such summary remarks are Ptolemy's usual way of introducing a major change of subject matter. The remaining sections of the work will treat the construction of $r$-$\beta$-circles and practical issues that arise in actually carrying out geometric constructions for the purposes of making instruments. This section marks a transition in the mathematical methods of the treatise, as well as its subject matter. While the first part centered around computation, the latter part focuses on geometric problem solving.

At the beginning of *Planis.* 14, Ptolemy situates the problem as arising in the context of instrument building. He tells us that the construction of the equator and the $r$-$\delta$-circles inside a given circle will be particularly useful for setting out "the spider" (العنكبوت) in the class of instrument known as "the instruments of hours" (آلات الساعات). Although it is probably not possible to say with certainty what Ptolemy meant by "the spider" based on his brief remarks, we need not follow Neugebauer's [1975, 866] claim that it must be "the movable network that carries the pointers which indicate the positions of stars that are defined by ecliptic coordinates," as found on a plane astrolabe.[184] The system of ecliptic coordinates, to which Neugebauer refers, will not be introduced until the next section and here Ptolemy only makes reference to the $\delta$-circles of the equatorial coordinates. Moreover, an "instrument of hours" is not necessarily an ancient or medieval plane astrolabe.

Although a plane astrolabe can be used in various ways for telling time, the Arabic expression *ālāt al-sāʿāt* could also be a translation of one of the Greek idioms denoting a clock. Moreover, we have both textual and archaeological evidence that the Greeks made clocks that included a plane, disk map of the celestial sphere. The anaphoric clock, which is described by Vitruvius (*Arch.* IX 8.8–10), featured a planispheric face upon which a marker for the sun could be placed in various positions along the ecliptic [Granger 1934, v. 2, 260–262; Rowland and Howe 1999, 117]. This planisphere was then rotated hydraulically behind a brass grill that represented the local coordinates and allowed the observer to read the time by the position of the sun-marker against the grill. According to Vitruvius, the disk contained images of

[184] Both Neugebauer [1949] and Drachmann [1954] were of the opinion that the *Planisphere* was a treatise on the astrolabe.

the twelve signs of the zodiac. In such a device, the grill may have been called "the spider."

We know of fragments of the disks of two anaphoric clocks.[185] On one of these, found near the town of Salzburg, we find the names and images of the constellations as well as regularly spaced holes along the ecliptic that would have carried the sun-marker.[186] The northern hemisphere has been depicted as it would appear to an observer looking at the celestial sphere from the outside, as on a star globe.[187] It is clear that the Salzburg disk was produced using techniques equivalent to those described in the *Planisphere*.

It is not certain whether this offhand remark about the spider and the instruments of hours is a reference to astrolabes or anaphoric clocks, but it hardly matters. Ptolemy's project is not to describe the construction of a particular instrument, but rather to develop a body of mathematical techniques, many of which he knows will be of interest to instrument makers. Moreover, instrument making presents its own set of problems, some of which have mathematical solutions.

The most obvious practical consideration in drawing a planispheric image of the celestial sphere is that the plane of any actual diagram will be finite, whereas the whole of the celestial sphere can only be stereographically mapped onto an infinite expanse. Hence, it will be convenient to be able to draw the part of the celestial sphere north of some arbitrary, southernmost bounding circle within any given circle. The problem then is to draw the circles representing more northerly parallel circles in the proper positions on the plate. *Planis.* 14 solves this problem.

Although the structure of *Planis.* 14 is confused and the text may have undergone some corruption, the section appears to broadly follow the pattern of an ancient problem. It begins with a method of construction that assumes the problem has been solved in the mode of an analysis. This is followed by a synthesis that solves the problem for the paradigm case of the equator. Finally, we are given the proof that the stated construction solves the problem.

Ptolemy's treatment is obscured by the fact that he solves the problem once for only a single example, giving the construction and then showing that the construction holds. Moreover, the chosen example of the equator has some features that are

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[185] See Neugebauer [1975, 870, n. 5 & 6] for descriptions of the fragments.

[186] If the holes were continued regularly in the missing part of the disk, there would be 182 or 183 of them. Images of the Salzburg disk are reproduced by Evans [1999, 252].

[187] Drachmann [1948, 25] claims that the orientation of the constellations is such that the motion of the sun-marker on face of the clock will simulate the motion of the real sun across the sky. Any such correspondence, however, would also depend on the overall orientation of the clock. Sleeswyk and Huldén [1991, 40–41] try to account for the orientation of constellations on the Salzburg disk by claiming that the projection was made from the north pole, somehow not noticing that the northern constellations are depicted inside the ecliptic.

unique from a geometric standpoint and detract from its status as a general case. In Figure 24, Ptolemy shows how to construct the equator, $SLM$, about center $E$, drawn with a radius equal to $TK$. As Anagnostakis [1984, 129–130] points out, however, the construction and proof that Ptolemy gives can be used to produce any circle north of the bounding circle.

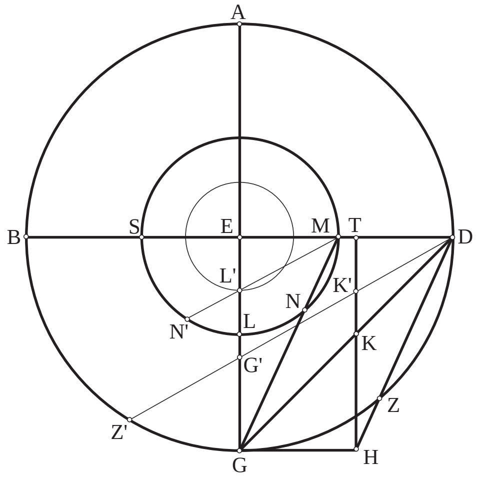


**Figure 24**: Extension of *Planis.* 14.

In Figure 24, let $ABGD$ be the bounding circle of the plate and $\overset{\frown}{DZ}$ the arc-distance of an arbitrary, southernmost bounding circle from the south pole. Line $DZ$ is joined and extended to meet $GH$, the tangent to $ABGD$ at point $G$. The line $HT$ is dropped perpendicular to $ED$. Line $HT$ can then be used to determine the radius of any $r$-$\delta$-circle on the plate as follows.

Let $\overset{\frown}{GZ'}$ be the declination of an arbitrary $\delta$-circle. We join $DZ'$ such that it intersects line $HT$ at point $K'$. We draw a circle about $E$ with radius $EL' = TK'$. It will be the $r$-$\delta$-circle corresponding to the original $\delta$-circle. The proof follows the final paragraph of *Planis.* 14. We show that $\triangle MEL'$ is congruent to $\triangle DTK'$, so that $MN' \parallel DZ'$ and $\overset{\frown}{DZ'}$ is similar to $\overset{\frown}{MN'}$. Again, we see how Ptolemy gives the solution of a specific case as a paradigmatic treatment of a more general problem.

## *Planisphere* **15–17**

The next three sections, introduced by *Planis.* 15.1, provide a basic treatment of the circles of the ecliptic system, particularly the $\beta$-circles. *Planis.* 15.2 provides the construction of the point corresponding to the pole of the $\beta$-circles, while *Planis.* 15.3 briefly discusses circles representing the great circles through the poles of the ecliptic, which we call $\lambda$-circles, because they are circles of constant celestial longitude. *Planis.* 16 gives the construction of the $r$-$\beta$-circle corresponding to a given $\beta$-circle, and *Planis.* 17 shows that no two $r$-$\beta$-circles are concentric.

*Planis.* 15.2, is a problem so simple that it requires no proof of its validity. Although unusual for a problem, this lack of a proof, gives us insight into Ptolemy's assumptions. *Planis.* 15 makes it certain that Ptolemy considers the point on the planisphere corresponding to a given point on the sphere to be determined when a line is joined between the given point and the south pole through the cutting plane. As usual, Ptolemy represents two planes in the plane of the figure, the cutting plane and the solstitial colure.

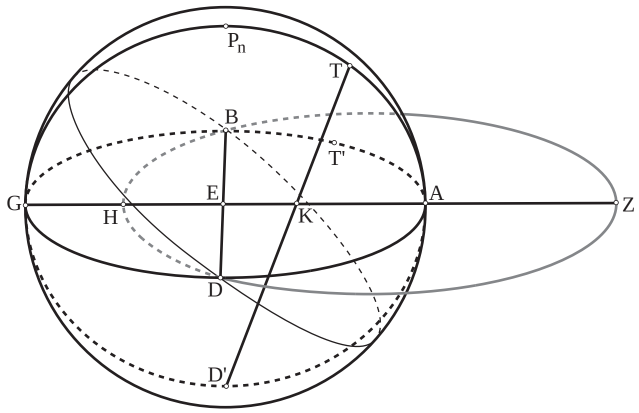


**Figure 25**: Perspective diagram of *Planis.* 15.

In Figure 25, we see the two planes in their solid configuration. Circle $ABGD$ is the equator, circle $TGD'A$ the solstitial colure, and $\widehat{P_nT}$ is set out equal to the obliquity of the ecliptic, $\varepsilon$. Then point $T$ is the pole of the ecliptic and of the $\beta$-circles, so that when line $TD'$ is joined, its intersection with the diameter $AG$ at point $K$ is the $r$-pole of the ecliptic. Ptolemy's claim that, "this is in accordance with what we described," underscores his assumption, from the beginning, that the reader understands that the correlates of individual points are to be found in this way.

*Planis.* 15.3, gives a sketch of the construction of the circles representing great circles through the poles of the ecliptic, the $r$-$\lambda$-circles. As Maslama points out, this paragraph can be used to draw the $r$-$\lambda$-circle that passes through a star given in ecliptic coordinates, $S(\lambda, \beta)$ [Kunitzsch and Lorch 1994, 20]. This may be done using either descriptive geometry or trigonometric computation, as follows.

Since the longitude of the star is given, the declination of the intersection of the $\lambda$-circle and the equator, $\delta(\lambda)$, may be found by *Alm.* I 15. In Figure 26, let it be set out as $\delta(\lambda) = \widehat{GW}$. Then by the method given in the commentary to *Planis.* 1, we determine the intersection of the $r$-$\lambda$-circle and the $r$-ecliptic, say at point $X$.[188] We join point $X$ through $E$ and extend it to the opposite point of the ecliptic at

[188]There are two intersections, but only one of them corresponds to the star's longitude.

$Y$. The three points $X$, $Y$ and $K$, the $r$-pole of the ecliptic, then determine the position of the $r$-$\lambda$-circle through the given star. As can readily be seen, all of the lengths and angles set out in this construction can also be determined numerically using chord table methods.

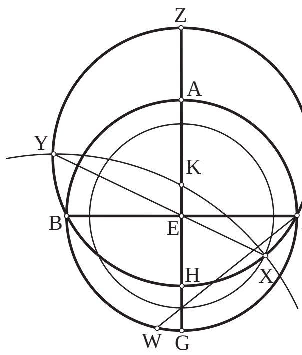


**Figure 26**: Diagram for *Planis.* 15.3.

*Planis.* 16 is structured like a typical problem in Greek mathematics. It gives the construction for an $r$-$\beta$-circle and then demonstrates that the circle so constructed satisfies certain criteria, that is, that it bisects the $r$-$\delta$-circle that intersects it in the equinoctial colure. Again, Ptolemy depicts both the solstitial colure and the cutting plane folded into a single plane.

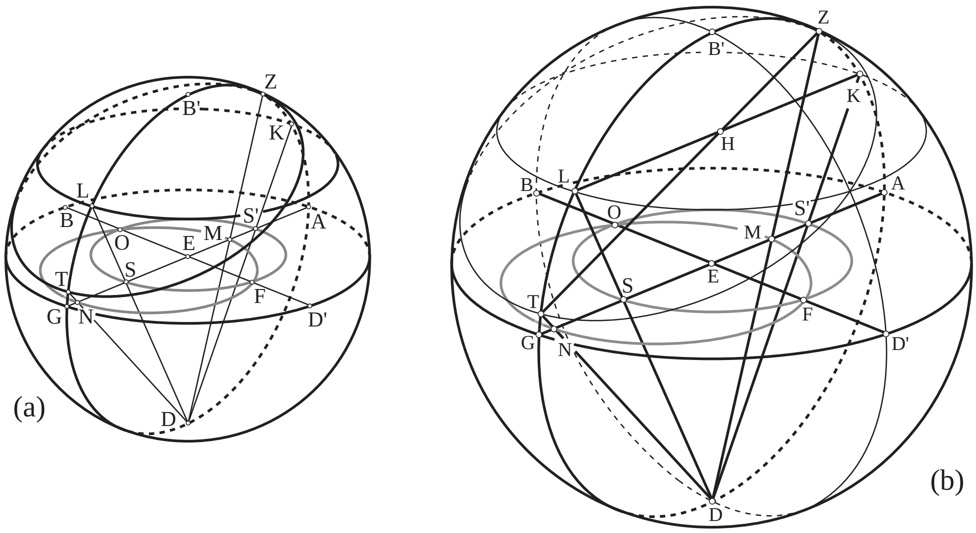


**Figure 27**: (a) Principal objects in *Planis.* 16. (b) Perspective diagram of *Planis.* 16.

In Figure 27 (b), let $ABGD'$ be the equator, line $ZT$ the diameter of the given $\beta$-circle in the solstitial colure, and $KL$ the diameter in the same plane of the $\delta$-circle that the $\beta$-circle about $ZT$ bisects. The corresponding $r$-$\beta$-circle, $MONF$, and $r$-$\delta$-circle, $OSFS'$, are then drawn in the usual manner. It remains to show that circle $MONF$ bisects circle $OSFS'$.

Following Commandino [1558, f. 25v; Sinisgalli 1993, 146–147], some scholars have maintained that the proof in *Planis.* 16 is a proof of circle preservation [Anagnostakis 1984, 133; Lorch 1995, 277]. In fact, however, Ptolemy simply assumes the objects in question are circles. The proof is the usual complement to the construction, standard in any ancient problem, which shows that the construction is mathematically satisfactory. In this case, it shows that the circle drawn about $MN$ intersects the circle drawn about $SS'$ at the two points $O$ and $F$ on the diameter $OEF$. This amounts to showing that key arcs of the $r$-$\delta$-circle are similar to those of the $\delta$-circle, a case of orthogonal angle preservation.

It is worth noting that the construction of *Planis.* 16 can be used to compute the size and location of the circle representing any given $\beta$-circle. Ptolemy begins with the line $ZT$, a diameter of the $\beta$-circle, as arbitrary. In the context of Greek geometry, this means we may take this line as given, either chosen at the mathematician's discretion or determined by the prior conditions of the problem. Since it is a $\beta$-circle, "given" presumably means "given in celestial latitude," $\beta$, but there is no object corresponding to $\beta$ in the figure. The key to understanding this situation lies in noting that in constructing the $r$-$\beta$-circles, Ptolemy again uses the tangential $r$-$\delta$-circles.

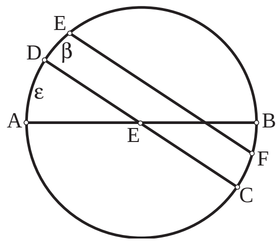


**Figure 28**: Extension for *Planis.* 16.

In Figure 28, we see that if both $\beta$ and the obliquity of the ecliptic, $\varepsilon$, are given, the declinations of the tangential $\delta$-circles are determined by the sum,

$$\varepsilon + \beta = \delta.$$

Hence, $\delta_1 = \overset{\frown}{AE} = \varepsilon + \beta$, and $\delta_2 = \overset{\frown}{BF} = \beta - \varepsilon$. Then, in Figure 27, we can understand line $ZT$ as given in terms of $\overset{\frown}{AZ}$ and $\overset{\frown}{GT}$. Moreover, by the methods set out in *Planis.* 4, we can calculate the size of the two tangential $r$-$\delta$-circles. Hence, we can determine the size and position of any $r$-$\beta$-circle, given in celestial latitude. Here again, we see how the requirements of computation underly Ptolemy's geometric presentation.

*Planis.* 17 is a proof that no two $r$-$\beta$-circles are concentric. Ptolemy constructs the diameters of two $r$-$\beta$-circles in the solstitial colure and then shows that their midpoints are not one and the same. It should be noted that the proof is about the actual centers of the $r$-$\beta$-circles, not the points corresponding to the centers of the $\beta$-circles.

### *Planisphere* **18**

*Planis.* 18 solves the special problem of constructing an $r$-$\beta$-circle that intersects the bounding circle of a given plate. The version of this theorem in the Arabic text and Hermann's translation are somewhat different [Heiberg 1907, 255–257]. Since Hermann's treatment introduces some objects which are also mentioned in Maslama's notes, it seems probable that the Latin translation contains alterations that were made in the version of the text with which Maslama worked. Maslama believed that this problem lacked a full proof and provided one modeled on the proof in *Planis.* 16.[189] We believe, however that our reading of the text shows why Ptolemy would have considered the very brief argument he gives sufficient to demonstrate that the problem has been solved.

Although Ptolemy calls the bounding circle the "always hidden circle" (الدائرة الخفية ابدًا), the discussion makes it clear that mathematically this means the southernmost bounding circle introduced in *Planis.* 14. This circle is depicted, in Figure 13, using an analemma construction, in what is the most explicit case of Ptolemy's practice of folding multiple planes into the plane of the figure.

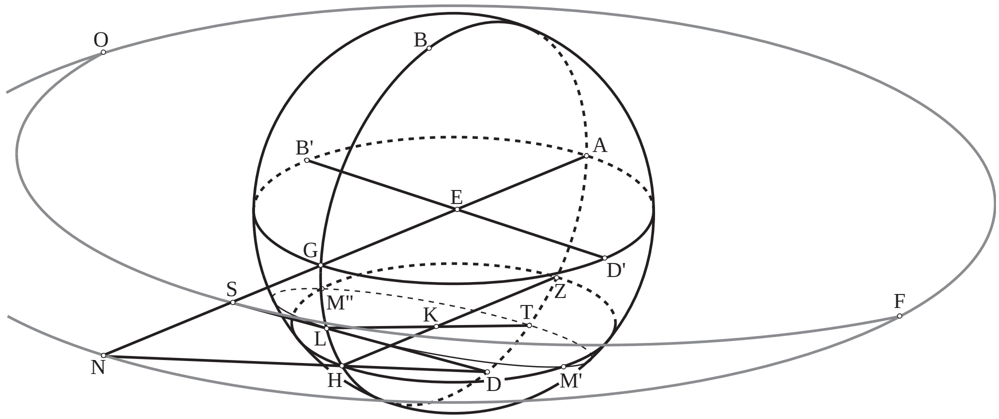


**Figure 29**: Perspective diagram of *Planis.* 18.

In Figure 29, we find the three different planes that Ptolemy depicts in a single plane in Figure 13, in their solid configuration. The southernmost circle, $ZM'HM''$,

[189]In fact, Maslama provides two proofs for this section. The first shows that three points, one of which is not found in the surviving Arabic version, lie on a circle, while the second shows that the property demonstrated in *Planis.* 16 also applies for $\beta$-circles that intersect the bounding circle of the plate [Kunitzsch and Lorch 1994, 24–28; Lorch 1995, 278–280]. The latter is unnecessary, however, since Ptolemy's proof in *Planis.* 16 is valid for any $\beta$-circle.

has been folded at diameter $ZH$ and both sides of it have been rotated into the plane of the solsitial colure to the south, forming the semicircle $ZMH$ in Figure 13. Next the entire plane of the solstitial colure, $ABGD$, including the semicircle $ZMH$, has been rotated around diameter $AG$ into the cutting plane.

Ptolemy then proceeds as follows. He constructs the $r$-southernmost-circle in the usual manner and finds point $S$, which corresponds to the northern endpoint of the diameter $DL$ of the $\beta$-circle. He then cuts off $\overset{\frown}{NF}$ and $\overset{\frown}{NO}$ on circle $FNO$ similar to $\overset{\frown}{HM'}=\overset{\frown}{HM''}$ and points out that the circle through the three points $F$, $N$ and $O$ is the necessary $r$-$\beta$-circle. He does not bother with the proof because it is obvious. Since $\overset{\frown}{NF}$ is similar to $\overset{\frown}{HM'}$ and $\overset{\frown}{NO}$ is similar to $\overset{\frown}{HM''}$, points $F$ and $O$ will represent points $M'$ and $M''$ respectively. Hence, the $r$-$\beta$-circle is determined by three points and obviously satisfies a basic case of conformality, since it intersects the bounding circle at the appropriate places. In this version of the treatise, there is no need for Maslama's proof that the $r$-$\beta$-circle passes through the appropriate point.

## *Planisphere* **19**

*Planis.* 19 solves the problem of constructing the line that corresponds to the $\beta$-circle through the south pole. Both the construction and the proof are simple but this is the only section in the work where we find Ptolemy working entirely in solid geometry. Moreover, it supplies the kind of argument that must have been used to show that any circle through the point of projection will be represented by a line.

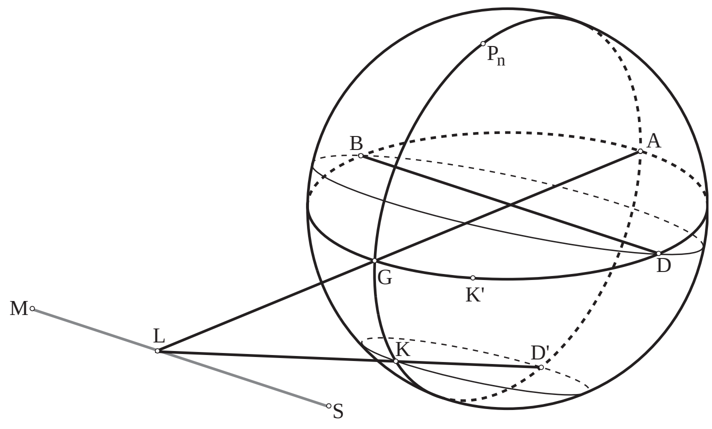


**Figure 30**: Perspective diagram of *Planis.* 19.

In Figure 30, let $ABGD$ be the equator, $AGKD'$ the solstitial colure and $D'K$ the diameter of the $\beta$-circle through the south pole. Let $D'K$ be extended to $L$ and erect $MS$ perpendicular to $AL$. Ptolemy then uses the solid geometry of *Elem.* XI to point out that all of the lines joining the points of the $\beta$-circle with $D'$ are in a plane that intersects the plane of the equator in a line perpendicular to the solstitial colure.

The diagram accompanying this section, Figure 14, contains some objects related to the bounding circle that are not mentioned in the text. Maslama's note, on the other hand, provides a discussion of the bounding circle that includes, among others, these objects, although differently named [Kunitzsch and Lorch 1994, 28–30]. There are a number of possibilities that could explain these circumstances – some of the original text may have been lost, a scribe may have added these objects in consultation with Maslama's notes or independently, and so forth.

### *Planisphere* **20**

The final section returns to the interests of instrument makers by discussing the practical construction of a grid of lines representing both the equatorial and ecliptic coordinates. This is done so that the stars can be located on the planisphere, whether they are given in equatorial or ecliptic coordinates.

Following a general description of the project, Ptolemy summarizes the results that will be used for drawing the equator, its $r$-pole, the meridians, the $r$-ecliptic and its $r$-pole, the $r$-$\beta$-circles, and the $r$-$\lambda$-circles. All of these constructions are fully explained in the text, except that for the $r$-$\lambda$-circles. Since Ptolemy shows how to locate the $r$-poles of the ecliptic, in order to draw an $r$-$\lambda$-circle, it suffices to locate the two opposite points on the $r$-ecliptic corresponding to the longitude.

There are four known medieval methods for finding the $r$-$\lambda$-circles, three of which are discussed by Maslama in his completion of the *Planisphere* [Vernet and Catalá 1965, 22–24; Anagnostakis 1987]. All four of these methods are exact as computational procedures, but the two that are most apparent from Ptolemy's text would present practical difficulties for instrument making. For this reason, Maslama advances a third procedure, which is easier to implement using a compass and straight edge. We will discuss the first three methods, since they shed light on Ptolemy's project.[190]

The first method is the only one explicitly used by Ptolemy in the *Planisphere*.[191] It consists in taking the declinations corresponding to each degree of celestial longitude (given in *Alm.* I 15) and using these to construct $r$-$\delta$-circles, following the method explained in the commentary to *Planis.* 1 (see page 115). Although this method can be used to compute the size of the $r$-$\delta$-circles exactly (see page 119), it presents a number of practical difficulties when used for finding the corresponding $r$-$\lambda$-circles. In order to carry out the construction accurately, the instrument maker would need to use a highly precise protractor, since the difference between the nec-

[190]The fourth method, although the simplest for instrument makers, is not attested until later in the medieval period. Hence, we will not consider it here [Anagnostakis 1987, 138–139]. Kunitzsch [1981] dates the treatise containing this method to between 1246 and 1263.

[191]See *Planis.* 1, 4–6.

essary declinations is sometimes quite small. Moreover, if one wished to draw every $r$-$\lambda$-circle, this method would involve constructing, on the plate, 90 points between about 24° of arc. Finally, in the region around the solstices, the variation in declination is so small that it would be almost impossible to use this method to mark the divisions of the ecliptic with reasonable accuracy.[192] As Maslama points out, this method will only be approximate [Vernet and Catalá 1965, 22].

The second method is to set out the right ascensions corresponding to each degree of celestial longitude (given in *Alm.* I 16) and use these to construct $r$-meridians.[193] This method is much more practical, but there are still some difficulties. The right ascensions in the *Almagest* are calculated at 10° intervals, with the expectation that lesser intervals can be derived from these using linear interpolation. Unless the right ascensions are recalculated at shorter intervals, some accuracy will be lost through this interpolation. Moreover, in order to mark the right ascensions accurately on the equator, one would once again need a protractor with a fine scale. Nevertheless, as Maslama remarks, this method is better than the first [Vernet and Catalá 1965, 22].

In order to avoid the practical difficulties involved in the first two methods, Maslama shows how to construct circles representing the great circles that intersect the equator and the ecliptic such that the right ascension is equal to the longitude [Vernet and Catalá 1965, 22–23; Anagnostakis 1987, 136–138]. In this way, any division that can be carried out on the equator using compass and straight edge, or a protractor marked with degrees, can also be carried out on the ecliptic.

There is no indication in the text as to which method Ptolemy intended his readers to follow, or if he had considered any of these practical issues. In fact, the only practical suggestion that Ptolemy makes involves setting out stars that are given in equatorial coordinates, $S(\alpha, \delta)$, as they probably were in the older star

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[192]For the 20° on either side of the solstices the total difference in declination is $1;31,9°$.

[193]Vernet and Catalá [1965, 29, n. 41] give a different interpretation of this construction, which although possible, is not the simplest way of understanding Maslama's remarks. According to Maslama the procedure is, "that we produce straight lines that go through the center of the equator and we produce them from the equator at the right ascensions, degree by degree,"

> أن تجيز خطوطًا مستقيمة تمر على مركز دائرة معدل النهار وتجيزها من معدل النهار على مطالع درجة درجة من الكرة المستقيمة [Vernet and Catalá 1965, 22],

> ut protrahamus lineas rectas per centrum circuli equatoris diei, et protrahemus eas ab equatore diei super ascensiones unius gradus et unius gradus de spera recta [Kunitzsch and Lorch 1994, 55].

These passages describe the construction of $r$-meridians using right ascensions.

catalogs [Duke 2002]. For this case, Ptolemy explains how to set out the stars using a simple division of the equator and a special ruler marked with the lengths of the radii of the $r$-$\delta$-circles at every degree. In this way, no guide circles need be drawn on the plate. One simply rotates the ruler to the star's right ascension and uses the ruler to mark the position by its declination. As Ptolemy points out, however, this method will not work for stars cataloged in ecliptic coordinates, as in the *Almagest*. The fact that Ptolemy is so vague about the technical issues involved in working with ecliptic coordinates may indicate that in his time, for practical purposes, star positions were still generally handled in equatorial coordinates.

Ptolemy's only suggestion for working with ecliptic coordinates, is to draw all the $r$-$\beta$-circles and $r$-$\lambda$-circles and use these as guides. Since drawing a circle at every degree is overly intricate, he suggests approximating this by drawing the circles at every $2^{\text{nd}}$, $3^{\text{rd}}$, or $6^{\text{th}}$ degree. Because these are the only common factors of 30 and 24, in this way the lines for the tropics and the meridians through the beginnings of the each of the signs will be given in the diagram.

The text appears to end abruptly, which lead a medieval commentator to write completing chapters and modern scholars to speculate on the content of the missing sections. Whatever the case, both for the purposes of instrumentation and from the perspective of mathematical theory, the *Planisphere* leaves considerable room for improvement and supplementation. Nevertheless, it is a challenging and illuminating text, and it stimulated a considerable body of work in the medieval and early modern periods.

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