SCIAMVS 14 (2013), 3-39

Shadow-Length Schemes in Babylonian Astronomy

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Introduction

Several schemes which model the variation in the length of the shadow cast by a gnomon at different times of day over the course of the year are presented in astronomical cuneiform texts. The early astronomical compendium known as MUL.APIN contains a scheme which assumes a simple reciprocal relationship between the length of the shadow and the time after sunrise. The MUL.APIN scheme, which is the earliest known shadow-length scheme, has been studied by several scholars over the past century,[1] but most of these previous studies incorporate assumptions about the Babylonian model for the change in the length of daylight throughout the year that have been shown to be erroneous in the light of more recently published texts.[2]

Three later sources dating from the late Neo-Babylonian and Achaemenid periods contain additional schemes for the length of the shadow: BM 45721 is previously unpublished and contains mathematical examinations of the noon-shadow and the change over the course of the year in the time of day at which the shadow reaches specific lengths. W 23273 is a large metrological tablet which contains a short section at the end dealing with shadows and a catchline which refers to another shadow-length scheme. BM 29371 contains a scheme for the length of daylight and a shadow-length scheme which has previously been studied by Brown, Fermor and Walker (1999–2000); I offer here an alternative interpretation of this text. In addition, BM 33564, a fragment from a previously unpublished atypical procedure text, refers in part to a shadow length scheme; unfortunately, too little of the scheme is preserved to fully understand it (for completeness, the text is edited here).

The aim of this paper is to examine the various shadow-length schemes. One interesting result is that all of the late schemes (except perhaps that found on BM 33564) ultimately derive from the MUL.APIN scheme. The texts containing these schemes, therefore, provide evidence of a late tradition of schematic astronomy, an area of Babylonian astronomy that has largely been ignored until now.[3] At the heart of this schematic astronomy is the 360-day calendar in which the solstices and equinoxes are

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[1] See in particular Weidner (1924), van der Waerden (1951), Neugebauer (1975), Pingree in Hunger and Pingree (1989), Bremner (1993), Fermor (1997) and Falk (2000).

[2] Gehlken (1991), Brown, Fermor and Walker (1999–2000).

[3] Some sections of the text TU 11 (Brack-Bernsen and Hunger 2002) dealing with the moon also attest to a late schematic astronomy which uses mathematical techniques and parameters from MUL.APIN.

placed on day 15 of Months I, IV, VII and X. This schematic calendar is used in all of the shadow-length schemes,[4] and I suggest that the use of the schematic calendar is typical of what I term "schematic astronomy" as opposed to the more broadly empirical astronomy of the observational and predictive texts such as the Astronomical Diaries and the Goal-Year Texts and the mathematical astronomy of the Ephemerides and associated Procedure Texts, both of which operate with the civil luni-solar calendar.

I begin this study with a discussion of Babylonian models for the variation in the length of day and night as these models are linked with the shadow-length schemes. I then discuss each of the shadow-length schemes in turn, beginning with the MUL.APIN scheme which, as I will show, underlies the other schemes. In the final part of this paper I discuss the relationship between these different texts and the phenomenon of "schematic astronomy" in the late period.

The Length of Day and Night in Babylonian Astronomy

A variety of cuneiform sources dating from the early second millennium BC down to the end of the first millennium BC contain schemes for the variation in the length of day and night over the course of the year.[5] All but the very latest of these schemes model this variation using a linear zigzag function operating within the framework of the ideal 360-day year. In this ideal calendar, the year is assumed to have twelve 30-day months making a total of 360 days. The solstices and equinoxes are placed on the 15th day of every third month: in the very earliest sources the spring equinox is on the 15th of Month XII with the summer solstice, autumnal equinox and winter solstice on the 15th of Months III, VI and IX; in later sources, including all of the texts discussed in this paper, the equinoxes and solstices are placed on the 15th of Months I, IV, VIII and X.

The length of day and night on any day of the year must add up to 360 UŠ (= 24 hours). Several units are used in the various schemes for the length of day and night, including *mina*, a unit of weight, and *bēru* and UŠ (1 *bēru* = 30 UŠ), which are units of time. In all cases it is clear that weight (presumably the weight of water in a waterclock) and time are taken to be directly proportional.[6] All of the preserved schemes for the length of day and night are based upon one of two values for the ratio of the length of the longest day (i.e. at summer solstice) to the shortest day (i.e. at winter solstice):[7] 2:1 or 3:2.

[4] In two cases, data is presented only for Month IV (summer solstice) to Month X (winter solstice), presumably because the scheme for the other half of the year from winter solstice to summer solstice simply mirrors that from summer solstice to winter solstice.

[5] For an overview of the most important sources with translations, see Hunger (1999).

[6] See, for example, Gehlken (1991) and Brown, Fermor and Walker (1999–2000).

[7] The ratio can also be seen as that between the longest night (ie at winter solstice) and the shortest night (ie at summer solstice) or between the length of day and the length of night (at either solstice).

Table 1 shows the lengths of day and night on the 15th of each month according to the 2:1 and the 3:2 ratios.

	2:1		3:2	
Month	Day	Night	Day	Night
I	3,0	3,0	3,0	3,0
II	3,20	2,40	3,12	2,48
III	3,40	2,20	3,24	2,36
IV	4,0	2,0	3,36	2,24
V	3,40	2,20	3,24	2,36
VI	3,20	2,40	3,12	2,48
VII	3,00	3,0	3,00	3,00
VIII	2,40	3,20	2,48	3,12
IX	2,20	3,40	2,36	3,24
X	2,00	4,00	2,24	3,36
XI	2,20	3,40	2,36	3,24
XII	2,40	3,20	2,48	3,12

Table 1. The length of day and night in UŠ for the 15th day of each month in the ideal calendar according to the 2:1 and 3:2 schemes

As has long been recognized, the 2:1 ratio is very inaccurate for the latitude of Babylon (or anywhere else in Babylonia or Assyria). Nevertheless, the ratio is attested across a wide variety of astronomical texts including all texts from the pre-Late Babylonian period such as *Enūma Anu Enlil* 14, MUL.APIN and the so-called Astrolabes, as well as many later tablets. Brown has suggested that the 2:1 ratio was used purely for divinatory purposes in the early texts, as an ideal against which observation could be judged with agreement being a positive omen and disagreement a negative omen.[8] It is possible that the ratio was sometimes used in this fashion, but I think it is wrong to see this as its only use. In MUL.APIN, for example, the 2:1 ratio was used to generate other astronomical schemes, in particular the scheme for the duration of visibility of the moon in MUL.APIN II ii 43 – iii 14.[9] We will find that the 2:1 ratio is used in some of the shadow-length scheme texts in a similar fashion.

[8] Brown (2000).

[9] The section of MUL.APIN is commonly referred to as the "water-clock scheme" but this name is misleading as the purpose of this section is not to provide data for how to use a water clock but instead to give a scheme for the change in the length of night and the duration of lunar visibility throughout the year.

In contrast to the many texts which used the 2:1 ratio, the 3:2 ratio only appears in the System A and System B lunar theories and on the tablets BM 29371 and BM 33564 (both discussed below).[10] Brown, Fermor and Walker (1999–2000) suggest that the 3:2 ratio replaced the 2:1 ratio in the eighth century BC, but the earliest evidence of the 3:2 ratio is the late sixth century BC text BM 29371, and we should be cautious in ascribing an earlier date for this ratio. In any event, it is clear that the 2:1 ratio continued to be used in certain contexts until well after the 3:2 ratio was known.

MUL.APIN

MUL.APIN is a two-tablet compendium of astronomical and astrological material. The text is known from about 40 copies from Assyria and Babylonia on the basis of which it has been possible to reconstruct almost the complete text. An edition of MUL.APIN with English translation by Hermann Hunger and an astronomical commentary by David Pingree is published in Hunger and Pingree (1989). The date of composition of MUL.APIN is not known. Only two of the known manuscripts of MUL.APIN preserve colophons which provide information on the date of the copy: a tablet in Assyrian script dating to 687 BC and a tablet in Babylonian script dating to the Hellenistic period.[11] Several attempts have been made to date the composition of MUL.APIN by an astronomical analysis of its contents.[12] These analyses must be treated with some caution because of the assumptions about the nature of the data in MUL.APIN that are required, but their results, a date in the late second millennium BC, is compatible with what is known from other texts about the development of astronomy in Mesopotamia.[13] The date of the compilation of MUL.APIN itself (as opposed to the material which is collected in the text) probably lies somewhere in the late second millennium or early first millennium BC.

The shadow-length scheme in MUL.APIN is found towards the middle of the second tablet (II ii 21–42 in Hunger's edition). It follows the so-called second intercalation scheme and precedes the night-length scheme to which the shadow-length scheme is related. As discussed in the preceding section, the night-length scheme uses a zigzag function to represent the length of night on the first and fifteenth of each month of the year assuming that the ratio for the longest to the shortest night is 2:1. The lengths of day and night at the solstices and equinoxes are also stated in the list of the dates of the heliacal risings of stars at MUL.APIN I ii 36 – iii 12 and are explicitly mentioned in the shadow-length scheme.

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[10] It is possible that the 3:2 ratio also appears on the school tablet BM 29440 (Leichty and Walker 2004: 209–211), but the interpretation of the numbers of this tablet is not certain.

[11] Hunger and Pingree (1989: 9).

[12] For example, Hunger and Pingree (1989: 10–12), de Jong (2011).

[13] Watson and Horowitz (2011: 3–6).

I reproduce below the part of MUL.APIN which presents the shadow-length scheme:[14]

MUL.APIN II ii

21 DIŠ *ina* $^{\text{itu}}$BÁR UD-15-KAM 3 MA.NA EN.NUN u_4-*mi* 3 MA.NA EN.NUN GE$_6$

22 1 *ina* 1 KÙŠ $^{\text{giš}}$MI 2 1/2 DANNA u_4-*mu*

23 2 *ina* 1 KÙŠ $^{\text{giš}}$MI 1 DANNA 7 UŠ 30 NINDA u_4-*mu*

24 3 *ina* 1 KÙŠ $^{\text{giš}}$MI 2/3 DANNA 5 UŠ u_4-*mu*

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25 DIŠ *ina* $^{\text{itu}}$ŠU UD-15-KAM 4 MA.NA EN.NUN u_4-*mi* 2 [MA.NA] EN.NUN GE$_6$

26 1 *ina* 1 KÙŠ $^{\text{giš}}$MI 2 DANNA u_4-*mu* 2 *ina* 1 KÙŠ $^{\text{gi[š}}$MI 1 DAN]NA u_4-*mu*

27 3 *ina* 1 KÙŠ $^{\text{giš}}$MI 2/3 DANNA u_4-*mu* 4 *ina* 1 KÙŠ $^{\text{giš}}$MI 1/2 DANNA u_4-*mu*

28 5 *ina* 1 KÙŠ $^{\text{giš}}$MI 12 UŠ u_4-*mu* 6 *ina* 1 KÙŠ $^{\text{giš}}$MI 10 UŠ u_4-*mu*

29 8 *ina* 1 KÙŠ $^{\text{giš}}$MI 7 UŠ 30 NINDA u_4-*mu*

30 9 *ina* 1 KÙŠ $^{\text{giš}}$MI 6 UŠ 40 NINDA u_4-*mu* 10 *ina* 1 KÙŠ $^{\text{giš}}$MI 6 UŠ u_4-*mu*

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31 DIŠ *ina* $^{\text{itu}}$DU$_6$ UD-15-KAM 3 MA.NA EN.NUN u_4-*mi* 3 MA.NA EN.NUN GE$_6$

32 1 *ina* 1 KÙŠ $^{\text{giš}}$MI 2 1/2 DANNA u_4-*mu*

33 2 *ina* 1 KÙŠ $^{\text{giš}}$MI 1 DANNA 7 UŠ 30 NINDA u_4-*mu*

34 3 *ina* 1 KÙŠ $^{\text{giš}}$MI 2/3 DANNA 5 UŠ u_4-*mu*

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35 DIŠ *ina* $^{\text{itu}}$AB UD-15-KAM 2 MA.NA EN.NUN u_4-*mi* 4 MA.NA EN.NUN GE$_6$

36 1 *ina* 1 KÙŠ $^{\text{giš}}$MI 3 DANNA u_4-*mu* 2 *ina* 1 KÙŠ $^{\text{giš}}$MI 1 1/2 DANNA u_4-*mu*

37 3 *ina* 1 KÙŠ $^{\text{giš}}$MI 1 DANNA u_4-*mu* 4 *ina* 1 KÙŠ $^{\text{giš}}$MI 2/3 DANNA 2 UŠ 30 NINDA u_4-*mu*

38 5 *ina* 1 KÙŠ $^{\text{giš}}$MI 18 UŠ u_4-*mu* 6 *ina* 1 KÙŠ $^{\text{giš}}$MI 1/2 DANNA u_4-*mu*

39 8 *ina* 1 KÙŠ $^{\text{giš}}$MI 11 UŠ 15 NINDA u_4-*mu* 9 *ina* 1 KÙŠ $^{\text{giš}}$MI 10 UŠ u_4-*mu*

40 10 *ina* 1 KÙŠ $^{\text{giš}}$MI 9 UŠ u_4-*mu*

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41 BE-*ma nap-pal-ti* 1 KÙŠ $^{\text{giš}}$MI *ana* IGI-*ka* 40 *nap-pal-ti* u_4-*m*[*i*]

42 *u* GE$_6$ *ana* 7,30 ÍL-*ma* 5 *nap-pal-ti* $^{\text{giš}}$MI 1 KÙŠ *tam*-[*mar*]

21 ¶ On the 15th of Month I, 3 minas is a daytime watch, 3 minas is a nighttime watch.

22 1 cubit of shadow 2 1/2 *bēru* of daytime

23 2 cubits of shadow 1 *bēru* 7 UŠ 30 NINDA daytime

24 3 cubits of shadow 2/3 *bēru* 5 UŠ daytime

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25 ¶ On the 15th of Month IV, 4 minas is a daytime watch, 2 [minas] is a nighttime watch.

[14] The transliteration and translation follows Hunger and Pingree (1989: 96–101) except I have replaced the Babylonian names of the months with "Month I", "Month II", etc and I have translated the DIŠ sign where it is used as a textual marker as "¶" (Hunger does not translate these initial DIŠ signs).

26 1 cubit of shadow 2 *bēru* daytime 2 cubits of sha[dow 1] *bēru* daytime

27 3 cubits of shadow 2/3 *bēru* daytime 4 cubits of shadow 1/2 *bēru* daytime

28 5 cubits of shadow 12 UŠ daytime 6 cubits of shadow 10 UŠ daytime

29 8 cubits of shadow 7 UŠ 30 NINDA daytime

30 9 cubits of shadow 6 UŠ 40 NINDA daytime 10 cubits of shadow 6 UŠ daytime

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31 ¶ On the 15th of month VII, 3 minas is a daytime watch, 3 minas is a nighttime watch.

32 1 cubits of shadow 2 1/2 *bēru* daytime

33 2 cubits of shadow 1 *bēru* 7 UŠ 30 NINDA daytime

34 3 cubits of shadow 2/3 *bēru* 5 UŠ daytime

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35 ¶ On the 15th of month X, 2 minas is a daytime watch, 4 minas is a nighttime watch.

36 1 cubit of shadow 3 *bēru* daytime 2 cubits of shadow 1 1/2 *bēru* daytime

37 3 cubits of shadow 1 *bēru* daytime 4 cubits of shadow 2/3 *bēru* 2 UŠ 30 NINDA daytime

38 5 cubits of shadow 18 UŠ daytime 6 cubits of shadow 1/2 *bēru* daytime

39 8 cubits of shadow 11 UŠ 15 NINDA daytime 9 cubits of shadow 10 UŠ daytime

40 10 cubits of shadow 9 UŠ daytime

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41 If you are to find the difference for 1 cubit of shadow, you multiply 40, the difference for daytime and

42 nighttime, by 7,30, and you find 5, the difference for 1 cubit of shadow.

The shadow length scheme is presented over five sections, each separated by a horizontal ruling. The first four sections contain a series of statements of the time after sunrise (given in *bēru*, UŠ and NINDA where there are 30 UŠ in a *bēru* and 60 NINDA in an UŠ) for respectively the 15th day of Month I, Month IV, Month VII and Month X. These dates are the days of the equinoxes and solstices in the 360-day ideal calendar used throughout MUL.APIN; this fact is reinforced by statements of the length of day and night on those days. The fifth section gives a short procedure for calculating the monthly change in time for a 1 cubit shadow length. The first four sections begin with a DIŠ sign acting as a textual marker (translated ¶ above) but no initial DIŠ is used in the fifth section which contains the procedure. Watson and Horowitz (2011) have argued that an initial DIŠ is usually used in MUL.APIN only in those sections where the core data (often in list format) of a multi-section unit of the text is presented. An initial DIŠ is not usually found in either introductory or concluding sections which present either a description of the data or a procedure that utilizes the data given in the DIŠ-marked sections. Thus the first four sections of the shadow-length scheme can be seen as the core data for the scheme and the fifth as a supplementary procedure based upon that data. In order to understand the shadow-length scheme it is necessary therefore to propose an interpretation of the data that allows the supplementary procedure to also be explained.

	Month I	Month IV	Month VII	Month X
1 cubit	1,15	1,0	1,15	1,30
2 cubit	37;30	30	37;30	45
3 cubit	25	20	25	30
4 cubit		15		22;30
5 cubit		12		18
6 cubit		10		15
8 cubit		7;30		11;15
9 cubit		6;40		10
10 cubit		6		9

Table 2. Summary of the shadow-length data given in MUL.APIN.

The shadow-length data from the first four sections is summarized in table 2. For convenience I have converted all of the times into UŠ. The same data is given for the two equinoxes (Months I and VII), as we would expect. As recognized already by van der Waerden (1951: 34) and Neugebauer (1975: 544–545), the length of the shadow s multiplied by the time after sunrise t is equal to a constant c whose value depends upon the month. Thus,

$$t = c / s.$$

For Months I and VII (the equinoxes) $c = 75$, for Month IV (the summer solstice) $c = 60$ and for Month X (the winter solstice) $c = 90$. This simple mathematical rule explains why a shadow length of 7 cubits does not appear in the scheme: dividing 60, 75 or 90 by 7 leads to a non-terminating sexagesimal fraction. The basic rule works surprisingly well at the latitude of Babylon as can be seen from figure 1 which compares the Babylonian scheme with modern computation of the length of the shadow.[15] The modern computation of shadow length in figure 1 assumes that the height of the gnomon is 1 cubit. The close agreement between the MUL.APIN scheme and the computed data demonstrates that the gnomon in MUL.APIN is taken to be 1 cubit in height.[16]

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[15] Weidner (1924: 202) reaches a similar conclusion by calculating the length of shadows on the summer solstice.

[16] Weidner (1924) and van der Waerden (1951) interpret the phrase *ina* 1 KÙŠ after the numerical value for the length of the shadow to be a statement that the gnomon was 1 cubit in height. However, as noted by Sachs (quoted by Neugebauer 1975: 544), *ina* 1 KÙŠ simply means “reckoning in cubits” or just “cubits”.

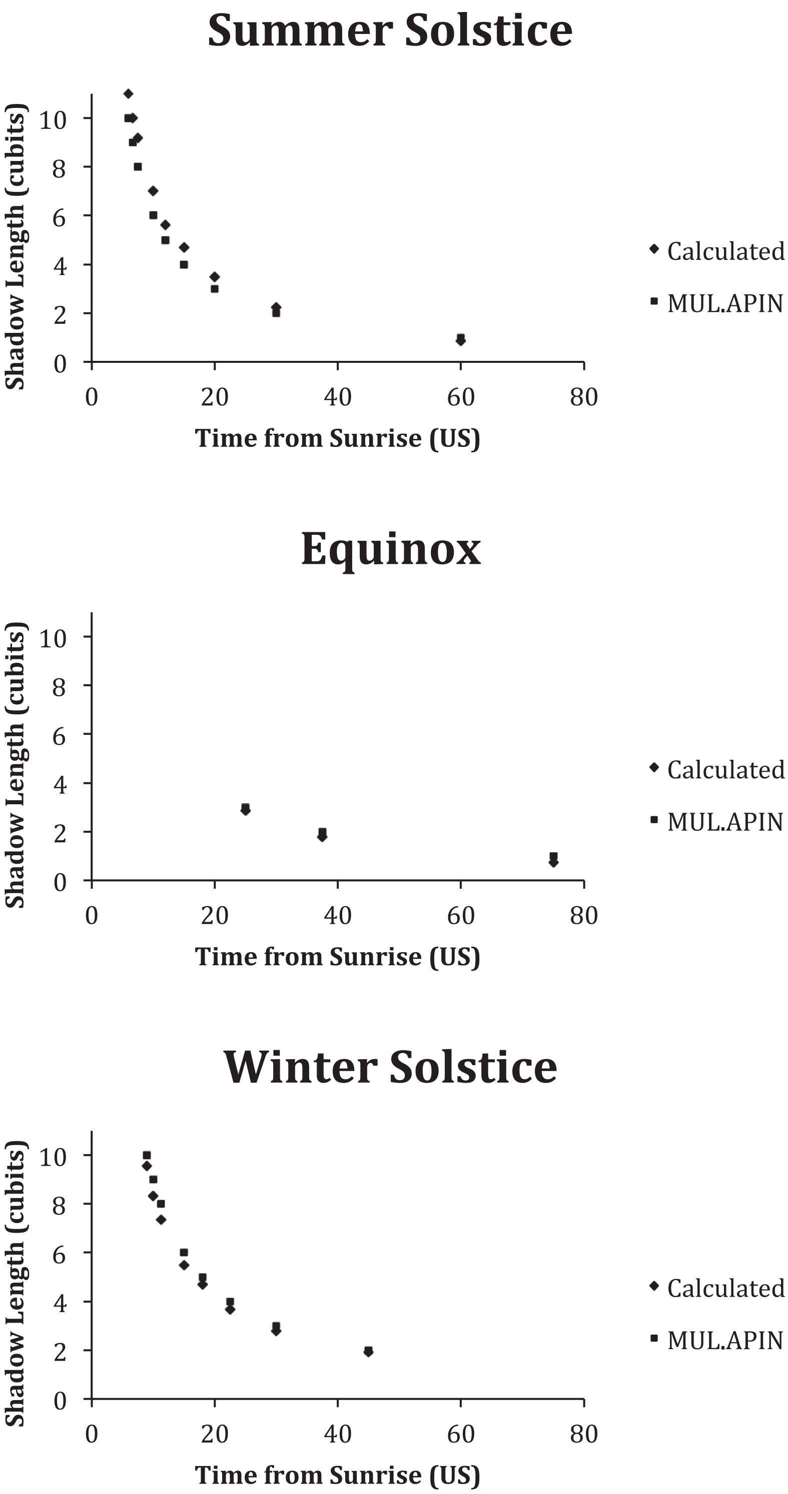

Figure 1. The MUL.APIN scheme for the length of shadow. The winter solstice data for 1 cubit shadow is omitted because the shadow can never reach so short a length on that day at the latitude of Babylon.

In his discussion of the mathematical basis of the shadow-length scheme Neugebauer commented that the ratio of the constants 60 and 90 for the two solstices is equal to 3:2, the same ratio he (incorrectly) believed represented the ratio of the longest to the shortest day given in MUL.APIN.[17] But he comments: “The satisfaction with this result is spoiled by the implication that the noon shadow is always 5/6 cubits long, independent of the seasons”, which does not correspond to reality at the latitude of Babylon. Neugebauer’s conclusion here, however, is based upon two false assumptions, first that the 3:2 ratio for the longest to the shortest day is used in MUL.APIN,[18] and secondly that the data for summer and winter solstice have been switched in the text. Assuming instead that the 2:1 ratio underlies this scheme, as is clear from the statement of the lengths of day and night which appears at the beginning of each section of the scheme, and that the scheme as written in the text is correct and has not been switched between summer and winter solstice, we can calculate that the length of the shadow at noon is equal to 0;50 cubits at the equinoxes, 0;30 cubits at the summer solstice and 1;30 cubits at winter solstice. Figure 2 compares the lengths of shadow at noon calculated from the MUL.APIN scheme and from modern computation. Although the agreement between the MUL.APIN scheme and modern data is not exact there is a general qualitative agreement, and the noon shadow at winter solstice is very close to the true value. This might suggest that empirical data for the length of shadow at winter solstice was combined with a mathematical scheme to produce the complete shadow-length scheme.

As just discussed, the noon shadow at the winter solstice is equal to 1;30 cubits according to the MUL.APIN scheme. The length of the shadow can therefore never be as short as 1 cubit at the winter solstice, even though the scheme assigns a shadow of 1 cubit at 90 UŠ after sunrise on this day. However, according to the 2:1 ratio for the length of day, at the winter solstice daylight will last 120 UŠ, and so noon will be at 60 UŠ after sunrise. 90 UŠ after sunrise is therefore in the afternoon, when the length of the shadow is already increasing. This entry in the scheme is therefore purely an artefact of the underlying mathematical rule and is presumably included in the text either simply for the sake of completeness or perhaps because it is the value of the constant *c* for that month and so is useful in calculation.

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[17] Neugebauer (1975: 545). In his commentary to the edition of MUL.APIN Pingree follows Neugebauer in his interpretation of the shadow-length scheme. Neugebauer and Pingree’s interpretation of the shadow-length scheme is shown to be incorrect by Friberg, Hunger and al-Rawi (1990: 498–499), but is largely repeated in Hunger and Pingree (1999: 80–81). Falk (2000) follows Neugebauer and Pingree and makes several further incorrect statements about both the mathematics and the interpretation of the shadow-length scheme. The papers by Bremner (1993) and Gleßmer (1996) add further confusion by suggesting that time is measured horizontally along the horizon, an idea that for which there is no textual evidence.

[18] See already Friberg, Hunger and al-Rawi (1990: 498–499).

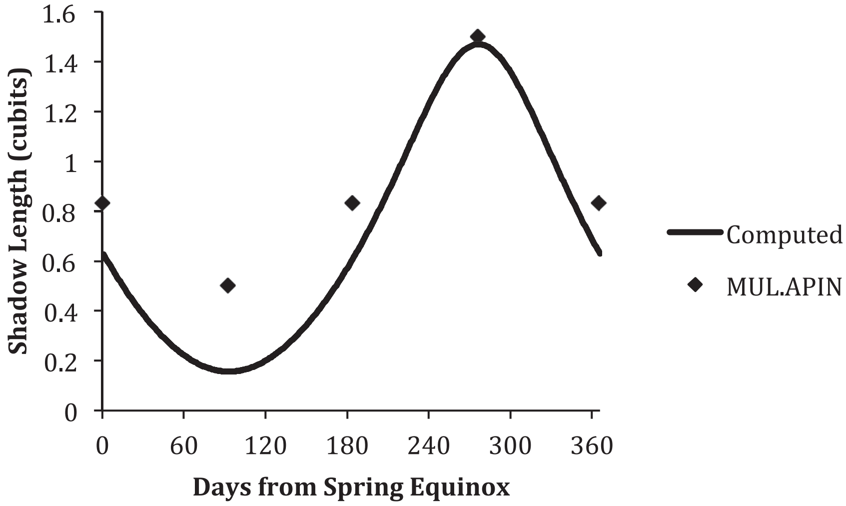

Figure 2. Noon shadows derived from the MUL.APIN scheme.

The short procedure in the section following the shadow-length scheme data provides further confirmation that the scheme should be understood in the context of the 2:1 ratio for the length of day. The procedure reads as follows:

> If you are to find the difference for 1 cubit of shadow, you multiply 40, the difference for daytime and nighttime, by 7,30, and you find 5, the difference for 1 cubit of shadow.

Pingree in his MUL.APIN commentary suggests that the number 40 should be interpreted as 0;0,40 *mina* and represents the daily change in the weight of water corresponding to one watch. This figure is then divided by 8 (equivalent to multiplying by 0;7,30) on the grounds that the shadow decreases by approximately 1 cubit in 1/8th of a watch. Unfortunately, Pingree's explanation is based upon the assumption that MUL.APIN means one-third of a day or night when it refers to a "watch", but we now know that this expression is used to indicate the whole of the day or night.[19] Furthermore, there is no basis for assuming that the shadow decreases by approximately 1 cubit in 1/8 of a watch, however the watch is defined, as can be seen by inspecting table 2. Pingree proposes a modification to his interpretation in Hunger and Pingree (1999: 81–82), but some of the same problems remain.

A much more plausible interpretation of this passage has been proposed by Friberg, Hunger and al-Rawi (1990: 498–499) and independently by Fermor (1997). According to the 2:1 ratio, at summer solstice day lasts for 240 UŠ and at winter solstice day lasts 120 UŠ. Therefore the daily change in the length of day is equal to

[19] al-Rawi and George (1991). See also Hunger (2001).

$$(240 - 120) / 180 = 0;40 \text{ UŠ}$$

because there are 180 days between the solstices in the ideal calendar. Thus 40 “the difference for daytime and nighttime” is the daily change in the length of day or night. Multiplying by 30 gives a monthly change in the length of day or night of 20 UŠ. Now according to the shadow-length scheme in MUL.APIN, the shadow is equal to 1 cubit in length 60 UŠ after sunrise at summer solstice, 75 UŠ after sunrise at the equinoxes and 90 UŠ after sunrise at winter solstice. Thus the daily change in the time after sunrise when the shadow equals 1 cubit is equal to

$$(90 - 60) / 180 = 0;10 \text{ UŠ}$$

This is equivalent to a monthly change of 5 UŠ, which is stated in the procedure to be the “difference for 1 cubit of shadow”. Thus, the monthly change in the time after sunrise at which the shadow is 1 cubit is one-quarter of the monthly change in the length of day or night. To convert the daily change in the length of day or night into the monthly change in the time after sunrise when the shadow is 1 cubit we must multiply by 30 and by ¼, which equals 7;30 as given in the procedure. The procedure is therefore simply a statement illustrating how the monthly change in the time after sunrise when the shadow is 1 cubit can be derived from the daily change in the length of day or night. This latter parameter is a well-known constant found elsewhere in MUL.APIN and other astronomical as well as in several coefficient lists.[20]

To summarize: the shadow-length scheme in MUL.APIN is derived from a simple mathematical relationship in which the length of shadow and the time after sunrise are inversely proportional. Multiplying the length of shadow by the time after sunrise gives a constant which is defined by a linear zigzag function with minimum $m = 60$ in Month IV (summer solstice) and maximum $M = 90$ in Month X (winter solstice) and monthly difference $d = 5$. The short procedure at the end of the shadow-length scheme confirms that the time for a shadow of a given length varies linearly between the two solstices. It is therefore possible to reconstruct the shadow-length scheme for the whole year, as I have done in table 3. The MUL.APIN scheme is fully compatible with the 2:1 ratio for the length of daylight at the two solstices, as is shown in the short procedure which follows the shadow-length scheme.

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[20] Robson (1999: 129). See also Brown, Fermor and Walker (1999–2000: 131).

	Month I	Month II	Month III	Month IV	Month V	Month VI	Month VII	Month VIII	Month IX	Month X	Month XI	Month XII
c	1,15	1,10	1,5	1,0	1,5	1,10	1,15	1,20	1,25	1,30	1,25	1,20
1 cubit	1,15	1,10	1,5	1,0	1,5	1,10	1,15	1,20	1,25	1,30	1,25	1,20
2 cubit	37;30	35	32.30	30	32.30	35	37;30	40	42;30	45	42;30	40
3 cubit	25	23;20	21;40	20	21;40	23;20	25	26;40	28;20	30	28;20	26;40
4 cubit	18;45	17;30	16;15	15	16;15	17;30	18;45	20	21;15	22;30	21;15	20
5 cubit	15	14	13	12	13	14	15	16	17	18	17	16
6 cubit	12;30	11;40	10;50	10	10;50	11;40	12;30	13;20	14;10	15	14;10	13;20
8 cubit	9;22,30	8;45	8;7,30	7;30	8;7,30	8;45	9;22,30	10	10;37,30	11;15	10;37,30	10
9 cubit	8;20	7;46,40	7;13,20	6;40	7;13,20	7;46,40	8;20	8;53,20	9;26,40	10	9;26,40	8;53,20
10 cubit	7;30	7	6;30	6	6;30	7	7;30	8	8;30	9	8;30	8

Figure 3. Reconstruction of the complete MUL.APIN shadow-length scheme.

BM 45721

BM 45721 (= 81-7-6, 128) is a substantial fragment written in a cramped script (figures 3 and 4). A small part of the left edge is preserved on the obverse. From consideration of the tablet's contents at least ten lines must be lost at the beginning of the obverse and the end of the reverse if, as seems likely, the first part of the obverse contained material for Month IV to Month X. Four lines must be lost between the end of the obverse and the beginning of the reverse, at least three of which, perhaps all four, are lost from the reverse. Obv. 13′ – Rev. 10′ contains a numerical scheme which may be restored fully. The restoration of missing text in this scheme implies that around three-quarters of the width of the tablet is preserved at its greatest extent.

The tablet is divided by horizontal rulings into at least eighteen sections (several of these sections are only one line in length). Many of these sections are subsections of multi-section units, each containing data for a specific month. Typically seven of these subsections form the multi-section unit. Four of these larger units are partially preserved (Obv. 1′–5′, Obv. 6′–12′, Obv. 13′ – Rev. 10′ and Rev. 13′–17′). Each textual unit contains information for one month running from Month IV (summer solstice) to Month X (winter solstice).[21] This is presumably because the length of shadow in the other half of the year mirrors that in these months.

The date when BM 45721 was written is unknown. Some of the material on BM 45721 relates to the shadow length scheme on W 23273, which dates to the late fifth century BC. The appearance of both the older, nine-wedge form for the number 9 in Obv. 9′ and the newer, cursive form in Rev. 5′ also suggests a pre-Seleucid date.

[21] Compare W 23273 Rev. IX 45–51 which also gives information for the length of the shadow for only half of the year.

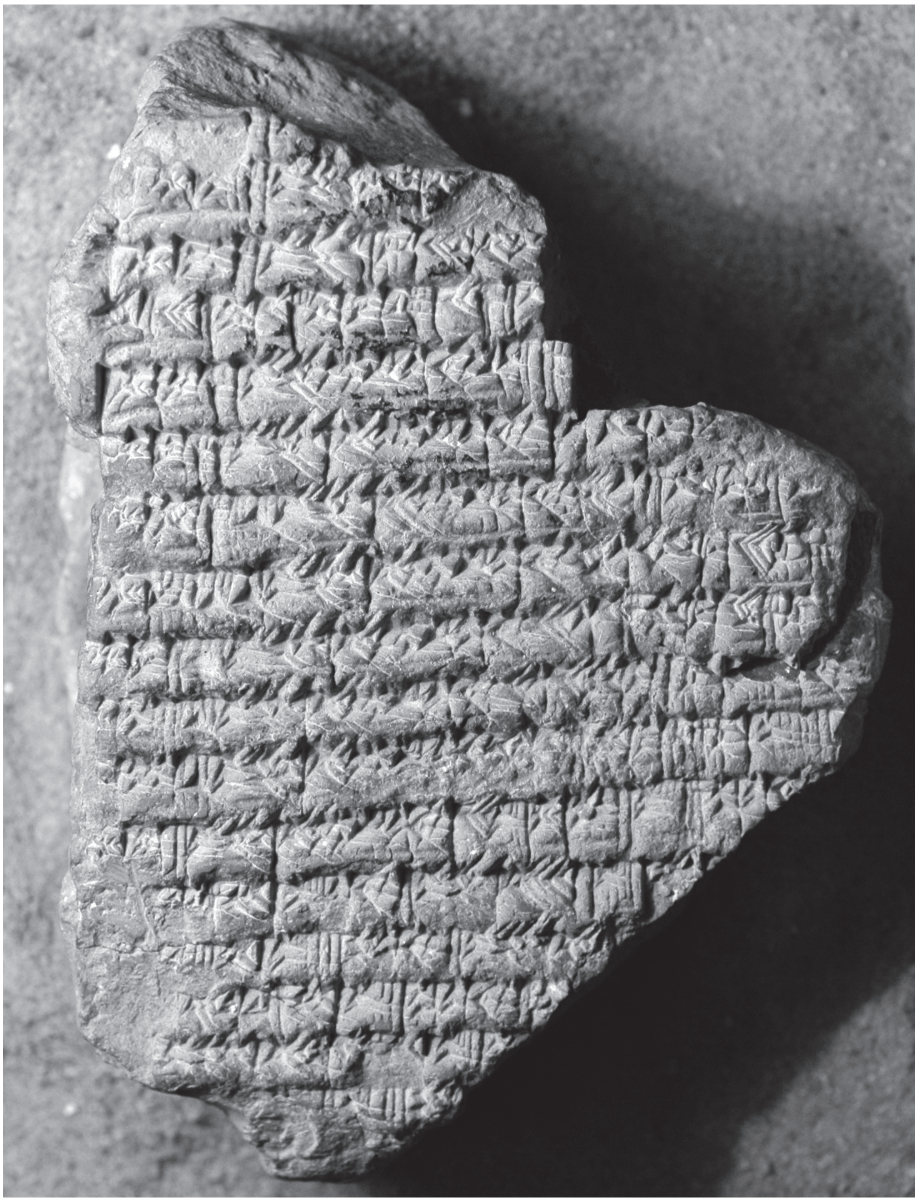

Figure 3. BM 45721 Obv.

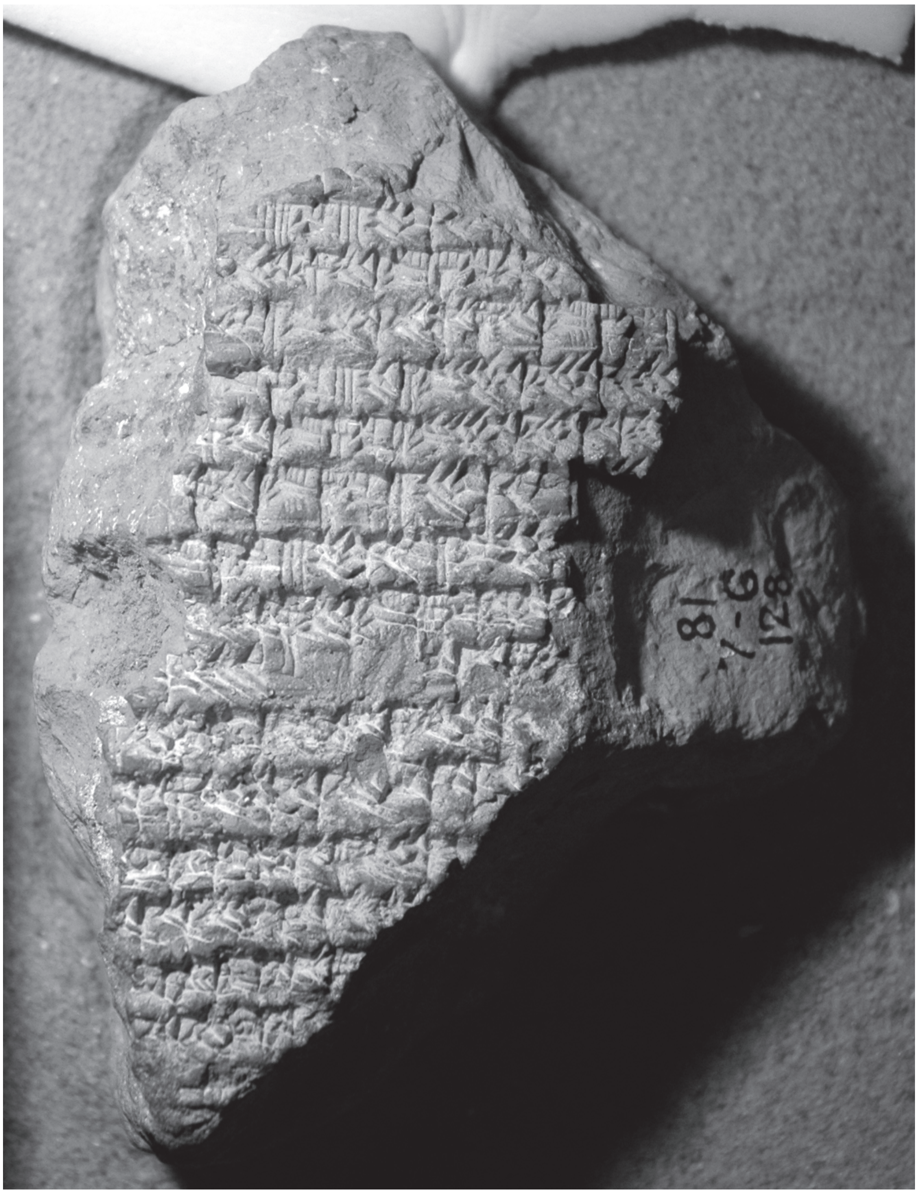

Figure 4. BM 45721 Rev.

Obv.

1′ […] x […]

2′ […] ⸢$^{\text{itu}}$APIN⸣ *šá* 1 DANNA 10 ⸢UŠ u_4-*mu*⸣ Ḫ[É-GÁL $^{\text{giš}}$MI …]

3′ DIŠ *ina* $^{\text{itu}}$GAN *šá* 1 DANNA 12 UŠ 30 ⸢u_4-*mu* ⸣ […]

4′ 42,30 A-RÁ 24 DU-*ma* 17 1,25 A-R[Á …]

5′ DIŠ *ina* $^{\text{itu}}$AB *šá* 1 DANNA ḪÉ-GÁL $^{\text{giš}}$MI 10$^{?}$ ME$^{?}$ 2,5 […]

6′ DIŠ *ina* $^{\text{itu}}$ŠU *šá* 4 DANNA u_4-*mu* ḪÉ-GÁL $^{\text{giš}}$MI 1 ⸢*ina* 1 NIM-*ma*⸣ […]

7′ ⸢DIŠ⸣ *ina* $^{\text{itu}}$IZI *šá* 3 2/3 DANNA u_4-*mu* ḪÉ-GÁL $^{\text{giš}}$MI 50 *ina* 1,5 ⸢NIM-*ma* 15 ZAL⸣ [3 SI $^{\text{giš}}$MI …]

8′ [DIŠ] *ina* $^{\text{itu}}$KIN *šá* 2/3$^{!}$ 1/3 DANNA u_4-*mu* ḪÉ-GÁL $^{\text{giš}}$MI 40 ⸢*ina* 1,10 NIM-*ma*⸣ 30 ZAL 6 [SI $^{\text{giš}}$MI …]

9′ [DIŠ] *ina* $^{\text{itu}}$DU$_6$ *šá* 3 DANNA u_4-*mu* ḪÉ-GÁL $^{\text{giš}}$MI 30 *ina* 1,15 NIM-*ma* 45 ZAL ⸢9⸣ [SI $^{\text{giš}}$MI …]

10′ [DIŠ *ina* $^{\text{it}}$]$^{\text{u}}$APIN 2 2/3 DANNA u_4-*mu* ḪÉ-GÁL $^{\text{giš}}$MI 20 *ina* 1,20 NIM-*ma* 1 ZAL ⸢1 KÙŠ $^{\text{giš}}$⸣MI […]

11′ [DIŠ *ina* $^{\text{i}}$]$^{\text{tu}}$GAN 2 1/3 DANNA u_4-[*mu*] ḪÉ-GÁL $^{\text{giš}}$MI 10 *ina* 1,25 NIM-*ma* 1,15 ZAL 1 KÙŠ 3 [SI $^{\text{giš}}$MI …]

12′ [DIŠ *ina*] $^{\text{itu}}$AB 2 DANNA u_4-*mu* ḪÉ-GÁL $^{\text{giš}}$MI 1,30 KÙŠ GIM KA 1 KÙŠ 6 SI $^{\text{giš}}$M[I …]

13′ [DIŠ *ina* $^{\text{itu}}$ŠU U]D-15-KAM 1 KÙŠ $^{\text{giš}}$MI 2 DANNA u_4-*mu* 21 1 KÙŠ 1 ŠE $^{\text{giš}}$[MI …]

14′ [DIŠ *ina* $^{\text{itu}}$IZI] ⸢UD-3⸣-KAM 1 KÙŠ 3 ŠE $^{\text{giš}}$MI 2 DANNA u_4-*mu* […]

15′ [… u_4]-*mu* UD-15-KAM 1 KÙŠ 1 SI $^{\text{giš}}$MI 2 DA[NNA …]

16′ […] DANNA u_4-*mu* UD-⸢27-KAM⸣ 1 KÙŠ 1 S[I …]

17′ [DIŠ *ina* $^{\text{itu}}$KIN UD-3-KAM 1] ⸢KÙŠ⸣ 2 SI 3 Š[E …]

Rev.

1′ […] x […]

2′ [DIŠ *ina* $^{\text{itu}}$APIN UD-3-KAM 1] KÙŠ 3$^{?}$ SI 3 ŠE $^{\text{giš}}$M[I …]

3′ [… u_4] -⸢*mu*⸣ UD-15-KAM 1 KÙŠ 4 SI $^{\text{giš}}$M[I …]

4′ […$^{\text{giš}}$]MI 2 DANNA u_4-*mu* UD-27-KAM 1 KÙŠ 4 SI ⸢2⸣ [ŠE …]

5′ [DIŠ *ina* ^{itu}GAN UD-3-KAM 1 K]ÙŠ 4 SI 3 ŠE 2 DANNA u_4-*mu* UD-9-KAM […]

6′ [… U]D-15-KAM 1 KÙŠ 5 SI 2 DANNA u_4-*mu* UD-21-KAM […]

7′ [… UD]-27-KAM 1 KÙŠ 5$^?$ SI 2 ŠE ^{giš}MI ⌜2 DANNA⌝ […]

8′ [DIŠ *ina* ^{itu}AB UD-3-KAM 1] KÙŠ 5 SI 3 ŠE ^{giš}MI 2 DANNA […]

9′ [… u_4]-*mu* UD-15-KAM 1 KÙŠ 6 SI x ^{giš}[MI …]

10′ [… u_4]-*mu* UD-⌜27-KAM⌝ […]

11′ [… x]+6$^?$ ŠE$^?$ GIM$^?$ UD-27$^?$-KAM UD-10[+x-KAM …]

12′ […] UD-15-KAM UD-21-KAM x UD-9-KA[M …]

13′ […] GIM ^{itu}GAN$^?$ UD-27-KAM UD-10[+x-KAM …]

14′ […] UD-21-KAM GIM UD-9-KA[M …]

15′ […] GIM ^{itu}DU$_6^?$ UD-27-[KAM …]

16′ [… U]D-21-KAM GIM UD-9-[KAM…]

17′ […] x […]

Obv.

1′ […] ... […]

2′ […] Month VIII at 1 *bēru* 10 UŠ of day […]

3′ ¶ In Month IX at 1 *bēru* 12;30 UŠ of day, the Ḫ[É-GÁL of the shadow is …]

4′ 42;30 multiplied by 24 is 17. 1,25 multip[lied by…]

5′ ¶ In Month X at 1 *bēru* the ḪÉ-GÁL of the shadow is ... 2,5 […]

6′ ¶ In Month IV at 4 *bēru* of day, the ḪÉ-GÁL of the shadow is 1 subtracted from 1 is […]

7′ ¶ In Month V at 3 2/3 *bēru* of day, the ḪÉ-GÁL of the shadow is 0;50. Subtract it from 1;5 and 0;15 is delayed [3 fingers of shadow …]

8′ [¶] In Month VI at 3 1/3 *bēru* of day, the ḪÉ-GÁL of the shadow is 0;40. Subtract it from 1;10 and 0;30 is delayed 6 [fingers of shadow …]

9′ [¶] In Month VII at 3 *bēru* of day, the ḪÉ-GÁL of the shadow is 0;30. Subtract it from 1;15 and 0;45 is delayed 9 [fingers of shadow …]

10′ [¶ In] Month VIII, 2 2/3 *bēru* of day, the ḪÉ-GÁL of the shadow is 0;20. Subtract it from 1;20 and 1;0 is delayed 1 cubit of shadow […]

11′ [¶ In] Month IX, 2 1/3 *bēru* of day, the ḪÉ-GÁL of the shadow is 0;10. Subtract it from 1;25 is 1;15 is delayed 1 cubit 3 [fingers of shadow …]

12′ [¶ In] Month X, 2 *bēru* of day, the ḪÉ-GÁL of the shadow is 1;30 cubits which corresponds to 1 cubit 6 fingers of shad[ow …]

13′ [¶ In Month IV,] 15th [d]ay 1 cubit of shadow at 2 *bēru* of day. 21<st day> 1 cubit 1 barleycorn […]

14′ [¶ In Month V,] 3rd day 1 cubit 3 barleycorn of shadow at 2 *bēru* of day. […]
15′ [… of d]ay. 15th day 1 cubit 1 finger of shadow at 2 *bē*[*ru* …]
16′ […] 2 *bēru* of day. 27th day 1 cubit 1 fin[ger …]

17′ [¶ In Month VI, 3rd day 1] cubit 2 fingers 2 barley[corn …]

Rev.
1′ […] ... […]

2′ [¶ In Month VIII, 3rd day, 1] cubit 3 fingers 3 barleycorn of shad[ow …]
3′ [… of d]ay. 15th day 1 cubit 4 fingers of shad[ow …]
4′ [… sha]dow 2 *bēru* of day. 27th day 1 cubit 4 fingers 2 [barleycorn …]

5′ [¶ In Month IX, 3rd day, 1 cu]bit 4 fingers 3 barleycorn at 2 *bēru* of day. 9th day […]
6′ […] 15th [d]ay 1 cubit 5 fingers at 2 *bēru* of day. 21st day […]
7′ […] 27th [day] 1 cubit 5 fingers 2 barleycorn of shadow at 2 *bēru* […]

8′ [¶ In Month X, 3rd day, 1] cubit 5 finger 3 barleycorn of shadow at 2 *bēru* […]
9′ [… of d]ay. 15th day 1 cubit 6 finger … sha[dow …]
10′ [… of d]ay. 27th day […]

11′ […] … Month XII$^{?}$ corresponds to 27$^{?}$th day [xth] day […]
12′ […] 15th day 21st day … 9th day […]

13′ […] corresponds to Month IX$^{?}$ 27th day [xth] day […]
14′ […] 21st day corresponds to 9th day […]
15′ […] corresponds to Month VII 27[th] day […]
16′ […] 21st [d]ay corresponds to 9th day […]
17′ […] … […]

Critical Apparatus

Obv. 5′–12′: The sign read GÁL in ḪÉ-GÁL could equally be EN; see the discussion in the excursus.
Obv. 4′: The number 1,25 is unclearly written and might possibly be 1,28.
Obv. 5′: The signs 10 ME could also be IGI, perhaps with the meaning "reciprocal".
Obv. 8′: The number 2/3 1/3 is an error for 3 1/3.

Excursus: The Meaning of ḪÉ-GÁL

The combination of signs read above as ḪÉ-GÁL appears in Obv. 5′–12′ and is probably to be restored at the end of Obv 3′. The identification of the first sign is certain but the cramped script makes it difficult to identify the second sign; indeed, this second sign looks more like EN than GÁL. However, I am unable to make sense of the combination ḪÉ-EN, whereas ḪÉ-GÁL is at least a well attested logographic combination for various Akkadian words including *ḫegallu* and *ṭuḫdu*, both of which have meanings such as "abundance" and "plenty". By extension ḪÉ-GÁL can mean "climax" or "greatest point", which leads also to the meaning "midday". In the text under discussion, ḪÉ-GÁL normally appears after a phrase giving a time ending in *u*$_4$*-mu* "of day" (the only exception is at Obv. 6′ where the *u*$_4$*-mu* is missing and ḪÉ-GÁL follows immediately after the time-unit *bēru*, but the omission of *u*$_4$*-mu* is probably just a scribal error) and immediately before ^{giš}ME "shadow" (in Obv. 3′ the text is broken after Ḫ[É-GÁL] but restoring ^{giš}ME here would make sense). Thus, we would expect that ḪÉ-GÁL is either the end of the phrase about the time or the beginning of the phrase about the shadow. It is worth noting, however, that both *u*$_4$*-mu* and ^{giš}ME appear elsewhere in the text without ḪÉ-GÁL.

In Obv. 6′–12′ the time preceding ḪÉ-GÁL is the time of midday. This might suggest, therefore, that ḪÉ-GÁL should be translated as "midday"; this interpretation, however, does not fit the use of ḪÉ-GÁL in Obv. 3′ and 5′ where the time does not correspond to midday but rather to the moment when the shadow is 2 or 3 cubits in length respectively. Thus, it would appear that ḪÉ-GÁL does not refer to the time preceding it. ḪÉ-GÁL must therefore be connected to the length of the shadow; it cannot, however, have the meaning "greatest" as the shadow lengths being discussed in these sections are not the greatest lengths of shadow: Obv. 3′ refers to a 2-cubit length shadow, Obv. 5′ refers to a 3-cubit length shadow and Obv. 6′–12′ to the shadow at midday which is when the shadow is shortest. Perhaps ḪÉ-GÁL simply means "length" here, but if so this would represent a new meaning of the term.

In the light of the uncertainties of the meaning of ḪÉ-GÁL I leave it untranslated in my translation.

Commentary

Obv. 1′–5′

Parts of three sections are preserved in this unit of text.[22] Obv. 2′ and 3′–4′ concern the time at which the shadow equals 2 cubits in Months VIII and IX respectively according to the MUL.APIN scheme. In Obv. 3′, which concerns Month IX, the time is given as 1 *bēru* 12 UŠ and 30 (NINDA). In the next line, this time is restated sexagesimally as 42;30 (UŠ). The time is then multiplied by (0);24 giving the result 17. Multiplication by 0;24 is equivalent to dividing by 2;30 and 2;30 UŠ is the monthly change in the time after sunrise when the shadow reaches 2 cubits, but I do not understand the reason for dividing by this number. The line continues with another multiplication of which only the first number 1,25 is preserved. This number is the value of the constant *c* of the MUL.APIN scheme for Month IX which equals the time when the shadow equals 1 cubit. One might speculate that this number is multiplied by 0;12, which is equivalent to dividing by 5 which is the monthly change in the time when the shadow reaches 1 cubit (as stated in the procedure at the end of the MUL.APIN shadow-length scheme). The result is again 17. Following this line of speculation, similar multiplications can be made for the other months. The result of the multiplications would be 12 in Month IV, 13 in Month V, 14 in Month VI, 15 in Month VII, 16 in Month VIII, 17 in Month IX, and 18 in Month X (with the same values for the other half of the year). Thus, the multiplications produce another zigzag function with minimum 12 in Month IV and maximum 18 in Month X. This zigzag function also corresponds to the time when the shadow equals 5 cubits. If this speculation is correct then this section appears to be an investigation of the mathematical structure of the MUL.APIN shadow-length scheme.

Obv. 5′ is puzzling. We would expect a continuation of the sections interested in shadows of 2 cubits in length giving the data for Month X, but the time at the beginning of this line (1 *bēru*) corresponds to a shadow of 3 cubits. I do not understand the numbers at the end of this line.

Obv. 6′–12′

This unit contains a series of seven parallel sections for Months IV–X. The form of each section is as follows (the entry for Month X is shortened and does not include *b* or *d*):

DIŠ *ina* MN *a* DANNA u_4-*mu* ḪÉ-GÁL ^{giš}MI *b ina c* NIM-*ma d* ZAL *e* KÙŠ ^{giš}MI […]

¶ In Month MN, at *a bēru* of day, the ḪÉ-GÁL of the shadow is *b*. Subtract it from *c* and *d* is delayed *e* cubits of shadow […]

[22] It is probable that the traces in Obv. 1′ represent a fourth section, but too little is preserved to be certain.

MN	*a*	*b*	*c*	*d*	*e*
IV	4 bēru	1,(0)	1,0	[0]	[0]
V	3 2/3 bēru	50	1,5	0,15	[3 fingers of shadow]
VI	3 1/3 bēru	40	1,10	0,30	6 [fingers of shadow]
VII	3 bēru	30	1,15	0,45	9 [fingers of shadow]
VIII	2 2/3 bēru	20	1,20	1,0	1 cubit of shadow
IX	2 1/3 bēru	10	1,25	1,15	1 cubit 3 [fingers of shadow]
X	2 bēru	–	–	1,30	1 cubit 6 fingers of shad[ow]

Table 4. Summary of the scheme in BM 45721 Obv. 6′–12′.

Table 4 summarizes the entries in these sections. All of *a*, *b*, *c*, *d* and *e* are zigzag functions; *a* and *b* decrease from a maximum in Month IV to a minimum in Month X whereas *c*, *d* and *e* increase from a minimum in Month IV to a maximum in Month X.

The time *a bēru* of day is equal to half the length of daylight and therefore corresponds to the time of noon according to the 2:1 ratio. The remainder of the line performs a mathematical operation to determine the length of shadow. The numerical relationship between the different numbers is straightforward: $c - b = d = e$, but the rationale behind the calculation is less clear. *c* is equal to the constant *c* in the MUL.APIN scheme. Thus *c* can be interpreted either as the length of shadow at 60 UŠ after sunrise or as the time after sunrise at which the shadow reaches 1 cubit in length. *b* equals the time of noon in UŠ minus 60 UŠ. The result, *d*, is therefore equal to the length of the shadow at 60 UŠ after sunrise minus the number of UŠ between 60 UŠ after sunrise and noon. This result, *d*, is given sexagesimally and is followed by the sign ZAL. The same zigzag function *d* is also found on W 23273 where it is followed by $^{\text{giš}}$MI ZAL-*ra*. ZAL is therefore probably to be read as Akkadian *uḫḫuru* "to be late".[23] *d* is then equated with *e* which transforms the sexagesimal number *d* into cubits and fingers where there are 12 fingers in a cubit (see below on this metrology); the equation $d = e$ is implied in the entries for Months IV–IX but stated explicitly in the entry for Month X (presumably because this line has extra space as $b = 0$ and therefore $c = d$; as a result the text *b ina c* NIM-*ma d* ZAL is replaced by simply *c* KÙŠ making space for GIM KA "which corresponds to"). Thus it is clear that *d* is to be interpreted as the length of the shadow in

[23] This reading was suggested (for ZAL-*ra* in W 23273) by Hermann Hunger.

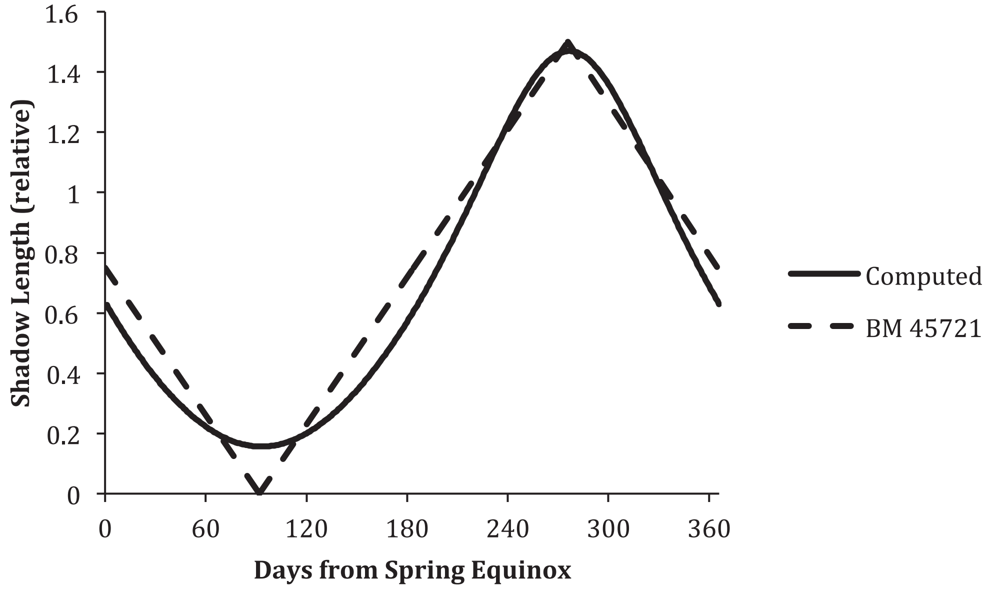

Figure 5. Noon shadow from the BM 45721 scheme.

cubits. Since each section begins with the time of noon, it seems likely that it is the noon shadow that is calculated in these sections.

In figure 5 I compare the shadow lengths given in this section with computation of the length of the shadow at noon for the latitude of Babylon (for convenience I plot the zigzag function for the whole year). The zigzag function is in remarkable agreement with the actual variation of the shadow length at noon, better than noon shadows deduced from the MUL.APIN scheme (see figure 2 above and table 5). Indeed, I do not think it would have been possible to find a better-fitting zigzag function that uses such simple values for the maximum, minimum and difference. The assumption that the shadow is equal to zero at the summer solstice is of course incorrect, but was probably seen as a small price to pay for an otherwise excellent and simple function for the length of noon shadow.

Although the mathematical rule for the determination of the shadow is clear, I can offer no explanation for the reasoning behind it. I do not understand why a time difference should be subtracted from a shadow length to produce another shadow length (shadow lengths and time are not proportional). The only explanation I can offer is that the scribe is attempting to justify the zigzag function for the noon shadow, which is in very good agreement with nature, using numbers taken from the MUL.APIN scheme which is itself in not so good agreement with nature. The justification is forced and incorrect, however, as the scribe must have known,[24] even though the resulting zigzag function for the noon shadow is very good.

Month	Noon Shadow (cubits)		
	MUL.APIN	BM 45721	Modern calculation
IV	0;30	0;00	0;9
V	0;35,30	0;15	0;12
VI	0;42	0;30	0;22
VII	0;50	0;45	0;37
VIII	1;00	1;00	0;56
IX	1;12,50	1;15	1;17
X	1;30	1;30	1;28

Table 5. Comparison of the MUL.APIN and BM 45721 schemes for the length of the noon shadows. Shadow lengths from the MUL.APIN scheme have been rounded to the nearest 0;0,10 cubit.

Obv. 13′ – Rev. 10′

These sections contain statements of the length of the shadow at 2 *bēru* after sunrise every six days from summer solstice to winter solstice. The length of shadow is assumed to increase linearly from summer solstice to winter solstice as shown in table 6. The scheme assumes that there are 5 barleycorn in a finger and 12 fingers in a cubit (see the metrological note below). According to this scheme the maximum shadow length (at winter solstice) at 2 *bēru* after sunrise is 1;30 cubits and the minimum shadow length (at summer solstice) 2 *bēru* after sunrise is 1 cubit and the variation between these extremes is given by a linear zigzag function. Both the extreme shadow lengths and the use of the linear zigzag function are fully compatible with the MUL.APIN shadow-length scheme.

Rev. 13′–17′

This unit appears to be stating that the dates given in the preceding unit for Months IV to X are equivalent to dates in the other half of the year. In other words, a mirror image of the scheme presented in Table 5 applies to Months XI to III.

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[24] The situation may be similar to the fudged attempt to harmonize the two lunar visibility tables of *Enūma Anu Enlil* 14 that appear in the commentary texts BM 45821+46093+46215 and BM 45900. See Steele and Brack-Bernsen (2008).

Day	Month IV	Month V	Month VI	Month VII
3		1 KÙŠ 3 ŠE	[1 K]ÙŠ 1 SI 3 Š[E]	[1 KÙŠ 2 SI 3 ŠE]
9		[1 KÙŠ 4 ŠE]	[1 KÙŠ 1 SI 4 ŠE]	[1 KÙŠ 2 SI 4 ŠE]
15	1 KÙŠ	[1 KÙŠ 1 SI]	[1 KÙŠ 2 SI]	[1 KÙŠ 3 SI]
21	1 KÙŠ 1 ŠE	1 KÙŠ 1 SI 1 ŠE	[1 KÙŠ 2 SI 1 ŠE]	[1 KÙŠ 3 SI 1 ŠE]
27	[1 KÙŠ 2 ŠE]	1 KÙŠ 1 S[I 2 ŠE]	[1 KÙŠ 2 SI 2 ŠE]	[1 KÙŠ 3 SI 2 ŠE]

Day	Month VIII	Month IX	Month X
3	[1] KÙŠ 3 SI 3 ŠE	[1 K]ÙŠ 4 SI 3 ŠE	[1] KÙŠ 5 SI 3 ŠE
9	[1 KÙŠ 3 SI 4 ŠE]	[1 KÙŠ 4 SI 4 ŠE]	[1 KÙŠ 5 SI 4 ŠE]
15	1 KÙŠ 4 SI	1 KÙŠ 5 SI	1 KÙŠ 6 SI
21	[1 KÙŠ 4 SI 1 ŠE]	[1 KÙŠ 5 SI 1 ŠE]	[1 KÙŠ 5 SI 4 ŠE]
27	1 KÙŠ 4 SI 2 [ŠE]	1 KÙŠ 5 SI 2 ŠE	[1 KÙŠ 5 SI 3 ŠE]

Table 6. Summary of the scheme in BM 45721 Obv. 13′ – Rev. 10′.

Metrological Note

The final results of the mathematical operations in Obv. 6′–12′ and the shadow lengths in Obv. 13′ – Rev. 10′ are given using a previously unattested metrology. The cubit (KÙŠ), finger (SI) and barleycorn (ŠE) are widely used units of length in Babylonian metrologies. Various relationship between these units, however, are attested in different sources. For example, the so-called "Standard System" of length measures, used from the third millennium down to the first millennium, equates 30 fingers with 1 cubit.[25] In the Late Babylonian period, however, an alternate system with 24 fingers in a cubit is also attested (sometimes in the same text as the 30-finger cubit).[26] Late Babylonian astronomical texts use exclusively the relationship 24 fingers equals 1 cubit.[27] Evidence for the subdivision of the finger into barleycorn is very scarce. It is usually assumed that 6 barleycorn equaled a finger, but some metrological texts provide an alternate system in which there

[25] Powell (1987–90).

[26] Powell (1987–90: 470), Friberg (1993).

[27] Steele (2003: 283–286).

are 5 barleycorn in a finger,[28] and the 5-barleycorn finger is also attested in two Uruk texts which contain a scheme for the growth of an unborn baby.[29]

The present text, however, implies a previously unattested metrology in which there are 12 fingers in a cubit and 5 barleycorn in a finger. It is possible that this metrology was restricted to measurements of the lengths of shadows; perhaps this is even directly implied in the text by the consistent writing of $^{\text{giš}}$MI "shadow" after the smallest unit of cubits, fingers and barleycorn of a shadow length.

W 23273 (SpTU IV 172)

W 23273 is a large metrological tablet from Uruk dating to the late 5th century BC. The tablet, said to be a copy of an old tablet of Uruk, was owned by Rīmūt-Anu, son of Šamaš-iddin, descendent of Šangû-Ninurta, a well known scribe.[30] A copy of the tablet was published by von Weiher (1993) as SpTU IV 172. A translation of the tablet may be found in Robson (2007: 160–166); for discussions of the metrological sections, see also Friberg (1993: 400) and Robson (2008: 232). At the end of the tablet, just before the colophon, is a short section (or perhaps two sections, see below) which refers to shadows. The first line of the colophon contains a catchline which also refers to shadows. I give below a transliteration and translation of these thirteen lines of Rev. IX:

39 […] *am-mat* ŠU
40 […] KÙŠ IZI *u* SIG$_4$ KI.MIN
41 [… KÙŠ KIN] *u* GU$_4$ KI.MIN
42 [… KÙŠ DU$_6$ *u*] ⌜BÁRA⌝ KI.MIN
43 [… KÙŠ APIN *u* ŠE] KI.MIN
44 […]
45 […]
46 [IZI 15] ⌜$^{\text{giš}}$⌝MI ZAL-*ra*
47 KIN 30 $^{\text{giš}}$MI ZAL-*ra*
48 DU$_6$ 45 $^{\text{giš}}$MI ZAL-*ra*
49 APIN 1 ⌜$^{\text{giš}}$⌝MI ZAL-*ra*
50 GAN 1,15 $^{\text{giš}}$MI ZAL-*ra*
51 AB 1,30 $^{\text{giš}}$MI ZAL-*ra*

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52 DIŠ 1,12 $^{\text{giš}}$MI 1;40 DANNA *u*$_4$-*mu* EGIR-*šú*

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[28] Friberg (1993). The only appearance of barleycorns in Babylonian astronomy other than the present text is in the ACT lunar theory, where 6 barleycorn are probably taken to be equal to 1 finger—see Neugebauer (1945).

[29] Hunger (1994) and (1996).

[30] On this scribe, see Robson (2008: 232).

39 […] cubit Month IV
40 […] cubits Month V and Month III the same.
41 [… cubits Month VI] and Month II the same.
42 [… cubits Month VII and] Month I the same
43 [… cubits Month VIII and Month XII] the same
44 […]
45 […]
46 [Month V 15] the shadow is delayed
47 Month VI 30 the shadow is delayed
48 Month VII 45 the shadow is delayed
49 Month VIII 1(,0) the shadow is delayed
50 Month IX 1,15 the shadow is delayed
51 Month X 1,30 the shadow is delayed

----

52 ¶ 1,12 shadow at 1;40 *bēru* of day after it.

Lines 44–45 are completely lost but it is possible that a dividing line separated lines 39–44 from lines 45–46 into two sections. Lines 39–44 are fragmentary but seem to be sets of statements that the length of shadow is the same in months equidistant from the summer solstice. Each line gives a number of cubits associated with the pair of months. The numbers are all lost at the beginning of the lines, but it seems that in line 39, the value for Month IV is 1 cubit because "cubit" is written syllabically without an ending indicating plurality. I suggest that the 1 cubit found here corresponds to the value of the constant *c* for Month IV in the MUL.APIN shadow scheme. The constant *c* can be interpreted as the length of the shadow at 60 UŠ after sunrise. Thus, lines 39–44 probably list the values of *c* from the MUL.APIN scheme.

Lines 45–51 are a series of statements concerning the length of shadow in certain months. Although line 45 is completely lost, it is likely that it should be restored with an entry for Month IV, providing a full sequence from summer solstice (Month IV) to winter solstice (Month X). There is no room to restore entries for the other months, but the shadow lengths from winter solstice to summer solstice mirror those for summer solstice to winter solstice given in lines 45–51. Thus, line 43 would state that the shadow in Month VII and Month XII is the same, line 44 would state that the shadow in Month VIII and Month XI is the same, and line 45 would give the shadow length for Month V.

The shadow lengths in lines 45–51 follow a simple zigzag scheme with minimum m = 0 in Month IV, maximum M = 1,30 in Month X and monthly difference d = 15.[31] This

[31] Hunger and Pingree (1999: 81) incorrectly state that the zigzag function has m = 15 and M = 1,45, presumably because they considered a shadow of zero length to be impossible. Their interpretation would, however, place the solstices in one month too late, in contradiction to lines 35–44.

zigzag function is the same as the zigzag function for the noon shadow found in Obv. 6′–12′ of BM 45721 discussed above.

I now turn to the catchline at the beginning of the colophon. From von Weiher's copy it appears that this line begins 2,12 not DIŠ 1,12, and this is how the line was read in Robson's translation. But the difference between 2 and DIŠ 1 is only a question of the spacing of the 2 vertical wedges. I propose to read the catchline as follows:[32]

DIŠ 1,12 $^{\text{giš}}$MI 1;40 DANNA u_4-*mu* EGIR-*šú*

¶ 1;12 shadow at 1;40 *bēru* of day after it.

A similar statement is found in part of the first line of BM 29371 (see below), a text that presents the length of shadow at 1;40 *bēru* after sunrise at different dates during the year.[33] Although BM 29371 cannot be the text referred to in this catchline as the wording is slightly different, this statement must refer to the same scheme.

BM 29371

BM 29371 (= 98-11-14, 4) is an almost completely preserved tablet containing statements of weights and the length of shadow for every five days in the ideal 360-day calendar. A photograph of the reverse of the tablet was published in Britton and Walker (1996: 47). A translation and brief discussion of the text was published by Hunger (1999) and a photograph, transliteration and discussion of the text was published by Brown, Fermor and Walker (1999–2000: 145–148).[34] According to the colophon the tablet was copied by Nabû-apla-iddin, son of Nabû-nādin-šumi (or alternatively Nabû-šuma-iddin), descendant of Ešguzi-mansum, "in order to read it".[35] According to Waerzeggers (2012: 296), Nabû-apla-iddin lived in Borsippa and was active during the mid to late 6th century BC.

Below I transliterate the tablet based upon the edition in Brown, Fermor and Walker (1999–2000). I have not collated the tablet, and have followed Brown, Fermor and Walker in not marking damaged signs. For the colophon, see Brown, Fermor and Walker's edition.

[32] H. Hunger (personal communication) confirms that the reading DIŠ 1 is perfectly possible from inspection of a photograph of the tablet.

[33] This point was noted already by Hunger (1999: 135).

[34] Neither Hunger's translation nor Brown, Fermor and Walker's transliteration indicate which text is restored. Enough of the text is preserved, however, that there can be no doubt about the restorations.

[35] Brown, Fermor and Walker (1999–2000: 145).

Obv.

1	DIŠ *ina* ^{itu}ŠU	UD-15-KAM	1 KI.LÁ	1;12	DIŠ *ina* ^{itu}ŠU	UD-15-KAM	1 KI.LÁ	1;12	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
2		UD-20-KAM	1;0,50	1;13		UD-10-KAM	1;0,50	1;13	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
3		UD-25-KAM	1;1,40	1;14		UD-5-KAM	1;1,40	1;14	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
4		UD-30-KAM	1;2,30	1;15	DIŠ *ina* ^{itu}SIG$_4$	UD-30-KAM	1;2,30	1;15	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
5	DIŠ *ina* ^{itu}IZI	UD-5-KAM	1;3,20	1;16		UD-25-KAM	1;3,20	1;16	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
6		UD-10-KAM	1;4,10	1;17		UD-20-KAM	1;4,10	1;17	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
7		UD-15-KAM	1;5	1;18		UD-15-KAM	1;5	1;18	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
8		UD-20-KAM	1;5,50	1;19		UD-10-KAM	1;5,50	1;19	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
9		UD-25-KAM	1;6,40	1;20		UD-5-KAM	1;6,40	1;20	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
10		UD-30-KAM	1;7,30	1;21	DIŠ *ina* ^{itu}GU$_4$	UD-30-KAM	1;7,30	1;21	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
11	DIŠ *ina* ^{itu}KIN	UD-5-KAM	1;8,20	1;22		UD-25-KAM	1;8,20	1;22	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
12		UD-10-KAM	1;9,10	1;23		UD-20-KAM	1;9,10	1;23	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
13		UD-15-KAM	1;10	1;24		UD-15-KAM	1;10	1;24	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
14		UD-20-KAM	1;10,50	1;25		UD-10-KAM	1;10,50	1;25	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
15		UD-25-KAM	1;11,40	1;26		UD-5-KAM	1;11,40	1;26	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
16		UD-30-KAM	1;12,30	1;27	DIŠ *ina* ^{itu}BAR	UD-30-KAM	1;12,30	1;27	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
17	DIŠ *ina* ^{itu}DU$_6$	UD-5-KAM	1;13,20	1;28		UD-25-KAM	1;13,20	1;28	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
18		UD-10-KAM	1;14,10	1;29		UD-20-KAM	1;14,10	1;29	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
19		UD-15-KAM	1;15	1;30		UD-15-KAM	1;15	1;30	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
20		UD-20-KAM	1;15,50	1;31		UD-10-KAM	1;15,50	1;31	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
21		UD-25-KAM	1;16,40	1;32		UD-5-KAM	1;16,40	1;32	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
22		UD-30-KAM	1;17,30	1;33	DIŠ *ina* ^{itu}ŠE	UD-30-KAM	1;17,30	1;33	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*

Rev.

1	DIŠ *ina* ^{itu}APIN	UD-5-KAM	1;18,20	1;34		UD-25-KAM	1;18,20	1;34	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
2		UD-10-KAM	1;19,10	1;35		UD-20-KAM	1;19,10	1;35	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
3		UD-15-KAM	1;20	1;36		UD-15-KAM	1;20	1;36	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
4		UD-20-KAM	1;20,50	1;37		UD-10-KAM	1;20,50	1;37	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
5		UD-25-KAM	1;21,40	1;38		UD-5-KAM	1;21,40	1;38	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
6		UD-30-KAM	1;22,30	1;39	DIŠ *ina* ^{itu}ZÍZ	UD-30-KAM	1;22,30	1;39	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
7	DIŠ *ina* ^{itu}GAN	UD-5-KAM	1;23,20	1;40		UD-25-KAM	1;23,20	1;40	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
8		UD-10-KAM	1;24,10	1;41		UD-20-KAM	1;24,10	1;41	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
9		UD-15-KAM	1;25	1;42		UD-15-KAM	1;25	1;42	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
10		UD-20-KAM	1;25,50	1;43		UD-10-KAM	1;25,50	1;43	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
11		UD-25-KAM	1;26,40	1;44		UD-5-KAM	1;26,40	1;44	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
12		UD-30-KAM	1;27,30	1;45	DIŠ *ina* ^{itu}AB	UD-30-KAM	1;27,30	1;45	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
13	DIŠ *ina* ^{itu}AB	UD-5-KAM	1;28,20	1;46		UD-25-KAM	1;28,20	1;46	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
14		UD-10-KAM	1;29,10	1;47		UD-20-KAM	1;29,10	1;47	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*
15		UD-15-KAM	1;30	1;48		UD-15-KAM	1;30	1;48	1 KÙŠ ^{giš}MI 1 2/3 DANNA *u*$_4$*-mu*

The tablet contains a series of entries for each fifth day in the ideal calendar, starting with the 15th day of Month IV (the summer solstice). The entries are read down the left hand side of the tablet, onto the reverse and then back up the right hand side of the tablet from reverse to obverse. Because the first line on the obverse and the last line on the reverse contain doubled entries for the solstices, the numbers given after the day numbers are identical in the left and right columns. At the end of each line is written the phrase 1 KÙŠ $^{\text{giš}}$MI 1 2/3 DANNA u_4-*mu*.

Let us consider the first line of the text as an example of text's structure:

> DIŠ *ina* ITU.ŠU UD-15-KAM 1 KI.LÁ 1;12 DIŠ *ina* ITU ŠU UD-15-KAM 1 KI.LÁ 1;12 1 KÙŠ $^{\text{giš}}$MI 1 2/3 DANNA u_4-*mu*

The line begins with the DIŠ sign used as a textual marker followed by a date in the ideal calendar. Following the date we find a number, the signs KI.LÁ which mean "weight" and then a second number. The statement "weight" suggests that the number preceding it is to be understood as the weight of water in a waterclock.[36] The line continues with another date, and two numbers, the first of which is again marked as being a weight. At the end of the line we have the phrase 1 KÙŠ $^{\text{giš}}$MI 1 2/3 DANNA u_4-*mu*. The subsequent lines have the same structure, except that the designation "weight" for the first of the two numbers following the dates is omitted.

I begin by discussing the two numbers that follow the dates in each entry. Both sets of numbers form linear zigzag functions with minimum at summer solstice and maximum at winter solstice. The first has a minimum of 1;00 and maximum of 1;30 with a difference for five days of 0;0,50 which corresponds to a daily difference of 0;0,1. These numbers are designated as "weights", but no unit is given. As recognized by Hunger (1999: 134) and Brown, Fermor and Walker (1999–2000: 146), these weights almost certainly relate to the length of night in the ratio 3:2 for the longest to shortest night. Brown, Fermor and Walker (1999–2000: 146) further show that the weights suggest a different relationship between time and weight than is found in other texts.

The second set of numbers form a linear zigzag function with minimum 1;12, maximum 1;48 and five-day difference 1. Brown, Fermor and Walker (1999–2000: 146) interpret this zigzag function as a second statement of the length of night, this time the length of half of the night given in UŠ. This interpretation raises the question, however, of why two statements relating to the length of night, albeit given in different units, are written side-by-side on this tablet. I propose an alternative interpretation of the second set of numbers below.

At the end of every line appears the phrase 1 KÙŠ $^{\text{giš}}$MI 1 2/3 DANNA u_4-*mu*, which Brown, Fermor and Walker (1999–2000: 145) and others have translated as "1 cubit of shadow, 1 2/3 *bēru* of day". Britton and Walker (1996) point out that the length of

[36] Hunger (1999: 134); Brown, Fermor and Walker (1999–2000: 145).

shadow should change throughout the year and suggest that the scribe mistakenly copied this piece of text after every line. Brown, Fermor and Walker (1999–2000: 146–147), however, propose to take the statement seriously. From modern astronomical theory they calculate the length of shadow at Babylon for a time of 1 2/3 *bēru* after sunrise throughout the year and conclude that "the length of shadow varies but little from early spring (declination -10°) through to mid-summer (declination 23°) and back to late autumn. Only in winter is the shadow cast 3 hours and 20 minutes after sunrise at this latitude noticeably longer than one unit". Brown, Fermor and Walker draw their conclusion from a plot of shadow length against solar declination, which does indeed show that for most of the curve the shadow length is close to 1.2 cubits for a 1 cubit high gnomon. However, this plot is misleading because the sun's motion in declination is not uniform over the year. By plotting the data in this fashion, the graph emphasizes the summer months, when the shadow is shortest. If we plot the length of shadow at 1 2/3 *bēru* after sunrise against day of the year (see the solid line in figure 6) we get a better idea of how good an approximation a constant 1 cubit length shadow is. For roughly half the year, from autumnal equinox to spring equinox, the shadow is considerably longer than 1 cubit, reaching a maximum of about 1 3/4 cubits.

I suggest that instead of translating the phrase 1 KÙŠ $^{\text{giš}}$MI 1 2/3 DANNA u_4-*mu* as "1 cubit of shadow, 1 2/3 *bēru* of day", we instead take 1 KÙŠ simply to mean "cubits", as it does in many other texts (including MUL.APIN (above) and BM 33564 (below)). "Cubits" must then refer to the preceding number which is the zigzag function ranging from 1;12 to 1;48. By comparison, the same zigzag function in the left-hand column would also be for the length of the shadow. The resulting zigzag function is shown with the dashed line in figure 4. The maximum and minimum of this zigzag function are in very close agreement with the actual length of shadow at 1 2/3 *bēru* after sunrise on at the solstices. Agreement is poorer in the months before and after summer solstice, but the fit is still significantly better than an assumption of a shadow of 1 cubit for the whole year.

Two pieces of evidence can be adduced in support of my interpretation. First is the catchline on W 23273 which reads "¶ 1;12 shadow at 1;40 *bēru* of day after it". This catchline states exactly the same shadow length at 1 2/3 *bēru* after sunrise as the first line of BM 29371 in my interpretation. Secondly, the zigzag function on BM 29371 is in agreement with the mathematical rule underlying the shadow-length scheme of MUL.APIN. According to this rule, the length of shadow in cubits multiplied by the time after sunrise in UŠ is equal to a constant c where $c = 60$ at the summer solstice, $c = 75$ at the equinoxes and $c = 90$ at the winter solstice. From the zigzag function on BM 29371 we find at summer solstice a shadow length of 1;12 cubits at a time of 50 UŠ after sunrise, giving $c = 60$; at equinox a shadow of 1;30 cubits at 50 UŠ after sunrise, giving $c = 75$; and at winter solstice a shadow of 1;48 cubits at 50 UŠ after sunrise, giving $c = 90$, in full agreement with the MUL.APIN scheme. My interpretation of the second zigzag function on BM 29371 as the length of shadow also removes the apparent redundancy of having two statements of the length of (part of) night side by side.

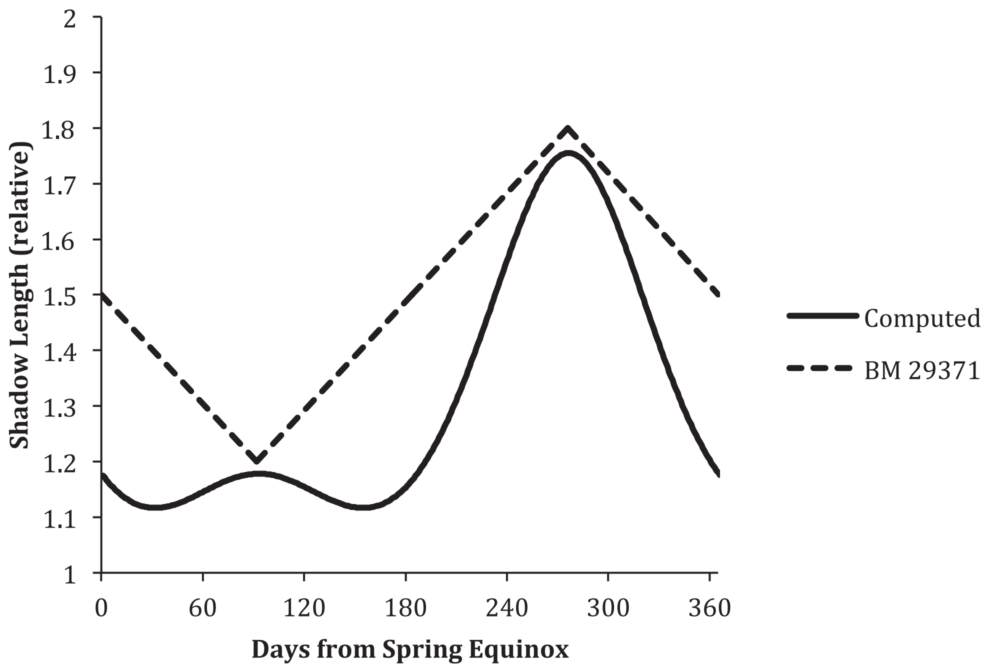

Figure 6. The variation in the length of shadow at 1 2/3 *bēru* after sunrise.

BM 29371 can now be seen to contain two zigzag functions, one for the length of night, and the other for the length of the shadow at 1 2/3 *bēru* after sunrise. Both zigzag functions have a ratio of 3:2 for maximum to minimum. To my knowledge this tablet is the only pre-Hellenistic source which uses this ratio for the length of night. It is interesting, therefore, that the 3:2 ratio for length of night appears alongside the 3:2 ratio for the length of shadow. It is tempting, if ironic, to see this as the origin of the 3:2 daylength ratio: a Babylonian scribe rationalizing that length of day/night should vary in the same ratio as length of shadow, exactly as Neugebauer did in his incorrect interpretation of the shadow-length table of MUL.APIN.

BM 33564

Unlike the texts discussed above, BM 33564 (= Rm. 4, 120) does not contain a shadow-length scheme as such. Instead, the text is a procedure text containing several short schemes concerned with shadows, the length of day and the calendar. When complete, BM 33564 was a small rectangular tablet in landscape orientation with 9 lines on each of the observe and reverse (figure 7). The tablet is broken roughly down the middle; BM 33564 preserves the right half of the tablet. The last four lines of the reverse contain a colophon which identifies the scribe as the son of Bēl-apla-idinna, descendent of Mušēzib; although the scribes name is broken away, from comparison with other texts the scribe is almost certainly Marduk-šapik-zēri. The Mušēzibs are a well-known family of scholars who were active in Babylon during the fourth and early third century BC.[37] A wide range of astronomical tablets are attributed to members of this family including ACT ephemerides, several of the so-called "atypical" texts, collections of celestial omens and observational texts.

Figure 7. BM 33564 Obv. (top), Rev. (bottom left) and right edge (bottom right)

[37] See, for example, Oelsner (2000) and Robson (2008: 223–225).

Obv.

1 […$^{\text{gi}}$]$^{\text{š}}$⌜MI⌝ AN.⌜NE⌝ […]

2 […] ⌜*a*$^{?}$⌝-*na* 1,6 1 KÙŠ-⌜*ma*$^{?}$⌝ […]

3 […] KÙŠ 48 *ma-ṭu* E-*bi*

4 […] x *la* 1 KÙŠ ITU LAL-*ú*

5 […] x 48 *te-ez-zib-ma*

6 […] ⌜*u*$_{4}$⌝-*mu šá* $^{\text{itu}}$ŠU E-*bi*

7 […] x ŠÚ DU-*ma* 3,36 LAL

8 […] ŠU TA $^{\text{itu}}$ŠU EN $^{\text{itu}}$AB

9 […] ⌜x⌝ KÙŠ ITU TAB *lu-u* LAL

Rev.

1 […] ⌜x⌝ $^{\text{giš}}$MI *šá* ŠU *šá* KA 3 ŠÚ

2 […] $^{\text{itu}}$ŠU

---

3 […] ⌜x⌝ 2,24 GE$_{6}$ *u*$_{4}$-*mu* 24 *taš-pil-tum*

4 […] ⌜x⌝ MU *ana* MU 10,30 KI LAL

5 […] ⌜x⌝ KI DU ITU ⌜DIR⌝

---

6 […] A *šá* $^{\text{Id}}$EN-A-MU DUMU $^{\text{lú}}$*mu-še-zib*

7 […] x [DU]MU $^{\text{lú}}$*mu-še-zib*

8 [… *a*]*na* ŠU$^{\text{II}}$ *nu ú-še-ṣi*

9 […] ŠU$^{?}$ *ú*

Obv.

1 […] midday shad[ow …]

2 […] to 1,6 cubits, and […]

3 […] cubits(?) 48 is lacking you predict.

4 […] … cubits (per?) month it decreases.

5 […] you leave (behind) 48 and

6 […] day of Month IV you predict.

7 […] … goes forward and 3,36 it decreases.

8 […] Month IV. From Month IV to month X

9 […] x cubits (per) month increase or decrease.

Rev.

1 […] shadow of month IV. According to a 3rd method

2 […] Month IV

---

3 […] 2,24 night. 0;24 (UŠ per) day is the difference

4 […] Year by year 10;30 degrees it goes back.

5 […] degrees it goes forward. Intercalary month.

---

6 [… Marduk-šapik-zēri] son of Bēl-apla-idinna descendent of Mušēzib

7 [… des]cendent of Mušēzib.

8 […] He must not let (the tablet) go out of hand

9 […] …

Critical Apparatus

Obv. 2: Only the two stacked wedges of *a* remain. Is the reading *a-na*, correct, or do we have x NA? If so, the following DIŠ sign could be *ana* so we would have x NA *ana* 6 1 KÙŠ-*ma* (but a shadow of 6 cubits is too big to be a noon shadow). The last sign of the line looks line -*ma*.

Obv. 3: The reading *ma-ṭu* was suggested by H. Hunger on the basis of a photograph.

Obv. 5: The number at the beginning of this line might be either 48 or 38. I tentatively read 48 in parallel with line 3. H. Hunger suggested the reading *te-ez-zib-ma*.

Obv. 6: The reading u_4 for the first sign is very tentative.

Obv. 7: The first sign looks like [x+]1.

Commentary

The tablet is divided into two sections, a long section extending over the whole of the obverse and the first two lines of the reverse, and a shorter section of just three lines on the reverse, plus the colophon. Unfortunately, because only the second half of each line is preserved, it is difficult to understand the procedures in the text. I am therefore only able to make only a few preliminary remarks about the text.

Obv. 1 – Rev. 2

This section mentions the noon shadow and the length of day in Month IV, and their change between Month IV and Month X (cf BM 45721 and W 23273 which also confine themselves to the half of the year between summer and winter solstice). The end of Rev. 1 appears to contain the phrase "According to a 3rd method", which if read correctly indicates that the section contains three alternate methods concerned with the same phenomenon. This complicates the interpretation of this section because the corresponding "According to a 2nd method" must be located somewhere in the missing text, and it is not obvious where the 1st method ends and the 2nd method begins. I can make little out of the numbers in this section, other than 3,36 in Obv. 7 which is the length of daylight in Month IV according to the 3:2 ratio.

Rev. 3–5

The section begins with a statement of the length of night on the summer solstice, 2,24 UŠ as given by the 3:2 ratio. 24 refers to the daily change in length of day/night: the 3:2 ratio implies that between Month IV and Month VII, night has changed from 2,24 UŠ to 3,0 UŠ, a change of 36 UŠ in 90 days, or 0;24 UŠ per day assuming the 360-day calendar. The next line states that after 1 year, the sun goes back 10;30°. Here we are dealing with the real year of 12 synodic months. The figure 10;30 degrees is implied in the so-called Text S and is important for constructing ACT lunar theories but the present text is the first explicit attestation of the parameter.[38]

The Place of the Shadow-Length Schemes in Babylonian Astronomy

The Late Babylonian shadow length schemes provide a clear example of the continued practice of MUL.APIN-style astronomy in the late period. The late texts contain mathematical explorations of the MUL.APIN scheme and expand the scheme to cases not mentioned in MUL.APIN. Central to these late schemes are the schematic calendar of 360 days and mathematical rules and parameters taken from MUL.APIN (some of which also appear in other early astronomical texts such as *Enūma Anu Enlil* 14). For simplicity, I will call astronomy that relies upon MUL.APIN and the schematic calendar "schematic astronomy".

Schematic astronomy in the late period has received very little attention, in part because the sources are often fragmentary and difficult to understand. But there is a considerable body of texts that include schematic astronomical material. Some of these texts, such as BM 45721 and BM 29371 discussed above are devoted wholly to schematic astronomy but other texts mix schematic astronomy with other types of astronomy. The shadow-length material in BM 33564, for example, is probably schematic (in that it used the 360-day calendar), whereas the second section is related to the non-schematic tradition of mathematical astronomy. TU 11, an important collection of material about the calculation of the lunar six, month lengths, eclipses, as well as astrological material, includes a section which relies purely on MUL-APIN type schematic astronomy.[39]

Schematic astronomy in the tradition of MUL.APIN continued to be practiced alongside the observational, predictive and mathematical astronomical traditions of the late period. It is wrong, therefore, to see these other traditions as replacing MUL.APIN-style astronomy. Furthermore, interest in MUL.APIN-style astronomy in the late period was not confined to the genre of commentaries. Indeed, no texts are known which identify themselves as commentaries to MUL.APIN (in contrast to the many that were

[38] Britton (2002: 36).

[39] Section 19 in the edition of Brack-Bernsen and Hunger (2002).

written about *Enūma Anu Enlil*),[40] and although it is possible that some of these late texts (eg BM 45721) were identified as commentaries in now lost colophons, the inclusion of sections which build upon MUL.APIN within texts such as TU 11 indicates that MUL.APIN-style schematic astronomy was not purely of antiquarian scholarly interest but continued to play a role in contemporary astronomical thinking during the late period.

Acknowledgements

I thank the Trustees of the British Museum for permission to study and publish tablets in their collection. I am grateful to Hermann Hunger, Mathieu Ossendrijver, Matthew Rutz and to the anonymous referees for their suggestions during the writing of this paper.

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[40] Frahm (2011: 166).

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(Received: October 16, 2012)