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The *Bālabodhāṅkavṛtti*: Śambhudāsa's Old-Gujarātī Commentary on the Anonymous Sanskrit Arithmetical Work *Pañcaviṃśatikā*

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Abstract

The *Pañcaviṃśatikā* is a small arithmetical work written in twenty-five Sanskrit verses before CE 1429. It is available in three manuscripts, one with Śambhunātha's commentary (between CE 1562 and 1730) and two with Śambhudāsa's (CE 1428/29), both in Old Gujarātī. I edited here the *Pañcaviṃśatikā* with Śambhudāsa's commentary, *Bālabodhāṅkavṛtti*, based on those manuscripts. This is the first critical edition of a mathematical work written in Old Gujarātī. The commentary is important for two reasons, among others. First, since it contains far more numbers in Old-Gujarātī numerals than can be expected of literary works, it will contribute to the study of the Old-Gujarātī language with respect to the numerals. Second, it offers us important information on the multiplication methods in medieval India, about which we so far have only scanty knowledge. The *Pañcaviṃśatikā* refers to nine methods of multiplication, two of which occur for the first time in this work. The commentary briefly explains the working processes of the nine methods by using illustrative examples. Based on those explanations, I have reconstructed the details of each of the nine methods and, by comparing them with the multiplication methods mentioned in other mathematical texts, clarified part of the tangled history of multiplication methods in India.

Contents

I Introduction

The anonymous Sanskrit arithmetical work of unknown date, *Pañcaviṃśatikā* ("⟨Book⟩ of Twenty-Five ⟨Verses⟩," abbr. PV), is available in three manuscripts, one each from Ahmedabad, Baroda, and Jaipur (see "Manuscripts" below). The Ahmedabad manuscript contains a commentary written in Old Gujarātī by a certain Śambhunātha some time between CE 1562 and 1730, while the Baroda and Jaipur manuscripts contain another Old Gujarātī commentary written by a certain Śambhudāsa in CE 1428/29 (see Hayashi 1991, 399).

The PV commented on by Śambhunātha is different in many points from the one commented on by Śambhudāsa. I have already studied the difference and tried to reconstruct a common ancestor of the two versions by means of the Ahmedabad and Baroda manuscripts (Hayashi 1991); the Jaipur manuscript was not available to me at that time. I included in that study some information from the two commentaries also but there remained obscure passages. Later, thanks to the late Professor David Pingree, the Jaipur manuscript became available to me and it helped me to understand those obscure passages. This made it possible for me to edit Śambhudāsa's commentary here. For my understanding of the Old Gujarātī, I am indebted to the pioneering studies of Baumann (1975), Bender (1951, 1992), and Dave (1935).

Śambhudāsa's commentary on the PV is called *Bālabodhāṅkavṛtti* (abbr. BBA) in the colophonic verse. The word *aṅka* used in mathematics usually means "a digit" (numerical figure) and by extension "the number" designated by the digit. Presumably, the word in the present title means the science (or art) of number, i.e. arithmetic—cf. the compounds *aṅka-tantra* in Sanskrit and *aṃka-gaṇita* in Hindi. The title may therefore be translated as "A Commentary on the Arithmetic ⟨entitled PV⟩ Awakening the Youth" or "An Introductory Commentary on the Arithmetic ⟨entitled PV⟩."

The PV is a small textbook of arithmetic and mensuration (*pāṭī*) for beginners. Written in the traditional style of *pāṭī* books like Śrīdhara's *Triśatikā* (alias *Gaṇitasāra*, ca. CE 800), it deals with the "fundamental operations" (*parikarmāṇi*) and "procedures" (*vyavahārāḥ*) for practical (or applied) mathematics, although it omits some of the ordinary topics of *pāṭī* books such as the cube root, the operations on fractions besides multiplication, and the procedures of mathematical series (except the sum of the natural series which is treated as the first topic) and of sawing.[1]

It is inferred from the wide circulation of the manuscripts that the *Triśatikā* (in about 180 verses) was used as a textbook of arithmetic and mensuration by beginners all over the Indian subcontinent until, at least, the appearance of Bhāskara II's

[1] For more information about the genre of Indian mathematics called *pāṭī*, see Hayashi (2002, 2014).

Līlāvatī (in about 270 verses) in CE 1150. After that it was gradually replaced by the *Līlāvatī*, but that notwithstanding the *Triśatikā* seems to have continued to be used owing to its conciseness, as is suggested by the fact that more than one dozen of its manuscripts have survived to this day. This signifies that concise textbooks of mathematics for beginners were in demand even in the presence of the *Līlāvatī*. The PV seems to have been one such book.

An anonymous Sanskrit arithmetical work without a title, whose contents are closely connected with the BBA, is found in three manuscripts: Nos. 4660 (= O1) and 3211 (= O2) of the Oriental Institute, Baroda, and No. 9550 (= H1) of Hemacandrācārya Jaina Jñāna Mandira, Pattan. I have edited the latter with the provisional title *Parikarmacatuṣṭaya* ("Quartet of Fundamental Operations," abbr. PC) as it deals with only the first four fundamental operations (Hayashi 2006a). It is remarkable that Śambhudāsa's examples for the first four operations show a complete numerical agreement with those of the PC, although his examples are given in Old-Gujarātī prose while the examples of the PC in Sanskrit verse, and the things treated and the weights and measures employed in the former are different from those of the latter. See Tables 1 and 2 for the examples of multiplication and division, respectively. In each table, column 1 shows the numerical content of each problem with citations of their appearances in earlier texts, if any; columns 2 and 3 show the verse number and the paragraph number, respectively, in which the problem appears in the relevant text, along with the material context and the units of measure employed for it in that problem.[1]

In all three manuscripts, the PC is immediately followed by the *Natvāśivam* (also called *Natvā-gaṇitasāram*, *Gaṇitasāra-natvāśivam*, etc.), which is an anonymous Old-Gujarātī commentary on the *paribhāṣā* section (weights and measures) of the *Triśatikā*. The manuscript O2 has no date but the manuscripts O1 and H1 are dated respectively 30 April 1391 and 23 October 1600. This means that the PC antedates the BBA by more than 37 years. It is therefore likely that Śambhudāsa borrowed the examples of the PC and adapted them to his environment.

In the first verse of the PV in Śambhudāsa's version the anonymous author declares that he divides all the rules into five groups called *sūtra* ("thread"). The grouping is extremely unbalanced because the first four groups (verses 2–9) treat the first four arithmetical operations and the fifth group (verses 10–26) the remaining topics from the square to the noon-shadow lengths. See the Contents of the *Bālabodhāṅkavṛtti* below. As Śambhunātha's version has no such grouping, the original PV also seems not to have had such a grouping.

The PV prescribes four methods of multiplication in verses 4–8. Each method has two or three varieties and eventually we have nine methods in total. It is noteworthy that two of them, namely, the methods called *go-mūtrikā* and *tatstha-koṣṭha-bheda*

[1] For more details see Hayashi (2006a).

Table 1: Examples for multiplication in the PC and the BBA.

	Examples[*1]	PC	BBA
1.	$1195 \cdot 18 = 21528$	22: gold pieces (*dīnāras*)	8.5: silver (*gadīyāṇa-dramma*)
2.	$865 \cdot 32 = 27680$	23: gold pieces (*dīnāras*)	8.9: gold (*tola-ṭaṅka*)
3.	$196 \cdot 35 = 6860$	24: gold pieces (*dīnāras*)	8.10: madder (*maṇa-ṭaṅka*)
4.	$4865 \cdot 36 = 175140$	25: gold pieces (*dīnāras*)	8.11: ivory (*maṇa-ṭaṅka*)
5.	$38327 \cdot 81 = 3104487$	26–27: cows	8.12: sugar (*maṇa-dramma*)
6.	$152207 \cdot 73 = 11111111$ (GSS 2.15)	28: numerical	8.16: numerical
7.	$1767 \cdot 64 = 113088$	29: *anuḍuha* (num.-*purāṇa*)	8.13: sandal wood (*maṇa-ṭaṅka*)
8.	$3707 \cdot 188 = 696164$	30–31: beans (*vāha-purāṇa*)	8.14: *āchī* (*maṇa-dramma*)
9.	$1859 \cdot 308 = 572572$	32–33: flax (*vāha-purāṇa*)	8.15: threads (*maṇa-ṭaṅka*)
10.	$33333366666 \cdot 33 =$ 11000011000011 (GSS 2.11)	34: numerical	8.17: numerical
11.	$142857143 \cdot 7 =$ 1000000001 (GSS 2.13)	35: numerical	8.18: numerical

[*1] This order of examples is according to the MS O1 of the PC. In the MSS O2 and H1, No. 6 is placed between Nos. 9 and 10 as in the BBA.

occur for the first time in this work.

Śambhudāsa supplies eleven examples for multiplication, presumably from the PC (see Table 1), together with his own answers. For the first example, he also shows how to work with each of the nine methods (BBA 8.5–8). His explanation is very brief but he has left with us the arrangements of numerical figures at several crucial steps of each working process. From them we can reconstruct the whole procedure of each method. See the Notes for BBA 8.5–8 in the Annotated Translation. For the correspondence between the names given to the various multiplication methods in mathematical works and for my brief remarks on their history, see the Note for PV 4.

The BBA contains a number of Old-Gujarātī numerals. See Index 3. This is quite natural as a mathematical text but is exceptional in the Old-Gujarātī literature. This

Table 2: Examples for division in the PC and the BBA.

	Examples[*1]	PC	BBA
1.	$488 \div 4 = 122$	39: *dīnāras*/servant	9.3: *drammas*/part
2.	$327 \div 3 = 109$	40: *dīnāras*/servant	9.4: *drammas*/part
3.	$4096 \div 16 = 256$	41: *dīnāras*/man	9.5: *drammas*/part
4.	$30276 \div 87 = 348$	42: *dīnāras*/man	9.7: *drammas*/part
5.	$11664 \div 108 = 108$	43: *dīnāras*/servant	9.6: *drammas*/part
6.	$156025 \div 395 = 395$	44: *dīnāras*/man	9.8: *drammas*/part
7.	$1466521 \div 1211 = 1211$	45: *dīnāras*/man	9.9: *drammas*/part
8.	$193454600 \div 1808 = 10700$	46: *purāṇas*/man	9.10: *drammas*/part
9.	$10000003 \div 13 = 769231$	51[*2]: *paṇas*/man	9.11: *drammas*/part
10.	$100001 \div 11 = 9091$	54[*2]: oil/*brāhmaṇa* (*māṣa* per *brāhmaṇa*)	9.12: *drammas*/part

[*1] This order of examples is according to the MS O1 of the PC. In the MSS O2 and H1, No. 4 is placed between Nos. 5 and 6 as in the BBA.

[*2] These two examples are preserved only in the MS O2. The MS O1 has eight exs. (39–46), H1 two more exs. (49–50), and O2 eight further exs. (51–58).

commentary will no doubt contribute to the study of the Old-Gujarātī numerals.

The Language of the Commentary

Śambhudāsa writes his commentary on the first verse (PV 1) both in Sanskrit (BBA 1.1) and in Old Gujarātī (BBA 1.2): they are almost parallel. Most of the other parts of the commentary are written in Old Gujarātī. But the language is characterized by a number of Sanskrit loan words for mathematical terms.

The language of the first (introductory) and the last (concluding) paragraphs of the commentary on each verse is grammatically close to Sanskrit but, from the viewpoint of regular Sanskrit, they contain irregular *sandhis*, grammatical anomalies (such as disagreement of gender), and even Old Gujarātī words. The language of these paragraphs is therefore Sanskrit blended with Old Gujarātī.

Manuscripts

Manuscript A

LD Institute, Ahmedabad, No. 5325. Title: Gaṇitasāra. Author: unknown. Language: Sanskrit. Script: Devanāgarī. Extent: complete. Fols. 1–5. 17 lines to a

page. About 55 to 60 *akṣaras* to a line. Material: paper. With a commentary. Title: Gaṇitasāra. Author: Śambhunātha. Language: Old Gujarātī.

Manuscript B

Oriental Institute, Baroda, No. 5283. Title: Pañcaviṃśatikā. Author: unknown. Date: unknown. Language: Sanskrit. Script: Devanāgarī (with *pṛṣṭha-mātrā* e). Extent: Complete. Fols. 1b–5a (8 pages). 17 lines to a page. About 55 to 60 *akṣaras* to a line. Material: paper. With a commentary. Title: *Bālabodhāṅkavṛtti*. Author: Śambhudāsa. Language: Old Gujarātī, which is slightly different from that of Śambhunātha. Date: Saṃ 1485 (= CE 1428/29). Place of composition: Ahmedabad.

Manuscript J

Rajasthan Oriental Research Institute, Jaipur, No. 8039. Title: Pañcaviṃśatikā. Author: unknown. Date: unknown. Language: Sanskrit. Script: Devanāgarī. Extent: Incomplete. Extant fols.: 1a–1b and 3a–6b. Several lines of the last part of the work must have been on the lost fol. 7. 15 lines to a page. About 52 to 58 *akṣaras* to a line. Material: paper. With the commentary of Śambhudāsa. No information about the commentary or the scribe is preserved.

Editorial Principles

The present edition is primarily based on the manuscript B, which is complete, and is collated with the manuscript J, which is incomplete. The verses of the PV are also collated with the manuscript A. The PV verses in these manuscripts contain a number of irregularities, from the viewpoint of the regular classical Sanskrit, with respect to the phonetics and orthography but I refrained from normalizing them in this edition unless they have affected the grammar, the meter, or the meaning as I am not familiar with the state of the Sanskrit language in the time of the anonymous author of the PV.

Editorial Conventions

The verse numbers are those given in the manuscripts B and J. When the first and the second lines (i.e., halves) of verse n are separately commented on by Śambhudāsa, I supply the line numbers as n_1 and n_2. When I divide the commentary on verse n (designated PV n) into paragraphs, I assign them sequential numbers, BBA $n.0$ (for introductory phrase), BBA $n.1$, BBA $n.2$, etc. In the edited text, I put them at the end of each paragraph, in roman script with a pair of angular brackets: $\langle n.0\rangle$, $\langle n.1\rangle$, $\langle n.2\rangle$, etc. I also assign sequential numbers to the five quoted supplementary rules (S1, S2, etc.) and to the thirty-two figures (Figure 1, Figure 2, etc.), both within a pair of angular brackets in the text. I add a pair of quotes to indicate the passages quoted by the commentator from the PV verses.

Notation

In the apparatus:

x M_1] y M_2: x in M_1, which is accepted in this edition, reads y in M_2.
x M_1] ∅ M_2: x in M_1 is omitted in M_2.
x M_1] y M_2(cor.): For x in M_1, M_2 first writes down y but corrects it to x.
x] y_1 M_1 y_2 M_2: M_1 and M_2 read y_1 and y_2 resp. but I propose to read x.

In the Translation:

Śambhudāsa sometimes explains or paraphrases the word(s) (*a*) in the verse by using other word(s) (*b*). In my translation, I express this by a pair of long hyphens as "'A'–B–," where A and B are translations respectively of *a* and *b*. If *b* is simply a synonym of *a*, I express it as "'A' (*a*: *b*)."

A pair of angular brackets, ⟨ ⟩, indicates the word(s) added to the translation to complete the syntax of the sentence; a pair of parentheses, ( ), encloses either the original Sanskrit word(s) or my explanation of the immediately preceding word(s).

Abbreviations of Titles

GK	*Gaṇita-kaumudī* of Nārāyaṇa
GT	*Gaṇita-tilaka* of Śrīpati
GM	*Gaṇita-mañjarī* of Gaṇeśa
GL	Gaṇeśa's commentary on the L
GSK	*Gaṇita-sāra-kaumudī* of Ṭhakkura Pherū (GS in Hayashi 1991)
GSS	*Gaṇita-sāra-saṃgraha* of Mahāvīra
Tr	*Triśatikā* of Śrīdhara
PG	*Pāṭī-gaṇita* of Śrīdhara
PC	*Parikarma-catuṣṭaya*, anonymous
PV	*Pañcaviṃśatikā*, anonymous
BG	*Bīja-gaṇita* of Bhāskara
BBA	*Bāla-bodha-aṅka-vṛtti*, Śambhudāsa's commentary on the PV
BSS	*Brāhma-sphuṭa-siddhānta* of Brahmagupta
L	*Līlāvatī* of Bhāskara
SGT	Siṃhatilaka's commentary on the GT

See bibliographical works, such as Hayashi (2000), for editions of these texts except GM, GSK, PC, and BG, for which see respectively Hayashi (2013a), SaKHYa (2009), Hayashi (2006a), and Hayashi (2009) in the References of this paper.

Contents of the *Bālabodhāṅkavṛtti*

II Text of the *Bālabodhāṅkavṛtti* with *Pañcaviṃśatikā*

∀∀ओं नमः ।[1] श्रीगणेशाय नमस्कारः ॥[2] ⟨1.0⟩ B1b J1a

महादेवं प्रणम्यादौ बालानां बुद्धिवृद्धये ।[3]
पंचसूत्रैरहं वक्ष्ये पंचविंशतिकामिमाम् ॥ १ ॥[4]

[5]अहं आदौ पूर्वं महादेवं श्रीसर्वज्ञं प्रणम्य बालानां बुद्धिवृद्धये पंचसूत्रैरिमां पंचविंशतिकां वक्ष्ये ॥[6] ⟨1.1⟩ हूं आदि पहिलू महादेव श्रीसर्वज्ञ प्रणमी नमस्करी नइ बाल अज्ञान नी बुद्धि नी वृद्धि नइ अर्थि पांचसूत्रि करी गणितसार पंचवीसी बोलूं ॥[7] ⟨1.2⟩

प्रथम संकलितसूत्रं ॥[8] ⟨2.0⟩

सैकाद्यपदघातार्द्धं तथा एकोत्तरेण च ।[9]
पदवर्ग्राद्ययुक्तार्द्धं सैकाद्यार्द्धवधे फलं ॥ २ ॥[10]

एवं संकलित चिहु परि ।[11] प्रथमपरि 'सैकाद्यपदघातार्द्धं' [PV 2a] ।[12] 'सैक' । प्रस्नपद एकसहित कीजइ ।[13] 'आद्यपदघात' । पच्छइ ते आद्य पहिला पद सिउं गुणीइ ।[14] 'अर्द्धं' ।[15]

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[1]ओं नमः B] ∅ J.

[2]नमस्कारः B] नमः J.

[3]वृद्धये A] वृद्धाये B(cor.), वृद्धिये J(cor.).

[4]पंचसूत्रैरहं BJ] स्वीयसूत्रैरहं A.

[5]∅ B] व्याख्या ॥ J.

[6]सूत्रैरिमां] सूत्रै इमां BJ; विंशतिकां B] विंशकां J.

[7]हूं आदि पहिलू B] हुं आदौ पहिलुं J; नमस्करी नइ B] नमस्कारकरी नइं J; नी (1st) B] ∅ J; पांचसूत्रि B] पंचसूत्र J; पंचवीसी बोलूं B] पचवीसे बोलुं J.

[8]प्रथम B] प्रथमं J; संकलित B] संकलिते J.

[9]तथा एकोत्तरेण च BJ] भवेदेकोत्तरेण तत् A(better).

[10]वर्ग्र B] वर्ग J(hereafter also); युक्ता BJ] योगा A(better); वधे BJ] वधेः A; २ BJ] १ A.

[11]चिहु] विहु B, चिहुं J.

[12]J repeats प्रथमपरि; र्द्धं B] र्द्ध J.

[13]प्रस्नपद B] प्रश्न J; सहित J] सहि B.

[14]पच्छइ B] पछइ J(hereafter also); सिउं B] सुं J; गुणीइ B] गुणीयइ J(hereafter also).

[15]अर्द्धं J] अर्द्धीं B.

पच्छइ ते पद अर्द्धी कीजइ।[1] संकलित हुइ॥[2] ⟨2.1⟩

[3]द्वितीयपरि।[4] 'तथा एकोत्तरेण च'[PV 2b]। 'तथा' वली वाध नुं आंक प्रस्न जांण लिषी सिरवालु करीइ।[5] संकलित हुइ॥ ⟨2.2⟩

[6]तृतीयपरि।[7] 'पदवर्ग्राद्ययुक्तार्द्धं'[PV 2c]।[8] 'पदवर्ग्र'। प्रस्नपद नु वर्ग्र करीइ।[9] 'आद्य-युक्त'।[10] अनइ आद्यपदयुक्त कीजइ।[11] 'अर्द्धं'। पच्छइ ते पद अर्द्ध कीजइ। संकलित हुइ॥ ⟨2.3⟩

[12]चतुर्थपरि।[13] 'सैकाद्यार्द्धवधे फलं'[PV 2d]। 'सैक'।[14] प्रस्नपद एकसहित कीजइ। 'आद्य'।[15] पच्छइ आद्य अंक जमलु लिषीइ।[16] 'अर्द्धवधे'।[17] पच्छइ ते अंक बिहु माहि पूरा पद नूं अर्द्ध करी गुणीइ।[18] संकलित हुइ॥[19] ⟨2.4⟩

उदा०।[20] दस नी संकलित पंचावन १०। ५५।[21] प्रथम। प्रस्नपद १० 'सैक' एकसहित जात ११।[22] 'आद्यपद' १० 'घात' गुणा ११०। 'अर्द्धं' ५५॥ ⟨2.5⟩

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[1]पच्छइ] पच्चइ B(cor.); अर्द्धी B] अर्द्ध J.

[2]हुइ B] हुयइ J(hereafter also).

[3]∅ B] अथ J.

[4]परि B] प्रकार J.

[5]नुं B] तउ J; प्रस्न] घस्न B, प्रश्न J(mostly hereafter also); लिषी B] लिषीयइ J; वालु B] वालउ J; करीइ B] करीयइ J(hereafter also).

[6]∅ B] अ J.

[7]परि B] प्रकार J.

[8]र्द्धं B] र्द्ध J.

[9]प्रस्न B] प्रश्न J(hereafter also); नु B] नुं J.

[10]आद्य J] अद्य B(cor.).

[11]अनइ B] अनइं J.

[12]∅ B] अथ J.

[13]परि B] प्रकार J.

[14]सैक B] सैकक J.

[15]आद्य B] आद्य अपन॰इं J.

[16]आद्य B] आद्यं J; लिषीइ B] लिषीयइ J(hereafter also).

[17]वधे] वधा B, वध J.

[18]नूं B] नुं J;

[19]हुइ B] हुयइ J.

[20]उदा॰ B] यथोदाहरणं J.

[21]दस B] दश J; संकलित B] संख्याकलित J(cor.); वन B] वन्न J.

[22]पद J] द B; सहित B] सहितं J; जात B] जातं J.

द्वितीय।[1] प्रश्नपद १०। 'एकोत्तरेण'।[2] एक थिकी दस जांण सिरवालु की[∀] जइ।[3] जात ५५॥[4] ⟨2.6⟩ B2a

तृतीय।[5] प्रस्नपद १०। वर्ग्रे कृते जात १००।[6] आद्यपद १० युक्त जात ११०।[7] अर्द्धं ५५॥[8] ⟨2.7⟩

चतुर्थपरि।[9] प्रस्नपद १०। सैक जात ११।[10] आद्यपद १० अर्द्धं ५ गुणा जात ५५॥[11] ⟨2.8⟩

वीस नी संकलित बि सइ दहोत्तर २०। २१०।[12] तीस नी संकलित च्यारि सइ पांसठि ३०। ४६५।[13] च्यालीस नी संकलित आठ सइ वीसां ४०। ८२०।[14] पंचास नी संकलित बार सइं पंचहुत्तरि ५०। १२७५।[15] साठि नी संकलित अढार सइं त्रीसां ६०। १८३०।[16] सित्तिरि नी संकलित चुवीस सइं पंच्यासी ७०। २४८५।[17] असी नी संकलित बत्रीस सइ च्यालीसां ८०। ३२४०।[18] नउ ना संकलित च्यालीस सइ पंचाणू[∀] ९०। ४०९५।[19] सु नी संकलित पंचास पंचासां १००। ५०५०॥[20] ⟨2.9⟩ J1b

संकलितमूलं।

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[1]द्वितीय B] अथ द्वितीयप्रकार J.

[2]एकोत्तरेण B(*kta* after *eko* canceled)] ∅ J.

[3]थिकी B] थकी J; दस B] दश J; जांण B] सीम J; वालु B] वालउ J.

[4]जात B] जातं J.

[5]तृतीय B] अथ तृतीयप्रकार J.

[6]जात B] जातं J.

[7]जात B] जातं J.

[8]५५ B] ५५ इति J.

[9]चतुर्थ J] चचतुर्थ B(cor.); परि B] प्रकार J.

[10]जात B] जाते J.

[11]अर्द्धं B] अर्द्धं J; जात B] जातं J.

[12]दहोत्तर B] दाहोत्तर J.

[13]Two illegible *akṣaras* between *pāṃ* and *saṭhi*, J.

[14]About 10 *akṣaras* after *saṃkalita* canceled, J; सइ B] सइ नइं J.

[15]सइं B] सइ J; पंचहुत्तरि B] पच्योत्तरि J.

[16]साठि B] साठ J; सइं B] सइ J; त्रीसां B] त्रीस J; ६० B] ∅ J.

[17]सित्तिरि B] सत्तरि J; चुवीस B] चउवीस J; सइं B] सइ J.

[18]च्यालीसां B] च्यालीस J.

[19]नउ B] निउ J; ना B] नी J; पंचाणू B] पंचांणू J.

[20]सु B] सउ J; पंचास पंचासां B] ∅ J.

संकलितद्विगुणान्मूलसमो गच्छः॥[1] ⟨S1⟩

संकलित मूल पद बिमणू करी वर्ग्रमूल लीजीइ।[2] अनइ समां रूप हुइ। तु एक गछ कीजइ।[3] मूल आवइ॥ ⟨2.10⟩

इति संकलितं समाप्तं॥[4] ⟨2.11⟩

द्वितीयं व्यवकलितसूत्रं॥[5] ⟨3.0⟩

संकलितोत्पन्नद्युम्नात् व्ययं त्यक्त्का धनं भवेत्।[6]
तत् धनं व्यवकलितं कथितं मुनिभिः पुरा॥ ३॥[7]

संकलित पदि उत्पन्न द्रव्य तेह तु वरु पाडी बाकी काढीइ।[8] ते धन मुनि रषीश्वर व्यवकलित कहइं॥[9] ⟨3.1⟩

उदा॰।[10] शतसंकलित थिकी दस वीस त्रीस च्यालीस पंचास साठि सत्तरि असी नउ सु नां संकलित वरइ करी बाकी धन किम हुइ।[11] न्यासः।[12]

५०५० सं १००	५०५० सं १००	५०५० सं १००	५०५० सं १००	५०५० सं १००
५५ व्य १०	२१० व्य २०	४६५ व्य ३०	८२० व्य ४०	१२७५ व्य ५०
४९९५ बाकी	४८४० बाकी	४५८५ बाकी	४२३० बाकी	३७७५ बाकी

[1]संकलित B] संकलितं J; मूलसमो] मूलूं समं B($\bar{u}$ of *lūṃ* canceled), मूलसम J; गच्छः] गच्छ BJ. The same content is found also in verse 3cd of A but its verbal expression is different.

[2]पद B] ∅ J; बिमणू B] बिमणु J; करी B] कीजइ J; लीजीइ B] लीजइ J.

[3]तु B] तउ J.

[4]संकलितं J] संकलित B.

[5]द्वितीयं B] अथ द्वितीय J; व्यवकलित J] व्यकवलित B.

[6]तोत्पन्न BJ] तोन्नत A; द्युम्नात् J] द्युम्नात B, द्रम्मात् A; त्यक्त्का B] त्यक्ता J, कृत्वा A; भवेत् BA] हरेत् J.

[7]तत् धनं B] तद्धनं JA; मुनिभिः BA] मुनभिः J; ३ BJ] २ A.

[8]तु B] नु J; वरु B] वरउ J; काढीइ B] काढीयइ J(hereafter also).

[9]रषीश्वर B] ऋषीश्वरे J; व्यवकलित B] व्यकलित J; कहइं J] कहिइ B.

[10]उदा॰ B] उ॰ J.

[11]शत B] सत J; थिकी B] थकी J; दस B] दश J; नउ B] निउ J; सु नां B] सउ ना J.

[12]B places this table, which consists of 5 cells × 2, at the top of fol.2b; J places it between *tathā* and *khaṃḍa* in the 4th *pāda* of verse 4. सं B(all)] स J; व्य ३० J(in the 3rd cell)] द्य ३० B.

५०५० सं १००	५०५० सं १००	५०५० सं १००	५०५० सं १००	५०५० सं १००
१८३० व्य ६०	२४८५ व्य ७०	३२४० व्य ८०	४०९५ व्य ९०	५०५० व्य १००
३२२० बाकी	२५६५ बाकी	१८१० बाकी	९५५ बाकी	०००० बाकी

एतलां ए व्ययपद तथा शेषधनपद।[1] अनइ एणं रीति आयवराइ बाकी काढीइ॥[2] ⟨3.2⟩ इति व्यवकलितं समाप्तं॥[3] ⟨3.3⟩

तृतीयं प्रत्युत्पन्नसूत्रं॥[4] ⟨4.0⟩

द्विधा कपाटसंधिश्च तथा गोमूत्रिका द्विधा।[5]
तस्थो द्विधा पुनः प्रोक्तस्तथा षंडस्त्रिधा स्मृतः॥ ४॥[6]

कपा$^{\forall}$टसंधिः॥[7] ⟨5.0⟩ B2b

प्रस्नोपरि न्यसेन्मूल्यं मूल्येन गुणयेत्क्रमात्।[8]
अनुलोमविलोमाभ्यां कपाटाख्यं द्विधा भवेत्॥ ५॥[9]

कपाटसंधि बिहु परि।[10] प्रथमं अनुलोमगतिः।[11] अनुलोमगतिं प्रस्नपद ऊपरि कपाट-संधिक्रमिं धुरि मूल्य मांडीइ।[12] पच्छइ मूल्य सिउं क्रमिं प्रस्नपद गुणी मेलीइ।[13] फल आवइ॥ ⟨5.1⟩

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[1]एतलां B] एला J; ए B] ∅ J; तथा B] ∅ J.

[2]अनइ B] अनइं J; एणं B] इणी J; वराइ B] वराई J.

[3]व्यवकलितं B] व्यवकलित J.

[4]तृतीयं B] अथ तृतीय J.

[5]द्विधा (1st) BA] द्वितीय J; *udā°* after *tathā* canceled, B.

[6]तस्थो (for *tatstho*) A] तस्थौ BJ; षंडस्त्रिधा B] खंडविधि J, खंडस्त्रिधा A; स्मृतः BJ] मतः A; ४ BJ] ६ A.

[7]संधिः J] संधि B.

[8]प्रस्नोपरि BJ] प्रश्नोपरि A; मूल्यं मूल्येन J] मूलं मूलोन B, मूलं मौल्येन A.

[9]५ BJ] ६ (2nd) A.

[10]संधि B] सिंधि J; बिहु B] बिहुं J.

[11]*trya* after *prathamaṃ* canceled, B.

[12]गतिं B] गति J; प्रस्न B] प्रश्न J; संधि J] सधि B; क्रमिं B] क्रमइं J; धुरि] अंति B, अति J; मांडीइ B] मांडीयइ J. For the emendation of *aṃti* see *dhuri* in 6.1.

[13]सिउं B] सुं J; क्रमिं B] क्रमइं J; मेलीइ B] मेलीयइ J.

[1]तथा द्वितीया विलोमगति।[2] विलोमगति प्रस्नपद ऊपरि कपाटसंधिक्रमइं अंति मूल्य मांडीइ।[3] पच्छइ मूल्य सिउं क्रमिं प्रस्नपद गुणी मेलीइ।[4] फल आवइ॥ ⟨5.2⟩

अथ गोमूत्रिका॥ ⟨6.0⟩

प्रश्नादधो न्यसेन्मूल्यं गुणयेत्सरलं मिथः।[5]
अनुलोमविलोमाभ्यां गोमूत्राख्यं द्विधा भवेत्॥ ६॥[6]

J2a B3a ∀ [7]गोमूत्रिका बिहु परि। प्रथमं अनुलोमगति। अनुलोमगति प्रस्नपद हे∀ठि जमलू धुरि मूल्य मांडीइ। पच्छइ ते पाधरूं अनइ अन्योन्यि अनइ वली अंति पाधरूं गुणी मेलीइ। फल आवइ॥ ⟨6.1⟩

तथा द्वितीया विलोमगतिः।[8] विलोमगतिं प्रस्नपद हेठि जमलू अंति मूल्य मांडीइ। पच्छइ पाधरू अन्योन्यि गुणी मेलीइ। फल आवइ॥ ⟨6.2⟩

अथ तस्थभेदः॥ ⟨7.0⟩

एकैकगुणनाद्राशेः शीर्षभेदो निगद्यते।[9]
कोष्टाभेदः पुनः प्रोक्तस्तस्थोऽपि द्विविधः स्मृतः॥ ७॥[10]

तस्थ बिहु परि। प्रथमं शीर्षभेदि प्रस्नपद ऊपरि मूल्य माथइ मांडीइ। अनइ मूल्य नु एक एक आंक लेई प्रस्नपद गुणीइ।[11] अलग मेलीइ। फल आवइ॥[12] ⟨7.1⟩

तथा द्वितीयः कोष्टाभेदः। कोष्टाभेदि कोठा लिषी चीरीइं। प्रस्नपद माथइं मांडीइं। अनइ

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[1]Before *tathā* B has a canceled passage: तथा द्वितीया विलोमगेति। प्रस्नपद ऊपरि। अथ गोमूत्रिका।

[2]गति B] गतिः J.

[3]कपाट B] कापाट J; क्रमइं B] क्रमइ J; अंति मूल्य मांडीइ B] ∅ J.

[4]पच्छइ मूल्य सिउं क्रमिं B] ∅ J; मेलीइ B] मेलीयै(sic) J.

[5]मूल्यं JA] मूल्य B; सरलं J] स॥ श्रीरलं B, सकलं A; मिथः BJ] ततः A.

[6]मूत्राख्यं BA] मूत्रिख्यं J(cor.); ६ BJ] ७ A.

[7]Fol. 2 of J is missing.

[8]गतिः] तिः B(with *ga* in margin).

[9]शीर्ष A] शीर्षा B(cor.); निगद्यते A] निगद्यतः B.

[10]कोष्टाभेदः] कोष्टाभेद B, पृष्टभेदे A; तस्थोऽपि] तस्थौऽपि B, तस्थोपि A; विधः] विधेः B(cor.), विधा A; स्मृतः BJ] पुनः A; ७ B] ८ A.

[11]लेई] लेइ B.

[12]फल] कल B(cor.).

मूल्यपद आगलि लिषीइ।[1] अनइ मूल्य एक एक आंक लेई प्रस्नपद सिउं गुणी कोठा माहि लिषीइ। अनइ मेलीइं। फल आवइ॥ ⟨7.2⟩

अथ षंडभेदः॥ ⟨8.0⟩

क्वचित् रूपविभागश्च स्थानभागः क्वचिद्भवेत्।[2]
क्वचित् हीनाधिको भागः षंडो पि त्रिविधः स्मृतः॥ ८॥[3]

षंडभेद त्रिहु परि। प्रथम रूपविभागः। रूपविभागि प्रस्नपद द्विभाग त्रिभाग चतुर्भाग करी मूल्यपद सिउं गुणी एकत्र जोडीइ।[4] फल आवइ। तिम मूल्यपद भाग करी प्रस्नपद सिउं गुणीइ। तु तेह जि फल आवइं॥ ⟨8.1⟩

द्वितीयः स्थानभागः। स्थानविभागि प्रस्नपद एक दश शत सहस्रादि स्थानक जूजूया करी मूल्य सिउं गुणी एकत्र जोडीइ। फल आवइ। तिम मूल्यपद स्थानविभाग करी प्रस्नपद सिउ गुणीइ। तु तेह जि फल आवइ॥[5] ⟨8.2⟩

तृतीय हीनाधिको भागः। हीनाधिकि ∀ भागि प्रस्नापद द्विभाग चतुर्भाग कीजइ।[6] तिम B3b
मूल्यपद बिमणूं चुमणूं करी गुणीइ। फल आवइ। तिम मूल्यपद भाग करी प्रस्न सिउ गुणीइ।[7] तु तेह जि फल आवइ। एतल प्रत्युत्पन्न नव परि॥ ⟨8.3⟩

परसूत्रं।

शून्येन गुणितं शून्यं शून्यमर्ग्रे नियोजयेत्।[8]
तदेवैकेन गुणितं भवेदेवं हि सर्वतः॥[9] ⟨S2⟩

शून्य शून्य सिउं गुणीइ शून्य जि हुइ। अनइ मूल्यपद नां शून्य प्रस्नपद आगलि जेतलां हुइ

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[1]अनइ] अग्नइ B(cor.); पद] पकद B(cor.).

[2]क्वचित् रूप B] क्वचिद्रूप A;

[3]क्वचित् हीना B] क्वचिद्धीना A; षंडो] षंहडो B(cor.), खंडो A; त्रि B] द्वि A; ८ B] ९ A.

[4]विभागि] त्रिभागि B; मूल्यपद] मूल्यदप B(cor.).

[5]तेह जि] तेहठेजि B(cor.).

[6]हीनाधिकि] हीनाद्विकि B.

[7]बिमणूं चुमणूं] बिमंणू वुमणूं B.

[8]गुणितं B] गुणेनं A(cor.); शून्यमर्ग्रे B] शून्यमार्गे A.

[9]तदेवैकेन] तदैवैकेन B, तद्वदेकेन A; भवेदेवं B] भवेदेव A; सर्वतः A] ससर्वतः B(cor.). In A, this verse is numbered as 10.

तेतलां लिषीइ। अनइ एक नूं गुणिउ तेतलू जि। इम गुणाकरइ हुइ॥ ⟨8.4⟩

प्रथमोदा उदाहरणं।[1] रूपा गदीयाणा एक सहस्र एक सु छनू। प्रति द्रम्मा १८ अढार। किं फलं भवति। न्यासः। कपाटसंधि अनुलोमगति [2] गुणने रूपं [3] लब्धं द्रम्माः [4] तथा कपाटसंधि विलोमगतिः [5] गुणने रूपं [6] लब्ध द्रम्माः [7] ॥

⟨8.5⟩

अथ गोमूत्रिका अनुलोमगति [8] सरल अन्योन्य गुणने रूपं लब्धं द्रम्माः [9] तथा गोमूत्रिका विलोमगति

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[1] उदाहरणं] उदारणं B(with *ha* in margin).

[2] B.

[3] B.

[4] B.

[5] B, which places this box between *tathā gomūtri* and *kā* two lines below.

[6] B places this box between *gomūtrikā* and *anulomagati* in the next sentence.

[7] B, which places this box between *tathā gomūtri* and *kā* in the next line.

[8] B.

१	१	९	६	
		१	८	

[1] सरल अन्योन्य गुणी ने रूपं

१	१	७	४	८	
		९	७	८	

[2] लब्धं द्रम्माः

२१५२८

[3] ॥[4] ⟨8.6⟩

अथ शीर्षाभेदः

	१	८	
१	१	९	६

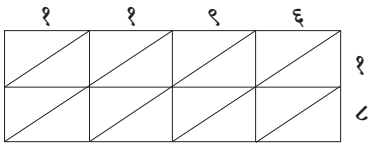
[5] एकैकगुणने रूपन्यासः[6]

१	१	९	६	८	
	८	८	२		
		७	४		

[7] लब्धं द्रम्मा

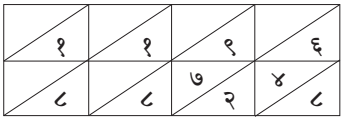
२१५२८

। तथा तस्य कोष्टाभेदः।



⟨Figure 1⟩[8]

एकैकगुणने रूपं।



⟨Figure 2⟩[9]

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[1]

१	१	९	६	
	X	१		

B.

[2]

१	९	७	८	८	
	१	७	४		

B.

[3] ∅ B.

[4] अन्योन्य] अनोन्य B.

[5] B puts "18" above "119" and places this box between *drammā* and the box for 21528 in the next line.

[6] गुणने] गुने B(with *ṇa* in margin); रूपन्यासः] रूपं न्यासः B.

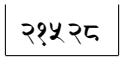
[7]

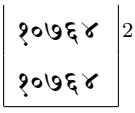
१	१	९	६	८	
	८	८	७		

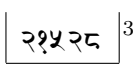
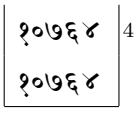
B.

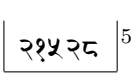
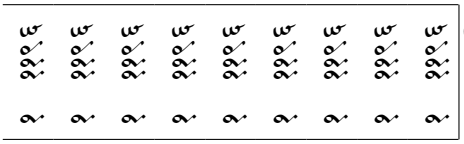
[8] Of this diagram, B has only the upper digits "1196" here and places the rest, together with "1" and "8," between *la* and *bdhaṃ* after *niyojane* following the next diagram.

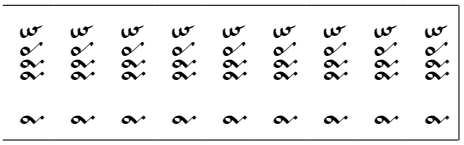
[9] B repeats the same diagram in a slightly larger size in the bottom margin.

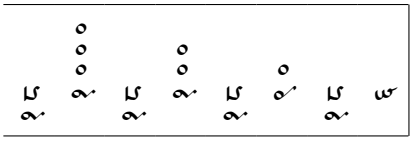
नियोजने लब्धं द्रम्माः  ॥ ⟨8.7⟩

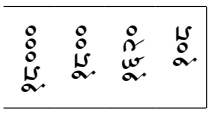
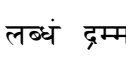
अथ षंड रूपविभाग द्विधा। कपाटसंधिवत् उभयो गुणने अर्धः रूपं[1]  [2]।

B4a लब्धं द्रम्मा  [3] द्वितीय मूल्यभागः। प्रथक गुणने ∀ रूपं  [4]

लब्धं द्रम्मा  [5]  [6] तथा षंड स्थानविभागः।



एकदशशतसहस्रादि। रूपं  [7] कपाटसंधिवत् सर्वेषां गुणने

रूपं  [8] लब्धं द्रम्मा  तथा षंड हीनाधिको भागः। अर्धः

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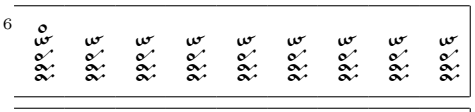
[1] गुणने] गुण B(with *ne* in margin); अर्धः] अर्धःने B(*ne* canceled).

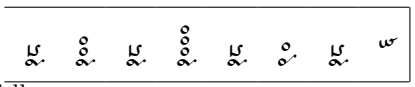
[2] ∅ B.

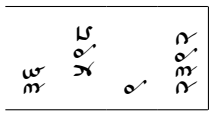
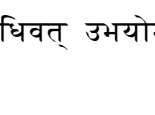
[3] ∅ B.

[4] B places this box between *mmā* and *tathā*.

[5] B places this box between *dra* and *mmā*.

[6]  B, which places this two-column table in the "*kapāṭasaṃdhivat*" immediately after the next table, that is, the left (lower) column between *pā* and *ṭa* and the right (upper) column between *saṃ* and *dhi*. (Here and hereafter, I rotated the tall boxes through 90 degrees for saving space.)

[7]  B, which puts this box below *kapā* of *kapāṭasaṃdhivat* that follows.

 [1] कपाटसंधिवत् उभयोर्गुणने एकमेव फलं लब्धं द्रम्मा  [2] ।[3] एणं रीति त्रिभाग चतुर्भाग करी गुणीइ। तु तेह जि फल आवइ। अथ प्रथम उदाहरण नी रीती सर्वत्रा॥[4] ⟨8.8⟩

द्वितीयोदा॰। हेम तोला आठ सइ पांसठि। प्रति टंका बत्रीस। न्यासः। ८६५ गुण ३२।[5] लब्धं टंका २७६८०॥ ⟨8.9⟩

तृतीयोदा॰। मंजीठ मण एक सु छनूं। प्रति टंका पांत्रीसः। न्यासः। १९६ गुणा ३५।[6] लब्धं टंक ६८६०॥ ⟨8.10⟩

चतुर्थोदा॰। दांत मण च्यारि स[∀] हस्र आठ सइ पांसठि।[7] प्रति टंका ३६ च्छत्रीस।[8] J3a
न्यासः। ४८६५ गुणा ३६।[9] लब्धं टंका राज्य १७५१४०॥ ⟨8.11⟩

पंचमोदा॰।[10] षांड मण अठत्रीस सहस्र त्रिणि सइ सतावीस।[11] प्रति द्रम्मा एक्याशी।[12] न्यासः। ३८३२७ गुणा ८१।[13] लब्धं द्रम्मा ३१०४४८७॥[14] ⟨8.12⟩

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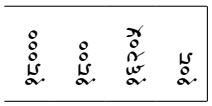
[8]  B.

[1] B places this box between *e* and *ṇaṃ* in the next line.

[2] B places this box below the preceding *drammā*.

[3] संधिवत्] संधिकंत् B(cor.); एकमेव] एकामेव B(cor.).

[4] प्रथम] प्रम B; उदाहरण] उदाणर B(*ha* in margin and *ṇara* cor. by "1" and "2" placed respectively over *ra* and *ṇa*).

[5] ८६५] ८६ B.

[6] गुणा ३५।] गुणा॥ ३५ B.

[7] पांसठि B] पांसट्ठि J.

[8] च्छत्रीस B] छत्रीस J.

[9] गुणा B] गुणाः J.

[10] पंचमोदा॰ B] पं॰ J.

[11] त्रिणि] त्रित्रिणि B(cor.), त्रिण्ह J; सतावीस B] सत्तावीस J.

[12] द्रम्मा B] दांम J; एक्याशी B] एक्यासी J.

[13] गुणा B] गुणाः J.

[14] द्रम्मा B] द्रमा J; ३१०४४८७ J] ३१४४८७ B.

षष्टोदा॰।[1] सूकडि मण एक सहस्र सात सइं सतसठि।[2] प्रति टंका चुसठि।[3] न्यास।[4] १७६७ गुणा ६४।[5] लब्धं टंका ११३०८८॥[6] ⟨8.13⟩

सप्तमोदा॰।[7] आछी मण त्रिणि सहस्र सात सइ त्रिडोत्तर।[8] प्रति द्रम्मा एक सु अद्यासी।[9] न्यासः। ३७०३ गुणा १८८।[10] लब्धं द्रम्माः ६९६१६४॥[11] ⟨8.14⟩

B4b अष्टमोदा॰।[12] पडसूत्र मण एक सहस्र आठ सइं उगणसठि।[13] प्रति टं$^{\forall}$का त्रिणि सइं अट्ठोत्तर।[14] न्यासः। १८५९ गुणा ३०८। लब्धं टंका ५७२५७२॥ ⟨8.15⟩

नवमोदा॰।[15] एक लाष बावन सहस्र बि सइं सत्तोत्तर गुणा त्रिहुत्तरि।[16] न्यासः। १५२२०७ गुणा ७३। लब्धं एकावलिरूपं १११११११११॥[17] ⟨8.16⟩

दशमोदा॰।[18] त्रिणि षर्व तेत्रीस अर्व तेत्रीस कोडि छत्रीस लाष छासठि सहस्र च्छ सि सतसठि गुणा तेत्रीसे।[19] न्यासः। ३३३३३३६६६६६७ गुणा ३३। लब्धं कंठाभरणरूपं ११००००११००००११॥[20] ⟨8.17⟩

एकादशमोदा॰।[21] चौद कोडि अठावीस लाष सत्तावन सहस्र एक सु त्रयतालीस गुणा

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[1]षष्टोदा॰ B] ष॰ J.

[2]सूकडि B] सूकड J; सइं B] सइ J; सतसठि B] सतसट्ठि J.

[3]चुसठि B] चउसट्ठि J.

[4]न्यास B] न्यासः J.

[5]गुणा ६४ B] गुणाः ६५ J.

[6]लब्धं B] लब्ध J.

[7]सप्तमोदा॰ B] सप्तमोदाहरणं J.

[8]त्रिणि B] त्रिण्हि J; सहस्र B] सहस्र J(hereafter also); त्रिडोत्तर B] तिडोत्तर J.

[9]द्रम्मा B] दांम J; सु B] सौ J.

[10]३७०३ B] ३००३ J.

[11]द्रम्माः B] द्रम्मा J; ६९६१६४] ६९६१४ B, ६९६४ J.

[12]अष्टमोदा॰ B] अष्टमोदाहरणं J.

[13]पडसूत्र B] पटसूत्र J; सइं B] सइ J; उगणसठि B] उगणसट्ठि J.

[14]त्रिणि B] त्रिण्ह J; सइं B] सइ J; अट्ठोत्तर J] अट्ठोत्त B.

[15]नवमोदा॰ B] नवमोदाहरणं J.

[16]सहस्र B] हजार J; सइं B] सइ J; गुणा B] गु J.

[17]एकावलि J] एकोवलि B.

[18]दशमोदा॰ B] दशमोदाहरणं J.

[19]त्रिणि B] त्रिण्ह J; तेत्रीस अर्व J] ∅ B; तेत्रीस कोडि B] तेत्तीस कोड J; छत्रीस लाष J] तेत्रीस लाष B; छासठि सहस्र] च्छत्तीसग्-हस्र B, छासठि सहस्र J; च्छ सि B] छ सइ J; तेत्रीस J] तेत्रीसे B.

[20]कंठाभरण B] टंकाभरण J,

[21]एकादशमोदा॰ B] एकादशमोदाहरणं ६० J.

सात।[1] न्यासः। १४२८५७१४३ गुणा ७। लब्धं हाररूपं १०००००००१॥[2] ⟨8.18⟩

एवं प्रत्युत्पन्नः समाप्तः॥[3] ⟨8.19⟩

चतुर्थं भागाहारसूत्रं॥[4] ⟨9.0⟩

प्रश्नादधो हरं न्यस्य प्रश्नं च्छित्वा हरेण च।[5]
भागो हार्यः क्रमान्नूनं भागाहारविधिः स्मृतः॥ ९॥[6]

प्रस्नपद हेठि भाग लिषीइ।[7] अनइ भागि सिउं प्रस्नपद च्छेदीइ।[8] अनइ क्रमि भाग हरीइ।[9] ते निश्चिं भागाहारविधि कहीइ॥[10] ⟨9.1⟩

परसूत्रं।

भागो नास्ति लब्धं शून्यं॥[11] ⟨S3⟩

॥ ⟨9.2⟩

प्रथमोदा॰।[12] द्रम्मा च्यारि सइं अद्याशी भाग च्यारि।[13] न्यास पूर्णगतिः[14]

४	८	८
४	भाग	

[15] भागे पातिते लब्धं द्रम्माः १२२।[16] तथा षंडगति।[17] प्रस्नराशि तथा

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[1]चौद कोडि B] चऊद कोड J; लाष B] लाषं J; सत्तावन B] सतावन J; सु B] सउ J.

[2]१०००००००१ B] १०००००००१ J.

[3]एवं] एव B, एतं J; प्रत्युत्पन्नः B] प्रत्युन्नः J; समाप्तः B] समाप्त J.

[4]चतुर्थं B] अथ चतुर्थं J.

[5]प्रश्नादधो JA] प्रस्नाधो B; प्रस्नं B] प्रश्नं JA; च्छित्वा B] छित्वा JA.

[6]भागो JA] भागा B; विधिः BA] विधि J; ९ B] १० J, ११ A.

[7]हेठि B] हेठइ J; लिषीइ B] लिखीयइ J.

[8]सिउं J] सउं B; च्छेदीइ B] छेदीयइ J.

[9]क्रमि B] क्रमइ J; हरीइ B] हुइ J.

[10]निश्चिं B] निश्चइ J; कहीइ B] कहीयइ J.

[11]नास्ति B] नास्तइ J. A does not have this quarter verse.

[12]प्रथमोदा॰ B] प्रथमोदाहरणं J.

[13]द्रम्मा B] द्रांम J; च्यारि B] च्यार J(twice); सइं B] सइ J; अद्याशी B] अद्यासी J.

[14]न्यास B] न्यासः J; पूर्ण B] पूर्णं J.

[15]J puts this table, without the frame, between *pāti* and *te* of *pātite* after the next box.

[16]भागे पातिते B] भागो पातइते J; द्रम्माः B] द्रम्मा J.

[17]षंड B] खम्द्अ J.

भागाराशि अर्द्ध न्यास[1] $\left|\begin{array}{c}२४४\\२भा\end{array}\right|$ [2] प्रस्न अर्द्ध भाग अर्द्ध।[3] भागे पातिते लब्धं द्रम्माः १२२।[4] एणि रीती त्रिभाग चतुर्भाग करी भाग दीजइ।[5] तेह जि फल आवइ॥ ⟨9.3⟩

द्वितीयोदा॰।[6] द्रम्मा त्रिणि सइ सत्तावीस भाग त्रिणि।[7] न्यासः $\left|\begin{array}{c}३२७\\३भा\end{array}\right|$ लब्धं द्रम्मा १०९॥ ⟨9.4⟩

तृतीयोदा॰।[8] द्रम्मा च्यारि सहस्र छनूं भागे सोल।[9] न्यासः। ४०९६ भाग १६। लब्धं द्रम्मा २५६॥ ⟨9.5⟩

चतुर्थोदा॰।[10] द्रम्मा अग्यार सहस्र छ सइ चुसठि भाग एक सु अट्ठोत्तरर।[11] न्यासः।[12]
B5a ११६६४ भाग १०८। लब्धं $^{\forall}$ भागे द्रम्मा १०८॥[13] ⟨9.6⟩

J3b पंचमोदा॰।[14] द्रम्मा त्रीस सहस्र $^{\forall}$ बि सइ च्छहुत्तरि भाग सत्याशी।[15] न्यासः। ३०२७६ भाग ८७। लब्धं भागे द्रम्मा ३४८॥[16] ⟨9.7⟩

षष्टोदा॰।[17] द्रम्मा एक लाष छपन सहस्र पंचवीस भाग त्रिणि सइं पंचाणूं।[18] न्यासः। १५६०२५ भाग ३९५। लब्धं भागे द्रम्मा ३९५॥ ⟨9.8⟩

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[1]प्रस्नराशि B] प्रथमो रासइ J; भागाराशि B] भागारासइ J; न्यास B] न्यासः J.

[2]भा J] ∅ B.

[3]भाग अर्द्ध J] ∅ B.

[4]पातिते] पाततें B(with *i* added later); लब्धं] ब्धं B(with *la* added later); द्रम्माः B] ∅ J.

[5]एणि] एण B(with *i* added later), इणी J; रीती B] रीतिं J; भाग B] ∅ J; दीजइ J] दीज्जइ B(cor.).

[6]द्वितीयोदा॰ B] द्वितीयो॰ J.

[7]त्रिणि B(1st)] त्रिण्हं J; सत्तावीस B] सत्तावीसं J; त्रिणि B(2nd)] त्रीने J.

[8]तृतीयोदा॰ B] तृतीयो॰ J.

[9]च्यारि B] च्यार J; छनूं B] छिन्नूं J; भागे B] भागो J; सोल B] सोल्हे J.

[10]चतुर्थोदा॰ B] चतु॰ J.

[11]द्रम्मा B] द्राम J; अग्यार] आग्यार B(cor.) इग्यार J; चुसठि B] चउसट्ठि J; एक सु B] १०८ एक सउ J; अट्ठोत्तर J] अछोत्तर B(cor.).

[12]न्यासः B] न्यास J.

[13]लब्धं B] लब्ध J; भागे B] भागो J; द्रम्मा J] द्रमाः B.

[14]पंचमोदा॰ B] पंच॰ J.

[15]द्रम्मा B] द्रम्म J; च्छहुत्तरि B] छिहुत्तरि J; सत्याशी B] सत्यासी J.

[16]द्रम्मा ३४८ B] द्रमा ३८४ J.

[17]षष्टोदा॰ B] षष्टा J.

[18]छपन सहस्र B] छप्पन्न सहस J; त्रिणि B] त्रिण्ह J; सइं B] सइ J; पंचाणूं B] पंचाणू J.

सप्तमोदा॰ ।[1] द्रम्म चौद लाष छासठि सहस्र पांच सइं एकवीसां भाग बार सइ अग्यार ।[2] न्यासः । १४६६५२१ भाग १२११ । लब्धं भागे द्रम्मा १२११॥ ⟨9.9⟩

अष्टमोदा॰ ।[3] द्रम्मा एक कोडि त्राणू लाष पंचितालीस सहस्र च्छ सइ भाग अढार सइ अठोत्तर ।[4] न्यासः ।[5] १९३४५६०० भाग १८०८ । लब्धं भागे द्रम्मा १०७००॥[6] ⟨9.10⟩

नवमोदा॰ ।[7] द्रम्म एक कोडि अनइ त्रिणि भागे तेरह ।[8] न्यासः । १०००००००३ भागे १३ ।[9] लब्धं भागे द्रम्मा ७६९२३१॥[10] ⟨9.11⟩

दशमोदा॰ ।[11] द्रम्मा एक लाष एकोत्तर भाग अग्यार ।[12] न्यासः । १००००१ भाग ११ । लब्धं भागे द्रम्मा ९०९१॥ ⟨9.12⟩

एवं भागाहारः समाप्तः॥[13] ⟨9.13⟩

अथ पंचमं अनेकार्थसूत्रं ।[14] अनेकार्थि वर्ग घन वर्ग्रमूल भिन्नप्रत्युत्पन्न त्रैराशिक विस्त-त्रैराशिक प्रक्षेपकरण सुवर्णमान क्षेत्रवस्त्रमान षातकाष्टपाषाणकोष्टागारचितिमान वर्त्तुलकाष्ट-पाषाणस्तंभकूपमानं गोलकमान कणराशिमान च्छायामान दिनमान मध्यपादमान इत्यादि व्यवहार क्रमि कहीसिं॥[15] ⟨10-26.0⟩

वर्ग्रे पूर्वार्द्धं॥ ⟨10_1.0⟩

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[1]सप्तमोदा॰ B] सप्तमो॰ J.

[2]द्रम्म B] द्रम्मा J; चौद B] चऊद J; छासठि B] छासट्ठि J; सइं B] सइ J; एकवीसां] पकवीसां B(cor.), इकवीस J; बार B] १२ बारइ J; सइ B] सो J; अग्यार B] इग्यार J.

[3]अष्टमोदा॰ B] अष्ट॰J.

[4]पंचितालीस B] पचतालीस J; अठोत्तर J] अठोत्तरः B.

[5]न्यासः J] न्यासाः B.

[6]भागे] ∅ BJ.

[7]नवमोदा॰ B] नवमो J.

[8]द्रम्म B] द्रम्मा J; कोडि J] केडि B; अनइ B] अनइं J; भागे J] भाग B; तेरह J] तेरे B.

[9]भागे J] भाग B.

[10]भागे B] ∅ J; द्रम्मा B] द्रमा J.

[11]दशमोदा॰ B] दसमो॰ J.

[12]अग्यार B] इग्यार J.

[13]भागाहारः B] भागाहार J.

[14]अथ पंचमं J] पंचमो B.

[15]अनेकार्थि B] अनेकोर्थ J; वर्ग घन वर्ग्रमूल B(*rvargra* for the 2nd *varga*)] वर्गमूल घन वर्ग J; विस्त J] विस B; सुवर्णमान B] सुवर्णमानं J; चिति B] चित J; कूपमानं B] कूपमान J; गोलक B] गोल J; कणराशि J] कणशशि B(cor.); क्रमि B] क्रमइं J; कहीसिं B] कहीस्यइं J.

तुल्यराशिद्वयोर्घाते वर्ग्रो भवति केवलं॥ १०_१॥[1]

तुल्य शरीषी बि राशि गुणीइ।[2] ते वर्ग्र हुइ॥[3] ⟨10_1.1⟩

उदा०।[4] एक बि त्रिणि च्यारि पांच पनर पंचवीस बि सइ पांत्रीस ना वर्ग्र किम हुइ।[5]

न्यासः

१	२	३	४	५	१५	२५	२३५
१	२	३	४	५	१५	२५	२३५

[6] लब्धं वर्ग्राः १। ४। ९। १६। २५। २२५। ६२५। ५५२२५॥[7] ⟨10_1.2⟩

B5b ∀ इति वर्ग्रः॥[8] ⟨10_1.3⟩

घने अपरार्द्धं॥ ⟨10_2.0⟩

पदत्रयाणां तुल्यानां वधे नूनं घनो भवेत्॥ १०_२॥[9]

तुल्य शरीषां त्रिणि पद गुणीइ।[10] घन हुइ॥[11] ⟨10_2.1⟩

उदा०।[12] एक बि त्रिणि चारि पांच पंनर पंचवीस ना घन किम हुइं।[13] न्यासः

१	२	३	४	५	१५	२५
१	२	३	४	५	१५	२५
१	२	३	४	५	१५	२५

[14] लब्ध घना।[15] १। ८। २७। ६४। १२५। ३३७५।

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[1]राशिद्वयोर्घाते] राशिद्वयोघाते BJ, राशियोद्वयोर्घाते A; ॥ १०_१॥] ॥ ९॥ B, ११ J, ∅ A.

[2]शरीषी B] सरीषी J; बि B] छि J.

[3]हुइ] हुइ न्यासः B, हुयइ J.

[4]उदा० B] उदाहरणं J.

[5]एक B] −वं J(1st letter illegible); त्रिणि B] त्रि J; पंचवीस B] पचवीस J; वर्ग्र] कर्ग्र B(cor.), वर्ग J.

[6]J puts this table, with a top line but without the medial line, at the end of 10_1.3.

[7]वर्ग्राः १।] वर्ग्रः॥ छ॥ १। B, वर्गाः॥ १। J.

[8]इति B] इतइ J.

[9]त्रयाणां JA] त्रियाणां B; १०_२] १० B, ∅ J, १२ A.

[10]शरीषां B] सरीषां J.

[11]घन हुइ] ∅ B, घन हुयइ J.

[12]उदा० B] उदाहरणं J.

[13]त्रिणि B] त्रिण J; चारि B] च्यार J; पंनर B] पनर J; ना B] नी J; हुइं B] हुयइ J.

[14]J puts this table, with a top line, at the bottom right corner of fol. 3b between *dviguṇenā* and *paraṃ* in verse 11b.

[15]घना B] घनाः J.

१५६२५ ॥[1] ⟨10_2.2⟩

इति घनः ॥[2] ⟨10_2.3⟩

वर्गमूलं ॥[3] ⟨11-12.0⟩

वर्गं संत्यज्य विषमात् द्विगुणेन परं भजेत् ।[4]
लब्धं निवेशयेत्पंक्त्यां तद्वर्गं परिशोधयेत् ॥ ११ ॥[5]
पूर्ववत् द्विगुणीकृत्य तदुत्सार्य परं भजेत् ।[6]
एवं परे पि कर्त्तव्यं दलयेत् द्विगुणीकृतं ॥ १२ ॥[7]

विषम सम विषम । विषमपद थिकी वर्ग पाडीइ ।[8] प$^{\forall}$ छइ ते वर्गमूल बिमणु करी पर आगिला आंक हेठि लिषीइ ।[9] [10]अनइ तिहां पुहतु जोइ भाग पाडीइ ।[11] पच्छइ ते लब्ध हुइ । ते पंक्ति नु लिषीइ ।[12] अनइ तेह नु वर्ग्र पाडीइ ।[13] पछइ पहिली परि लब्ध नु आंक बिमणु करी तिहां थिकु ऊपाडी पर आगिला आंक हेठि लिषीइ ।[14] एणं रीति आगलि इमि जि चलावीइ ।[15] पछइ बिमणा कीधाइ हु तेह नूं अर्द्ध कीजइ ।[16] वर्ग्रमूल हुइ ॥ ⟨11–12.1⟩ J4a

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[1]१५६२५ J] ११५६२५ B.

[2]घनाः B] घना J.

[3]वर्ग B] अथ वर्ग J.

[4]वर्गं BJ] वंर्गं A; संत्यज्य A] सत्यज्य B, संत्यज J; विषमात् द्विगुणेन परं] विषमात् द्विगुणानपरं B, विषमात् द्विगुणेनापरं J, विषमाद्विगुणेन परं A.

[5]निवेशयेत् BJ] विनिवेशयेत् A; तद्वर्गं JA] तद्वंर्ग B; परिशोधयेत् BJ] परतस्त्यजेत् A; ११ BJ] १३ A.

[6]पूर्ववत् द्विगुणी BJ] पूर्ववद्विगुणं A; तदुत्सार्य A] तदुत्थार्थं B, तदुत्थार्य J; भजेत् BJ] भवेत् A.

[7]परे A] परो BJ; दलयेत् द्वि BJ] दलयेद्धि A; १२ BJ] १४ A.

[8]थिकी B] थकी J; पाडीइ B] पाडीयइ J(hereafter also).

[9]वर्गमूल] वर्ग BJ; बिमणु B] बिमणुं J; लिषीइ B] लिखीयइ J.

[10]J omits the passage up to the next *liṣīi*.

[11]जोइ] जोई B(cor.).

[12]नु] नुलि B.

[13]अनइ B] अनइं J; नु B] नुं J.

[14]पछइ B] अनइं J; नु B] नउ J; बिमणु B] बिमणुं J; थिकु B] थकी J; ऊपाडी B] ऊपाडीयइ J; लिषीइ B] लिखीयइ J.

[15]एणं रीति B] एणी रीतिं J; इमि जि B] इम ज J; चलावीइ B] चलावीयइ J.

[16]कीधाइ हु B] कीधा हुयइं J; नूं B] नुं J.

उदा॰ ।[1] प्रथमोक्तवर्ग्रफलानां न्यासः । १ । ४ । ९ । १६ । २५ । २२५ । ६२५ । ५५२२५ ।[2] लब्धानि मूलानि १ । २ । ३ । ४ । ५ । १५ । २५ । २३५ ॥ ⟨11–12.2⟩

इति वर्ग्रमूलं ॥ ⟨11-12.3⟩

भिन्नप्रत्युत्पन्नः ॥[3] ⟨13.0⟩

रूपं रूपैः समं गुण्यं छेदांशावपि तैः समं ।[4]
परस्परं तदंशौ च भिन्नोत्पन्नफलं लभेत् ॥ १३ ॥[5]

भिन्नप्रत्युत्पन्नि रूप रूप सिउं गुणीइ ।[6] पछइ छेद अंश रूप सिउं गुणीइ ।[7] पच्छइ छेद अंश अन्योन्यइ गुणीइ ।[8] फल आवइ ॥ ⟨13.1⟩

प्रथमोदा॰ ।[9] अत्र जेष्टांगुल गजमानं ।[10] गजि अंगुल चुवीस हुइ ।[11] क्रीत ध्वज १ दीर्घगज
B6a दस अंगुल बार विस्तरगज एक अंगुल आठ ।[12] न्या$^{\forall}$सः | १० ॥ दीर्घ / १ ।२ विस्तर |[13] दश गजे

[1]उदा॰] त्रदा B(cor.), उदाहरणं J.

[2]२५ । J] २२ । B(cor.); ५५२२५ । J] ५५२०५ । B.

[3]भिन्नप्रत्युत्पन्नः B] भिन्नप्रत्युत्पन्नप्रत्युत्पन्नसूत्रं J.

[4]छेदांशावपि] च्छिदांशावपि B, छेदांशाः वप J; तैः समं] तेः सम B, तैसञ ? J.

[5]परस्परं] परस्पर BJ; तदंशौ B] तदंशो J; भिन्नोत्पन्न J] भिन्नोत्पन्ने B. In A, the corresponding verse (15) gives a different rule. See the Note for PV 13 in the Annotated Translation.

[6]प्रत्युत्पन्नि B] प्रत्युत्पन्नं J; रूप (1st) B] रूपं J.

[7]छेद अंश] छेद BJ.

[8]पच्छइ B] पछइ J; अन्योन्यइ B] अन्योन्य J.

[9]प्रथमोदा] प्रथमोउदा B(cor.), प्रथमोदाहरणं J.

[10]मानं B] मान J.

[11]गजि B] गंजर J; अंगुल J] अंगुले B; चुवीस B] चउवीस J.

[12]ध्वज] गज BJ; बार B] १२ J; विस्तरगज J] विस्तरगजग B(cor.); एक] ए B(with *ka* in margin), १ J; आठ B] ८ J.

[13]J puts only "10 dīrgha" here and the entire table after *gaja arddha* in the 2nd line below in place of "0‖." The number of the *akṣaras* between the table and "0‖" is 46, which must be approximately the length of the line of writing of the parent manuscript. This number is less than another estimation (49) obtained from a similar situation in 16.2 by only 3. ॥ J] । B; २ B] ∅ J; विस्तर J] विस्त B.

एक गज गुणीइ।[1] जात गज १०।[2] तथा बार आंगुले एक गज गुणीइ।[3] जात अंगुल १२।[4] तेषां जात गज अर्द्ध ०॥ ।[5] तथा अठ आंगुले दश गज गुणीइ।[6] जात अंगुल असी ८०।[7] तेषां जात गज त्रिणि अंगुल आठ।३। ।२।[8] तथा बार आंगुले आठ आंगुल गुणीइ।[9] जात व्यंगुल छनूं ९६।[10] तेषां जात अंगुल च्यारि ०।४।[11] एकत्र नियोजने लब्ध गजाः १४॥[12] ⟨13.2⟩

द्वितीयोदा०।[13] अत्र विसा गजमानं।[14] गजिं विसा वीस २० हु।[15] क्रीत भूमि दीर्घगज पंचवीस विसा चौद।[16] तथा विस्तरगज सोल विसा दस।[17] न्यासः

२५ ॥४ दीर्घ
१६ ॥ विस्तर

[18]

पंचवीस गजे सोल गज गुणीइ। जात गजा ४००।[19] तथा चौद विसे सोल गज गुणीइ।[20] जात विसा बि सइं चुवीस २२४।[21] तेषां गज जातं ११ । ४।[22] तथा पंचवीस गजे दस

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[1]दश B] दशे J; गज J] गड B.

[2]जात B] जातं J.

[3]बार आंगुले] बार आगुले B, आंगुल बारी? J; एक J] गक B(cor.).

[4]जात B] जातं J.

[5]जात B] जातं J. J replaces "0‖" with the above table.

[6]अठ B] आठ J; आंगुले J] आंगुणे B(cor.).

[7]जात B] जातं J; अंगुल J] अंगुणी B(cor. to *aṃgule*); असी B] ∅ J.

[8]आठ।३॥२ B] ८ ३ २ J. After this B inserts: तथा बार आंगुले आठ।३॥२.

[9]बार आंगुले B] बारे J.

[10]जात B] जाछं J; छनूं B] छिन्नू J.

[11]तेषां जात B] तेषा जातं J; अंगुल J] अंगुण B; च्यारि ०।४। B] ४ J.

[12]लब्ध गजाः B] लब्धं गज J. After नियोजने , J has the table

२.	०.	८.	१६.	२.

[13]द्वितीयोदा० B] द्वितीयो० J.

[14]विसा] चिशा B, विश्वा J(hereafter also).

[15]वीस B] ∅ J; हु B] हुयइ J.

[16]चौद B] १४ J.

[17]सोल B] १६ J; दस B] १० J.

[18]४ B] ∅ J.

[19]जात गजा B] जातं गज J.

[20]चौद B] चऊद J; विसे B] विश्वा J; गज B] गजे J.

[21]जात B] जातं J; सइं B] सइ J; चुवीस B] चउवीस J.

[22]तेषां गज जातं ११ । ४] ∅ B, तेषां गज जातं ११८४ J.

विसा गुणीइ।[1] जात विसा बि सइं पंचास २५०।[2] तेषां जातं गज १२॥।[3] तथा चौद विसे दश विसां गुणीइ।[4] जात विसांसा एक सु च्यालीस १४०।[5] तेषां जात विसा ०।२ ।[6] एकत्र नियोजने लब्धं गजा ४२४ ।१।[7] इणी रीति वस्त्र तथा क्षेत्र मवीइं॥[8] ⟨13.3⟩

इति भिन्नप्रत्युत्पन्नः॥[9] ⟨13.4⟩

त्रैराशिके पूर्वार्द्धं॥[10] ⟨14_1.0⟩

मध्याद्ययोर्वधः पूर्वमन्त्यभागस्त्रिराशिके॥ १४_१॥[11]

J4b त्रैराशिकि आदि अंक मध्य सिउं गुणीइ।[12] अनइ अंत्य सिउं ∀ भाग दीजइ।[13] फल आवइ॥ ⟨14_1.1⟩

प्रथमोदा०।[14] व्रीह पांच मण एकवीस रामि लाभइ।[15] तिहां नौ मण ना द्राम केतला हुइ।[16] न्यासः।[17] ५ । २१ । ९० । लब्ध द्रम्मा | ३७८ | ॥ ⟨14_1.2⟩

द्वितीयोदा०।[18] मुग त्रेवीसि रामि च्छ मण लाभइ।[19] तिहां एकासी राम ना केतला

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[1]तथा ... गुणीइ] ∅ B, तथा पंचवीस २५ गज दशे विश्वे गुणीयइ J.

[2]जात ... २५०] ∅ B, जातं विश्वा २५० J.

[3]तेषां B] ∅ J; जातं J] ∅ B.

[4]चौद B] चऊद J; विसे B] विश्वे J; दश B] दस J.

[5]जात B] जातं J; विसांसा (for this word see BBA 17_1.3)] विसा B, विश्वा J; सु B] सउ J.

[6]तेषां B] का तेषां J; जात B] जातं J; ०।२ B] सात ७ खरा। स? ०।२ J.

[7]गजा B] गज J; ४२४ ।१] ४२४१ B, ४२४८७ J.

[8]मवीइं B] मवीइ J.

[9]भिन्न B] भन्न J; प्रत्युत्पन्नः B] प्रत्युत्पन्न J.

[10]त्रैराशिके] त्रैशशिके B(cor.), अथ त्रैराशिके J; पूर्वार्द्धं B] पूर्वार्द्ध J.

[11]मध्याद्ययोर् B] मध्यद्ययोर् J; पूर्वमन्त्य B] पूर्व अंत्य J; दीजइ। फ after भाग B(canceled), ∅ J; १४_१] १३ BJ, ∅ A. A (verse 16ab) reads: मध्यांत्ययोर्वधः पूर्वमादिभागस्त्रिराशिके।

[12]अंक B] आंक J; सिउं J] सिउ B.

[13]अनइ B] अनइं J; सिउं] सिउ B, सुंभा J; दीजइ B] दीजि J.

[14]प्रथमोदा० B] प्रथ० J.

[15]रामि] राशि B, राषि J; लाभइ B] लाभि J.

[16]नौ B] निऊ J; द्राम B] इम J.

[17]न्यासः B] न्यास J.

[18]द्वितीयोदा० B] द्विती० J.

[19]मुग B] मूंग J; त्रेवीसि B] २३ J; रामि B] रामिं J; च्छ B] छ J; लाभइ B] लाभि J.

हुइ।[1] न्यासः। २३। ६। ८१। लब्धं मण २१ सेर ५ भागश्च[2] $\left|\begin{array}{c}५\\२३\end{array}\right|^{3}$ ॥ ⟨14_1.3⟩

इति त्रैराशिकं॥ ⟨14_1.4⟩

विस्तत्रैराशि$^{\forall}$ के अपरार्द्ध॥[4] ⟨14_2.0⟩ B6b

मध्यांत्ययोर्वधो व्यस्त आदिभागः फलं लभेत्॥ १४_२॥[5]

विस्तत्रैरासिकि अंत्य अंक मध्य सिउं गुणीइ।[6] अनइ आदि सिउं भाग दीजइ।[7] फल आवइ॥ ⟨14_2.1⟩

प्रथमोदाहरणं।[8] अशी वरस नी कमारी च्यालीस टंके लाइ।[9] तिहां सोल वरस नी केतला लहिइ।[10] न्यासः। ८०। ४०। १६। लब्धं षोडशवर्षा दासी मूल्यं टंका २००॥[11] ⟨14_2.2⟩

द्वितीयोदा॰।[12] मोक्तीक सु चडतु टांक पंचवीसि रामि लाभइ।[13] तिहां साठि चडतु टांक कसिउ लहइ।[14] न्यासः। १००। २५। ६०।[15] लब्धं द्रम्मा ४१ जथल २॥[16] ⟨14_2.3⟩

इति विस्तत्रैराशिकं॥[17] ⟨14_2.4⟩

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[1]हुइ J] इहु B(cor.).

[2]२१ J] १२ B(cor.); सेर J] सेरे B.

[3]J has ० above ५.

[4]त्रैराशिके B] त्रैराशिकं J; अपरार्द्ध B] ∅ J.

[5]J omits this hemistich. व्यस्त] व्यस्ते B; भागः] भाग B; १४_२] १४ B. A (verse 16cd) reads: मध्यादयोव्यधो व्यस्ते प्रांत्यभक्तः फलं भवेत्। १६।

[6]विस्तत्रैरासिकि B] ∅ J; सिउं B] स्युं J; गुणीइ] गुणीइं B, गुणीयइ J.

[7]सिउं] सिउ B, सुं J. J has 4 illegible letters before भाग.

[8]प्रथमोदाहरणं] प्रथमोदारणं B, प्रमो J.

[9]अशी B] असी J; टंके B] टंका J; लाइ B] लाभइ J.

[10]लहिइ B] लाभइ J.

[11]दासी J] द्राशी B; मूल्यं J] मूलं B.

[12]द्वितीयोदा॰ B] द्वियोः J.

[13]मोक्तीक B] मौक्ति नु J; सु B] सउ J; चडतु B] चडतउ J; टांक B] टंका J; रामि B] रामइ J; लाभइ B] लाभि J.

[14]साठि J] सावि B; चडतु B] चडतो J; कसिउ लहइ B] किम लाभइ J.

[15]६० J] ६ B.

[16]जथल B] जयथल J.

[17]विस्त B] व्यस्त J.

प्रक्षेपकरणं॥ ⟨15.0⟩

नानाप्रक्षेपरूपाणामुत्पन्नेन वधः क्रमात्।[1]
युतिभागाद्वयी लब्धी फलं स्यात्तत्पृथक्पृथक्॥ १५॥[2]

नाना अनेकविध प्रक्षेप भाग रूप नइं उत्पन्न द्रव्य सिउं वधु गुणाकारु कीजइ।[3] अनइ प्रक्षेपरूप नी युति कीजइ।[4] अनइ तेणं सिउं क्रमतु भाग दीजइ।[5] जूजूयां फल आवइ॥[6] ⟨15.1⟩

तथा द्वितीय परि। नानाप्रक्षेपरूप नी युति कीजइ।[7] अनइ तेणं सिउं उत्पन्न द्रव्य नइं भाग दीजइ।[8] एकविभाग नू फल आवइ।[9] पच्छइ जेह नइं जेतला विभाग हुइ तेह नइ तेतला गुणा कीजइ।[10] एतलिइ जूजूयां फल आवइं॥[11] ⟨15.2⟩

प्रथमोदा॰।[12] क्यारि चिहु बीज सेई बि त्रिणि च्यारि पांच वावी बि सइ दस ऊपनी।[13] किहां केतला हऊआ।[14] न्यासः। बीज सेइं २। ३। ४। ५।[15] उत्पंन्न[16] | २१० | प्रथम परि। बि सइ दस बिगुणा जात च्यारि सइ वीसां ४२०।[17] बि सइ दस त्रिगुणा जात छ सइ त्रीसां

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[1]वधः क्रमात् BJ] फलेन ते A.

[2]भागाद्वयी लब्धी] भागाद्वया लब्धी B, भागावया लब्धि J; पृथक्पृथक्] प्रथक् पृथक् B, पृथक् २ J. A (verse 17cd) reads: गुण्याः स्वयोगभक्तास्ते खंडानि स्युः पृथक्पृथक्। १७।

[3]विध B] विधि J; प्रक्षेप B] प्रक्षोप J; रूप B] रूपा J; नइं B] नइ J; सिउं B] संउ J; वधु गुणाकारु कीजइ] ∅ J.

[4]अनइ ... कीजइ B] ∅ J; युति] मुति B(cor.).

[5]अनइ तेणं सिउं B] ∅ J; क्रमतु B] क्रमनउ J.

[6]जूजूयां] हूजूयां B, जूजूआ J.

[7]युति B] युक्ति J.

[8]तेणं सिउं] ∅ BJ; नइं] सिउं BJ; दीजइ B] दीजि J.

[9]नू B] नुं J; आवइ B] अरवइ J.

[10]पच्छइ B] पचइ J; नइ B] नइं J.

[11]एतलिइ B] एतलइ J; जूजूयां B] जूजूआ J; आवइं B] आवइ J.

[12]प्रथमोदा॰] प्रथमो॰ B, प्रथः J.

[13]चिहु B] चिहुं J; च्यारि J] च्यारे B.

[14]हऊआ B] हूआ J.

[15]सेइं B] सेई J.

[16]उत्पंन्न B] उत्पन्न बि सइ ने दस J.

[17]जात B] जातं J; वीसां B] बीस J.

६३०।[1] बि सइं दस चुगुणा जात आठ सइ च्यालीसां ८४०।[2] बि सइ दस पांचगुणा जातं दस सइं पंचासां १०५०।[3] ∀ पछइ बीज नी युति सेइ चौद १४।[4] तेणं सिउं भाग।[5] भागे B7a
कृते लब्धं पृथक्पृथक्फलं ३०। ४५। ६०। ७५॥[6] ⟨15.3⟩

द्वितीय परि। बीज नी युति सेई चौद १४।[7] उत्पन्न सेई बि सई दस २१०।[8] भागे लब्धं एकविभाग फल सेई १५।[9] बिगुणा ३० त्रिगुणा ४५ चुगुणा ६० पांचगुणा ७५॥[10] ⟨15.4⟩

द्वितीयोदा०।[11] ग्रामिज मइ टंका सोल सइं।[12] विजेदार त्रिणि सइ नु पांचसइ नु सहस्र नु।[13] ऊपनां बि सइ टंका।[14] कहि नइ केतला हुइ।[15] न्यासः।[16] विजेहदार[17]
| १०० | ५०० | १००० | ∀ उत्पन | २०० |[18] प्रथम परि। बि सइं सइगुणा जात वीस सहस्र[19] J5a
| २०००० |[20] बि सइ पांचसइगुणा जात लाष १००००००।[21] बि सइ सहस्रगुणा जात बि लाषः

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[1]जात B] जातं J.

[2]सइं B] सइ J; चु B] चउ J; जात B] जातं J; आठ सइ च्यालीसां ८४० J] ८४० B(in margin).

[3]बि सइ दस पांचगुणा जातं J] बे से द पांचगुणा B(in margin); दस सइं] दसइं B, दस सइ J; पंचासां] पंचांसों B, पचासां J.

[4]सेइ B] सेई J; चौद B] ∅ J.

[5]तेणं सिउं B] तिणां सुं J.

[6]भागे कृते] भागोकृते B, भागोत्तर J; पृथक्पृथक्] पृथक२ BJ; फलं B] फल J.

[7]चौद B] सउदइ J, which has त्तऊदइ above these letters.

[8]सई B] सइ J; दस J] ∅ B.

[9]भागे B] भागो J; लब्धं B] लब्ध J; एक J] पक B; फल B] ∅ J.

[10]चु B] चउ J.

[11]द्वितीयोदा० B] द्वितीं० J.

[12]ग्रामिज B] ग्रामइज J; मइ B] म J; सइं B] सइ J.

[13]विजेदार B] विजैदारु J; त्रिणि B] त्रिणइ J; सइ B] सउप J; नु (3rd) J] तु B.

[14]ऊपनां B] ऊपना J.

[15]कहि B] कि J.

[16]न्यासः B] न्यास J.

[17]विजेहदार B] विजइदारु J.

[18]| १०० | ५०० | १००० | उत्पन | २०० |] | १०० | ५०० | १००० | उत्पन | २०० | B, १००। ५००। १०००। उत्पन्न २००। J.

[19]सइं B] सइ J; सइगुणा B] सउगुणा J; वीस J] विस B; सहस्र] सहास्र B(cor.), सहस्र J.

[20]Without a bottom for the box, J.

[21]पांचसइ B] पांचसउ J.

२००००० ।[1] विजेहदार ना भाग नी युति जात टंका सोल सइ १६०० ।[2] तेहे भाग ।[3] भागे कृते पृथक् लब्धं टंका[4] | १२॥ | ६२॥ | १२५ |[5] ॥ ⟨15.5⟩

द्वितीय परि ।[6] विजेहदार ना भाग नी युति टंका १६०० ।[7] उत्पन टंका २०० ।[8] तेसां जयस्थला ९६०० ।[9] षोडशशतेन भाग । लब्धं टंका प्रति जयथल ६ ।[10] सितगुणिते टंका १२॥ ।[11] पंचसितगुणिते टंका ६२॥ ।[12] सहस्रगुणिते जात १२५॥[13] ⟨15.6⟩

इति प्रक्षेपकरणं॥[14] ⟨15.7⟩

सुवर्णलेष्यकं॥[15] ⟨16.0⟩

नानासुवर्णवर्णानामैक्यं स्वर्णेन भाज्यते ।[16]
लब्धांको वर्णिका ज्ञेया सुवर्णं वर्णिकाहृतं॥ १६॥[17]

जूजूया सुवर्ण ना वाना एकत्र करी सुवर्ण सिउं भाग दीजइ ।[18] तु वानी आवइ ।[19] वानी

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[1]जात B] जातं J; लाषः B] लाष J.

[2]विजेदार B] विजैदार J; जात टंका सोल सइ B] ∅ J.

[3]तेहे B] तेह J.

[4]कृते B] कृत् J.

[5]With a top horizontal line for the box, J. १२॥ J] १२ ॥ B.

[6]परि B] ∅ J.

[7]विजेहदार B] विजेदार J; ना B] ∅ J; १६०० B] १६० J.

[8]उत्पन B] उत्पन्न J.

[9]तेसां B] तेषां J.

[10]लब्धं B] लब्ध J.

[11]सित B] शत J. After टंका B repeats: प्रति जयथल ६ सितगुणिते टंका

[12]पंच J] पंचं B; सित B] शत J; ६२॥ B] ६२ J.

[13]गुणिते J] गुणितं B; जात B] जाते J.

[14]करणं B] करण J.

[15]सुवर्ण B] अथ सुवर्ण J; लेष्यकं B] लेषकं J.

[16]वर्णानाम् B] वण्णानाम् J(reduplication of *ṇ* after *r* hereafter also); स्वर्णेन] स्वण्र्येन B(cor.), स्वण्र्णेन J. A (verse 18ab) reads: सुवर्णवर्णघात्यैक्यं स्वर्णयोगेन भाजयेत् ।

[17]लब्धांको] लब्धांके B, लब्धांकै J; वर्णिका J(*rṇṇi*)] चर्णिकार B(*ra* canceled); ज्ञेया J] ज्ञैया B; वर्णिकाहृतं] वर्ण्णिकातूडतं B, वर्ण्णिकाद्वितं J. A (verse 18cd) reads: लब्धांके वर्णका ज्ञेया वर्णोद्धृत्ते स वर्णकः । १८ ।

[18]जूजूया B] जूजूआ J; सुवर्ण (1st) B] सोवर्ण्ण J; एकत्र करी] एकरी B, कीजइ J; सिउं B] सुं J.

[19]तु वानी आवइ ।] ∅ BJ.

सिउं भाग दीजइ।[1] तु सुवर्ण लाभइ॥[2] ⟨16.1⟩

उदा॰।[3] गदीयाणा आठ वानी बारही वाना च्छनो।[4] तथा गदीयाणा च्यारि वानी चौदी वाना छपन।[5] न्यासः

गदी ८	वानी १२	वाना ९६
गदी ४	वानी १४	वाना ५६

[6] नियो$^{\forall}$जने गदीयाणा १२ वाना B7b

१५२।[7] वर्णानां सुवर्णेन भागः।[8] लब्धं वानी १२ भागश्च[9]

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३

[10] तथा पङ्क वर्ण तेर १३।[11]

तेन भागः।[12] लब्धं सुवर्ण गदी॰ ११ भागश्च[13]

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१३

॥ ⟨16.2⟩

इति सुवर्णलेष्यकं॥[14] ⟨16.3⟩

क्षेत्राणां मानं। क्षेत्ररूप नव।[15] चतुरश्रे पूर्वार्द्धं॥[16] ⟨17_1.0⟩

चतुरश्रे समे दीर्घे भुजकोटिवधः फलं॥ १७_१॥[17]

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[1]वानी सिउं भाग दीजइ।] ∅ BJ.

[2]तु B] तउ J.

[3]उदा॰ B] उदाहरणं J.

[4]गदीयाणा] गदीयायाणा B, गदीआणा J; आठ B] अठ J; बारही J] बाहरी B; च्छनो B] छन्नू J.

[5]गदीयाणा B] गदीआणा J; च्यारि J] चारि B; चौदी B] चऊदई J; छपन] थपन B(cor.), सपन J.

[6]J places the first row here and the second row between पद्यु and वर्ण in the next line (both without a frame). It means that the passage from ग ८ (for गदी ८) to पद्यु (for पङ्क or पङ्क), consisting of 49 *akṣaras*, constituted a line of writing in the parent manuscript of J. For another similar estimation see 13.2 above. गदी B] ग J(twice); वानी (1st) B] वांनी J; वाना (2nd) B] वानी J.

[7]नियोजने B] नियोने J; गदीयाणा B] गदीआणा J; वाना J] वानी B; १५२ B] ११५२ J.

[8]भागः B] भाग J.

[9]लब्धं B] लब्ध J; भागश्च B] भागाश्च J.

[10]३ B] ∅ J.

[11]पङ्क] पद्यु BJ; तेर १३] तेर१३ B, तेरा१३ J.

[12]भागः B] भाग J.

[13]गदी॰ B] गदीआणा J.

[14]लेष्यकं B] लेषकं J.

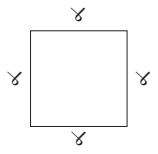
[15]नव B] नेथ J(error?).

[16]चतुरश्रे B] चतुरस्रे J; पूर्वार्द्धं B(in margin), पूर्वार्द्ध J.

[17]चतुरश्रे J] ∅ B(added in margin), चतुरस्रे A; १७_१] १६ BJ, ∅ A.

समचुरसि लांबइ दीर्घ विस्तर सिउं गुणीइ।[1] फल आवइ॥ ⟨17_1.1⟩

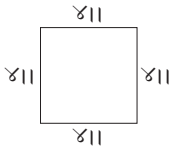
प्रथमोदाहरणं।[2] सम दीर्घ गज च्यारि विस्तर गज च्यारि।[3] रूपन्यासः



[4]⟨Figure 3⟩[5]

च्यारि चुकू सोल।[6] लब्धं गज १६॥[7] ⟨17_1.2⟩

भिन्नः। दीर्घ गजा साढा च्यारि विस्तर गज साढा च्यारि।[8] रूपन्यासः[9]



⟨Figure 4⟩[10]

भिन्नप्रत्युत्पन्नवत्। विसा गजमानं। च्यारि चुकू १६।[11] तथा दस विसा चुगुणा जात विसा ४०।[12] तेषां जात गज २।[13] [14]तथा दस विसा चुगुणा जात विसा ४०।[15] तेषां जात गज

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[1]चुरसि] वुरसि B, चउरंस J.

[2]दाहरणं J] दारणं B(with *ha* in margin).

[3]च्यारि (twice) B] ४ J.

[4]Here and hereafter, B puts every figure in an open box (⌴); J puts Figs. 3–8 and Figs. 10–12 in closed boxes (□) and the rest in open boxes. See Appendix.

[5]J places this figure between सि and हु of सिहु (for चिहु) in the next example.

[6]च्यारि चुकू सोल। B] च्यारइ चिहुं। १६। J.

[7]गज १६ J] गजा १४ B.

[8]गजा B] गज J; च्यारि (1st) B] च्यार ४॥ J; विस्तर गज साढा च्यारि B] ∅ J.

[9]न्यासः B] न्यासः॥ ४॥। J.

[10]J places this figure between तेषां ४॥ and जाते गज २ in the next line of writing.

[11]च्यारि चुकू] व्यारि वुकू B, च्यारइ सिहु J(with an irregular letter like *ma* without a *mātrā* line after *hu*).

[12]तथा J] था B(with *ta* between lines); दस B] वस J; विसा B] वइश्वा J; चुगुणा] चु।गुणा B(cor.), चउणा J; जात B] जेते J; ४० J] ४ B(with "0" between lines).

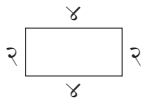
[13]तेषां जात B] तेषां। ४॥ जाते

[14]B omits the next two sentences, तथा ...। ... गज २।

[15]तथा J] ∅ B; दस विसा] ∅ B, दस विश्वा J; चुगुणा] ∅ B, चउगुणा J; जात विसा ४०] ∅ BJ.

२।[1] दस विसा दश विसा सिउं गुणीइ।[2] जात विसांसा १००।[3] तेषा जात विसा पांच ५।[4] संयोजने जात गज सवा वीस २०। ॥[5] ⟨17_1.3⟩

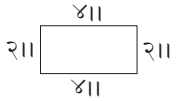
द्वितीयोदा॰।[6] दीर्घ गज च्यारि विस्तर गज २।[7] रूपन्यासः



⟨Figure 5⟩

च्यारि दू आठ।[8] लब्धं गजा ८॥[9] ⟨17_1.4⟩

भिन्नः।[10] दीर्घ गज साढा च्यारि विस्तर गज साढा बि २॥ ।[11] रूपन्यासः



⟨Figure 6⟩

पूर्वभिन्नवत् गुणने लब्धं गजा सवाग्यारः $\boxed{११।}$॥[12] ⟨17_1.5⟩

विषमचतुरस्रे अपरार्द्धं॥[13] ⟨17_2.0⟩

वसुधामुखयोगार्द्धं लंबघ्नं विषमे फलं॥ १७_२॥[14]

विषमि चुरसि भूमि अनइ मुख ना गज एकत्र करी अर्द्ध कीजइ।[15] पच्छइ ते लंब सिउं

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[1]तेषां] ∅ BJ; जात] ∅ B, जाते J; गज J] ∅ B; २] ∅ B, १ J.

[2]दश विसा सिउं B] ∅ J.

[3]जात B] जाते J; विसांसा B] विश्वासुं J.

[4]तेषा B] तेषां J; जात B] जाते J; पांच B] ∅ J.

[5]जात B] जाते J; सवा J] संवा B(cor.); २०। J] २० B.

[6]द्वितीयोदा॰ B] द्वितीयोदाहरणं J.

[7]च्यारि B] च्यार J; २ B] बि J.

[8]च्यारि B] च्यार J.

[9]गजा B] गज J.

[10]भिन्नः] ∅ BJ.

[11]दीर्घगज J] दीर्घग B; च्यारि B] च्यार J; साढा बि J] ∅ B.

[12]गजा B] गजः J; सवाग्यारः B] ८। इग्यार २ J; $\boxed{११।}$] $\boxed{११॥}$ B(cor.), ॥११॥। J.

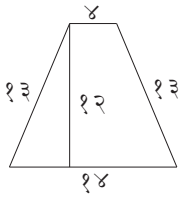
[13]विषम J] चिषम B(cor.); चतुरस्रे] चतुस्रे B(with *ra* in margin), चतुरश्रे J.

[14]घ्नं JA] ध्नं B; १७_२] १७ BJ, १९ A.

[15]विषमि B] विषम J; चुरसि B] चउरसइ J.

J5b गुणीइ ।[1] फल $^{\forall}$ आवइ॥ ⟨17_2.1⟩

उदा॰ ।[2] भुज एक गज तेर ।[3] बीजु गज तेर ।[4] मुख गज च्यारि ।[5] भूमि गज चौद ।[6] लंब गज बार ।[7] रूपन्यासः[8]



⟨Figure 7⟩

लब्धं गजा १०८॥[9] ⟨17_2.2⟩

B8a त्रिको$^{\forall}$णे पूर्वार्द्धं॥ ⟨18_1.0⟩

वसुधार्द्धलंबघातः फलं क्षेत्रे त्रिकोणके॥ १८_१॥[10]

त्रिकोणि क्षेत्रि वसुधा भणीइ भूमि ।[11] तेह नूं अर्द्ध कीजइ ।[12] अनइ लंब सूं गुणीइ ।[13] फल आवइ॥ ⟨18_1.1⟩

उदा॰ ।[14] भुज एक गज तेर बीजु गज तेर भूमि गज दस लंब गज बार ।[15] रूपन्यासः

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[1]पच्छइ B] पछइ J; लंब J] लब B; सिउं] सित्रं B, सुं J.

[2]उदा॰ B] उदाहरणं J.

[3]तेर B] तेरे । १३ । J.

[4]बीजु B] बीउ J; तेर B] तेरे J.

[5]मुख B] मुष J; च्यारि B] । ४ । J.

[6]भूमि B] भूम J; चौद B] । १४ । J.

[7]बार B] । १२ । J.

[8]रूप B] रूपं J.

[9]लब्धं B] लब्ध B; गजा B] गज J.

[10]वसुधार्द्धलंबघातः BJ] वसुधार्द्धं लंबनिघ्नं A; क्षेत्रे JA] क्षेत्रि B; १८_१] १७ BJ, ∅ A.

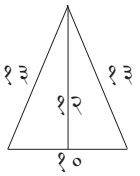
[11]त्रिकोणि B] त्रिकोण J; क्षेत्रि B] त्रिं J; भणीइ भूमि B] भणीइ । भूमइ J.

[12]नूं B] नुं J.

[13]अनइ B] अनइं J; सूं B] सुं J.

[14]उदा॰ B] उदाहरणं J.

[15]भुज B] सुन J; तेर (twice) B] । १३ । J; बीजु B] बीजउ J; भूमि B] भूम J; दस B] । १० । J; बार B] । १२ J.



⟨Figure 8⟩

लब्धं गजा ६०॥[1] ⟨18_1.2⟩

वृत्तक्षेत्रे अपरार्द्धं॥[2] ⟨18_2.0⟩

व्यासस्य तुर्यभागेन वधो वृत्तेश्च वर्तुलः॥ १८_२॥[3]

वाटलइ क्षेत्रि परिधि नुं आंक व्यास नइं चुथइ भागि सिउं गुणीइ।[4] वृत्तफल आवइ॥[5] ⟨18_2.1⟩

तथा परिधिमानं।

व्यासस्य त्रिगुणा परिधिः षट्भागसमन्विता॥[6] ⟨S4⟩

व्यास नु अंक त्रिगणु करी च्छठ नु भाग माहि घातीइ।[7] तु परिधिफल आवइ॥[8] ⟨18_2.2⟩

[9]उदा०। परिधि गज उगणीस व्यास गज च्छ। रूपन्यासः

---

[1]गजा B] गज J.

[2]वृत्त B] वत्त J.

[3]भागेन A] भागन B, भागोन J; वृत्तेश्च JA] वर्त्तेश्च B; वर्तुलः] वर्तुलेः B(cor.), वर्तुले JA; १८.२] १८ BJ, २० A.

[4]क्षेत्रि B] क्षेत्र J; नुं B] नु J; नइं B] नइ J; चुथइ B] चउथा J; भागि B] भाग J; सिउं B] सइउ J; गुणीइ B] गणी J.

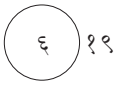
[5]वृत्तफल आवइ B(with *pha* between lines)] रूपं न्यासः X J(symbol "X" seems to indicate that this phrase should be placed after *paridhiphala āvai* at the end of 18_2.2, where the same symbol occurs).

[6]समन्विता J] समंन्विता B(cor.). A (verse 21ab) reads: व्यासत्रिगुणिता वृत्तिर्व्यासषड्भागसण्युतो।

[7]नु B] नुं J; अंक B] आंक J; गणु B] गुणउ J: च्छठ नु B] उठ उ J; घातीइ B] घातीयइ J(hereafter also except in 25.1).

[8]तु B] तउ J. After आवइ J puts a symbol like "X" and Fig.9.

[9]J omits the passage up to रूपन्यासः



⟨Figure 9⟩

लब्धं गजा $\boxed{\text{२८॥}}$ ॥[1] ⟨18_2.3⟩

लंब विना तिस्रं चतुरस्रं॥[2] ⟨19.0⟩

भुजैक्यार्द्धं चतुर्द्धा च भुजहीनं च तद्वधात्।[3]
मूलं फलं भवेन्नूनं तिस्रे वापि चतुर्मुखे॥ १९ ॥[4]

[5]चुरसि तिस्रि क्षेत्रि भुज नूं एक्य करी अर्द्ध कीजइ।[6] पच्छइ ते चिहु ठामि लिषीइ।[7] अनइ तेह माहि भुज हीन कीजइ।[8] पच्छइ ते रहित आंक क्रमिं गुणी कीजइ।[9] अनइ पच्छइ मूल लीजइ।[10] एतलइ चतुमुख त्रिकोण नू फल आवइ॥[11] ⟨19.1⟩

उदा॰।[12] पूर्वोक्तचतुरस्ररूपन्यासः।[13]

---

[1]गजा B] गजे J; $\boxed{\text{२८॥}}$ B] २८ J.

[2]तिस्रं B] तिस्रं J(hereafter also); चतुरस्रं B] चतुरस्रं J(hereafter also).

[3]भुजैक्यार्द्धं J] भुजेक्याद्धं B; चतुर्द्धा B] चतुर्द्धा J; च तद्वधात्] चतघात् B, तथाहति J.

[4]तिस्रे वापि B] तिस्रोयापि J; मुखे B] मुखः J; A does not have this verse.

[5]Immediately after the verse number "19," J puts परिधि ना आंक ६ गज १९ भा॰ व्यास लाभइ , which seems to be a corrupt version of the example in 18_2.3.

[6]चुरसि B] चतुरःसिं J; तिस्रि B] तिस्रि J; क्षेत्रि B] क्षेत्र J; भुज नूं B] भुज J(with an illegible letter before and after *bhuja*); एक्य B] ऐक्य J.

[7]पच्छइ B] पछइ J; चिहु] विहु B, चिहुं J; ठामि B] ठामइं J; लिषीइ B] लिखीयै J.

[8]अनइ B] अनइं J; तेह B] चिहुं J; माहि B] मांहि J.

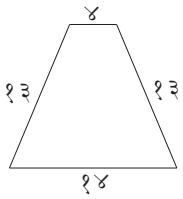
[9]पच्छइ B] पछइ J; रहित B] रह J; क्रमि B] क्रमइ J; गुणी B] गुणा J.

[10]अनइ B] ∅ J; पच्छइ B] पछइ J.

[11]चतुमुख B] चतुर्मुख J; नू B] नुं J.

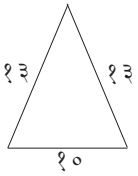
[12]उदा॰ B] उदाहरणं J.

[13]पूर्वोक्त J] पूर्वो B.



⟨Figure 10⟩ [1]

[2]लब्धं गजा १०८ । पूर्वोक्तिसरूपन्यासः



⟨Figure 11⟩ [3]

लब्धं गजा ६०॥[4] ⟨19.2⟩

[5]चापक्षेत्रे पूर्वार्द्धं॥ ⟨20_1.0⟩

[6]ज्याशरैक्यदलेषुघ्नं भागाष्टादशसंयुतं॥ २०_१॥[7]

चापक्षेत्रि जीवा अनइ शर एकत्र करी अर्द्ध कीजइ ।[8] अनइ ते आंक शर सिउं गुण[∀]इ ।[9] B8b पच्छइ तेह नु अढारमु भाग माहि घातीइ ।[10] फल आवइ॥[11] ⟨20_1.1⟩

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[1] J places this figure between *labdhaṃ* and *gaja* (for *gajā*) at the end of this paragraph.

[2] J omits these two sentences.

[3] J places this figure between *ghātī* and *yai* (for *i*) in 20_1.1.

[4] गजा B] गज J.

[5] J omits this sentence.

[6] J omits this hemistich.

[7] ज्याशरैक्यदलेषुघ्नं A] य्याशरैक्यदलेय्युघ्नं B; भागाष्टादश] भागाष्टः दश B, सागाष्टादश A; २०_१] ∅ BJA.

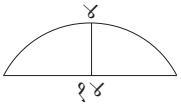
[8] क्षेत्रि B] क्षेत्रइं J; जीवा B] जी·· J(the letter after *jī* is illegible).

[9] अनइ B] अनिं J; गुणइ B] गुणीयइ J.

[10] पच्छइ B] पछइ J; नु B] नुं J; अढारमु B] अढारमुं J; माहि] ∅ B, मांहि J.

[11] आवइ J] आवइं B.

उदा॰ ।[1] जीवा गज चौद शर गज च्यारि ।[2] रूपन्यासः[3]



⟨Figure 12⟩[4]

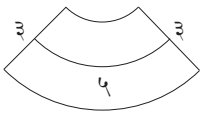
लब्धं गजाः ३८ ॥[5] ⟨20_1.2⟩

नेम्याकारे परार्द्धं ॥[6] ⟨20_2.0⟩

नेम्याकृतौ च विषमे व्यासलंबवधः फलं ॥ २०_२ ॥[7]

नेम्याकारि गढ नइ कोठइ विषमक्षेत्रि व्यास लंब सिउं गुणीइ ।[8] फल आवइ ॥ ⟨20_2.1⟩

उदा॰ ।[9] लंब गज पांच विस्तर गज त्रिणि ।[10] रूपन्यासः[11]



⟨Figure 13⟩[12]

लब्धं गजाः १५ ॥[13] ⟨20_2.2⟩

तथा मुरजयवाकारवज्राकारअर्द्धचंद्रपंचभुजक्षेत्राणां ज्ञासः ।[14] [15]मुरजक्षेत्राणां ज्ञासः । मुरजक्षेत्र

---

[1]उदा॰ B] उदाहरणं J.

[2]चौद B] १४ J; च्यारि B] ४ J.

[3]रूप B] रूपं J.

[4]४ J] ३ B(cor.).

[5]गजाः B] गज J.

[6]नेम्याकारे B] नेम्याकररे J; परार्द्धं B] अपरार्द्धं J.

[7]व्यासलंब J] व्योसंब B, व्यासदीर्घ A; २०_२] २० B, ∅ J, २५ A.

[8]नेम्याकारि] मेनेम्याकारि B(cor.), नेनेम्याकारइ J; क्षेत्रि B] क्षेत्रिं J; व्यास J] च्यारु B.

[9]उदा॰ B] उदाहरणं J.

[10]लंब B] ५ लब्धं J; पांच B] ५ J; त्रिणि B] त्रिण्णि ३ J.

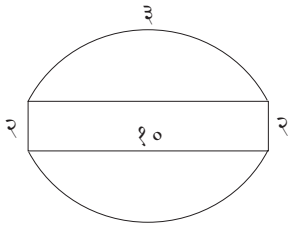
[11]रूपन्यासः B] रूपं न्यास J.

[12]J places this figure between *muraja* and *yavākāra*, after Fig.14, at the beginning of 20_2.3.

[13]गजाः B] गज J.

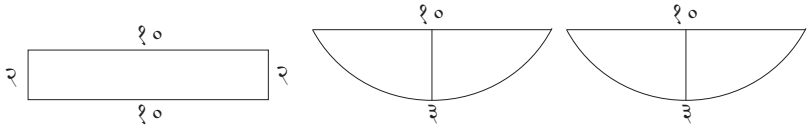
[14]वज्राकार J] वज्रचाकार B; अर्द्धचंद्र B] अर्द्धं चहः J; ज्ञासः B] न्यासः J.

[15]J omits this sentence.



⟨Figure 14⟩ [1]

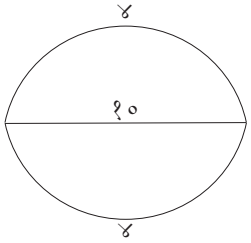
जा[∀]त चतुरस्र एक धनुष बि।[2] रूपं[3] J6a



⟨Figure 15⟩ [4]

[5] 'भुजकोटिवध' [PV 17b] 'ज्याशरैक्यदल' [PV 20a] इत्यादिना लब्धं गजाः ६१८४॥[6] ⟨20_2.3⟩

यवाकारक्षेत्रं[7]



⟨Figure 16⟩ [8]

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[1] J places this figure between *muraja* and *yavākāra*, before Fig.13, at the beginning of this paragraph (20_2.3). J rotates the figure clockwise through 90 degrees and puts "2" on the left arc (opposite to "3"). See Appendix.

[2] जात B] जातं J; एक B] एके J; धनुष J] धनष B.

[3] रूपं B] रूप दिनालब्धंत्राणां न्यासः मरजक्षेत्र २ J.

[4] J places these figures between *muraja* and *yavākāra*, after Fig.13, at the beginning of this paragraph (20_2.3).

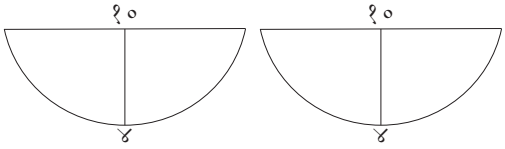
[5] J omits this sentence.

[6] लब्धं] लजा B(*jā* canceled and *bdhaṃ* in margin).

[7] यवाकार B] ∅ J.

[8] J places a corrupt figure for Fig.16 and its revision respectively immediately before and after

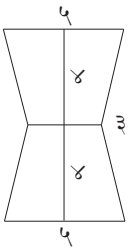
जात धनुष बि।[1]



⟨Figure 17⟩ [2]

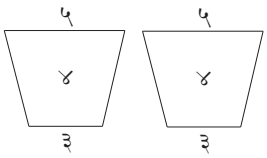
'ज्याशरैक्यदल' [PV 20a] इत्यादिना लब्धं गजा ५९८२॥४॥[3] ⟨20_2.4⟩

वज्राकारक्षेत्रं



⟨Figure 18⟩ [4]

जात चतुर्भुज बि।[5] रूपन्यासः



⟨Figure 19⟩ [6]

'वसुधामुखयोगार्द्धं' [PV 17c] इत्यादिना लब्धं पृथक्पृथक्फलं गजा १६ तथा १६।[7] द्वयोर्युतिः

३२ ॥ ⟨20_2.5⟩

Fig.19. See "16j(canceled)+19j+16j" in the Appendix.

[1]जात B] जातं J.

[2]J places these figures between *muraja* and *yavākāra*, after Fig.15, at the beginning of 20_2.3.

[3]गजा B] गजः J.

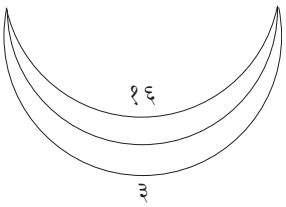
[4]B places this figure between *ityādinā* and *labdhaṃ* in the next line. J omits this figure.

[5]जात B] जातं J.

[6]B places this figure between *phalaṃ* and *gajā* of the same line.

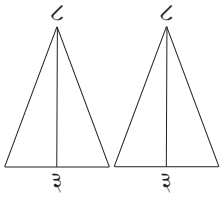
[7]पृथक्पृथक् B] पृथक् २ J; फलं B] फल J; गजा B] गज J; तथा १६ B] ∅ J.

अर्द्धचंद्रक्षेत्रः[1]



⟨Figure 20⟩[2]

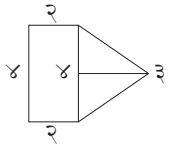
जात त्रिणिभुज बि।[3]



⟨Figure 21⟩[4]

'वसुधार्द्धलंबघातः' [PV 18a] इत्यादिना लब्धं गजा १२ तथा १२।[5] द्वयोर्युति | २४ | ॥ ⟨20_2.6⟩

पंचभुजक्षेत्रः[6]



⟨Figure 22⟩[7]

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[1]क्षेत्रः B] क्षेत्रं J.

[2]J places this figure between *gajā* and *12* at the end of this paragraph.

[3]जात B] जाते J; त्रिणि B] त्रिण्णि J.

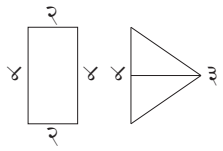
[4]B places this figure between *la* and *bdhaṃ* in the next sentence. J places this figure between *vasudhārddhaṃ* (for *-rddha*) and *laṃba* in the next sentence.

[5]वसुधार्द्धलंबघातः] वसुद्धार्द्धलंधातश्च B(with *ba* in margin), वसुधार्द्धं लंबघातश्च J; इत्यादिना] इत्यादिः। ना B, इत्यादिनर J; १२ (2nd) J] | ३२ | B(cor. in margin).

[6]क्षेत्रः B] क्षेत्र J.

[7]J omits this figure.

जात चतुरश्र एक त्रिकूण एक।[1] न्यासः



⟨Figure 23⟩[2]

B9a ‘भु$^{\forall}$जकोटिवधः’ [PV 17b] ‘वसुधार्द्धलंबघातः’ [PV 18a] इत्यादिना लब्धं गजाः | १४ | ॥[3] ⟨20_2.7⟩

इणं रीति विषमक्षेत्र इम षंड करी पहिला क्षेत्र नी परे मवीइ॥[4] ⟨20_2.8⟩

इति क्षेत्रव्यवहारः॥[5] ⟨20_2.9⟩

षातकाष्टपाषाणकोष्टागारचितिमानं॥[6] ⟨21.0⟩

षाते काष्टे च पाषाणे कोष्टागारे चितौ तथा।[7]
व्यासदीर्घवधस्तत्र पिंडघातः समे फलं॥ २१॥[8]

षामइ कोठि पाषाणि कोठारि चेजइ एक्क करी व्यास दीर्घ सिउं गुणीइ।[9] पछइं पिंड सिउं गुणीइ।[10] फल आवइ॥ ⟨21.1⟩

---

[1]जात B] जाते J; त्रिकूण J(with a medial *o* for *kū* in addition to the medial *ū*)] त्रिकू B; एक (2nd) J] ∅ B.

[2]B places this figure between *ke* (for *ko*) and *ṭi* at the beginning of fol. 9a.

[3]भुज B] भुजः J; कोटि J] केटि B; वसुधार्द्ध] वसुधार्द्धं B, वेसद्धार्द्ध J; लंब B] लंबं J; घातः] घातश्च BJ; लब्धं गजाः B] गज लब्धं J.

[4]इणं B] इणी J; विषम B] पहिल J; षंड J] षड B; परे] परिः B, पार J; मवीइ B] मवीयइ J(hereafter also).

[5]क्षेत्र J] क्षेत्रे B.

[6]षात B] खात J; कोष्टागार J] कोष्टागर B; चिति B] चित J; मानं B] मान J.

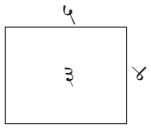
[7]षाते B] खाते JA; काष्टे BA] कोष्टे J; च JA] ∅ B; कोष्टागारे J(with an illegible letter like *ṇa* between *ṣṭā* and *gā*)A] कोष्टारे B.

[8]वधस्तत्र A] वधास्तत्र BJ; पिंडघातः समे] पिंडघाते समं BJ, पिंडघातवधः A; २१ BJ] २६ A.

[9]J has three letters erased before *ṣāmaï*; कोठि B] कोठइ J; पाषाणि B] पाषाण J; कोठारि B] कोठारइ J; चेजइ B] त्रेजइ J; एक्क B] एक J.

[10]सिउं J] सिउ B.

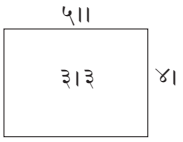
उदा॰ ।[1] दीर्घ गज पांच विस्तर गज च्यारि पिंड गज त्रिणि ।[2] साधारणं रूपं



⟨Figure 24⟩[3]

लब्धं गजा ६०॥[4] ⟨21.2⟩

द्वितीय भिन्नोदा॰ ।[5] अत्र विसा गजमानं ।[6] दीर्घ गज पांच विसा दस १० विस्तर गज च्यारि विसा पांच ५ पिंड गज त्रिणि विसा आठ ।[7] साधारणरूपं[8]



⟨Figure 25⟩[9]

भिन्नप्रत्युत्पन्नवत् लब्धं गजा ७९ ।४॥[10] ॥ ⟨21.3⟩

तथा वक्रं ।[11]

विषमं यदि चेत् तेन विषमयुतिभागकः॥[12] ⟨S5⟩

---

[1]उदा॰ B] यथोदाहरणं J.

[2]पांच B] ५ J; च्यारि B] ४ J; पिंड J] पंड B; त्रिणि B] त्रिणि ३ J.

[3]J places this figure between *sādhārṇṇaṃ* (for *-raṇa*) and *rūpaṃ* in the next paragraph (21.3).

[4]गजा B] गज J.

[5]द्वितीय भिन्नोदा॰] द्वितीय भिन्नो उदा॰ B(*u* canceled), द्वितीयोदाहरणं J.

[6]गज] ∅ BJ.

[7]पांच (1st) B] ५ J; विसा B] विस्वा J; दस] ∅ J; च्यारि B] ४ J; पांच (2nd) B] ∅ J; त्रिणि B] त्रिणि ३ J; आठ B] ८ J.

[8]साधारण B] साधाण्णं J.

[9]J places this figure after "79|4" (for "79|4||") at the end of this paragraph (21.3).

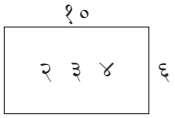
[10]भिन्नप्रत्युत्पन्नवत् J] भिन्नप्रत्युत्पंन्नंवत् B; ७९ ।४॥] ४९ ।४॥ B, ७९ ।४ J(without a frame).

[11]वक्रं J] च । क्रं B.

[12]विषमं ... भागकः] बिषमं यदे चेत विषमयुति चैततः कुर्यात् B; विषमं यदि चेत् विषमयुति वैततः कुर्यात् J. A does not have this hemistich.

वांकइ विसमइ ऊंडपणि विस्तारि दीर्घि विषमयुति कीजइ।[1] जेतले थाहरे मवीइ तेतमु भाग लीइ।[2] एतलइ ते पाधरू थाइ।[3] वेधि विस्तारि दीर्घि एक रीति विषमयुति फलानयने पुष्करणी षात॥[4] ⟨21.4⟩

उदा॰।[5] द्वि त्रि चतुर्गज वेधे दश दीर्घि षट् गजा विपुले।[6] न्यासः



⟨Figure 26⟩[7]

वेध विषमयुति गज १।[8] तेषां तृतीयभागे गज ३ वेधः।[9] विस्तर गज ६। दीर्घ गज १०।[10] पूर्ववत् लब्धं गजा | १८० | ॥[11] ⟨21.5⟩

इति षातकाष्ठपाषाणकोष्ठागारचितिव्यवहारः॥[12] ⟨21.6⟩

B9b अथ वर्त्तुलकाष्टपाषाणस्तंभकूप$^{\forall}$मानं॥[13] ⟨22.0⟩

काष्टपाषाणयोः स्तम्भे कूपे च सदृशं फलं।[14]

J6b वृत्तक्षेत्रफलं तत्र $^{\forall}$ तेन पिंडवधः फलं॥ २२॥[15]

---

[1]वांकइ B] वाकइ J; विसमइ J] विसइ B; ऊंडपणि B] ऊंडइ पिणि J; विस्तारि B] विस्तरइ J; कीजइ J] कइ B.

[2]जेतले B] तेतले J; थाहरे B] पाहरे J; तेतमु B] तेतिम J; लीइ B] दीजइ J.

[3]ते B] ∅ J; पाधरू B] पाधरउ J.

[4]विस्तारि B] विस्तारइ J; दीर्घि B] दीर्घइं J; विषम B] विष J; फलानयने] फालानयने B, फलानयेन J; पुष्करणी B] पुष्करिणी J; षात] षत B, खातः J.

[5]उदा॰ B] यथोदाहरणं J.

[6]चतुर्गज B] चतुरगज J; वेधे J] वधे B; दश B] दस J; दीर्घि B] दीर्घे J.

[7]J places this figure, in a corrupt form, between *ta* and *i* of *citaï* (for *citi*) in 21.6; १० B] १० दीर्घ J.

[8]विषम B] विवम J.

[9]तेषां B] वेषा J.

[10]दीर्घ गज १०।] ∅ BJ.

[11]१८० B] १८ J.

[12]षात B] खात J; कोष्ठागार B] कोष्टागार J; चिति B] चितइ J.

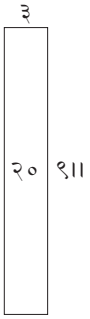
[13]वर्त्तुल J] वर्त्तल B.

[14]पाषाणयोः] पाषाणयो BJA; च JA] व B; सदृशं JA] सद्रशं B.

[15]वृत्त BA] वर्त्तु J; क्षेत्र B] क्षेत्रे JA; तेन BA] ततः J; वधः] वाधः B, वधे J, समं A; २२ BJ] २७ A.

काष्ट पाषाण स्तंभि कूइ वाटला क्षेत्र नूं फल करी पिंड सिउं गुणीइ।[1] फल आवइ॥ ⟨22.1⟩

उदा॰।[2] अत्र जेष्टांगुल गजमानं।[3] दीर्घ गज वीस विस्तर गज त्रिणि परधि गज साढा नव।[4] रूपन्यासः[5]



⟨Figure 27⟩[6]

परिधि नु आंक व्यास नाइं चतुर्थ भागि गुणीइ।[7] वृत्तफल हुइ।[8] लब्धं गजा ७ऽ३[9] पछइ पिंड सिउं गुणीइ।[10] लब्धं गजाः १४२॥ ॥[11] ⟨22.2⟩

इति वृत्तव्यवहारं॥[12] ⟨22.3⟩

गोलकमाने पूर्वार्द्धं॥[13] ⟨23_1.0⟩

गोलव्यासघनार्द्धं च भागाष्टादशसंयुतं॥ २३_१॥[14]

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[1]कूइ B] कूयइ J; वाटला J] वटला B; नूं B] नुं J; सिउं J] सित्रं B.

[2]उदा॰ B] यथोदाहरणं J.

[3]जेष्टांगुल B] ज्येष्टांगुल J.

[4]वीस B] वीस। २०। J; विस्तर J] विस्त B(with *ra* between lines); त्रिणि B] ३। J; परधि B] परिधा J; साढा J] सार्ड B(*r* canceled); नव B] नव॥ ९॥॥ J.

[5]रूप B] रूपं J.

[6]३ B] ३ विस्तर J; २० B] २० दी J; ९॥ B] पि॰ ९॥ J.

[7]नु B] नइ J; नाइं B] नइ J; भागि B] भाग J.

[8]हुइ B] आव(?)इ हुयइ J.

[9]७ऽ३ B] ७ऽ२ J(placed before the next *labdhaṃ*).

[10]सिउं J] सिउ B.

[11]गजाः १४२॥ B] गज। (here is part of Figure 27) । ज १४२॥ J("*ja* 142 ||" circled).

[12]वृत्तव्यवहारं B] वत्तअवहारः J.

[13]पूर्वार्द्धं B] पूर्वार्द्ध J.

[14]गोल BA] गोले J; घनार्द्धं JA] नार्द्धं B(only *mātrā* line for *gha*); संयुतं BA] संजुतं J; ॥ २३_१॥] २२॥ B, । J, ∅ A.

गोला नु व्यास त्रिणि वार गुणीइ।[1] [2]अर्द्ध कीजइ। [3]पछइ तेह नु अढारमु भाग माहि घातीइ।[4] फल आवइ॥ ⟨23_1.1⟩

उदा॰।[5] व्यास गज त्रिणि।[6] रूपन्यासः[7]

⟨Figure 28⟩

लब्धं गजा १४। ॥[8] ⟨23_1.2⟩

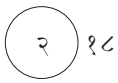
इति गोलकः॥ २३_१.३॥

कणराशिमान अपरार्द्धं॥[9] ⟨23_2.0⟩

वृत्तषडंशवर्ग्रस्तदुदयेन हतः फलं॥ २३_२॥[10]

वृत्त ना च्छठा भाग नु वर्ग्र ऊंचपण सिउं गुणीइ।[11] फल आवइ॥[12] ⟨23_2.1⟩

उदा॰।[13] परधि गज अढार उदय गज बि।[14] रूपं



⟨Figure 29⟩

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[1]गोला B] गोल J; नु B] नुं J; त्रिणि B] त्रित्रि J.

[2]J omits this sentence.

[3]J omits this sentence.

[4]अढारमु] अढामु B(with *ra* between lines).

[5]उदा॰ B] यथोदाहरणं J.

[6]त्रिणि B] त्रिणि ३ J.

[7]रूप B] रूपं J.

[8]१४। B] १४ J("14" circled).

[9]मान B] मानं J.

[10]वृत्त B] वृर्त्तः J, वृत्तेः A; षडंशवर्ग्रस्तदुदयेन] षंडशवर्ग्र। स्तदुदयेन B, षडंशवर्गास्त। दुदयेन J, खडंशस्तद्वर्ग उदयेन A; हतः JA] हत्तः B; २३_२] २३ BJ, २८ A.

[11]च्छठा] च्छवा B(cor. in margin), साढा J; भाग B] भागु J; नु B] नुं J; वर्ग्र B] वर्ग J; पण B] फण J; सिउं B] स्यउं J.

[12]फल J] फ B(with *la* between lines).

[13]उदा॰ B] यथोदाहरणं J.

[14]परधि B] परिधि J; अढार B] अढारं J; बि B] । २। J.

लब्धं गजा १८॥[1] ⟨23_2.2⟩

अत्र दीर्घ गज एक विस्तर गज एक वेध गज एक मापइ गोधूम मण सोल १६ हु।[2] आरिचारी मण १५ मापइ।[3] शेष थाकतां अन्नव्यवहारिं जाणिवां॥[4] ⟨23_2.3⟩

एणीं रीतिं भिद्याइं अर्द्ध पडइ।[5] कूणइ त्रिणि भाग पडइ।[6] बहि:कोणि चतुर्थं पडइ। भिद्या रूपं[7]

⟨Figure 30⟩

लब्धं गजा[8] ⌊ ९ ⌋ कोण रूपं

⟨Figure 31⟩

लब्धं गजा ⌊ ४॥ ⌋ बहि:कोण रूपं[9]

⟨Figure 32⟩[10]

लब्धं गजा:[11] ⌊ १३॥ ⌋ ॥ ⟨23_2.4⟩

इति राशिव्यवहार॥[12] ⟨23_2.5⟩

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[1]गजा १८ B] गज। १८। J.

[2]गज एक (1st) B] गजः १ J; एक (2nd) B] । १। J; वेध गज एक B] मापइ J; गोधूम J] गोधू B(*go* between lines); सोल B] ∅ J.

[3]आरिचारी B] आर J; मापइ] माइ B, ∅ J.

[4]थाकतां B] थाकता J; व्यवहारिं B] व्यवहार J; जाणिवां B] जांणिवा J.

[5]एणीं B] एणी J; रीतिं B] रीति J; भिद्याइं] ∅ BJ.

[6]कूणइ] पूंणइ B, पूणि J; त्रिणि भाग B] ∅ J.

[7]भिद्या B] भित्या J.

[8]गजा B] गज J.

[9]J has गोलकफलं between *rūpaṃ* and the next figure.

[10]For this figure, J draws a circle with "2" at the end of line 8 of fol. 6b and writes "13 ||" at the beginning of the next line. See Appendix.

[11]गजाः B] गजा J.

[12]व्यवहार B] व्यकहारत J.

च्छायामानं॥[1] ⟨24.0⟩

च्छाया सप्तयुता कार्या मध्यच्छायां ततस्त्यजेत्।[2]
B10a दिनार्धात्सप्तभिर्गुण्यात् भागान्ना[∀]ड्यो गताः स्थिताः॥ २४॥[3]

छायापाद माहि सात घातीइ।[4] अनइ मध्यपाद हीन कीजि।[5] अनइ दिनार्द्ध सातगणू करी छायापाद सिउं भाग हरीइ।[6] फल आवइ। मध्याह्न जांण गत घडी लाभइ।[7] अनइं अपराह्नि रहित घडी लाभइ॥[8] ⟨24.1⟩

दिनमानं॥ ⟨25.0⟩

[9]मकरादिदिनाः त्रिघ्नाः षत्रिपंचैकसंयुताः।[10]
षष्टिभागाः कला लब्धिः कर्क्कार्द्धात्रिस्तथैव हि॥ २५॥[11]

मकरसंक्रांति थिकी प्रस्न जांण दिन एकत्र करी त्रिगुण कीजइ।[12] पच्छइ पनर सइ त्रीसां मांहि घातीइ।[13] अनइ साठि ६० भाग दीजइ।[14] दिनमान लाभइ। इण रीति कर्क्कसंक्रांति

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[1]च्छाया B] अथ च्छाया J.

[2]च्छाया B] छाया J; च्छायां BJ] च्छाया A; ततस्त्यजेत् JA] ततस्तजेत् B(in margin).

[3]दिनार्धात्] दिनार्धं B(in margin)JA; सप्तभिर् JA] सप्तभि B(in margin); गुण्यात् J] त् B(*guṇyā* in margin), गुण्यं A; भागान्नाड्यो गताः स्थिताः B] भागान्नाड्यो गता स्थिताः J, तेनाप्तं घटिकादिकं A; २४ BJ] ३३ A.

[4]पाद B] पद J.

[5]अनइ B] अनइं J; कीजि B] कीजइ J.

[6]अनइ B] अनइं J; दिनार्द्ध] दिनार्थ B, दिनमान J; गणू B] गुणो J; करी J] री B; हरीइ B] हरीयइ J.

[7]जांण] जाण BJ; घडी B] ते घटी J.

[8]अपराह्नि B] अपराह्नइ J; घडी B] घटी J.

[9]This verse does not occur in A.

[10]दिनाः त्रि J] दिनाः। त्रि B; ष B] ख J; संयुताः J] संयुता B.

[11]षष्टिभागाः] षषेर्भागात् B, षष्टभागात् J; लब्धिः B] लब्धि J; कर्क्कार्द्धात्रिस् B] कर्कार्द्धात्रस् J; हि B] हिः J.
J records a variant (*pāṭhāntara*) of this verse after verse 26. A does not have this verse.

[12]मकरसंक्रांति J] मकसक्रांति B(with *ra* between lines); थिकी B] थी। J; जांण J] जांजा B; त्रिगुण B] त्रिगुणा J.

[13]पच्छइ B] पछइ J; मांहि J] मा B.

[14]६० B] ∅ J.

थिकी रात्रि लाभइ॥[1] ⟨25.1⟩

मध्यपादमानं॥[2] ⟨26.0⟩

[3]त्रिद्व्येकखेंदुपक्षाग्निनयुगेषुषट्शरा युगाः।[4]
क्रमान्मेषादिसंक्रांतौ मध्यपादाः प्रकीर्त्तिताः॥ २६॥[5]

[6][पाठांतरे॥

अयनादिकवासररामहता गगनानलबाणशशांकयुता।[7]
षष्टिविभाजितलब्धकला मकरादि दिनं कर्कादि निशा॥ २६^{a}॥[8]

१५३०९६० भा॥[9] ⟨26.1⟩]

मेषसंक्रांति थिकी ३।२।[10] ∀ १।०।१।२।३।४।५।६।५।४। मध्यपाद क्रमिं हुइ॥[11] J7a
⟨26.2⟩

उदा°। चैत्र मासि मेषसंक्रांति च्छाया पाद ११ मध्यपाद ३ दिनार्द्धघटी १५। किं फलं भवति। न्यासः। ⟨lacuna⟩ लब्धं दिनगतघटी ७॥ ⟨26.3⟩

इति च्छायाव्यवहारः॥ ⟨26.4⟩

एवं पंचमं अनेकार्थसूत्रं समाप्तं॥ ⟨26.5⟩

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[1]इण B] इणी J; कर्क्क B] कर्क J; थिकी] थिकीकी B, थी J; लाभइ B] लाभि J.

[2]मध्य B] अथ मध्य J.

[3]This verse corresponds to verse A1° of Śambhunāta's version (Hayashi 1991), which however gives different values.

[4]षेंदु B] खेंदु J; युगेषु J] युगेय्यु B; युगाः] युगा BJ.

[5]पादाः J] पादक्रमिं B(*kramiṃ* canceled).

[6]This paragraph occurs only in J, and not in B and A. Here occurs a verse numbered again as 26 and introduced as "another (variant) reading." In fact, however, this is another verbal expression of the rule of verse 25.

[7]बाण] बांण J.

[8]कला] काला J(cor.); मकरादि] मकारादि J(cor.); २६^{a}] २६ J. The meter of this verse is irregular: the 1st and 2nd *pādas* in Toṭaka, the 3rd in Citragati, and the 4th unknown to me.

[9]६०] ६ J.

[10]J is available up to here.

[11]थिकी B] का। मध्यपाद। J.

बाणाहिवेदचंद्राब्दे ऽहम्मदावादपत्तने ।[1]
बालबोधांकवृत्तिश्च शंभुदासेन निर्म्मिता॥ २७॥[2]

इति मन्त्रशंभुदासविरचिता पंचविंशतिकाबालबोधांकवृत्तिः समाप्ता॥[3] ⟨27.1⟩

[1]वेद] देव B.

[2]वृत्तिश्च] पृस्तिश्च B; शंभु] शुंभु B. Of course, this verse belongs to the commentary (BBA) and not to the PV but I leave the verse number 27 as it is in the manuscript (B).

[3]विरचिता] विरिचिता B; समाप्ता] समाप्तः B.

III Annotated Translation of the *Bālabodhāṅkavṛtti* with *Pañcaviṃśatikā*

Om, Salute! Salutation to the auspicious Gaṇeśa!/BBA 1.0/

> Having first bowed down to Mahādeva (Great Lord, i.e., Śiva), I shall tell this *Pañcaviṃśatikā* by means of five *sūtras* in order to enhance the intelligence of the beginners./PV 1/

···Note

PV 1. The word *sūtra* originally means "a thread" and by extension "a short aphorism" and then "a rule." In mathematics it is often used in the sense of "a versified rule." But none of these meanings seems to fit in the present context. Probably it is used here in the sense of "a group of *sūtras* or versified rules," although the grouping is extremely unbalanced. See the Contents of the *Bālabodhāṅkavṛtti* at the end of the Introduction. It is noteworthy that Śambhunātha's version of the PV reads *svīya* ("my own") instead of *paṃca* ("five"). In that case, the ordinary meaning, "a versified rule," would fit in the context.[1]

Having first (ādau: pūrvaṃ) bowed down to Mahādeva, the Omniscient, in order to enhance the intelligence of the beginners, by means of five *sūtras*, I shall tell this *Pañcaviṃśatikā*. /BBA 1.1/

Having first (ādi: pahilū) bowed down (praṇamī: namaskarī nai) to Mahādeva, the Omniscient, in order to enhance the intelligence of the beginners (bāla: ajñāna), by means of five *sūtras*, I shall tell *Paṃcavīsī*, the essence of mathematics. /BBA 1.2/

The first *sūtra* for *saṃkalita*./BBA 2.0/

> Half the product of the first term ⟨and the same⟩ plus one, or also ⟨that which is obtained⟩ by increasing one-by-one, or half the square of the term increased by the first, or ⟨that which is obtained⟩ in the product of half the first ⟨and the same⟩ plus one, is the fruit. /PV 2/

···Note

PV 2. *saṃkalita* or the sum of the natural series. Four methods are prescribed here. Let $S = S(n)$ be defined as the sum of the first n terms of the natural numbers beginning with 1. Then, 1. $S = \frac{n(n+1)}{2}$. 2. $S = (((1 + 2) + 3) + \cdots + (n - 1)) + n$. 3. $S = \frac{n^2+n}{2}$. 4. $S = \frac{n}{2} \cdot (n + 1)$. The

[1] In this Annotated Translation I do not mention the correspondence between the rules of the PV and those of other Sanskrit works, for which see the Translation with Mathematical Commentary of Hayashi (1991).

last one also implies: 4′. $S = n \cdot \frac{n+1}{2}$ to be used when n is odd. Note that the words "first term" (*ādya-pada*) or simply "first" (*ādya*) denote "the first one among the given numbers." Also note that the word "fruit" (*phala*) in this verse as well as in many other verses in this work means "the result to be obtained by the calculation."

Thus, four methods for the *saṃkalita* (sum of the natural series) ⟨is taught here⟩. First method: "Half the product of the first term ⟨and the same⟩ plus one" [PV 2a]. "Plus one": The term in question is increased by one. "The product of the first term": Then, ⟨the result is⟩ multiplied by that first (ādya: pahilā) term. "Half": Then, that term ⟨in the last form⟩ is made half. It (the term in question) becomes the *saṃkalita* ⟨by this procedure⟩. /BBA 2.1/

Second method: "Also ⟨that which is obtained⟩ by increasing one-by-one" [PV 2b]. "Also" (tathā: valī): Having written down the increasing digits up to the ⟨number in⟩ question, mental work (? *siravālu*) ⟨of successive addition⟩ is made. It becomes the *saṃkalita*. /BBA 2.2/

···Note

BBA 2.2. I cannot identify the term *siravālu* but presumably it is a compound of *sira* (Skt. *śiras*, head) and *vālau* (from the verb *vālai*, "turns" or "manipulates") and means "manipulation by head" or mental work. The same word occurs also in BBA 2.6.

Third method: "Half the square of the term increased by the first" [PV 2c]. "The square of the term": The square of the term in question is made. "Increased by the first": Then, ⟨the result is⟩ increased by the first term. "Half": Then, that term ⟨in the last form⟩ is made half. It becomes the *saṃkalita*. /BBA 2.3/

Fourth method: "⟨that which is obtained⟩ in the product of half the first ⟨and the same⟩ plus one, is the fruit" [PV 2d]. "Plus one": The term in question is made increased by one. "The first": Then, the first digit is written down together. Then, having made half the full (i.e., even) term between these two digits, ⟨the result is⟩ multiplied ⟨by the other⟩. It becomes the *saṃkalita*. /BBA 2.4/

···Note

BBA 2.4. It is noteworthy that the word *pūrauṃ* (full) is used to denote an "even" number. I have not seen its Skt. counterpart *pūrṇa* used in that sense.

Ex. The *saṃkalita* of ten is fifty-five: 10, 55. First ⟨method⟩. The term in question is 10. "Plus one": Increased by one, it becomes 11. "The first term" is 10. "The product": multiplied, 110. "Half" is 55. /BBA 2.5/

···Note

BBA 2.5. Ex. 1 for *saṃkalita*: $n = 10$. By the 1st method, $10 + 1 = 11$, $11 \cdot 10 = 110$, $110/2 = 55$.

Second ⟨method⟩. The term in question is 10. "One-by-one": From one to ten, mental work ⟨of successive addition⟩ is made. Produced is 55. /BBA 2.6/

···Note

BBA 2.6. By the 2nd method, $1 + 2 = 3$, $3 + 3 = 6$, $\cdots$, $45 + 10 = 55$.

Third ⟨method⟩. The term in question is 10. When square is made, it becomes 100. Increased by the first term, 10, it becomes 110. "Half" is 55. /BBA 2.7/

···Note

BBA 2.7. By the 3rd method, $10^2 = 100$, $100 + 10 = 110$, $110/2 = 55$.

Fourth method. The term in question is 10. Increased by one, it becomes 11. Multiplied by half, 5, of the first term, 10, it becomes 55. /BBA 2.8/

···Note

BBA 2.8. By the 4th method, $10 + 1 = 11$, $10/2 = 5$, $11 \cdot 5 = 55$.

The *saṃkalita* of twenty is two hundred and ten: 20, 210. The *saṃkalita* of thirty is four hundred and sixty-five: 30, 465. The *saṃkalita* of forty is eight hundred and twenty: 40, 820. The *saṃkalita* of fifty is twelve hundred and seventy-five: 50, 1275. The *saṃkalita* of sixty is eighteen hundred and thirty: 60, 1830. The *saṃkalita* of seventy is twenty-four hundred and eighty-five: 70, 2485. The *saṃkalita* of eighty is thirty-two hundred and forty: 80, 3240. The *saṃkalita* of ninety is forty hundred and ninety-five: 90, 4095. The *saṃkalita* of one hundred is fifty ⟨hundred and⟩ fifty: 100, 5050. /BBA 2.9/

···Note

BBA 2.9. Exs. 2–10 for *saṃkalita*. Ex. 2: $n = 20$, $S = 210$. Ex. 3: $n = 30$, $S = 465$. Ex. 4: $n = 40$, $S = 820$. Ex. 5: $n = 50$, $S = 1275$. Ex. 6: $n = 60$, $S = 1830$. Ex. 7: $n = 70$, $S = 2485$. Ex. 8: $n = 80$, $S = 3240$. Ex. 9: $n = 90$, $S = 4095$. Ex. 10: $n = 100$, $S = 5050$. Note that the last number, 5050, is expressed as "fifty fifty" (*paṃcāsa paṃcāsāṃ*) without "hundred" (*saï*).

The root (*mūla*) of the *saṃkalita*.

> The step (*gaccha*, the number of terms) is equal to ⟨the integer part of⟩ the ⟨square⟩ root of twice the *saṃkalita*. /S1/

Having doubled the value (*pada*) of the root (*mūla*) of the *saṃkalita*, the square root (*varga-mūla*) ⟨of the result⟩ is taken. There will be an integer equal to it. One step (i.e., the number of terms) is made of it. The root (*mūla*) is obtained. /BBA 2.10/

···Note

BBA 2.10. From the third method of PV 2,

$$\sqrt{2S} = \sqrt{n^2 + n} = n + \text{fractional part.}$$

Therefore, the integer part of $\sqrt{2S}$ is equal to the number of terms of the natural series summed up.

The word *gaccha* (step) used in mathematical contexts means the number of terms of a mathematical series. The word *pada* (foot), too, usually means the number of terms as well as the square root but here and elsewhere in this work it seems to denote the given term or value. Most confusing in this paragraph is the word *mūla* (root). In the cited verse (S1), it obviously means the square root and the commentator employs the explicit term *varga-mūla* for it. But it cannot be the square root in the other three instances. Two of them, in the first and the last sentences, suggest that the "root" of the *saṃkalita* is the aim of this paragraph, which must be the number of terms of the *saṃkalita*, but this interpretation is not applicable to the remaining instance, which seems to suggest that "the root of the *saṃkalita*" means S itself.

Thus, the *saṃkalita* is completed. /BBA 2.11/

The second *sūtra* for *vyavakalita*./BBA 3.0/

> When one has subtracted the expenses from the property produced as a *saṃkalita* (sum), there will be a property. This property has been called the difference (*vyavakalita*) by ancient sages. /PV 3/

···Note

PV 3. *vyavakalita* or the difference between two *saṃkalitas*. The *vyavakalita*, $V_{n,m}$, is defined as

$$V_{n,m} = S(n) - S(m),$$

and calculated by the same formula.

Having made an expenditure (*varau*) drop from that which is the property produced by the term (*pada*) of the *saṃkalita*, the remainder (*bākī*) is taken out. The sages (muni: raṣīśvara) call this property *vyavakalita* (difference). /BBA 3.1/

Ex. When one has made an expenditure of the *saṃkalita* of ten, twenty, thirty, forty, fifty, sixty, seventy, eighty, ninety, or one hundred, from the *saṃkalita* of one hundred, what will be the remaining property? Setting-down:

5050 sum 100 55 exp. 10 4995 rem.	5050 sum 100 210 exp. 20 4840 rem.	5050 sum 100 465 exp. 30 4585 rem.	5050 sum 100 820 exp. 40 4230 rem.	5050 sum 100 1275 exp. 50 3775 rem.
5050 sum 100 1830 exp. 60 3220 rem.	5050 sum 100 2485 exp. 70 2565 rem.	5050 sum 100 3240 exp. 80 1810 rem.	5050 sum 100 4095 exp. 90 955 rem.	5050 sum 100 5050 exp. 100 0000 rem.

These much are the values of expenditures (subtrahends) and the values of the remaining properties. In this manner, the remainder is taken out of the income and expenses. /BBA 3.2/

···Note·····

BBA 3.2. Examples 1–10 for *vyavakalita*. Ex. 1: $V_{100,10} = S(100) - S(10) = 5050 - 55 = 4995$. Ex. 2: $V_{100,20} = S(100) - S(20) = 5050 - 210 = 4840$. Ex. 3: $V_{100,30} = S(100) - S(30) = 5050 - 465 = 4585$. Ex. 4: $V_{100,40} = S(100) - S(40) = 5050 - 820 = 4230$. Ex. 5: $V_{100,50} = S(100) - S(50) = 5050 - 1275 = 3775$. Ex. 6: $V_{100,60} = S(100) - S(60) = 5050 - 1830 = 3220$. Ex. 7: $V_{100,70} = S(100) - S(70) = 5050 - 2485 = 2565$. Ex. 8: $V_{100,80} = S(100) - S(80) = 5050 - 3240 = 1810$. Ex. 9: $V_{100,90} = S(100) - S(90) = 5050 - 4095 = 955$. Ex. 10: $V_{100,100} = S(100) - S(100) = 5050 - 5050 = 0000$.

Thus, the *vyavakalita* is completed. /BBA 3.3/

The third *sūtra* for the multiplication./BBA 4.0/

> There are two kinds of Kapāṭasandhi. Likewise, there are two kinds of Gomūtrikā. Tatstha has also been declared to be of two kinds, and Khaṇḍa has been laid down as of three kinds. /PV 4/

···Note·····

PV 4. Multiplication. Four methods are mentioned here. Each method has two or three varieties, and therefore there are nine methods in total. They are explained in PV 5-8 and illustrated by an example (1196×18) in BBA 8.5–8.

1. Kapāṭasandhi (door junction) → PV 5.

Direct order → BBA 8.5.

Inverse order → BBA 8.5.

2. Gomūtrikā (cow's urine) → PV 6.

Direct order → BBA 8.6.

Inverse order → BBA 8.6.

3. Tatstha (being there) → PV 7.

Śīrṣa (head) variety → BBA 8.7.

Koṣṭha (cell) variety → BBA 8.7.

4. Khaṇḍa (part) → PV 8.

Rūpa-vibhāga (integer division) → BBA 8.8.
Sthāna-bhāga (place division) → BBA 8.8.
Hīnādhika-bhāga (lesser-greater division) → BBA 8.8.

Tables N1-1 and N1-2 show the correspondence between the names given to the various multiplication methods in mathematical works. For the abbreviations in the names see under Table N1-2. The references given under the names are to the verse numbers in the respective works.

Table N1-1

Texts	BSS	Tr/PG	GSS	GT	L
Dates	628	ca. 800	ca. 850	ca. 1040	1150
1	—	*kavāṭa-sa.*	*kavāṭa-sa.*	*kapāṭa-sa.*	nn
		5-6ab/18-19ab	2.1ab	17abc	14ab
2	—	—	—	—	—
3	—	*tatstha*	*tatstha*	*tatstha*	—
		6cd/19cd	2.1cd	17d	
4	—	—	—	—	—
5	*gosūtrikā*[*1]	*rū.vi.kha.*	*khaṇḍa*	*rū.vi.kha.*	*rū.vi.kha.*
	12.55d	7ab/20ab	2.1cd	18	14cd
6	*gosūtrikā*	*sthā.vi.kha.*	*khaṇḍa*	*sthā.vi.kha.*	*sthā.vi.kha.*
	12.55abc	7ab/20ab	2.1cd	18	15d
7		(*rū.vi.kha.*)[*2]	—	—	*rū.vi.kha.*
		7ab/—			15ab
8	nn[*3]	—	—	—	nn
	12.56				16

[*1] The reading *gosūtrikā* (cows' rope) is based on Pṛthūdaka's commentary (ca. CE 864; I used the three MSS mentioned in the References, which include a copy of the manuscript used by S. Dvivedin). The editions of S. Dvivedin and of R. S. Sarma both read *gomūtrikā* (cow's urine) for it. The term *gosūtrikā* most probably meant the multiplication by parts after dividing the multiplier into either integers or notational places. See the Note for BBA 8.6 below. Pṛthūdaka (*op.cit.*) also mentions the *tatstha* and the *kapāṭa-sandhi*.

[*2] This is of the type: $nm = \frac{n}{a} \cdot (ma)$ or $nm = (na) \cdot \frac{m}{a}$. According to an anonymous commentator of the Tr (MS: LD Institute, 6967, fol. 5b, lines 11–14), the *rūpavibhāga* of the Tr includes this case also. Pṛthūdaka (*op.cit.*) ascribes this method to Skandasena.

[*3] This is of the type: $nm = n(m \pm a) \mp na$.

Table N1-2

	SGT ca. 1269	GSK ca. 1315	GK 1356	PV/BBA —/1428	GL 1545	GM ca. 1570
1	*kapāṭa-sa.* 17–18.2	*kavāḍa-sa.* 1.27	*kapāṭa-sa.* 1.13	*kapāṭa-sa.* 5	*rū.gu2.* on L 14	*akh.gu2.* 16a
2	—	—	—	*gomūtrikā* 6	*tatsthāna-gu2.* on L 17	—
3	*tatstha* 17–18.3	—	—	*tatstha-śī.* 7	—	—
4	—	—	—	*tatstha-ko.* 7	*kapāṭa-sa.* on L 17	*kapāṭa-sa.* 17-18
5	*rū.vi.kha.* 17–18.4–5	*khaṃḍa* 1.29(?)	*rū.vi.kha.* 1.14ab	*rū.vi.kha.* 8	*rū.vi.gu2.* on L 14	nn 16b
6	*sthā.vi.kha.* 17–18.6–7	*khaṃḍa* 1.29(?)	*sthā.vi.kha.* 1.14cd	*sthā.vi.kha.* 8	*sthā.gu2.* on L 15–16	nn 16cd
7	—	—	nn 1.15ab	*hī.a.kha.* 8	*rū.vi.gu2.* on L 15–16	—
8	*gu1.a/lī.kha.* 17–18.8–9	—	—	—	nn on L 15–16	—

Abbreviations of terms: *a.* = *adhika*, *a/lī.* = *adhikakāri/līnatākāri*, *akh.* = *akhaṇḍa*, *ko.* = *koṣṭha*, *kha.* = *khaṇḍaguṇana*, *gu1.* = *guṇaka*, *gu2.* = *guṇana*, *rū.* = *rūpa*, *vi.* = *vibhāga*, *śī.* = *śīrṣa*, *sa.* = *sandhi*, *hī.* = *hīna*; nn = no-name.

These tables reveal several points of the history of multiplication methods in India.

1) The two kinds of the "multiplication with parts" (*khaṇḍa-guṇana*), i.e., "by division ⟨of the multiplier⟩ into integers" (*rūpa-vibhāga*) and "by division ⟨of the multiplier⟩ into notational places" (*sthāna-vibhāga*), are mentioned or explained in all these arithmetical works but in their first occurrence in the BSS they were called the "cows' rope" (*go-sūtrikā*). See the Note for BBA 8.6.

2) The method called "door junction" (*kapāṭa-sandhi* or *kavāṭa-* or *kavāḍa-*) is not mentioned in the BSS but I agree with Datta and Singh (2001, I.135) who maintain that the common and well-known method of *kapāṭa-sandhi* has been omitted by him (i.e., Brahmagupta). Since the Tr/PG it is treated at the beginning of the section for multiplication in all the works surveyed here, although its name was replaced with "integer multiplication" (*rūpa-guṇana* or *akhaṇḍa-*) in the sixteenth century by the two Gaṇeśa's (i.e., the authors of the GL and of the GM), and the old name, "door junction," was given by them to the lattice method, which was previously (in the PV) called the "cell variety of being there" (*tat-stha-koṣṭha-bheda*). Śambhunātha (between CE 1562 and 1730), another commentator of the PV, too calls the lattice multiplication *kapāṭa-sandhi* but, unlike the two Gaṇeśa's, he gives the name *tatstha* to the previous *kapāṭa-sandhi*. The reason of these name changes is not known.

3) As far as is known, the PV is the first book in India that treats the lattice multiplication, while in the Arabic world and in England it occurs earlier (see Chabert 1999, 21–26).

4) The method called "cow's urine" (*go-mūtrikā*), too, appeared for the first time in the PV. But it seems to have originally been invented in astronomy for the multiplication of numbers with sexagesimal fractions. See the Note for BBA 8.6. That is probably the reason why the first appearance of the method in arithmetical works is as late as in the fifteenth century.

Kapāṭasandhi (door junction)./BBA 5.0/

> One should place the price above the ⟨term in⟩ question and multiply ⟨each place of the term in question⟩ one-by-one by the price. With the direct and the inverse ⟨order⟩, the ⟨multiplication⟩ called Kapāṭa shall be of two kinds. /PV 5/

Kapāṭasandhi has two methods. The first is the direct-order ⟨method⟩. In the direct-order ⟨method⟩, the price is arranged above the term in question, at the beginning (the highest notational place) in the manner of the door junction (*kapāṭasandhi*). Then, having multiplied ⟨each place of⟩ the term in question one-by-one by the price, ⟨the results of all steps⟩ are added up. The fruit (product) is obtained. /BBA 5.1/

···Note

BBA 5.1. Kapāṭasandhi—Direct order. For an example see BBA 8.5 below. Note that Śambhudāsa's terminology for the order of the notational places such as "direct," "inverse," "the beginning," and "the end" is contrary to the ordinary one. The same terminology is used by Ṭhakkura Pherū in his GSK (see SaKHYa 2009, 49, fn. 12).

Likewise, the second is the inverse-order ⟨method⟩. In the inverse-order ⟨method⟩, the price is arranged above the term in question, at the end (the units' place) in the manner of the door junction (*kapāṭasandhi*). Then, having multiplied ⟨each place of⟩ the term in question one-by-one by the price, ⟨the results of all steps⟩ are added up. The fruit (product) is obtained. /BBA 5.2/

···Note

BBA 5.2. Kapāṭasandhi—Inverse order. For an example see BBA 8.5 below.

Now, Gomūtrikā (cow's urine)./BBA 6.0/

> One should place the price below the ⟨term in⟩ question and multiply ⟨each place of both numbers⟩ straightly and alternately. With the direct and the inverse ⟨order⟩, the ⟨multiplication⟩ called Gomūtrikā (cow's urine) shall be of two kinds. /PV 6/

Gomūtrikā has two methods. The first is the direct-order ⟨method⟩. In the direct-order ⟨method⟩, the price is arranged together with, and below, the term in question, at the beginning (the highest notational place). Then, having multiplied them (each place of both numbers) straightly and alternately and further straightly at the end, ⟨all the results⟩ are added up. The fruit (product) is obtained. /BBA 6.1/

···Note

BBA 6.1. Gomūtrikā—Direct order. For an example see BBA 8.6 below.

Likewise, the second is the inverse-order ⟨method⟩. In the inverse-order ⟨method⟩, the price is arranged together with, and below, the term in question, at the end (the units' place). Then, having multiplied ⟨each place of both numbers⟩ straightly and alternately, ⟨the results at each step⟩ are added up. The fruit (product) is obtained. /BBA 6.2/

···Note

BBA 6.2. Gomūtrikā—Inverse order. For an example see BBA 8.6 below.

Now, the Tatstha (being there) variety./BBA 7.0/

> Since the multiplication is made by each ⟨digit⟩ of the quantity ⟨placed on the top of the other quantity, it is⟩ called the Śīrṣa (head or top) variety. Further, the Koṣṭha (cell) variety has been declared ⟨in the Tatstha variety⟩. Tatstha too, ⟨therefore,⟩ has been laid down as of two kinds. /PV 7/

Tatstha has two methods. First, in the Śīrṣa (head) variety, the price is arranged above the term in question at the head. Then, taking each digit of the price, the term in question is multiplied ⟨by it⟩. ⟨Then the results are⟩ separately (i.e., at each notational place) added up. The fruit (product) is obtained. /BBA 7.1/

···Note

BBA 7.1. Tatstha—Śīrṣa variety. For an example see BBA 8.7 below.

The second is the Koṣṭha (cell) variety. In the Koṣṭha variety, having drawn cells, ⟨they are⟩ split ⟨diagonally⟩. The term in question is arranged on the top, and the term of the price is written in front. Then, having taken each digit of the price and multiplied ⟨it⟩ by the term in question, ⟨the results⟩ are written into the cells and then added up. The fruit (product) is obtained. /BBA 7.2/

···Note

BBA 7.2. Tatstha—Koṣṭha variety. For an example see BBA 8.7 below.

Now, the Khaṇḍa (part) variety./BBA 8.0/

> There may be the integer division in one case, the place division in another case, and the lesser-greater division still in another case. Khaṇḍa too, ⟨therefore,⟩ has been laid down as of three kinds. /PV 8/

Khaṇḍa has three methods. The first is the integer division. In the integer-division, having divided the term of the question into two, three, or four parts and multiplied ⟨each part⟩ by the term of the price, ⟨the results are⟩ added together at one place. The fruit (product) is obtained. Likewise, having divided the term of the price into parts, ⟨each part may⟩ be multiplied by the term of the question. From that, in the same way, the fruits (products) are obtained. /BBA 8.1/

...Note.....

BBA 8.1. Khaṇḍa—Integer division. For an example see BBA 8.8 below.

The second is the place division. In the place-division, having divided the term of the question into the places of one, ten, hundred, thousand, etc. and multiplied ⟨each place⟩ by the price, ⟨the results⟩ are added up at one place. The fruit (product) is obtained. Likewise, having divided the places of the term of the price, ⟨each place may⟩ be multiplied by the term of the question. From that, in the same way, the fruit (product) is obtained. /BBA 8.2/

...Note.....

BBA 8.2. Khaṇḍa—Place division. For an example see BBA 8.8 below.

The third is the lesser-greater division. In the lesser-greater division, the term of the question is made into one half, one fourth, ⟨etc.⟩. Likewise, having made the term of the price twice, four times, ⟨etc., it⟩ is multiplied ⟨by the former⟩. The fruit (product) is obtained. Likewise, having divided the term of the price, ⟨it⟩ is multiplied by the ⟨term of⟩ question. From that, in the same way, the fruit (product) is obtained. That is, the multiplication has nine methods. /BBA 8.3/

...Note.....

BBA 8.3. Khaṇḍa—Lesser-greater division. For an example see BBA 8.8 below.

Others' sūtra:

> ⟨A quantity⟩ multiplied by zero is zero. One should employ zero in front. ⟨A quantity⟩ multiplied by unity remains the same. Thus is indeed ⟨the multiplication by zero and by unity⟩ everywhere./S2/

Zero is multiplied by zero; there will be only zero. Then, zero for the term of the price ⟨to be multiplied by each place of the term of the question⟩ is written so many times as there are ⟨notational places⟩ in front in the term of the question. Then, ⟨a quantity⟩ multiplied by one is just the same. ⟨The operation⟩ in multiplication is ⟨made⟩ in this way. /BBA 8.4/

···Note

BBA 8.4. The commentator regards S2 as "others' sūtra" and does not give a verse number. But, as it occurs also in Śambhunātha's version and is given the verse number 10 by him, it is likely to have belonged to the original PV. The rule is concerned with the treatment of zero and unity in multiplication. Note that the Tr and the PG too place a verse for zero (Tr 8 and PG 21) immediately after the verses for multiplication. 1) $a \cdot 0 = 0$. (The statement, $0 \cdot 0 = 0$, at the beginning of BBA 8.4 seems to be a written error either by the author himself or by the scribe.) 2) Put so many zero's in front of the term of the price, when multiplied by each digit of the term of the question, as many notational places there are in front of that digit. 3) $a \cdot 1 = a$. For the second rule see BBA 8.8 below.

First example. Silver, one thousand one hundred and ninety-six *gadīyāṇas*, for (unit price) eighteen 18 *drammas*. What is the fruit (price)? Setting-down. Kapāṭasandhi in the direct order:

1	8			
	1	1	9	6

After multiplying, the form ⟨of the penultimate step is⟩

1	8	8	2	8
	1	9	6	
		7	4	

Obtained is | 21528 | *drammas*. In the same way, Kapāṭasandhi in the inverse order:

			1	8
1	1	9	6	

After multiplying, the form ⟨of the penultimate step is⟩

1	1	9	6	8
	8	7	4	
		8	2	

Obtained is | 21528 | *drammas*. /BBA 8.5/

···Note

BBA 8.5. Ex. 1 for multiplication: What is the price of 1196 *gadīyāṇas* of silver when the unit price is 18 *drammas*?

Solution by Kapāṭasandhi (door junction) method Both the direct-order and the inverse-order methods can be reconstructed as follows. At each step, the newly written digits are printed in bold face. Note that the three tables given for each variety in BBA 8.5 represent steps 1), 12) without the multiplier, and 13).

1. Direct order (cf. BBA 5.1):

1) Write down the multiplier (18) above the multiplicand (1196) in such a way that the first place of the former is just above the highest place of the latter:

	1	**8**			
		1	**1**	**9**	**6**

2) $1 \cdot 1 = 1$:

	1	8			
	1	1	1	9	6

3) $1 \cdot 8 = 8$:

	1	8			
	1	**8**	1	9	6

4) Move “18” to the right by one place:

		1	**8**		
	1	8	1	9	6

5) $1 \cdot 1 = 1$:

		1	8		
	1	8	1	9	6
		1			

6) $1 \cdot 8 = 8$:

		1	8		
	1	8	**8**	9	6
		1			

7) Move “18” to the right by one place:

			1	**8**	
	1	8	8	9	6
		1			

8) $9 \cdot 1 = 9$:

			1	8	
	1	8	8	9	6
		1	**9**		

9) $9 \cdot 8 = 72$:

			1	8	
	1	8	8	**2**	6
		1	9		
			7		

10) Move “18” to the right by one place:

				1	**8**
	1	8	8	2	6
		1	9		
			7		

11) $6 \cdot 1 = 6$:

				1	8
	1	8	8	2	6
		1	9	**6**	
			7		

12) $6 \cdot 8 = 48$:

				1	8
	1	8	8	2	**8**
		1	9	6	
			7	**4**	

13) Add up the numbers except the multiplier (18):

	2	**1**	**5**	**2**	**8**

2. Inverse order (cf. BBA 5.2):

1) Write down the multiplier (18) above the multiplicand (1196) in such a way that the highest place of the former is just above the first place of the latter:

				1	**8**
1	**1**	**9**	**6**		

2) $6 \cdot 8 = 48$:

				1	8
1	1	9	6	**8**	
			4		

3) $6 \cdot 1 = 6$:

				1	8
1	1	9	**6**	8	
			4		

4) Move “18” to the left by one place:

			1	**8**	
1	1	9	6	8	
			4		

5) $9 \cdot 8 = 72$:

			1	8	
1	1	9	6	8	
		7	4		
			2		

6) $9 \cdot 1 = 9$:

			1	8	
1	1	**9**	6	8	
		7	4		
			2		

7) Move “18” to the left by one place:

	1	**8**			
1	1	9	6	8	
		7	4		
			2		

8) $1 \cdot 8 = 8$:

	1	8			
1	1	9	6	8	
		7	4		
		8	2		

9) $1 \cdot 1 = 1$:

	1	8			
1	**1**	9	6	8	
		7	4		
		8	2		

10) Move “18” to the left by one place:

1	**8**				
1	1	9	6	8	
		7	4		
		8	2		

11) $1 \cdot 8 = 8$:

1	8			
1	1	9	6	8
	8	7	4	
		8	2	

12) $1 \cdot 1 = 1$:

1	8			
1	1	9	6	8
	8	7	4	
		8	2	

13) Add up the numbers except the multiplier (18):

2	**1**	**5**	**2**	**8**

As has been pointed out by Datta and Singh (2001, I.137), the name of this method, *kapāṭa-sandhi* ("door junction"), seems to originate from "the relative positions of the multiplicand and the multiplier."[1] At every step, the multiplier and the multiplicand, representing the two doors seen from above, make a junction at one notational place. In the case of the direct order it may be illustrated as follows.

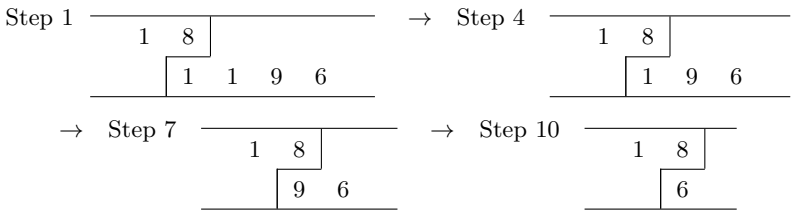


Figure N1: The relative positions of the multiplicand and the multiplier in the direct order of Kapāṭa-sandhi.

Śambhunātha calls the inverse-order method of Kapāṭa-sandhi with the multiplier above the multiplicand *śīrṣa* ("head") variety of Tatstha and the direct-order method of the same with the multiplier below the multiplicand *pṛṣṭha* ("back") variety of Tatstha (cf. Hayashi 1991, 420).

Now, Gomūtrikā in the direct order:

1	1	9	6
1	8		

After multiplying straightly and alternately, the form ⟨of the penultimate step is⟩

1	9	7	8	8
	1	7	4	

Obtained is 21528 *drammas*. In the same way, Gomūtrikā in the inverse order:

1	1	9	6
		1	8

After multiplying straightly and alternately, the form ⟨of the penultimate step is⟩

1	1	7	4	8
	9	7	8	

Obtained is 21528 *drammas*. /BBA 8.6/

[1] I am indebted to Professor S. R. Sarma for my understanding of Datta and Singh's intention.

...Note.....

BBA 8.6. Solution of Ex. 1 by Gomūtrikā (cow's urine) method. Both the direct-order and the inverse-order methods can be reconstructed as follows. Note that the three tables given for each variety in BBA 8.6 represent steps 1), 8) without the multiplicand and the multiplier, and 9).

1. Direct order (cf. BBA 6.1):

1) Write down the multiplier (18) below the multiplicand (1196) in such a way that the highest place of the former is just below the highest place of the latter:

1	**1**	**9**	**6**
1	**8**		

2) $1 \cdot 1 = 1$ (straightly):

1	1	9	6
1	8		
1			

3) $1 \cdot 8 + 1 \cdot 1 = 9$ (alternately):

1	1	9	6
1	8		
1	**9**		

4) Move '18' to the right by one place:

1	1	9	6
	1	**8**	
1	9		

5) $1 \cdot 8 + 9 \cdot 1 = 17$ (alternately):

1	1	9	6
	1	8	
1	9	**7**	
	1		

6) Move '18' to the right by one place:

1	1	9	6
		1	**8**
1	9	7	
	1		

7) $9 \cdot 8 + 6 \cdot 1 = 78$ (alternately):

1	1	9	6
		1	8
1	9	7	**8**
	1	**7**	

8) $6 \cdot 8 = 48$ (straightly):

1	1	9	6	
		1	8	
1	9	7	8	**8**
	1	7	**4**	

9) Add up the digits obtained:

2	**1**	**5**	**2**	**8**

2. Inverse order (cf. BBA 6.2):

1) Write down the multiplier (18) below the multiplicand (1196) in such a way that the units' place of the former is just below the units' place of the latter:

	1	**1**	**9**	**6**
			1	**8**

2) $6 \cdot 8 = 48$ (straightly):

1	1	9	6
		1	8
		4	**8**

3) $6 \cdot 1 + 9 \cdot 8 = 78$ (alternately):

1	1	9	6
		1	8
	7	4	8
		8	

4) Move '18' to the left by one place:

1	1	9	6
	1	**8**	
	7	4	8
		8	

5) $9 \cdot 1 + 1 \cdot 8 = 17$ (alternately):

1	1	9	6
	1	8	
1	7	4	8
	7	8	

6) Move '18' to the left by one place:

1	1	9	6
1	**8**		
1	7	4	8
	7	8	

7) $1 \cdot 1 + 1 \cdot 8 = 9$ (alternately):

1	1	9	6
1	8		
1	7	4	8
9	7	8	

8) $1 \cdot 1 = 1$ (straightly):

	1	1	9	6
	1	8		
1	1	7	4	8
	9	7	8	

9) Add up the digits obtained:

2	**1**	**5**	**2**	**8**

The name of this method, *gomūtrikā* (cow's urine), seems to originate from its procedure expressed as "straightly and alternately" (*saralaṃ mithaḥ* in PV 6b, *pādharūṃ anai anyonyi* in BBA 6.1, *pādharū anyonyi* in BBA 6.2, and *sarala anyonya* in BBA 8.6). For brevity, let two integers be $(a_2, a_1) = 10a_2 + a_1$ and $(b_2, b_1) = 10b_2 + b_1$. Then the basic pattern of the procedure of Gomūtrikā for their product may be expressed algebraically as follows.

$$\begin{pmatrix} a_2 & a_1 \\ b_2 & b_1 \end{pmatrix} \quad \rightarrow \quad (a_2 b_2, a_1 b_2 + a_2 b_1, a_1 b_1)\,,$$

where a_2b_2 and a_1b_1 are obtained by "straight" or vertical multiplications and $a_1b_2 + a_2b_1$ by "alternate" or cross multiplications.

Datta and Singh (2001, 147, fn. 4) say, "The method of multiplication of astronomical quantities is called *gomûtrikâ* even upto the present day by the paṇḍits." Presumably, those "astronomical quantities" were partly expressed in sexagesimal notation. In Sumatiharṣa's commentary (CE 1621) on the *Karaṇakutūhala* of Bhāskara II, multiplications of numbers with sexagesimal fractions are said to be carried out "by means of Gomūtrikā" (*gomūtrikayā*), which most probably refers to the above method with base 60 in place of 10 (see Sumatiharṣa 1991, 31 & 116). It is called *catuṣpadī-nyāya* ("quadruped principle") in an anonymous commentary on Mañjula's *Laghumānasa*, which was written after Śrīpati in Karnataka (Shukla 1990, 40–44). According to Shukla (1990, 49), it puts the sexagesimal fractions below the integral parts. Let the two numbers be $\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = a_1 + a_2/60$ and $\begin{pmatrix} b_1 \\ b_2 \end{pmatrix} = b_1 + b_2/60$. Then, their product is obtained by

$$\begin{pmatrix} a_1 & b_1 \\ a_2 & b_2 \end{pmatrix} \quad \rightarrow \quad \begin{pmatrix} a_1b_1 \\ a_1b_2 + a_2b_1 \\ a_2b_2 \end{pmatrix},$$

where the denominator of a_2b_2 is 60^2. Cf. the multiplication of fractions in PV 13 below. It is likely that the Gomūtrikā method of multiplication was originally designed for numbers with sexagesimal fractions.

Gaṇeśa's version of this method, which he calls "that-place multiplication" (*tat-sthāna-guṇana*), does not move the multiplier but does use the "alternate" or cross multiplication, which is the main characteristic of the Gomūtrikā. He refers to it as "like the thunderbolt multiplication" (*vajrābhyāsavat*). At the end of his commentary on L 17, he describes this method as follows.

> The multiplication of those digits having that place, which are located at that same place, is "that-place multiplication." It is as follows. Having placed the multiplier below the multiplicand and multiplied the units' place by the units' place, ⟨the result⟩ should be placed below. Then, having multiplied the units' places by the tens' places like the thunderbolt multiplication and summed ⟨the products⟩, ⟨the result⟩ should be placed in the row (*paṅkti*) ⟨of the product⟩ previously placed. Then, having multiplied the units' place by the hundreds' place, the hundreds' place by the units' place, and the tens' place by the tens' place, and summed ⟨the products⟩, ⟨the result⟩ should be placed like before. The other places too ⟨should be treated⟩ in the same manner. This having been done, the row will be the fruit of the multiplication. This ⟨method⟩ is of the nature of great curiosity (*mahad-āścarya-rūpa*) and cannot be understood by dull persons without traditional teaching (*pāramparya-upadeśa*). In like manner, other methods of multiplication too should be considered by intelligent ones. (Gaṇeśa on L 17)

The difficulty mentioned here may have been caused by the traditional teaching's omission of the move of the multiplier. By the phrase "like the thunderbolt multiplication" Gaṇeśa was probably referring to the cross multiplication of the same name that occurs in the *varga-prakṛti* (square nature), that is, a solution of the quadratic indeterminate equations of the type, $px^2 + t = y^2$. See BSS 18.65 (where occurs *vajra-vadha*) and BG 42 and 43 (*vajra-abhyāsa*). In his commentary on the latter verses, Kṛṣṇa (before CE 1601) explains the compound as follows.

> Thunderbolt multiplication (*vajra-abhyāsa*) is the name of oblique multiplication (*tiryag-guṇana*), for thunderbolt (*vajra*) has the nature of striking obliquely (*tiryak-prahāra*). (Kṛṣṇa on BG 41–43)

Śambhunātha's version of Gomūtrikā does not even use the cross multiplication. His initial arrangement of the given digits is

a_1	a_2
b_1	
b_2	b_1
	b_2

He does not show the rest of the calculation but it can be easily reconstructed as follows. →

a_1	a_2
a_1b_1	
a_1b_2	a_2b_1
	a_2b_2

→

a_1	a_2
a_1b_1	
$a_1b_2 + a_2b_1$	
a_2b_2	

(cf. Hayashi 1991, 419).

The use of the word *gomūtrikā* in mathematics is in conformity with that in Sanskrit rhetoric, where it is the name of a type of verses in which every alternate syllable of the first hemistich is identical with those of the second. Therefore, if one reads the syllables of the first and the second hemistichs zigzag, i.e. alternately, one obtains exactly the same verse as the original. An example occurs in Bhāravi's *Kirātārjunīya* (6th century).

> nāsuro 'yaṃ na vā nāgo dharasaṃstho na rākṣasaḥ/
> nā sukho 'yaṃ navābhogo dharaṇistho hi rājasaḥ//15.12//

In this verse, the even syllables of the first hemistich are exactly the same as those of the second. Figure N2 shows two zigzag lines, one commencing from the frist syllable *nā* of the first hemistich and the other from the first syllable *nā* of the second. Reading the syllables along the first or second zigzag line will bring us the first or second hemistich, respectively.

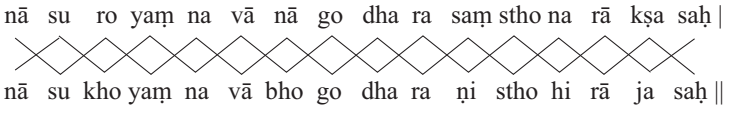


Figure N2: An example of gomūtrikā in Sanskrit rhetoric

The name of the multiplication method, *gomūtrikā*, should not be confused with *gosūtrikā* of BSS 12.55, which reads:

guṇakārakhaṇḍatulyo guṇyo gosūtrikākṛto guṇitaḥ/
sahitaḥ pratyutpanno guṇakārakabhedatulyo vā//12.55//

Colebrooke translated this verse as follows.

> The multiplicand is repeated like a string for cattle, as often as there are integrant portions in the multiplier; and is severally multiplied by them, and the products are added together: it is multiplication. Or the multiplicand is repeated as many times as there are component parts in the multiplier. (BSS 12.55, trans. by Colebrooke 2005, 319)

Datta and Singh (2001) disagreed with Colebrooke's reading *gosūtrikā* and adopted *gomūtrikā* of S. Dvivedin's edition. But Colebrooke based his reading on the commentary of Pṛthūdakasvāmin (fl. CE 864; MS: India Office Library, Eggeling 2769, fol. 93) and in a footnote explains the compound as follows.

> *Gosūtrikā*; a rope piqueted (picketed) at both ends; with separate halters made fast to it for each ox or cow.

Sanskrit dictionaries also record similar meanings. The arrangement of the cows and the rope intended here may be illustrated as follows.

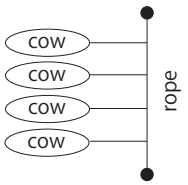


Figure N3: Illustration of gosūtrikā seen from above

The expression *gosūtrikā* then is indeed appropriate for visualizing the two kinds of multiplication with parts later called *sthāna-vibhāga-khaṇḍa-guṇana* ("multiplication with parts by division into notational places") and *rūpa-vibhāga-khaṇḍa-guṇana* ("multiplication with parts by division into integers"). Cf. L 17. In this simile, the "rope" is the sequence of the parts into which the multiplier is divided and the "cow" is the multiplicand. See Figure N4. The following translation of BSS 12.55, which is more literal than Colebrooke's, will more clearly describe the intended procedure.

> The multiplicand, equal ⟨in number⟩ to the parts (*khaṇḍa*, notational places) in the multiplier, ⟨when⟩ arranged in the form of cows' rope, multiplied ⟨by each part severally⟩, and added together, ⟨becomes⟩ the product. Or, otherwise, ⟨the multiplicand⟩, equal ⟨in number⟩ to the splits (*bheda*, component integers) in the multiplier, ⟨is treated likewise⟩. (BSS 12.55, trans. by myself)

					1196	2	→	2392	
1196	1	→	1196		1196	4	→	4784	
1196	2	→	2392		1196	6	→	7176	
			14352					14352	
Sthāna-vibhāga					Rūpa-vibhāga				

Figure N4: Illustration of two kinds of khaṇḍa-guṇana (by $1196 \times 12 = 14352$)

The commentator Pṛthūdaka in the three manuscripts that I used does not strictly follow this rule. While illustrating the Gosūtrikā with the example 235×288, he arranges the multiplicand (235) and the partial products (470[00]/1880[0]/1880) vertically and the "parts" of the multiplier (2/8/8) horizontally in the Sthānavibhāga; in the Rūpavibhāga, he puts both the "splits" of the multiplier (9/8/151/120) and the partial products (2115/1880/35485/28200) horizontally. This has probably resulted either from Pṛthūdaka's loose application of the BSS rule or from scribal errors, or from both.

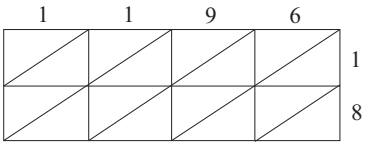
Now, the Śīrṣa variety ⟨of Tatstha⟩:

	1	8	
1	1	9	6

After multiplying one-by-one, the setting-down of the form ⟨of the penultimate step is⟩

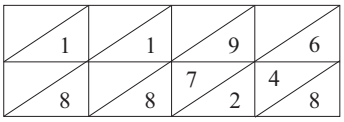
1	1	9	6	8
	8	8	2	
		7	4	

Obtained is | 21528 | *drammas*. Likewise, the Koṣṭha variety of Tatstha:



⟨Figure 1⟩

After multiplying one-by-one, the form ⟨of the penultimate step is⟩



⟨Figure 2⟩

Obtained in adding up ⟨the results⟩ is | 21528 | *drammas*. /BBA 8.7/

...Note.....

BBA 8.7. Solution of Ex. 1 by Tatstha. The two varieties of Tatstha may be reconstructed as follows.

1. Śīrṣa variety of Tatstha (cf. BBA 7.1).

1) Write down the multiplier (18) on top of the multiplicand (1196):

	1	**8**		
1	**1**	**9**	**6**	

2) $1 \cdot 1 = 1$:

	1	8		
1	1	9	6	
1				

3) $1 \cdot 1 = 1$:

	1	8		
1	1	9	6	
1	**1**			

4) $9 \cdot 1 = 9$:

	1	8		
1	1	9	6	
1	1	**9**		

5) $6 \cdot 1 = 6$:

	1	8		
1	1	9	6	
1	1	9	**6**	

6) $1 \cdot 8 = 8$:

	1	8		
1	1	9	6	
1	1	9	6	
	8			

7) $1 \cdot 8 = 8$:

	1	8		
1	1	9	6	
1	1	9	6	
	8	**8**		

8) $9 \cdot 8 = 72$:

	1	8		
1	1	9	6	
1	1	9	6	
	8	8	**2**	
		7		

9) $6 \cdot 8 = 48$:

	1	8		
1	1	9	6	
1	1	9	6	**8**
	8	8	2	
		7	**4**	

10) Add up the digits obtained:

2	**1**	**5**	**2**	**8**

BBA 7.1 specifies the position of "18" as "at the head" (*māthai*) in addition to "on top of the term of the question" (*prasnapada ūpari*) but probably both have the same meaning because it does not matter if "18" is placed at the units' place or at the highest place of "1196." The first three tables in BBA 8.7 represent steps 1), 9) without the multiplier and the multiplicand, and 10).
Śambhunātha does not treat this method and gives the name *tatstha-śīrṣa-bheda* to the inverse-order method of Kapāṭa-sandhi.

2. Koṣṭha variety of Tatstha (cf. BBA 7.2).
1) Draw as many cells as the number of places of the multiplicand times that of the multiplier; draw a diagonal line in each cell; and write down the multiplicand on the top of the box and the multiplier to the right side:

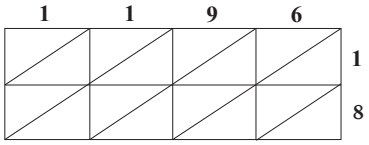


Figure N5-1: Koṣṭha variety—drawing cells with the multiplicand and multiplier

2) Fill each cell with the product of the corresponding digits of the multiplicand and the multiplier:

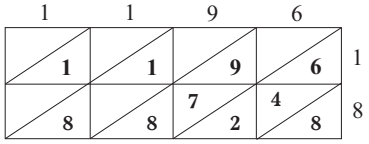


Figure N5-2: Koṣṭha variety—writing the partial products

This diagram in the manuscript B omits the multiplicand (1196) and the multiplier (18).
3) Add up the digits along the diagonal lines: | **2 1 5 2 8** |

Śambhunātha, like the authors of the GL and of the GM, calls this method *kapāṭa-sandhi*.

Now, the integer-division of Khaṇḍa is ⟨made⟩ in two ways. ⟨The first is the division of the term of the question into, for example, two halves, 598 and 598.⟩ After multiplying both ⟨by the price 18⟩ like Kapāṭasandhi, the form of ⟨each⟩ half is: | 10764 / 10764 | Obtained is | 21528 | *drammas*. The second is the division of the price, ⟨for example into two halves, 9 and 9⟩. After multiplying each ⟨by the term of the question⟩, the form is | 10764 / 10764 | Obtained is | 21528 | *drammas*. ⟨When the

price is divided into eighteen unities,⟩

1	1196	1	1196	
1	1196	1	1196	⟨Obtained is 21528
1	1196	1	1196	
1	1196	1	1196	
1	1196	1	1196	
1	1196	1	1196	
1	1196	1	1196	
1	1196	1	1196	
1	1196	1	1196	

drammas.⟩

Likewise, the place-division of Khaṇḍa. One, ten, hundred, thousand, etc. The form ⟨of the first step of each division is⟩

1 8
1 0 0 0
1 8
1 0 0
1 8
9 0
1 8
6

After multiplying all ⟨parts⟩ like Kapāṭasandhi, the form ⟨of the results is⟩

18000
1800
1620
108

Obtained is 21528 *drammas.* Likewise, the lesser-greater division. ⟨The form of⟩ half ⟨and double⟩:

36
598
9
2392

After multiplying both like Kapāṭasandhi, only one fruit obtained is 21528 *drammas.* In the same way, having made one third or one fourth ⟨of one of the two numbers and three times or four times of the other, both are⟩ multiplied. From that in the same way the fruit (product) is obtained. Now, the methods for the first example ⟨shown above are to be employed⟩ everywhere ⟨in the examples that follow⟩. /BBA 8.8/

···Note·····

BBA 8.8. Solution of Ex. 1 by Khaṇḍa.

1. Integer-division. By dividing the multiplicand into two halves, $1196 \cdot 18 = (598 + 598) \cdot 18 = 10764 + 10764 = 21528$. Or, otherwise, by dividing the multiplier into two haves, $1196 \cdot 18 = 1196 \cdot (9 + 9) = 10764 + 10764 = 21528$. Or, otherwise, by dividing the multiplier into eighteen

unities, $1196 \cdot 18 = 1196 \cdot (1 + 1 + \cdots + 1) = 1196 + 1196 + \cdots + 1196 = 21528$. This is, so to speak, an ultimate integer division.
2. Place-division. By dividing the multiplicand into the decimal places, $1196 \cdot 18 = 1000 \cdot 18 + 100 \cdot 18 + 90 \cdot 18 + 6 \cdot 18 = 18000 + 1800 + 1620 + 108 = 21528$. An application of the second rule of verse S2 cited in BBA 8.4 can be seen here in 18000, 1800, and 1620.
3. Lesser-greater division. $1196 \cdot 18 = \frac{1196}{2} \cdot (18 \cdot 2) = 598 \cdot 36 = 21528$. Or, otherwise, $1196 \cdot 18 = (1196 \cdot 2) \cdot \frac{18}{2} = 2329 \cdot 9 = 21528$.
Presumably, the multiplications of parts in all these cases were made either by Kapāṭasandhi or by Gomūtrikā or by Tatstha.

Second example. Gold, eight hundred and sixty-five *tolas*, for (unit price) thirty-two *ṭaṅkas*. Setting-down: 865 multiplier 32. Obtained is 27680 *ṭaṅkas*. /BBA 8.9/

···Note·····

BBA 8.9. Ex. 2 for multiplication: What is the price of 865 *tolas* of gold when the unit price is 32 *ṭaṅkas*? Answer: 27680 *ṭaṅkas*.

Third example. Madder, one hundred and ninety-six *maṇas*, for (unit price) thirty-five *ṭaṅkas*. Setting-down: 196 multiplier 35. Obtained is 6860 *ṭaṅkas*. /BBA 8.10/

···Note·····

BBA 8.10. Ex. 3 for multiplication: What is the price of 196 *maṇas* of madder when the unit price is 35 *ṭaṅkas*? Answer: 6860 *ṭaṅkas*.

Fourth example. Ivory, four thousand eight hundred and sixty-five *maṇas*, for (unit price) thirty-six 36 *ṭaṅkas*. Setting-down: 4865 multiplier 36. Obtained is 175140 royal *ṭaṅkas*. /BBA 8.11/

···Note·····

BBA 8.11. Ex. 4 for multiplication: What is the price of 4865 *maṇas* of ivory when the unit price is 36 *ṭaṅkas*? Answer: 175140 royal *ṭaṅkas*.

Fifth example. Sugar, thirty-eight thousand three hundred and twenty-seven *maṇas*, for (unit price) eighty-one *drammas*. Setting-down: 38327 multiplier 81. Obtained is 3104487 *drammas*. /BBA 8.12/

···Note·····

BBA 8.12. Ex. 5 for multiplication: What is the price of 38327 *maṇas* of sugar when the unit price is 81 *drammas*? Answer: 3104487 *drammas*. 38327 is a prime number.

Sixth example. Sandal wood, one thousand seven hundred and sixty-seven *maṇas*, for (unit price) sixty-four *ṭaṅkas*. Setting-down: 1767 multiplier 64. Obtained is 113088 *ṭaṅkas*. /BBA 8.13/

···Note

BBA 8.13. Ex. 6 for multiplication: What is the price of 1767 *maṇas* of sandal wood when the unit price is 64 *ṭaṅkas*? Answer: 113088 *ṭaṅkas*.

Seventh example. Āchī, three thousand seven hundred and three *maṇas*, for (unit price) one hundred and eighty-eight *drammas*. Setting-down: 3703 multiplier 188. Obtained is 696164 *drammas*. /BBA 8.14/

···Note

BBA 8.14. Ex. 7 for multiplication: What is the price of 3703 *maṇas* of *āchī* when the unit price is 188 *drammas*? Answer: 696164 *drammas*. The *āchī* has not been identified.

Eighth example. Threads for cloth, one thousand eight hundred and fifty-nine *maṇas*, for (unit price) three hundred and eight *ṭaṅkas*. Setting-down: 1859 multiplier 308. Obtained is 572572 *ṭaṅkas*. /BBA 8.15/

···Note

BBA 8.15. Ex. 8 for multiplication: What is the price of 1859 *maṇas* of threads for cloth when the unit price is 308 *ṭaṅkas*? Answer: 572572 *ṭaṅkas*.

Ninth example. One *lakṣa* fifty-two thousand two hundred and seven, multiplier seventy-three. Setting-down: 152207 multiplier 73. What is obtained has the form of a sequence of one's (*ekāvali*): 11111111. /BBA 8.16/

···Note

BBA 8.16. Ex. 9 for multiplication: $152207 \cdot 73 = 11111111$. This example is numerically the same as GSS 2.15, which calls the product "a necklace" (*kaṇṭhābharaṇa*).[1]

Tenth example. Three *ṣarva* thirty-three *arva* thirty-three *koṭi* thirty-six *lakṣa* sixty-six thousand six hundred and sixty-seven, multiplier thirty-three. Setting-down: 333333666667 multiplier 33. What is obtained has the form of a necklace (*kaṇṭhābharaṇa*): 11000011000011. /BBA 8.17/

···Note

BBA 8.17. Ex. 10 for multiplication: $333333666667 \cdot 33 = 11000011000011$. This example is numerically the same as GSS 2.11. Both GSS 2.11 and PC 34 call the product "a necklace" (*kaṇṭhābharaṇa*).

[1] For other such amusing examples of multiplication in the *Pāvulūrigaṇitamu*, see Sarma (1987, 170–71).

Here in BBA 8.17 the numerals, *ṣarva* and *arva*, are used to denote respectively 10^{11} and 10^{9}. The corresponding Sanskrit terms, *kharva* and *arbuda*, mean respectively 10^{10} and 10^{8} in the standard Hindu list of decimal notation, and 10^{12} and 10^{10} in the Jaina list of GSS 1.65–66. It is noteworthy that Ṭhakkura Pherū (ca. CE 1315) uses the corresponding Apabhraṃśa words, *khavva* and *avva*, in exactly the same senses as BBA 8.17.

Eleventh example. Fourteen *koṭi* twenty-eight *lakṣa* fifty-seven thousand one hundred and forty-three, multiplier seven. Setting-down: 142857143 multiplier 7. What is obtained has the form of a necklace (*hāra*): 1000000001. /BBA 8.18/

···Note

BBA 8.18. Ex. 11 for multiplication: $142857143 \cdot 7 = 1000000001$. This example is numerically the same as GSS 2.13, which calls the product "a royal necklace" (*rāja-kaṇṭhikābharaṇa*), while PC 35 calls it "Śiva's necklace" (*harassa kaṃṭhalliyā* = Skt. *harasya kaṇṭhālikā*).

Thus, the multiplication is completed. /BBA 8.19/

Now, the fourth *sūtra* for division./BBA 9.0/

> Having put down the divisor below the ⟨term in⟩ question and divided the ⟨term in⟩ question by the divisor, ⟨the greatest multiple of⟩ the part (divisor) should be taken away in order ⟨from each place of the dividend⟩. The operation of division has been surely laid down ⟨in this way⟩. /PV 9/

···Note

PV 9. Division. The verse does not specify the place where the quotient at each step is written down. It is usually put on top of the dividend but any place on the calculating board could also be used. For the actual procedure see Ex. 1 below.

The ⟨number of⟩ parts is written down below the term in question. Then, by the ⟨number of⟩ parts, the term in question is divided. Then, in order, the part is taken away. This is surely called the operation of division. /BBA 9.1/

Others' sūtra:

> ⟨When⟩ there is not the part ⟨in the term in question⟩, the ⟨partial⟩ quotient is zero./S3/

/BBA 9.2/

···Note

BBA 9.2. A supplementary rule (S3) is cited here. It is to be used when the number at a place of

the dividend is too small to be divided by the divisor during the division procedure. This occurs in Exs. 2, 4, 8, and 10.

First example. Four hundred and eighty-eight *drammas* into four parts. Setting-down in full condition:

4	8	8
4	parts	

The part being dropped ⟨from the question, i.e., the dividend⟩, obtained is 122 *drammas*. Likewise, in partial condition, i.e., setting-down of halves of both the quantity in question and the quantity of parts:

244
2 parts

⟨These are⟩ half of the question and half of the ⟨number of⟩ parts. The part being dropped ⟨from the question⟩, obtained is 122 *drammas*. In this manner, having reduced ⟨both the question and the number of parts⟩ into one third, one fourth, ⟨etc.⟩, the ⟨number of⟩ parts is given, and in the same way ⟨as above⟩ one obtains the fruit (quotient). /BBA 9.3/

···Note·····

BBA 9.3. Ex. 1 for division: 488 *drammas* ÷ 4 = 122 *drammas*. This calculation may be reconstructed as follows.

1) Write down the divisor below the highest place of the dividend:

4	**8**	**8**
4		

2) Divide the upper 4 by the lower 4, that is, take away the greatest multiple of the divisor from the corresponding place of the dividend ($4 - 4 \cdot 1 = 0$), and write down the quotient (1) above the divisor (4):

1		
	8	8
4		

3) Move the divisor to the right by one place:

1		
	8	8
	4	

4) Divide the upper 8 by the divisor ($8 - 4 \cdot 2 = 0$), and write down the quotient (2) above the divisor (4):

1	**2**	
		8
	4	

5) Move the divisor to the right by one place:

1	2	
		8
		4

6) Do the same as in step 4:

1	2	**2**
		4

Hence the quotient is 122.

The division may be made after the dividend and the divisor are reduced by a common factor.

Second example. Three hundred and twenty-seven *drammas* into three parts. Setting-down: | 327 / 3 parts | Obtained is 109 *drammas*. /BBA 9.4/

···Note·····

BBA 9.4. Ex. 2: 327 *drammas* ÷ 3 = 109 *drammas*.

Third example. Four thousand and ninety-six *drammas* into sixteen parts. Setting-down: 4096 into 16 parts. Obtained is 256 *drammas*. /BBA 9.5/

···Note·····

BBA 9.5. Ex. 3: 4096 *drammas* ÷ 16 = 256 *drammas*.

Fourth example. Eleven thousand six hundred and sixty-four *drammas* into one hundred and eight parts. Setting-down: 11664 into 108 parts. Obtained is 108 *drammas* per part. /BBA 9.6/

···Note·····

BBA 9.6. Ex. 4: 11664 *drammas* ÷ 108 = 108 *drammas*.

Fifth example. Thirty thousand two hundred and seventy-six *drammas* into eighty-seven parts. Setting-down: 30276 into 87 parts. Obtained is 348 *drammas* per part. /BBA 9.7/

···Note·····

BBA 9.7. Ex. 5: 30276 *drammas* ÷ 87 = 348 *drammas*.

Sixth example. One *lakṣa* fifty-six thousand twenty-five *drammas* into three hundred and ninety-five parts. Setting-down: 156025 into 395 parts. Obtained is 395 *drammas* per part. /BBA 9.8/

···Note·····

BBA 9.8. Ex. 6: 156025 *drammas* ÷ 395 = 395 *drammas*.

Seventh example. Fourteen *lakṣa* sixty-six thousand five hundred and twenty-one *drammas* into twelve hundred and eleven parts. Setting-down: 1466521 into 1211 parts. Obtained is 1211 *drammas* per part. /BBA 9.9/

···Note·····

BBA 9.9. Ex. 7: 1466521 *drammas* ÷ 1211 = 1211 *drammas*.

Eighth example. One *koṭi* ninety-three *lakṣa* forty-five thousand and six hundred *drammas* into eighteen hundred and eight parts. Setting-down: 19345600 into 1808 parts. Obtained is 10700 *drammas* per part. /BBA 9.10/

···Note·····

BBA 9.10. Ex. 8: 19345600 *drammas* ÷ 1808 = 10700 *drammas*.

Ninth example. One *koṭi* and three *drammas* into thirteen parts. Setting-down: 10000003 into 13 parts. Obtained is 769231 *drammas* per part. /BBA 9.11/

···Note·····

BBA 9.11. Ex. 9: 10000003 *drammas* ÷ 13 = 769231 *drammas*. This is a prime number.

Tenth example. One *lakṣa* and one *drammas* into eleven parts. Setting-down: 100001 into 11 parts. Obtained is 9091 *drammas* per part. /BBA 9.12/

···Note·····

BBA 9.12. Ex. 10: 100001 *drammas* ÷ 11 = 9091 *drammas*. This is a prime number.

Thus, the division is completed. /BBA 9.13/

Now, the fifth *sūtra* for various purposes. The procedures for various purposes, such as square, cube, square root, multiplication of fractions, three-quantity operation, inverse three-quantity operation, investment, measurement of gold, measurement of fields and clothes, measurement of excavations, timbers, stones, storehouses, and piling ⟨of bricks⟩, measurement of circular timbers, stones, pillars, and wells, measurement of spheres, measurement of the heaped-up grains, measurement of shadows, measurement of daylight, measurement of the noon ⟨shadow lengths in⟩ feet, etc., will ⟨hereafter⟩ be told in order. /BBA 10–26.0/

The former half ⟨of verse 10⟩ on the square./BBA 10_1.0/

> In the product of two like quantities, only the square will be produced. /PV 10_1/

···Note·····

PV 10_1. Square. $n^2 = n \times n$.

Two like (tulya: śarīṣī) quantities are multiplied ⟨with each other⟩. It becomes the square. /BBA 10_1.1/

Ex. What are the squares of one, two, three, four, five, fifteen, twenty-five, and two hundred and thirty-five? Setting-down:

1	2	3	4	5	15	25	235
1	2	3	4	5	15	25	235

Obtained is the squares, 1, 4, 9, 16, 25, 225, 625, and 55225. /BBA 10_1.2/

···Note

BBA 10_1.2. Examples for square. 1. $1^2 = 1$. 2. $2^2 = 4$. 3. $3^2 = 9$. 4. $4^2 = 16$. 5. $5^2 = 25$. 6. $15^2 = 225$. 7. $25^2 = 625$. 8. $235^2 = 55225$.

Thus the square. /BBA 10_1.3/

The latter half ⟨of verse 10⟩ on the cube./BBA 10_2.0/

> In the product of three like terms the cube will surely be produced. /PV 10_2/

···Note

PV 10_2. Cube. $n^3 = n \times n \times n$.

Three like (tulya: śarīṣāṃ) terms are multiplied ⟨with each other⟩. It becomes the cube. /BBA 10_2.1/

Ex. What are the cubes of one, two, three, four, five, fifteen, and twenty-five? Setting-down:

1	2	3	4	5	15	25
1	2	3	4	5	15	25
1	2	3	4	5	15	25

Obtained are the cubes, 1, 8, 27, 64, 125, 3375, and 15625. /BBA 10_2.2/

···Note

BBA 10_2.2. Examples for cube. 1. $1^3 = 1$. 2. $2^3 = 8$. 3. $3^3 = 27$. 4. $4^3 = 64$. 5. $5^3 = 125$. 6. $15^3 = 3375$. 7. $25^3 = 15625$.

Thus the cube. /BBA 10_2.3/

Now, the square root./BBA 11–12.0/

> Having abandoned a square from the ⟨highest⟩ odd ⟨place⟩, one should divide the next ⟨place⟩ by twice ⟨the square root⟩, put the quotient into the line ⟨of the roots⟩, and subtract its square ⟨from the next⟩. /PV 11/ Having doubled ⟨the square root⟩ as before, and slid that ⟨line of the quotients to the next place⟩, one should divide the next. This ⟨procedure⟩ should be done at the succeeding ⟨places⟩ also. ⟨Finally⟩, one should halve what has been doubled. /PV 12/

···Note

PV 11–12. Square root. For the actual procedure see the Note for the next paragraph (BBA 11–12.1) below.

Odd, even, odd. From the ⟨highest⟩ odd term a square is dropped. Then, having doubled the square root, ⟨the result is⟩ written down below the next (*para: āgilā*) digit. And after seeing ⟨the operation⟩ reached there part is dropped (i.e., the upper digits are divided by the lower). Then, it is the quotient. It is written down in the line ⟨of the quotients⟩. And its square is dropped ⟨from the next place⟩. Then, having doubled the digit obtained ⟨as a quotient⟩ through the previous procedure, ⟨the result⟩ is written down below the digit next (*para: āgilā*) starting from there. This procedure is further carried out in exactly the same way. Then, half of those which have been doubled is made. It becomes the square root. /BBA 11–12.1/

···Note·····

BBA 11–12.1. This procedure may be illustrated by Ex. 8 of the next paragraph (BBA 11–12.2) as follows.

1) Write down the number whose square root is required:

5	**5**	**2**	**2**	**5**

2) Subtract a square number (4) from the highest square place (5), and write down its root below it:

1	5	2	2	5
2				

3) Double the root (2) and move the result (4) to the right by one place:

1	5	2	2	5
	4			

4) Divide the upper digits (15) by the lower (4), and write down the quotient (3) below the next place (2):

3	2	2	5
4	**3**		

5) Subtract the square of the quotient (3) from the upper (32):

2	**3**	2	5
4	3		

6) Double the quotient (3) just obtained:

2	3	2	5
4	**6**		

7) Move the lower digits (46) to the right by one place:

2	3	2	5
	4	**6**	

8) Divide the upper digits (232) by the lower (46), and write down the quotient (5) in the next place:

		2	5
4	6	**5**	

9) Subtract the square of the quotient (5) just obtained from the upper digits (25):

		0	
4	6	5	

10) Halve the digits obtained by doubling:

		0	
2	**3**	5	

Hence follows $\sqrt{55225} = 235$.

Exs. Setting-down of the results of the square told first: 1, 4, 9, 16, 25, 225, 625, 55225. The roots obtained are 1, 2, 3, 4, 5, 15, 25, 235. /BBA 11–12.2/

···Note·····

BBA 11–12.2. Examples for the square roots. 1. $\sqrt{1} = 1$. 2. $\sqrt{4} = 2$. 3. $\sqrt{9} = 3$. 4. $\sqrt{16} = 4$. 5. $\sqrt{25} = 5$. 6. $\sqrt{225} = 15$. 7. $\sqrt{625} = 25$. 8. $\sqrt{55225} = 235$. Cf. the examples for square in BBA 10_1.2.

Thus the square root. /BBA 11-12.3/

Multiplication of fractions./BBA 13.0/

> The integer should be multiplied by the ⟨other⟩ integer. Both fractional parts for the denominator too ⟨should be multiplied⟩ by them (i.e., by the integers). And those fractional parts are mutually ⟨multiplied with each other⟩. ⟨In this way,⟩ one can obtain the fruit (product) produced from fractions. /PV 13/

···Note·····

PV 13. Multiplication of numbers with fractional parts. Let a_i and b_i ($i = 1, 2$) be the integral and fractional parts of two numbers for a certain common denominator d. That is, $(a_1; a_2) = a_1 + a_2/d$ and $(b_1; b_2) = b_1 + b_2/d$. Then, $(a_1; a_2) \times (b_1; b_2) = a_1b_1 + (a_1b_2 + a_2b_1)/d + a_2b_2/d^2 = (a_1b_1; a_1b_2 + a_2b_1, a_2b_2)$. This is basically the same as the Gomūtrikā method of multiplication. See the Note for BBA 8.6. Śambhunātha's version (verse 15) prescribes the ordinary multiplication and division of fractions, that is, $\frac{b}{a} \times \frac{d}{c} = \frac{bd}{ac}$ and $\frac{b}{a} \div \frac{d}{c} = \frac{b}{a} \times \frac{c}{d} = \frac{bc}{ad}$.

In the multiplication of fractions, the integer is multiplied by the ⟨other⟩ integer. Then, the fractional parts for the denominator are multiplied by the ⟨opposite⟩ integer. Then, the fractional parts for the denominator are mutually multiplied. The fruit (product) is obtained. /BBA 13.1/

···Note·····

BBA 13.1. The second sentence, as it occurs in both MSS, reads, "Then, the denominator is multiplied by the integer," but this does not make sense. I supplied the word *aṃśa* (fractional part) after the *cheda* (denominator) according to PV 13.

First example. Here, the standard for *gaja* is the greater *aṅgula*. There are twenty-four *aṅgulas* in one *gaja*. Bought is 1 banner. Length ten *gajas* and twelve *aṅgulas*. Width one *gaja* and eight *aṅgulas*. Setting-down:

10 ||	length
1 | 2	width

One *gaja* is multiplied by ten *gajas*; produced is ten *gajas*. Likewise, one *gaja* is multiplied by twelve *aṅgulas*; produced is 12 *aṅgulas*. Of these ⟨*aṅgulas*⟩, a half *gaja* is produced: 0||. Likewise, ten *gajas* is multiplied by eight *aṅgulas*; produced is eighty

aṅgulas. Of these ⟨*aṅgulas*⟩, three *gajas* and eight *aṅgulas* are produced: 3|2. Likewise, eight *aṅgulas* is multiplied by twelve *aṅgulas*; produced is ninety-six *vyaṅgulas*. Of these ⟨*vyaṅgulas*⟩, four *aṅgulas* is produced: $0, 4$. Obtained in the summation ⟨of these results⟩ in one place is 14 *gajas*. /BBA 13.2/

...Note.....

BBA 13.2. Ex. 1 for the multiplication of fractions: Area of a rectangular banner, whose length and width are respectively 10 *gajas* 12 *aṅgulas* and 1 *gaja* 8 *aṅgulas*, where 1 *gaja* = 24 *aṅgulas*. According to the commentator, the *aṅgula* employed here is the "greater" (*jeṣṭha*, Skt. *jyeṣṭha*) one. In the calculation, another, smaller unit *vyaṅgula* is also used. It is obviously one twenty-fourth of the *aṅgula*, although no definition is given here. The naming with the prefix *vi-* for the smaller unit with the same denominator (*d* in the Note for PV 13 above) was popular in India: 60^2 *vighaṭikās* = 60 *ghaṭikās* = 1 day and night, 60^2 *vikalās* = 60 *kalās* = 1 degree of arc, etc.

Solution: 1 *gaja* ×10 *gajas* = 10 *gajas*, 1 *gaja* ×12 *aṅgulas* = 12 *aṅgulas*, 10 *gajas* ×8 *aṅgulas* = 80 *aṅgulas* = 3 *gajas* 8 *aṅgulas*, 8 *aṅgulas* ×12 *aṅgulas* = 96 *vyaṅgulas* = 4 *aṅgulas*; $10 + 0; 12 + 3; 8 + 0; 4 = 14$ *gajas*.

A short vertical stroke used in the table indicates a quater of the unit. Since the *gaja* of this example is defined as 24 *aṅgulas*, one stroke represents 6 *aṅgulas*.

Between the words, *niyojane* ("in the summation") and *labdha* ("obtained"), J has the table,

10
0‖
3|2
S4
14

The first four rows of this table list the four products obtained by partial multiplication and the last row the sum of them. It is noteworthy that the two symbols, *śūnya* (a small circle) in the second row and *avagraha* (an S-like symbol) in the fourth row, are used for the same purpose, that is, for indicating that the integer part of *gaja* does not exist. Originally, the *avagraha* is a symbol used in Devanāgarī script for indicating the omission of the initial *a* of words after words ending in *-e* or *-o* (including *-o* from *-as*).

Second example. Here, the standard for *gaja* is the *visā*. There are twenty, 20, *visās* in one *gaja*. Bought is a piece of land. Length twenty-five *gajas* and fourteen *visās*. Width sixteen *gajas* and ten *visās*. Setting-down:

25	‖ 4	length
16	‖	width

Sixteen *gajas* is multiplied by twenty-five *gajas*; produced is 400 *gajas*. Likewise, sixteen *gajas* is multiplied by fourteen *visās*; produced is two hundred and twenty-four *visās*, 224. Of these ⟨*visās*⟩, 11,4 *gajas* is produced. Likewise, ten *visās* is multiplied by twenty-five *gajas*; produced is two hundred and fifty *visās*, 250. Of these ⟨*visās*⟩, 12‖ *gajas* is produced. Likewise, ten *visās* is multiplied by fourteen

visās; produced is one hundred and forty *visāṃsas*. Of these ⟨*visāṃsas*⟩, 0| 2 *visās* is produced. Obtained in the summation ⟨of these results⟩ in one place is 424,1 *gajas*. In this way, cloth and land are measured. /BBA 13.3/

···Note·····

BBA 13.3. Ex. 2 for the multiplication of fractions: Area of a piece of land, whose length and width are respectively 25 *gajas* 14 *visās* and 16 *gajas* 10 *visās*, where 1 *gaja* = 20 *visās*. In the solution, another, smaller unit *visāṃsa* is used in the sense of one twentieth of the *visā*. Although the unit name *visāṃsa* is corrupted in this paragraph in both manuscripts, B and J, it can be confirmed by another occurrence in BBA 17_1.3.

Solution: 25 *gaja* ×16 *gajas* = 400 *gajas*, 16 *gaja* ×14 *visās* = 224 *visās* = 11 *gajas* 4 *visās*, 25 *gajas* ×10 *visās* = 250 *visās* = 12 *gajas* 10 *visās*, 10 *visās* ×14 *visās* = 140 *visāṃsas* = 0 *gaja* 7 *visās*; $400 + 11; 4 + 12; 10 + 0; 7 = 424; 1$ *gajas*.

Here also, a short vertical stroke indicates a quater of the unit. Since the *gaja* of this example is defined as 20 *visās*, one stroke represents 5 *visās*. Cf. the previous Note (BBA 13.2).

J uses the *avagraha* twice in this paragraph but it is not for indicating non-existence as in the table that J adds in the previous paragraph but simply for separating the two notational places, *gaja* and *visā*.

Thus the multiplication of fractions. /BBA 13.4/

Now, the former half ⟨of verse 14⟩ for the three-quantity operation. /BBA 14_1.0/

> The product of the middle and the first ⟨digits is⟩ first ⟨made, and then⟩ divided by the last in the three-quantity operation. /PV 14_1/

···Note·····

PV 14_1. Three-quantity operation. If $a : b = c : x$, then $x = (bc)/a$. The rule of this hemistich is applied to the three numbers, a, b, and c arranged in a horizontal row: $a/\ b/\ c/$. In the verse, these are called respectively "the last," "the middle," and "the first." The same nomenclature is used also for the rule of the inverse three-quantity operation (PV 14_2), but it is most unusual. Most other works on *pāṭī* use the same terms in the reverse order. Śambhunātha's version (verse 16) follows the latter.

In the three-quantity operation, the first digit is multiplied by the middle and then part is given (i.e., the product is divided) by the last. The fruit is obtained. /BBA 14_1.1/

First example. Rice: Five *maṇas* are obtained for twenty-one *drammas*. Then, how many *drammas* are ⟨necessary⟩ for ninety *maṇas*? Setting-down: 5, 21, 90. Obtained is 378 *drammas*. /BBA 14_1.2/

···Note

BBA 14_1.2. Ex. 1 for the three-quantity operation: rice (quantity-price). 5 *maṇas* : 21 *drammas* = 90 *maṇas* : x. Answer: 378 *drammas*.

Second example. Kidney-beans: Six *maṇas* are obtained for twenty-three *drammas*. Then, how much is ⟨obtained⟩ for eighty-one *drammas*? Setting-down: 23, 6, 81. Obtained is 21 *maṇas* 5 *seras* and part ⟨of *sera*⟩, $\frac{5}{23}$. /BBA 14_1.3/

···Note

BBA 14_1.2. Ex. 2 for the three-quantity operation: Kidney-beans (price-quantity). 23 *drammas* : 6 *maṇas* = 81 *drammas* : x. Answer: 21 *maṇas* $5\frac{5}{23}$ *seras*.

Thus the three-quantity operation. /BBA 14_1.4/

The latter half ⟨of verse 14⟩ on the inverse three-quantity operation./BBA 14_2.0/

> In the inverse ⟨three-quantity operation⟩, the product of the middle and the last ⟨digits⟩ is divided by the first. One shall obtain the fruit. /PV 14_2/

···Note

PV14_2. Inverse three-quantity operation. If the fruit (x) decreases when the requisite (c) increases, then the rule of this hemistich is applied to: a/ b/ c/. That is, $x = (ab)/c$.

In the inverse three-quantity operation, the last digit is multiplied by the middle and part is given (i.e. divided) by the first. The fruit is obtained. /BBA 14_2.1/

First example. A woman eighty years old is obtained for forty *ṭaṅkas*. Then, how much is ⟨a woman⟩ of sixteen years old? Setting-down: 80, 40, 16. Obtained is the price of a female servant of sixteen years old, 200 *ṭaṅkas*. /BBA 14_2.2/

···Note

BBA 14_2.2. Ex. 1 for the inverse three-quantity operation: women (age-price). 80 years/ 40 *ṭaṅkas*/ 16 years/. Answer: 200 *ṭaṅkas*.

Second example. A pearl is obtained for twenty-five *drammas* of one hundred *caḍatu ṭaṅkas*. Then, for how many ⟨*drammas* of⟩ sixty *caḍatu ṭaṅkas* is ⟨the same⟩ obtained? Setting-down: 100, 25, 60. Obtained is 41 *drammas* 2 *jathalas*. /BBA 14_2.3/

···Note

BBA 14_2.3. Ex. 2 for the inverse three-quantity operation: coins (quality-number). 100 *caḍatu ṭaṅkas*/ 25 *drammas*/ 60 *caḍatu ṭaṅkas*/. Answer: 41 *drammas* 2 *jathalas*. As this answer must be

equal to $\frac{100 \cdot 25}{60} = 41\frac{2}{3}$ *drammas*, the monetary unit *jathala* is equivalent to 1/3 *dramma*.

Thus the inverse three-quantity operation. /BBA 14_2.4/

Investment./BBA 15.0/

> Multiplication in order, of the ⟨number of⟩ units of ⟨each of⟩ the various investments by the produce ⟨in partnership is made⟩. From the division ⟨either of each product or of the produce⟩ by the sum ⟨of the investments⟩, two quotients ⟨are obtained. In the latter case, the quotient is multiplied by each investment.⟩ It (the result) shall be severally the fruit (share). /PV 15/

···Note·····

PV 15. Investment. Let a_i be the investment of the i-th person and M the total profit (or production) obtained from the investments. Then, the share of each person is,

$$p_i = \frac{a_i M}{A}, \quad \text{or} \quad p_i = \frac{M}{A} \cdot a_i,$$

where $A = a_1 + a_2 + \cdots + a_n$. Not only the words "two quotients" (*dvayī labdhī*) in verse 15 but also the commentary (BBA 15.1–2) suggest that verse 15 prescribes the above two formulas. But the verse does not mention the multiplication by a_i in the latter formula. Śambhunātha's version (verse 17) prescribes the former only.

"Multiplication" (*vadha: guṇākāra*) of "various"–many kinds of–"investments"–parts–by the "produced" thing is made. And "the sum of the units of the investments" is made. And a part is given (i.e., a division is made), in order, ⟨of each product⟩ by it. The fruit (share) is obtained severally. /BBA 15.1/

Likewise, the second method. "The sum of the units of the various investments" is made. And by it is given a part (i.e., is made a division) of "the produced" thing. The fruit for one division (i.e., for one unit of investment) is obtained. Then, a multiplication ⟨of the result⟩ is made by so much part as one has ⟨in the investment⟩. In this manner, the fruits (shares) are obtained severally. /BBA 15.2/

First example. Having sowed seeds two, three, four and five *seīs* severally on four seed-plots, two hundred and ten ⟨*seīs* of seeds⟩ were produced ⟨in total⟩. How much was produced where (on each seed-plot)? Setting-down: Seeds, 2, 3, 4, 5 *seīs*. Produce is 210. First method. Two hundred and ten are multiplied by two. Four hundred and twenty are produced: 420. Two hundred and ten are multiplied by three. Six hundred and thirty are produced: 630. Two hundred and ten are multiplied by four. Eight hundred and forty are produced: 840. Two hundred and ten are multiplied by five. Ten hundred and fifty are produced: 1050. And the sum

of the seeds is fourteen *seīs*, 14. Division by this: When the divisions are made, the quotients are severally the fruits (shares), 30, 45, 60, and 75. /BBA 15.3/

···Note

BBA 15.3. Ex. 1 for the investment: seeds. Given: $a_i = (i+1)$ *seīs* $(i = 1, 2, 3, 4)$, $M = 210$ *seīs*. Solution by the 1st method: $a_1M = 2 \cdot 210 = 420$, $a_2M = 2 \cdot 210 = 630$, $a_3M = 2 \cdot 210 = 840$, $a_4M = 2 \cdot 210 = 1050$. $A = a_1 + a_2 + a_3 + a_4 = 14$ *seīs*. $a_1M/A = 30$, $a_2M/A = 45$, $a_3M/A = 60$, $a_4M/A = 75$. The unit is *seī*.

Second method. The sum of the seeds is fourteen *seīs*, 14. The produce is two hundred and ten *seīs*, 210. The quotient, 15 *seīs*, of the division is the fruit (share) for one part. Multiplied by two, 30. Multiplied by three, 45. Multiplied by four, 60. Multiplied by five, 75. /BBA 15.4/

···Note

BBA 15.3. Ex. 1 for the investment solved by the 2nd method. $M/A = 210/14 = 15$ *seīs*/*seī*. Therefore, $(M/A) \cdot a_1 = 30$, $(M/A) \cdot a_2 = 45$, $(M/A) \cdot a_3 = 60$, $(M/A) \cdot a_4 = 75$. The unit is *seī*.

Second example. There are sixteen hundred *ṭaṅkas* in ⟨the hand of⟩ a village-born man. ⟨His⟩ investments in three ⟨parts⟩ are one hundred, five hundred, and one thousand. The ⟨total⟩ produce (i.e., profit) is two hundred *ṭaṅkas*. How much is ⟨the share⟩ for which ⟨part⟩? Setting-down: Investments 100, 500, 1000. Produce 200. First method. Two hundred multiplied by one hundred become twenty thousand, 20000. Two hundred multiplied by five hundred become one *lakṣa*, 100000. Two hundred multiplied by one thousand become two *lakṣas*, 200000. The sum of the ⟨three⟩ parts of the investment is sixteen hundred *ṭaṅkas*, 1600. Division by this: When the division is made, the quotients are severally $12\|, 62\|$, and 125 *ṭaṅkas*. /BBA 15.5/

···Note

BBA 15.5. Ex. 2 for the investment: money. Given: $a_1 = 100$, $a_2 = 500$, $a_3 = 1000$. $M = 200$. The unit is *ṭaṅka*. Solution by the 1st method: $a_1M = 100 \cdot 200 = 20000$, $a_2M = 500 \cdot 200 = 100000$, $a_3M = 1000 \cdot 200 = 200000$. $A = a_1 + a_2 + a_3 = 1600$. $a_1M/A = 12\frac{1}{2}$, $a_2M/A = 62\frac{1}{2}$, $a_3M/A = 125$.

Second method. The sum of the ⟨three⟩ parts of the investment is 1600 *ṭaṅkas*. The produce is 200 *ṭaṅkas*. The ⟨number of⟩ *jayasthalas* for these is 9600. Division by sixteen hundred ⟨is made⟩; Obtained is 6 *jayathalas* per *ṭaṅka*. ⟨This,⟩ when multiplied by one hundred, is $12\|$ *ṭaṅkas*; when multiplied by five hundred, is $62\|$ *ṭaṅkas*; and when multiplied by one thousand, is 125 ⟨*ṭaṅkas*⟩. /BBA 15.6/

···Note

BBA 15.6. Ex. 2 for the investment solved by the 2nd method. $M = 200$ *ṭaṅkas* $= 9600$ *jayasthalas*. $M/A = 9600/1600 = 6$ *jayasthalas*/*ṭaṅka*. Therefore, $(M/A) \cdot a_1 = 600$ *jayasthalas* $= 12\frac{1}{2}$ *ṭaṅkas*,

$(M/A) \cdot a_2 = 3000$ *jayasthalas* $= 62\frac{1}{2}$ *ṭaṅkas*, $(M/A) \cdot a_3 = 6000$ *jayasthalas* $= 125$ *ṭaṅkas*.

Thus the investment. /BBA 15.7/

Now, the calculation of gold./BBA 16.0/

> The sum of ⟨the products of⟩ the various gold pieces and their colors is divided by ⟨the sum of⟩ the gold pieces. The digit obtained should be known as the color ⟨of the mixture⟩. ⟨The same sum,⟩ divided by the color, is the gold ⟨mixture⟩. /PV 16/

···Note·····

PV 16. The word "gold" (*suvarṇa/svarṇa*) here means the weight of a gold piece, and "color" (*varṇa/varṇikā*) its purity. The highest purity is 16. This verse in its present form lacks words for "product" and "sum," which are indispensable for the rule. When v_i and w_i denote respectively the purity and the weight of the i-th gold piece ($i = 1, 2, ..., n$) and V and W those of the mixture,

$$V = \frac{v_1 w_1 + v_2 w_2 + \cdots + v_n w_w}{W},$$

$$W = \frac{v_1 w_1 + v_2 w_2 + \cdots + v_n w_w}{V}.$$

The latter formula is meaningful only when the gold pieces lose their weights in the process of purification, because otherwise W is simply the sum of w_i.[1]

Having made the *vānās* of individual gold pieces into one, ⟨the sum is⟩ divided by the gold. Then, the color (*vānī*) is obtained. ⟨The same sum⟩ is divided by the color. Then, the gold is obtained. /BBA 16.1/

···Note·····

BBA 16.1. The word *vānī* (Skt. *varṇikā*, lit. "color") means purity, whereas the word *vānā* (Skt. *varṇaka*, lit. "color") here means the product of the weight and the purity of a gold piece.

Example. ⟨First gold piece: Weight is⟩ eight *gadīyāṇas*, color is twelve, and *vānā* is ninety-six. Likewise, ⟨another gold piece: Weight is⟩ four *gadīyāṇas*, color is fourteen, and *vānā* is fifty-six. Setting-down:

gadī 8 color 12 *vānā* 96
gadī 4 color 14 *vānā* 56

When added together, ⟨the weight is⟩ 12 *gadīyāṇas* and *vānā* 152. Division of the colors (here in the sense of *vānā*) by the gold ⟨is made⟩. Obtained is color 12 and $\frac{2}{3}$ part. Likewise, ⟨let⟩ the color of the cooked (i.e., refined) ⟨alloy⟩ be thirteen, 13. Division ⟨of the same *vānā*⟩ by it ⟨is made⟩. Obtained is gold 11 *gadīyāṇas* and $\frac{9}{13}$ part. /BBA 16.2/

[1] For the Indian systems of expressing the purity of gold, see Sarma (1983).

···Note

BBA 16.2. Ex. for the calculation of gold: purity and weight of the mixture. Given: $v_1 = 12$ *varṇas*, $v_2 = 14$ *varṇas*, $w_1 = 8$ *gadīyāṇas*, $w_2 = 4$ *gadīyāṇas*. Solution: $v_1 w_1 = 96$ *vānās*, $v_2 w_2 = 56$ *vānās*. If the two gold pieces are simply smelted into one alloy, then $W = w_1 + w_2 = 12$ *gadīyāṇas*, and $V = (v_1 w_1 + v_2 w_2)/W = 152/12 = 12\frac{2}{3}$ *varṇas*. If they are smelted with refinement into one alloy having the purity $V = 13$, then $W = (v_1 w_1 + v_2 w_2)/V = 152/13 = 11\frac{9}{13}$ *gadīyāṇas*. When I was writing my paper of 1991, I could not understand the two syllables *padyu* in the manuscript and gave an inaccurate comment on this calculation (Hayashi 1991, 431), but now I think that the *padyu* is a misspeling of *pakka* or *pakva* ("cooked"), which means "refined" in the calculation of gold. See PG 53, E63, E64; GSS 6.182–91; GSK 3.18 (*pakkha* for *pakva*) (cf. SaKHYa 2009, 138).

Thus the calculation of gold. /BBA 16.3/

Measurement of fields. There are nine forms of fields. The former half ⟨of verse 17⟩ on quadrilaterals. /BBA 17_1.0/

···Note

BBA 17_1.0. Verses 17–20 treat the following plane figures.

17_1: *sama-* and *dīrgha-caturasra* (square and oblong, i.e., rectangle).

17_2: *viṣama-caturasra* (irregular quadrilateral) with *lamba* (perpendicular).

18_1: *trikoṇa* (triangle) with *lamba* (perpendicular).

18_2 (+ S4): *vṛtta* (circle).

19: *tryasra* and *caturasra* (tri- and quadri-lateral) without *lamba* (perpendicular).

20_1: *cāpa* (bow, i.e. circle segment).

20_2: *nemi* (rim, i.e. annular figure) and *viṣama* (irregular figures). The latter include *muraja* (drum), *yava* (barleycorn), *vajra* (thunderbolt), *ardhacandra* (half moon), and *pañcabhuja* (five-armed).

The "nine forms" mentioned in 17_1.0 seem to be the following. 1. Rectangle (17_1). 2. Quadrilateral with a perpendicular (17_2). 3. Trilateral with a perpendicular (18_1). 4. Circle (18_2). 5. Trilateral without a perpendicular (19). 6. Quadrilateral without a perpendicular (19). 7. Circle segment (20_1). 8. Rim (20_2). 9. Irregular figures (20_2).

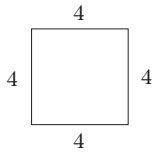
In the case of equal and long quadrilaterals, the product of the arm and the upright is the fruit (area). /PV 17_1/

···Note

PV 17_1. Equal and long quadrilaterals (square and oblong). Let a and b be the two orthogonal sides ($a = b$ for a square). Then, the area is: $A = ab$.

In an equi-quadrilateral (i.e., a square) and an oblong, the length is multiplied by the width. The fruit (area) is obtained. /BBA 17_1.1/

First example. Equal. Length in *gajas* four. Width in *gajas* four. Setting-down of the form:



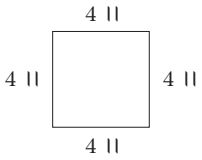
⟨Figure 3⟩

Four by four make sixteen. Obtained is 16 *gajas*. /BBA 17_1.2/

···Note·····

BBA 17_1.2. Ex. 1 for squares. 1.1. Given integers: $a = b = 4$ *gajas*. Solution: $A = 4 \cdot 4 = 16$ *gajas*.

Fractions. Length in *gajas* four and a half. Width in *gajas* four and a half. Setting-down of the form:



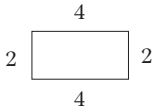
⟨Figure 4⟩

Like multiplication of fractions. The standard for *gaja* is the *visās*. Four by four make 16. Likewise, ten *visās*, multiplied by four, become 40 *visās*. Of these, 2 *gajas* are produced. Likewise, ten *visās*, multiplied by four, become 40 *visās*. Of these, 2 *gajas* are produced. Ten *visās* are multiplied by ten *visās*. 100 *visāṃsas* are produced. Of these, five *visās* are produced, 5. In the summation, twenty and a quarter *gajas* are produced, 20|. /BBA 17_1.3/

···Note·····

BBA 17_1.3. Ex. 1 for squares (cont.). 1.2. Given fractions: $a = b = 4\frac{1}{2}$ *gajas*. Solution (as in the multiplication of fractions, PV 13): $A = 4\frac{1}{2} \cdot 4\frac{1}{2} = (4; 10) \cdot (4; 10) = (16; 40 + 40, 100) = (16 + 2 + 2; 5) = (20; 5) = 20|$ *gajas*.

Second ex. Length in *gajas* four. Width in *gajas* two. Setting-down of the form:



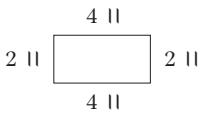
⟨Figure 5⟩

Four by two make eight. Obtained is 8 *gajas*. /BBA 17_1.4/

···Note

BBA 17_1.4. Ex. 2 for oblongs. 2.1. Given integers: $a = 4$ *gajas* and $b = 2$ *gajas*. Solution: $A = 4 \cdot 2 = 8$ *gajas*.

Fractions. Length in *gajas* four and a half. Width in *gajas* two and a half. Setting-down of the form:



⟨Figure 6⟩

Obtained by the multiplication, as in the previous "Fractions" (17_1.3), is eleven and a quater *gajas*, 11|. /BBA 17_1.5/

···Note

BBA 17_1.5. Ex. 2 for oblongs (cont.). 2.2. Given fractions: $a = 4\frac{1}{2}$ *gajas*, $b = 2\frac{1}{2}$ *gajas*. Solution (as in BBA 17_1.3): $A = 4\frac{1}{2} \cdot 2\frac{1}{2} = (4;10) \cdot (2;10) = (8;40+20,100) = (8+2+1;5) = (11;5) = 11|$ *gajas*.

The latter half ⟨of verse 17⟩ on inequi-⟨lateral⟩ quadrilaterals./BBA 17_2.0/

> In inequi⟨-lateral quadrilaterals⟩, half the sum of the earth (base) and the face, multiplied by the perpendicular, is the fruit (area). /PV 17_2/

···Note

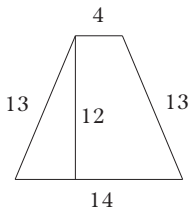
PV 17_2. Area of an inequilateral quadrilateral. Let a, b, c, and d be the four sides (earth, arm, face, arm) of an inequilateral quadrilateral, and h its perpendicular (height). Then, its area is:

$$A = \frac{a+c}{2} \cdot h.$$

If the quadrilateral is a trapezium, the area obtained is exact; otherwise the result is only approximate.

In inequi-⟨lateral⟩ quadrilaterals, having made the sum of the *gajas* of the earth and the face, half ⟨of the result⟩ is made. Then, ⟨the half⟩ is multiplied by the perpendicular. The fruit (area) is obtained. /BBA 17_2.1/

Ex. One arm in *gajas* thirteen. The second ⟨arm⟩ in *gajas* thirteen. Face in *gajas* four. Earth in *gajas* fourteen. Perpendicular in *gajas* twelve. Setting-down of the form:



⟨Figure 7⟩

Obtained is 108 *gajas*. /BBA 17_2.2/

···Note·····

BBA 17_2.2. Ex. for inequilateral quadrilaterals. Given: $a = 14$, $b = 13$, $c = 4$, $d = 13$, and $h = 12$ *gajas*. Solution: $A = \frac{14+4}{2} \cdot 12 = 108$ *gajas*. The figure intended here is a traditional trapezium (see Hayashi 2013b, 319).

The former half ⟨of verse 18⟩ on a triangle./BBA 18_1.0/

> The product of half the *vasudhā* (earth, i.e. base) and the perpendicular is the fruit (area) in a field having three corners. /PV 18_1/

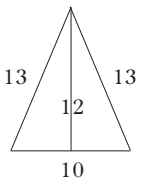
···Note·····

PB 18_1. Area of a triangle. Let a, b, and c, be the three sides (earth, arm, arm) of a triangle, and h its perpendicular (height). Then, its area is:

$$A = \frac{a}{2} \cdot h.$$

In a field having three corners, the *vasudhā* is called *bhūmi* (earth, i.e. base). Its half is made. And ⟨the result⟩ is multiplied by the perpendicular. The fruit (area) is obtained. /BBA 18_1.1/

Ex. One arm in *gajas* thirteen. The second ⟨arm⟩ in *gajas* thirteen. Earth in *gajas* ten. Perpendicular in *gajas* twelve. Setting-down of the form:



⟨Figure 8⟩

Obtained is 60 *gajas*. /BBA 18_1.2/

···Note

BBA 18_1.2. Ex. for triangles. Given: $a = 10$, $b = 13$, $c = 13$, and $h = 12$ *gajas*. Solution: $A = \frac{10}{2} \cdot 12 = 60$ *gajas*. This is also a traditional example (see Hayashi 2013b, 319).

The latter half ⟨of verse 18⟩ on a circular field./BBA 18_2.0/

> The product of the circumference by one fourth of the diameter is ⟨the area of⟩ the circle. /PV 18_2/

···Note

PV 18_2. Area of a circle. Let d and C be the diameter and the circumference of a circle. Then, its area is:

$$A = C \cdot \frac{d}{4}.$$

In a circular field, the digit of the circumference is multiplied by a fourth part of the diameter. The fruit (area) of the circle is obtained. /BBA 18_2.1/

Likewise, the measurement of the circumference.

> Three times the diameter, increased by its sixth part, is the circumference. /S4/

Having made three times the digit of the diameter, ⟨the result is⟩ added into one sixth part ⟨of the diameter⟩. Then, the fruit (length) of the circumference is obtained. /BBA 18_2.2/

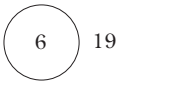
···Note

BBA 18_2.2. A formula for the circumference of a circle is cited here (S4).

$$C = 3d + \frac{d}{6}.$$

It has been conjectured that this value, $3\frac{1}{6}$ or $\frac{19}{6}$, originates from an approximation to the Jaina value of π, $\sqrt{10}$, that is, $\sqrt{10} = \sqrt{3^2 + 1} \approx 3 + \frac{1}{2 \cdot 3}$.[1]

Ex. Circumference in *gajas* nineteen. Diameter in *gajas* six. Setting-down of the form:



⟨Figure 9⟩

[1] See, for example, Gupta (1975, 43).

Obtained is 28|| *gajas*. /BBA 18_2.3/

···Note

BBA 18_2.3. Ex. for circles. Given: $C = 19$, $d = 6$ *gajas*. Solution: $A = 19 \cdot \frac{6}{4} = 28\frac{1}{2} = 28||$ *gajas*.

⟨The areas of⟩ tri- and quadri-laterals without a perpendicular./BBA 19.0/

> Half the sum of the arms, ⟨placed⟩ fourfold, is ⟨severally⟩ decreased by ⟨one of ⟩ the arms. The square root of the product of them shall indeed be the fruit (area) in a trilateral and also in a four-faced ⟨figure⟩ (i.e., a quadrilateral). /PV 19/

···Note

PV 19. Areas of tri- and quadri-laterals without a perpendicular. Let a, b, c, and d be the four sides of a quadrilateral or of a trilateral (in which case one of the sides, say d, is zero). Then, its area is:

$$A = \sqrt{(s-a)(s-b)(s-c)(s-d)}, \quad \text{where} \quad s = \frac{a+b+c+d}{2}.$$

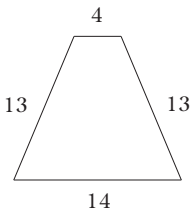
This is the so-called Brahmagupta's formula, which gives the exact area when the quadrilateral is cyclic; otherwise the result is only approximate. Śrīdhara prescribes the same formula for "inequi-perpendicular" (*asamalamba*) quadrilaterals, which are also called "non-straight-face" (*anṛjumukha*) quadrilaterals (see Hayashi 2013b, 316–18).
This rule is not found in Śambhunātha's version.

In quadri- and tri-lateral figures, having made the sum of the arms, half ⟨of the result⟩ is made. Then, it is written down in four places. And from them ⟨severally⟩, subtraction of the arms is made. Then, those remaining digits are multiplied ⟨with each other⟩ in order. And then, one should take the square root ⟨of the product⟩. In this way, the fruit (area) of the four-faced ⟨figure⟩ or of the triangle is obtained. /BBA 19.1/

···Note

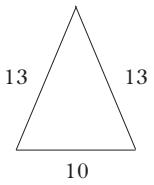
BBA 19.1. The word *rahita* in regular Sanskrit means "deprived of" and, in mathematics, "decreased by" but in this and another (24.1) paragraphs it is used in the sense of "remaining." The same usage is found in a Sanskrit arithmetical work, *Iṣṭāṅkapañcaviṃśatikā*, of Tejasiṃha (d. CE 1686), a Jaina scholar who wrote in Gujarātī as well as in Sanskrit (see Hayashi 2006b, 136).

Ex. Setting-down of the form of the quadrilateral told above (17_2.2 with Fig. 7).



⟨Figure 10⟩

Obtained is 108 *gajas*. Setting-down of the form of the trilateral told above (18_1.2 with Fig. 8).



⟨Figure 11⟩

Obtained is 60 *gajas*. /BBA 19.2/

···Note·····

BBA 19.2. Exs. for the formula without a perpendicular. 1. The "inequilateral quadrilateral" treated in BBA 17_2.2. Given: $a = 14$, $b = 13$, $c = 4$, $d = 13$, and $h = 12$ *gajas*. Solution: $s = (a+b+c+d)/2 = 22$. $A = \sqrt{(22-14)(22-13)(22-4)(22-13)} = \sqrt{8 \cdot 9 \cdot 18 \cdot 9} = \sqrt{11664} = 108$ *gajas*. This "inequilateral quadrilateral" is an isosceles trapezium and therefore cyclic. 2. The trilateral treated in 18_1.2. Given: $a = 10$, $b = 13$, $c = 13$, and $h = 12$ *gajas*. Solution: $s = (a+b+c)/2 = 18$. $A = \sqrt{(18-10)(18-13)(18-13)(18-0)} = \sqrt{8 \cdot 5 \cdot 5 \cdot 18} = \sqrt{3600} = 60$ *gajas*.

The former half ⟨of verse 20⟩ on the bow field./BBA 20_1.0/

> The product of the arrow and half the sum of the chord and the arrow, increased by one eighteenth part ⟨of itself, is the fruit (area) of a bow field.⟩ /PV 20_1/

···Note·····

PV 20_1. Area of a bow field (i.e., a circle segment). Let a and h be respectively the chord and the arrow (height) of a bow field. Then, its area is:

$$A = \frac{a+h}{2} \cdot h + \frac{1}{18} \cdot \left(\frac{a+h}{2} \cdot h \right).$$

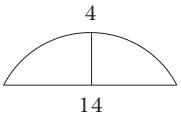
This formula can be rewritten as:

$$A = \frac{19}{6} \cdot \frac{(a+h)h}{6},$$

and therefore it is surmised that, like the formula of S4 cited in BBA 18_2.2 above, this formula uses 19/6 for π (see Hayashi (1990, 1–2).[1]

In a bow field, having brought together the chord and the arrow in one place, half ⟨of the result⟩ is made. And one multiplies this digit by the arrow. Then, ⟨the result is⟩ added to its one eighteenth part. The fruit (area) is obtained. /BBA 20_1.1/

Ex. Chord in *gajas* fourteen. Arrow in *gajas* four. Setting-down of the form:



⟨Figure 12⟩

Obtained is 38 *gajas*. /BBA 20_1.2/

···Note·····

BBA 20_1.2. Ex. for bow figures. Given: $a = 14$ *gajas*, $h = 4$ *gajas*. Solution: $\frac{a+h}{2} \cdot h = 36$, $A = 36 + 36/18 = 38$ *gajas*.

The latter half ⟨of verse 20⟩ on the rim figure./BBA 20_2.0/

> In a rim shape and in ⟨any⟩ irregular ⟨figure⟩, the product of the width and the length[2] is the fruit (area). /PV 20_2/

···Note·····

PV 20_2. Area of a rim figure. Consider a "rim figure" bordered by two concentric circles with radii, r_1 and r_2 $(0 \leq r_1 < r_2)$. Let w and L be respectively the width of the rim figure and the circumference of another concentric circle with radius $(r_1 + r_2)/2$. That is,

$$w = r_2 - r_1 \quad \text{and} \quad L = \pi(r_1 + r_2).$$

Then, the area A_L of the rim figure is:

$$A_L = \pi r_2^2 - \pi r_1^2 = \pi(r_2 + r_1)(r_2 - r_1) = Lw.$$

Let l be such a portion of L that is cut by the sector OAB. Then, the area A_l of the partial rim figure ABCD is:

$$A_l = \frac{l}{L} \cdot A_L = lw.$$

Therefore, the rule of verse 20_2 can be applied for both the whole and the partial rim figures.

---

[1] For an insightful discussion on the origin of this and related formulas, see Gupta (2011).

[2] *laṃba*. Śambhunātha's version reads *dīrgha* instead of *laṃba*.

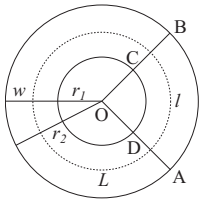
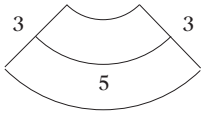


Figure N6: Rim figure

For other irregular figures the present rule only mentions the basic principle of calculation, "the product of the width and the length" (cf. Keller 2006, I.42–49, II.40–45).

In a rim shape, in a surrounding wall of a castle, and in ⟨any⟩ irregular figure, the width is multiplied by the length. The fruit (area) is obtained. /BBA 20_2.1/

Ex. Length in *gajas* five. Width in *gajas* three. Setting-down of the form:



⟨Figure 13⟩

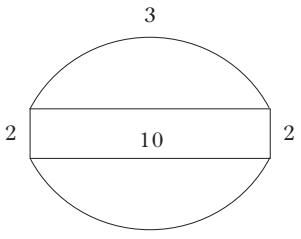
Obtained is 15 *gajas*. /BBA 20_2.2/

···Note·····

BBA 20_2.2. Ex. of a rim figure. Given: $l = 5$ *gajas* and $w = 3$ *gajas*. Solution: $A_l = 5 \cdot 3 = 15$ *gajas*.

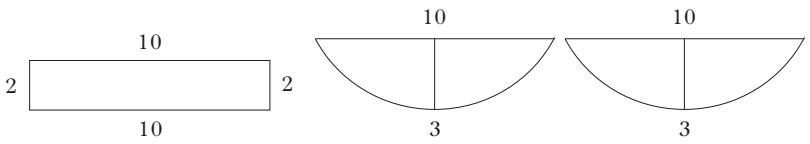
Note that, when $l = 5$ and $w = 3$, we have $r_1 = \frac{900}{\pi} \cdot \frac{1}{\theta} - \frac{3}{2}$, where θ is the central angle of the sector OAB in Figure N6 above. Since $r_1 \geq 0$, we have $\theta \leq \frac{600}{\pi} \approx 191$; and when $\theta = 180$, $r_1 \approx 0.09$, when $\theta = 90$, $r_1 \approx 1.68$, when $\theta = 60$, $r_1 \approx 3.27$, when $\theta = 45$, $r_1 \approx 4.87$, and when $\theta = 30$, $r_1 \approx 8.05$. As r_1 for $\theta = 180$ is too small, the θ intended by the author was most probably smaller enough than that, say $\theta = 90$, which is the case shown in Fig.13.

Likewise, investigation of ⟨irregular⟩ figures such as a drum, a barleycorn shape, a thunderbolt shape, a half moon, and a five-armed. ⟨First of all⟩, investigation of a drum figure. ⟨Setting-down of the form of⟩ a drum figure:



⟨Figure 14⟩

Produced are one quadrilateral and two bows. Form:



⟨Figure 15⟩

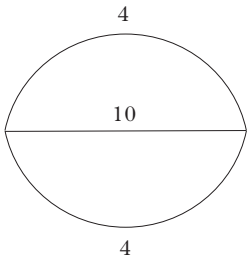
Obtained by means of ⟨the rules⟩ "the product of the arm and the upright ..." [PV 17_1] and "half the sum of the chord and the arrow ..." [PV 20_1] is 61;4 *gajas*. /BBA 20_2.3/

···Note·····

BBA 20_2.3. Exs. for irregular figures. Irregular figures are reduced to one or more regular figures for which formulas have already been given.

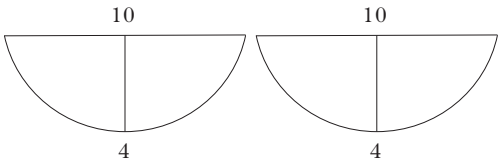
1. Drum (*muraja*). Reduced to one rectangle and two equal circle segments. Given: $a = 10$, $b = 2$, and $h = 3$ *gajas*. Solution: The area (A_1) of the rectangle by 17_1: $A_1 = 20$ *gajas*. The area (A_2) of each circle segment by 20_1: $A_2 = 20; 14$ *gajas*. Hence follows: $A = A_1 + 2A_2 = 61; 4$ *gajas*.

⟨Setting-down of the form of⟩ a barleycorn shaped figure:



⟨Figure 16⟩

Produced are two bows.



⟨Figure 17⟩

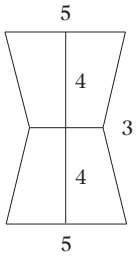
Obtained by means of ⟨the rule⟩, "half the sum of the chord and the arrow ..." [PV 20_1] is $59; 2||, 4$ *gajas*. /BBA 20_2.4/

···Note·····

BBA 20_2.4. Exs. for irregular figures (cont.).

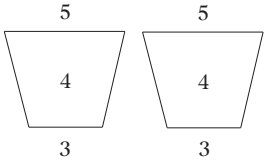
2. Barleycorn (*yava*). Reduced to two equal circle segments. Given: $a = 10$ and $h = 4$ *gajas*. Solution: The area (A_1) of each circle segment by 20_1: $A_1 = 29\frac{5}{9}$ *gajas*. Hence follows: $A = 2A_1 = 59\frac{1}{9} = 59; 2, 16 = 59; 2||, 4$ *gajas*.

⟨Setting-down of the form of⟩ a thunderbolt shaped figure:



⟨Figure 18⟩

Produced are two quadrilaterals. Setting-down of the form:



⟨Figure 19⟩

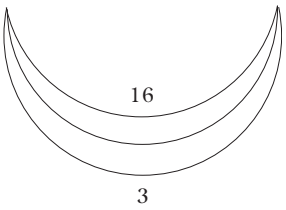
Obtained by means of ⟨the rule⟩, "half the sum of the earth and the face ..." [PV 17_2] is 16 and 16 *gajas*. The sum of the two is 32 ⟨*gajas*⟩. /BBA 20_2.5/

...Note.....

BBA 20_2.5. Exs. for irregular figures (cont.).

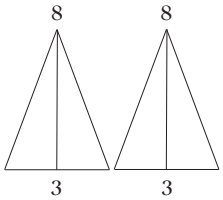
3. Thunderbolt (*vajra*). Reduced to two equal isosceles trapezia. Given: $a = 5$, $c = 3$, and $h = 4$ *gajas*. Solution: The area (A_1) of each isosceles trapezium by 17_2: $A_1 = 16$ *gajas*. Hence follows: $A = 2A_1 = 32$ *gajas*.

Half moon figure.



⟨Figure 20⟩

Produced are two trilaterals.



⟨Figure 21⟩

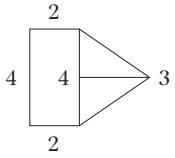
Obtained by means of ⟨the rule⟩, "the product of half the earth and the perpendicular ..." [PV 18_1] is 12 and 12 *gajas*. The sum of the two is 24 ⟨*gajas*⟩. /BBA 20_2.6/

···Note·····

BBA 20_2.6. Exs. for irregular figures (cont.).

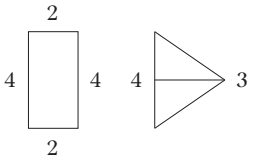
4. Half moon (*ardha-candra*). Reduced to two equal isosceles triangles. Given: $a = 3$ and $h = 8$ *gajas*. Solution: The area (A_1) of each isosceles trapezium by 18_1: $A_1 = 12$ *gajas*. Hence follows: $A = 2A_1 = 24$ *gajas*. The nomenclature, "half moon," sounds rather strange. For such figures, a more appropriate name, "young moon" (*bāla-indu*), occurs in GSS 7.18.

Five-armed figure.



⟨Figure 22⟩

Produced are one quadrilateral and one triangle. Setting-down:



⟨Figure 23⟩

Obtained by means of ⟨the rules⟩, "the product of the arm and the upright ..." [PV 17_1] and "the product of half the earth and the perpendicular" [PV 18_1] is 14 *gajas*. /BBA 20_2.7/

···Note·····

BBA 20_2.7. Exs. for irregular figures (cont.).

5. Five-armed (*pañcabhuja*) or a pentagon. Reduced to one quadrilateral (rectangle in this example) and one triangle. Given: $a = 4$, $b = 2$, and $h = 3$ *gajas*. Solution: The area (A_1) of the rectangle by 17_1: $A_1 = 8$ *gajas*. The area (A_2) of the triangle by 18_1: $A_2 = 6$ *gajas*. Hence follows: $A = A_1 + A_2 = 14$ *gajas*.

In this manner, having divided irregular figures into parts like these, ⟨each part⟩ is measured by the methods for the first (component) figures. /BBA 20_2.8/

Thus the procedure for ⟨plane⟩ figures. /BBA 20_2.9/

Measurement of the excavation, timber, stone, storehouse, and ⟨brick-⟩piling. /BBA 21.0/

> In excavation, timber, stone, storehouse, and piling ⟨of bricks⟩, when the product of the width and the length is multiplied by the thickness, the fruit (volume) for the uniform ⟨space or solid is obtained⟩. /PV 21/

···Note

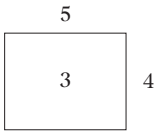
PV 21. Volumes of constructions having the form of a rectangular parallelepiped. Let a, b, and c be the length, the width, and the thickness of a construction. Then, its volume is:

$$V = (ab)c.$$

This is said to be the formula for the "uniform" (*sama*) figure that has a "uniform" length in all the three directions.

In excavation, timber, stone, storehouse, and piling ⟨of bricks⟩, having made one (that is, in all of them), the width is multiplied by the length, and then multiplied by the thickness. The fruit (volume) is obtained. /BBA 21.1/

Ex. Length in *gajas* five. Width in *gajas* four. Thickness in *gajas* three. The common form:



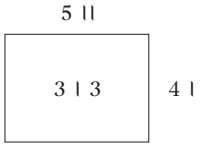
⟨Figure 24⟩

Obtained is 60 *gajas*. /BBA 21.2/

···Note

BBA 21.2. Ex. of a uniform excavation. Given: $a = 5$, $b = 4$, $c = 3$ *gajas*. Solution: $V = (5 \cdot 4) \cdot 3 = 20 \cdot 3 = 60$ *gajas*.

Second ex. for fractions. Here, the standard for *gaja* is the *visā*. Length five *gajas* and ten *visās*, 10. Width four *gajas* and five *visās*, 5. Thickness three *gajas* and eight *visās*. The common form:



⟨Figure 25⟩

Obtained, as in the multiplication of fractions, is 79|4|| *gajas*. /BBA 21.3/

···Note

BBA 21.3. Ex. 2 for a uniform excavation with the three dimensions accompanied by fractions. Given: $a = 5$ *gajas* 10 *visās*, $b = 4$ *gajas* 5 *visās*, $c = 3$ *gajas* 8 *visās*. 20 *visās* = 1 *gaja*. Solution: $(5; 10) \cdot (4; 5) = (23; 7, 10)$. $(23; 7, 10) \cdot (3; 8) = (79; 9, 10) = 79\frac{1}{4}$ *gajas* $+ 4\frac{2}{4}$ *visās* $= 79|; 4||$ *gajas*.

Likewise, nonuniform ⟨figures⟩.

> If ⟨the measurements in a direction are⟩ nonuniform, one should divide the sum of the nonuniform ⟨measurements⟩ by that (i.e., the number of the measured places). ⟨The result is the mean length in that direction.⟩/S5/

In the case of nonuniform ⟨figures⟩ (*vāṃkau: visamau*) with regard to the depth or width or length, the sum of the nonuniform ⟨measurements⟩ is made. As many places there are where it is measured, so many parts ⟨of the sum⟩ are taken. By this, it becomes straight. When one calculates the fruit (volume) from the sum of the nonuniform ⟨measurements⟩ by means of the same method with regard to the depth, width or length, the pool or excavation ⟨is known⟩. /BBA 21.4/

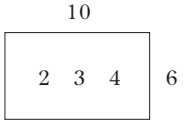
···Note

BBA 21.4. A rule (S5) for calculating the mean length is cited here. When the rectangular parallelepiped is not uniform (*viṣama*) in a direction, say in the length, then the length is measured at several places. Let a_i be the measurements at n places. Then, the mean length is:

$$\bar{a} = \frac{a_1 + a_2 + \cdots + a_n}{n}.$$

For the other directions also, if nonuniform, the mean values are obtained in the same way and the volume is calculated with them: $V = (\bar{a}\bar{b})\bar{c}$.

Ex. Two, three, and four *gajas* for the depth, ten ⟨*gajas*⟩ for the length, and six *gajas* for the width. Setting-down:



⟨Figure 26⟩

The sum of the nonuniform ⟨measurements⟩ with regard to the depth is 9 *gajas*. One third part of this is 3 *gajas*, which is the ⟨mean⟩ depth. The width is 6 *gajas*. The length is 10 *gajas*. Obtained as before is 180 *gajas*. /BBA 21.5/

···Note

BBA 21.5. Ex. of a nonuniform construction. Given: $a = 10$ *gajas*, $b = 6$ *gajas*, and $c_1 = 2$, $c_2 = 3$, $c_3 = 4$ *gajas*. Solution: $\bar{c} = (2 + 3 + 4)/3 = 3$ *gajas*. Hence follows: $V = (ab)\bar{c} = 180$ *gajas*.

Thus the procedure for the excavation, timber, stone, storehouse, and ⟨brick-⟩ piling. /BBA 21.6/

Now, measurement of the circular timber, stone, pillar, and well./BBA 22.0/

> In the ⟨circular⟩ timber, stone, pillar, and well, the fruit (volume) is ⟨obtained⟩ in the same way. In that case, the fruit (area) of the circular figure ⟨is first obtained⟩. The product of the depth by that is the fruit (volume). /PV 22/

···Note·····

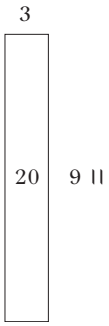
PV 22. Volume of a cylindrical construction. Let A and h be the area of a cross section and the depth (or height) of the cylinder. Then, its volume is:

$$V = Ah.$$

As the cross section is a circle, A is calculated by the formula of 18_2.

In the case of ⟨circular⟩ timber, stone, pillar, and well, having made the fruit (area) of the circular figure ⟨of the cross section⟩, ⟨the result is⟩ multiplied by the depth. The fruit (volume) is obtained. /BBA 22.1/

Ex. Here, the standard for *gaja* is the greater *aṅgula*. Length in *gajas* twenty. Width in *gajas* three. Circumference in *gajas* nine and a half. Setting-down of the form:



⟨Figure 27⟩

The digit of the circumference is multiplied by one fourth part of the width. It become the fruit (area) of the circle. Obtained is 7;3 *gajas*. Then, it is multiplied by the depth. Obtained is 142|| *gajas*. /BBA 22.2/

···Note·····

BBA 22.2. Ex. of a cylindrical construction. Given: $h = 20$ *gajas*, $d = 3$ *gajas*, $C = 9\frac{1}{2}$ *gajas*. 24 *aṅgulas* $= 1$ *gaja*. Solution: $A = C \cdot \frac{d}{4} = 9\frac{1}{2} \cdot \frac{3}{4} = 7\frac{1}{8} = 7;3$. $V = Ah = 7;3 \cdot 20 = 142;12 = 142||$. The values of C and d given in this example follow the formula of S4 cited in 18_2.2.

Thus the procedure for the circular ⟨constructions⟩. /BBA 22.3/

The former half ⟨of verse 23⟩ on the measurement of a sphere./BBA 23_1.0/

Half the cube of the diameter of a sphere increased by one eighteenth part ⟨of it is the volume of the sphere⟩. /PV 23_1/

···Note·····

PV 23_1. Volume of a sphere.

$$V = \frac{d^3}{2} + \frac{1}{18} \cdot \frac{d^3}{2}.$$

This formula can be rewritten as:

$$V = \frac{19}{6} \cdot \frac{d^3}{6},$$

and therefore it is conjectured that here also $\frac{19}{6}$ is used for π. See PV 20_1 above.

The diameter of a sphere is multiplied ⟨by itself⟩ three times (in fact, twice). Half ⟨of the result⟩ is made. Then, ⟨the result⟩ is added into one eighteenth part of it. The fruit (volume) is obtained. /BBA 23_1.1/

Ex. Diameter in *gajas* three. Setting-down of the form:

⟨Figure 28⟩

Obtained is 14| *gajas*. /BBA 23_1.2/

···Note·····

BBA 23_1.2. Ex. of a sphere. Given: $d = 3$ *gajas*. Solution: $d^3/2 = \frac{27}{2}$. $V = \frac{27}{2} + \frac{1}{18} \cdot \frac{27}{2} = 14\frac{1}{4} = 14|$ *gajas*.

Thus the sphere. /BBA 23_1.3/

The latter half ⟨of verse 23⟩ on the measurement of the heaped-up grains./BBA 23_2.0/

The square of one sixth part of the circle (i.e., the circumference), multiplied by its height, is the fruit (volume). /PV 23_2/

···Note·····

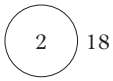
PV 23_2. Volume of the heaped-up grain. Let C and h be respectively the circumference and the height of grain heaped up in the shape of a cone on a level surface. Then,

$$V = \left(\frac{C}{6}\right)^2 \cdot h.$$

Most of other *pāṭī* works provide the relationship, $h = C/\beta$, where β is 9 or 10 or 11 according to the fineness etc. of grain.

The square of one sixth part of the circle is multiplied by the height. The fruit (volume) is obtained. /BBA 23_2.1/

Ex. Circumference in *gajas* eighteen. Height in *gajas* two. Form:



⟨Figure 29⟩

Obtained is 18 *gajas*. /BBA 23_2.2/

···Note·····

BBA 23_2.2. Ex. for heaped-up grain. Given: $C = 18$, $h = 2$. The unit is *gaja*. Solution: $V = \left(\frac{18}{6}\right)^2 \cdot 2 = 18$ *gajas*. Note that, in this example, $\beta = C/h = 9$.

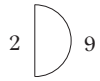
⟨When⟩ one measures one *gaja* for length, one *gaja* for width, and one *gaja* for depth, the ⟨weight in⟩ *maṇas* of wheat (*godhūma*) ⟨of that volume⟩ is sixteen, 16. Āricārī ⟨of that volume⟩ measures 15 *maṇas*. The remaining ⟨values for the⟩ rest should be known in ⟨the books on⟩ the procedure for foods. /BBA 23_2.3/

···Note·····

BBA 23_2.3. Specific gravities of grains.

Grains	Specific gravities
godhūma (wheat)	16 *maṇas*/*gaja*^{3}
āricārī (unidentified)	15 *maṇas*/*gaja*^{3}

In this manner, half is dropped (subtracted) at ⟨the flat side of⟩ a wall; three parts (quarters) are dropped at ⟨the inside of⟩ a ⟨rectangular⟩ corner; and a fourth (quarter) is dropped at the outside of a ⟨rectangular⟩ corner. The form of ⟨the flat side of⟩ a wall:



⟨Figure 30⟩

Obtained is 9 *gajas*. The form of ⟨the inside of⟩ a corner:

⟨Figure 31⟩

Obtained is 4|| *gajas*. The form of the outside of a corner:

⟨Figure 32⟩

Obtained is 13|| *gajas*. /BBA 23_2.4/

···Note·····

BBA 23_2.4. Ex. of partial cones. When the heaped-up grain in the shape of a cone is reduced into a half, or a quarter, or three quarters because it is piled up against a side of a wall, or against the inside of a corner, or against the outside of a corner, then respectively a half, or three quarters, or a quarter is subtracted from the complete cone.

$$V_1 = V - \frac{V}{2}, \quad V_2 = V - 3 \cdot \frac{V}{4}, \quad V_3 = V - \frac{V}{4}.$$

In the above example (23_2.2), $V = 18$ *gajas* has been obtained. Therefore, $V_1 = 9$ *gajas*, $V_2 = 4\frac{1}{2} = 4||$ *gajas*, $V_3 = 13\frac{1}{2} = 13||$ *gajas*.

Thus the procedure for the heaped-up ⟨grain⟩. /BBA 23_2.5/

Measurement of shadows./BBA 24.0/

The shadow is increased by seven. From it one should subtract the noon shadow. From the division of half the daylight multiplied by seven ⟨by that remainder⟩, the *nāḍīs* passed ⟨from sunrise⟩ or remaining ⟨until sunset are obtained⟩. /PV 24/

···Note·····

PV 24. Time from shadow. Let t be the time from sunrise before noon or the time remaining until sunset afternoon, d the length of daylight, s the shadow length, and s_n the length of the noon shadow on that day. Then,

$$t = \frac{7(d/2)}{s + 7 - s_n} \quad nāḍīs.$$

The values of d and s_n are given in the next verses, 25 and 26, respectively. The unit for the shadow lengths is *pāda* (foot) according to the commentator (BBA 24.1).[1]

Seven is added into the *pādas* of the shadow. And subtraction of the *pādas* of the noon ⟨shadow from the result⟩ is made. And, having multiplied half the daylight by seven, part is taken away ⟨from the result⟩ by the *pādas* of the shadow. The fruit (time) is obtained. Up to the midday, the past *ghaḍīs* are obtained. And in the afternoon, the remaining *ghaḍīs* are obtained. /BBA 24.1/

[1] For the linear unit *pāda* (foot) for shadow, see GSS 9.20 and Stone (1985). For similar formulas for time from shadow, see SaKHYa (2009, 160–62).

···Note

BBA 24.1. The word form *ghaḍī* in the sense of *ghaṭī* (= *nāḍī*) is rare but it occurs also in the *Natvāśivam* (see Hayashi 2017, 23).

Measurement of daylight./BBA 25.0/

> The ⟨number of⟩ days from ⟨the beginning of ⟩ Makara (Capricorn) is multiplied by three, increased by sky-three-five-one (i.e., 1530), and divided by sixty. The quotient is the *kalās* ⟨of daylight⟩. From ⟨the beginning of⟩ Karka (Cancer), ⟨the *kalās* of⟩ night ⟨are obtained⟩ in the same way. /PV 25/

···Note

PV 25. The length of daylight.

$$d = \frac{3n + 1530}{60} \quad kalās.$$

When n is the number of days from the beginning of Makara or Capricorn (winter solstice), d is the length of daylight of that day. When n is the number of days from the beginning of Karka(ṭa) or Cancer (summer solstice), d is the length of night of that day. The variable range of n seems to be [0, 180] and

$$d_{\text{mini}} = \frac{3 \cdot 0 + 1530}{60} = 25\frac{1}{2} \; kalās \text{ (at winter solstice)},$$

$$d_{\max} = \frac{3 \cdot 180 + 1530}{60} = 34\frac{1}{2} \; kalās \text{ (at summer solstice)}.$$

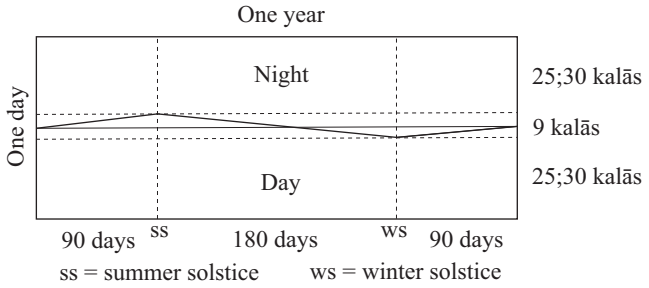


Figure N7: Annual change of the lengths of the day and the night

It is noteworthy that the word *kalā* is here used as a time unit in place of the ordinary *ghaṭikā* (or *ghaṭī*) or *nāḍikā* (or *nāḍī*), that is, 1 *kalā* = 1 *ghaṭikā* = 1 *nāḍikā* = 1/60 day (i.e., day and night). In the traditional time measuring systems of India, the *kalā* is variously defined as 1/10800, or 1/2400, or 1/1800, or 1/900, or 1/600 of one day and night (see Hayashi 2017). But the nomenclature of the author of the PV seems to be based on the analogy of the divisions of arc. In the ancient Indian astronomical works, the following parallelism between time and arc is often mentioned (see Hayashi 2017, sec.30).

Table N2

Time	day	Arc	degree
vighaṭikā	0;0,1	*vikalā*	0;0,1
ghaṭikā	0;1	*kalā*	0;1
divasa	1	*aṃśa*	1
māsa	30	*rāśi*	30
varṣa	360	*bha-gaṇa*	360

The above formula is a variation of the formula of the *Vedāṅgajyotiṣa* (Sarma 1985, 66; Ôhashi 1993, 205-06).[1]

Having summed up the days from Makara-saṃkrānti (the sun's entrance into Capricorn) to the ⟨day in⟩ question, ⟨the result is⟩ multiplied by three. Then, ⟨the result⟩ is added into fifteen hundred and thirty. And ⟨the sum⟩ is divided by sixty, 60. The measurement of daylight is obtained. In the same way, from Karka-saṃkrānti (the sun's entrance into Cancer), the ⟨length of⟩ night is obtained. /BBA 25.1/

Measurement of the noon⟨-shadow in⟩ *pādas*./BBA 26.0/

> Three, two, one, sky (0), moon (1), wings (2), fires (3), *yugas* (4), arrows (5), six, arrows (5), and *yugas* (4) have been declared to be the noon *pādas* (i.e., the noon shadows measured in *pādas*) at the ⟨sun's⟩ entrance ⟨into each zodiacal sign⟩, in order, beginning with Meṣa (Aries). /PV 26/

···Note·····

PV 26. *madhya-pādas*: noon-⟨shadow⟩-*pādas*. The lengths of the noon shadows measured in *pādas* are given here for the days of the sun's entry (*saṃkrānti*) into each zodiacal sign beginning with Meṣa or Aries.

Table N3

Entry into:	Meṣa	Vṛṣa	Mithuna	Karkaṭa	Siṃha	Kanyā
s_n	3	2	1	0	1	2
Entry into:	Tulā	Vṛścika	Dhanus	Makara	Kumbha	Mīna
s_n	3	4	5	6	5	4

It is surmised that the same value of s_n was used in the formula of PV 24 for one month covering the preceding and the succeeding fifteen days of each entry (*saṃkrānti*).

[1] For the so-called word numerals (*bhūta-saṃkhyā* in Sanskrit) such as "sky" for 0, "moon" for 1, etc. used in this and the next three verses see, for example, Sarma (2002) and Hayashi (2012, 178–87).

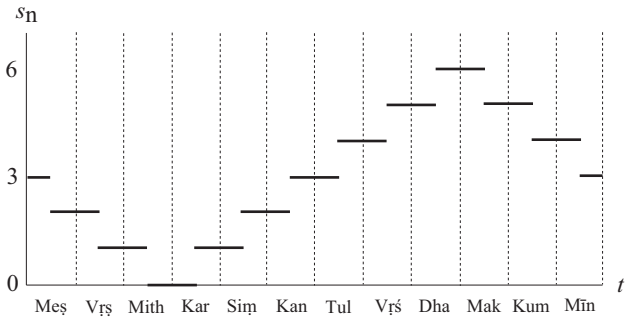


Figure N8: The lengths of the noon shadows in *pādas*

Śambhunātha's corresponding verse (A1° in Hayashi 1991) gives different values.

[Another reading ⟨of verse 25⟩.

> The ⟨number of⟩ days from the beginning of the ⟨sun's northern or southern⟩ course are multiplied by Rāma (3), increased by sky-fires-arrows-moon (1530), and divided by sixty. The quotient is the *kalās* of daylight if ⟨the course⟩ begins with Makara (Capricorn), and of night if ⟨the course⟩ begins with Karka (Cancer). /PV 26^a/

⟨Multiplier 3, additive⟩ 1530 and div. 60. /BBA 26.1/]

···Note·····

BBA 26.1. This paragraph occurs only in J. Moreover, it seems to have been misplaced because this second 26th verse prescribes the same rule as verse 25 (not 26) in a different wording.

From the Meṣasaṃkrānti (the sun's entrance into Aries, i.e., spring equinox), 3, 2, 1, 0, 1, 2, 3, 4, 5, 6, 5, and 4 are in order the noon-⟨shadow⟩-feet. /BBA 26.2/

Ex. In the month of Caitra, on ⟨the day of⟩ Meṣasaṃkrānti, the shadow feet 11, noon feet 3, half the length of daylight 15 *ghaṭīs*. What is the fruit? Setting-down: ⟨lacuna⟩. Obtained is the passed ⟨time in⟩ *ghaṭī* of the daylight, 7. /BBA 26.3/

···Note·····

BBA 26.3. Example of calculation of time from shadow length. Given: On the day of Meṣasaṃkrānti, $s = 11$ *pādas*. Solution: According to verse 26, $s_n = 3$ *pādas*, which is already given in the statement of the problem. Since $n = 90$ days, we have $d = 30$ *kalās* according to verse 25. Hence follows $d/2 = 15$ *kalās*, which is also given in the statement of the problem. Hence, according to verse 24,

$$t = \frac{7 \cdot (30/2)}{11 + 7 - 3} = 7 \text{ } nāḍīs.$$

Note that the time unit *kalā* used for d in verse 25 is replaced by *ghaṭī* in this example.

Thus the procedure for shadow. /BBA 26.4/

Thus the fifth *sūtra* for various purposes is completed. /BBA 26.5/

> In the year of arrows-serpents-Vedas-moon (1485), in the town of Ahammadāvāda, the commentary *Bālabodhāṅka* was composed by Śambhudāsa. /PV 27/

Thus the commentary *Bālabodhāṅka* on the *Pañcaviṃśatikā*, composed by the wise man Śambhudāsa, is completed. /BBA 27.1/

...Note.....

BBA 27.1. The modifier *maṃtra* of the author's name "Śaṃbhudāsa" seems to be used in the sense of *mantrin*, that is, "a wise man" or "a minister," although I can not so far attest these meanings of the word *mantra* elsewhere.

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Indexes

The references in Index 1 are to the verse numbers and those in Indexes 2 and 3 to the paragraph numbers. "B" and "J" attached to the paragraph numbers indicate the manuscripts. The numbers without them refer to both. Paragraphs 6.1 to 8.10 are missing in J and therefore accompanied by "B." In Index 1, I use parentheses for indicating the numbers intended by word numerals (*bhūta-saṃkhyā*) and the irregular word forms that occur in the edited text. In Indexes 2 and 3, I use the following abbreviations.

∅ = omitted, canc.= canceled, cor. = corrected, corr. = corrupted,
fn. = footnote, intp. = interpolation, P. = Persian, w. = with.

Index 1: Sanskrit mathematical terms in the *Pañcaviṃśatikā*

Index 2: Old Gujarātī words in the *Bālabodhāṅkavṛtti*

Index 3: Old Gujarātī numerals in the *Bālabodhāṅkavṛtti*

tera, 9.11B, 17_2.2 (tere J), 18_1.2 ("13" J)
teraha, 9.11J

14:

cauda (caūda J), 8.18, 9.9, 13.3, 15.3 (∅ J), 15.4 (corr. J), 17_2.2 ("14" J), 20_1.2 ("14" J)
caudī (caūdaī J), 16.2

15:

panara, 10_1.2, 10_2.2 (paṃ- B), 25.1

16:

ṣoḍaśa, 15.6
sola, 9.5 (solhe J), 13.3, 14_2.2, 15.5, 17_1.2 ("16" J), 23_2.3 (∅ J)

18:

aḍhāra, 2.9, 8.5, 9.10, 23_2.2 (-aṃ J)

19:

ugaṇīsa, 18_2.3

20:

vīsa, 2.9, 3.2, 13.3 (∅ J), 15.5, 17_1.3, 22.2
vīsāṃ, 2.9, 15.3 (bīsa J)

21:

ikavīsa, 9.9J
ekavīsa, 14_1.2
ekavīsāṃ, 9.9B

23:

trevīsa, 14_1.3 ("23" J)

24:

cuvīsa (caüvīsa J), 2.9, 13.2, 13.3

25:

paṃcavīsa, 9.8, 10_1.2 (pacavīsa J), 10_2.2, 13.3, 14_2.3 (-si), 10_2.2, 13.3, 14_2.3
paṃcavīsī, 1.2 (pacavīse J)

27:

sattāvīsa, 8.12 (satā- B), 9.4 (-aṃ J)

28:

aṭhāvīsa, 8.18

30:

tīsa, 2.9
trīsa, 2.9J, 3.2, 9.7
trīsāṃ, 2.9B, 15.3, 25.1

32:

batrīsa, 2.9, 8.9

33:

tetrīsa, 8.17 (also tettīsa J)

35:

pāṃtrīsa, 8.10, 10_1.2

36:

chatrīsa, 8.11, 8.17

38:

aṭhatrīsa, 8.12

40:

cyālīsa, 2.9, 3.2, 13.3, 14_2.2
cyālīsāṃ, 2.9B, 15.3 ("40" B)

43:

trayatālīsa, 8.18

45:

paṃcitālīsa, 9.10 (pacatā- J)

50:

paṃcāsa, 2.9, 3.2,
paṃcāsāṃ, 2.9B, 15.3 (paṃcāsoṃ B, pacāsāṃ J)

52:

bāvana, 8.16

56:

chapana, 9.8 (chappanna J), 16.2 (sapana J)

57:

sattāvana, 8.18 (satā- J)

59:

ugaṇasaṭhi, 8.15 (-saṭṭhi J)

60:

sāṭhi, 2.9 (sāṭha J), 3.2, 14_2.3, 25.1

64:

cusaṭhi (caüsaṭṭhi J), 8.13, 9.6

65:

pāṃsaṭhi (-saṭṭhi J), 8.9, 8.11

66:

chāsaṭhi (-saṭṭhi J), 8.17 (corr. B), 9.9

67:

satasaṭṭhi, 8.13J

satasaṭhi, 8.13B, 8.17

70:

sattari, 2.9J, 3.2

sittiri, 2.9B

73:

trihuttari, 8.16

75:

paṃcahuttari, 2.9 (pacyottari J)

76:

chahuttari, 9.7 (chi- J)

80:

asī, 2.9, 3.2, 13.2 (∅ J), 14_2.2 (aśī B)

81:

ekāsī, 14_1.2

ekyāsī, 8.12 (-śī B)

85:

paṃcyāsī, 2.9

87:

satyāsī, 9.7 (-śī B)

88:

aṭhyāsī, 8.14, 9.3 (-śī B)

90:

nau (niu/niū J), 2.9, 3.2, 14_1.2

93:

trāṇū, 9.10

95:

paṃcāṃṇū, 2.9J (corr.?)

paṃcāṇū, 2.9B, 9.8 (-ūṃ B)

96:

chanū (-ū, -ūṃ, -o), 8.5B, 8.10B, 13.2B, 16.2B

channū, 16.2J

chinnū, 13.2J

100:

ʹsata, 3.2B, 8.8B, 15.6

saï, 2.9, 8.9B, 8.11, 8.13, 8.14, 8.15J, 8.17 (si B), 9.3J, 9.4J, 9.8J, 9.9, 9.10, 10_1.2, 13.3J, 15.3, 15.4 (saī B), 15.5, 25.1

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saü, 2.9J, 3.2J, 8.14J, 9.6J, 8.18J, 13.3J, 14_2.3J, 15.5J

sata, 3.2J

sita, 15.6

su, 2.9B, 3.2B, 8.5B, 8.10B, 8.14B, 8.18B, 9.6B, 13.3B, 14_2.3B

so, 9.9J

140:

eka su cyālīsa, 13.3 (saü for su J)

196:

eka su chanūṃ, 8.10B

200:

bi saï, 15.5 (also saïṃ B)

210:

bi saï dasa, 15.3 (also saïṃ B), 15.4J

bi saï dahottara, 2.9B

bi saï dāhottara, 2.9J

bi saī, 15.4B (omits dasa)

224:

bi saï caüvīsa, 13.3J

bi saïṃ cuvīsa, 13.3B

250:

⟨bi saïṃ paṃcāsa⟩, 13.3

327:

triṇi saï sattāvīsa, 9.4B

triṇhaṃ saï sattāvīsaṃ, 9.4J

395:

triṇi saïṃ paṃcānūṃ, 9.8B

triṇha saï paṃcānū, 9.8J

420:

cyāri saï vīsāṃ, 15.3 (bīsa J)

465:

cyāri saï pāṃsaṭhi, 2.9

488:

cyāra saï aṭhyāsī, 9.3J

cyāri saïṃ aṭhyāśī, 9.3B

500:

paṃca śata, 15.6J

paṃcaṃ sita, 15.6B

pāṃca saï, 15.5B

pāṃca saü, 15.5J

8.16J

156025:

eka lāṣa chapana sahasra paṃcavīsa, 9.8 (chappanna J)

200000:

bi lāṣaḥ, 15.5 (lāṣa J)

1466521:

cauda lāṣa chāsaṭhi sahaśra pāṃca saïṃ ekavīsāṃ, 9.9B

caūda lāṣa chāsaṭṭhi sahaśra pāṃca saï ikavīsa, 9.9J

10000000 ($= 10^7$):

koḍa, 8.17J, 8.18J

koḍi, 8.17B, 8.18B, 9.10, 9.11

10000003:

eka keḍi anaï triṇi, 9.11B

eka koḍi anaïṃ triṇi, 9.11J

19345600:

eka koḍi trāṇū lāṣa paṃcitālīsa sahasra ccha saï, 9.10 (pacatālīsa J)

142857143:

cauda koḍi aṭhāvīsa lāṣa sattāvana sahaśra eka su trayatālīsa, 8.18B

caūda koḍa aṭhāvīsa lāṣaṃ satāvana sahaśra eka saü trayatālīsa, 8.18J

1000000000 ($= 10^9$):

arva, 8.17J (∅ B). Skt. arbuda. Cf. avva in GSK 1.12.

100000000000 ($= 10^{11}$):

ṣarva, 8.17. Skt. kharva. Cf. khavva in GSK 1.13.

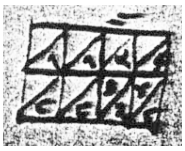
333333666667:

triṇi ṣarva tetrīsa koḍi tetrīsa lāṣa cchattīsag-hasra ccha si satasaṭhi (sic), 8.17B

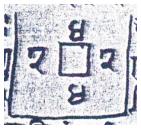
triṇha ṣarva tetrīsa arva tettīsa koḍa chatrīsa lāṣa chāsaṭhi sahaśra cha saï satasaṭhi, 8.17J

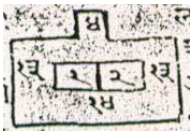
Appendix: Figures in the manuscripts

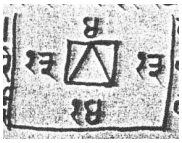
The letters “b” and “j” attached to the figure numbers denote respectively the manuscripts B and J, and “m” of “2b_m” margin.

1b  2b  2b_m  1j–2j missing

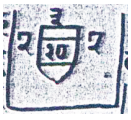
3b  3j  4b  4j 

5b  5j  6b  6j 

7b  7j  8b  8j 

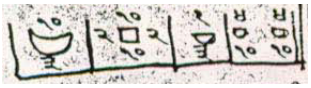
9b  9j  10b  10j 

11b  11j  12b  12j 

13b  13j  14b  14j 

15b  15j  16b  16j 

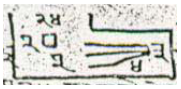
17b  17j 

15j+17j 

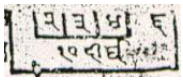
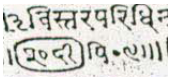
18b  18j missing 19b  19j 

16j(canceled)+19j+16j 

20b  20j  21b  21j 

22b  22j missing 23b  23j 

24b  24j  25b  25j 

26b  26j  27b  27j 

28b  28j  29b  29j 

30b  30j  31b  31j 

32b  32j  (fol.6b, end of l.8 + beginning of l.9)

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