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Jābir b. Aflaḥ
On the limits of solar and lunar eclipses[1]

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Abstract

The limits of solar and lunar eclipses were computed by Ptolemy in *Almagest* VI.5 to establish the maximum interval in the argument of latitude in which it was possible for an eclipse to occur when the moon's mean position in the argument of latitude at each mean syzygy falls within the limits of this interval. To determine these limits, Ptolemy first obtained the true nodal distance of the Moon in the lunar inclined orbit at the apparent syzygy for a minimum possible eclipse. He then added to this true position the maximum difference in the argument of latitude between the mean and true syzygies. The interval obtained, after taking into account the argument of latitude of the lunar nodes, was slightly wider than the maximum interval in the argument of latitude of lunar mean positions at mean syzygies for solar eclipses. This can be seen, either, as a suitable but rough estimate of the correct value or as an inaccurate procedure for deducing the lunar mean position in the argument of latitude at mean syzygies from apparent syzygies.

Jābir b. Aflaḥ, the twelfth-century Andalusian astronomer, understood Ptolemy's procedure in this second sense. He noticed this point and showed the accurate procedure for obtaining lunar mean positions in the argument of latitude at the mean syzygy from the apparent syzygy and thus provided more accurate estimations of the eclipse limits.

1. Introduction

In this paper I will discuss the first criticism of Ptolemy appearing in Book V of Jābir b.

[1] This paper has been prepared as part of the research programme "La evolución de la ciencia en la sociedad de al-Andalus desde la Alta Edad Media al pre-Renacimiento y su repercusión en las culturas europeas y árabes (siglos X-XV)", sponsored by the Spanish Ministry of Education and Science (FFI2008-00234/FILO) and FEDER.

Aflaḥ's *Iṣlāḥ al-Majisṭī*.[2] Jābir b. Aflaḥ was an Andalusian mathematician and astronomer, probably from Seville, known in the Latin world as Geber.[3] He was active during the first part of the 12th century. His most notable work was the *Iṣlāḥ al-Majisṭī* or *Correction of the Almagest*, in which he rewrote the *Almagest* to simplify its mathematics. He also introduced some criticisms of the original *Almagest*, although these were mainly from a mathematical perspective. The *Iṣlāḥ al-Majisṭī* was an astronomical handbook in circulation until the 18th century, above all in the Latin world.[4] It was

[2] For a general introduction to Jābir b. Aflaḥ, see R.P. Lorch (1975), "The Astronomy of Jābir b. Aflāḥ", *Centaurus*, Vol. 19, pp. 85-107 (reprint in R.P. Lorch (1995a), *Arabic Mathematical Sciences: Instruments, Text, Transmission*, Aldershot, VI) an abridgement of his doctoral thesis read at Manchester University in 1971: *Jābir ibn Aflaḥ and his Influence in the West*; José Bellver (2008), "On Jābir b. Aflaḥ's Criticisms of Ptolemy's *Almagest*" in E. Calvo *et al.* (2008), *A Shared Legacy: Islamic Science East and West. Homage to professor J.M. Millás Vallicrosa*, Barcelona, pp. 230-8; and J. Bellver (2009), "El lugar del *Iṣlāḥ al-Maŷisṭī* de Ŷābir b. Aflaḥ en la llamada «rebelión andalusí contra la astronomía ptolemaica»", *al-Qanṭara*, Vol. 30, fasc. 1 (2009), pp. 83-136. Lorch has written other papers on the work of Jābir b. Aflaḥ, such as R.P. Lorch (1976), "The Astronomical Instruments of Jābir ibn Aflaḥ and the Torquetum", *Centaurus*, Vol. 20, pp. 11-34 (reprint in R.P. Lorch (1995a), XVI); R.P. Lorch (1995c), "Jābir ibn Aflaḥ and the Establishment of Trigonometry in the West" in Lorch (1995a), VIII; R.P. Lorch (1995b), "The Manuscripts of Jābir's Treatise" in Lorch (1995a), VII; R.P. Lorch (2001), *Thābit ibn Qurra, On the Sector-Figure and Related Texts. Edited with Translation and Commentary*, Frankfurt am Main, pp. 387-90. Other scholars have studied aspects of Jābir b. Aflaḥ's work, such as N.M. Swerdlow (1987), "Jābir ibn Aflaḥ's interesting method for finding the eccentricities and direction of the apsidal line of superior planets" in D.A. King and G. Saliba (eds.) (1987), *From Deferent to Equant. A Volume of Studies in the History of Science in the Ancient and Medieval Near East in Honour of E.S. Kennedy*, New York, pp. 501-12; H. H. Hugonnard-Roche (1987), "La théorie astronomique selon Jābir ibn Aflaḥ", in G. Swarup, A.K. Bag and K.S. Shukla (1987), *History of Oriental Astronomy. Proceedings of an International Astronomical Union Colloquium nº 91 (1985)*, Cambridge, pp. 207-8; J. Samsó (2001), "Ibn al-Haytham and Jābir b. Aflaḥ's Criticism of Ptolemy's Determination of the Parameters of Mercury", *Suhayl*, Vol. 2 (2001), pp. 199-225 (reprint in J. Samsó (2007), *Astronomy and Astrology in al-Andalus and the Maghrib*, Aldershot - Burlington, VII); J. Bellver (2006), "Jābir b. Aflaḥ on the four-eclipse method for finding the lunar period in anomaly", *Suhayl*, Vol. 6 (2006), pp. 159-248; J. Bellver (2007a), "Ŷābir b. Aflaḥ en torno a la inclinación de los eclipses en el horizonte", *Archives Internationales d'Histoire des Sciences*, Vol. 57, Fasc. 158 (2007), pp. 3-25; and J. Bellver (2007b), "Jābir b. Aflaḥ on lunar eclipses", *Suhayl*, Vol. 8 (2008), pp. 47-92.

[3] Not to be confused with the other Latin Geber, the Arabic alchemist Jābir b. Ḥayyān.

[4] In Oxford, it was an introductory handbook for Ptolemaic astronomy, as established in the Statutes of Savile of 1619 for the Chair of Professor of Astronomy; cf. Strickland Gibson (ed.) (1931), *Statuta Antiqua Universitatis Oxoniensis*, Oxford, p. 529. It played the same role in the University of Salamanca according to its canons of 1561; cf. Victor Navarro Brotons (1995), "The Reception of Copernicus in Sixteenth-Century Spain: The Case of Diego de Zuniga", *Isis*, Vol. 86, No. 1. (Mar., 1995), p. 55.

translated into Latin by Gerard of Cremona (1114-1187) and published in 1534 by Petrus Apianus (1495-1552).[5] The *Iṣlāḥ al-Majisṭī* was also translated into Hebrew twice: in 1274 by Moshe ibn Tibbon (fl. 1240-1283), and by Jacob ben Maḥir ibn Tibbon (1236-1304). This second translation was corrected by Samuel ben Jehuda of Marseille (fl. 2nd quarter XIV c.) in 1335.

The first criticism appearing in Book V of Jābir b. Aflaḥ's *Iṣlāḥ al-Majisṭī* deals with the limits of the inclined lunar orbit in which an eclipse can take place. Jābir refers to this error in his introduction to the *Iṣlāḥ al-Majisṭī* stating that "there is another error in the limits of solar eclipses".[6] Ptolemy treats this issue in *Almagest* VI.5.[7]

First we briefly describe Ptolemy's approach to eclipse limits. Then we show Jābir b. Aflaḥ's description of the error committed by Ptolemy and his solution to it. The last two sections are devoted to the edition and translation of this criticism in the *Iṣlāḥ al-Majisṭī*.

2. On the solar and lunar eclipse limits according to the *Almagest*

Ptolemy intends to predict whether an eclipse may occur given the nodal distance of the mean syzygy. If the mean syzygy falls within the maximum nodal distances determined by minimum eclipses, then an eclipse can occur. Knowing these limits, Ptolemy does not need to compute the possibility of an eclipse occurring for all mean syzygies.[8] By 'limit' he understands the greatest nodal distance along the inclined lunar orbit of a mean syzygy which is related to an apparent syzygy in which an eclipse may occur.

Consequently, Ptolemy must consider those variables that maximize the eclipse limits. This happens when the eclipses are the minimum possible; that is when the Sun and Moon, for solar eclipses, or the Sun and the Earth's shadow, for lunar eclipses, have a minimum contact at the mid-eclipse. However, Ptolemy simplifies this minimum eclipse in order to make the computations easier (see Figure 1). First, he considers the arcs of the ecliptic and the inclined orbit as straight lines in order to apply plane trigonometry. Secondly, he considers the apparent syzygy instead of the mid-eclipse.

[5] Petrus Apianus, *Instrumentum primi mobilis. Accedunt iis Gebri filii Affla Hispalensis Astronomi vetustissimi pariter et peritissimi, libri IX de astronomia, ante aliquot secula Arabice scripti, et per Giriardum Cremonensem latinitate donati, nunc vero omnium primum in lucem editi*, Nuremberg, 1534.

[6] Carmody edited Gerard of Cremona's Latin translation of the list of errors found in Jābir b. Aflaḥ's introduction to the *Iṣlāḥ al-Majisṭī*. *Vid.* F.J. Carmody (1952), pp. 29-32. See also Bellver (2009) for an edition of the introduction.

[7] Cf. G.J. Toomer (1984), *Ptolemy's Almagest*, London, pp. 282-7 [henceforth referred to as Toomer]; O. Pedersen (1974), *A Survey of the Almagest*, Odense, pp. 227-30 [henceforth referred to as Pedersen]; and O. Neugebauer (1975), *A History of Ancient Mathematical Astronomy*, 3 vols., Berlin, Heidelberg & New York, pp. 125-129 and p. 1240 [henceforth referred to as HAMA].

[8] See Toomer, 282-3.

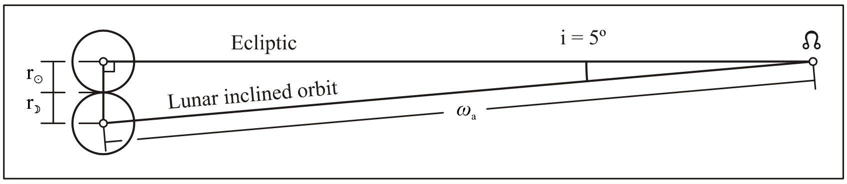

Figure 1. Minimum solar eclipse according to Ptolemy.

The maximum nodal distances of a minimum eclipse can be obtained when considering the maximum apparent radii of the Sun and the Moon (for solar eclipses) or of the Moon and the Earth's shadow (for lunar eclipses). A first approximation to the computation of nodal distances in the inclined orbit of the apparent Moon during a minimum eclipse is, for solar eclipses:

$$\omega_{\mathrm{a}} = \frac{r_\odot + r_☽}{\sin i} \tag{1a}$$

and for lunar eclipses:

$$\omega_{\mathrm{a}} = \frac{r_☽ + r_{\mathrm{s}}}{\sin i} \tag{1b}$$

where ω_{a} refers to the nodal distance of the Moon at a minimum eclipse at the apparent syzygy, $r_\odot$, $r_☽$ and r_{s} are the radius of the Sun, the Moon and the shadow cone, and i is the angle of inclination between the inclined orbit and the ecliptic. Ptolemy wants to obtain an estimation of lunar mean positions at mean syzygies in order to know whether it is possible for an eclipse to occur. He follows two steps:

i. first, he obtains the true position of the Moon at the apparent syzygy and
ii. second, he estimates the maximum lunar mean position for the lunar true position at the apparent syzygy.

In the first step, the parallax effect must be considered. For lunar eclipses, given that there is no effect due to parallax,[9] the apparent syzygy is equal to the true one. So for lunar eclipses

$$\omega_{\mathrm{t}} = \omega_{\mathrm{a}} = \frac{r_☽ + r_{\mathrm{s}}}{\sin i} \tag{2a}$$

where ω_{t} refers to the nodal distance of a minimum eclipse at the apparent syzygy which, in the case of lunar eclipses, is equivalent to the true syzygy. For solar eclipses, the apparent syzygy depends mainly on the lunar parallax. So in order to maximize the eclipse limits, Ptolemy considers those maximum parallaxes in latitude and longitude depending on whether the eclipse occurs to the north of the node or to the south of it. So the true position of the Moon at the apparent syzygy, for solar eclipses, is

[9] Cf. Toomer, 174.

$$\omega_t = \frac{r_{\odot} + r_{☽} + p_\beta}{\sin i} + p_\lambda \qquad (2b)$$

where now ω_t refers to the true nodal distance of a minimum eclipse at the apparent syzygy, p_β and p_λ refer to the maximum parallax in latitude and longitude of the apparent Moon, once the solar parallax is subtracted.[10] The following table shows the Ptolemaic nodal distances of the eclipse limits given the true lunar position at the apparent syzygy, which for lunar eclipses is equivalent to the true syzygy:[11]

Eclipse limits given the true lunar position at the apparent syzygy in nodal distance		
	To the south of the node	To the north of the node
Solar eclipse	8;22°	17;41°
Lunar eclipse	12;12°	12;12°

The next step is to estimate for the above true values lunar mean positions maximizing the eclipse limits. Since we are looking for the maximum limits, the difference between the mean and the true syzygies must be the greatest possible. The maximum difference between true and mean syzygies is approximately 3°.[12] That is

$$[\ \lambda_t - \lambda_m\ ]_{max} \cong 3^o$$

where λ_t is the longitude of the true syzygy and λ_m is the longitude of the mean syzygy. In order to apply this maximum difference between true and mean syzygies, Ptolemy approximates nodal distances by longitudes. Therefore, the maximum nodal distance (ω_m) of a minimum lunar eclipse given a mean syzygy, that is the nodal distance of the lunar eclipse limits according to Ptolemy, is

$$\omega_m = \frac{r_{☽} + r_s}{\sin i} + 3^o \qquad (3a)$$

while the maximum nodal distance (ω_m) of a minimum solar eclipse given the maximum lunar mean position for the apparent syzygy, that is the nodal distance of the solar eclipse limits according to Ptolemy, is

$$\omega_m = \frac{r_{\odot} + r_{☽} + p_\beta}{\sin i} + p_\lambda + 3^o\,. \qquad (3b)$$

So we can tabulate the different values provided by Ptolemy of the eclipse limits.

[10] See Pedersen, 229 and Toomer, 174 for the parallax values applied by Ptolemy depending on whether the eclipse is to the north or to the south of a node.

[11] See Toomer, 285-6 for the computation of these values.

[12] See Toomer, 286 and HAMA, 125-126 for the computation of this difference.

Maximum nodal distance of the limits		
	To the south of the node	To the north of the node
Solar eclipse	11;22º	20;41º
Lunar eclipse	15;12º	15;12º

And assuming that the descending node is at 90º and the ascending one at 270º, the arguments of latitude of the limits are:

Arguments of latitude of the limits				
	Descendant Node		Ascendant Node	
	North side	South side	South side	North side
Solar eclipse	69;19º	101;22º	258;38	290;41º
Lunar eclipse	74;48º	105;12º	254;48º	285;12º

This is Ptolemy's procedure for obtaining the maximum limits in the inclined orbit in which an eclipse can occur estimated from mean syzygies.

3. Jābir b. Aflaḥ's criticism

Before considering the actual method for obtaining the limits, Jābir b. Aflaḥ stresses the need to know the value of the maximum apparent diameters of the Sun, the Moon and the shadow cone, and points out several minor variations with respect to Ptolemy's remarks:

> So [Ptolemy] needed to determine the measure of the arc subtended by the lunar diameter when, at the syzygies, it is at the perigee. As previously, he determined that [value] relying upon two observed lunar eclipses. In both eclipses, the Moon was near the perigee of the epicycle. He found that [its value] was $0;35\frac{1}{3}$º. With this, he obtained the measure of the arc subtended by the diameter of the shadow circle at this distance itself; and found that [its value] was 1;32º. He did [it] on the condition that the measure of this circle, i.e. the circle of the shadow, does not differ at the same distance of the Moon from the Earth, but in reality, it differs because of the solar eccentricity relative to the centre of the world. However, this difference (*ikhtilāf*) is small, since the value of this eccentricity is not big. And for this reason, he did not calculate this difference (*ikhtilāf*). [In addition], he had previously determined the measure of the arc, which the diameter of the Sun subtends, of the circle passing through it [i.e. the Sun], i.e. $0;31\frac{1}{3}$º. But

> in like manner, in reality, it differs because of the solar eccentricity, although this difference is imperceptible.[13]

So Jābir b. Aflaḥ notes that Ptolemy did not take into account the solar eccentricity to compute the apparent diameter of the shadow cone and of the Sun. He remarks that the variations introduced by the solar eccentricity are negligible. These technical minutiae bear witness to Jābir b. Aflaḥ's extremely critical approach.

After these preliminary remarks, Jābir b. Aflaḥ abridges Ptolemy's method to obtain the limits in which an eclipse can occur. Next, he continues by stating his criticism:

> [Ptolemy] committed a mistake (*wahm*) when he added these three degrees, which correspond to the maximum [difference] between the positions of the [mean and true] syzygies, to the [true] nodal distance of the lunar body at the moment of the apparent syzygy. However, it is only appropriate to add [these three degrees] to the position of the Moon at the moment of the true syzygy, since these three degrees only correspond to the maximum [difference] between the positions of the mean and true syzygies, and not to the [maximum difference] between the mean and apparent syzygies.[14]

That is, the maximum difference in longitude between the mean and the true syzygy can only be added to the true syzygy. Instead, Ptolemy only obtains the true position of the Moon at the apparent syzygy and not the true syzygy.

Jābir b. Aflaḥ corrects Ptolemy in two steps:

i. First, given the true Moon at the apparent syzygy, he obtains the true syzygy.
ii. Second, he chooses the parallax that maximizes the limits in which an eclipse can occur.

Finally, because he is working with true syzygies, Jābir b. Aflaḥ adds, as Ptolemy does, the maximum difference between the mean and the true syzygy, i.e. 3º, to the result obtained in the previous steps.

3.1 *First correction*

Jābir b. Aflaḥ bases his first correction upon Figure 2 where point B is a node, the arc of great circle AB is a section of the ecliptic, the arc of great circle BG a section of the inclined lunar orbit, point A is the apparent position of the centre of the solar body, point E is the apparent position of the centre of the lunar body and point D is the true position

[13] Cf. *infra* p. 23.

[14] Cf. *infra* p. 25.

of the Moon. Circle AE is perpendicular to the ecliptic. Therefore, the arc of great circle ED is the lunar parallax at the apparent syzygy and thus arc GD is approximately the lunar parallax in longitude and arc EG is the lunar parallax in latitude. Arc GD can be considered as the lunar parallax in longitude since the inclination of the inclined orbit relative to the ecliptic is relatively small.

Jābir b. Aflaḥ, in the first step of his correction, wants to obtain the true syzygy from the true position of the Moon at the apparent syzygy, i.e. point D. In order to obtain the true syzygy he relies upon the mean motions of both luminaries in the ecliptic. These are[15]

$$\bar{v}_{\odot} = 0;59,8^{o/d}$$

$$\bar{v}_{☽} = 13;10,34^{o/d}\,.$$

Jābir b. Aflaḥ considers the following proportion between both mean motions

$$\frac{\bar{v}_{\odot}}{\bar{v}_{☽}} \cong \frac{1}{13}\,. \qquad (4)$$

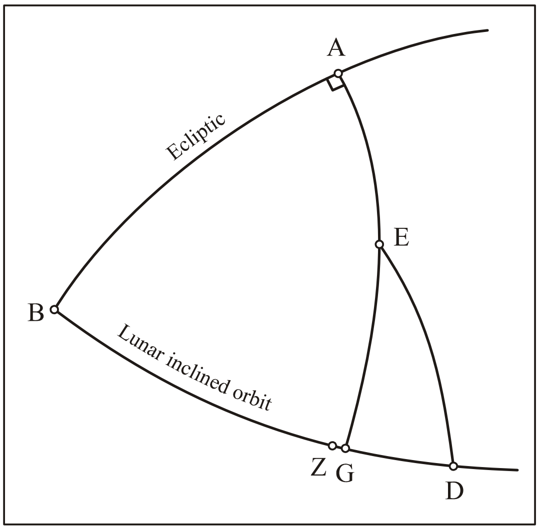

Figure 2. First approach to obtain the eclipse limits according to Jābir b. Aflaḥ.

We will now obtain the true syzygy from the apparent one, according to Jābir b. Aflaḥ's first correction.

Let us suppose that the apparent syzygy occurs before the true one and that, at the apparent syzygy, the Sun is on point A and the Moon on point D, as illustrated in Figure 2 where BD > BA. Under these conditions, the Sun and the Moon must approach to the node and the true syzygy must be in between the node, point B, and the nodal distance in

[15] Cf. Pedersen, 188.

the inclined orbit of the apparent syzygy, point G. Should the Moon and the Sun withdraw from the node with the apparent syzygy taking place before the true one and BD > BA, the Moon will move away from the Sun and the next syzygy will not take place until a return of the Moon in its inclined orbit.

Whereas, in Figure 2 with BD > BA, if the true syzygy occurs before the apparent syzygy and the true syzygy is in between the node, point B, and the nodal distance in the inclined orbit of the apparent syzygy, point G, the Moon and the Sun must withdraw from the node.

In order to obtain the true syzygy, Jābir b. Aflaḥ presents his method without any demonstration. He first divides the arc of great circle GD in twelve parts. Next, he endeavours to obtain a point, Z, in the inclined orbit to the left of point G, whose distance from point G is a twelfth part of arc GD.

Therefore,

$$\mathrm{DZ} = \frac{13}{12}\mathrm{DG}\,. \tag{5}$$

Jābir b. Aflaḥ considers that point Z is the position of the Moon at the true syzygy, as in Figure 3, where the true syzygy corresponds to the dashed line.

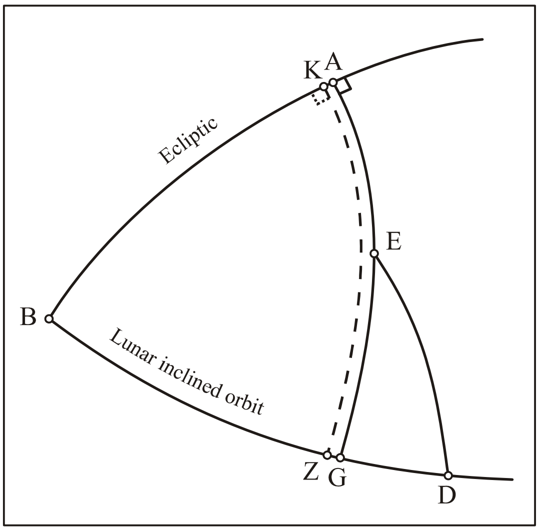

Figure 3. Resolution of the apparent syzygy from the true one.

At the apparent syzygy, the true Sun is at point A while the true Moon is at point D. During the time interval, Δt, the Sun traverses $\Delta\lambda_{\odot} = v_{\odot}\,\Delta t$ and the Moon traverses

$$\Delta\lambda_{☽} = v_{☽}\Delta t \cong \frac{\bar{v}_{☽}}{\bar{v}_{\odot}}\Delta\lambda_{\odot} \cong 13\Delta\lambda_{\odot} \tag{6}$$

where true motions have been approximated by mean motions. The difference in longitude traversed by the Moon and the Sun during the time elapsed between the apparent and the true syzygies amounts to the lunar parallax in longitude once the solar parallax is subtracted. Hence,

$$p_\lambda = \Delta\lambda_☽ - \Delta\lambda_\odot \cong \frac{\bar{v}_☽ - \bar{v}_\odot}{\bar{v}_\odot}\Delta\lambda_\odot \cong 12\Delta\lambda_\odot \quad (7)$$

where p_λ is the lunar parallax once the solar parallax is subtracted. From (6) and (7), the longitude traversed by the Moon ($\Delta\lambda_☽$) during the time elapsed between the apparent syzygy and the true one (Δt) is

$$\Delta\lambda_☽ = v_☽ \Delta t \cong \frac{\bar{v}_☽}{\bar{v}_☽ - \bar{v}_\odot} p_\lambda .$$

Jābir b. Aflaḥ then approximates nodal distances by longitudes and hence considers that the nodal distance traversed by the Moon ($\Delta\omega_☽$) during the time elapsed between the apparent syzygy and the true one (Δt) is approximately equal to the longitude traversed by the Moon ($\Delta\lambda_☽$) during that time interval. Therefore he obtains

$$\Delta\omega_☽ \cong \frac{\bar{v}_☽}{\bar{v}_☽ - \bar{v}_\odot} p_\lambda ,$$

which corresponds to the geometric relation expressed in (5).

So Jābir b. Aflaḥ's procedure is based upon two approximations: first, he considers longitudes as distances in the inclined orbit; and second, he considers mean motions instead of true motions. As to this second approximation, since the maximum possible values of the solar and lunar equations are considered, the true solar and lunar motions are approximately equal to the mean ones.

In short, after this first correction, Jābir b. Aflaḥ's solution to equation (3b) is

$$\omega_m = \frac{r_\odot + r_☽ + p_\beta}{\sin i} - \frac{p_\lambda}{12} + 3^\circ . \quad (8)$$

3.2. *Second correction*

After the first correction and his new procedure for computing the eclipse limits, Jābir b. Aflaḥ discusses a second correction (see Figure 4):

> In consequence, there must be for the position in which he placed the Moon in this figure a decrement in the [eclipse] limit in the measure of arc DZ.
>
> However, in reality the [correct solution for this] matter is not like that, since [Ptolemy], in addition to the error by which he added these three degrees to arc DB, committed another mistake in the position of the Moon at the moment of the apparent syzygy, as he placed it further away from the node than point G, but the position of the Moon must be nearer the node than

point G, such as at point H of this figure; and thus arc HE would be the total [lunar] parallax, arc GH the [lunar] parallax in longitude and arc GE the [lunar] parallax in latitude.[16]

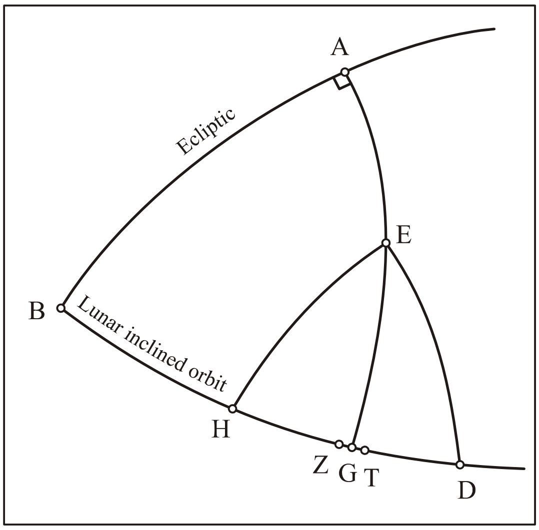

Figure 4. Second approach to obtain the eclipse limits according to Jābir b. Aflaḥ.

In this second correction, Jābir b. Aflaḥ considers that the true Moon, at the apparent syzygy, must be nearer to the node, as in point H, than the true Sun in order to maximize the eclipse limits. Hence the appropriate parallax in longitude, according to Jābir b. Aflaḥ, must be subtracted from the apparent Moon, not added to it, as is done by Ptolemy. Jābir b. Aflaḥ can choose this new maximum parallax in longitude of the opposite sign to Ptolemy's, since an eclipse can occur in whatever ecliptic longitude and whatever horizon, and hence its parallax in longitude can be either positive or negative. Under this condition, if the Sun and the Moon withdraw from the node and the apparent syzygy occurs before the true one, the true syzygy, point T, is farther away from the node, point B, than the apparent syzygy, point G, as illustrated in Figure 5. A similar situation is found if the Sun and the Moon approach to the node and the true syzygy occurs before the apparent one.

Although Jābir b. Aflaḥ considers that Ptolemy has committed a second mistake, it does not seem fully justified to follow him on this point, since this is a logical consequence of Ptolemy's rough estimate pointed out in first place.

[16] Cf. *infra* p. 26.

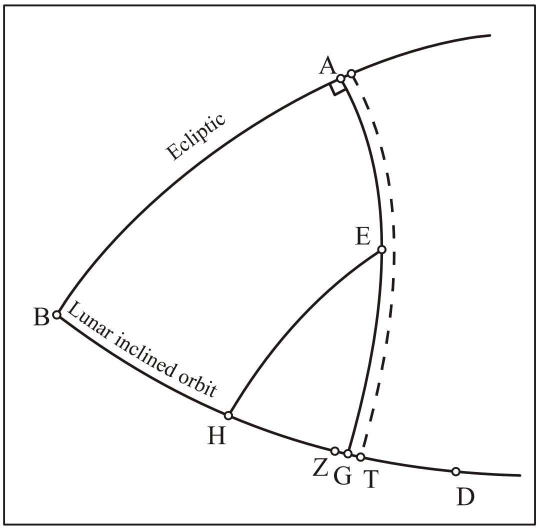

Figure 5. Changes due to the second correction

Therefore, after this second correction, Jābir b. Aflaḥ's final solution to the eclipse limits is

$$\omega_{\mathrm{m}} = \frac{r_{\odot} + r_{☽} + p_{\beta}}{\sin i} + \frac{p_{\lambda}}{12} + 3^{\circ}\cdot \qquad (9)$$

Finally, Jābir b. Aflaḥ provides the numerical corrections to the Ptolemaic magnitudes.

> Hence, in reality there must be a decrement in the [eclipse] limit that he obtained in the measure of arc DT which for the greater limit, in which the parallax in longitude is 0;15º, is $0;13\frac{3}{4}$º; and which for the smaller limit, in which the parallax in longitude is 0;30º, is twice this value [i.e. twice $0;13\frac{3}{4}$º], that is $0;27\frac{1}{2}$º; and that is what we wanted to demonstrate.[17]

Jābir b. Aflaḥ's corrections subtract

$$|\mathrm{DT}| = \frac{11}{12}|\mathrm{GD}| = \frac{11}{12}p_{\lambda}$$

from Ptolemy's values. These corrections amount to:

$0;27,30° = \frac{11}{12}0;30°$ (to the south of the node) and $0;13,45° = \frac{11}{12}0;15°$ (to the north of the node).

The new values, according to Jābir b. Aflaḥ, are:

[17] Cf. *infra* p. 26.

Solar eclipses		
	To the south of the node	To the north of the node
Maximum parallax in longitude	0;30º	0;15º
Limits according to Ptolemy	11;22º	20;41º
Correction according to Jābir b. Aflaḥ	– 0;27,30º	– 0;13,45º
Limits according to Jābir b. Aflaḥ	10;54,30º	20;27,15º

And the percentage correction of Jābir b. Aflaḥ's values relative to Ptolemy's is:

Solar eclipses		
	To the south of the node	To the north of the node
Percentage correction according to Jābir b. Aflaḥ	– 4.032%	– 1.108%

And assuming the descending node to be at 90º and the ascending one at 270º, the arguments of latitude of the limits according to Jābir b. Aflaḥ are:

Arguments of latitude of the limits according to Jābir b. Aflaḥ				
	Descendant Node		Ascendant Node	
	North side	South side	South side	North side
Solar eclipse	69;32,45º	100;54,30º	259;5,30º	290;27,15º
Lunar eclipse	74;48º	105;12º	254;48º	285;12º

The limits of the lunar eclipses are the same as Ptolemy's since Jābir b. Aflaḥ has not applied any correction to this kind of eclipse.

4. Conclusion

In this paper, we have considered Jābir b. Aflaḥ's first criticism of Ptolemy appearing in Book V of Jābir b. Aflaḥ's *Iṣlāḥ al-Majisṭī*. His criticism is aimed at Ptolemy's method for obtaining the limits of the inclined orbit in which an eclipse can occur given mean syzygies. Therefore, Jābir b. Aflaḥ's criticism is of a mathematical character.

Jābir b. Aflaḥ's criticism does not invalidate the Ptolemaic method for obtaining the limits of the eclipses since the limits obtained by Ptolemy are slightly greater than the ones corrected by the author of the *Iṣlāḥ al-Majisṭī*, which amount to decreases of 4% and 1%. We stress that Jābir b. Aflaḥ's concern, in addition to his pedagogical purpose, is theoretical consistency, not a substantial improvement in the procedures applied.

We should also consider the possibility that Ptolemy was aware that these detailed computations would not lead to any increase in accuracy. There are a number of places in the *Almagest* where it can be shown that Ptolemy did more detailed, computationally heavy, work before settling on the simplified presentation we find in the finished text.

5. Edition[18]

[Es^{1} f. 56v, Es^{2} f. 67v, B. f. 58v]

فيما ينبغي أن يتقدم العلم به من أحوال الكسوفات

ولما تبين له جميع ما تقدم من أحوال النيرين أخذ بعد ذلك في تبيين أمر كسوفاتهما فنظر في تبيين حدود الكسوفات[19] الشمسية والقمرية أعني تحديد المواضع [Es^{2} f. 68r] من الفلك المائل التي إذا كان موضع الاتصال الوسطي فيما بينهما وبين إحدى العقدتين كان[20] الكسوف[21] ممكنا وإذا كان فيما[22] بينهما وبين إحدى النهايتين كان ممتنعا فبين ذلك على هذه الجهة[23] وذلك أنه قد كان تبين له فيما تقدم مقدار القوس التي يوترها قطر القمر من الدائرة المارة به وهو في أبعد بعده من الأرض في الاتصالات وهذه الحدود إنما ينبغي أن تطلب والقمر في أقرب قربه من {الأرض في الاتصالات أعني إذا كان في أقرب قربه من}[24] فلك التدوير فاحتاج أن يبين مقدار القوس التي[25] يوترها قطر القمر إذا كان في البعد الأقرب في الاتصالات فيبين[26] ذلك بمثل ما تقدم بكسوفين قمريين رصدهما والقمر في كل واحد منهما

[18] What follows is not a critical edition but a working one, based only on the Arabic manuscripts extant in Arabic script. We have not used the Arabic manuscripts in Hebrew script, or the Hebrew or Latin manuscripts, although Apianus' Latin edition published in 1534 was consulted during the preparation of this study. The three extant Arabic manuscripts in Arabic script on which the edition is based are Mss. Escorial 910 henceforth referred to as Es^{1}, Escorial 930 henceforth referred to as Es^{2} and Berlin 5653 henceforth referred to as B. Whenever a variant appears in an annotation relating to a particular manuscript, the published version considered correct is that of the manuscripts that do not appear in the annotation. Whenever a variant affects more than one word, its extension has been indicated with braces. Whenever a variant is an addition to the text, it has been indicated with a + sign. Lastly, whenever two pairs of braces are intertwined, the correspondence is indicated with a number subscripted.

[19] Ms. Es^{1} كسوفان.

[20] Ms. B. كانت.

[21] Ms. Es^{2} كسوفات.

[22] Not in Ms. B.

[23] Ms. B., Es^{2} صفة.

[24] Not in Mss. B., Es^{2}.

[25] Ms. B., Es^{2} الذي.

[26] Ms. Es^{1} يبين.

قريب من بعده[27] الأقرب من فلك التدوير فوجدها[28] خمس[29] وثلاثين دقيقة وثلث دقيقة وبذلك {علم أيضا}[30] مقدار القوس التي {يوترها قطر}[31] دائرة الظل في ذلك البعد بعينه فوجدها جزءا واحدا[32] واثنتين وثلاثين دقيقة وعمل على أن مقدار هذه الدائرة أعني دائرة الظل لا تختلف في البعد الواحد للقمر من الأرض وهي[33] في الحقيقة تختلف من أجل خروج مركز دائرة الشمس عن مركز [Es[1] f. 57r] العالم إلا أن الاختلاف فيها يسير من أجل أن[34] خروج هذا المركز ليس بالكثير ومن أجل ذلك لم يعتد هذا[35] الاختلاف وقد كان يبين له فيما تقدم مقدار القوس التي يوترها قطر الشمس من الدائرة المارة بها وذلك إحدى وثلاثون دقيقة وثلث وكذلك {تختلف أيضا}[36] هذه القوس في الحقيقة من أجل خروج مركز دائرة الشمس إلا أن اختلافها أيضا غير محسوس فالمجتمع[37] من نصفي قطري النيرين يكون ثلاثا وثلاثين دقيقة وعشرين ثانية فلذلك إذا كان في كسوف الشمس بعد ما بين مركزي القمر والشمس الذين يريان[38] ثلاثا[39] وثلاثين دقيقة وعشرين ثانية فحينئذ أول ما يمكن أن يكون [B. f. 59r] وضع[40] القمر الذي يرى على مماسة[41] الشمس وخط لذلك مثالا على هذه الصفة.

[انظر شكل 6] لتكن قطعة من دائرة البروج عليها ألف باء وقطعة من الفلك المائل عليها جيم دال وجعل مسيرات الكسوفات منهما[42] متوازية وليكن مركز جرم[43] القمر في الدائرة المائلة في زمان الاجتماع المرئي نقطة دال وموضعه[44] المرئي نقطة هاء [Es[2] f. 68v] وتكون قوس دال هاء اختلاف منظره الكلي ولتكن نقطة ألف مركز الشمس ولتكن قوس ألف هاء جيم من دائرة عظيمة قائمة على الفلك المائل على زوايا قائمة وهي عند الحس قائمة أيضا على فلك البروج فتكون قوس هاء جيم اختلاف المنظر في العرض وقوس جيم دال اختلاف المنظر في الطول ولتكن النقطة التي تتماس[45] عليها جرما النيرين في ذلك الاجتماع المرئي نقطة زاي فقوس ألف زاي هاء التي هي[46] مجموع نصفي قطري النيرين يمكن أن تبلغ على ما تبين ثلاثا وثلاثين دقيقة وعشرين ثانية وقوس هاء جيم التي هي اختلاف المنظر في العرض أكثر ما يمكن أن تبلغ في جميع المعمور من الأرض أعني من أقصى البلاد التي أطول نهارها ثلاث عشرة ساعة إلى أقصى البلاد التي أطول نهارها ست عشرة ساعة في أقرب أبعاد القمر في الاتصالات

[27] Ms. Es[2] قربه.

[28] Ms. Es[2] فوجدها تخرج. In the margin فوجدها.

[29] Ms. Es[1], Es[2] خمسة.

[30] Ms. Es[2] أيضا علم.

[31] Ms. Es[1] توترها قوس.

[32] Ms. B. واحد.

[33] Ms. B. فهي.

[34] In the margin in Ms. Es[1].

[35] Ms. B. بهذا.

[36] Ms. B. أيضا تختلف. Ms. Es[2] أيضا تخلف.

[37] Ms. B. والمجتمع

[38] Ms. Es[2] يران.

[39] Ms. Es[1] ثلاث.

[40] Ms. B. موضع .

[41] Ms. Es[1] مسامتة.

[42] Ms. Es[2] منها.

[43] Ms. Es[1] جيم.

[44] Ms. Es[1] موضع .

[45] Ms. B. تماسا. Ms. Es[2] تماسها.

[46] In the margin in Ms. B.

بعد أن يحتسب باختلاف منظر الشمس أما مما يلي الجنوب منه فثمان وخمسون دقيقة وأما مما يلي الشمال منه[47] {فثماني دقائق}[48] وقوس جيم دال التي هي اختلاف المنظر في الطول أكثر ما يكون أما[49] إذا كانت قوس جيم هاء الثماني[50] والخمسون دقيقة فخمس عشرة دقيقة وأما إذا كانت {ثماني دقائق}[51] فثلاثون دقيقة فقوس ألف هاء جيم أكثر ما يمكن أن تبلغ أما إذا كان القمر شمالا[52] عن الشمس وكان على أكثر ما يمكن من اختلاف منظره فيما يلي الجنوب جزءا واحدا وإحدى وثلاثين دقيقة وأما إن كان جنوبا[53] عنها وكان على أكثر ما {يمكن أن}[54] يكون من اختلاف منظره فيما[55] يلي الشمال منه فإحدى وأربعون دقيقة ثم إنه ضاعف هذه القوس أعني قوس ألف هاء جيم إحدى عشرة مرة ونصفا[56] من أجل أن [Es^{1} f. 57v] نسبتها إلى القوس التي من العقدة إليها هي على التقريب نسبة واحد إلى أحد[57] عشر ونصف فكان[58] مبلغ القوس التي من العقدة إليها أما إذا كانت قوس ألف[59] هاء جيم الجزء الواحد والإحدى والثلاثين دقيقة فسبعة عشر جزءا وست وعشرون دقيقة ويكون مع قوس جيم دال التي تكون {حينئذ خمس عشرة دقيقة سبعة عشر جزءا وإحدى وأربعون دقيقة وأما إذا كانت قوس ألف[60] هاء جيم الإحدى والأربعين[61] الدقيقة فسبعة أجزاء واثنتان وخمسون دقيقة ويكون مع قوس جيم دال التي تكون مبلغها}[62] حينئذ ثلاثين دقيقة ثمانية أجزاء {واثنتين وعشرين}[63] دقيقة[64] [Es^{2} f. 69r] فلذلك إذا كان بعد موضع القمر الحقيقي في الدائرة المائلة من إحدى العقدتين أما إذا كان[65] شمالا عن الشمس فالسبعة {عشر جزءا}[66] والإحدى والأربعين الدقيقة وأما إذا كان جنوبا عن الشمس فالثمانية[67] الأجزاء والاثنتان والعشرون الدقيقة فحينئذ في البلاد المفروضة أول ما يمكن أن يكون وضعه[68] الذي يرى على مماسة [B. f. 59v] الشمس.

[47] Not in Mss. B., Es^{2}.

[48] Mss. Es^{1} and B. ثمان حقائق.

[49] Not in Mss. B., Es^{2}.

[50] Mss. Es^{1} and B. الثمان.

[51] Mss. Es^{1}, Es^{2} and B. ثمان حقائق.

[52] Ms. B. شماليا.

[53] Ms. B. جنوبيا.

[54] Not in Ms. Es^{1}.

[55] Ms. B. مما.

[56] Ms. B., Es^{2} ونصف.

[57] Ms. B., Es^{2} إحدى.

[58] Ms. B. فكان ذلك.

[59] In the margin in Ms. Es^{1}.

[60] ألف in the margin corrects باء in the text in Ms. Es^{1}. Not in Ms. Es^{2}.

[61] Ms. B. والأربعون.

[62] In the margin in Ms. B.

[63] Mss Es^{1}, Es^{2} and B. واثنتان وعشرون.

[64] Interlineal addition in Ms. Es^{2}.

[65] Ms. B. كانت.

[66] Ms. B. عشر الجزء. Ms. Es^{2} العشر الجزء.

[67] Ms. B. والثمانية.

[68] Ms. B. موضعه.

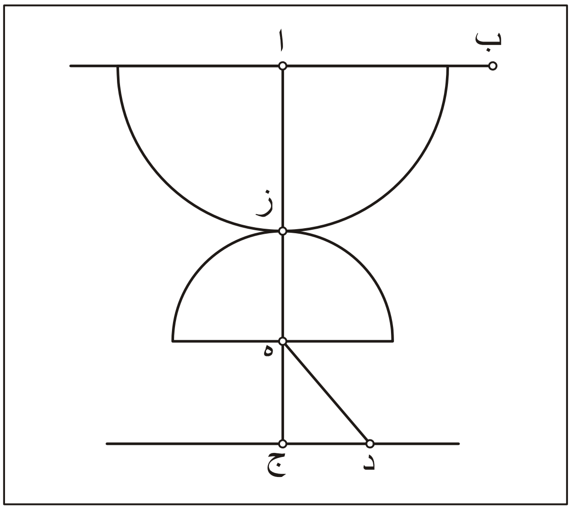

Figure 6. Determination of the eclipse limits according to Ptolemy (Ms. Es[1] 57v).

ثم إنه بعد ذلك أخذ أكثر ما يكون من الاختلاف لكل واحد من النيرين فجمعهما وأخذ جزء ذلك من ثلاثة عشر وهو ما تقطعه الشمس في الزمان الذي يقطع فيه القمر أجزاء الاختلافين على التقريب وحمل على ذلك الجزء جزءه من ثلاثة عشر جزءا[69] أيضا وهو ما تقطعه الشمس أيضا في الزمان الذي يقطع فيه القمر ذلك الجزء فما كان فهو على التقريب ما تسير الشمس حتى يلحقها القمر وذلك سبع وثلاثون دقيقة فحمل ذلك على أكثر ما يكون من {اختلاف الشمس فما كان فهو أكثر ما يكون بين[70]} [71] الاتصالات {الوسطي والحقيقي} [72] في الطول ومثل ذلك على التقريب يكون في العرض وذلك ثلاثة أجزاء فحمل هذه ثلاثة الأجزاء على نهاية بعد القمر من العقدة في الفلك المائل في وقت الاتصال المرئي الذي بصر[73] فيه وضع[74] القمر المرئي على مماسة[75] الشمس أعني بعد نقطة دال[76] من العقدة فما كان من أجزاء الفلك المائل فهو نهاية بعد[77] موضع الاتصال الوسطي من إحدى العقدتين الذي بصر[78] فيه وضع[79] القمر الذي يرى على مماسة[80] الشمس وذلك أما[81] متى كان القمر شمالا عن الشمس فعشرون جزءا وإحدى وأربعون دقيقة {وأما متى} [82] [كان] جنوبا عنها [Es[1] f. 58r] فأحد[83] عشر جزءا واثنتان وعشرون دقيقة.

[69] Not in Ms. Es[1].
[70] Ms. B. من.
[71] Not in Ms. Es[1].
[72] Ms. Es[1] الوسطي والحقيقة. Ms. B. الوسطية والحقيقية.
[73] Ms. Es[2] يصير.
[74] Ms. B. موضع.
[75] Ms. Es[1] مساحة.
[76] Ms. Es[2] ح.
[77] In Ms. Es[2] + القمر crossed out.
[78] Ms. Es[2] يصير.
[79] Ms. B. موضع.
[80] Ms. Es[1] مساحة. Ms. B. مسامتة.
[81] Ms. B. أنه.
[82] Ms. Es[2] ومتى.

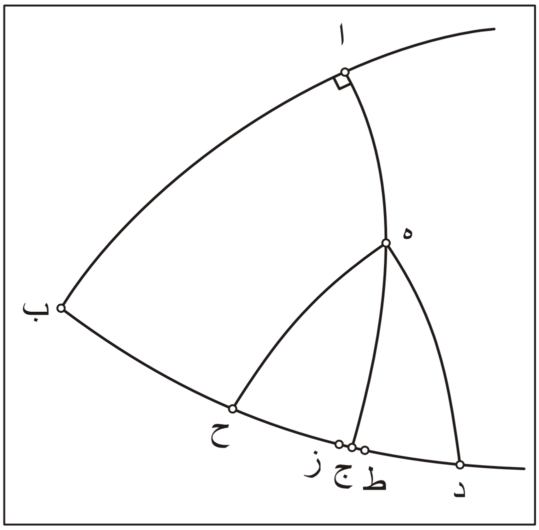

Figure 7. Determination of the eclipse limits according to Jābir b. Aflaḥ (Ms. Es^1 58v).

ووهم في حمله الثلاثة الأجزاء[84] التي هي أكثر ما يكون بين موضعي[85] الاتصالين على بعد جرم القمر من العقدة في وقت الاتصال المرئي وإنما كان ينبغي أن يحملها على موضع القمر في وقت الاتصال الحقيقي لأن هذه الثلاثة الأجزاء[86] إنما هي أكثر ما يكون بين موضع الاتصال الوسطي وموضع[87] الاتصال الحقيقي {لا ما يكون[88] بين موضع} [89] الاتصال الوسطي والاتصال المرئي.

[انظر شكل 7] فلنضع[90] الشكل على ما هو عليه [Es^2 f. 69v] في الحقيقة أعني أن تكون قطعة[91] فلك البروج قوس ألف باء ومركز الشمس عليها $\{_1$ نقطة ألف $\{_2$ وقطعة الفلك المائل قوس دال باء ومركز القمر عليه $\}_1$ [92] نقطة دال $\}_2$ [93] ولتكن قوس دال هاء [قطعة] من الدائرة المارة به وبسمت الرأس[94] وليكن مركز القمر المرئي نقطة هاء فتكون قوس دال هاء اختلاف منظره الكلي في دائرة الارتفاع ولتكن قوس ألف هاء جيم المارة بمركز[95] الشمس ومركز القمر المرئي قائمة على فلك البروج على زوايا قائمة فتكون نقطة جيم من الفلك المائل هي موضع القمر في وقت الاتصال المرئي وتكون قوس دال جيم هي على التقريب اختلاف منظره في الطول وقوس جيم هاء على التقريب اختلاف منظره في العرض ولتكن قوس جيم زاي الجزء من اثني عشر من قوس دال جيم [B. f. 60r] فيلزم أن

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83 Ms. Es^2 وإحدى.

84 Ms. B. أجزاء.

85 Ms. Es^1 موضي.

86 Mss. B., Es^2 أجزاء.

87 Not in Ms. B.

88 Not in Ms. Es^1.

89 Not readable in Ms. Es^2.

90 Ms. B. فلنبين ذلك فإن نضع.

91 Ms. B. قطعتين من. Ms. Es^2 قطعة من.

92 In the margin in Ms. Es^2.

93 Ms. B. ومركز القمر على قوس بد التي هي الفلك المائل. Follows الشمس crossed out.

94 In the margin in Ms. B. corrects الدائرة in the text.

95 Ms. B. لمركز.

تكون نقطة زاي هي موضع القمر في وقت الاتصال الحقيقي فعلى قوس زاي باء ينبغي أن تحمل الثلاثة الأجزاء لا على قوس دال باء[96] كما فعل فيجب على هذا الوضع الذي جعله للقمر في هذا الشكل أن يكون في الحدود زيادة بمقدار قوس دال زاي.

وليس الأمر في الحقيقة[97] كذلك لأنه مع خطئه في حمل الثلاثة الأجزاء[98] على قوس دال باء أخطأ أيضا في وضع[99] القمر في وقت الاتصال المرئي وذلك أنه جعله[100] أبعد من العقدة من نقطة جيم وإنما كان يجب أن يكون وضع[101] القمر أقرب إلى العقدة من نقطة جيم كأنه على نقطة حاء من هذا الشكل وتكون قوس حاء هاء اختلاف منظره الكلي وقوس جيم حاء اختلاف منظره في {الطول وقوس جيم هاء اختلاف منظره في}[102] العرض فيكون لذلك موضع القمر في وقت الاجتماع الحقيقي أبعد من العقدة من نقطة جيم بمقدار الجزء من اثني عشر من قوس جيم حاء كأنها[103] نقطة طاء وكأن قوس جيم طاء هي الجزء من اثني عشر من قوس جيم حاء فتحمل الثلاثة الأجزاء[104] على قوس طاء باء[105] لا على قوس زاي باء كما يلزم من الوضع[106] الذي وضع عليه القمر فيجب على الحقيقة أن يكون في الحدود التي استخرجها زيادة مقدار قوس دال طاء وهي في الحد الأكبر الذي فيه اختلاف المنظر في الطول[107] خمس عشرة[108] دقيقة ثلاث عشرة دقيقة وثلاثة أرباع دقيقة وفي الحد الأصغر وهو الذي فيه اختلاف المنظر [Es[1] f. 58v] في الطول هي[109] ثلاثون دقيقة ضعف ذلك وهو[110] سبع وعشرون دقيقة ونصف وذلك ما أردنا بيانه[111].

[Es[2] f. 70r] وأما حدود الكسوفات القمرية فإنه استخرجها على هذه الصفة وذلك أنه أضاف القوس التي يوترها نصف قطر القمر في قربه الأقرب من فلك تدويره وذلك {سبع عشرة}[112] دقيقة وأربعون ثانية إلى القوس التي يوترها نصف قطر دائرة الظل لذلك البعد الأقرب وذلك خمس وأربعون دقيقة وست وخمسون ثانية وأخذ ما يجب لذلك من الدائرة المائلة فكان ذلك نهاية بعد جرم القمر من العقدة في وقت[113] وسط زمان الكسوف وذلك اثنا عشر جزءا واثنتا عشرة دقيقة فحمل على ذلك ثلاثة الأجزاء المذكورة التي هي أكثر ما يكون بين موضعي[114] الاتصالين

[96] Interlineal addition in Ms. Es[1].

[97] Ms. B. الحقيقي.

[98] Ms. B. أجزاء.

[99] Ms. B. موضع.

[100] Ms. B., Es[2] جعل موضع القمر.

[101] Ms. B. موضع.

[102] Not in Ms. Es[2].

[103] Ms. Es[1] كأنه.

[104] Ms. B., Es[2] أجزاء.

[105] In the margin in Ms. Es[2] +فيتحمل.

[106] Ms. B. الموضع.

[107] Ms. Es[2] العرض.

[108] Ms. B., Es[2] عشر.

[109] Not in Ms. Es[1].

[110] Ms. Es[2] وهي.

[111] Ms. B. أن نبين.

[112] Ms. Es[2] سبعة عشر.

[113] Not in Ms. Es[1].

[114] Ms. B. موضع.

في الطول {وهو على التقريب يكون}[115] في العرض فتكون [B. f. 60v] من ذلك غاية بعد موضع الاتصال الوسطي من إحدى العقدتين الذي يكون فيه[116] القمر على مماسة[117] دائرة الظل وذلك خمسة عشر جزءا واثنتا[118] عشرة دقيقة فهذا هو الحد بين الاتصالات الوسطية التي يمكن فيها أن ينكسف القمر والاتصالات التي لا يمكن فيها ذلك[119].

6. Translation[120]

[Es^1 f. 56v, Es^2 f. 67v and B. f. 58v]

On what is appropriate to know in the first place regarding the configurations of eclipses

Once [Ptolemy] determined all that had to be considered in the first place,[121] regarding the configurations of the two luminaries [i.e. the Sun and the Moon], he began to determine the matter of the two eclipses [i.e. the solar and lunar]. [First of all,] he first determined the limits of solar and lunar eclipses; that is, to define the positions [Es^2 f. 68r] of the inclined orbit [such as]:

- [i.] when the mean syzygy is between them [i.e. these positions] and one of the nodes, the eclipse is possible; and
- [ii.] when the mean syzygy is between them [i.e. these positions] and one of the extremes (*nihāyatayn*) [of the inclined orbit],[122] it is impossible.

He determined this issue in the following manner:

That is, he had previously determined the measure of the arc,[123] which the diameter of the Moon subtends, of the circle passing through it [i.e. the Moon], when it is at its apogee[124] at the syzygies. But those limits must, rather, be sought when the Moon is, at

[115] Ms. B. وهي على التقريب تكون .

[116] Not in Ms. Es^1.

[117] In the margin in Ms. B. correcting مسامتة in the text.

[118] Ms. B. وثلاثة.

[119] Ms. Es^2 وذلك ما أردنا بيانه.

[120] In the following translation, brackets are used to clarify anaphoric references within the text, while parentheses are used to provide transliterations of Arabic terms. Brackets are also used to reference the folios of the edited manuscripts.

[121] A more literal translation may be: 'all that had to come in first place became clear to him'.

[122] These are the points of the lunar inclined orbit with nodal distance ± 90º.

[123] A more literal translation may be: 'the measure of the arc had become clear to him'.

[124] Even though Jābir b. Aflaḥ's term for apogee in his *Iṣlāḥ al-Majisṭī* is usually *buʿd abʿad*, in this occasion we find *abʿad al-buʿd*.

the syzygies, at its perigee,[125] i.e. when the Moon is at the perigee of the epicycle. So [Ptolemy] needed to determine the measure of the arc subtended by the lunar diameter when, at the syzygies, it is at the perigee. As previously, he determined that [value] relying upon two observed lunar eclipses. In both eclipses, the Moon was near the perigee of the epicycle. He found that [its value] was $0;35\frac{1}{3}^{\circ}$. With this, he obtained the measure of the arc subtended by the diameter of the shadow circle at this distance itself; and found that [its value] was 1;32º. He did [it] on the condition that the measure of this circle, i.e. the circle of the shadow, does not differ at the same distance of the Moon from the Earth, but in reality, it differs because of the solar eccentricity relative to the centre [Es[1] f. 57r] of the world. However, this difference (*ikhtilāf*) is small, since the value of this eccentricity is not big. And for this reason, he did not calculate this difference (*ikhtilāf*). [In addition], he had previously determined the measure of the arc,[126] which the diameter of the Sun subtends, of the circle passing through it [i.e. the Sun], i.e. $0;31\frac{1}{3}^{\circ}$. But in like manner, in reality, it differs because of the solar eccentricity, although this difference is imperceptible. The sum of the radii of the two luminaries [i.e. the Sun and the Moon] is 0;33,20º. Therefore, when, in a solar eclipse, the distance between the apparent [positions of the] centres of the Moon and the Sun is 0;33,20º, [B. f. 59r] the apparent position of the Moon can be tangent to the Sun for the first time. For this reason, [Ptolemy] traced a figure in the following manner.

[See Figure 6.] Let there be a segment of the ecliptic containing points A and B and a segment of the inclined orbit containing points G and D. [Ptolemy] set that the trajectories of the eclipses of both [luminaries] were parallel. Let point D be the [true position of the] centre of the lunar body in the inclined orbit during the time of the apparent conjunction (*ijtimāʿ*) and point E its apparent position. [Es[2] f. 68v] Arc DE is the total parallax [of the Moon]. Let point A be the centre of the Sun. Let the arc of great circle AEG be perpendicular to the inclined orbit. [This arc] is also perceived by the senses as perpendicular to the ecliptic. Arc EG is the parallax in latitude and arc GD the parallax in longitude. Let point Z be the point in which the bodies of the two luminaries [i.e. the Sun and the Moon] touch [tangentially] at this apparent conjunction. Therefore, arc AZE, which is the sum of the radii of the two luminaries [i.e. the Sun and the Moon], can reach, according to what has been determined, to 0;33;20º; and arc EG, which is the maximum parallax in latitude that can be reached in all the inhabitable [latitudes] of the Earth, i.e. [those ranging] from the remote countries in which their longest day is thirteen

[125] Jābir b. Aflaḥ's term for perigee in his *Iṣlāḥ al-Majisṭī* is usually *qurb aqrab*. However, throughout this text, Jābir b. Aflaḥ uses different variants to express this concept, such as *aqrab al-qurb*, *buʿd aqrab*, *aqrab al-buʿd* and *qurb aqrab*, all of which have been rendered as perigee.

[126] A more literal translation may be: 'the measure of the arc had become clear to him'.

hours to the remote countries in which their longest day is sixteen hours, when the Moon is in its perigee at the syzygies once the solar parallax is taken into consideration, [I say again arc EG] to the south is 0;58º and to the north is 0;08º. When arc GE is 0;58º, the maximum value of arc GD, which is the parallax in longitude, is 0;15º and, when [arc GE] is 0;08º, [the maximum value of arc GD] is 0;30º. The maximum possible value of arc AEG, when the Moon is to the north of the Sun and the [lunar] parallax is the maximum possible to the south, is 1;31º and, when [the Moon] is to the south [of the Sun] and the [lunar] parallax is the maximum possible to the north, [the maximum possible value of arc AEG] is 0;41º. After that, [Ptolemy] multiplied this arc, i.e. arc AEG, by $11\frac{1}{2}$[127] since [Es[1] f. 57v] the ratio of [arc AEG] to the arc from the node to [point G] is approximately equal to the ratio of 1 to $11\frac{1}{2}$. The resulting [length] of the arc from the node to [point G], when arc AEG is 1;31º, is 17;26º. And [the sum of this arc] plus arc GD, being [arc GD] 0;15º, is 17;41º. When arc AEG is 0;41º, [the length of the arc from the node to point G] is 7;52º. And [the sum of this arc] plus arc GD, being [arc GD] 0;30º, is 8;22º. [Es[2] f. 69r] For this reason, [the time] when the nodal distance of the true position of the Moon in the inclined orbit is 17;41º, if [the Moon] is to the north of the Sun, or 8;22º, if [the Moon] is to the south of the Sun, is the first time that in the given countries the [lunar] apparent position can be touching tangentially [B. f. 59v] the Sun.

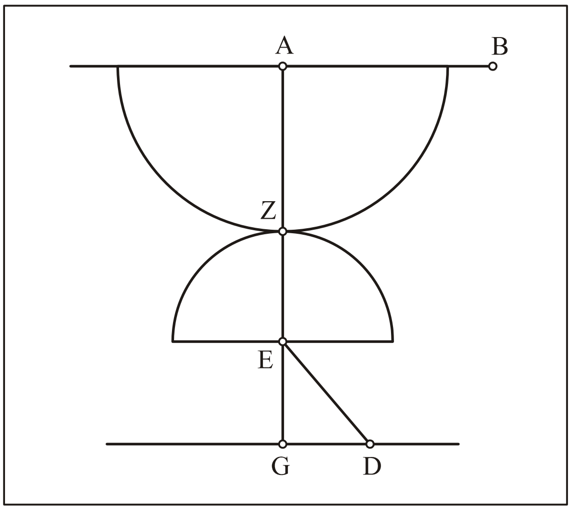

Figure 6. Determination of the eclipse limits according to Ptolemy (Ms. Es[1] 57v).

[127] The value of 1/sin i is approximately 11½, assuming i = 5º, where i is the angle of the inclined orbit relative to the ecliptic.

After that, [Ptolemy] took the maximum possible values of the equations of anomaly of the two luminaries [i.e. the Sun and the Moon] and added both values. He then took a thirteenth part [of the sum], which amounts approximately to the distance traversed by the Sun during the time that the Moon traverses [the sum of] both equations of anomaly. He added to this [thirteenth part], its thirteenth part,[128] which is the distance traversed by the Sun during the time the Moon traverses the [thirteenth part of the sum of both equations of anomaly]. The total sum amounts approximately to the distance traversed by the Sun until it is reached by the Moon, i.e. 0;37º. He added this result to the maximum solar equation of anomaly. The result is the maximum [difference] in longitude between mean and true syzygies. This [result] is approximately the same for the [argument of] latitude, i.e. 3º. He added this 3º to the maximum value (*nihāya*) of the nodal distance of the Moon in its inclined orbit at the moment of the apparent syzygy in which the apparent position of the Moon is touching tangentially the Sun , i.e. [this maximum nodal distance being] the nodal distance of point D. Therefore, the resulting [arc in] degrees of the inclined orbit [after this sum] is the maximum value of the nodal distance of the position of the mean syzygy in which the apparent position of the Moon is touching tangentially the Sun. [This value,] when the Moon is to the north of the Sun, is 20;41º and, when it is to the south of it, [Es[1] f. 58r] is 11;22º.

[Ptolemy] committed a mistake (*wahm*) when he added these three degrees, which correspond to the maximum [difference] between the positions of the [mean and true] syzygies, to the [true] nodal distance of the lunar body at the moment of the apparent syzygy. However, it is only appropriate to add [these three degrees] to the position of the Moon at the moment of the true syzygy, since these three degrees only correspond to the maximum [difference] between the positions of the mean and true syzygies, and not to the [maximum difference] between the mean and apparent syzygies.

[See Figure 7.] So let us set the figure as it must be [Es[2] f. 69v] in reality; that is that arc AB be a segment of the ecliptic, point A be the centre of the Sun in this [segment], arc DB be a segment of the inclined orbit and point D be the centre of the Moon in this [segment]. Let arc DE be [a segment] of the [great] circle passing through the [centre of the Moon] and through the zenith (*samt al-ra's*). Let point E be the apparent centre of the Moon. Therefore, arc DE is the total [lunar] parallax of the circle of altitude. Let arc AEG, which passes through the centre of the Sun and through the apparent centre of the Moon, be perpendicular to the ecliptic. Hence, point G of the inclined orbit is the [apparent] position of the Moon [in the inclined orbit] at the time of the apparent syzygy, while arc DG is approximately the [lunar] parallax in longitude and arc GE is approximately the [lunar] parallax in latitude. Let arc GZ be a twelfth part of arc DG. [B.

[128] I.e. $\frac{1}{13}\left(1+\frac{1}{13}\right)$.

f. 60r] Therefore, point Z must be the position of the Moon at the moment of the true syzygy. Hence, the three [aforementioned] degrees must be added to arc ZB, not to arc DB as [Ptolemy] did. In consequence, there must be for the position in which he placed the Moon in this figure a decrement (*ziyāda*)[129] in the [eclipse] limit in the measure of arc DZ.

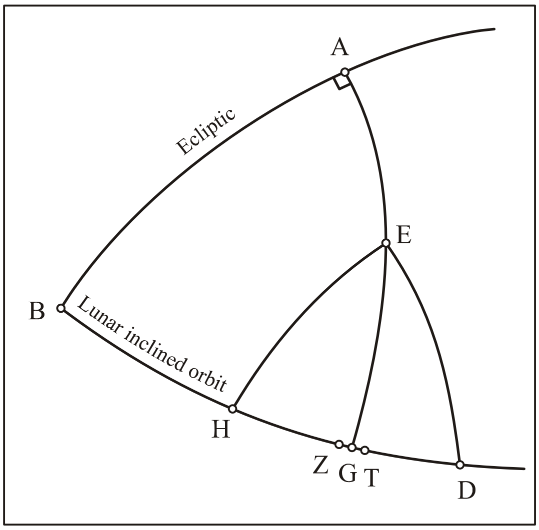

Figure 7. Determination of the eclipse limits according to Jābir b. Aflaḥ (Ms. Es[1] 58v).

However, in reality the [correct solution for this] matter is not like that, since [Ptolemy], in addition to the error by which he added these three degrees to arc DB, committed another mistake in the position of the Moon at the moment of the apparent syzygy, as he placed it further away from the node than point G, but the position of the Moon must be nearer the node than point G, such as at point H of this figure; and thus arc HE would be the total [lunar] parallax, arc GH the [lunar] parallax in longitude and arc GE the [lunar] parallax in latitude. For this reason, the position of the Moon at the moment of the true conjunction (*ijtimāʿ*) is farther away from the node than point G in the measure of a twelfth part of arc GH as if it were on point T [of this figure] and as if arc GT were a twelfth part of arc GH. So these three degrees are added to arc TB, not to arc ZB as follows from the position where he placed the Moon. Hence, in reality there must be a decrement (*ziyāda*)[130] in the [eclipse] limit that he obtained in the measure of arc DT which for the greater limit, in which the parallax in longitude is 0;15º, is 0;13$\frac{3}{4}$º; and which for the smaller limit, in which the parallax [Es[1] f. 58v] in longitude is 0;30º, is

[129] Literally 'an increment'.

[130] Literally 'an increment'.

twice this value [i.e. twice $0;13\frac{3}{4}$°], that is $0;27\frac{1}{2}$°; and that is what we wanted to demonstrate.

[Es[2] f. 70r] As to the lunar eclipse limits, [Ptolemy] obtained them in the following manner: He added the arc subtended by the lunar radius when the Moon is at the perigee of its epicycle, i.e. 0;17,40º, to the arc subtended by the radius of the circle of the shadow for this perigee, i.e. 0;45,56º, and took the [nodal distance] in the inclined orbit that is deduced from this [latitude], being [this nodal distance] the maximum value (*nihāya*) of the nodal distance of the lunar body at the moment of the mid-eclipse, i.e. 12;12º. He added to it the three aforementioned degrees, which correspond to the maximum [possible difference] in longitude between the positions of the [mean and true] syzygies. This [difference of three degrees in longitude] is approximately the same in [the argument of] latitude. Therefore, [B. f. 60v] from that, it is [obtained] the maximum [value] (*ghāya*) of the nodal distance of the position of the mean syzygy in which the Moon is touching tangentially the circle of the shadow; and that is 15;12º. This is the limit between those mean syzygies in which a lunar eclipse is possible, and those [mean] syzygies in which it is not possible.

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