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Solving problems by algebra in late antiquity: New evidence from an unpublished fragment of Theon's commentary on the *Almagest*

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I. Introduction

Theon of Alexandria wrote in the second half of the fourth century AD a full-scale commentary on Ptolemy's *Almagest*. Of this commentary, originally composed in thirteen books, corresponding to the books of the *Almagest* itself, most parts have been preserved. What is lost is the entire book XI, while, with regard to book V and the last books of the work there are lacunae in the transmitted text. Furthermore book VII and portions of other books are preserved only in a late Byzantine recension.

As Adolph Rome has explained, in the prolegomena to his edition (Rome 1931, xxi ff.), the direct manuscript tradition of Theon's commentary has three branches: one is formed by the early ninth century manuscript *Laur. Plut.* 28.18, "le manuscrit le plus fidèle" as Rome says (1931, xxiii), and it is available only for books I–IV, already published by him (1936; 1943), and VI; a second branch transmits also a 'genuine' text, possibly not coinciding with the one of the first branch, whose sole testimony for books VIII–X and XII–XIII (and of these, only partially) is *Vat. gr.* 1087; and a third branch is formed by a group of manuscripts – among which are *Vat. gr.* 198, *Marc. gr.* 310, *Norimb. Cent*. V, app. 8 – that carry the text of the aforementioned Byzantine recension; the printed version of the last manuscript is the Basel edition of 1538.[1]

On the other hand, it is also known that at some time – most likely in Late Antiquity, but, at any rate, not later than the first half of the ninth century – portions of Theon's main text were copied in some manuscripts in the margins of the *Almagest*. These portions have been transmitted as scholia appended to the text of the *Almagest* either in

[1] *Claudii Ptolemaei Magnae Constructionis id est Perfectae caelestium motuum pertractationis lib. XIII. Theonis Alexandrini in eosdem commentariorum lib. XI*. Basileae, Apud Joannem Vvalderum, an. MDXXXVIII.

manuscripts of the Byzantine recension, such as *Vat. gr.* 198 and *Marc. gr.* 310[2], or in some manuscripts which carry the text of the *Almagest*, accompanied by its *marginalia*, but not Theon's running text; such manuscripts are, for example, V*at. gr.* 1594, *Vat. gr.* 180, and *Vat. gr.* 184.

Unfortunately, as said before, only the first four books of Theon's commentary have been critically edited.[3] The scholium we publish here comes from the unpublished book XIII. It is a fragment in which Theon, commenting on a mensuration problem discussed by Ptolemy in *Almagest* XIII.3, proposes two different solutions to it. In our sole 'genuine' witness for Theon's book XIII, the codex *Vat. gr.* 1087, the fragment appears on f. 145r-v. This codex forms a unit with *Par. gr.* 2396, which carries the text of the commentaries on books I–II and IV, since the two manuscripts, as Rome himself already discovered, "sont le commencement et la fin d'un même codex, dont le milieu est perdu" (Rome 1931, xxi n. 1). The first part of this unit (the actual *Par. gr.* 2396, or, to be more precise, the most part of it since the last part of the commentary on book IV was added later) was copied in the years around the end of the thirteenth century in the entourage of Maximus Planudes, who not only supervised the whole project but also participated in the writing.[4] The portion of *Vat. gr.* 1087 containing the fragment we publish here was copied in the same period and in the same *milieu*, as we deduce from the presence on f. 145v of a tabular set-up of Theon's second solution, very much in the style of those presented in Planudes' commentary on Diophantus' *Arithmetica* I–II; it was obviously included in Maximus Planudes' lifetime, probably by Planudes himself.[5]

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[2] A well-known example of material originally belonging to the main text of Theon's commentary and transmitted as *marginalia* of the *Almagest* is that of book V, most of which has been found by A. Tihon (1987) in the margins of the fifth book of the *Almagest* in *Vat. gr.* 198. Cf. (Rome 1953).

[3] Besides the aforementioned Basel edition, Halma (1821) has also edited and translated into French the first two books. His edition was based on the fifteenth century manuscript *Par. gr.* 2398, a manuscript containing the text of the Byzantine recension (Halma 1821, viii).

[4] See (Mondrain 2005, 17). According to Mondrain (2007, 161) *Par. gr.* 2396 must have been written in the years 1292–1293.

[5] The abbreviated presentation of the solution is included in an empty place that was left in f. 145v. In the transcription below we are using the signs S and ⩚ that appear in the manuscript. The meaning of these signs will be explained at the beginning of section II.

Δ γ´ <S> $\overline{\alpha}$ $\overline{\theta}$ $\overline{\zeta}$

μ^{o} Δ γ´ ⩚ S $\overline{\alpha}$ μ^{o} $\overline{\zeta}$ ⩚ S $\overline{\alpha}$

μ^{o} $\overline{\rho\iota\zeta}^{\iota\varepsilon'}$ ⩚ S$^{o\tilde{\upsilon}}$ $\overline{\kappa\zeta}^{\iota\varepsilon'}$ μ^{o} $\overline{\rho\varepsilon}^{<\iota\varepsilon'>}$ ⩚ S$^{o\tilde{\upsilon}}$ $\overline{\iota\varepsilon}^{\iota\varepsilon'}$

μ^{o} $\overline{\rho\iota\zeta}^{\iota\varepsilon'}$ καὶ S$^{o\tilde{\upsilon}}$ $\overline{\iota\varepsilon}^{\iota\varepsilon'}$ γίν μ^{o} $\overline{\rho\varepsilon}^{\iota\varepsilon'}$ καὶ S$^{o\tilde{\upsilon}}$ $\overline{\kappa\zeta}^{\iota\varepsilon'}$

μ^{o} $\overline{\iota\beta}^{\iota\varepsilon'}$ ἴσαι S$^{o\tilde{\upsilon}}$ $\overline{\iota\beta}^{\iota\varepsilon'}$

καὶ γίνεται ὁ S$^{\grave{o}\varsigma}$ μ^{o} $\overline{\alpha}$

Besides *Vat. gr.* 1087, the fragment appears in some of the manuscripts of the Byzantine recension, in which the text of Theon's commentary is partially 'dismembered' and portions of it are relocated, formatted as scholia, in the margins of the text of the *Almagest* contained in the same manuscripts. Such manuscripts are *Vat. gr.* 198 and *Marc. gr.* 310.[6] The modalities of this 'dismemberment' are really amusing and deserve to be mentioned.

In *Vat. gr.* 198, for example, we see that in several places of the last books, just in-between Theon's words, is written a phrase referring the reader to certain scholia (ζήτει τὸ ἑξῆς ἐν τοῖς σχολίοις or ζήτει τὸ ἑξῆς ἐν τοῖς σχολίοις μέχρι τέλους). That is, the text is interrupted – only the first phrase of the text coming next is present – and the reader is referred for the rest to "the scholia". Each reference to "the scholia" is accompanied by a clause describing exactly where the corresponding missing fragment is located, and, furthermore, by a diacritical sign helping the reader to find it. So, by following the diacritical signs we find the corresponding missing passages from Theon's commentary to be relocated many folios earlier, in the margins of the *Almagest* included in the same codex. In the case of the fragment we publish here, the text of the commentary on book XIII.3 is interrupted in f. 484v, immediately after the phrase ἔστω δὴ πρότερον διὰ τῶν ἐκ τῶν γραμμικῶν ἐφόδων ἐπιλογισμῶν, and what we find next to it is the reference ζήτει τὸ ἑξῆς ἐν τοῖς σχολίοις, accompanied by the instruction ζήτει ἐν Ι$\Gamma^{ου}$ βιβλίου Γ´ κεφαλαίῳ ἐν ση … (the dots standing for the corresponding diacritical sign). The reference leads us back to f. 306r, where we find the same diacritical sign and, next to it, the fragment we are seeking.[7]

Besides the aforementioned manuscripts of the direct tradition of Theon's commentary, manuscripts belonging to the indirect tradition provide evidence for the fragment at issue as well. The most important of these manuscripts is *Vat. gr.* 1594, the magnificent codex of the late third quarter of the ninth century[8] containing one of the oldest copies of some of Ptolemy's works and probably copied in the same scriptorium in which, in the same period, were copied the codices that constituted the famous "collection

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[6] *Vat. gr.* 198 and *Marc. gr.* 310 belong to a group of manuscripts containing scholarly recensions of ancient texts of the quadrivial sciences, that were composed in Byzantium during the Palaiologan renaissance. *Vat. gr.* 198 is a true quadrivial codex, gathering in more than 500 folios a rich collection of treatises on astronomy, arithmetic and harmonic. Since the late 1980s the scribe who copied this codex has been identified with the *anonymus aristotelicus*, the erudite scribe who worked among others under the emperor John VI Cantacuzenus (1347–1354) and was recently identified with the monk Malachias (Mondrain 2005, 22–25). *Marc. gr.* 310, on the other hand, is a purely astronomical codex, carrying the text of the *Almagest* accompanied by Pappus's and Theon's commentaries on it. It was copied by Isaak Argyros, the leading Byzantine scholar in Ptolemaic astronomy in the 1360s and 1370s (Mondrain 2007, 165).

[7] The partial 'dismemberment' of Theon's commentary in the text contained in manuscripts of the Byzantine recension was already discussed by Heiberg (1907, xxiv) and Rome (1931, vi; 1953, 512).

[8] See (Follieri 1977, 145–146) and (Ronconi 2013).

philosophique”. The text of the *Almagest* in this manuscript is accompanied by a rich set of scholia, written either in the margins or between the columns, among which is found, in f. 248v, the fragment we publish here. *Vat. gr.* 180, dated in the second half of the tenth century (Orsini 2005, 317–322 and 340–342), is another indirect witness, providing for our fragment (f. 268r) the same text as *Vat. gr.* 1594. Furthermore, our fragment appears on ff. 75v–76r of *Vat. gr.* 184, a codex which carries a significant part of the scattered scholia on the *Almagest* of the codex *Vat. gr.* 1594, gathered onto ff. 25–80 under the title Θέωνος Ἀλεξανδρέως σχόλια πάνυ χρήσιμα εἰς τὴν Μεγάλην Σύνταξιν.

II. The text and its translation

For the edition of Theon’s fragment *Vat. gr.* 1087 (= V) has been used as main text and *Vat. gr.* 1594 (= B) to correct some of the readings of V. All corrections are shown in the notes. Since neither V nor B have any diagram accompanying the fragment, we reproduce the diagram of *Vat. gr.* 198. This diagram differs from the diagram of Heiberg’s edition of Ptolemy’s text (Heiberg 1903, 538) in that in the latter the line EO is not drawn. In the part of the text presenting the Diophantine solution of the problem at issue, the scribe uses repeatedly the signs Ƨ and ⋀. The same signs appear in our transcription too. The sign Ƨ stands for the word ἀριθμός, when used with the technical meaning of the name assigned to a number the finding of which the enunciation of the problem calls for. The sign ⋀ stands for λεῖψις, the word used for expressing when a term is ‘wanting’ (lacking).[9]

II.1 The text

ἔστω δὲ πρότερον διὰ τῶν ἐκ τῶν γραμμικῶν ἐφόδων ἐπιλογισμῶν τὰς εἰρημένας πηλικότητας ἀποδεῖξαι συναγομένας. ἐπεὶ οὖν ἐκ τῶν τηρήσεων κατείληπται ἡ μὲν ὑπὸ ΑΕΚ μ^{o} $\overline{\delta}$ γ´ οἵων εἰσὶν αἱ $\overline{\delta}$ ὀρθαὶ $\overline{\tau\xi}$, ἡ δὲ ὑπὸ ΒΕΞ τῶν αὐτῶν $\overline{\zeta}$, μείζων ἄρα ἡ ὑπὸ ΒΕΞ τῆς ὑπὸ ΑΕΚ. οὐκ ἄρα ἐπ’ εὐθείας ἐστὶν ἡ ΕΞ τῇ ΕΚ. διήχθω οὖν ἐπ᾽ εὐθείας τῇ ΕΞ ἡ ΕΟ.[10] ἡ ἄρα ὑπὸ ΒΕΞ μείζων ἐστὶ τῆς ὑπὸ ΑΕΚ τῇ ὑπὸ ΚΕΟ[11]. οἵων ἄρα ἡ ὑπὸ ΚΕΑ $\overline{\delta}$ γ´, τοιούτων ἡ ὑπὸ ΚΕΟ[12] δύο διμοίρου· τοσαύτη γὰρ ἡ ἐκ τῶν τηρήσεων ὑπεροχή.

[9] In the preface to the *Arithmetica* Diophantus refers explicitly to the use of signs (σημεῖα) for representing the terms ἀριθμός and λεῖψις. See (Tannery 1893–95, i, 6.3–5, 12.19–21).

[10] ΕΟ] ΕΘ V.

[11] ΚΕΟ] ΚΕΘ V.

[12] ΚΕΟ] ΚΕΘ V.

πάλιν, ἐπεὶ[13] τῆς ὑπὸ ΓΕΚ πρὸς τὴν ὑπὸ ΔΕΞ λόγος ἐστὶ τῶν ε̅ πρὸς θ̅, ὡς ἐκ τοῦ τῆς ἀνωμαλίας κανόνος γέγονε δῆλον, ἴση δὲ ἡ ὑπὸ ΔΕΞ τῇ ὑπὸ ΟΕΓ, λόγος ἄρα τῆς ὑπὸ ΓΕΚ πρὸς τὴν ὑπὸ ΓΕΟ ὁ τῶν ε̅ πρὸς[14] θ̅[15]· καὶ ἀνάπαλιν· καὶ διελόντι[16] λόγος τῆς ὑπὸ ΟΕΚ πρὸς τὴν ὑπὸ ΚΕΓ ὁ τῶν δ̅ πρὸς ε̅· ὥστε καὶ οἵων ἡ ὑπὸ ΚΕΓ ε̅ τοιούτων ἡ ὑπὸ ΚΕΟ δ̅. ἐπεὶ οὖν οἵων ἡ ὑπὸ ΓΕΟ θ̅ τοιούτων ἡ μὲν ὑπὸ ΓΕΚ ε̅, ἡ δὲ ὑπὸ ΚΕΟ δ̅, καὶ οἵων ἄρα ἡ ὑπὸ ΚΕΟ τῆς ἐκ τῶν τηρήσεων ὑπεροχῆς μ^{o} δύο διμοίρου· ἡ δὲ ὑπὸ ΚΕΑ[17] ἐκ τῆς τηρήσεως δ̅ γʹ, τοιούτων καὶ ἡ μὲν ὑπὸ ΓΕΚ γ̅ γʹ· ἡ δὲ ὑπὸ ΓΕΟ τουτέστιν ἡ ὑπὸ ΔΕΞ μ^{o} ϛ̅· δέδονται ἄρα αἱ ὑπὸ ΓΕΚ ΔΕΞ γωνίαι τῷ μεγέθει ὧν ὁ λόγος ἦν δεδομένος. ἢ καὶ οὕτως· ἐπεὶ ἡ ὑπὸ ΚΕΟ, τῆς ἐκ τῶν τηρήσεων ὑπεροχῆς τυγχάνουσα, μ^{o} ἐστὶ δύο διμοίρου, ἀλλὰ καὶ τῆς τοῦ λόγου πηλικότητος ὑπεροχὴ τυγχάνουσα,[18] μονάδων ἐστὶ δ̅· καὶ ἔστιν οἵων ἡ ὑπὸ ΟΕΚ δ̅, τοιούτων ἡ ὑπὸ ΚΕΓ ε̅· καὶ οἵων ἄρα ἡ ὑπὸ ΟΕΚ β̅ διμοίρου, τοιούτων ἡ ὑπὸ ΚΕΓ γ̅ γʹ· καὶ ἐπεί ἐστιν[19] ὡς τὰ δ̅ πρὸς ε̅ οὕτως τὰ β̅ δίμοιρα πρὸς γ̅ γʹ, καὶ ἐναλλὰξ καὶ ἀνάπαλιν, ὃ μέρος ἐστὶ τὰ δύο δίμοιρα τῆς τῶν πηλικοτήτων ὑπεροχῆς, τῶν δ̅ τῆς τοῦ λόγου ὑπεροχῆς, τοσούτων ἔσται καὶ τῶν ε̅ τὰ τρία γʹ. ὡσαύτως δὲ ὃ μέρος ἐστὶ τὰ δύο δίμοιρα τῶν δ̅, τὸ αὐτὸ ἔσται καὶ τὰ ϛ̅ τῶν θ̅.

διό φησι· ἐὰν ὅσον μέρος ἐστὶν ἡ ὑπεροχὴ τῶν ὅλων πηλικοτήτων τῆς ὑπεροχῆς τῶν λόγων τὸ τοσοῦτον μέρος ἑκάστου τῶν λόγων λάβωμεν, ἕξομεν τὴν ὑπὸ τὸν οἰκεῖον λόγον πηλικότητα.

καὶ ἐπεὶ συνάγεται ἐκ τῶν εἰρημένων τοιαύτη τις πρότασις· ὅτι ἐὰν ὦσι δύο ἀριθμοὶ δεδομένοι καὶ ἀπ᾽ αὐτῶν ἀφαιρεθῶσι τινὲς ἴσοι καὶ τῶν καταλειπομένων ὁ λόγος δοθῇ, δοθήσονται καὶ αὐτοὶ ὧν ὁ λόγος δέδοται· καὶ λοιποὶ δηλαδὴ οἱ ἴσοι. ἔστω ἐπὶ τῶν προκειμένων ἀριθμῶν, τοῦ τε δ̅ γʹ καὶ τοῦ ζ̅ καὶ τοῦ τῶν ε̅ πρὸς τὰ θ̅ λόγου, τὸ τοιοῦτον ἐφοδεῦσαι διὰ τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς.[20]

ἔστω ὁ ἀφαιρούμενος ἀφ᾽ ἑκατέρου τοῦ τε δ̅ γʹ καὶ τοῦ ζ̅ ƨ α̅. καὶ ἐὰν μὲν ἀπὸ μ^{o} δ̅ γʹ ἀφαιρεθῇ, λοιπαὶ μ^{o} δ̅ γʹ λείπουσαι ƨ α̅, ἐὰν δὲ ἀπὸ μ^{o} ζ̅, λοιπαὶ μ^{o} ζ̅ λείπουσαι[21] ƨ α̅. δεήσει ἄρα μ^{o} δ̅ γʹ λείπουσαι ƨ α̅[22] πρὸς μ^{o} ζ̅ λειπούσας ƨ α̅ λόγον ἔχειν ὃν ε̅ πρὸς θ̅. ἀλλὰ τὰ ε̅ τῶν θ̅ λείπουσι τέτρασιν ἑαυτῶν πέμπτοις. ὥστε[23] καὶ μ^{o} δ̅ γʹ λειπούσαις ƨ α̅ λείπουσι μ^{o} ζ̅ ⋀ ƨ α̅ τέτρασιν ἑαυτῶν πέμπτοις. ἐὰν ἄρα μ^{o} δ̅ γʹ ⋀ ƨ α̅ προσθῶμεν δ̅ αὐτῶν πέμπτα ἔσονται ἴσαι μ^{o} ζ̅ ⋀ ƨ α̅. ἀλλὰ μ^{o} δ̅ γʹ ⋀ ƨ α̅ προσλαβοῦσαι τὰ δ̅ ἑαυτῶν

[13] ἐπεὶ] ἐπὶ V.

[14] πρὸς] bis V.

[15] θ̅] ante θ add. τὸν V.

[16] διελόντι] δῆλον ὅτι V.

[17] ΚΕΑ] ΚΕΟ V.

[18] ὑπεροχὴ τυγχάνουσα] ὑπεροχῆς τῶν δ̅ τῆς τοῦ λόγου ὑπεροχῆς V.

[19] ἐστιν] ἐ V.

[20] τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς] τῆς τῶν διοφαντίων ἀριθμητικῆς ἀγωγῆς V.

[21] λείπουσαι] scripsimus: λειπούσας V: ⋀ B.

[22] δεήσει ἄρα μ^{o} δ̅ γʹ λείπουσαι ƨ α̅] om. V | λείπουσαι] scripsimus: ⋀ B.

[23] ὥστε] ὧντε V.

πέμπτα γίνεται $\overline{\text{ριζ}}$ πεντεκαιδέκατα μονάδος λείπουσαι $\overline{\text{θ}}$ πέμπτα ἀριθμοῦ τουτέστιν $\overline{\text{κζ}}$ πεντεκαιδέκατα ὡς ἑξῆς δείξομεν.

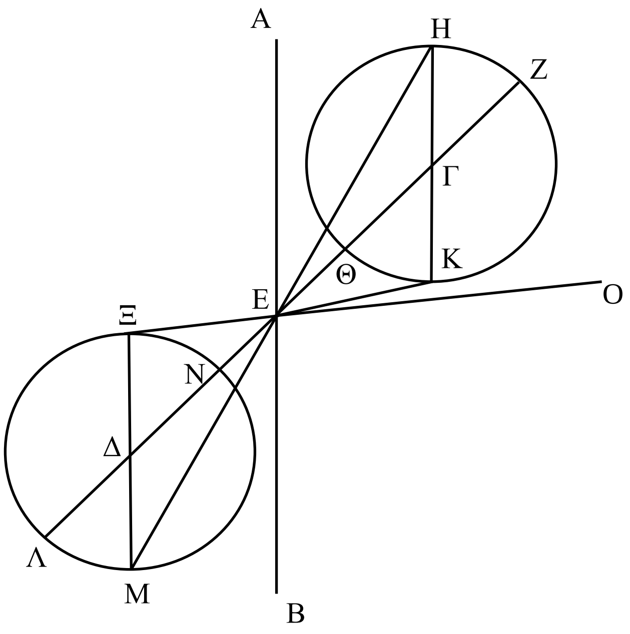
μονάδος ἄρα ριζ$^{\text{ιε}´}$ ⋀ κζ$^{\text{ιε}´}$ ἀριθμοῦ ἴσα[24] ἐστὶ μ^{o} $\overline{\text{ζ}}$ ⋀ ϛ $\overline{\text{α}}$ τουτέστι $\overline{\text{ρε}}$ πεντεκαιδεκάτοις μονάδος[25] ⋀ $\overline{\text{ιε}}$ πεντεκαιδεκάτοις ἀριθμοῦ.

κοινὴ προσκείσθω ἡ ⋀ τῶν μβ$^{\text{ιε}´}$ τοῦ ἀριθμοῦ· μονάδος ἄρα ριζ$^{\text{ιε}´}$ καὶ ἀριθμοῦ ιε$^{\text{ιε}´}$ ἴσα ἐστὶ μονάδος ρε$^{\text{ιε}´}$ καὶ ἀριθμοῦ κζ$^{\text{ιε}´}$. ἀπὸ ὁμοίων ὅμοια. λοιπὸν ἄρα μονάδος ιβ$^{\text{ιε}´}$ ἴσα ἐστὶν ἀριθμοῦ ιβ$^{\text{ιε}´}$. καὶ πάντα πεντεκαιδεκάκις. ὁ ἄρα ἀριθμὸς ἔσται μ^{o} μιᾶς.

ἐπὶ τὰς ὑποστάσεις. ἔταξα τὸν ἕνα τῶν τὸν δεδομένον λόγον ἐχόντων μ^{o} $\overline{\text{δ}}$ γ´ ⋀ ϛ $\overline{\text{α}}$. ἔσται μ^{o} $\overline{\text{γ}}$ γ´. τὸν δὲ λοιπὸν μ^{o} $\overline{\text{ζ}}$ ⋀ ϛ $\overline{\text{α}}$. ἔσται μ^{o} $\overline{\text{ς}}$. καὶ λοιπὸς δηλονότι ἑκάτερος τῶν ἴσων ὁ ἀφαιρούμενος ἀφ᾽ ἑκατέρου ἔσται τῆς τοῦ ἀριθμοῦ μ^{o} $\overline{\text{α}}$.

ὅτι δὲ μ^{o} $\overline{\text{δ}}$ γ´ ⋀ ϛ $\overline{\text{α}}$ προσλαβοῦσαι ἑαυτῶν τὰ τέσσαρα πέμπτα γίνεται ριζ$^{\text{ιε}´}$ μονάδος λείπουσαι[26] κζ$^{\text{ιε}´}$ ἀριθμοῦ οὕτω γίνεται δῆλον.

ἐπεὶ γὰρ τῶν $\overline{\text{δ}}$ γ´ τὸ ε´ γίνεται ἑξηκοστῶν $\overline{\text{νβ}}$, τὰ ἄρα $\overline{\text{δ}}$[27]ε´ ἔσται $\overline{\text{ση}}$ ἑξηκοστῶν τουτέστι νβ$^{\text{ιε}´}$. εἰσὶ δὲ καὶ αἱ μονάδες $\overline{\text{δ}}$ γ´ ξε$^{\text{ιε}´}$. αἱ ἄρα $\overline{\text{δ}}$ γ´ προσλαβοῦσαι ἑαυτῶν τὰ $\overline{\text{δ}}$ ε´ συνάγουσιν ριζ$^{\text{ιε}´}$. ἔστι δὲ καὶ ἡ ⋀ τοῦ ἀριθμοῦ μετὰ τῶν $\overline{\text{δ}}$ ε´ ἑαυτοῦ ποιῶν λεῖψιν $\overline{\text{θ}}$ πέμπτα ἀριθμοῦ· τουτέστιν κζ$^{\text{ιε}´}$. ὥστε μ^{o} $\overline{\text{δ}}$ γ´ ⋀ ϛ $\overline{\text{α}}$ προσλαβοῦσαι[28] ἑαυτῶν $\overline{\text{δ}}$ ε´ γίνονται ριζ$^{\text{ιε}´}$ μονάδος λείπουσαι κζ$^{\text{ιε}´}$ ἀριθμοῦ.[29]



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[24] ἴσα] ὅσα V.

[25] μονάδος] scripsimus: μονάδες V: μονα$^{\Delta}$ B.

[26] λείπουσαι] scripsimus: καὶ ἔτι V: ⋀ B.

[27] $\overline{\text{δ}}$] om. V.

[28] προσλαβοῦσαι] om. V.

[29] ἀριθμοῦ] -οῖς V: ϛ$^{\text{οῦ}}$ B.

II.2 The translation

So, first let it be to demonstrate obtaining the stated values with the calculations from the geometrical methods.

For, since from the observations angle[30] AEK was determined to be 4 ⅓ degrees, of such <degrees> that the 4 right angles are 360, while angle BEΞ is 7 of the same, therefore angle BEΞ is greater than angle AEK. Hence, EΞ is not in a straight [line] with EK. So, let EO be extended in a straight [line] with EΞ. Therefore, angle BEΞ is greater than angle AEK by angle KEO. Accordingly, of such <degrees> that angle KEA is 4 ⅓, angle KEO is 2 ⅔ of the same. For thus much was the excess from the observations.

Again, since the ratio of angle ΓEK to angle ΔEΞ is 5 to 9 – as has been clear from the table of anomaly – and angle ΔEΞ is equal to angle ΓEO, then, the ratio of angle ΓEK to angle ΓEO is 5 to 9. And *invertendo*; and *separando*; the ratio of angle OEK to angle KEΓ is 4 to 5. And so, of such <parts> that angle KEΓ is 5, angle KEO is 4 of the same. So, since of such <parts> that angle ΓEO is 9, angle ΓEK is 5 of the same, while angle KEO is 4, therefore, of such <degrees> that angle KEO, the excess from the observations, is 2 ⅔, while from the observation angle KEA is 4 ⅓, angle ΓEK is 3 ⅓ of the same, while angle ΓEO, that is angle ΔEΞ, is 6 degrees. Therefore, the angles ΓEK, ΔEΞ, the ratio of which was given, are given in magnitude.

Or in this manner: Since angle KEO, being, on the one hand, the excess from the observations, is 2 ⅔ degrees, and being, on the other hand, the excess between the values <of the terms> of the ratio, is 4 units, and <since> of such <parts> that angle OEK is 4, angle KEΓ is 5 of the same, therefore, of such <degrees> that angle OEK is 2 ⅔, angle KEΓ is 3 ⅓ of the same. And since it is as the 4 to the 5 so the 2 ⅔ to the 3 ⅓, and *alternando* and *invertendo*, whatever part of the 4, of the excess <between the terms> of the ratio, is the 2 ⅔, of the excess of the values, the same part will also be the 3 ⅓ of the 5. In like manner, whatever part of the 4 is the 2 ⅔, the same <part> will be the 6 of the 9.

For this reason he says: if, as much the excess of the whole values is of the excess of the ratios, that much we take of each of the ratios, we shall have the value connected with the corresponding ratio.

And since from the aforesaid a proposition like the following is deduced: if two numbers are given, and some equal <numbers> are removed from them, and the ratio of the remainders is given, those, the ratio of which was given, will also be given, and the others, that is, the equal ones, <will be given too>, <so> let in the case of the proposed numbers, the 4 ⅓ and the 7, and the ratio 5 to 9, work out this methodically by the process of the Diophantine numbers.

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[30] The text has "the by AEK", meaning, "the <angle contained> by AE, EK". This is the standard expression for angles in Greek geometry. For clarity's sake we have adopted in our translation the familiar formula "the angle AEK".

Let the subtracted from each of the two, the 4 ⅓ and the 7, <be set> 1 *arithmos*. Now, if subtracted from 4 ⅓ units the remaining will be 4 ⅓ units wanting 1 *arithmos*. And if from 7 units the remaining will be 7 units wanting 1 *arithmos*. Therefore, 4 ⅓ units wanting 1 *arithmos* must have to 7 units wanting 1 *arithmos* the ratio that 5 has to 9. But the 5 fall short of the 9 by 4 fifths of themselves. Therefore 4 ⅓ units wanting 1 *arithmos*, likewise, fall short of 7 units wanting 1 *arithmos* by four fifths of themselves. If therefore we add to 4 ⅓ units wanting 1 *arithmos* 4 fifths of themselves they will be equal to 7 units wanting 1 *arithmos*. But 4 ⅓ units wanting 1 *arithmos* when receiving the 4 fifths of themselves become 117 fifteenths of unit wanting 9 fifths of *arithmos*, that is to say 27 fifteenths, as we will show later.

Therefore, 117 15^{ths} of unit wanting 27 15^{ths} of *arithmos* are equal to 7 units wanting 1 *arithmos*; that is to say 105 fifteenths of unit wanting 15 fifteenths of *arithmos*.

Let the wanting 42 15^{ths} of *arithmos* be added in common: consequently, 117 15^{ths} of unit and 15 15^{ths} of *arithmos* are equal to 105 15^{ths} of unit and 27 15^{ths} of *arithmos*. And <we remove> like from like; it remains therefore that 12 15^{ths} of unit are equal to 12 15^{ths} of *arithmos*. And all fifteen times. Therefore, the *arithmos* will be 1 unit.

To the numerical values: I have set the one of those having the given ratio 4 ⅓ units wanting 1 *arithmos*; it will be 3 ⅓ units. And the other, 7 units wanting 1 *arithmos*; it will be 6 units. And the rest, that is to say each of the equal ones, which is the subtracted from each <of the two>, will be the 1 unit of the *arithmos*.

Now, the fact that 4 ⅓ units wanting 1 *arithmos* receiving the four fifths of themselves become 117 15^{ths} of unit wanting 27 fifteenths of *arithmos*, is made manifest in this manner: For, since the 5^{th} of the 4 ⅓ becomes 52 sixtieths, then the 4 5^{ths} will be 208 sixtieths, that is 52 15^{ths}; the 4 ⅓ units, on the other hand, are 65 15^{ths}. Therefore, the 4 ⅓ <units>, when receiving the 4 5^{ths} of themselves, sum up 117 15^{ths}. There is also the wanting <part> of the *arithmos* together with the 4 5^{ths} of itself, which makes 9 fifths of *arithmos* wanting, that is to say, 27 15^{ths} <of *arithmos*>. Therefore, 4 ⅓ units wanting 1 *arithmos*, when receiving 4 5^{ths} of themselves, become 117 15^{ths} of unit falling short by 27 15^{ths} of *arithmos*.

III. Comments and remarks

III.1 Comments on the content

In the above published text Theon solves the mensuration problem that Ptolemy discusses in the third chapter of Book XIII of the *Almagest*, bearing on the latitudinal motions of planets. The fragment we published belongs to the commentary on section 538.17–540.18 of Ptolemy's text (in Heiberg's edition), treating specifically the latitudinal motion of Mars.

For his commentary Theon uses Ptolemy’s diagram without entering into its setting-out, already done by Ptolemy (537.15–538.16). In our reproduction of the diagram we have added the line EO on the authority of Theon’s text and of the diagram of the Byzantine recension (*Vat. gr.* 198, f. 306r).[31] So, let the plane of the diagram represent a plane orthogonal to the plane of the ecliptic, let AB be the intersection of this plane with the plane of the ecliptic, and ΓΔ be the intersection of the same plane with the plane of the deferent circle. Let E be the centre of the ecliptic, where the observer is, and Γ and Δ the apogee and the perigee, respectively, of the deferent. Let on the orthogonal plane, about Γ and Δ, the equal circles ZHΘK and ΛMNΞ be drawn, representing the circles through the poles of the epicycles. On these circles let the planes of the epicycles be drawn on lines HΓK and MΔΞ respectively. Finally, let the straight lines EH and EM, which join E with the apogees of the epicycles, and the straight lines EK and EΞ, which join E with the perigees of the epicycles, be drawn.

For the study of the latitudinal motion of Mars one has to find the inclination with respect to the ecliptic (called ἔγκλισις) of various circles. More specifically, one has to determine the equal angles AEΓ and BEΔ, which describe the inclination of the deferent, and the angles HΓZ and MΔΛ (represented by the arcs ΘK and NΞ), which describe the inclination of the epicycle. Taking into account the observational data, the problem of finding the inclination of the deferent is reduced, by a simple geometrical argument, to the problem of finding the two angles ΓEK and ΔEΞ. Thus, from a purely geometrical viewpoint the problem that Ptolemy sets for solution with regard to the determination of the inclination of the deferent is to find the values of the angles ΓEK and ΔEΞ, when we know:

(1) their ratio (namely ∡ ΓEK : ∡ ΔEΞ :: 5 : 9);
(2) that ∡ ΓEK = ∡ AEK – ∡ AEΓ and ∡ ΔEΞ = ∡ BEΞ – ∡ BEΔ, where ∡ AEΓ = ∡ BEΔ;
(3) the values of ∡ AEK and ∡ BEΞ (namely ∡ AEK = 4 ⅓° and ∡ BEΞ = 7°).

Ptolemy solves this problem, and refers to an “arithmetical lemma” by means of which the solution can be “demonstrated” (δείκνυται): “If, as much the excess of the whole values [i.e. the values of ∡ AEK and ∡ BEΞ] is of the excess of the ratios [i.e. 5 and 9] that much we take of each of the ratios, we shall have the value connected with the corresponding ratio. This can be demonstrated by means of an arithmetical lemma” (540.3–7).[32] So, since 7 – 4 ⅓ = 2 ⅔, and 9 – 5 = 4, and since 2 ⅔ is two-thirds of 4, if we

[31] The role of the line EO is to show that the three points Ξ, E, K are not lying on the same line. The diagram follows closely Theon’s text, in which we read the phrase διήχθω οὖν ἐπ᾽ εὐθείας τῇ EΞ ἡ EO (see lines 4–5 of the text).

[32] In his English translation of the *Almagest* G. J. Toomer discusses this passage and suggests a reconstruction for the lemma and its proof: “Given two magnitudes *A* and *B*, and the ratio *l* : *m* of two other magnitudes, *C*, *D* such that $A = x + C$, $B = x + D$, the lemma states that

$$C = l \times \frac{B-A}{m-l},\ D = m \times \frac{B-A}{m-l}.$$

take the two-thirds of 5 and 9 we will get, respectively, the values of the sought-after angles, namely ∡ ΓΕΚ = 3 ⅓° and ∡ ΔΕΞ = 6°. This is the solution, and through the values thus found Ptolemy can determine the inclination sought-for (ie. the angles ΑΕΓ and ΒΕΔ), which is 1°.

Commenting upon this passage Theon proposes two ways by which the algorithm stated by Ptolemy could be "demonstrated"; in fact, he proposes two solutions to the problem. The first solution is described as a solution by ἐπιλογισμοὶ ἐκ τῶν γραμμικῶν ἐφόδων (calculations from the geometrical methods). The description is concise, yet the meaning is clear: ἐπιλογισμὸς ἐκ τῶν γραμμικῶν ἐφόδων is a method for solving mensuration problems, in which one makes the appropriate calculations by following closely – one might say, exaggerating a bit, step by step – a geometrical working out, be it a demonstration, a mere argument, or an ἔφοδος in the broad sense of the term. Concerning the second solution, it is described by Theon as a solution "by the process of the Diophantine numbers";[33] this is nothing but an algebraic solution.[34]

a) *The solution by "ἐπιλογισμὸς ἐκ τῶν γραμμικῶν ἐφόδων"*

Since ∡ ΑΕΚ ≠ ∡ ΒΕΞ, and ΑΕ, ΕΒ are in the same line, therefore ΕΞ and ΕΚ are not in the same line. Let ΕΞ be produced to ΕΟ. So, ∡ ΒΕΞ = ∡ ΑΕΚ + ∡ ΚΕΟ. (Indeed, ∡ ΒΕΞ = ∡ ΑΕΟ and ∡ ΑΕΟ = ∡ ΑΕΚ + ∡ ΚΕΟ.)

Now, since ∡ ΒΕΞ = 7° and ∡ ΑΕΚ = 4 ⅓°, by subtraction ∡ ΚΕΟ = 2 ⅔°.

From the data we have ∡ ΓΕΚ : ∡ ΔΕΞ :: 5 : 9, while ∡ ΔΕΞ = ∡ ΓΕΟ. Therefore ∡ ΓΕΚ : ∡ ΓΕΟ :: 5 : 9. *Invertendo* we will have ∡ ΓΕΟ : ∡ ΓΕΚ :: 9 : 5. And s*eparando*, (∡ ΓΕΟ – ∡ ΓΕΚ) : ∡ ΓΕΚ :: (9 – 5) : 5, i.e. ∡ ΟΕΚ : ∡ ΓΕΚ :: 4 : 5; therefore, the angle ΓΕΚ is 3 ⅓°, while the angle ΓΕΟ, and accordingly, the ΔΕΞ, is 6°.

In the last part of this first solution Theon presents a verification that the values found for the sought-after angles do satisfy the conditions of the lemma stated by Ptolemy: since the difference between the whole angles, that is the angle ΟΕΚ, is 2 ⅔°, the value 3 ⅓° of the angle ΓΕΚ does satisfy the proportion ∡ ΟΕΚ : ∡ ΓΕΚ :: 4 : 5. On the other hand, since 4 : 5 :: 2 ⅔ : 3 ⅓, after *alternando* and *invertendo* we find 2 ⅔ : 4 :: 3 ⅓ : 5, or 2 ⅔ :

Proof: Since $\frac{D}{C} = \frac{m}{l}$, $\frac{D-C}{C} = \frac{m-l}{l}$. But $D - C = B - A$. Therefore $C = l \times \frac{B-A}{m-l}$, $D = C \times \frac{m}{l} = m \times \frac{B-A}{m-l}$. (Toomer 1998, 604 n. 26)

[33] In contrast with *Vat. gr.* 1594, *Vat. gr.* 180, *Vat. gr.* 184, and the manuscripts carrying the Byzantine recension (*Vat. gr.* 198 and *Marc. gr.* 310), which have the reading διὰ τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς, *Vat. gr.* 1087 has instead the reading διὰ τῆς τῶν διοφαντίων ἀριθμητικῆς ἀγωγῆς (see line 29 of the text). This reading of *Vat. gr.* 1087 does not make sense. The reading ἀριθμητικῆς ἀγωγῆς appears also in the marginal annotation on f. 145v, accompanying the tabular set-up of the solution. But this time the words τῶν διοφαντίων preceding it have been replaced by the words τοῦ Διοφάντου.

[34] See below, footnote 36.

4 :: 3 ⅓ : 5 :: (2 ⅔ + 3 ⅓) : 9. Thus, 2 ⅔ : 4 :: 6 : 9, and therefore the value of the angle ΔΕΞ, which has been found 6°, is the same part of 9 as the value 2 ⅔ (of the angle ΟΕΚ) is of the number 4 (the difference of the terms of the ratio 5 : 9).

As said, the method by which this solution is conducted is an ἐπιλογισμὸς ἐκ τῶν γραμμικῶν ἐφόδων, that is, a sequence of arithmetical calculations modeled on a geometrical reasoning. This method was amply used in Antiquity in solving mensuration problems, as witnessed in the works of Heron of Alexandria, Ptolemy, and Theon himself. There are several applications of the method in those works, which, despite the stylistic variations, are similar to the one we see in the present text. A very lucid description of this method is made in the following excerpt from Theon's commentary on the sixth book of the *Almagest*: "And it is clear that if we prefer to obtain them (*i.e.* the 'directions', προσνεύσεις) accurately, we calculate them *by following faithfully* the approaches set forth previously, by the geometrical demonstrations".[35] This description makes clear something which, in our text, is only implied by the genitive ἐκ τῶν γραμμικῶν ἐφόδων (from the geometrical methods), namely the verbal expression κατακολουθοῦντες (following faithfully, following step by step). An ἐπιλογισμὸς, in this context, is a sequence of calculations which κατακολουθεῖ the development of a geometrical reasoning (be it a proof in the strict sense of the term, or a mere argument, a working out, or an approach).

b) *The solution by the "process of the Diophantine numbers"*

The "process of the Diophantine numbers" is, of course, the method that Diophantus teaches and practices in his *Arithmetica*. It is a method of problem solving that entails (a) naming the unknown(s), the finding of which the enunciation of the problem stipulates, (b) working through the operations stated in the enunciation, (c_1) setting up an equation as the outcome of the two aforesaid processes, (c_2) manipulating and solving of the equation, and, finally, (d) answering the problem by means of the solution to the equation. Since the time of medieval Islam, this method of problem solving is called algebraic.[36] We will see now that the same method is used by Theon in the second solution to the problem we are discussing.

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[35] δῆλον δὲ ὅτι κἂν ἀκριβῶς αὐτὰς προαιρώμεθα λαμβάνειν, ἐπιλογιούμεθα αὐτὰς κατακολουθοῦντες ταῖς διὰ τῶν γραμμικῶν δείξεων προεκτεθειμέναις ἡμῖν ἐφόδοις. See *Laur. Plut.* 28.18 (f. 258v).

[36] It should be stressed at this point that a clear distinction should be made between premodern and modern (post-Vietan) algebra. The word "algebra" when used in contexts like the one we discuss here has always to be understood with its premodern meaning, that is, as a method of problem solving, a method, however, which follows the above structure, and is paired with a conceptual background with regard to the key notions of polynomial and equation which differ profoundly from ours. For a recent discussion of all these matters see (Christianidis and Oaks, 2013).

For the sake of simplicity, instead of using three letters for representing the angles we will use in our discussion the one letter representation. Thus, in the followings, the letters α and β stand respectively for the sought-after angles ΓΕΚ and ΔΕΞ, and the letter φ for the also unknown equal angles ΑΕΓ and ΒΕΔ. In addition, the following notation will be used[37]: x will be used to represent the word ἀριθμός when it appears in the text with the technical meaning of a name assigned to an unnamed sought-after term; the double arrow "⇉" will be used to indicate a prescription stated in the enunciation; and the single arrow "→" to indicate the outcome of an operation. Finally, the sign ":=" will be used to indicate the action of assigning a name to an unnamed term, while we reserve the sign "=" only for stating equations.

Now, with the agreed conventions for the symbolism, the enunciation of the problem can be stated as follows: To find three angles α, β, and φ, such that, $\alpha : \beta \rightrightarrows 5 : 9$, $\alpha \rightrightarrows 4\ ⅓ - \varphi$, $\beta \rightrightarrows 7 - \varphi$. But this problem is similar to Diophantus' problem I.9, the enunciation of which asks "From two given numbers to subtract the same number so as to make the remainders have to one another a given ratio" (Tannery 1893–95, i, 26.13–15). The difference between the two problems, besides the numerical values of the data, is the context within which they are formulated. The Diophantine problem is arithmetical, the Theonine is an astronomically motivated problem of mensuration. But the difference in the context does not prevent Theon from recognizing that the same method of solution can be applied in both cases. This is not at all unexpected since the scope of algebra is by no means restricted to 'pure' arithmetical problems. Medieval algebraists were accustomed in solving problems of mensuration by algebra. This is the case with the present Theonine solution.

The solution is summarized in the following tables:

1. Set up of the equation

To find three angles α, β, and φ, such that, $\alpha : \beta \rightrightarrows 5 : 9$, $\alpha \rightrightarrows 4\ ⅓ - \varphi$, $\beta \rightrightarrows 7 - \varphi$.		
Assignment of names	Operations with named terms	Equation
$\varphi := 1x$		
	$4\ ⅓ — 1x \rightarrow 4\ ⅓ - 1x$[38]	
	$7 — 1x \rightarrow 7 - 1x$	

[37] This notation was proposed in (Bernard and Christianidis, 2012; Christianidis and Oaks, 2013).

[38] The left part in this expression indicates the operation of subtraction announced with the verb ἀφαιρεθῇ in line 31 of the text. The sign "—" (elongated –) in the left part indicates the subtraction which *is to be performed*. The short "–" in the right part does not indicate subtraction; it stands for the term λείπουσαι (wanting) appearing in the same line 31, and it is used to link the 'present' term 4 ⅓ and the 'lacking' term $1x$ in the expression $4\ ⅓ - 1x$, which describes the *result* of the subtraction. For more on this subtle difference see (Christianidis and Oaks, 2013).

		$(4\ ⅓ – 1x) + \frac{4}{5} \times (4\ ⅓ – 1x) = 7 – 1x$
	$(4\ ⅓ – 1x) + \frac{4}{5} \times (4\ ⅓ – 1x) \rightarrow \frac{117}{15} - \frac{9}{5}x$ [The working-out of this complex operation is postponed for the end]	
	$\frac{9}{5} \rightarrow \frac{27}{15}$	
		$\frac{117}{15} - \frac{27}{15}x = 7 – 1x$

2. Manipulation of the equation

Initial equation	Simplification	Simplified equation and solution
$\frac{117}{15} - \frac{27}{15}x = 7 – 1x$		
	add the wanting in common	$\frac{117}{15} = \frac{105}{15} + \frac{12}{15}x$
	remove like from like	$\frac{12}{15} = \frac{12}{15}x$
	all 15 times	<12 = 12x>
		$x = 1$

3. Answer to the problem and proof

α was set as 4 ⅓ – 1x, therefore its value is 3 ⅓; β was set as 7 – 1x, therefore its value is 6. And it is manifest that 1, i.e. the difference between 4 ⅓ and 3 ⅓, as well as the difference between 7 and 6, is the value of φ.

4. The last part of the text explains in every detail how the operations (multiplication and addition) involved in the expression (4 ⅓ – 1x) + $^4/_5$ × (4 ⅓ – 1x) are to be conducted so as to get $^{117}/_{15} - {}^9/_5x$. This part is carried out entirely within what Diophantus calls, in the introduction to the *Arithmetica*, “the arithmetical theory”. The calculations involved have nothing to do with the prescriptions stated in the enunciation of the problem; they are pure algebraic calculations.

III.2 Historiographical comments

In the preceding pages we published a fragment from Theon’s commentary on chapter XIII.3 of the *Almagest*, in which Ptolemy discusses a mensuration problem emanating from the study of the latitudinal motion of Mars, and we discussed the two solutions that Theon provides to it. Our main testimonies for the fragment are, first and foremost, the codex *Vat. gr.* 1087, which contains the running text of Theon’s thirteenth book, and the marginal note in the *Almagest* found in *Vat. gr.* 1594, among other manuscripts. That Theon is the author of the fragment cannot be disputed, taking into account its inclusion in the main text of Theon's commentary in *Vat. gr.* 1087, a manuscript which, incidentally, does not contain the *Almagest*.

What is particularly interesting in Theon's text, and deserves to be commented upon, is the phrase διὰ τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς by which Theon refers to the second solution. What might the expression "Diophantine numbers" mean? And what might designating something as the "process of the Diophantine numbers" as distinguished from other problem-solving techniques signify? We will conclude this article with some thoughts on these questions.

It is well known that in the introduction to the *Arithmetica* Diophantus makes a clear distinction between two series of terms, both referring to numbers. The first comprises words of everyday language, namely the words 'square', 'cube', and simple 'number'. These words function in Diophantus' text as common nouns, and they are used in the enunciations of the problems. As he himself says, it is "from the addition, subtraction or multiplication of *these numbers* [that is, the numbers considered from the point of view of their qualification as 'squares', 'cubes' or simply 'numbers'] or from the ratios which they bear to one another or to their own sides respectively that most arithmetical problems are formed" (Tannery 1893–95, i, 4.7–10, our emphasis). Obviously, the Theonine expression "Diophantine numbers" cannot refer to the numbers of this series. There is nothing specifically "Diophantine" in them. The other series is composed of the numbers of the "arithmetical theory" (ἀριθμητικὴ θεωρία), i.e. the unknown, its powers, and its reciprocal powers. These numbers have specific technical designations (ἀριθμός, δύναμις, κύβος, δυναμοδύναμις …), they are never used in the enunciations, they are only used in the solutions, and they function as proper names assigned to the unnamed numbers denoted by the terms of the first series. Briefly put, they are the numbers through which the solutions to the problems are conducted, according to Diophantus' method of solution. In a sense, the whole issue in a Diophantine solution to a problem could be stated as a game of transition from the first set of terms to the second. Indeed, in a Diophantine solution, the problem stated in the common language is gradually 'translated' into the technical language of the 'arithmetical theory', thus being gradually converted to an equation, entirely framed in the technical language. Hence, there is little doubt that with the expression "Diophantine numbers" Theon refers to the technical terms that constitute the 'building blocks' of the solution according to Diophantus' method. "Diophantine numbers" cannot be but the numbers of the 'arithmetical theory'.[39]

But what does the designation "Diophantine" mean for these numbers? It is well attested that, besides Diophantus, some of the terms occur in other authors as well. For instance, the word δυναμοδύναμις occurs in the *Metrica* of Heron of Alexandria (Schöne

[39] Diophantus' method of solution is discussed in depth in (Christianidis 2007; Bernard and Christianidis 2012; Christianidis and Oaks, 2013).

1903, 48.11;19;21), though not with the technical meaning of the name assigned to an unnamed sought-after number produced by a square multiplied by itself, with which this term occurs in Diophantus.[40] On account of this, our answer to the above question is that it was not Diophantus who introduced these terms; at least not all of them. Most likely, Diophantus was the first, or among the first, to use them with the technical meaning they bear in a solution according to the method he was practicing, i.e. the method of algebra.

The characterization "Diophantine numbers" is not the only interesting point in Theon's phrase introducing the second solution. Even more important for the early history of algebra is the phrase διὰ τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς used by Theon to describe the second solution. For, the expression "the process of the Diophantine numbers"[41] could not be used unless it was intended to indicate a concrete, unique way of problem-solving, a way that Theon recognized as distinguished from other methods practiced in his time. We therefore understand that, in the period following Diophantus' death, algebra had already become an acknowledged method of problem solving, with its own identity, and, presumably, with its teachers and practitioners. Our tentative guess is that this method was diffused through the world of late antiquity, before it was appropriated in a new cultural environment, the Islamic world, within which it greatly flourished.

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[40] In Diophantus δυναμοδύναμις is the name assigned, as we would say today, to the fourth power of *an unknown number*; it names its numerical value as long as the latter is unmanifest (ἄδηλος, Tannery 1893–95, i, 78.19). Heron, by contrast, uses this term for the fourth power of *a known quantity* (cf. the phrase ἔστι δοθεῖσα ἡ ἀπὸ ΒΓ δυναμοδύναμις, *ibid*. 48.21–22).

[41] We translate the Greek word ἀγωγή by "process" following Tannery, who writes in his "Index Graecitatis apud Diophantum": "ἀγωγή, processus (ad solutionem problematum)" (Tannery 1893–95, ii, 261).

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