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BM 42282+42294 and the Goal-Year Method

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Introduction

Our research into Babylonian Astronomy is at the moment focussed on early texts. The far off goal of investigation is to understand how the elegant numerical mathematical (ACT) astronomy of the Seleucid era was developed. We hope that it will be possible to find a way from genuine Babylonian observations over their simple prediction rules and early formation of theory to the ACT astronomy. Therefore the so called atypical astronomical cuneiform texts and other texts classified as intermediate texts are in the focus of interest.[1]

In this paper we shall concentrate on lunar phenomena and on the so called "Goal-Year" Method for the prediction of time intervals between risings and settings of sun and moon. Analysis of lunar data, of the kind collected on the Goal-Year tablets, had resulted in the construction of six, mutually similar, rules for the prediction of lunar phases. We called these proposed procedures the "Goal-Year Method".[2] That the Babylonians, indeed, knew and used the (reconstructed) method, was proved by two passages in the procedure text TU 11.[3]

Section 14 and 16 of TU 11 give advice to how such data, as collected on the Goal-Year tables, can be used for finding time intervals between risings and settings of sun and moon for a month to come and for determining the length of that month. Section 14 and 16 of TU 11 are, however, written in a rather unclear and abbreviated form, and only three out of the six rules are attested. Now we are in the lucky position to have found another and much older[4] text, BM 42282+42294, on which the Goal-Year method is presented much clearer and in every detail. (We thank Chr. Walker for drawing our attention to this text.) The text includes five of the

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[1]A. Sachs introduced the term "intermediate astronomy", explaining it as referring "to stages later than MUL.APIN and earlier than ACT. The boundaries in both directions are not sharp". See Pinches et al. p. xxxv

[2]See Lis Brack-Bernsen 1997, pp. 115–133 and 1999

[3]Brack-Bernsen and Hunger 2002, pp. 44–46 and 63–64.

[4]Probably from the Achaemenid period, see Finkel 2000, p 140f.

six rules and in addition it gives some correcting rules, which had been postulated in Brack-Bernsen 1997, p. 121.

I BM 42282 + 42294

Transliteration

Obverse

1) [D]UB *ni-ṣir-tu*$_4$ AN-*e pi-riš-tú* DINGIR$^{\text{meš}}$ GAL$^{\text{meš}}$ ana ŠU$^{\text{II}}$ NU BÍ.È *ana* DUMU-*šú šá i-ra-am-mu li-šá-hi-iz* **§ 1**

2) [*ana la* DU]MU TIN-TIR$^{\text{ki}}$ *u la* DUMU *bar-zip*$^{\text{ki}}$ *u la* DUMU EN *ih-zu šu-hu-zu* NÍG.GIG $^{\text{d}}$AG *ù* $^{\text{d}}$NISABA

3) [x x *l*]*a*(?) DUMU TIN.TIR$^{\text{ki}}$ *ù la* DUMU *bar-zip*$^{\text{ki}}$ *ù la* DUMU EN *ih-zu šá la* GAR-*ma u ma-am-mu i-dab-bu-*[*bu*]

4) [$^{\text{d}}$AG *u*] $^{\text{d}}$NISABA *ina ih-zu uh-hu-zu la ú-kan-nu-uš ina lu-up-nu ù tam-ṭa-a-t*[*u*$_4$]

5) [x x]-*šú lu-qa-at-tu-*⌈*u'*⌉ *-i-ma ina a-ga-nu-til-le-e li-du-ku-*[*šu*]

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6) GIN *u* GUR *ana* DÙ-*ka* MU-*ka ina* ŠU$^{\text{II}}$-*ka tu-kal* 18 MU$^{\text{meš}}$ *ana* EGIR-*ka* GUR-*ma ina* MU-18-K[ÁM x x x x] **§ 2**

7) 6 ITU *ana* EGIR-*ka* GUR-*ma* ŠÚ *u* NA *šá* $^{\text{itu}}$DU$_6$ GIŠ-*ma ana* UGU *a-ha-meš* GAR-GAR-*ma* ⌈ki(?) x⌉ [....]

8) TA NA *šá* UD-1-KÁM *šá* BAR *šá* MU-18-tum(*sic*) ZI-*ma mi-nu-ú ki-i ina* NA ⌈x x⌉ [....]

9) *ki-i ana-la* 10 UŠ *i-ṣa ana* GIN E *ki-i* ⌈NA⌉ *šá* UD-1-KÁM *šá* BAR *šá* MU-*k*[*a* ....]

10) *ana* GIN E *ki-i* LAL-*ú ana* GUR E *šal-šú* ⌈*šá*⌉ ŠÚ *u* NA *šá ana* UD-1-⌈KAM⌉ [....]

11) *šá* MU-*ka* BE-*tum šá mim-ma* TA ŠÀ NU ZI TAB-*ma* [....]

12) *šá-niš* ŠÚ *u* NA *gab-bi šá ana* UD-1-KAM DÙ-*šú ana* UGU N[A(?) ....]

13) GAR-GAR-*ma ana* UD-1-KAM *šá* BAR *šá* MU-*ka eš-še-t*[*i* ....]

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14) GABA-RI 36 *ana* DÙ-*ka* TA BAR <*šá*> 36 6 ITU GUR-*ma* [40 *šá* ŠÚ *u* NA *šá* $^{\text{itu}}$DU$_6$ GIŠ-*ma* TA NA *šá* UD-1-KAM *šá* BAR] **§ 3**

15) *šá* 36 ZI-*ma ki-i ana-la* 10 UŠ LAL-*ú* ŠÚ *u* N[A *gab-bi ana* UGU TAB 40 *šá* ŠÚ *u* NA TA NA *šá* MURUB$_4$ ITU]

16) ZI-*ah* 40 *šá* ME *u* GE$_6$ TA G[E$_6$ ZI-*ah*]

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17) [x x 1]8 MU$^{\text{meš}}$ 6 ITU *ana* EGIR-*ka* G[UR ....] **§ 4**

18) [x x] ⌈*šá*(?) x⌉ ZI-*ma* BE-*ma ana-la* ⌈10 UŠ⌉ [....]

19) [x x x x] ⌈*šá*⌉ *eli-šú* IGI-*ma* BE-*ma* [....]

20) [x x x x] ⌈x⌉ 40 LAL BAR *šá* MU-*ka* [....]
21) [x x x x] ⌈x⌉ *šá eli-šú* IGI-*ma* B[E-*ma* ....]
22) [x x x x] ⌈*ú*⌉ *ki* ⌈12⌉ +[x ....]
23) [x x x x x] ⌈x⌉ TA ⌈12⌉ +[x ....]

'Reverse

1') ⌈BE-*ma* 30 UD-*mu ana*⌉ [IGI-*ka* ....] **§ 1'**
2') GUB-*ma šamáš* ŠÚ UD-22-K[AM ....]
3') GUB-*ma šamáš* KUR-*ma* UD-[....]

4') BE-*ma* 1 UD-*mu ana* IGI-*ka* UD ⌈3+x KAM⌉ [....] **§ 2'**
5') GUB-*ma šamáš* ŠÚ UD-23-KAM *ina* KUR [....]
6') GUB-*ma šamáš* KUR-*ma* UD-*mu* [....]

7') 12 NA *ana* DÙ-*ka* UD-NÁ-A *šá* ŠE *šá e*[*li* BAR *šá* MU-*ka* ....] **§ 3'**
8') *šá eli* BAR *šá* MU-*ka* BE-*tum ana-la* ⌈x⌉ [....]
9') TA NA *šá* BAR *šá* MU-*ka eš-še-tú* Z[I-*ma* ....]
10') KI NA *šá* BAR *šá* MU-*ka eš-še-tú* [TAB ....]

11') ŠÚ *u* NA *ana* DÙ-*ka* 18 MU$^{\text{meš}}$ *ana* EGIR-*k*[*a* GUR-*ma* .... ŠÚ *u* NA *ana muhhi ahameš takammar*] **§ 4'**
12') *šal-šá-šú-nu* GIŠ-*ma* TA NA *šá* MURUB$_4$ BAR [ZI-*ma šá ina* NA .... NA *šá* MU-*ka eš-še-ti* E]
13') *ù šal-šú šá* ŠÚ *u* NA KI ŠÚ TAB-*m*[*a* ŠÚ *šá* MU-*ka eš-še-ti* E]
14') *ki-i šal-šú šá* ŠÚ *u* NA TA NA ZI-[*ma ina* UGU NA *i-ti-qu mim-ma šá i-ti-qu* ŠÚ *šá* MU-*ka*]
15') *eš-še-tú* E *ù mim-ma šá ina* UGU NA [*i-ti-qu mi-nu-u ki-i a-na* ŠÚ *u* NA *šá a-na a-ha-meš*]
16') *tak-mu-ru* LAL-⌈*u a*⌉ -*na* NA *šá* MU-*k*[*a eš-še-ti* E *ki-i ina* UGU NA]
17') *la i-ti-qu* ⌈ŠÚ *u* NA⌉ *šá* IGI *šámaš* [....]
18') ŠÚ *u* NA *la i*-⌈*šu u šal*⌉ -*šú* [*šá* ŠÚ *u* NA ....]

19') ME *u* GE$_6$ *ana* DÙ-*ka* 18 MU$^{\text{meš}}$ *ana* ⌈EGIR-*ka* GUR⌉ ME *u* [GE$_6$ *ana muhhi ahameš takammar*] **§ 5'**
20') *šal-šá-šú-nu* GIŠ-*ma* TA G[E$_6$ *šá*(?)] BAR ZI-*ma šá ina* GE$_6$ ⌈x⌉ [.... GE$_6$ *šá* MU-*ka eš-še-ti* E]
21') *ù šal-šú šá* ME *u* ⌈GE$_6$⌉ [K]I ME TAB-*ma ana* <<*ana*>> ME [*šá* MU-*ka eš-še-ti* E]
22') ⌈*ki*⌉ -*i šal*-⌈*šú šá* M[E *u* G]E$_6$ ⌈TA⌉ GE$_6$ ZI-*ma ina* UGU GE$_6$ ⌈*i*⌉ -*ti*-⌈*qu* x x⌉ [.... ME *šá* MU-*ka*]

23') [*eššeti*(?)] ⌈E(?) *ù*(?)⌉ *mim-ma šá ina* UGU GE$_6$ *i-te-qa mi-nu-u ki-i* ⌈*ana* ME⌉ [*u* GE$_6$]

24') [*šá a-na*] *a-*⌈*ha*(?)*-meš tak-mu-ru* LAL-*u ana* GE$_6$ *šá* MU-*ka eš-še-tú* KA [x x (x)]

25') [x x x] ⌈x⌉ [x t]u(?) *ka ma mim!-ma ina* UGU *la i-te-qa u ana* ME *u* GE$_6$ *ul i-pi*(?)-x

26') [x x x x x] MEŠ [0] *he-pí he-pí*

27') [x x x x] GIŠ(?) ŠÚ *šá* ME ⌈GAR-*an*⌉ *he-pí ù* 2-*ta* ŠU$^{\text{II meš}}$ *ana* GE$_6$ GAR-⌈*an*⌉

28') [x x x] ana ⌈DÙ-*ka*⌉ 1[8 MU]⌈$^{\text{meš}}$⌉ *ù* 6 ITU *ana* EGIR-*ka* GUR-*ma*

Translation

Obverse

1) Tablet of the secret of heaven, the hidden thing of the great gods. He must not give it out of hand; let him teach (it) to his son whom he loves. **§ 1**

2) To teach (it) to a non-citizen of Babylon or a non-citizen of Borsippa or any one who is not learned, is a taboo of Nabû and Nisaba.

3) .... a non-citizen of Babylon or a non-citizen of Borsippa or any one who is not learned who does not .... and speaks anything,

4) may Nabû and Nisaba not confirm him in the knowledge he learned, in poverty and loss

5) may they bring his [life(?)] to an end, and kill him with dropsy.

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6) In order for you to calculate full or hollow (month), you hold in your hand your year, you return 18 years behind you, and in the 18th year [....] **§ 2**

7) you return 6 months behind you, and you take ŠÚ and NA of month VII and add (them) together, and [you take one-third of it, and]

8) you subtract (it) from NA of the 1st day of month I of the 18th year, and whatever [....] when in .... [....]

9) if it is less than 10 UŠ, predict (the month) as full. If NA of the 1st day of month I of your year [....]

10) predict (the month) as full. If it is less, predict (the month) as hollow. One-third of ŠÚ plus NA which for the first day [....]

11) of your old year from which nothing was subtracted, you add, and [....]

12) Secondly, the whole of ŠÚ plus NA which you calculated for the first day, to the N[A(?) ....]

13) you add, and to the first day of month I of your new year [....]

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14) In order for you to calculate the correspondence of 36 (years), you return **§ 3**

from month I of your 36 (years) by 6 months, and [you take 0;40 of ŠÚ plus NA of month VII, and from the NA of the 1st day of month I]

15) of 36 (years) you subtract (it); if (the result) is less than 10 UŠ, [you add the entire ŠÚ plus NA. 0;40 of ŠÚ plus NA from NA of the middle of the month]

16) you subtract. 0;40 of ME plus GE_6 [you subtract] from GE_6.

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17) [....] you return [1]8 years and 6 months behind you [....] **§ 4**

18) [....] .... you subtract, and if it is [less/more] than 10(?) UŠ(?) [....]

19) [....] which is above of it, you examine(?), and if [....]

20) [....] less [than(?)] 40, month I of your [....] year [....]

21) [....] which is above of it, you examine(?), and if [....]

22) [....] .... with(?) 12+[x ....]

23) [....] .... from 12+[x ....]

Reverse

1') If [you are to see] the 30th day [....] **§ 1'**

2') stands(?), and the sun sets. The 22nd day [....]

3') stands(?), and the sun rises, and the day [....]

4') If you are to see the 1st day, the 3+xth day [....] **§ 2'**

5') stands(?), and the sun sets. The 23rd day in(?) the KUR [....]

6') stands(?), and the sun rises, and the day [....]

7') In order for you to calculate 12, the NA, [you look] at the day of invisibility of month XII which is above [month I of your .... year ....] **§ 3'**

8') which is above month I of your old year; if it is more than [....]

9') you subtract from the NA of month I of your new year, [and ....]

10') you [add] it to the NA of month I of your new year [....]

11') In order for you to calculate ŠÚ and NA, [you return] 18 years behind you [.... you add ŠÚ and NA together,] **§ 4'**

12') you take one-third of them, and [you subtract] it from the NA of the middle of month I, [and what remains of NA you predict as the NA of your new year;]

13') and you add one-third of ŠÚ and NA to the ŠÚ, and [predict it as the ŠÚ of your new year.]

14') If you subtract one-third of ŠÚ and NA from the NA [and it goes beyond the NA, (whatever goes beyond the NA,) as the ŠÚ]

15') [of your] new [year] you predict. And whatever [it goes] beyond the NA [(of your old year), whatever] (this result) is less [than ŠÚ and NA which]

16') you added [to each other, you predict it] as the NA of your [new] year. [If it]

17') does not go beyond [the NA (of your old year)], ŠÚ and NA in front of the sun .... [....]

18') it has no ŠÚ and NA; and one-third [of ŠÚ and NA ....]

19') In order for you to calculate ME and GE_6, you return 18 years behind you. ME and [GE_6 (of your old year) you add together,] **§ 5'**

20') you take one-third of them, and you subtract it from the G[E_6 of] month I, and what [remains(?)] of GE_6 [you predict as the GE_6 of your new year.]

21') And one-third of ME and GE_6 you add to the ME, and [you predict it] as the ME [of your new year.]

22') If you subtract one-third of ME and GE_6 from the GE_6, and it goes beyond the GE_6 .... [whatever goes beyond the GE_6]

23') you predict [as the ME (of your new year).] And whatever goes beyond the GE_6, what (this result) is less than the ME [and GE_6]

24') [which] you added together, you predict as the GE_6 of your new year. [....]

25') [....] .... does not go anything beyond it, and it does not .... to ME and GE_6.

26') [....] (broken) (broken)

27') [....] .... of ME you place (broken) and two-thirds as the GE_6 you place.

catch-line:

28') [....] for you to calculate, you return 18 years and 6 months behind you, and.

Notes to the text

1-2: these lines are quoted by I. L. Finkel, in: Wisdom, Gods and Literature. Studies in Assyriology in Honour of W. G. Lambert (Winona Lake, 2000), p. 141 n. 11.

5: a writing similar to *lu-qa-at-tu-*⌈*u'*⌉ *-i-ma* occurs in a Late Babylonian legal document: *ip-tu-ú-u'-i*, AfO 50 (2005) 256 BM 25098:14. This form too is 3rd person plural of a verb III inf.

6: "to hold in your hand" meaning "to keep in mind" is frequent in procedure texts, see CAD K kullu mng. 3f.

14-15: restored from TU 11 0bv., 37f.

rev. 7'-10': this is almost a duplicate to TU 11 rev. 20-22, except that one expects "your new year" in line 8' instead of "your old year". For this reason we have not restored the missing part. Note that more than half of the lines is broken off so that there is probably more to restore than is written in TU 11.

rev. 12'-14': restorations in these lines are tentative and based on the parallel lines 20'-23'.

rev. 20': just where the tablet breaks off, a word for "(what) remains" is to be expected.

rev. 22’: restoration of the end of this line is only based on context, i. e. the calculation.

rev. 23’ff.: these lines must have been damaged on the original, as can be seen from the word “broken” in lines 26’f.

rev. 23’: the context suggests the word “new” at the beginning, but space is not sufficient for a writing *eš-še-ti*. Maybe a logogram was used.

rev. 25’: the verb at the end is obscure to me.

Let us start with an overview of the content of BM42282+:

- 1: secret tablet
- 2: Goal-Year Method utilized for finding full or hollow months. (This Section is paralleled by Section 14 of TU 11.)
- 3: Goal-Year Method used for finding NA_N, NA and GE_6 after 36 years. (Parallel to TU 11 Section 16)
- 4: Goal-Year Method applied for 18 years.
- 1’: Month short if moon culminates at sunrise and sunset at early day.
- 2’: Month long if moon culminates at sunrise and sunset at late day.
- 3’: the month length is found by a primitive schematic method. (This Section is parallel to TU 11 Section 22)
- 4’: G-Y-Rules for finding *ŠÚ* and *NA*, all cases considered.
- 5’: G-Y-Rules for finding *ME* and GE_6, all cases considered.

II The Goal-Year Method

The time-intervals used and predicted by the Goal-Year method are called the “Lunar Six” - two of these can be observed around new moon (conjunction), while four can be observed around full moon (opposition). The four observable, occurring around opposition, are referred to as the “lunar four”. Below we repeat the Babylonian names and their meaning.

The Lunar Six:

NA_N was observed on the evening when the new crescent was visible for the first time after conjunction, indicating the first day of the month:

NA_N= the time between sunset and the first visible setting of the new moon.[5]

At sunrise and sunset in the days around opposition, the following "Lunar Four" time intervals were regularly measured:

$\check{S}\acute{U}$ = time from moonset to sunrise, measured at last moonset before sunrise.
NA = time from sunrise to moonset, measured at first moonset after sunrise.
ME = time from moonrise to sunset, measured at last moonrise before sunset.
GE_6= time from sunset to moonrise, measured at first moonrise after sunset.

At the end of the month the event KUR took place and could be measured:

KUR = the time from last visible moonrise before conjunction to sunrise.

These time intervals are obvious and easy to observe. From a modern theoretical astronomical point of view however, the intervals are very complicated quantities. They depend on the time, Δt, when in comparison to sunset the conjunction (or the opposition) takes place. They also depend on $\lambda_☾$, the position of the full (or new) moon in the ecliptic, and on the lunar velocity, $v_☾$, and latitude, $\beta_☾$:

$$NA = NA(\Delta t, \lambda_☾, v_☾, \beta_☾).$$

Therefore it was a big surprise, that the Babylonians found an elegant, easy, and astonishingly precise "short cut" leading directly from known Lunar Six to future Lunar Sixes.

The "Goal-Year" method for predicting Lunar Six

The Goal-Year Method utilizes the Saros (1 Saros = 223 synodic months ≈ 239 anom. months ≈ 241 sid. months ≈ 242 drac. months ≈ 18 years). It is based on the fact that except for Δt, the time from sunset to setting or rising of the moon, all the quantities, $\lambda_☾$, $v_☾$, and $\beta_☾$, which determine the Lunar Six, do repeat almost exactly after a Saros. The change in Lunar Sixes, situated 1 Saros apart therefore is caused by the change in Δt alone – it is only the result of the change in Δt.

[5]In the texts with which we are working, this interval is called *NA*, but it occurs always together with an indication that it is the *NA* of the first day or the *NA* at the beginning of the month. We put this identification into the name, calling it NA(of the new crescent), or NA_N. We do this in order to be as precise as the Babylonian texts. There the term *NA* is also used for a time interval in the middle of the month, but always identified by calling it the *NA* of day 14 or the *NA* opposite the sun.

After a Saros, the time Δt will have changed by 1/3 day, since the Saros equals 6585 1/3 day. A series 1, 2, ... ,i,. of consecutive lunar months is considered. The "Lunar Six" stemming from month i are identified by the index i. Below, we render the Babylonian predicting rules in mathematical form as equation (1) – (6).[6] In all equations we drop the subscript from GE_6 for clarity.

The Goal-Year Formulae

$$(NA_N)_i = (NA_N)_{i-223} - 1/3\,(\check{S}\acute{U} + NA)_{i-229}. \tag{1}$$

$$\check{S}\acute{U}_i = \check{S}\acute{U}_{i-223} + 1/3\,(\check{S}\acute{U} + NA)_{i-223}, \tag{2}$$

$$NA_i = NA_{i-223} - 1/3\,(\check{S}\acute{U} + NA)_{i-223}, \tag{3}$$

$$ME_i = ME_{i-223} + 1/3\,(ME + GE)_{i-223}, \tag{4}$$

$$GE_i = GE_{i-223} - 1/3\,(ME + GE)_{i-223}. \tag{5}$$

BM 42282+ has traces of these 5 formulas, but nothing on *KUR*. However, corresponding to (1), the reconstructed formula for calculating *KUR* must be:

$$KUR_i = KUR_{i-223} + 1/3(ME + GE)_{i-229}. \tag{6}$$

We have, however, also found textual evidence for this case: one section of the fragment BM 37110 is indeed concerned with this rule.[7] As the equations show, the rules for predicting the Lunar Six for a specific month(i) are based on their values (known) in the months(i-223) and (i-229) situated 1 Saros and 1 Saros plus 6 months earlier, respectively. Exactly these quantities were collected on the Goal-Year table for the year of month i. The rules are more or less implicitly given in Sections 14 and 16 of TU 11, and formulated more clearly on BM 42282+42294. In order to understand the texts on these fragments we shall introduce a concept which has often been utilized by the Babylonians:

[6] For a more thorough explanation and for astronomical comments, see Brack-Bernsen 1999 or 1997 pp. 113–121. The Goal-Year method was also utilized for predicting if a coming month would become full (i.e. 30 days long) or hollow (i.e. 29 days long), see Brack-Bernsen and Hunger 2002 pp. 40–54.

[7] See Brack-Bernsen, Hunger and Walker, 2007

The daily retardation of the moon

In the average and with respect to sunset, the moon rises and sets 48 minutes later from day to day. In reality, the daily retardation of the moon varies and it is different for the setting and the rising moon. In the days around opposition, $\check{S}\acute{U}$+NA is a good measure for the daily retardation of the setting full moon, and ME+GE_6 for the daily retardation of the rising full moon. Both quantities can be measured directly, whenever an opposition takes place at good weather, so that in the days around opposition no clouds obscure the sky near the horizon.

The equations (2) and (3) use the third of $(\check{S}\acute{U}{+}NA)_{i-223}$. It has been demonstrated that the observable $(\check{S}\acute{U}{+}NA)_{i-223}$ gives the daily retardation of the setting moon in month i-223 as well as in month i (control calculations have shown that $(\check{S}\acute{U}{+}NA)_{i-223} \approx (\check{S}\acute{U}{+}NA)_i$). The Goal-Year Method utilizes that the time of opposition with respect to sunset is shifted by 1/3 day after a Saros. As a result, the observable $\check{S}\acute{U}$ will be enlarged by 1/3 × the daily retardation of the setting moon and NA is reduced by the same amount after 1 Saros.

Corrections to the Goal-Year formulae

Sometimes, the results found by the calculations according to equation (1)–(6) are preliminary. A correction to procedure (1) is found in cuneiform texts. In TU 11 Obv. 37. and in Section 3 of BM 42282+ we read: If NA_{Ni} happens to become too small (the text says smaller than 10 UŠ), then the moon will only become visible the next day, and hence the value of NA_{Ni} will become $(\check{S}\acute{U}{+}NA)$ larger: corrected $(NA_N)_i$ = preliminary $(NA_N)_i$ + $(\check{S}\acute{U}{+}NA)_{i-229}$.

$$(NA_N)_i = (NA_N)_{i-223} + 2/3\,(\check{S}\acute{U} + NA)_{i-229}. \qquad (1c)$$

These cuneiform texts show us that the Babylonian astronomers knew and utilized, that after one day, the setting of the new moon was retarded by $(\check{S}\acute{U}{+}NA)_{i-229}$.[8] Sometimes similarly, the calculated Lunar Four need a correction: At some lunation, i, the NA_i calculated by formula (3) can happen to become negative, and for the same lunation, $\check{S}\acute{U}_i$ would become larger than $(\check{S}\acute{U}{+}NA)_{i-223}$ - indicating that the full moon also the next day would set before sunrise. This means that for this full moon formula (2) and (3) calculates the quantities looked for one day too early. Hence, the results are preliminary and need a correction: The calculated $(\check{S}\acute{U})_i$ must be reduced by $(\check{S}\acute{U}{+}NA)_{i-223}$ while the calculated (negative) $(NA)_i$ must be enlarged by the same amount. In analogy to procedure (1c), which was found in

[8]See Brack-Bernsen (1999) where it was shown, that$(\check{S}\acute{U}{+}NA)_{i-229}$ is an excellent - and observable - approximation to the daily retardation of the crescent setting at the beginning of month (i - 223) as well as the one setting at the beginning of month (i).

cuneiform texts, we propose below in formula (2c) and (3c) how the corrected values of $\check{S}\acute{U}_i$ and NA_i can be found:

$$\check{S}\acute{U}_{iC} = \check{S}\acute{U}_{i-223} - 2/3\,(\check{S}\acute{U} + NA)_{i-223}, \tag{2c}$$

$$NA_{iC} = NA_{i-223} + 2/3\,(\check{S}\acute{U} + NA)_{i-223}, \tag{3c}$$

In analogy, we propose the following procedure for finding the corrected values of ME_i and GE_{6i}:

$$ME_{iC} = ME_{i-223} - 2/3\,(ME + GE)_{i-223}, \tag{4c}$$

$$GE_{iC} = GE_{i-223} + 2/3\,(ME + GE)_{i-223}. \tag{5c}$$

That the Babylonians were concerned with different procedures, and hence with special cases, is supported by our text BM 42282 + 42294. The lines 11'– 18' and 19'–28' on the reverse side of this text are concerned with the Goal-Year rules and their eventual corrections. We will, however, have to transform our equations a bit in order to show it.

III BM 42282+42294 on the Goal-Year Method

The Sections 2, 3, 4, 4', and 5' of BM 42882+42994 are concerned with the Goal-Year Method. Sections 2 and 3 are parallel to TU 11 Sections 14 and 16 and need no further comments, and of Section 4 so little is readable that we can only say that it is concerned with the Goal-Year Method. We shall concentrate on the sections 4' and 5' which contain important new insights.

Section 4' lines 11'–13' and Section 5' lines 19'–21'

Section 4' deals with the calculation of $\check{S}\acute{U}$ and NA while 5' treats ME and GE_6. In order to understand the text better, we have changed our equations (2) - (5) in that we use the terminology of the text. Instead of writing NA_i we write NA_{new} since the text call it NA of the new year while NA_{i-223}, which is the NA measured one Saros earlier than NA_{new}, is written as NA_{old}.

$$\check{S}\acute{U}_{new} = \check{S}\acute{U}_{old} + 1/3\,(\check{S}\acute{U} + NA)_{old}, \tag{2'}$$

$$NA_{new} = NA_{old} - 1/3\,(\check{S}\acute{U} + NA)_{old}, \tag{3'}$$

$$ME_{new} = ME_{old} + 1/3\,(ME + GE)_{old}, \tag{4'}$$

$$GE_{new} = GE_{old} - 1/3\,(ME + GE)_{old}. \tag{5'}$$

We point at the close connection between the text and the method as rendered in our formulae: Evidently, lines rev. 11' – 13', give advice for calculating *NA* and *ŠÚ* by the Goal-Year Method.

In order for you to calculate ŠÚ and NA, [you return] 18 years behind you [...you add ŠÚ and NA together,] you take one-third of them and [you subtract] it from the NA of the middle of month I [and what remains of NA you predict as the NA of your new year] and you add one-third of ŠÚ+NA to the ŠÚ, and [predict it as the ŠÚ of your new year.]

In complete analogy are the rules for finding *ME* and GE_6 given in lines rev. 19'–21'.

In order for you to calculate ME and GE_6*, return 18 years behind you. ME and [*GE_6 *(of your old year) you add together,] you take one-third of them, and you subtract it from the G[*E_6 *of] month I, and what [remains] of* GE_6 *[you predict as the* GE_6 *of your new year.] And one-third of ME and* GE_6 *you add to the ME, and [you predict it] as the ME[of your new year.]*

We have here clear textual evidence for (the normal case of) the Goal-Year Method, but we have even more: A glance on (3') or (5') reveals that using the Goal-Year method, one might consider three different cases:

	for finding *ŠÚ* and *NA*	or	for finding *ME* and GE_6
1.	$1/3\ (\check{S}\acute{U}+NA)_{old} < NA_{old}$	or	$1/3\ (ME+GE_6)_{old} < GE_{6old}$
2.	$1/3\ (\check{S}\acute{U}+NA)_{old} = NA_{old}$	or	$1/3\ (ME+GE_6)_{old} = GE_{6old}$
3.	$1/3\ (\check{S}\acute{U}+NA)_{old} > NA_{old}$	or	$1/3\ (ME+GE_6)_{old} > GE_{6old}$,

And our text BM 42282+42294 does, indeed, consider all three cases. The first case is the "normal" case which was treated above. In the second case, where the third of the daily retardation of the setting or rising moon equals *NA* or GE_6, respectively, the Goal-Year Method will predict that the sun will rise at the moment of moonset (or set at the moment of moonrise). Therefore it is a matter of taste to decide if e.g. *ŠÚ* (or *ME*) is zero and *NA* (or GE_6) equals the whole *ŠÚ*+*NA* (or *ME*+GE_6, respectively) or vice versa.

In the third case the method delivers the time differences for a day later than intended. Therefore one has to introduce a correction, using the daily retardation of the moon, in order to find the correct Lunar Four.

As we have seen above, the first and "normal" case is treated in the first part of the sections 4' and 5': in lines 11'–13' and 19'–21', respectively. The second case is treated in lines 17'–18' and 25'–26', while the third case can be found in lines 14'–16' and 22'–24', respectively.

Section 4', special cases

Case 3: $\mathbf{1/3(SU+NA)}_{old} > \mathbf{NA}_{old}$

Line 14'– 16' is concerned with the special cases where $NA - 1/3(\check{S}\acute{U}+NA)$ becomes negative. Rev.14' writes: if $1/3(\check{S}\acute{U}+NA)$ which should be subtracted from NA is greater than NA, then calculate the difference $1/3(\check{S}\acute{U}+NA) - NA$. This way of coping with differences which might become negative is known from other texts (e.g. TU11), and will result in slightly different arithmetic procedures than ours. We have learned how to handle negative numbers as normal numbers while the Babylonians just calculated the opposite (i.e., positive) difference and continued working with that.

Instead of calculating as we proposed in equation (2c):

$$\check{S}\acute{U}_{newC} = \check{S}\acute{U}_{old} - 2/3\,(\check{S}\acute{U} + NA)_{old}, \qquad (2c')$$

the Babylonian text gives another equivalent, and hence correct, procedure for finding the new and corrected $\check{S}\acute{U}$:

$$\check{S}\acute{U}_{newC} = 1/3\,(\check{S}\acute{U} + NA)_{old} - NA_{old}, \qquad (7)$$

That the procedures are equivalent is easy to see. Just add and subtract NA_{old} on the right side of equation (2c') and you will get equation (7):

$$\begin{aligned}\check{S}\acute{U}_{newC} &= \check{S}\acute{U}_{old} + NA_{old} - 2/3\,(\check{S}\acute{U}+NA)_{old} - NA_{old}\\ &= 1/3\ (\check{S}\acute{U}+NA)_{old} - NA_{old}\end{aligned}$$

In words: *If you subtract 1/3 of ŠÚ+NA from NA [and it goes beyond NA, (whatever goes beyond the NA,) as the ŠÚ of your] new [year] you predict.*
We are convinced that this part of the text (i.e., Rev. 14') gives the advice for finding $\check{S}\acute{U}$, so we have completed the text accordingly. The argument behind this procedure is evident: NA_{old} tells us how long **after** sunrise the full moon sets in our old year, while $\check{S}\acute{U}_{old}$ is the time from moonset to sunrise measured the day before. It is known that the moon in the new year will set earlier by the amount of $1/3(\check{S}\acute{U}+NA)$ with respect to sunrise. If, therefore, $1/3\ (\check{S}\acute{U}+NA)_{old}$ is larger than the NA_{old}, then it just means that the full moon in the new year will set at the time of $1/3\ (\check{S}\acute{U}+NA)_{old} - NA_{old}$ **before** sunrise, and this quantity is evidently equal to $\check{S}\acute{U}_{new}$.

In other words rev. 14' just uses the fact, that:

NA_{old} - 1/3($\check{S}\acute{U}$+NA)$_{old}$ equals NA_{new}, when $NA_{old} > 1/3(\check{S}\acute{U}+NA)_{old}$

while

NA_{old} - 1/3($\check{S}\acute{U}$+NA)$_{old}$ equals $\check{S}\acute{U}_{new}$, when $NA_{old} < 1/3(\check{S}\acute{U}+NA)_{old}$

Also in case of the corrected NA, the text uses another way than we would do. Instead of calculating

$NA_{new} = NA_{old}$ - $1/3(\check{S}\acute{U}+NA)_{old} + (\check{S}\acute{U}+NA)_{old} = NA_{old} + 2/3(\check{S}\acute{U}+NA)_{old}$,
as proposed in equation (3c), the text utilizes the fact that the daily retardation of the setting moon repeats after a Saros, $(\check{S}\acute{U}+NA)_{new} = (\check{S}\acute{U}+NA)_{old}$, and calculates NA_{newC} as $(\check{S}\acute{U}+NA)_{old}$ - $\check{S}\acute{U}_{newC}$ ending up with the following procedure (written in Rev. 15' and 16'):

$$NA_{new} = (\check{S}\acute{U} + NA)_{old} - [1/3\,(\check{S}\acute{U} + NA)_{old} - NA_{old}], \tag{8}$$

which is equivalent to our formula (3c) .

Case 2: $\mathbf{NA}_{old} = \mathbf{1/3(SU+NA)}_{old}$

Rev. 17' and 18' refers to the situation where NA calculated by equation (3) happens to become equal to zero. This means that the moon is expected to set exactly at sunrise. In this case it is a matter of choice to say: $\check{S}\acute{U}$ is large, equal to the whole $\check{S}\acute{U}$+NA while NA equals zero, or to say $\check{S}\acute{U}$ is zero and NA equals $\check{S}\acute{U}$+NA. The Babylonians seem to say they are both zero. This situation occurs whenever $NA = 1/3(\check{S}\acute{U}+NA)$. Or in words similar to the text: *if 1/3(ŠÚ+NA) does not go beyond the NA, then there is no ŠÚ or NA.* The reading of "no $\check{S}\acute{U}$ or NA" as "moonset at the moment of sunrise" i.e. as $\check{S}\acute{U} = 0 = NA$ is supported by other texts. The words "there is no $\check{S}\acute{U}$ or NA", occur e.g. in a Goal-Year text (LBAT 1285) while "there is no ME or GE_6" is written in Diary (No. -328). In both cases the interpretation of moonset at the moment of sunrise (or moonrise at sunset, respectively) has been confirmed by control calculations.

Section 5', special cases for ME and GE_6

Section 5' deals with the calculation of ME and GE_6 and is structured very similar to Section 4' - so similar that visible signs in one section have helped reconstructing hardly visibly signs in the other section. The corrections in case of $1/3(ME+GE_6)$ being bigger than or equal to GE_6 are completely analogous to those treated above where $\check{S}\acute{U}$+NA was larger than or equal to NA.

Case 3: $\mathbf{1/3(ME+GE)}_{old} > \mathbf{GE}_{old}$

Instead of calculating as we proposed in equation (4c) Rev. 22' gives advice to find ME_{new} as:

$$ME_{newC} = 1/3\,(ME + GE)_{old} - GE_{old}, \tag{9}$$

Evidently, the Babylonian procedure (9) for finding ME in case of $GE_6 < 1/3(ME{+}GE_6)_{old}$ is completely parallel to the procedure (7) for finding $\check{S}\acute{U}$, when $NA < 1/3(\check{S}\acute{U}{+}NA)_{old}$. The procedure for finding GE_6 in case of GE_{6old} being smaller than $1/3(ME{+}GE_6)_{old}$ is given in 23' and 24' and reproduced in equation (10) below. Procedure (10) is, as expected, analogous to procedure (8).

$$GE_{newC} = (ME + GE)_{old} - [1/3\,(ME + GE)_{old} - GE_{old}], \tag{10}$$

The theoretical explanations behind the procedures (9) and (10) for finding ME_{newC} and GE_{6newC} are analogous to those behind (7) and (8) for finding $\check{S}\acute{U}_{newC}$ and NA_{newC}. Therefore we refrain from repeating it in detail. In short: the idea behind these procedures is that - $GE_6 = ME$, and that the daily retardation of the rising moon repeats its actual value almost exactly after 1 Saros.

Case 2: $\mathbf{1/3(ME{+}GE)_{old} = GE_{old}}$

There is not much left on rev. 25' and 26'; but still enough to ensure that these lines treat the case where $1/3(ME{+}GE_6)_{old} = GE_{6old}$. The quantities ME and GE_6 are mentioned together with the words *"does not go beyond"*. We reconstruct the procedure: *There is no ME or GE_6.* Line 27' may mention 2/3 of $ME{+}GE_6$ (which we cannot explain - we would have expected to find the whole of $ME{+}GE_6$); but too little text is visible to enable us to read it.

Line 28' is the catch-line for the next tablet.

It says: *In order to calculate [...], return 18 years and 6 months behind you, and.* We see that the cuneiform tablet following BM42282 in the ancient collection again was concerned with the Goal-Year Method.

IV BM 42282+42294 on full or hollow month

Section 2, i.e., lines 6–13 deals with full and hollow months. It uses the Goal-Year Method for finding NA_N. As was the case in Section 14 of TU 11, a detail in the procedure is used for predicting full or hollow month: if NA_N is found by means of formula (1) - or if an addition is necessary formula (1C). We shall not repeat the argumentation but refer to our discussion of Section 14 in our edition of TU 11.[9]

[9]See Brack-Bernsen and Hunger 2002, pp.40–54.

Sections 1' and 2'

Sections 1' and 2' on the reverse of BM 42282+ are again concerned with the duration of the Babylonian month. It is indicated at the beginning of the sections: *If you are to see the 30th (or the 1st day), respectively.*

The structure of the predictions of month length seems to be like:
If you are to see a short (respectively long) month on day 6 (or 7, respectively) will ... stand while the sun sets,
and on day 22 (or 23, respectively) will ... stand while the sun rises.

The new (Babylonian) month begins the first evening after new moon, at which the new crescent becomes visible. In the readable part of the text only the sun is mentioned in spite of the fact that it is the moon which determines when the new month begins. We propose that "... stands" refers to the culminating (half) moon. The day numbers 22 or 23 of the culminating (waning half) moon are clearly written in the text, while the days, 6 and 7, of the (culminating waxing half) moon are only partly preserved. We reconstruct the text as follows:

If you are to see the 30th day [i.e. a short month] then on day 6 the (waxing half) moon will culminate at sunset, and on day 22 the (waning half) moon will culminate at sunrise.

If you are to see the 1st day [i.e., a long month] then on day 7 the (waxing half) moon will culminate at sunset, and on day 23 the (waning half) moon will culminate at sunrise.

A predicting rule like this can possibly be constructed from the ideal lunar schemes A (or B) from *Enūma Anu Enlil* XIV (EAE XIV from now on).[10] The rule does, however, not reflect empirical knowledge – nature behaves differently. This could seem to contradict our proposal; however, from TU 11 we know of two similar examples, where ideal schemes are used for predictions. In TU11 Section 18 is the schematic length of night used to find the change in NA_N after one Saros; and Section 19 derives the daily retardation of the moon from the "ideal" length of the day, according to table C of EAE.[11] None of the two procedures are confirmed by empirical data - nature behaves more complex. The procedures are, however, still important for us, since they teach us some early astronomical concepts and how the Babylonians in the early formation of lunar theory utilized the ideal schemes. Let us therefore speculate if the ideal schemes possibly may be used for finding full or hollow month.

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[10] See Al-Rawi and George 1991

[11] See Brack-Bernsen and Hunger 2002, pp. 70–73.

For the 30 days of the ideal (equinoctial) month, the "visibility-times" of the moon are given in Table A and B of EAE. Therefore, for such an "ideal" month of 30 days, we can find the times between sunset (respectively sunrise) and moonset for some days according to the scheme:

On the 7th day will the moon set 84 *uš* after sunset.
On the 8th day will the moon set 96 *uš* after sunset.
On the 14th day will the moon set 168 *uš* after sunset.
On the 15th day will the moon set 180 *uš* after sunset.
On the 22th day will the moon rise 96 *uš* before sunrise.
On the 23th day will the moon rise 84 *uš* before sunrise.

What do these times tell us, and how did the Babylonians treat the phenomena? Here the astrological reports written to Assyrian kings may give us some insight. According to EAE XIV, the moon will set 168 *uš* after sunset on the 14th day. This implies that the moon will rise 12 *uš* before sunset, the sun being still visible at the time of moonrise. In several reports we read about this event, e.g., in Report 15 (Hunger 1992 p. 11):

If the moon and sun are in balance: the land will become stable; reliable speech will be placed in the mouth of people; the king of the land will make his throne last long. If the moon and sun are in opposition: the king of the land will widen his understanding. If on the 14th day the moon and sun are seen together: reliable speech, the land will become happy. The gods will remember Akkad favorably; joy among the troops; the king will become happy; the cattle of Akkad will lie in the steppe undisturbed

We see, the text is concerned with the "two gods" being seen together. How is it in the days around at half moon?

Day 7: the moon, setting 84 *uš* after sunset, culminates 6*uš* **before** sunset.[12]
Day 8: the moon, setting 96 *uš* after sunset, culminates 6*uš* **after** sunset.

As a consequence: the sun will still be visible at the western horizon when the waxing half moon "culminates" on day 7, while the sun will have gone down already when the moon "culminates" on day 8. In case of the culminating half moon, considered here, on day 7 (but not on day 8) would the two gods both be visible at the moment of culminating moon. Similarly, only on day 23, but not on day 22 will the sun be visible at the horizon at the moment when the moon culminates, according to the ideal schemes:

[12] by "culmination" we will here understand the moon being 90 *uš* from setting or rising and use this "definition" to figure out when the moon culminates on day 7 and 8 according to the scheme.

Day 22: the moon, rising 96 *uš* before sunrise, culminates 6 *uš* **before** sunrise.
Day 23: the moon, rising 84 *uš* before sunrise, culminates 6 *uš* **after** sunrise.

Such considerations may lie behind the prediction, that on day 7 **in a long (and ideal) month** will the moon stand highest when the still visible sun is about to set, while on day 23 it will stand highest at the moment when the sun has just risen.

In case of the "imperfect" **short month of 29 days**, these events are assumed to take place one day earlier, which is on day 6 and 22, respectively.

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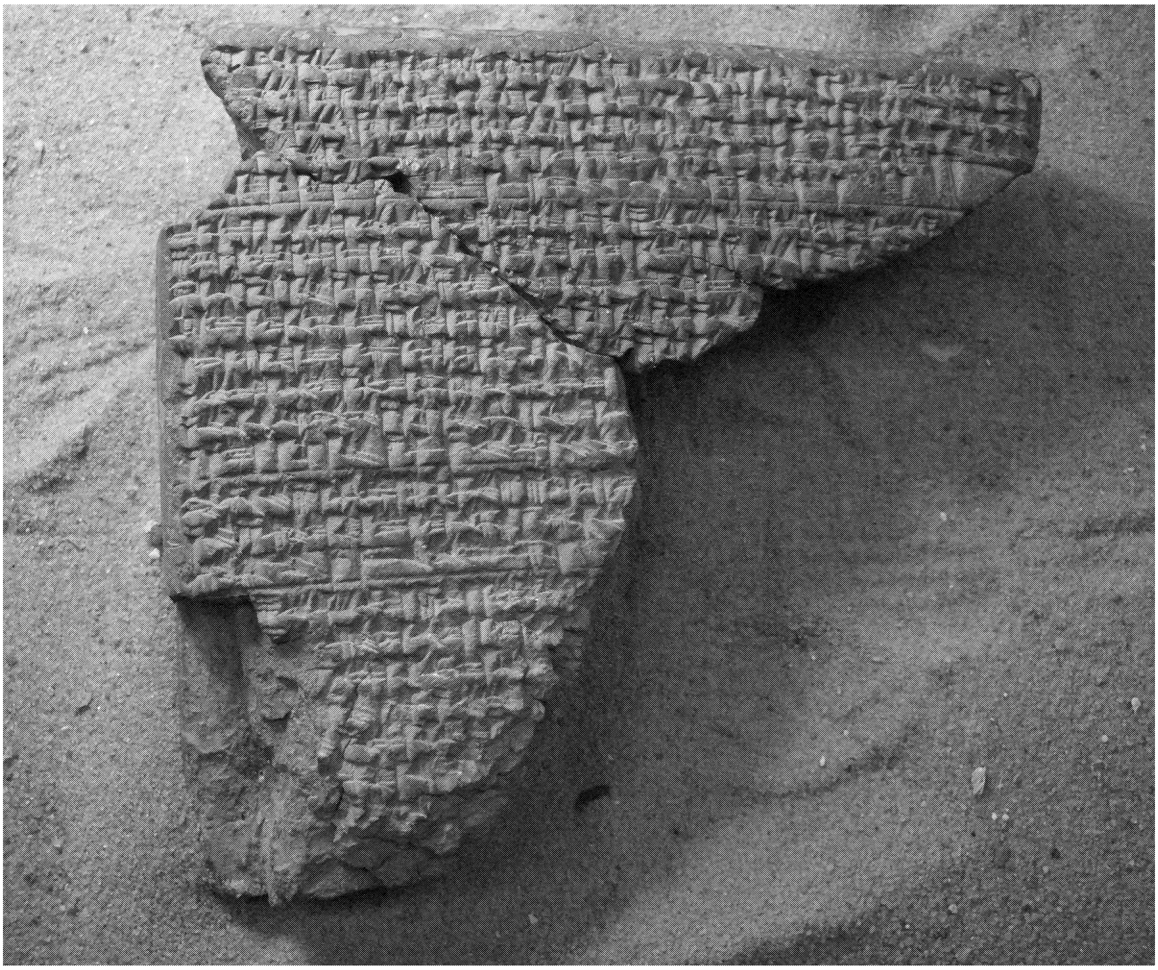
Appendix: Images of the Tablet



Obverse

Reverse

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