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The Mathematical *Scholia Vetera* to *Almagest* I.10–15 With a Critical Edition of the Diagrams and an Explanation of Their Symmetry Properties

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Abstract

The article contains a complete edition, with a translation and a commentary, of the *scholia vetera* to the "mathematical" chapters of Book I of Ptolemy's *Almagest*, along with an edition of the diagrams that represent the geometric configurations at issue in the selected chapters. A new explanation of the phenomenon of "overspecification" of geometric diagrams is finally provided.

1. Introduction

The set of *scholia vetera* to the *Almagest* (henceforth *Alm.*) has never been edited in its entirety.[1] The best occasion to do that would have been when Heiberg prepared his critical edition of Ptolemy's treatise: the great Danish scholar had in fact already published and thoroughly studied over 1450 scholia to the *Elements* (henceforth *El.*), and he could well have done the same for *Alm*. If Heiberg had restricted himself to the scholia preserved in the most authoritative witnesses—four manuscripts, as we shall presently see—and if he had eliminated the bare references to canonical texts as he had done in the case of *El.*, he probably would have reached to no more than 1500 items. We may only guess the reasons for his not undertaking this task: maybe he judged that an analysis of the scholia was not necessary to corroborate his reconstruction of the textual tradition of *Alm.*,[2] or he simply did not have enough time.

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[1] In particular, no one has so far redacted even a tool as simple as a concordance table of the scholia in the oldest manuscripts of *Alm*. For editions limited to few *specimina*, see Mogenet (1975), excerpts from 5 items; Tihon (1976), 1 item; Pingree (1994), 3 items, see n. 30 below; Jones (2003), 4 items, see n. 20 below; Christianidis and Skoura (2013), 1 item; Acerbi and Riedlberger (2014), 1 item; it is scholium **94** below; Acerbi (2015), 3 items, two of which are sch. **77** and **87** below; and Tihon (2015), 9 items. Mogenet (1975) also gave some preliminary indications as to the structure of the collection (see below).
[2] In the case of *El.*, the analysis of the scholia provided decisive clues in this regard; see Heiberg (1888, 236–242 and 297–298), and Vitrac (2003). Heiberg published about 1380 scholia in his critical edition of *El.* (*EOO* V), about 50 in his dedicated study of 1888, and finally 33, contained in Scorial. Φ.III.5, in the *Paralipomena* to his edition (1903, 338–344). *Sigla* such as *EOO* are explained in the Bibliography.

In this paper, a sub-collection of the *scholia vetera* to *Alm*. is edited in its entirety, namely, the annotations to the "mathematical" chapters of Book I. Including the tables, these are chapters I.10–11 and I.13–15 of *Alm*., which feature Ptolemy's construction of the Table of Chords, his proof of the Sector Theorem, and the first application of the Sector Theorem to the construction of the Table of Declination. Among all scholia to these chapters, I have selected those that I shall call "*scholia vetera*:" these are annotations in the hand of the main copyists of the relevant manuscripts. Such a selection proves necessary because *Alm*. was heavily annotated in these same codices by Byzantine scholars.

The choice of this sub-collection, which amounts to 107 items, has been dictated by two criteria. First, only scholia with astronomical content have been published so far, whereas those edited in the present paper have an exclusively mathematical character. Second, the selected chapters of *Alm*. really constitute self-contained expositions, fairly independent from the rest of the treatise.

The purpose of this paper is to provide a cross-section of Late-Antiquity scholarly activity on *Alm*.: its sources, its aims, its methods. We shall also get more detailed information on the structure of the entire collection of *scholia vetera* to *Alm*. For this reason, I have edited *all* scholia pertaining to the selected chapters, even when they are bare references to canonical texts. We must not underestimate the kind of mathematics involved in some of the scholia: they pertain to the domain of calculation techniques that developed in strict connection with the adoption of the sexagesimal system in mathematical astronomy. Since Ptolemy is invariably reticent as to the computational techniques to be employed, these are expounded, sparsely, in commentaries *in Alm*. such as Pappus' or Theon's; in a systematic form, in computational primers such as the anonymous *Prolegomena ad Almagestum* (henceforth *Prol*.); occasionally, in dedicated scholia. In some scholia we also find interesting pieces of mathematics or valuable historical information; in view of their importance, some of these annotations have deserved a more detailed study in separate publications.

The paper is organized as follows. Section 1.1 introduces, mainly in order to establish a terminology, a typology of mathematical scholia. Section 1.2 outlines the structure and the origin of the main collection of *scholia vetera* to *Alm*. Section 1.3 presents in some detail the mathematical content and background of the selected chapters of *Alm*., with special emphasis on computational techniques. The scholia pertaining to every single proposition or argument in the selected chapters are also indicated. Section 1.4 summarizes the general characters of the edited scholia, providing tabular overviews of their types and functions; the relationships between the collections contained in the main manuscripts of *Alm*. are also clarified. Section 1.5 describes the structure of the subsequent section. Section 2 contains the edition, a translation, and a commentary on the scholia. The Appendix presents a "critical edition" of the diagrams that represent the geometric configurations at issue in the selected chapters of *Alm*., and a new explanation of the phenomenon of their "overspecification."

1.1 A Typology of Mathematical Scholia

All mathematical scholia share some basic features:[3] their mutual disconnectedness, their metatextual character, their paratextual position. Several categorizations can be used to differentiate them: their script (whether majuscule or minuscule), date of composition, date of transcription (whether first or later hand in a given manuscript), origin (whether as extracts from other writings or not), format (length, shape, set-up), type of argument (whether formal or informal), content (technical, historical, textual), location with respect to the main text (liminar, marginal, interlinear). On the basis of some of these criteria, I offer in the following list a typology of the scholia to Greek mathematical texts that seems to me to capture the main species of this literary sub-genre:

- Liminar scholia: connected, non-marginal series of annotations placed immediately before the beginning of a treatise and normally discussing its principles and its deductive structure.
- Comment scholia: unconnected, marginal annotations containing a full-fledged argument aimed at completing, correcting, or supplementing a specific *locus* (ranging from a word to an entire proposition) of the main text.
- παραγραφαί (henceforth *paragraphai*, sing. *paragraphê*):[4] very short, non argumented marginal annotations transcribing a variant reading or filling a lacuna in the main text, or explaining a mathematical statement either by an operative indication (like «because a tangent cuts a circle at exactly two points»[5]), or by a reference to a canonical text (*El.*, *Data*, *Conica* …), such as διὰ τὸ ιθʹ τοῦ βʹ τῶν στοιχείων «by the 19th of the 2nd of the Elements»: proposition, book, treatise.
- Diagrammatic scholia: marginal annotations in the form of diagrams not accompanied by a discursive explanation.
- Schematic scholia: small text cells organized hierarchically as a flow diagram and offering a summary of the main text, or providing supplementary information.
- Tabular scholia: marginal annotations in the form of numerical tables or of calculations arranged in a tabular set-up; they can be supplemented by short textual units such as headings, identifications of numerical values with specific magnitudes in the main text, etc.
- Interlinear scholia: (short) interlinear annotations clarifying specific lexical or mathematical points.

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[3] In this and in the following Section I shall partly employ the material presented in Acerbi (2014).
[4] This denomination comes from Eutocius, *in Con. IV*, *AGE* II, 354.8. John Pediasimos, a Byzantine scholar well versed in mathematics, called them παρασημειώσεις (see *sch*. X.405 *in El.* X.91, *EOO* V, 563.27).
[5] I shall use French quotation marks for quotes or translation from Greek, usual quotation marks to emphasize words.

The borderline between comment scholia and *paragraphai* is not as sharply defined as those between the other categories, that are determined by requirements of format or of location; let us stipulate that a *paragraphê* becomes a comment scholium when it has at least two clauses. The scholia may be keyed to the relevant passage in the main text by means of a *signe de renvoi*. The scholia may be *figurata*, that is, the text itself may be written in the form of an object, such as a Latin cross, or an amphora. The disposition of the annotations in the page can provide us with valuable pieces of information about the *mise en page* of the models of the manuscripts in which we presently read them.

1.2 The *Scholia Vetera* to the *Almagest*: Their Structure and Their Origin

Heiberg knew of 36 manuscripts containing *Alm.* in its entirety;[6] he organized them into three families,[7] whose best (and oldest) representatives, identified by their *sigla*, are

(**A**) Par. gr. 2389 (in majuscule, 9th c. *in*., *siglum* **A**, *Alm.*);

(**B**, **C**) Vat. gr. 1594 [9th c. *p. m*., *siglum* **B**, *Prol*. incomplete, Ptolemy, *Alm*., *Phaseis*, *De judicandi facultate et animi principatu* (*Judic.*), *De hypothesibus planetarum* (*Hyp.*), Book I][8] and Marc. gr. 313 (9th c. *ex*.–10th c. *in*., *siglum* **C**, *Prol*. incomplete, *Alm*.);[9]

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[6] The codices containing only parts of *Alm*. are listed and shortly described at *POO* II, CXLIII–CXLVII.

[7] In the order of the following list, partial *stemmata* are given at *POO* II, LXXVI, LIII, CXXXVI. **G** is incomplete (des. *POO* I.2, 589.7 ἀνωμαλίας) and was employed by Heiberg only in tome I.2 (= *Alm*. VII–XIII); he reported the variant readings of the star catalog at *POO* II, CXXI–CXXVI.

[8] This codex of 284 folios contains: *Prol.*, ff. 1–8v; *Alm*., ff. 9r–263v (the books are distributed as follows: Book I, ff. 9r–28r; Book II, ff. 28r–57r; Book III, ff. 57r–76v; Book IV, ff. 77r–97r; Book V, ff. 97r–122r; Book VI, ff. 122r–145r; Book VII, ff. 145v–159v; Book VIII, ff. 160r–174r; Book IX, ff. 174r–195v; Book X, ff. 195v–209v; Book XI, ff. 209v–229r; Book XII, ff. 229r–244v; Book XIII, ff. 245r–263v); *Phaseis*, ff. 264r–272r.; *Judic*., ff. 272v–276v; *Tabula categoriarum*, *ad praecedens opus pertinens*, f. 277r; f. 277v *vacuum*; *Hyp*. I, ff. 278r–283r; *Tabulae variae*, ff. 283v–284r; *Adnotationes variae*, f. 284v. A very detailed description of **B** can be found in Acerbi (2018b). Here it is important to bear in mind that this codex, formerly included in the "collection philosophique" (on the issue see Ronconi 2013) was written by two copyists, called IIa (ff. 1r–277r) and IIc (ff. 278r–283r), and that *Alm*. was transcribed on two columns.

[9] This codex of 370 folios (ff. 1–2 and 6 are missing) is written on full page and contains: ff. 1r–30v, preliminary material; ff. 31r–370v, *Alm.* I–XIII, mutilated *in fine* (des. *POO* I.2, 593.23 ὅπου). According to N. Wilson (*per litteras* on 5/2/2005), it can be assigned to the 9th *ex*.–10th c. *in*. The sporadic accentuation, that Wilson took as a clue to anticipate the date «saec. X med.» proposed in Mioni (1985, 24), had prompted B. Fonkič (2005, *non vidi*) to assign this codex to the beginning of 9th century; see also Agati (1992, 141–142). The only known apograph of **C** is Marc. gr. 311, ff. 1, 3–11 26–58, 60–65, 67–112, 113 *pars superior*, 118–123, 125–165, 169–170, 172–184, 186–191 (it is the ancient part of the codex, in oriental paper); the rest belongs to the family of **G**, but presents earlier layers of text. The Greco-Latin translation of *Alm*.—made within the Sicilian school of translation, which flourished during the third quarter of 12th century (Haskins and Lockwood 1910; Heiberg 1910 and 1911; Haskins 1912) and to which we also owe the Greco-Latin translations of Euclid's *Data* and *El.* and of Proclus' *Elementatio physica*—derives from **C**. The two copyists of Marc. gr. 311 and the Greco-Latin translation share corrections with respect to the text of Marc. gr. 311 (Heiberg 1911). **C**, that contains some annotations in Latin assigned by Heiberg to the 14th century, was also the model of the Greco-Latin translation of *Prol*., made by the same translator as that of *Alm*. Of this translation of *Prol*., only the initial segment has so far been uncovered, containing the isagogic section and Zenodoros' treatise on isoperimetric figures; the latter is edited in Busard (1980), the former in Acerbi, Vinel, and Vitrac (2010, 90–91). This edition is based on the Florence, Bibl. Naz. Conv. Soppr. A V 2654, f. 120v,

(**D**, **G**) Vat. gr. 180 (10th c., *siglum* **D**, *Alm.*)[10] and Vat. gr. 184 (13th c. *p. m.*, *siglum* **G**, *varia arithmetica et astronomica*, *Prol.*, *scholia ad Alm.*, *Alm.*).[11]

Heiberg notes that the tradition represented by the third family, although less correct and often interpolated, allows very old textual layers to be reached. Overall, the structure of the *stemma* proposed by Heiberg makes it possible to go very far back in the tradition of *Alm*. As for the scholia, the situation can be summarized as follows.

(**A**) **A** is a *de luxe* exemplar and has no *scholia vetera.*

(**B**, **C**) A large amount of scholia transcribed by the main copyists can be found in **B** and **C**.[12] The sets of scholia contained in these two codices are almost identical. As a consequence, the two manuscripts are independent witnesses of a single collection, in the same way as they are of *Alm*. itself. These annotations were eliminated *en bloc* from all apographs of **B**.

(**G**) **G** is an apograph of **B** as for *Prol.* The earliest *marginalia ad Alm.* were transcribed by the main copyists themselves.[13] As the script of the main text in **G** is quite dense, the scholia are keyed by means of *signes de renvoi*; they are organized in clusters of often unrelated annotations; these are often separated by the standard graphic marker "*dicolon* + *paragraphos* + short blank space," but it frequently happens that the scholia in the cluster follow each other without separating marks. A preliminary assessment shows that the bulk of these annotations is a subset of those in **BC**. Their text shows strict affinities with the readings of **C**, and I take it as certain that **G**, or a model of it, is an apograph of **C** as far as these scholia are concerned (details in Section 1.4).[14]

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owned by Antonio Corbinelli. The preliminary material is not contained in the other witnesses of the Greco-Latin translation of *Alm.* (Vat. lat. 2056, owned by Coluccio Salutati; Pal. lat. 1371; Guelf. Gud. lat. 147).

[10] On the structure of this codex, written by four copyists, see *POO* II, LXXVII–LXXVIII and LXXX–LXXXII. Heiberg's analysis has been completed in Orsini (2005, 317–322 and 340–342). The ff. 1r–2r and 280v of Vat. gr. 180, in a hand of the 11th century, contain excerpts form Theodoretus' *Commentarius in Psalmos*.

[11] This manuscript (notes of one of the copyists dated 1269–1271; for the structure of the codex see Bianconi 2004, 330–331) contains annotations and corrections by John Pothos Pediasimos, John Catrarios, Nicolas Eudaimonoioannes (Tihon 2003). Bianconi (2005b, 150–151) identified one of the hands singled out by Tihon with that of John Catrarios; the identification of Pediasimos (*olim* Turyn's R, to be found also in Bodl. Dorv. 301: B. Mondrain, unpublished—and in Vat. gr. 2326: Bianconi 2014, 467–468) is in Pérez Martín (2010).

[12] There are just a handful of first-hand scholia to *Prol.*, nevertheless they provide us with one interesting piece of information: a possible alternative title of Pappus' *Collectio* (but see the subscription of Book IV at Hultsch 1876–1878, 303.18 *app.*). The text is ἰστέον ὅτι ὁ μέγας Πάππος ταῦτα ἐπέδειξεν ἐμμελῆ ἐν τῇ ε′ βίβλῳ τῶν ἀνθηρῶν προβλημάτων (**C**, f. 3v, and **B**, f. 5r, in minuscule, edited in Acerbi, Vinel, and Vitrac 2010, 132.24 *app.*).

[13] See Bianconi (2004, 331, n. 59): hand A both copied the text of, and apposed a few annotations to, *Alm.* I.10 (ff. 86r–87v); hand C copied all scholia to *Alm*. I.13–15 (ff. 90v–93r; f. 92 only contains scholia, f. 93r has the Table of Declination; the main text at ff. 90v–91v is due to hand B).

[14] On the fact that the copyists of **G** certainly had access to **B** see *POO* II, XXXII–XXXIII and CXVII–CXXI. Apparently, **B** was used as a reference manuscript in Constantinople's scholarly circles: extensive lacunae in **A** were supplied by one of the copyists of Theodoros Metochites, who directly relied on **B**; see ibid., XXXVII–XXXVIII, and Pérez Martín (2008, 436, and n. 177); the lacunae in **A** are at *POO* I.2, 10.5–28.8, 250.1–332.22 and 599.5–608.10. As to **C**, Heiberg surmised that the model of **G** was collated with **C** (*POO* II, CXXI).

(**K**) A further select collection of scholia was transcribed in Vat. gr. 184, before *Alm.* itself, at ff. 25r–80v, with the title Θέωνος ἀλεξανδρέως· σχόλια πάνυ χρήσιμα εἰς τὴν μεγάλην σύνταξιν Πτολεμαίου.[15] This collection, that will receive the *siglum* **K**, was certainly drawn from **B** since it also includes many annotations by a very active hand of the 12th century apposed in **B** itself (see point 6 below). Hence, we sometimes find that the same annotation is found twice in Vat. gr. 184, both in the margins of *Alm*. (for which *siglum* **G** will again be used) and in **K**. Scholia or groups of scholia in collection **K** are normally preceded by an appropriate citation from *Alm*.

(**D**) One of the main copyists[16] of **D** found in its model a rich scholiastic apparatus, totally disjoint from that of **BC**. As regards the chapters of *Alm*. in which we are interested, this corpus is made of 84 annotations: 41 comment scholia and 43 *paragraphai*. The former include 3 extensive excerpts from Theon's commentary, preceded by the indication ἐκ τῶν Θέωνος.[17] Of the *paragraphai*, 31 are simple citations by proposition, book, treatise (mostly *El.*) or references to previous results of *Alm*. Taking into account also these annotations would have made this article too long and complex; *diis faventibus*, I shall present this material in a future study. The later annotations we presently read in **D** were transcribed selectively, repeatedly and always by collation from **B**, by a series of hands arriving until as late as the 14th century. In particular, a rich scholiastic apparatus was transcribed, after collation of **B**, by a hand of the end 11th – beginning 12th century.

As a consequence of all of this, the denomination *scholia vetera* will be used in this paper to designate the huge collection transcribed by the main copyists of **B** and **C**; these scholia were surely contained in their common model. We shall presently see that we can also set a lower bound to their date of composition: the redaction of Theon's commentary *in Alm*., about 360. We are thus presented with a collection of scholia assembled not before Late Antiquity and contained in independent (albeit belonging to the same family) witnesses of the main text. But the situation is even more favorable, since we can both lower the upper bound and increase the lower bound. In order to see this, let us consider first some of the main features of this corpus as we have it in **B**, by far the best witness.

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[15] Mogenet (1975, 307) asserts that, for *Alm*. III, these are in all 121 annotations, of which 46 were originally written by the first hand of **B**, 75 by a second hand. The hand that transcribed in **K** the scholia here edited is that of copyist D: see Bianconi (2004, 331, n. 59).

[16] This very likely coincides with hand *c* in Heibergs's classification (*POO* II, LXXX). Hand *b* is responsible for most of the folia containing the chapters of *Alm*. we are interested in (ff. 15–24); hand *a* transcribed the beginning of *Alm*. I.10, on f. 14.

[17] The excerpted passages, to be found at ff. 16v, 17v, 23r, are at *in Alm. I.10*, *iA*, 486.15–487.4 and 501.4–502.5, *in Alm. I.13*, *iA*, 560.11–20, respectively; they present almost no variant readings with respect to the text edited by Rome. Theon is also mentioned in a further scholium.

1) *First-hand scholia in majuscule*. First-hand scholia are either in a majuscule of small module or in a minuscule identical with that of the main text but of reduced module: over the same length, the scholia contain about two-thirds of the signs of the main text. Scholia in majuscule[18] can be found at ff. 6r, 9r epigram (in *Auszeichnungs-majuskel*, of the same size as the titles of the chapters of *Alm*.), 12r (short schematic summary), 16r, 19r, 19v, 22v, 23r, 23v, 24r, 25r, 25v, 36r, 37v, 38v, 39r, 40r, 42r, 42v, 43r, 43v, 44r, 44v, 45r, 46v (schematic summary), 47r, 47v, 48r, 48v, 57v, 61v, 64v, 68v, 71r (short schematic summary), 75v, 81v–82r (annotations to the tables of the mean motions of the Moon), 92r, 97v, 100r, 102v, 106v, 110r, 116r, 118r, 120v, 122v, 123r, 123v, 125r (annotations to the tables of conjunctions), 126r, 127r, 128v, 132v, 133r, 138v, 140v, 141v, 148v, 154r, 155–164 (annotations to the star catalog), 168r, 168v, 169r (schematic summary), 169v, 170r (schematic summary), 174v, 176r, 178r, 179v, 181r, 18v, 184r (annotations to the tables of the mean motions in longitude and anomaly of the five planets), 185v, 193r (data concerning the Moon and Mercury, in tabular form), 196r, 196v, 197r, 199r, 207r, 208r, 209v, 211v, 213r, 215v, 216v, 217r, 221r, 222r, 222v, 223v, 224r, 228v, 229r, 257v. Schematic scholia normally have the first categorization in majuscule (see item 3). Numerical tables normally are in majuscule (see item 4).
2) *First-hand scholia in minuscule*. These can be found on almost every page, and sometimes are of considerable extent; as regards their content, see *infra*. Starting from f. 113v (*Alm*. V.4), a later hand marked these scholia selectively (the selection criterion appears to be related to length) by a slashed majuscule *gamma*. Starting from f. 140r (*Alm*. VI.10), the scholia are also numbered by a more recent hand, which at the beginning adds ση(μείωσαι) to the numeral letter, again selectively, and in fact over a subset of those receiving the first marking: numbers range from α′ to γ′ in Book VI (but including a β′ and a β′⁺), from α′ to ϙβ′ in Books VIII–IX, from α′ to κς′ in Book X, from α′ to ξβ′ in Books XI–XII, from α′ to ϙ′ in Book XIII: 274 scholia in all. Exactly the same numbering is apposed, only for Book XIII and by the same hand of the 14th century that transcribed the scholia, in **D**. Some scholia in minuscule have a "title" in majuscule. A small number of scholia are *figurata*: ff. 10v (amphora), 18v (Latin cross), 24v (altar), 26r (Latin cross having an amphora as basis, but only the final portion of the scholium, which begins in the upper margin), 36r (Latin cross), 36v (amphora with pointed basis), 38r (amphora), 68v (Latin cross having an amphora as basis), 205v (amphora having a Latin cross as basis). Even when they are not *figurata*, many first-hand scholia in minuscule have their last line(s) centered and showing progressively reduced length. Two long scholia at the end of *Alm*. V.7 and V.17 (ff. 106r and 117v) are set out, preceded and followed by decoration, as if they had a truly textual status, although they are still written with a script whose module is intermediate between that of the text and that of the marginal annotations.

[18] Short numerical scholia are excluded.

3) *Schematic scholia*. These can be found at ff. 1v,[19] 12r (entirely in majuscule), 12v, 24r, 46v (in majuscule), 47r, 71r (in majuscule), 119r, 145r, 169r (in majuscule), 170r (in majuscule), 260v. The schemes in majuscule are usually more condensed. All these schemes, except for the one at f. 145r, have portions in majuscule, normally the first row.
4) *First-hand tabular scholia*. Numerical tables (always without justification lines) can be found at ff. 27v, 37v, 38v, 42r, 44v, 45v, 48v, 59r, 62r, 70r, 74v, 75r, 75v, 76r, 76v, 78v, 79r, 81v, 82r, 86r, 86v, 87r, 88r, 89v, 92v, 93v, 95r, 95v (marg. sup., these are data from Hipparchus), 99r, 100r, 101r, 101v, 111r, 113r, 113v, 114v, 115r, 115v, 122v, 123r, 126v, 127r, 127v, 128r, 128v, 129r, 129v, 130r, 132v, 133r, 133v, 143r, 143v, 174r, 176v, 182v, 184r, 186r, 192r, 193r, 201v, 209v (comparison of the dates of observation reports by Dionysius and Ptolemy),[20] 210r, 216v, 217r, 222v, 235r, 235v, 236r, 236v, 245v (incomplete), 261r. Tabular calculations (normally fourth-proportional schemes required in interpolations or in applications of the Sector Theorem) can be found at ff. 23v, 26r, 27r, 29v, 30r, 42v, 44v, 46r, 48v, 70r, 71r, 76r, 90r, 90v, 91r, 94v, 102r, 102v, 103r, 103v, 104r, 104v, 105v, 108r, 108v, 111r, 111v, 112r, 114r, 114v, 115, 116r, 116v, 117r, 121v.
5) *First-hand diagrams attached to scholia*. They can be found at ff. 45r, 64v, 65r, 68r, 68v, 75v, 76r, 79v, 87v, 98v, 99v, 103v, 119r, 120r, 127v, 128v, 129r, 131r, 134r, 135r, 172r, 188r, 255r. The systematic presence of these diagrams shows that they were drawn at the same time as the scholia.
6) *Second-hand scholia and correctors*. These can be found at ff. 2r–5r (see n. 24 below), from f. 16v to f. 91r (*Alm*. I.9–IV.6), and at f. 112v (*Alm*. V.14). Heiberg identified four hands of correctors.[21] The hand of the first corrector (end 10th or beginning 11th century), whose *ductus* is nervous and angular, strongly bent on the right, supplements short pericopes omitted, usually by *saut du même au même*, by copyist IIa at ff. 18r–v, 19r–v.[22] By far the most frequent hand goes back to the 12th century[23] (an attentive reader: note the title at f. 169r): it also supplied ff. 66–67 and drew most of the diagrams omitted by the main copyist, relying for its interventions on a manuscript of the family of **D** and **G**.[24]

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[19] Edition in Acerbi, Vinel, and Vitrac (2010, 78.3 *app*.).
[20] The scholia edited in Jones (2003) report observations of passages of planets by bright stars, by the calendar "according to Dionysius" and by the calendar "according to the Chaldeans."
[21] *POO* II, XXXII–XXXIII.
[22] The supplemented passages are at *Alm*. I.10, *POO* I.1, 37.11–12, 37.15, 38.14, 39.16–18, 43.7, 43.13–16, 45.6–7. All these passages are also absent in **C**. Other interventions of this corrector are ibid., 13.9 (corr. μείζοναι in χωρῇ), 15.17 (restoration of the order of three words by means of superposed apices), 19.21 (trivial correction), 38.18 (add. τε οὐκ marg. et ras.), 44.6.13 (correction of denotative letters). Therefore, this hand only checked for the text of *Alm*. I.3–5 and I.10.
[23] *Expertise* of P. Canart *apud* Mogenet (1975, 303). Tihon (2015, 15–16) follows Giannelli (1950, 224), and assigns the hand to the late 13th century. Upon my request, Canart's *expertise* has been confirmed by I. Pérez Martín. The lacuna in **B** extends over *POO* I.1, 224.14–228.20.
[24] The first scholium written by this hand is of the utmost importance: in the margins of *Prol*. at ff. 2r–5r, it provides us with crucial pieces of information on the relationships between Byzantine and Arabic astronomy.

Both first-hand and second-hand scholia to *Alm*. can be divided into four categories (Mogenet 1975):

a) extracts from Theon's commentary *in Alm*.;
b) extracts from the "same" commentary but containing additions (or presenting variants) to be found nowhere else;[25]
c) annotations referring to periods after Theon's times;
d) annotations, often of minor import, whose date and origin cannot be determined.[26]

The first-hand scholia of categories a) and b) constitute almost the whole of the indirect tradition of Theon *in Alm*., and Mogenet, in his article of 1975, had already set up comparisons to the detriment of the editorial practices of A. Rome. Now, while looking at these scholia is partly made useless by the fact that Books I–IV and VI of Theon's commentary are transmitted by the *vetustissimus* Laur. Plut. 28.18 (9th c. *in*.; these are the books edited in *iA*, where the *scholia vetera* to *Alm*. were not collated), the only witness of Theon *in Alm*. VIII–X and XII–XIII[27] not containing a Byzantine recension happens to be a portion, dated to the end of the 13th century, of Vat. gr. 1087.[28] As for *in Alm*. VIII–X and XII–XIII, then, the earliest indirect tradition of Theon *in Alm*., represented by the first-hand scholia of **B**, is more than four centuries older than the direct tradition of the same work—the direct tradition being only available in a form in which we have every reason to suspect interventions of the renowned Byzantine scholar Maximus Planudes.[29]

Redacted *ca*. 1032 and transcribed in **B** from unknown sources, this scholium was therefrom copied in the margins of Vat. gr. 2326, ff. 26r–28r, and of Par. gr. 453 (Mogenet 1962, for the latter).

25 A tempting hypothesis (cf. Mogenet 1975, 307) on the origin of these extracts is that they come from Pappus' commentary *in Alm*., by all evidence the (very close) model of Theon's (see *iA*, LXXXIII–LXXXVI). Only Books V and VI of Pappus' *in Alm*. have survived, whereas Theon's equivalent commentary has been transmitted almost in its entirety (see below).

26 According to Mogenet (1975, 307), for *Alm*. III the collection **K** gives the following figures: of the 46 first–hand scholia, 31 belong in one of categories a) and b), whereas only 8 of the later scholia have this origin.

27 Book XI of Theon *in Alm*. is lost, Book VII can only be read in a Byzantine recension.

28 The relevant portion is at ff. 123–147. This manuscript must be completed with Par. gr. 2396 (Rome 1927), which contains Theon *in Alm*. I, II, IV and whose ff. 1–86 date back to the Planudean period and actually are in part (ff. 33v–76v) an autograph of Planudes himself (Mondrain 2002). The copyist of Par. gr. 2396, ff. 77r–86v, and of the said folios of Vat. gr. 1087, was an anonymous collaborator of Nicephoros Gregoras (Bianconi 2006, 147–151). Both codices also contain annotations by Gregoras (Bianconi 2005a, 414–415 and 417). On Vat. gr. 1087, see in the first place Pérez Martín (1997, 83), and, most recently, Menchelli (2013).

29 A proof of this is a calculation that Theon performs at *in Alm*. *XIII.3*, first διὰ τῶν ἐκ τῶν γραμμικῶν ἐφόδων ἐπιλογισμῶν, then διὰ τῆς τῶν διοφαντείων ἀριθμῶν ἀγωγῆς. We read this passage as an unassigned extract at **B**, f. 248v, and at **D**, f. 268r (copied from **B**: in both codices, it is marked by ordinal κς′; see item 2 above), written by a hand of the 14th century (starting exactly from *Alm*. XIII.3, the lower margin of each folio of **C** has been cut off, but what remains bears no annotations)—and obviously as a part of Theon's commentary at Vat. gr. 1087, ff. 145r–v, where it is followed, in the main text, by a carefully arranged tabular set-up of the diophantine-style solution, identical in form to those displayed in Planudes' commentary on Diophantus' *Arithmetica*. An edition of this text is published in Christianidis and Skoura (2013).

As far as the first-hand scholia belonging to category c) are concerned, an analysis of three such scholia led D. Pingree to conjecture that they were the *membra disiecta* of a commentary redacted between 537 and 637, probably by a representative of the Nestorian community based in Nisibis, now Nusaybin in south-eastern Turkey.[30] Pingree even ventured to propose a chain linking this unknown scholar with Theophilus of Edessa and his pupil Stephanus the Philosopher, who might have carried the model of **B** to Constantinople just before 775. As a matter of fact, Pingree's argument rests on poor evidence: the first scholium, from which he draws the window between 537 and 637, appears to forge fictitious data for the sake of example; the second scholium is a précis of *Hyp*. II, transmitted only in Arabic translation but probably available in its original form to Proclus and Simplicius;[31] the third scholium, of theological content, focuses on a point of doctrine (divine things are invisible to us only because of our weakness) that is not extremely specific—and Syria was not the only place where Nestorian ideas could be professed at the beginning of the 6th century.

A different suggestion as to the origin of the first-hand scholia in **B** come to the fore if we examine them more closely from a structural point of view. First of all, these scholia could not possibly have been composed or gathered by the copyist: as noted above, we find the same collection, with variations that will be studied in detail elsewhere, also in **C**, belonging to the same textual family of *Prol*. and *Alm*. as the Vatican codex.

A second, and crucial, remark is that the copyist of *Prol*. and *Alm.* in **B** apparently found two different layers of scholia in his model, which he differentiated by means of the graphic dichotomy majuscule/minuscule.[32] The most likely explanation for this is that the scholia he transcribes in majuscule could be identified as less recent in the model, the scholia he transcribed in minuscule being perceivably more recent, and perhaps traced in a more informal hand.[33]

A first clue corroborating this reconstruction comes from the epigram transcribed in a quite formal majuscule script in **B**, f. 9r marg. inf., whereas **C**, f. 30v (in a very formal majuscule; it is not clear whether the copyist intended to have the epigram in the text or

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[30] See Pingree (1994); the scholia can be read at **B**, ff. 169r (*Alm*. VIII.3), 174r (*Alm*. IX.1), and 10r (*Alm*. I.1); the first and the third have been recently edited, in order to refute Pingree's thesis, in Tihon (2015, texts 2 and 5), whose counter-arguments I have summarized in this paragraph. Other manuscripts carrying annotations to *Alm*. can prove useful for reconstructing ancient writings. Mogenet (1975) has shown that those of **B** can assist in filling lacunas in Book III of Theon *in Alm*. Tihon (1987) found Book V of the same treatise in the margins of Vat. gr. 198 (14th c. *p. m.*), after Rome (1953) had identified a long extract included in the main text of the same manuscript (ff. 421v–424v).

[31] Proclus, *in R*. II, 230.14–15 Kroll, and *in Ti*. III, 62.22–24 Diehl; Simplicius, *in Cael*., 456.22–27 Heiberg. These passages are also printed in *POO* II, 110. See, however, the reservations expressed in Bowen (2013, 211–212), as to the possibility that Proclus and Simplicius had access to a complete text of *Hyp*.

[32] The evidence shows that the argument in Irigoin (1957, 9–10), concerning the presence of majuscule *vs*. minuscule script in the scholia as a dating criterion, could not be right, simply because it was based on too reduced a sample (for instance, Irigoin does not mention Vat. gr. 190, whose scholia are all in minuscule).

[33] In particular, some schematic summaries could be older than others, the copyist having kept the majuscule only for the first line of the most recent ones. To the scholia in majuscule to *Alm*., four annotations in majuscule to *Judic*. must be added.

to keep it in the margins, since it partly occupies both; the presence of a decoration between summary and epigram suggests that the former alternative holds), and **D**, f. 3r, have it in the main text, just after the summary of *Alm*. I.[34] It is quite clear that the epigram was present in the form of a scholium in the common model of **B** and **C**, and that the incertitudes as to its placement in **C** must come from the fact that its copyist shifted the format from two columns to full page. Now, the epigram can also be read in Synesius, *De dono* 5 (no ascription; it is qualified as ἀρχαῖον), as *Anthologia palatina* IX.577 (Heidelb. Palat. gr. 23, page 455, lemma Πτολεμαίου εἰς ἑαυτόν), in **G**, f. 82r (minuscule script, in the intercolumnar space, no ascription and after an indication ἐπίγραμμα), in Leid. B.P.G. 78, f. 145r (Ptolemy, *Tabulae manuales*), where the epigram, written by the main copyist in a early 9th century majuscule, is included in the last table (!) of the catalog of the fixed stars, preceded by the title ἐπίγραμμα ὃ εἶπεν Πτολεμαῖος εἰς ἑαυτόν. All these testimonies had only access to a debased text, marred by three conspicuous *lectiones faciliores*. Thus, the best version of the epigram has only been transmitted by the model of **BC**, the debased version being witnessed by as early as Synesius. This suggests that the epigram was already contained as a scholium in an ancestor of the model itself: the formal *Auszeichnungsmajuskel* adopted in the surviving apographs of this codex can be taken to reflect this fact.

A second clue is that the scholia to *Prol*., a treatise which was redacted not before the end of the 5th century as we shall see, are all in minuscule, including the highly symmetric schematic summary on f. 1v.

A third clue, corroborated by what we shall argue about the origin of the hyparchetype of the textual family of *Alm*. led by **B**, is that the scholia in minuscule have the structure of a running commentary, made up of extracts from other commentaries: it is the work of a specialist, finding its natural *milieu* in well-organized scholarly circles, most notably from the point of view of access to sources.

A fourth clue suggesting the ancient origin of all first-hand scholia in Vat. gr 1594 is the notational practice of the signs representing sexagesimal parts: on the one side is the evidence of *Prol.*: apices; on the other stands the evidence of the main text of *Alm.* and of the scholia thereon in *all* the oldest manuscripts: bars. It seems as if our documents testify to a notational change that took place in the period between Theon (4th c. *p. m.*), whose *in Alm*. was the basis of the excerpted scholia, and the circles in which *Prol*. was redacted (6th c. *in*.), and the change may indeed have been triggered by these very circles. What is paradoxical is that in the same codex (**B**) one finds two different notations for the sexagesimal parts, and that one (apices) is systematically adopted in a work (*Prol.*) that was intended as a technical primer for the algorithmic practice of the treatise (*Alm.*) where the other (bars) is adopted (cf. Acerbi 2013, 148–150 and 156–159).

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[34] But in **D** the epigram is transcribed by the same hand of the end 11th–beginning 12th century that collated the scholia of **B**. The epigram is edited in *POO* I.1, 4.5 *app.* and, for the variant readings of the *recentiores*, *POO* II, CXLVII–CXLVIII; an edition taking into account the entire tradition is in Boll (1921/1950).

The strongest argument in favor of assigning the model of **BC** to the Alexandrine Neoplatonic school of the late 5th – early 7th centuries, is what we may call "preliminary material" (*scil*. to *Alm*.). This resource is made up of four texts, given in the following order: *Prol.*, the *Inscriptio Canobi* (a work by Ptolemy!), a list of seven observation reports of astronomic phenomena, the *Septem astrorum epitheta* of Dorotheus of Sidon (1st c.). Now, only the initial one-third of *Prol*. remains in **B**, but the original presence there of the preliminary material in its entirety is beyond doubt: two quires after the first quaternion have surely been lost in **B**, and we find the same material in its complete form, and immediately followed by *Alm*., in direct apographs of **B** such as Par. gr. 2390 and Vat. gr. 2326; what is more, we also find it in **C**, an independent copy of the same model of **B**.

Now, six of the observation reports contain the name of the observer: it is Heliodorus, son of Hermias and brother of Ammonius,[35] the Neoplatonic philosopher who held the chair of the School of Alexandria. To these six observations, expressly dated between 498 and 510, another was added, made in 475 in Athens, preceded and followed by the indication τοῦ θείου τήρησις.[36] The reports are worded in the first person and begin with εἶδον Ἡλιόδωρος. In one of them, made on 21 February 503, it is specified that «the beloved brother» of Heliodorus was also present (ἐγώ τε καὶ ὁ φιλώτατος ἀδελφός)—that is, Ammonius.[37]

Heliodorus' observations provide us with one crucial piece of information about the history of the text. In fact, the *incipit* of this short tract is ταῦτα ἀπὸ τοῦ ἀντιγράφου τοῦ φιλοσόφου ἔγραψα: it is the remark of a copyist, and the fact that, both in **C** and in the apographs of **B**, we find these words in isolation at the beginning of the text (still not identified as a title by some *Auszeichnungsschrift*) shows that they were contained in the common ancestor of the Vatican and Venetian codices. The rhetorical device of antonomasia and the fact that the epithet φιλόσοφος was, among contemporaries, exclusive to the chairholder of the Neoplatonic school make it almost certain that this unnamed «philosopher» is Ammonius or one of his immediate successors. Moreover, it goes without saying that the «exemplar of the philosopher» was a copy of *Alm*.: one is thus led to conclude that Heliodorus and Ammonius were an active part in the revision that has consigned to us a whole branch of the manuscript tradition of Ptolemy's *magnum opus*,

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35 See Zintzen (1967, 100.7–8, 101.2, 109.7–11) = Photius, *Bibliotheca*, codex 242, 341a7–9, and *Suda* E 3035, II, 412.22, and A 79, II, 162.13–17 Adler, respectively.

36 Heiberg (*POO* II, XXXVII) surmised that that «divine» was Proclus; Westerink (1971, 20, n. 27) recalled that in Late Antiquity θεῖος may simply mean «uncle» (this is the origin of the Italian word "zio"). From different sources (but they all can be traced back to Damascius' *Vita Isidori*), we know that the name of Hermias' brother was Gregorios (Zintzen 1967, 104.5 and 105.7 = Photius, *Bibliotheca*, codex 242, 341a33, and *Suda* Γ 453, I, 543.8 Adler, respectively).

37 Editions in *POO* II, XXXV–XXXVII; Jones (2005). An analysis of the astronomic content can be found in Neugebauer (1975, 1038–1041). The observations are: 1st May 498, conjunction Mars-Jupiter; 21 February 503, lunar occultation of Saturn; 18 November 475, lunar occultation of Venus; 27 September 508, conjunction Jupiter-Regulus (= α Leonis); 11 March 509, lunar occultation of the «bright [star] of the Hyades» (= α Tauri); 13 June 509, conjunction Mars-Jupiter; 20–21 August 510 (date not specified in the text), missed conjunction Venus-Jupiter.

enriched with the preliminary material. In this perspective, it is likely that the common model of **B** and of **C** was closely linked with the «exemplar of the philosopher». It should also be recalled that no more than 350 years passed between Heliodorus and the creation of **B**, and that a copy of *Alm.* was extremely expensive.[38] Consequently, Heiberg puts the hyparchetype of this branch of the tradition directly in the 6th century, arguing that this codex coincides with the exemplar of Heliodorus/Ammonius or—though he sees this as less likely—with an immediately subsequent copy.[39]

That *Prol.* was also produced in Neoplatonic circles is beyond doubt: it presents *Alm.* according to the isagogic schemes developed by late Neoplatonism;[40] it attaches the epithet «divine» to some of the authorities it names.[41] A *terminus post quem* is set by the ascription to Syrianus (died *ca.* 437) of a specific computational algorithm: λέγεται δὲ ἡ εὕρεσις Συριανοῦ τοῦ μεγάλου φιλοσόφου. The dubitative shade of meaning suggests that there was no strict doctrinal continuity between Syrianus' discovery and its mention in *Prol.*: it is therefore reasonable to shift our gaze from Athens to Alexandria and let a couple of generations pass. The text of *Prol.*, which—given the syntactic evidence and the types of arguments adduced—was in all probability based on lecture notes taken from oral teaching (redaction ἀπὸ φωνῆς) that were never the object of a final redaction in view of their ἔκδοσις, was thus composed in the Alexandrine Neoplatonic circles in order to serve as an introduction and a technical primer for the algorithmic practice of the recension of *Alm.* that circulated among Ammonius' pupils.

To sum up, the model of the model of **B** can probably be identified with the ἀντίγραφος τοῦ φιλοσόφου (Heliodorus' observation reports), which must have been a relatively clean exemplar (sparse first-hand scholia in majuscule in **B**). The model itself was at the same time the working exemplar of a top-level scholar (abundant and technically refined first-hand scholia in minuscule)[42] and an official copy, intended to assist teaching in the Alexandrian Neoplatonic school (preliminary material and recension of *Alm.*) but without the features of a *de luxe* exemplar (unfinalized state of *Prol.*).[43]

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[38] For the cost of a copy of a text in the times of Arethas (end 9th century) see Follieri (1973–1974).
[39] See *POO* II, XXXIV–XXXVII, and the *stemma* ibid., LIII.
[40] For a first orientation on this exegetic format, best exemplified by Simplicius' introduction to his *in Cat.*, 8.9–20.12 Kalbfleish, see Hadot (1990, 21–47 and 138–160), and the synthesis in Hoffmann (2006). A treatise was presented by clarifying in succession its goal, usefulness, position in the canonical sequence of readings, title, authenticity, division into "chapters," and assignment to a specific branch of Aristotelian philosophy.
[41] Eudoxus and Archimedes: οἱ θεῖοι ἄνδρες ἐκεῖνοι; Ptolemy: τοῦ θείου Πτολεμαίου and τοῦ μεγάλου Πτολεμαίου. We may add the epithet of Syrianus as τοῦ μεγάλου φιλοσόφου and that of Theon as τοῦ φιλοσόφου.
[42] A further indication of this is that these scholia are seldom accompanied by signs directing to the portion of text they are intended to clarify. As a consequence, only the circle of scholars who made a unitary collection of these annotations could profitably use a manuscript containing them. Accordingly, the scholia were eliminated from all apographs of **B**, while being copied selectively, surely on the initiative of scholars well acquainted with *Alm.*, in **D**, a manuscript belonging to another branch of the tradition.
[43] But the scholia to *Prol.* were composed within well-informed circles (see n. 12 above), and *Prol.* itself contains a version of Zenodoros' treatise on isoperimetric figures that is different from those we read in Pappus, *Coll.* V, and in Theon, *in Alm. I.10*.

Some factual data suggest that **B** is a (partially) conformal—and therefore direct—copy of this model (unlike **C**, in which all these characters disappear): the *mise en page* on two columns, the presence of a *tabula ansata* at f. 20v, the differentiation of the layers of scholia by means of the graphic dichotomy majuscule/minuscule.[44] Overall, the impression one draws from a careful study of **B** is that its content must be taken as a well-thought-out whole: *Prol*. and the rest of the "preliminary material," recension of *Alm*., scholia to *Alm*., other treatises of Ptolemy, all belong to a unitary exegetic enterprise, performed by a skilled and well-documented (circle of) scholars. The above discussion makes it almost certain that this project was carried out to assist the teaching activities within the Neoplatonic school of Alexandria.

We may proceed a step further in this argument and, in order to achieve a more exact indication of the date of redaction of *Prol*., discuss the pieces of information concerning the astronomic inclinations of the main scholars of the Neoplatonic school of Alexandria.[45] As we have seen, the «exemplar of the philosopher» was most likely the working copy of an Alexandrian *diadochus*, and by implication of someone giving official classes. We are thus left, in order of διαδοχία, with Ammonius, Eutocius,[46] Olympiodorus and Stephanos of Alexandria.

Damascius asserts that he was taught by Ammonius «the composition of the astronomic books of Ptolemy»,[47] and a passage in Simplicius depicts Ammonius who, in the presence of Simplicius himself, observes Arcturus with an armillary sphere, in order to determine the longitude of the star and thereby confirm the constant of precession given by Ptolemy.[48] A passage from a τέχνη μαθηματική of some Stephanos (who quotes Simeon Seth and is therefore later than the mid 11th century) mentions a «table» by Ammonius which, like those of Theon and Heraclius, employed the era of Philip and the Egyptian months. It may be that the text refers to a commentary by Ammonius *in Can*.[49] Finally, John Philoponus mentions, at the very beginning of his own treatise on the subject,[50] a work on the astrolabe redacted by Ammonius—and we have every reason to think that as in many other cases, Philoponus simply took up and reformulated lecture notes taken during a course held by Ammonius.

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[44] For such arguments of "kodikologische Stemmatik," see in the first place Kresten (1969), and the considerations in Cavallo (1999). A. Stramaglia pointed out to me the importance of the *tabula ansata*.

[45] Analysis of the technical aspects in Neugebauer (1975, 1037–1051).

[46] That Eutocius was the successor of Ammonius is suggested by the fact that, according to Elias, he gave classes on Porphyry's εἰσαγωγή: εἰ μέρος ἢ ὄργανον ἡ λογικὴ φιλοσοφίας, Εὐτόκιος μὲν ζητεῖ τῆς εἰσαγωγῆς ἀρχόμενος, in Westerink (1961, 134.4–5), within a fragment of a commentary of Elias on Aristotle's *APr.*, contained in the composite manuscript Par. suppl. gr. 678, ff. 131–138 (13th century this quire).

[47] Photius, *Bibliotheca*, codex 181, 127a8–10 = Zintzen 1967, 199.5–6.

[48] See *in Cael*., 462.20–30 Heiberg.

[49] The edition (by F. Cumont) of the passage is in Kroll and Olivieri (1900, 182.12–20); a discussion is in Tihon (1976, 178–179).

[50] *De usu astrolabii*, in Jarry (2015, 3.6–13).

Eutocius shows himself well at home with *Alm*. and its commentaries;[51] most notably, he mentions, in his commentary *in Conica*, his own non-inductive treatment of the theory of compounded ratios, developed in σχόλια to *Alm*. I.[52]

Between May and August 564, Olympiodorus held classes on Paul of Alexandria's εἰσαγωγή; we read a redaction of these lecture notes in incomplete form.[53] Other passages in Olympiodorus' Aristotelian commentaries show that he had remarkable astronomic skills.[54]

Stephanos of Alexandria, if we agree to identify[55] him with a number of other homonymous scholars circulating between end of the 6th and beginning of the 7th century, wrote *ca*. 619, in Constantinople, a commentary *in Can*.[56] duplicating Theon's "Little Commentary" but adapting it to the Byzantine world.[57] He also gave classes on several Aristotelian treatises (*Int*., maybe *de An*. III, the redactions of which are extant, *Cat*., and *APr*.); he may have written a tract on arithmetical matters[58] and, again if the identification is reliable, commentaries on Hippocrates and Galen. He taught first in Alexandria and then moved to Constantinople, where he became οἰκουμενικὸς διδάσκαλος, perhaps under the emperor Heraclios (*regn*. 610–641)—whose name is also attached to Stephanos' *in Can.*—and definitely before 617, when Alexandria was seized by the Persians.

The issue of the exact scholarly circles in which *Prol*. was redacted is not settled by this quite scanty evidence, but I would surmise that in *Prol.* we read the beginning of Ammonius' lecture notes on *Alm*. If we accept this, and the reconstruction of the ancestors of **B** given above, the model of **B** must have passed from Alexandria to Constantinople between the end of the 6th century and the mid 9th century, and we cannot help thinking of Stephanos of Alexandria as the likely vector.[59] This model was available for some decades after its arrival in Constantinople (transcription of **C**).

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[51] See *AOO* III, 260.1–5 and 232.13–17.
[52] See *AGE* II, 218.6–12.
[53] Edition in Boer (1962), attribution and discussion in Westerink (1971).
[54] See for instance *in Mete.*, 19.20–20.3, 52.24–53.2, 68.20–27, 72.14–16, 188.34–189.10, 261.34–262.13 Stüve. This commentary was redacted after 565.
[55] Wolska-Conus (1989), but see *contra* Roueché (2011 and 2012).
[56] See Lempire (2011): analytic study and discussion of the attribution issues, with overview of the other writings ascribed to Stephanos; and (2016): edition of the text. Stephanos' text contains some additions (chapters 1 and 28–30), whose author is the emperor Heraclios.
[57] That is, by changing latitude (the tables were recalculated for the latitude of Byzantium, taken as the arithmetic mean of the 5th and of the 6th *klima*) and by resorting to Julian months.
[58] See Philoponus (?), *in de An. III.1*, 457.24–25 Hayduck. Some manuscripts ascribe Book III of Philoponus' commentary to Stephanus: *status quaestionis* in Giardina (2012, 475–476).
[59] See Rashed (2002, 717), for the same hypothesis applied to Marc. gr. 226.

1.3 Mathematical Background

1.3.1 The Construction of the Table of Chords

The "Table of Chords" is a double list, organized in tabular form, of numerical values associating the size of an arc of circumference (first column) with the size of the chord subtending it (second column); all these values are expressed in the sexagesimal system (whose main features are outlined in sch. **2**–**3**). The arc of circumference is the tabulation value, tabulated by increments of ½º from ½º to 180º: in the table there are 360 lines. A third column contains the coefficients of linear interpolation within each half-degree. Each coefficient is calculated by dividing the partial increments between consecutive chords in the table by 30 minutes (= ½º = the difference of consecutive arcs in the table). This division simply amounts to taking the double of such a difference; actually, Ptolemy enters the 60th part of this quotient as the tabulated value (the heading of the column is «sixtieths»). The values of the chords are provided up to the second sexagesimal order, those of the partial increments up to the third order.

The Table of Chords is derived, described, and set out[60]. The derivation takes most of *Alm*. I.10: 15½ pages in Heiberg's edition; the description takes 11 lines, including the instructions for the use of the third column; the table itself extends over 16 pages.[61] The original *mise en rouleau* was different: as Ptolemy himself explains in his description, the 360 lines were distributed over 8 consecutive sub-tables of 45 lines each. The medieval manuscripts of *Alm*. respect this layout, the 8 sub-tables extending over 4 pages.[62] Given the reduced dimensions of a page of a Teubnerian volume, Heiberg was forced to split each sub-table: since 45 is odd and because of the presence of the critical apparatus, his Table of Chords does not retain anything of the original symmetry.[63]

In order to calculate the entries in the table, Ptolemy expounds seven self-contained mathematical arguments. I now describe these arguments, each according to the configuration adopted by Ptolemy;[64] see the Appendix for the associated diagrams.

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[60] I use here the denominations introduced in Sidoli (2014, 19). The "derivation" consists in the sequence of geometric and algorithmic arguments Ptolemy expounds in order to calculate the numerical values contained in a table. The "description" and the "setting-out" together make the "representation" of the table. It is irrelevant to our purposes that Ptolemy did not calculate any of his tables using only the sequence of geometric and algorithmic arguments he expounds to this purpose: see in the first place Newton (1985); best analysis and discussion in Van Brummelen (1993 and 1994). What is important is that everyone in antiquity thought he did so; accordingly, I shall formulate my commentary as if he really did.

[61] *Alm*. I.10, *POO* I.1, 31.10–46.20, 47.2–13, *Alm*. I.11, ibid., 48–63, respectively.

[62] We find them in **A**, ff. 17r–18v; **B**, ff. 20v–22r; **C**, ff. 46r–47v; **D**, ff. 18v–20r; **G**, ff. 88r–89v.

[63] The original layout is instead preserved in Toomer's translation.

[64] The descriptions of the seven arguments are repeated in the commentary on the relevant scholia. I shall use the sign *q*(AB) to denote the square on straight line AB, *r*(AB,ΓΔ) to denote the rectangle contained by straight lines AB, ΓΔ, *ch*(ΔE) the chord subtending arc ΔE, *arch*(EZ) the arc subtended by chord EZ.

1) Calculation of specific chords (*POO* I.1, 32.10–36.8). In a semicircle on diameter AΓ having a radius ΔB perpendicular to it, take the midpoint E of radius ΔΓ and join straight line EB; cut off from EA straight line EZ equal to EB. Then ZΔ is the side of the decagon inscribed in the full circle, BZ that of the pentagon. This result allows Ptolemy to calculate the chords associated to the arcs of 36º and 72º. The chords associated to the arcs of 60º, 90º, and 120º are also given by simple geometric arguments. Since the chord subtending an arc that is complementary to a semicircle of the arc subtended by a given chord is also given by *El*. I.47, one also gets the chords associated to the arcs of 144º (the complementary in the said sense of 36º) and 108º (complementary of 72º). See sch. **4–15** and Fig. 1.
2) The so-called "Ptolemy's Theorem" (ibid., 36.9–37.18). A quadrilateral ABΓΔ is inscribed in a circle of center E; diagonals AΓ, BΔ are joined. Then one proves that the rectangle contained by the diagonals is equal to the sum of the rectangles contained by the two pairs of opposite sides: *r*(AΓ,BΔ) = *r*(AB,ΓΔ) + *r*(AΔ,BΓ). This result provides the crucial step in the subsequent theorems 3 and 5. See sch. **16–18** and Fig. 2.
3) Theorem "by difference" (ibid., 37.19–39.3). It is formulated in the "language of the givens" (Acerbi 2011b). In a semicircle on diameter AΔ, if two chords AB, AΓ having a common endpoint A are given, the chord BΓ subtending the difference of the arcs subtended by the given chords is also given. This allows Ptolemy to perform the calculation of *ch*(72º – 60º) = *ch*(12º). See sch. **19–20** and Fig. 3.
4) Theorem "by bisection" and its consequences (ibid., 39.4–41.3). The theorem is partly formulated in the "language of the givens." If chord BΓ is given in a semicircle on diameter AΓ and arc BΓ is bisected at Δ, chord ΔΓ subtending half of arc BΓ is also given. By successive bisections from *ch*(12º), this allows Ptolemy to calculate both *ch*($1\frac{1}{2}$º) and *ch*($\frac{3}{4}$º). See sch. **21–25** and Fig. 4.
5) Theorem "by composition" and its consequences (ibid., 41.4–42.17). The theorem is formulated in the "language of the givens." In a circle with diameter AΔ and center Z, if two chords AB, BΓ having a common endpoint are given, the chord AΓ subtending the sum of the arcs subtended by the given chords is also given. This allows Ptolemy to calculate chords of arcs which are a multiple of $1\frac{1}{2}$º. See sch. **26–30** and Fig. 5.
6) Approximation Lemma (ibid., 42.18–45.8). In order to complete the table, one needs *ch*($\frac{1}{2}$º). Now, since trisection of an angle cannot be done by geometric methods, one must resort to an approximation. If in a circle ABΓΔ two unequal chords BΓ > BA having a common endpoint are drawn, chord ΓB to chord BA has a lesser ratio than arc BΓ to arc BA, that is, ΓB:BA < *arc*BΓ:*arc*BA. See sch. **31–37** and Fig. 6.
7) A double application of the previous lemma provides the estimate (ibid., 45.9–46.20) ($\frac{2}{3}$)*ch*($1\frac{1}{2}$º) < *ch*(1º) < ($\frac{4}{3}$)*ch*($\frac{3}{4}$º). Since *ch*($1\frac{1}{2}$º) = 1;34,15 and *ch*($\frac{3}{4}$º) = 0;47,8, multiplying by the coefficients and truncating to second sixtieths gives 1;2,50 both as a

lower and as an upper bound of *ch*(1º);[65] this entails that *ch*(1º) = 1;2,50 up to second sixtieths. By means either of theorem 3 or of theorem 4, one also gets *ch*(½º). See sch. **38–44**. The Table of Chords can in this way be completed. See sch. **45–46** and **47–53**, and Fig. 7.

1.3.2 The Sector Theorem

The most celebrated result of Greek spherical trigonometry is the Sector Theorem, known also as "Menelaos' Theorem" because of its being contained in *Sph*. (proposition III.1 in Abū Naṣr's redaction). It is a powerful mathematical tool, devised to determine arcs of a great circle on the surface of a sphere. It is the keystone of some of the most important technical results of *Alm*., where it is applied seventeen times. It comes as no surprise, then, that the Sector Theorem is also proved in *Alm*. I.13 and, with many more cases on offer, in Theon, *in Alm. I.13*, *iA*, 535.10–570.12.[66]

The Sector Theorem is proved by Ptolemy last of a series of seven propositions.

1) First rectilinear lemma, "by composition" (*POO* I.1, 68.23–69.20). From the endpoints B, Γ of two mutually intersecting straight lines AB, AΓ, two lines BE, ΓΔ are drawn across, meeting at Z and intersecting straight lines AΓ, AB at points E, Δ, respectively. This I shall call the "base" configuration. It is required to show that ΓA:AE = (ΓΔ:ΔZ)∘(ZB:BE).[67] The proof draws a suitable parallel to one of the straight lines and readily argues by similar triangles and substitutions in compounded ratios. See sch. **55** and **56–58** and Fig. 8.
2) Second rectilinear lemma, "by separation" (ibid., 69.21–70.16). In the same configuration as the first rectilinear lemma, one also has that ΓE:EA = (ΓZ:ZΔ)∘(ΔB:BA). See sch. **59–62** and Fig. 9.
3) First cyclic lemma (ibid., 70.17–71.13). In a circle ABΓ of center Δ, mark two consecutive arcs AB, BΓ, any of which is less than a semicircle, join ΔB and AEΓ intersecting at E, draw from A, Γ perpendiculars AZ, ΓH to radius ΔB. Then *ch*(2AB):*ch*(2BΓ)::AE:EΓ. See sch. **63–65** and Fig. 10.

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[65] Using the assumed values of *ch*(1½º) and *ch*(¾º) as if they were exact, the lower bound is exact; the non-truncated upper bound is 1;2,50,40.

[66] See Neugebauer (1975, 26–30) for a clear exposition of the mathematics involved; Krause (1936) for Abū Naṣr's redaction of an Arabic translation of Menelaos' *Sphaerica*; Björnbo (1902) and Sidoli (2006) for discussions of the issue of authenticity.

[67] The sign ∘ stands for "composition" of ratios: see Acerbi (2018a) on this notion. The two compounding ratios, in fact, are not "multiplied:" what is multiplied, *iuxta El*. VI.def.5, are the πηλικότητες «[numerical] values» of the two ratios, namely, the fractions corresponding to them (see further below). For simplicity's sake, the sign will sometimes be omitted in the commentary on the scholia. One must also note that even the sign "=" is misleading: a ratio is said to be «compounded» of two or more ratios, it is never said to be "equal to" or "the same as" something like their "composition." There is no operation of "composition" of ratios; on the other hand, the "inverse" operation of «removal» does exist, as we shall see at the end of this section.

4) Second cyclic lemma (ibid., 71.14–72.10). The theorem is formulated in the "language of the givens." Partly adopting the configuration of the first cyclic lemma, from center Δ draw a straight line ΔZ perpendicular to AEΓ. It is required to show that, once arc AΓ and ratio *ch*(2AB):*ch*(2BΓ) are given, each of arcs AB, BΓ is also given. See sch. **66–73** and Fig. 11.
5) Third cyclic lemma (ibid., 72.11–73.10). In a circle ABΓ of center Δ, mark two consecutive arcs AB, BΓ, any of which is less than a semicircle, join ΔA and ΓB intersecting at E once produced, draw from B, Γ perpendiculars BZ, ΓH to radius ΔA, possibly produced. Then *ch*(2ΓA):*ch*(2AB)::ΓE:BE. See sch. **74–75** and Fig. 12.
6) Fourth cyclic lemma (ibid., 73.11–74.8). It is formulated in the "language of the givens." Partly adopting the configuration of the third cyclic lemma, from center Δ join BΔ and draw ΔZ perpendicular to EBΓ. It is required to show that, if arc ΓB and ratio *ch*(2ΓA):*ch*(2AB) are given, arc AB is also given. See sch. **76–81** and Fig. 13.
7) The Sector Theorem (ibid., 74.9–76.9). From the endpoints B, Γ of two mutually intersecting arcs AB, AΓ of great circles on the surface of a sphere, two arcs BE, ΓΔ are drawn across, meeting at Z and intersecting arcs AΓ, AB at E, Δ, respectively; all these arcs must be less than a semicircle. Then

   $$ch(2\Gamma E):ch(2EA) = [ch(2\Gamma Z):ch(2Z\Delta)]\circ[ch(2\Delta B):ch(2BA)] \text{ ("by separation")},$$

   $$ch(2\Gamma A):ch(2AE) = [ch(2\Gamma\Delta):ch(2\Delta Z)]\circ[ch(2ZB):ch(2BE)] \text{ ("by composition")}.$$

   The proof introduces the configuration of a suitable rectilinear lemma (1 or 2 above): from the center H of the sphere, radii HB, HZ, HE are joined; HB is produced to meet AΔ produced at Θ; ΓΔ, ΓA are joined and they meet HZ, HE at K, Λ, respectively; one shows that points Θ, K, Λ are on one and the same straight line. Applying the preceding lemmas to the rectilinear configuration in which from the endpoints Θ, Γ of two mutually intersecting straight lines AΘ, AΓ, two lines ΘΛ, ΓΔ are drawn across, meeting at K and intersecting straight lines AΓ, AΘ at Λ, Δ, respectively, one readily obtains the result. There is, however, a case of the theorem "by composition" that cannot be covered by this proof; we shall take up the issue in the commentary on sch. **77** and **87**. Contrary to Ptolemy, both Theon[68] and *Sph*. III.1 derive the theorem "by composition" from that "by separation," by using the obvious fact that the same chord subtends the arc double of a given arc and the arc double of its complement to a semicircle. Sch. **82–91** refer to the configuration "by separation;" see Fig. 14.

[68] At *in Alm. I.13*, *iA*, 567.1–570.12.

The main technical ingredients of the Sector Theorem can be summarized as follows.

a) the relevant chords are those subtending the double of the associated arcs; this is the reason why the Sector Theorem can only be applied to arcs on the surface of a sphere that are less than a semicircle;
b) six chords are involved, distributed among three ratios; as we have seen, one of the ratios is said to be «compounded» of the other two;
c) the "path" in the compounded ratio from the first endpoint of the antecedent to the second endpoint of the consequent through the common endpoint almost always univocally determines the endpoints of the arcs in the compounding ratios,[69] as well as their order;
d) the relationship between an arc and the chord subtending it is provided by the Table of Chords.

The Sector Theorem is immediately applied, in *Alm*. I.14 (see sch. **92**–**104** and Fig. 15), in order to find the numerical values to be inserted in the Table of Declination I.15 (sch. **105**–**107**). The assumed geometric configuration is as follows. In a representation of the celestial sphere whose circular outline ΑΒΖΓΔ is the circle through pole Ζ of the equator and that of the ecliptic, arc ΑΕΓ is the equator, arc ΒΕΔ the ecliptic (and hence Β and Δ are the winter and summer solstices, respectively, and Ε is the spring equinox). Arc ΖΗΘ is drawn across from Ζ, meeting the ecliptic at Η and the equator at Θ. It is required to find ΗΘ (the declination of point Η of the ecliptic with respect to the equator)[70] given ΕΗ (the distance along the ecliptic of point Η from the spring equinox).

In any application of the Sector Theorem, five of the six arcs are numerically given, and one must find the sixth. The procedure, never worked out in detail by Ptolemy, amounts to using the Table of Chords to derive arcs from chords and chords from arcs, and to performing the operation of ἀφαίρεσις «removal» of a ratio.

Since Ptolemy never explains how to remove a ratio from a ratio, taking it as a matter of course (cf. *Alm*. I.1, *POO* I.1, 8.8–9), the gap was filled by the commentators on *Alm*. and by all subsequent generations of scholiasts.[71] This scholarly material is of two kinds: either general expositions trying to unify under a unitary treatment the several cases of the operation,[72] or applications to a specific configuration of the Sector Theorem.[73] The

[69] A trick to identify the endpoints of the compounding ratios is explained by Theon, *in Alm. I.13*, *iA*, 539.17–25; we shall take up the issue in the commentary on sch. **55**.
[70] I translate the term λόξωσις with «declination» when it designates the arc between a specific point on the ecliptic and its projection, along the meridian through the point, on the celestial equator; I translate λόξωσις with «inclination» when it designates the angle between the planes containing the ecliptic and the celestial equator. The only exception will be in sch. **104**, in order to preserve a linguistic feature of the original.
[71] See Acerbi and Pérez Martín (2015) for the scholia of Manuel Bryennios on the subject.
[72] See Acerbi (2018a) for a complete survey of ancient Greek and Byzantine evidence.
[73] See Theon, *in Alm.*, *iA*, 575.8–578.5 (*Alm*. I.14), 578.17–579.10 (*Alm*. I.14), 591.5–594.7 (*Alm*. I.16), 595.18–596.4 (*Alm*. I.16), 619.14–620.7 (*Alm*. II.2), 622.5–623.7 (*Alm*. II.3), 624.3–13 (*Alm*. II.3).

unitary treatment we shall find in one of the scholia (sch. **94**) can be summarized as follows.

A ratio is said to συγκεῖσθαι «be compounded» of two ratios when the πηλικότητες «[numerical] values» of the compounding ratios (that is, the fractions associated to them), if multiplied to each other, give the numerical value of the compounded ratio (*El.* VI.def.5, surely spurious). As a matter of fact, no one before Byzantine times applied the definition: composition and removal was always performed on ratios. Moreover, it was tacitly assumed that the "normal form" of a compounded ratio has the two compounding ratios sharing a common term (the verb συντιθέναι «to compound» is used both in the active and in the passive voice). Accordingly, the operation of «removal» is always performed on ratios that are explicitly provided in compounded form. Therefore, if from $a{:}b = (a{:}d)\circ(d{:}b)$ we want to remove ratio $d{:}b$, it is enough to literally ἀφαιρεῖν «remove» it from the right-hand side; ratio $a{:}d$ is, in a most concrete sense, the «remainder».

The problem is that, in the applications, a given compounded ratio is never presented in its "normal form." Two ratios are instead assigned, one of which must be removed from the other: as a consequence, the ratio from which the removal is going to occur must preliminarily be written in a suitable compounded form. This is done by operating a «fitting» (the related verb is ἐναρμόζειν). A ratio is «fitted» to a second ratio by producing a ratio identical to the first and having the πρόλογος «antecedent» or the ὑπόλογος «consequent» equal to the antecedent or to the consequent of the second; one has only to calculate the remaining term. For instance, let it be required to remove 4:3 from 12:6. Now, «to fit» 4:3 to 12:6 amounts to find a ratio identical to 4:3 with the following constraints:

a. either 12 is the new antecedent; thus one must calculate the new consequent by means of proportion $4{:}3{::}12{:}x$, yielding as a result[74] $x = (12\times3)/4 = 9$;
b. or 6 is the new consequent; thus one must calculate the new antecedent by means of proportion $4{:}3{::}x{:}6$, yielding as a result $x = (4\times6)/3 = 8$.

In case *a*, 4:3 «fitted» to 12:6 gives 12:9, which is a ratio identical to 4:3 but whose antecedent is the same as that of 12:6. In case *b*, instead, the ratio obtained by «fitting» 4:3 to 12:6 is 8:6.

Conversely, «to fit» 12:6 to 4:3 amounts to find a ratio identical to 12:6 with the following constraints:

c. either 4 is the new antecedent; thus one must calculate the new consequent by means of proportion $12{:}6{::}4{:}x$, yielding as a result $x = (6\times4)/12 = 2$;

[74] Of course, this is nothing but a taking of a fourth proportional. Contrary to what is suggested by the qualifier «fourth», the position in the proportion of the number to be determined is immaterial.

d. or 3 is the new consequent; thus one must calculate the new antecedent by means of proportion 12:6::*x*:3, yielding as a result $x = (12\times3)/6 = 6$.

In case *c*, 12:6 «fitted» to 4:3 gives 4:2; in case *d*, it gives 6:3.

If we want to remove 4:3 from 12:6, let us choose for instance procedure *a*: «fit» 4:3 to 12:6 to give 12:9; insert the middle term 9 between the terms of ratio 12:6 in order to write it as the compounded ratio (12:9)∘(9:6);[75] «remove» 12:9 and the «remainder» is 9:6. Since the ratio has been removed that contains the antecedent of ratio 12:6, the operation of removal is said to be performed «with respect to the antecedent»; otherwise, it is said to be performed «with respect to the consequent». Procedures *a* and *c* can only give rise to the first type of removal, *b* and *d* to the second.

Finally, it always happens that one is interested in calculating one of the terms of the «remainder», the other being given:[76] if the given term does not match the homologous term of the remainder calculated by any of the above procedures, a further taking of a fourth proportional must be performed. Suppose, as above, that we want to remove 4:3 from 12:6 in order to calculate the antecedent of the remainder, its consequent being given as 8. The operation of removal according to procedure *a* yields 9:6, that must be reduced to a ratio with consequent 8. This is done by setting out the proportion 9:6::*x*:8, yielding as a result $x = (9\times8)/6 = 12$.

The distribution of the scholia among the several arguments just listed is summarized in the following table.

Table of Chords		Sector Theorem	
introductory	**1**	introductory	**54–55**
sexagesimal system	**2–3**	first rectilinear lemma	**56–58**
specific chords	**4–15**	second rectilinear lemma	**59–62**
Ptolemy’s Theorem	**16–18**	first cyclic lemma	**63–65**
theorem “by difference”	**19–20**	second cyclic lemma	**66–73**
theorem “by bisection”	**21–25**	third cyclic lemma	**74–75**
theorem “by composition”	**26–30**	fourth cyclic lemma	**76–81**
approximation lemma	**31–37**	Sector Theorem	**82–91**
estimate of *ch*(1°)	**38–44**	specific declinations	**92–104**
Table of Chords	**45–53**	Table of Declination	**105–107**

[75] And in fact, resorting to πηλικότητες *iuxta El.* VI.def.5 and inserting number 9, we get $^{12}\!/_{6} = (^{12}\!/_{9})(^{9}\!/_{6})$, where we are entitled to write the πηλικότητες of the ratios as fractions.

[76] This is the reason why a «fitting» has four cases and not two (*a*, *b* or *c*, *d*): the position of the term to be calculated in the remainder can make it necessary to «fit» either of the assigned ratios to the other.

1.4 General Characteristics of the Edited Scholia

The main characteristics of the edited scholia can be described as follows.

1) 37 scholia out of 107 are taken more or less verbatim from Theon's commentary. The variant readings discussed by Mogenet (1975) are not found in our collection; it is also impossible to determine to which of the two ancient branches of the textual tradition of Theon *in Alm*. the scholiasts had access. The most interesting scholia are not excerpts from Theon, their content being sometimes at variance with his commentary.
2) **B** appears to reproduce the model more faithfully than **C**, as regards both the correctness of the transcription and the location of the scholia. There are just a handful of *signes de renvoi* in **B**, and none in **C**: therefore, the model was written on two columns, and that the scholia were not systematically keyed to the main text.
3) At least two, and maybe three, layers of annotations were present in the model of **B** and **C**. 15 scholia are in fact in majuscule in **B** and/or **C**; among those in minuscule, there are some that could not possibly have been written by the same reader (contrast for instance sch. **2** with **3**, **4** with **5**, **11** with **29**, **28** with **29**, and **33** with **34**).
4) Among the 107 annotations, there are 55 comment scholia, 48 *paragraphai*, 1 schematic scholium, 3 tabular scholia. 12 *paragraphai* are concise "bookish" references to a proposition (number, book, treatise in the case of *El*.); 9 comment scholia simply provide denominations of the theorems of *Alm*. they are related to; 5 scholia set out exclusively numerical values.
5) Four scholia are passages that other branches of the tradition of *Alm*. have in the text.

All these data are summarized in the following table.

comment scholia	**1**, **2**, **3**, **4**, **11**, **15**, **16**, **19**, **21**, **22**, **26**, **28**, **29**, **30**, **31**, **33**, **39**, **40**, **41**, **42**, **43**, **44**, **46**, **47**, **49**, **50**, **52**, **53**, **55**, **56**, **59**, **62**, **63**, **66**, **67**, **69**, **72**, **76**, **77**, **78**, **80**, **82**, **83**, **84**, **86**, **87**, **94**, **97**, **98**, **100**, **101**, **104**, **105**, **106**, **107**
paragraphai	**5***, **6***, **7***, **8***, **9***, **10**, **12**, **13**, **14***, **17**, **18**, **20**, **23**, **24**, **25**, **27**, **32**, **34**, **35**, **36**, **37***, **38**, **45**, **51**, **54**, **57**, **58**, **60**, **61**, **64**, **68**, **70**, **71**, **73**, **74**, **75**, **79**, **81**, **85**, **88**, **89**, **90**, **91**, **92**, **93**, **95**, **96**, **103**
schematic scholia	**65**
tabular scholia	**48**, **99**, **102**
bookish	**6**, **7**, **8**, **9**, **14**, **20**, **37**, **38**, **88**, **89**, **90**, **91**
denominations	**16**, **19**, **21**, **26**, **31**, **56**, **59**, **63**, **82**
numbers	**39**, **41**, **42**, **43**, **44**
from Theon's commentary	**10**, (**11**), (**12**), **13**, **15**, **25**, **28**, **32**, **33**, **34**, **35**, **36**, **42**, (**47**), **49**, **50**, **52**, **53**, **57**, **58**, **61**, **64**, **67**, **68**, **69**, **72**, **74**, (**75**), **78**, (**80**), **92**, **96**, **100**, **101**, (**104**), **106**, **107**
most interesting	**2**, **3**, **40**, **47**, **55**, **77**, **83**, **84**, **87**, **94**, **98**
in majuscule	**12**, **16**, **31**, **38**, **45**, **56**, **59**, **63**, **82**, **88**, **89**, **90**, **91**, **93**
main text	**12**, **17**, **85**, **95**

In the table of the preceding page, the non-verbatim excerpts from Theon's commentary are within parentheses. The references to *El.* are marked by an asterisk, those to previous results of *Alm.* are underlined. The upper half of the table realizes a partition of the scholia, the lower half does not; the two halves are kept distinct by a double separating line.

For the convenience of the reader, I give here a short description of the most interesting scholia; see the associated commentaries for further details.

- Sch. **2** summarizes the main features of multiplication and division between sexagesimal orders treated as Diophantine numerical species, that is, as indeterminate powers: *i*) degrees, when multiplied by any species, give the same species; *ii*) multiplying two species amounts to adding the numbers associated to their denominations (*first*, *second*, … sixtieths); *iii*) definition of division between species as the inverse operation of multiplication; *iv*) dividing one species by another results in a species whose denomination is the difference of the original denominations. The scholiast also adds a remark to the effect that greater (that is, with smaller denomination) species cannot be divided by lesser species.
- Sch. **3** explains that, if numbers are multiplied, the result is greater than any of the factors; if parts are multiplied, it is lesser than any of them. Examples are 2 times 3 making 6 and $\frac{1}{2}$ times $\frac{1}{2}$ making $\frac{1}{4}$. The scholium also contains the characterization of the degree as the identity element in the multiplication of species, the sum-of-denominations rule for multiplication, and the definition of division between species.
- Sch. **40** repeatedly applies and quotes *El.* V.8. The goal is to provide deductive steps omitted in, and to outline the rationale behind, Ptolemy's use of inequalities and lower/upper bounds in the calculation of $ch(1^{\circ})$. The scholiast implicitly charges Ptolemy with a mistake in his dealing with the inequalities, and holds that the mistake has to be corrected by adopting more accurate numerical values of one of the chords at issue: $1;2,50 < ch(1^{\circ}) < 1;2,50,12$.
- Sch. **47** explains how to interpolate between the tabulated numerical values of arcs/chords. This is a basic procedure of linear interpolation (set out in tabular form in sch. **48**) that resorts to the numbers listed in the third column of the Table of Chords, namely, the coefficients of linear interpolation within each half-degree: in order to find the sought chord (arc), it is enough to multiply (divide) by this coefficient the difference of the assigned arcs (chords) and add it to the lesser assigned chord (arc).
- Sch. **55** provides an *a priori* reckoning of the number of different configurations of the rectilinear lemmas pertaining to the Sector Theorem, and of the ways they can be proved. The parameters taken into account by the scholiast are the following: First, the number of ratios associated to each punctuated straight line in the "base" configuration of each lemma (that is, without the parallels introduced in the auxiliary construction): four ratios including trivial inversions in the case "by composition," two ratios in the case "by separation;" since four punctuated straight lines are involved, one gets sixteen and eight configurations, respectively. Second, the number of

parallels to straight lines of the base configuration that can be drawn in the auxiliary construction of each lemma; there are four of them in any instance, as the scholiast shows: two points are available for drawing parallels, two parallels can be drawn at each of these point.

- Sch. **77** shows that the result of the fourth cyclic lemma is also valid when ΑΔ and ΒΓ are parallel; this fact will prove crucial in the proof of the "parallel" configuration of the Sector Theorem.
- Sch. **83** points out the main differences between the configurations "by separation" and "by composition" of the Sector Theorem. The scholiast claims that the straight lines involved in the ratios that feature in corresponding rectilinear and spherical configurations hold the same position only in the configuration "by separation."
- Sch. **84** describes in the most general terms the construction of the rectilinear configuration associated to a generic case of the Sector Theorem.
- Sch. **87** characterizes the "parallel" configuration as a limiting case of the configuration actually assumed by Ptolemy. The scholiast also summarizes a correct proof in that case, not treated by Ptolemy and declared ἀσύστατος «unsolvable» by Theon. Sch. **77** and **87** constitute the first direct evidence that a proof of the "parallel" configuration of the Sector Theorem was elaborated in Greek (see Acerbi 2015).
- Sch. **94** provides general rules for performing the operation of removal of ratios. This is the Ur-text of the short treatise on the same subject ascribed to Domninus of Larissa (see Acerbi and Riedlberger 2014).
- Sch. **98** works out the first removal of ratios used by Ptolemy. It does so by describing a tabular arrangement allowing one to perform this operation in an orderly way. The tabular arrangement coincides with sch. **99** and amounts to a modification of the standard X-shaped scheme of calculation of a fourth proportional. The gist of the description in sch. **98** resides in the general indications about the places to be assigned, in the tabular set-up, to the relevant terms of the given ratios.

The main features of the exegetic work of our scholiast(s) can be summarized thus:

- Contrary to what happens with the scholiastic apparatus accompanying *El*. in some manuscripts (as for instance in Par. gr. 2344) and with the *Alm*. scholia in **D**, the "bookish" *paragraphai* are quite infrequent: 7 references to *El*., 5 of which are related to the calculation of specific chords; 6 to previous results of *Alm*., 4 of which are attached to the proof of the Sector Theorem. Note the absence of "bookish" *paragraphai* in the frequent references to *Data*: for instance, the enunciation of *Data* 7 in sch. **72** is entirely quoted.
- The oldest layer of annotations present in the model of **B** and **C** (15 scholia, as seen above) only provides short annotations, mainly consisting in *paragraphai* and denominations of theorems.

- As for the more recent scholia, most of them supply single deductive steps or short deductions. The longest annotations provide summaries of the deductive structure and expansions on specific points: computational issues such as the main features of the sexagesimal system and the operations of interpolation and of removal of ratios; deductive issues such as the complex case-structure of the Sector Theorem. Of particular interest is the scholiasts' insistence on the numerical values of the lower and upper bound in the approximation of *ch*(1°).
- The only identifiable source is Theon's commentary. The extent of these excerpts is quite variable: sometimes they are very short, even a single clause; sometimes they are modifications to *paragraphai* of more articulated references (as in the case of some of those to *El.*); sometimes they are centos of clauses, possibly permuted. There is just one long verbatim extract, namely, sch. **107**. Still, one must insist that the most interesting scholia are not excerpts from Theon, their content being sometimes at variance with his commentary.
- The style of the longest annotations is quite formal, but this does not mean that a lost source (such as for instance Pappus' *in Alm.*) is to be postulated for them.

As for the scholia in the apographs of **BC**, the distribution of those in collection **K** (= Vat. gr. 184, ff. 27v–33r), which is as said above a direct transcription from **B**, is set out in the following table.[77]

27v	28r	28v	29r	29v	30r
* **2**, **3**	**3** * **11** * * **22**, **28** *	(*), **29** * * * **40**	**40** * **47**, **50** *	(*) **52** * * * * *	* * * * * **55** *

30v	31r	31v	32r	32v	33r
(*) **57** * * * **69**, **66**, **67** * * *	(*) * **72** *	(*) * **77**, **78**, **84**	**84**, **87**, **86**, **83**, **94**	**94**, **98**, **106**, **107**	**107**

These are 57 scholia, of which 26 originally were in the first hand of **B**.

I also provide some details on the scholia found in Vat. gr. 184, ff. 86r–87v (*Alm*. I.10) and 90v–93r (*Alm*. I.13–15), and written by the main copyists (= **G**). Those coinciding with scholia in **BC**, and in fact directly transcribed from **C** as we shall presently see, are distributed as follows.

86r	86v	87r	87v	90v	91r
6, **7**, **8**, **13**, **11**	**19**	**29–32–33**	no scholia	**55**, **63–62**, **59**, **60**	**72**, **66–67**, **69**, **78–80–81**, **65**

91v	91v–92r	92r	92r–v	92v	93r
86, **83–84**	**94**	**99**	**98**	**102**, **104**	**105**, **106**, **107**

[77] An asterisk * stands for a scholium in the 12th-century layer of annotations of **B**. Repeated numerical values indicate that the referred to scholium extends over both pages; when this occurs to scholia in the 12th-century layer, a bracketed asterisk (*) is added.

The scholia connected by dashes in the previous table follow each other as if they were a continuous text. Sch. **19** is followed by a further explanation, probably a gloss to the original scholium. Sch. **94** is incomplete (des. line 27 ἀφαίρεσις). In sch. **99**, the denominations of the three ratios involved are separated from the tabular set-up, which in its turn is disintegrated by the copyist. In sch. **69** and **80** (both excerpts from Theon), **G** has the same text as **BC**; this shows that the scholia coinciding with extracts from Theon's commentary were not drawn directly from the original work. Excerpts from Theon's commentary in **G** not reproduced as scholia in **BC** are at ff. 87r, 91r (*bis*), 91v, 92v; they come from *in Alm. I.10*, *iA*, 486.15–17, *in Alm. I.13*, *iA*, 558.3–16 and 557.27–558.3,[78] *in Alm. I.14*, *iA*, 571.12–14 (heavily modified, and a subset of the subsequent excerpt) and 571.12–572.11, respectively. Scholia in a first hand of **G** that do not coincide with any of those in **BC**, nor with passages in Theon's commentary, are at ff. 86v (4 items), 87r (1), 90v (7), 91r (1), 91v (2 + 3 *paragraphai*), 92v (1).

A look at the apparatus shows that **G**, or a model of it, is a copy of **C** as far as the scholia are concerned;[79] unquestionably conjunctive variant readings are for instance those in sch. **8** (a *paragraphê* missing in **B**), **32**, **55** (missing ἔχομεν at line 19), **65** (omission of a key clause), **69** (aberrant denotation IA), **72** (omission of two articles), **84** (one denotation by letters and a couple of related wrong terminations), **94** (if we take into account an omission in **G** fully justified by the layout of **C**, **G** ends exactly where **C** passes from the lower to the upper margin: see line 27 *app*.), **99** (repetition of one denomination, in such a way as to include a seeming variant reading of **C**), **102** (several numbers are missing), **107** (meaningless variant for numeral 90). Since **G** is not an apograph of **C** as far as *Alm*. is concerned, this means that the copyist of **G**, or an antigraph of it, changed its model when he came to transcribe the scholia to Ptolemy's treatise. The presence in **G** of scholia that follow each other as if they were a continuous text decidedly suggests that the transcription took place, as Heiberg surmised, on a model of **G**; the copyist of **G** itself slavishly copied the marginal annotations without understanding their structure, thereby making a patchwork out of some of them.

As said above, one of the main copyists of **D** found in its model a scholiastic apparatus totally disjoint from that of **BC**: 84 annotations on *Alm*. I.10–11 and I.13–15, of which 41 are comment scholia[80] and 43 are *paragraphai* (31 of these are "bookish" quotations). A hand of the end 11th–beginning 12th century transcribed by collation from **B** some of the scholia here edited: these are sch. **94**, **98**, **107**, to be found at ff. 23v–24v of **D** (sch. **99** is also present, but written by an even later hand). This hand started its work at f. 22v; it transcribed no scholium for *Alm*. I.10–11; it also added two fairly articulated deductions

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[78] These are the general enunciation of the Sector Theorem (not provided by Ptolemy) and the clause introducing this very enunciation, respectively. In **G**, they are transcribed in this order, the one under the other in the outer margin.
[79] But note the anomaly of sch. **19**, witnessed by **G** but not by **C**.
[80] These include three extensive excerpts from Theon's commentary (see n. 17 above) and—preceded by ἐξ(ῆς) and written as if it were a part of the main text (σχο$^{\lambda}$ marg. add. m. 2) at f. 18r, lines 4–14—one scholium on the structure of the Table of Chords.

in the upper margins of ff. 22v and 23r (these were in their turn later transcribed in **C**, ff. 50v and 51r, respectively, and by the 12th-century hand of **B**, f. 25r), and two long texts on removal of ratios and on linear interpolation at ff. 24r and 24v, respectively (the latter is placed beside the Table of Chords).

1.5 Preliminaries to the Edition

For each scholium, its text, a translation, and a commentary are provided. The scholia are numbered, the sequence is ordered according to their placement in **B**.

The Greek text is edited according to modern conventions in matter of punctuation and accents; all compendia are tacitly resolved; scholia comprising one single clause will not have a full stop at the end. With the exceptions of value λʹ «$^{1}/_{30}$» and of the names of the books of *El.* and of theorems or lemmas in *Alm.*, the ordinals are written in full even if the manuscripts usually have them as numeral letters: I shall write πρῶτος instead of αʹ. Denotative letters are in majuscule; numeral letters are in minuscule and are not marked by a macron; the sexagesimal parts are followed by an appropriate number of apices. Relevant variant readings are recorded in a critical apparatus attached to the Greek text. The lines of a scholium are numbered whose text takes more than four lines and for which an apparatus is provided.

The translation is literal. Terms integrated in translation are included in brackets. The noun εὐθεῖα (lit. «straight line») is translated «chord» when it designates a chord in a circle. Possible awkwardnesses of the translation reflect a contrived syntax of the Greek text, and are explained in the commentary.

The commentary provides the following information. *a*) Exact location in the manuscripts of the scholium. *b*) Transcription and translation of the passage of *Alm.* to which the scholium refers (called "the *relatum*"), further identified by page.line(s) of the first tome of vol. I of *POO*. It is also specified whether the *relatum* coincides with the citation (if any) in **K** or not. In case it is possible to identify exactly the terms to which the scholium refers, or if the scholium is purposely (for instance, by means of a marginal sign) located beside a line of the text in **B**, the terms or the line are underlined. The translations of passages of *Alm.* are Toomer's, with modifications if necessary. *c*) Discussion of textual issues and of the mathematical context, with identification of likely sources or of similar passages in other authors. *d*) Graphic and codicological features. *e*) Lexical and syntactical remarks.

Except for point *a* of the commentary, the manuscripts are designated by the *sigla* assigned by Heiberg, namely, Vat. gr. 1594 = **B**, Marc. gr. 313 = **C**, Vat. gr. 180 = **D**. I shall instead distinguish between Vat. gr. 184, f. 81r sqq. = **G**, and Vat. gr. 184, ff. 25r–80v = **K**; the last three witnesses are strictly necessary only when **B** proves difficult to read because of its faded ink, nevertheless all their variant readings are recorded. These codices have been collated on color 300 dpi digital reproductions. Occasional references are made to Par. gr. 2389 = **A**. The variants with respect to Theon's commentary, mainly originating from the scholiasts' efforts to make an excerptum out of a continuous text, are followed in the apparatus by the *siglum* Th.

2. Edition, Translation, and Commentary

1

Text. συντομίᾳ γὰρ καὶ σαφηνείᾳ ἀεὶ κέχρηται

Transl. For he has always made use of conciseness and clarity

Comm. *a*) Vat. gr. 1594, f. 16v marg. int., Marc. gr. 313, f. 40v marg. ext. *b*) *Ad Alm*. I.10, 31.16–19 πρότερον δείξομεν, πῶς ἂν ὡς ἔνι μάλιστα δι’ ὀλίγων καὶ τῶν αὐτῶν θεωρημάτων εὐμεθόδευτον καὶ ταχεῖαν τὴν ἐπιβολὴν τὴν πρὸς τὰς πηλικότητας αὐτῶν ποιοίμεθα «first we shall show how one can undertake the calculation of their [*scil*. of the chords] [numerical] values by a simple and rapid method, using as few theorems as possible, the same set for all». *c*) It approves of Ptolemy’s statement, by paraphrasing the underlined terms. Sch. **1**–**51** refer to *Alm*. I.10. *d*) In **B**, sch. **1** is located beside the *relatum*; in **C** it is placed two lines before it (but the *relatum* lies in the subsequent page). As often happens, this short annotation is nicely shaped in **B**: it extends over 7 lines (corresponding to 4 lines of the main text), has them centered and showing progressively reduced length; this entails that the last three lines contain 2, 3, 2 characters [that is, κέ|χρη|τ(αι)], respectively. The scholium is further enriched by an ornamental motif, placed just below the last two signs. *e*) Theon calls Ptolemy φιλοσύντομος at *in Alm. I.10*, *iA*, 487.16, and *in Alm. III.1*, *iA*, 834.14. The substantive σαφήνεια is a key term in the ancient exegetic lexicon: see for instance Manetti and Roselli (1994, 1558–1559) and Decorps-Foulquier (1998). The expression σαφεστέρα ἑρμηνεία features in the definitions of σχόλιον, obviously stemming from a common source, contained in the three *Etymologica*; these definitions are conveniently collected in Lundon (1997, 76).

2

Text. ἡ μοῖρα, ἐφ’ ὃ ἂν εἶδος πολλαπλασιασθῇ, τὸ αὐτὸ εἶδος ποιεῖ, οἷον μοῖρα ἐπὶ μοίρας· ὁμοίως δὲ καὶ μοῖρα ἐπὶ πρῶτα ἑξηκοστὰ ποιεῖ πρῶτα ἑξηκοστά, καὶ ἐπὶ δεύτερα ποιεῖ δεύτερα, καὶ ἑξῆς ὁμοίως: — πᾶν εἶδος ἐπὶ πᾶν εἶδος πολυπλασιαζόμενον ἐκεῖνο τὸ εἶδος ποιεῖ τὸ ἐκ τῆς συνθέσεως τῶν ὀνομάτων αὐτῶν γινόμενον· οἷον πρῶτα ἐπὶ πρῶτα ποιεῖ δεύτερα, πρῶτα ἐπὶ δεύτερα ποιεῖ τρίτα, δεύτερα ἐπὶ δεύτερα τέταρτα, δεύτερα ἐπὶ τρίτα πέμπτα, τρίτα ἐπὶ τρίτα ἕκτα: — τὰ μείζονα εἴδη παρὰ ἐλάττονα μερίζεσθαι ἀδύνατον, ἐπειδὴ ὁ μερισμὸς ἔγγιον τῶν μοιρῶν φέρεται τὸ μεριζόμενον· εἰ γὰρ τὰ πρῶτα παρὰ πρῶτα μερίζομεν, ποιεῖ μοίρας· παρὰ τὰ δεύτερα μερίζεσθαι αὐτὰ πῶς δύναται; ἔλαττον πεσεῖται γὰρ καὶ τῆς μοίρας. πᾶν εἶδος παρὰ εἶδος μεριζόμενον ἐκεῖνο τὸ εἶδος ποιεῖ ὅπερ, πολλαπλασιαζόμενον μετὰ τοῦ παρ’ ὃ γίνεται ὁ μερισμός, ποιεῖ τὸ μεριζόμενον· οἷον τρίτα παρὰ δεύτερα ποιεῖ πρῶτα, ἐπειδὴ πρῶτα ἐπὶ δεύτερα ποιεῖ τρίτα. ἢ καὶ οὕτως· ἐπειδὴ τὰ μεριζόμενα μεγαλωνυμώτερά ἐστιν, ἄφελε τοῦ ὀνόματος αὐτῶν τὸ ὄνομα τοῦ παρ’ ὃν γίνεται ὁ μερισμός, καὶ εὑρήσεις τὸ ὄνομα τοῦ εἰς ὃν γίνεται ὁ μερισμός· οἷον εἰ δέοι πέμπτα παρὰ δεύτερα παραβαλεῖν, ἄφελε τὰ β ἀπὸ τῶν ε, καὶ λέγε τὰ ἐκ τοῦ μερισμοῦ γινόμενα τρίτα εἶναι.

1 μοῖρα] comp. ubique **BC** **2** post μοίρας expect. ποιεῖ μοίρας | ἑξηκοστὰ^{1}] εξῆς **C** **3–4** τὸ εἶδος ποιεῖ] ποιεῖ τὸ εἶδος **K** | ὀνομάτων] ὀνομα– comp. ὀ› **BC** : om. **K** spatio 2 litt. relicto **7** ἔγγιον τῶν μοιρῶν] ἐγγὺς μ^{o} **K** **8** πρῶτα^{2}] δʹ **K** **9** τῆς] τὰς codd. **11** πρῶτα^{1}] δʹ **K** **12** οὕτως comp. **B** : οὗ comp. **K** | μεγαλωνυμώτερά] μεγαλονυμότερα codd. | ὀνόματος] ὀνομα– comp. ὀ› **BC** : ἐλάττονος **K** **14** παραβαλεῖν legi nequit **C** : παραβαλών **K**

Transl. The degree, by whichever species be multiplied, makes the same species, like for instance a degree by degrees [makes degrees]; similarly, a degree by first sixtieths also makes first sixtieths, by seconds makes seconds, and so on similarly. — Every species, multiplied by any species, makes such species as results from the sum of their denominations; for instance, firsts by firsts make seconds, firsts by seconds make thirds, seconds by seconds, fourths, seconds by thirds, fifths, thirds by thirds, sixths. — Greater species cannot be divided by lesser [species], since the division carries the dividend closer to degrees. For if we divide firsts by firsts, they make degrees; how can they be divided by seconds? for they will fall less than a degree! Every species, divided by a species, makes such species as, multiplied with the [species] by which the division is going to occur, makes the dividend; for instance, thirds [divided] by seconds make firsts, since firsts [multiplied] by seconds make thirds. Or also as follows. Since the dividends have higher denominations, subtract from their denomination the denomination of the [number] by which the division is going to occur and you will find the denomination of the [number] upon which the division is going to occur; for instance, if there be needed to apply fifths to seconds, subtract 2 from 5 and say that the result of the division are thirds.

Comm. *a*) Vat. gr. 1594, f. 16v marg. ext., Marc. gr. 313, f. 41r marg. sup. et ext., Vat. gr. 184, f. 27v. *b*) *Ad Alm*. I.10, 32.3–5 καθόλου μέντοι χρησόμεθα ταῖς τῶν ἀριθμῶν ἐφόδοις κατὰ τὸν τῆς ἑξηκοντάδος τρόπον ‹διὰ τὸ δύσχρηστον τῶν μοριασμῶν› «in general we shall use the sexagesimal system for our arithmetical computations [because of the awkwardness of the fractional system]» (= citation in **K**, the bracketed clause excepted). *c*) The scholium summarizes, with examples, the main features of multiplication and division between sexagesimal orders treated as Diophantine numerical species, that is, as indeterminate powers: *i*) degrees, when multiplied by any species, give the same species; *ii*) multiplying two species results in a species whose «denomination» is the sum of the original «denominations» (*first*, *second* … sixtieths); *iii*) definition of division between species as the inverse operation of multiplication; *iv*) dividing one species by another results in a species whose denomination is the difference of the original denominations. The scholiast also adds *v*) a remark to the effect that greater species (that is, with smaller denomination) cannot be divided by lesser species (note that rule *iv* could not apply to this case). *Loci paralleli* in Theon are *in Alm. I.10*, *iA*, 452.14–462.17 (that refers to Diophantus' classification of numerical species), *GC* I, 109.10–111.24, *PC*, 200.9–201.4, even if the focus is on geometric justifications of the rules (but the characterization in rule *i* is identical with the clause at *iA*, 453.7, with the variant reading αὐτὸ τὸ εἶδος «the species itself»). In particular, Theon does not give definitions of the two operations. He does not address issue *v*, either, providing only examples of lesser species divided by greater species. The most complete ancient treatment of the subject is in *Prol.* (a text contained both in **B** and in **C**!), where we find definition *iii*, characterizations *i*, *ii*, and *iv*, and a lengthy discussion of issue *v*. In this discussion, the anonymous redactor of *Prol.* shows that, by means of the "analysis" of species (= reduction to the subsequent order after multiplication by 60), greater species can be divided by lesser species, even if only examples of consecutive species are adduced. Sch. **2**–**3** expound basics about multiplication and division in the sexagesimal system. *d*) In **B**, the scholium is preceded by a *signe de renvoi*, also placed in the inner margin beside the underlined pericope of the *relatum* (amounting to one line in **B**). In **C**, the scholium is in the upper and outer margins, no *signe de renvoi* being added (see sch. **4**). In the manuscripts, the three parts of the scholium are separated by the graphic marker "*dicolon* + *paragraphos* + blank space;" **BC** repeat the *paragraphos* beside the line at which each sub-scholium begins. In **K**, sch. **2** immediately precedes sch. **3**. Note that, in two instances in **BC**, the stem ὀνομα– is abbreviated with ὀ›. *e*) The unit in the sexagesimal system is usually called μοῖρα «degree»; the sexagesimal orders are examples of numerical εἴδη «species» and are called ἑξηκοστά «sixtieths» or λεπτά «minutes». The latter denomination came to be used only in Late Antiquity: in *Prol.*, there are 183 occurrences of the noun λεπτόν, 50 of ἑξηκοστόν. On the contrary, λεπτόν never appears in *Alm.* nor in Pappus, *in Alm.* V–VI. In Theon, we read twice the expression λεπτὰ ἤτοι ἑξηκοστά (*iA*, 463.6, *PC*, 200.9); otherwise, λεπτόν only features in the very short presentation of the sexagesimal system in the "Little Commentary" referred to above (5 occurrences), plus a handful of isolated occurrences in the same work: *PC*, 208.7–13 (*ter*), 243.15, 246.14. In these same writings, ἑξηκοστόν has about 1100 occurrences, of which about 270 are in *Alm.* In our corpus, the denomination λεπτά only features in sch. **47**. The several sexagesimal orders are identified by an ordinal, which is their ὄνομα «denomination». The operational terminology adopted in the scholium is also well-established. The «addition» is ἡ σύνθεσις, «to subtract» is ἀφαιρεῖν; the multiplication is ὁ πολλαπλασιασμός (graphic variant πολυ–), the associated verb πολλαπλασιάζειν; the division is ὁ μερισμός, the associated verb μερίζειν or παραβάλλειν «to apply», the latter according to the geometric interpretation of division as the application of areas. The dividend is ὁ μεριζόμενος «the [number] divided» or ὁ εἰς ὃν γίνεται ὁ μερισμός «the [number] upon which the division is going to occur»; ὁ μερίζων «the divisor» is also designed by the expression ὁ παρ' ὃν γίνεται ὁ μερισμός «the [number] by which the division is going to occur»; the neuter article is instead required for species. The prepositions denoting multiplication are ἐπί or (quite infrequently) μετά; division is identified by παρά or (quite infrequently) εἰς. In the technical corpus, the adjective μεγαλωνυμότερος «having a higher denomination» or related adverbial forms are only witnessed by Iamblichus, *in Ar.* II.11 (adv.), III.29, 53, 65, IV.117 (adv.), V.37, in Vinel 2014, 76.23, 110.18, 116.19, 118.33, 154.27, 178.12. At line 7, note, and see also sch. **3**, lines 20–21, the turn of phrase about division that φέρει «carries» the dividend ἔγγιον «closer» to degrees.

3

Text. ἐν τοῖς πολλαπλασιασμοῖς, ἐὰν μὲν μονάδα ἐπὶ μονάδας ποιήσωμεν, ὁ γινόμενος ἀριθμὸς μείζων ἐστὶ τῶν πολλαπλασιασθέντων ἀριθμῶν· οἷον ἐὰν μονάδας β ἐπὶ γ μονάδας ποιήσωμεν, γίνεται ϛ, ὃς μείζων ἐστὶ καὶ τῶν β καὶ τῶν γ, καὶ μάλα εἰκότως· ὁ γὰρ ϛ γέγονε τοῦ β κατὰ τὸ πλῆθος τῶν γ ληφθέντος (τουτέστι τοῦ β τρὶς ληφθέντος), ἢ τοῦ γ κατὰ τὸ πλῆθος τῶν β (τουτέστι τοῦ γ δὶς ληφθέντος), ὡς γίνεσθαι τοῦ γινομένου ἐκ τοῦ πολλαπλασιασμοῦ ἕνα τῶν πολλαπλασιασθέντων τοσαῦτα μέρη ὅσος ἐστὶν ὁ λοιπὸς ἀριθμὸς τῶν πολλαπλασιασθέντων· καὶ ἀεὶ εἷς τῶν πολλαπλασιασάντων καταμετρεῖ τὸν γενόμενον κατὰ τὸν λοιπόν. ἐὰν δὲ μόρια ἐπὶ μόρια ποιήσωμεν, ὁ γινόμενος ἀριθμὸς ἐλάττων ἐστὶ τῶν πολλαπλασιασθέντων· οἷον ἐὰν ∠ ἐπὶ ∠ ποιήσωμεν, τὸ γινόμενον μόριον, δʹ ὤν, ἔλαττόν ἐστι καὶ τοῦ ∠ καὶ τοῦ ἑτέρου ∠, καὶ τοῦτο εἰκότως· τὸ γὰρ δʹ γέγονε τοῦ ἑνὸς ∠ κατὰ τὸ λοιπὸν ∠ ληφθέντος (τουτέστι τοῦ ἑνὸς ∠ ἡμισάκις ληφθέντος), καὶ ἔστι τὸ ∠ τοῦ ∠ τέταρτον· ὡς γίνεται τὸ ἐκ τοῦ πολλαπλασιασμοῦ μόριον ἑνὸς τῶν πολλαπλασιασθέντων μορίων τοσοῦτον μέρος ὅσον ἐστὶ τὸ λοιπὸν τῶν πολλαπλασιασθέντων. ἐὰν δὲ μονάδες ὦσι καὶ μόρια, κατὰ δύο εἴδη γίνεται ὁ πολλαπλασιασμός, καὶ ὅρα πῶς· μειουμένη μὲν ἡ μονὰς καὶ γινομένη μόριον μονάδος ἢ μόρια ἐν τῷ πολλαπλασιασμῷ, μειοῖ τὸν συντεθέντα, αὐξηθεῖσα δὲ καὶ γενομένη αντὶ μιᾶς β ἢ γ, αὔξει αὐτό· ἐὰν δὲ μείνῃ μονάς, οὔτε αὔξει οὔτε μειοῖ· μονὰς γὰρ ἐπὶ μονάδα ‹μονάδα› ποιεῖ. τὰ δὲ αὐτὰ ἅπερ ἐπὶ μονάδος καὶ ἐπὶ μοίρας ἀξιωτέον ὡς ἀναλογούσης μονάδι, τὰ δ' ἑξηκοστὰ τοῖς μορίοις ἔοικε τῆς μονάδος. διὸ ἡ μὲν μοῖρα, ἐφ' ὃ ἂν γένηται, τὸ αὐτὸ εἶδος φυλάττει, τὰ δ' ἑξηκοστὰ ἐπὶ τὸ ἔλαττον εἶδος φέρει τὸν πολλαπλασιασμόν· πρῶτα γὰρ ἐπὶ πρῶτα ποιεῖ δεύτερα, καὶ δεύτερα ἐπὶ δεύτερα ποιεῖ τέταρτα· ἔσται γὰρ τὸ ἐκ τοῦ πολλαπλασιασμοῦ γινόμενον εἶδος παρώνυμον ἐκ τοῦ ἀριθμοῦ τοῦ γινομένου | ἐκ τῆς συνθέσεως τῶν ἀριθμῶν ἀφ' ὧν παρώνυμά ἐστι τὰ πολλαπλασιασθέντα ἐπ' ἄλληλα ἑξηκοστά. καὶ πᾶν εἶδος μεριζόμενον παρά τι εἶδος ἐκεῖνο τὸ εἶδος ποιεῖ ὅ, πολλαπλασιασθὲν ἐπὶ τὸ εἶδος τοῦ παρ' οὗ ὁ μερισμός, ποιεῖ τὸ εἶδος τοῦ μεριζομένου.

1 – 26 partim uix legitur **B** **2** μονάδας — **3** μονάδας] μ^{o} — μ^{o} **BC** : μοίρας — μοίρας **K** | ϛ, ὃς] ϛ ο ϛ **BC** : ἀριθμὸς ὁ ϛ **K** **4** γ ληφθέντος — **5** πλῆθος τῶν om. **K** | τρὶς] τρίτου **BC** **6** τοῦ] τ^{o} **K** | πολλαπλασιασμοῦ] –σάντων **K** **8** μόρια^{1} supra lineam **K** **10** ὥν] ὧν codd. | ἐστι] ἔσται comp. **BK** **11** γέγονε om. **K** **12** γίνεται codd. : expect. γίνεσθαι cfr. u. 5 **13** μέρος] μόνον codd. **15** πῶς] ὅπως **K** **16** συντεθέντα] συντιθ– **K** **17** μονὰς^{2}] μ^{o} **BC** : μοῖρα **K** | μονάδα] μ^{o} **BC** : μοίρας **K** **18** μονάδα om. codd. **19** μονάδος] μ^{o} **BC** : μοίρας **K** **23** ὧν om. **C** **25** ποιεῖ^{1} ob truncatum folium des. **C**

Transl. In multiplications, if we make a unit by units, the resulting number is greater than the numbers multiplied; for instance, if we make 2 units by 3 units, it results 6, which is greater than both 2 and 3—and quite reasonably so, for 6 came to be since 2 was taken according to the multiplicity of 3 (that is, 2 was taken thrice), or 3 according to the multiplicity of 2 (that is, 3 was taken twice), in such a way that one of the [numbers] multiplied is such parts of the [number] resulting from the multiplication as is the remaining number of those that are multiplied; and one of the [numbers] that has multiplied always measures the resulting [number] according to the remaining [number]. If, instead, we make parts by parts, the resulting number is less than the [parts] multiplied; for instance, if we make $\frac{1}{2}$ by $\frac{1}{2}$, the resulting part, being $\frac{1}{4}$, is less than both $\frac{1}{2}$ and the other $\frac{1}{2}$—and this [occurs] reasonably, for $\frac{1}{4}$ came to be since one $\frac{1}{2}$ was taken according to the remaining $\frac{1}{2}$ (that is, one $\frac{1}{2}$ was taken half a time), and $\frac{1}{2}$ of $\frac{1}{2}$ is a quarter: so that the part [resulting] from the multiplication is such part of one of the parts multiplied as is the remaining [part] of those that are multiplied. If, instead, there are units and parts, multiplication comes to be according to both species, and see how: the unit, if it is lowered in the multiplication and becomes a part or parts of a unit, lowers the [number] composed [*scil.* the product]; if, on the other hand, it has been increased and has become 2 or 3 instead of one, it increases it. If instead it remained a unit, neither increases nor lowers [the result], for a unit by a unit makes a unit. Exactly the same things, one must stress, that apply to a unit also apply to a degree, because the latter is analogous to a unit, and the sixtieths are like the parts of a unit. For this reason the unit, by whatever [species] is multiplied, keeps the same species, whereas the sixtieths carry multiplication towards a lesser species, for firsts by firsts make seconds and seconds by seconds make fourths—as a matter of fact, a species resulting from a multiplication will take its denomination from the number resulting from the sum of the numbers from which the denominations are taken of the sixtieths multiplied to each other. And every species, divided by some species, makes such species as, multiplied by the species of the [number] by which the division [is going to occur], makes the species of the dividend.

Comm. a) Vat. gr. 1594, f. 16v marg. ext. et inf., Marc. gr. 313, f. 41r marg. inf., Vat. gr. 184, ff. 27v–28r. *b*) *Ad Alm.* I.10, 32.5–6 ‹διὰ τὸ δύσχρηστον τῶν μοριασμῶν› ἔτι τε τοῖς πολυπλασιασμοῖς καὶ μερισμοῖς ἀκολουθήσομεν «[because of the awkwardness of the fractional system;] we will also carry out multiplications and divisions» (= citation in **K**). *c*) The *relata* of sch. **2** and **3** are consecutive. Sch. **3** explains that, if numbers are multiplied, the result is greater than any of the factors; if parts are multiplied, it is lesser than any of them. Examples are 2 times 3 making 6 and $\frac{1}{2}$ times $\frac{1}{2}$ making $\frac{1}{4}$. Both sch. **2** (statements *i–iii*) and **3** (the last three sentences) present the characterization of the unit (or the degree) as the identity element in the multiplication of species, the sum-of-denominations rule for multiplication, and the definition of division between species. The third statement is better formulated in sch. **3**, the other two have a clearer expression in sch. **2**. The characterizations, at lines 5–7 and 12–14 of sch. **3**, of the factors in a multiplication as parts of the product (or of the product as a part of one of the factors, if parts are multiplied) are not to be found elsewhere; they are formulated in strict parallelism. The property stated at lines 7–8 is a rewriting of a portion of the *definiens* in the definition of πολλαπλάσιος «multiple» at *El.* VII.def.5. At lines 18–21, sch. **3** carefully distinguishes between units and degrees; the distinction is blurred by the copyists' uncertainties in resolving the abbreviation M^o, almost ubiquitous in **BC** and apparently used throughout their common model. All of this suggests that sch. **2** and **3** come from independent sources. Note, however, the similar turn of phrase, at sch. **2** line 7 and at sch. **3** lines 20–21, about an operation or a species that φέρει «carries» the result towards the unit or towards lesser species (but note that the two drifts take place in opposite directions). *d*) In **B**, the scholium is preceded by a *signe de renvoi*, also placed in the inner margin beside the underlined pericope of the *relatum* (amounting to 1 line in **B**). In **C**, the scholium occupies the entire lower margin, no *signe de renvoi* being added. In **K**, sch. **3** immediately follows sch. **2**. *e*) The phrase κατὰ τὸ πλῆθος τῶν «according to the multiplicity of» at lines 4–5 has a parallel in the definition of multiplication given in *Prol.*, *DOO* II, 6.6–11. The use of the numeral adverbs δίς and τρίς again at lines 4–5 is reminiscent of the τοσαυτάκις in the definition of multiplication at *El.* VII.def.16 (*EOO* II, 186.15). At line 8, the scholium calls a part ἀριθμός «number». For γίνεσθαι ἐπί at line 19 with the meaning «to be multiplied by» (usually in the form of a participial clause) see *Prol.*, the anonymous commentator on the *Theaetetus* (col. XLII.7–8); Hero, *Metr.* I.8, in Acerbi and Vitrac (2014, 166.1); Diophantus, *Ar.* IV.38–39 and VI.1, in *DOO* I, 296.12, 302.14 and 392.13 (all with numeral adverbs), and *De polygonis numeris*, in Acerbi (2011a, 196.7); Domninus, *Ratio* 23, in Riedlberger (2013, 126.18–19). Note, at lines 22–23, the idiomatic phrase παρώνυμος εἶναι ἐκ/ἀπό «to take the denomination from», also found in *Prol.*, in *DOO* II, 8.8–15 (*ter*), 14.13 (adverb παρωνύμως), 14.19. Otherwise, but still in technical writers (Nicomachus, Iamblichus, Eutocius), the adjective παρώνυμος takes the genitive or the dative. Throughout the scholium and elsewhere in this article, the following criteria will be followed to translate the participial forms of γίνεσθαι indicating the «result» of an operation: plural neuter τὰ γινόμενα is «the result», τὸ γινόμενον «what results»; if specific substantives in the neuter have to be understood, as μέρος or μόριον in this scholium, or if the participle is in the masculine ("number" being understood), it is always rendered by a present participle and the understood substantives are supplied between brackets. The same is assumed for the aorist participle.

4

Text. ὅτι δὲ ἡ ἴση τῇ ΕΒ μεταξὺ τῶν Α Δ πίπτει δείξομεν οὕτως· ἐπὶ γὰρ τριγώνου τοῦ ΒΔΕ δύο αἱ ΒΔ ΔΕ τῆς ΕΒ μείζονές εἰσι· καὶ ἔστι ἡ ΒΔ τῇ ΔΑ ἴση· ἑκατέρα γὰρ ἐκ τοῦ κέντρου ἐστὶ τοῦ ΑΒΓ ἡμικυκλίου· αἱ ΑΔ ΔΕ ἄρα τῆς ΒΕ μείζονές εἰσι· ὥστε οὖν ἡ ἴση τῇ ΒΕ μεταξὺ τῶν Δ Α σημείων πίπτει.

Transl. And that the [straight line] equal to EB falls between A, Δ, we shall prove as follows: for in triangle BΔE the two [straight lines] BΔ, ΔE are greater than EB; and BΔ is equal to ΔA, for each of them is a radius of semicircle ABΓ; therefore AΔ, ΔE are greater than BE; so that, then, the [straight line] equal to EB falls between points Δ, A.

Comm. *a*) Vat. gr. 1594, f. 17r marg. sup., Marc. gr. 313, f. 41r marg. ext. *b*) *Ad Alm*. I.10, 32.15–16 καὶ κείσθω αὐτῇ ἴση ἡ EZ «and let EZ be made equal to it». *c*) In a semicircle on diameter AΓ having a radius ΔB perpendicular to it, take the midpoint E of radius ΔΓ and join straight line EB; cut off from EA straight line EZ equal to EB. Then ZΔ is the side of the decagon inscribed in the full circle, BZ that of the pentagon. The scholium shows that the endpoint Z of EZ falls between points A, Δ (Fig. 1). The deduction coincides with the first part of the proof of *El*. III.7 (*EOO* I, 180.6–10), this very proposition being directly cited in sch. **5**. The first step in the deduction applies *El*. I.20. Theon does not expand on this point. Sch. **4**–**15** mainly provide with *paragraphai* the long argument by means of which Ptolemy calculates the numerical values of some specific chords (sch. **11** summarizes his findings). *d*) In **B**, sch. **4** is located in the upper margin; in **C**, it is inside a partial indentation of sch. **2**, the location in the outer margin appearing to be accidental. In either manuscript, no sign is present to indicate the exact *relatum*. The placement in **BC** suggests that sch. **4** was originally transcribed in the upper or in the lower margin, near to sch. **2**, maybe closer to the *relatum* than the layout of **BC** can allow; this could explain the absence of a *signe de renvoi* in **BC**. One may also safely surmise that, in the common model of **BC**, sch. **4** was written after sch. **5** (*quod vide*). *e*) Note that the straight line at issue is correctly designated by the syntagm ἡ ἴση τῇ EB, not by its name ἡ EZ. A slight abuse of language makes the scholiast claim that a straight line falls between two points; otherwise he should have introduced the name of the straight line, ἡ EZ, or a circumlocutory phrase to designate its endpoint Z. The noun phrase ἡ ἐκ τοῦ κέντρου «radius» is without article because it is the predicate noun.

5

Text. μείζων γὰρ ἡ AE πασῶν διὰ τοῦ κέντρου οὖσα, ὡς δέδεικται ἐν τῷ ζʹ θεωρήματι τοῦ γʹ βιβλίου τῶν Εὐκλείδου

Transl. For AE is greater than all [straight lines] because it is through the center, as is shown in the 7th theorem of the 3rd book of those of Euclid.

Comm. *a*) Vat. gr. 1594, f. 17r interc., Marc. gr. 313, f. 41r marg. ext. *b*) The *relatum* is the same as sch. **4**. *c*) The scholium expands on the same point as sch. **4**; it does so by pointing out that AE (= AΔ + ΔE in the next-to-last step of sch. **4**) is *a fortiori* greater than EB because of *El*. III.7 (Fig. 1). *d*) Both in **B** and in **C**, sch. **5** is located beside the *relatum*. This shows that sch. **4** was written after sch. **5**; most likely, they belong to different layers of annotations. In **B**, the scholium is shaped as sch. **1**. *e*) Note the difference between the noun phrase ‹ἡ› ἐκ τοῦ κέντρου «radius» (see sch. **4**) and the locative adverbial qualifier διὰ τοῦ κέντρου «through the center», both in predicative position. See further sch. **13**.

6

Text. διὰ τὸ ϛʹ τοῦ βʹ στοιχείων

τὸ] τοῦ **G** | στοιχείων] –ου **G**

Transl. By the 6th of the 2nd of the Elements

Comm. *a*) Vat. gr. 1594, f. 17r marg. int., Marc. gr. 313, f. 41r marg. ext., Vat. gr. 184, f. 86r marg. ext. *b*) *Ad Alm*. I.10, 32.19–33.2 ἐπεὶ γὰρ εὐθεῖα γραμμὴ ἡ ΔΓ τέτμηται δίχα κατὰ τὸ Ε, καὶ πρόσκειταί τις αὐτῇ εὐθεῖα ἡ ΔΖ, τὸ ὑπὸ τῶν ΓΖ καὶ ΖΔ περιεχόμενον ὀρθογώνιον μετὰ τοῦ ἀπὸ τῆς ΕΔ τετραγώνου ἴσον ἐστὶν τῷ ἀπὸ τῆς ΕΖ τετραγώνῳ «in fact, since a straight line ΔΓ is bisected at E, and some straight line ΔΖ is adjacent to it, the rectangle contained by ΓΖ and ΖΔ, with the square on ΕΔ, is equal to the square on ΕΖ». *c*) The reference to the proposition of *El.* is correct. Theon, *in Alm. I.10*, *iA*, 464.10–465.2, both quotes a partially instantiated enunciation of *El*. II.6 and fully instantiates it according to the configuration adopted by Ptolemy (Fig. 1). *d*) Both in **B** and in **C**, sch. **6** is located beside the *relatum*. *e*) The construct διὰ τὸ «by the» + accusative or «because of» + infinitive is typical of the scholiastic jargon. The scholia alternate between the designations στοιχεῖα «Elements» and τῶν Εὐκλείδου «those [*scil*. books] of Euclid».

7

Text. ὡς δέδεικται ἐν τῷ κθʹ θεωρήματι τοῦ ϛʹ τῶν στοιχείων

κθʹ corr. sed legi nequit m. 2 **B** (fortasse λʹ scripsit) | post τοῦ legi nequit ob truncatum folium **C** | τῶν στοιχείων om. **G**

Transl. As is shown in the 29th theorem of the 6th of the Elements

Comm. *a*) Vat. gr. 1594, f. 17r marg. int., Marc. gr. 313, f. 41r marg. ext., Vat. gr. 184, f. 86r marg. ext. *b*) *Ad Alm*. I.10, 33.11–12 ἡ ΖΓ ἄρα ἄκρον καὶ μέσον λόγον τέτμηται κατὰ τὸ Δ «therefore [straight line] ΖΓ turns out to be cut in extreme and mean ratio at Δ». *c*) Ptolemy has just shown (Fig. 1) that $r(\Gamma Z,Z\Delta) = q(\Delta\Gamma)$. The scholiast does not refer to the definition of «straight line cut in extreme and mean ratio» at *El.* VI.def.3, as Heiberg correctly does in his edition of *Alm*., but to *El*. VI.29, which must coincide with the proposition VI.30, and which both we and the corrector read in our *El*. (This proposition is a “problem:” how to cut a given finite straight line in extreme and mean ratio.) If this is not a mistake, there is one natural candidate for the proposition of *El.* missing in the redaction that the scholiast had in his hands. It is VI.12, absent in the Arabo-Latin translation of Adelard of Bath, and in fact in the branch of the Arabic tradition stemming from the translation of al-Ḥajjāj, at least according to the pseudo-Ṭūsī, cited in Thaer (1936, 118–119). Theon does not add any references. *d*) Both in **B** and in **C**, sch. **7** is located beside the *relatum*.

8

Text. διὰ τὸ θʹ θεώρημα τοῦ ιγʹ βιβλίου τῶν Εὐκλείδου

τὸ θʹ θεώρημα] τοῦ θʹ θεωρήματος **G** | θʹ supra lineam corr. in ηʹ m. 2 **G** | τῶν] τοῦ **G**

Transl. By the 9th theorem of the 13th book of those of Euclid

Comm. *a*) Marc. gr. 313, f. 41r marg. ext., Vat. gr. 184, f. 86r marg. ext. If the scholium has not been erased in Vat. gr. 1594 in such a way as to leave no traces, the copyist missed this annotation. It has been added by the most active later scholiast (12th c.) of the Vatican codex, starting *supra lineam* at the beginning of the *relatum*; however, the later annotator refers to *El*. XIII.8, as does a corrector in **G**. *b*) *Ad Alm*. I.10, 33.12–18 ἐπεὶ οὖν ἡ τοῦ ἑξαγώνου καὶ ἡ τοῦ δεκαγώνου πλευρὰ τῶν εἰς τὸν αὐτὸν κύκλον ἐγγραφομένων ἐπὶ τῆς αὐτῆς εὐθείας ἄκρον καὶ μέσον λόγον τέμνονται, ἡ δὲ ΓΔ ἐκ τοῦ κέντρου οὖσα τὴν τοῦ ἑξαγώνου περιέχει πλευράν, ἡ ΔΖ ἄρα ἐστὶν ἴση τῇ τοῦ δεκαγώνου πλευρᾷ «then since the side of the hexagon and the side of the decagon, when both are inscribed in the same circle, cut the same straight line in extreme and mean ratio, and ΓΔ, since it is a radius, contains the side of the hexagon, therefore ΔZ is equal to the side of the decagon». *c*) The reference to the proposition of *El.* is not correct, since what is required is to prove one of its converses; the entire enunciation of *El*. XIII.9 is indeed quoted, in a modified formulation, as the assumption in the *relatum*. The later scholiast of **B** and a corrector of **G** (quite likely the latter on the basis of the former) invoke *El*. XIII.8; it is possible that they had access to a manuscript in which XIII.6 is absent, like Par. gr. 2344 or Bon. A 18–19 (see *EOO* IV, 246.20, 262.1 *app*., 263 *app*. I, 360.14–362.14, and 360.14 *app*. I, and the discussion at *EOO* V, LXXXII); in fact, XIII.6 is almost certainly spurious (see ibid.). Theon, *in Alm. I.10*, *iA*, 465.9–13, mentions (but by referring to Book XIII only) and quotes the enunciation of *El*. XIII.9, just to declare that what is required is to prove the converse. Accordingly, he proposes two proofs of this converse (ibid., 465.14–466.10). *d*) In **C**, sch. **8** is located beside the coassumption in the *relatum*.

9

Text. διὰ τὸ ιʹ θεώρημα τοῦ ιγʹ τῶν Εὐκλείδου

Transl. By the 10th theorem of the 13th of those of Euclid

Comm. *a*) Vat. gr. 1594, f. 17r marg. ext., Marc. gr. 313, f. 41v marg. ext. *b*) *Ad Alm*. I.10, 33.18–20 ὁμοίως δέ, ἐπεὶ ἡ τοῦ πενταγώνου πλευρὰ δύναται τήν τε τοῦ ἑξαγώνου καὶ τὴν τοῦ δεκαγώνου τῶν εἰς τὸν αὐτὸν κύκλον ἐγγραφομένων «similarly, since the square on the side of the pentagon equals in power the [side] of the hexagon and that of the decagon, when all are inscribed in the same circle». *c*) The reference to the proposition of *El.* is correct. Theon, *in Alm. I.10*, *iA*, 466.11–13, mentions (by referring to Book XIII only) and quotes the enunciation of *El.* XIII.10. *d*) Both in **B** and in **C**, sch. **9** is located beside the *relatum*.

10

Text. ἔστι δὲ καὶ ἡ ΔΕ λ

καὶ om. Th.

Transl. And ΔE also is 30 [parts]

Comm. *a*) Vat. gr. 1594, f. 17r interc., Marc. gr. 313, f. 41v marg. int. *b*) *Ad Alm*. I.10, 34.10–12 μήκει ἄρα ἔσται ἡ ΕΖ τμημάτων ξζ δʹ νεʺ ἔγγιστα, καὶ λοιπὴ ἡ ΔΖ τῶν αὐτῶν λζ δʹ νεʺ «therefore EZ will very nearly be of 67;4,55 parts in length, and ΔZ 37;4,55 of the same [parts], as a remainder». *c*) The scholium supplies the coassumption that allows, because ΔZ = EZ – ΔE, to immediately deduce the second statement of the *relatum* from the first; the fact that ΔE = 30 parts was proved just a few lines before (34.6–8). Heiberg, who correctly identified the clause as a scholium, and not as a stretch of text missed by some earlier copyist and subsequently integrated in the margin, only recorded its presence in **C** (cf. 34.11 *app*.). The text of the scholium coincides, except for the absence of καί, with Theon, *in Alm. I.10*, *iA*, 467.5, where it immediately follows a clause almost identical with the *relatum*. *d*) Both in **B** and in **C**, sch. **10** is located beside the *relatum*; in **C**, a *signe de renvoi* is placed just after ἔγγιστα; the sign is transformed by the copyist, by simple addition of breathing and accent, in the standard abbreviation of the first word in the scholium.

11

Text. διὰ τὸ αʹ θεώρημα τῶν ἐν κύκλῳ εὐθειῶν ὁ Πτολεμαῖος εὗρεν τὴν ὑπὸ τὰς λς μοίρας τοῦ δεκαγώνου πλευράν, καὶ τὴν ὑπὸ τὰς οβ τοῦ πενταγώνου, καὶ τὴν ὑπὸ τὰς ξ τὴν τοῦ ἑξαγώνου, καὶ τὴν ὑπὸ τὰς ϙ τὴν τοῦ τετραγώνου, καὶ τὴν ὑπὸ τὰς ρκ τὴν τοῦ τριγώνου, καὶ τὴν ὑπὸ τὰς ρμδ τὴν λείπουσαν εἰς τὸ ἡμικύκλιον τῇ τοῦ δεκαγώνου, καὶ τὴν ὑπὸ τὰς ρη τὴν λείπουσαν τῇ τοῦ πενταγώνου. εἶτα προεκθέμενος λημμάτιον καὶ ἐφεξῆς γ θεωρήματα – ὧν τὸ μὲν δείκνυσι τὰς ὑποτεινούσας τὰς ὑπεροχὰς τῶν διδομένων περιφερειῶν, τὸ δὲ τὰς διχοτομίας, τὸ δὲ τὰς συνθέσεις – καὶ προσευρεθείσης τῆς τὸ ἡμιμοίριον ὑποτεινούσης διὰ λημμάτων προληφθέντων, συμπληροῖ τὸν ὅλον κανόνα τῶν ἐν κύκλῳ εὐθειῶν.

1 τὸ α′ θεώρημα] τὸ α′ θεωρ/ **BC** : τοῦ α′ θεωρήματος **G** : τὴν πρώτην θεωρίαν **K** | ante κύκλῳ add. τῷ **G** **2** post μοίρας ac οβ expect. τὴν | τοῦ^{12}] τῆς **G** **3** τὴν^{5}] om. **G** **4** τοῦ δεκαγώνου] τῶν ι γωνιῶν codd. **5** καὶ] γ **G** **6** γ om. **G** | θεωρήματα] θεω^{ρ} **BC** : θεωρειῶν **G** : θεωριῶν **K** | ὑπεροχὰς comp. **BC** : γωνίας **G** **8** λημμάτων προληφθέντων] –ος –ος **G**

Transl. By means of the 1st theorem of the chords in a circle, Ptolemy discovered the side under the 36 degrees of the decagon, and that under the 72 of the pentagon, and that under the 60 of the hexagon, and that under the 90 of the square, and that under the 120 of the triangle, and that under 144, namely, the complement to a semicircle of the [arc subtended by the side] of the decagon, and that under 108, namely, the complement [to a semicircle] of the [arc subtended by the side] of the pentagon. After setting out preliminarily a short lemma and 3 theorems thereupon—one of which shows the [chords] subtending the differences of given arcs, another the bisections, another the sums—and the [chord] subtending a half-degree being found in addition by means of preliminarily established lemmas, he completes the whole table of the chords in a circle.

Comm. *a*) Vat. gr. 1594, f. 17r marg. inf., Marc. gr. 313, f. 41v marg. ext. et inf., Vat. gr. 184, ff. 28r et 86r marg. ext. et inf. *b*) No well-defined *relatum*. In **K**, it is not preceded by any citation. *c*) The scholium summarizes the deductive structure of *Alm*. I.10. For parallels to the first sentence, see Theon's statements (similar but better formulated) at *in Alm. I.10*, *iA*, 463.10–464.1, 469.10–15, and, in particular, 473.9–11, whose formulation is closely followed by the scholiast. The expression προεκθέμενος λημμάτιον is taken from Ptolemy's introductory statement to this very result, at 36.10–11; see also Theon, *in Alm. I.10*, *iA*, 473.12. On the designations of the propositions in *Alm*. I.10 (one lemma, three theorems, again one lemma, even if the scholiast alludes to the last lemma in the plural) see further sch. **16**, **19**, **20**, **21**, **26**, **29** (that adopts denominations different from those of the others), and **31**. *d*) Both in **B** and in **C**, sch. **11** is located under the long paragraph (34.5–35.16) containing the calculation of the numerical values of the listed chords. Since both **B** and **C** go to a new page at about the middle of the paragraph, we may safely assume that sch. **11** was also located in the lower margin in their common model. The mistake, at line 4, τῶν ι γωνιῶν for τοῦ δεκαγώνου shows that the original scholium was filled with abbreviations: δεκαγώνου must have been written ΙΓ^{ω}. As was the usage of later copyists, in **G** the δὲ's of the pronominal correlatives τὸ μὲν … τὸ δὲ … τὸ δὲ … carry a double grave accent. *e*) The designations of the listed chords are very compressed; their formulations (quite likely marred by a couple of omissions: see the apparatus) are set in strict parallel. For instance, the first designation means "the numerical value of the chord, that subtends an arc of 36°, coinciding with the side of an hexagon inscribed in a circle." The same must be said of the designations of the chords corresponding to the complementary arcs of the assigned chords. At lines 6–7, the plural nouns ὑπεροχάς, διχοτομίας, and συνθέσεις, instead of the singular nouns to be expected, are an idiomatic trait of ancient Greek language (see sch. **55** and **106**).

12

Text. πάλιν ἐπεὶ ἡ μὲν ΔΖ τμημάτων ἐστὶ λζ δ′ νε″

Transl. Again, since ΔZ is of 37;4,55 parts

Comm. *a*) Vat. gr. 1594, in textu, Marc. gr. 313, f. 41v marg. inf. *b*) It is *Alm*. I.10, 34.15–16. *c*) This is a clause missing in the common model of **BC** and **A** (**D** has the clause) and probably restored in the margins by very early collation; the copyist of **B** understood that it should be reintegrated in the main text. In *Alm*., a τμῆμα is one of the 120 «parts» in which the diameter of the reference circle is divided; the circumference is instead divided in 360 parts or μοῖραι «degrees» (cf. *POO* I.1, 77.6–13, with a few terminological inconsistencies). A statement quite similar to the scholium is contained in Theon's reformulation of Ptolemy's argument: ἐπεὶ οὖν τὴν ΔΖ ἐδείξαμεν τμημάτων οὖσαν λζ δ′ νε″ «then since we showed that ΔZ is of 37;4,55 parts» at *in Alm. I.10*, *iA*, 467.8. *d*) In **C**, the conventional sign for the Sun is placed after 34.15 διάμετρος ρκ as if it were included in the main text, the sign being repeated in the lower margin, followed by the explanation ἔστι σφαῖρα «it is a sphere»; sch. **12** is transcribed beside this indication, in majuscule. This is a further clue that the copyists of **BC** tried to graphically differentiate the layers of scholia contained in their common model: it is quite likely that the explanation ἔστι σφαῖρα was introduced by a scholiast of the model, who misinterpreted the sign and took it as a part of Prolemy's clause; since the Sun could not be mentioned in Ptolemy's sentence, and the most natural determinative of Ptolemy's διάμετρος ρκ, namely, "of the circle," would have been represented by another sign, the scholiast resorted to the less implausible object given the context, namely, a sphere.

13

Text. ἴσον γὰρ δύναται δυσὶ ταῖς ἐκ τοῦ κέντρου περιεχούσαις ἣν ὑποτείνει ὀρθὴν γωνίαν

ταῖς — περιεχούσαις] ταῖς — περιεχούσ(ης) **BC** : τῆς — περιεχούσης **G**

Transl. For it is equal in power to the two radii containing the [chord] that subtends a right angle

Comm. *a*) Vat. gr. 1594, f. 17v marg. ext., Marc. gr. 313, f. 41v marg. inf., Vat. gr. 184, f. 86r marg. ext. *b*) *Ad Alm*. I.10, 35.6–9 ὁμοίως δέ, ἐπεὶ ἡ μὲν τοῦ τετραγώνου πλευρά, ὑποτείνουσα δὲ μοίρας ϙ, <u>δυνάμει διπλασία ἐστὶν τῆς ἐκ τοῦ κέντρου</u> «similarly, since the side of the [inscribed] square, which subtends 90 degrees, is double in power of the radius». *c*) The explanation is quite obvious, and is identical with Theon, *in Alm. I.10*, *iA*, 467.19–20. *d*) In **B**, sch. **13** is located beside the *relatum*; in **C**, it is just under it. *e*) The expression ἡ πλευρὰ δυνάμει διπλασία ἐστὶ τῆς ἐκ τοῦ κέντρου «the side is double in power of the radius» in the *relatum* means "the square on the side is double of the square on the radius;" in the scholium, instead, it is question of a square which is equal to the sum of the squares on the two radii; as frequently happens, the sum of two objects is simply formulated by referring to them in the plural. On the language of the "power" in Greek mathematics see Vitrac (2008). The attraction of the relative in περιεχούσαις ἣν is rather wild; the participle must be corrected since **BC** have the standard compendium of termination –ης (but note that this compendium can simply be the sign abbreviating any termination: *POO* II, XCI). If ἡ ἐκ τοῦ κέντρου «radius» has to be the nominalization of an invariant prepositional syntagm, its plural must be αἱ ἐκ τοῦ κέντρου, as is in the scholium. Still, we find dozens of occurrences of a plural αἱ ἐκ τῶν κέντρων in Archimedes and Pappus, and four in *El.*, III.def.1 and III.26, 28, and 29 (*EOO* I, 164.3, 230.22, 236.6–7, 238.13, respectively). On the linguistic expression of the radius (ἡ ἐκ τοῦ κέντρου *vs*. διάστημα) see Federspiel (2005), which superseds all previous lucubrations.

14

Text. διὰ τὸ ιγʹ θεώρημα τοῦ ιγʹ βιβλίου τῶν Εὐκλείδου

Transl. By the 13th theorem of the 13th book of those of Euclid

Comm. *a*) Vat. gr. 1594, f. 17v interc. If the scholium has not been erased in Marc. gr. 313 in such a way as to leave no traces, the copyist missed this annotation, which has been added by a later scholiast, in the outer margin of f. 42r, beside the *relatum*. The later annotator correctly refers to *El*. XIII.12. *b*) *Ad Alm*. I.10, 35.9–10 ἡ δὲ τοῦ τριγώνου πλευρά, ὑποτείνουσα δὲ μοίρας ρκ, δυνάμει τῆς αὐτῆς ἐστιν τριπλασίων «and the side of the [inscribed] triangle, which subtends 120 degrees, is triple in power of the same [straight line] [*scil*. the radius]». *c*) The right proposition of *El.* to cite is XIII.12; the error is probably due to inadvertence or to a copyist's error: the only feature of the manuscript tradition of *El.* directly justifying the presence of an additional proposition before XIII.12 is the proof XIII.5 *aliter*, that, however, only Vat. gr. 190 has in the main text: see *EOO* IV, 260.25 *app*., 362.15–364.16, and 362.15 *app*. I. Theon, *in Alm. I.10*, *iA*, 467.20–468.2, mentions (by referring to Book XIII only) and quotes the enunciation of *El.* XIII.12. *d*) In **B**, sch. **14** is located beside the *relatum*; a *signe de renvoi* is added by a later hand and placed just above the initial τοῦ of the *relatum*. *e*) For the expression δυνάμει τριπλασίων «triple in power» see sch. **13**.

15

Text. καθ’ αὑτὰς μὲν λέγει ἐπεὶ ἑκάστη τούτων ἐξ οἰκείας καὶ μιᾶς προτάσεως δέδεικται, ἑξῆς δὲ μέλλει ἐκ μιᾶς προτάσεως πλείους πορίζεσθαι.

1 αὑτὰς] ἑαυτὰς Th. | μὲν] δὲ Th. | post ἑκάστη add. μὲν Th.

Transl. He says “individually” since each of these [chords] has been shown under a single and specific enunciation, whereas he next sets out to provide several [of them] under a single enunciation.

Comm. *a*) Vat. gr. 1594, f. 17v marg. ext., Marc. gr. 313, f. 42r marg. inf. *b*) *Ad Alm*. I.10, 35.17 καθ’ αὑτὰς «individually». *c*) This is a remark about the different deductive import of the first theorem on the one side and of those following it on the other; the former, and the subsequent calculations, deal with particular chords, the latter (chords associated to difference, bisection, and sum of arcs subtended by given chords) provide general rules to calculate chords. The text of the scholium coincides, except for two particles and the graphism ἑαυτάς, with Theon, *in Alm. I.10*, *iA*, 468.10–11. *d*) In **B**, sch. **15** is located beside the *relatum*; in **C**, it is in the lower margin, after a *signe de renvoi* (the sign for the Sun) that does not appear in the main text. *e*) Note the verb πορίζεσθαι, in the technical sense sanctioned in *Data*: it means “appointed given,” either by some superior instance, or by proof, or by calculation; the theorems to which the scholium alludes to are in fact formulated in the “language of the givens” (Acerbi 2011b). In the scholium, πορίζεσθαι is also rightly set in parallel with δέδεικται, since the first theorem and the subsequent calculations of the numerical values of particular chords do not resort to such a language. The enunciations pertaining to particular chords are said οἰκεῖαι since they are both «specific» and «appropriate». By shifting the particle μέν we read in Theon’s text, the scholiast modifies the commatic structure of the sentence.

16

Text. λῆμμα

Transl. Lemma

Comm. *a*) Vat. gr. 1594, f. 17v marg. int., Marc. gr. 313, f. 42r marg. ext. *b*) *Ad Alm*. I.10, 36.14 ἔστω γὰρ κύκλος ἐγγεγραμμένον ἔχων τετράπλευρον «let there be a circle having a quadrilateral inscribed» ff. *c*) In sch. **11**, this theorem is called λημμάτιον, and the same denomination is used by Ptolemy at 36.10–11 when he presents this very result, first proved in *Alm*. and known as “Ptolemy’s Theorem” (but *Data* 93 formulates a special case of it in the “language of the givens”). Sch. **16**–**18** provide three complements to this theorem. Theon declares that the proof set out by Ptolemy is σαφής «clear» (*iA*, 474.4; see sch. **1**) and goes on to prove the special case in which angles ΔΒΓ and ΑΒΔ are equal, and hence side ΑΕ of angle ΑΒΕ falls on diagonal ΒΔ, a case obviously covered by Ptolemy's proof (*iA*, 474.8–475.15; see Fig. 2 and sch. **17** for the construction). *d*) Both in **B** and in **C**, sch. **16** is in majuscule and is located beside the beginning of the *relatum*. *e*) As is customary with him, Ptolemy does not provide this theorem with a general enunciation; we read it in Theon, *in Alm. I.10*, *iA*, 474.1–3.

17

Text. ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ ΔΒΓ γωνία τῇ ὑπὸ ΑΒΕ

Transl. Then since angle ΔΒΓ is equal to ΑΒΕ

Comm. *a*) Vat. gr. 1594, f. 17v marg. int., Marc. gr. 313, f. 42r marg. ext. *b*) *Ad Alm*. I.10, 36.17–18 κείσθω γὰρ τῇ ὑπὸ τῶν ΔΒΓ γωνίᾳ ἴση ἡ ὑπὸ ΑΒΕ «in fact, let ΑΒΕ be made equal to angle ΔΒΓ». *c*) A quadrilateral ΑΒΓΔ is inscribed in a circle, whose center is Ε; diagonals ΑΓ, ΒΔ are joined, then the construction indicated in the *relatum* is performed. It is required to show (Fig. 2) that *r*(ΑΓ,ΒΔ) = *r*(ΑΒ,ΓΔ) + *r*(ΑΔ,ΒΓ). This is "Ptolemy's Theorem." The scholium, a part of the main text of *Alm*. in **D**, transforms the *relatum* in the antecedent of a paraconditional clause immediately subsequent to the *relatum* itself; accordingly, the οὖν (36.18) opening the protasis of the following conditional clause is absent in **D** and deleted by a later hand in **C**. The resulting sentence is a conditional clause nested in a paraconditional: ἐπεὶ οὖν ἴση ἐστὶν ἡ ὑπὸ ΔΒΓ γωνία τῇ ὑπὸ ΑΒΕ, ἐὰν ~~οὖν~~ κοινὴν προσθῶμεν τὴν ὑπὸ ΕΒΔ, ἔσται καὶ ἡ ὑπὸ ΑΒΔ γωνία ἴση τῇ ὑπὸ ΕΒΓ «then since angle ΔΒΓ is equal to ΑΒΕ, if ~~then~~ we add ΕΒΔ in common, angle ΑΒΔ will also be equal to ΕΒΓ». The syntax of the sentence is correct but quite contrived and non-canonical. Heiberg rightly regarded the clause as a scholium, and not as a part of the original text (that in this case would be attested in **D** only) simplified in the **ABC** hyper-branch but retained as an interlinear or marginal variant reading in **BC** (cf. 36.18 *app*.). *d*) Both in **B** and in **C**, sch. **17** is located beside the *relatum*; in **C**, a *signe de renvoi* is placed by a later hand just after ἡ ὑπὸ ΑΒΕ. *e*) The construct ἐπεὶ …, ἐὰν κοιν* …, + principal clause would be a syntactical *hapax* in Ptolemy.

18

Text. τὸ γὰρ αὐτὸ τμῆμα ὑποτείνουσιν

Transl. For they subtend the same segment

Comm. *a*) Vat. gr. 1594, f. 18r marg. int.; the annotation is missing in Marc. gr. 313. *b*) *Ad Alm*. I.10, 37.10–11 ἔστιν δὲ καὶ ἡ ὑπὸ ΒΑΕ ἴση τῇ ὑπὸ ΒΔΓ «and ΒΑΕ is also equal to ΒΔΓ». *c*) The scholium makes implicit reference to *El*. III.21; the common segment subtended by angles ΒΑΕ and ΒΔΓ is the one cut off by arc ΒΓ (Fig. 2). The clause is identical with that inserted in the main text at 37.1–2 to justify the same statement about the angles ἡ ὑπὸ ΒΔΑ and ἡ ὑπὸ ΒΓΕ. *d*) In **B**, sch. **18** is located beside the *relatum*.

19

Text. τοῦτο τὸ θεώρημα καθ’ ὑπεροχὴν λέγεται

Transl. This theorem is called “by difference”

Comm. *a*) Vat. gr. 1594, f. 18r marg. int., Vat. gr. 184, f. 86v marg. inf.; the annotation is missing in Marc. gr. 313. It has been added by a later scholiast, in the inner margin of f. 42v, beside the beginning of the *relatum*. *b*) *Ad Alm*. I.10, 37.19 τούτου προεκτεθέντος «having preliminarily set out this» ff. *c*) The scholium provides a rigid designator of the first theorem proved by Ptolemy (Fig. 3): in a semicircle on diameter ΑΔ, if two chords ΑΒ, ΑΓ having a common endpoint Α are given, the chord ΒΓ subtending the difference of the arcs subtended by the given chords is also given. The denomination is also used in Theon, *in Alm. I.10*; see for instance *iA*, 496.14. The three main theorems of *Alm*. I.10 are formulated in the “language of the givens.” Sch. **19–20** provide two complements to the theorem “by difference.” *d*) In **B**, sch. **19** is located beside the beginning of the *relatum*. *e*) As is customary with him, Ptolemy does not provide this theorem with a general enunciation. In this case, however, he states a general conclusion, preceded by καὶ φανερὸν ἡμῖν γέγονεν ὅτι «and it has become manifest to us that», at 38.15–17.

20

Text. διὰ τὸ προληφθὲν λῆμμα

Transl. By the preliminarily established lemma

Comm. *a*) Vat. gr. 1594, f. 18r marg. int., Marc. gr. 313, f. 42v marg. ext. *b*) *Ad Alm*. I.10, 38.7–11 ἐπεὶ οὖν ἐν κύκλῳ τετράπλευρόν ἐστιν τὸ ΑΒΓΔ, τὸ ἄρα ὑπὸ ΑΒ ΓΔ μετὰ τοῦ ὑπὸ τῶν ΑΔ ΒΓ ἴσον ἐστὶν τῷ ὑπὸ ΑΓ ΒΔ «then since ΑΒΓΔ is a cyclic quadrilateral, the [rectangle contained] by ΑΒ, ΓΔ with that [contained] by ΑΔ, ΒΓ is equal to that [contained] by ΑΓ, ΒΔ». *c*) This is a reference to Ptolemy’s Theorem, just proved, in whose configuration chords ΒΔ, ΓΔ have been joined (Fig. 3). *d*) Both in **B** and in **C**, sch. **20** is located beside the sentence immediately preceding the *relatum*: δεδομέναι ἄρα εἰσὶν δηλονότι καὶ αὗται διὰ τὸ λείπειν ἐκείνων εἰς τὸ ἡμικύκλιον «therefore, clearly, these too are given, because they are the complement to a semicircle of those [*scil.* the given chords]» (38.4–7). Actually, Ptolemy explains how to calculate the chords subtending supplementary arcs by means of a single sentence (35.18–36.2). *e*) See sch. **11** for the compressed designation of a chord subtending an arc supplementary to that subtended by a given chord. Again as in sch. **11**, note the etymological figure of speech in προληφθὲν λῆμμα. The sum of two objects is formulated in the *relatum* using the preposition μετά; it is the formulation preferred in *El.* II.

21

Text. τοῦτο κατὰ τὴν διχοτομίαν λέγεται

Transl. This is called "by bisection"

Comm. *a*) Marc. gr. 313, f. 43r marg. sup.; the annotation is missing in Vat. gr. 1594. It has been added by a later scholiast, in the intercolumnar space of f. 18r, beside the beginning of the *relatum*. *b*) *Ad Alm*. I.10, 39.4 πάλιν προκείσθω «again, let it be proposed» ff. *c*) The scholium provides a rigid designator of the second theorem proved by Ptolemy: the chord subtending half of the arc subtended by a given chord is also given. The denomination is also used in Theon, *in Alm. I.10*; see for instance *iA*, 485.13 and 490.2. Sch. **21**–**25** clarify some specific points of this theorem. *d*) In **C**, sch. **21** is located just above the beginning of the *relatum*, after the diagram associated to the preceding theorem. *e*) In this case, Ptolemy provides his theorem with a general enunciation, but this is formulated as the enunciation of a problem: δοθείσης τινὸς εὐθείας ἐν κύκλῳ τὴν ὑπὸ τὸ ἥμισυ τῆς ὑποτεινομένης περιφερείας εὐθεῖαν εὑρεῖν «given some chord in a circle, to find the chord under half of the subtended arc» (39.4–6). In geometric jargon, the διχοτομία is strictly speaking the «middle point» of a segment (cf. sch. **68**).

22

Text. πόθεν ὅτι ἡ κάθετος ἡ ΔΖ οὐχὶ ἐκτὸς τοῦ Ε πίπτει; οὐ πίπτει δὲ διότι ἰσοσκελές ἐστι τὸ ΔΕΓ τρίγωνον καὶ ἡ ὑπὸ ΑΕΔ γωνία ἀμβλεῖα· ἴση γάρ ἐστι τῇ ὑπὸ ΑΒΔ, ἡ δὲ ὑπὸ ΑΒΔ γωνία ἀμβλεῖα· ἐπὶ γὰρ περιφερείας βέβηκε μείζονος ἡμικυκλίου (τῆς τε τοῦ ἡμικυκλίου τῆς ἐπὶ τῆς ΑΓ καὶ τῆς ἐφεξῆς αὐτῇ τῆς ΓΔ) καὶ ἐν περιφερείᾳ ἐστὶν ἐλάττονι ἡμικυκλίου τῇ ΑΒΔ· ὥστε ἐπεὶ ἀμβλεῖά ἐστιν ἡ ὑπὸ ΑΕΔ, οὐ δύναται αὐτῆς ἐκτὸς πεσεῖν τὴν ὀρθήν. ἔτι καὶ λῆμμά ἐστιν ὡς ἐν τοῖς ἰσοσκελέσι τριγώνοις ἡ ἀπὸ τῆς κορυφῆς ἀγομένη κάθετος ἐπὶ τὴν βάσιν δίχα τέμνει τὴν βάσιν.

2 ὑπὸ[1]] ἀπὸ **K** **3** ἡμικυκλίου comp. **BK** : τριγώνου comp. **C** **4** ἡμικυκλίου comp. **BK** : τριγώνου comp. **C** | τῆς[4]] τῇ codd. | ἐλάττονι] ἔλαττον codd. **5** ἡμικυκλίου comp. **BK** : τριγώνου comp. **C** | ἐκτὸς codd. : expect. ἐντὸς **6** ὀρθὴν codd. : expect. κάθετον

Transl. Whence is it not the case that perpendicular ΔZ falls outside E? It does not [so] fall because triangle ΔΕΓ is isosceles and angle ΑΕΔ obtuse—for it is equal to ΑΒΔ, and angle ΑΒΔ is obtuse, for it stands upon an arc greater than a semicircle (the [arc made] of the semicircle—the [arc] on ΑΓ—and of ΓΔ adjacent to it) and it is in an arc ΑΒΔ less than a semicircle—so that, since ΑΕΔ is obtuse, it cannot be that the orthogonal [straight line] falls outside it. Again, there is also a lemma to the effect that in isosceles triangles the [straight line] drawn from the vertex perpendicular to the base bisects the base.

Comm. *a*) Vat. gr. 1594, f. 18v marg. ext., Marc. gr. 313, f. 43r marg. inf., Vat. gr. 184, f. 28r. *b*) *Ad Alm*. I.10, 39.21–40.1 ἐπεὶ οὖν ἰσοσκελοῦς ὄντος τριγώνου τοῦ ΔΕΓ ἀπὸ τῆς κορυφῆς ἐπὶ τὴν βάσιν κάθετος ἦκται ἡ ΔΖ, ἴση ἐστὶν ἡ ΕΖ τῇ ΖΓ «then since, there being an isosceles triangle ΔΕΓ, a [straight line] ΔZ has been drawn from the vertex perpendicular to the base, EZ is equal to ZΓ» (the citation in **K** ends at ἡ ΔΖ). *c*) A chord ΒΓ is given in a semicircle on diameter ΑΓ, arc ΒΓ is bisected at Δ and AB, ΑΔ, ΒΔ, ΔΓ are joined; perpendicular ΔZ is then drawn from Δ to ΑΓ; finally, AE = AB is cut off from diameter ΑΓ. One has to show that ΔΓ is also given (Fig. 4). The scholium sets out two arguments in order to justify a geometric fact implicit in the statement that EZ = ZΓ, namely, that the foot of perpendicular ΔZ must fall between points E and Γ. To show that angle ΑΒΔ is obtuse, the scholiast makes implicit reference to *El*. III.31. The semicircle on ΑΓ referred to in the scholium is the one that is not drawn in the diagram; note that, *iuxta El*. III.def.8, one should more properly speak of angles "in a segment," not "in an arc." The first argument is flawed, since the condition that ΑΕΔ is obtuse is of no use in proving the required property; the difficulties of the scholiast are revealed by the highly contrived syntax, by the slip ἐκτός for ἐντός, and by his resorting to a second explanation; as a matter of fact, Ptolemy himself had just shown (39.15–21) that triangle ΔΕΖ is isosceles without assuming that perpendicular ΔZ falls between points E and Γ. The final λῆμμα is an immediate consequence of *El*. I.26. Theon, *in Alm. I.10*, *iA*, 480.3–10, judiciously offers a different argument, a part of which might have prompted the scholiast to introduce angle ΑΕΔ. *d*) In **B**, sch. **22** is *figuratum* (a Latin cross upon a basement) and its beginning is located just above the beginning of the *relatum*; in **C**, the scholium is located in the lower margin, without a *signe de renvoi*, just below the diagram of the theorem. In **K**, sch. **22** immediately precedes sch. **28**. The model of **BC** almost certainly had an uncommon pointed sign for "semicircle;" the copyist of **C** took it as the sign for "triangle" (lines 3–5). *e*) The adverb πόθεν at line 1 is a canonical way to begin a comment scholium: one counts, for instance, 33 occurrences in the scholia to *El*. Again at line 1, note the emphatic οὐχί, whose scope is the whole clause (cf. sch. **28** and **50**). The construct οὐ δύναται «it cannot be that …» + aorist infinitive at line 5 is not canonical in mathematical texts. The adjective ὀρθήν at line 6 is most likely a misreading of the sign ⊥ for κάθετος. Note, again at line 6, the denomination λῆμμα for a result that is not a theorem of *El*.

23

Text. καὶ διὰ τοῦτο δέδοται

Transl. And it is given because of this

Comm. *a*) Vat. gr. 1594, f. 18v interc., Marc. gr. 313, f. 43r marg. ext. *b*) *Ad Alm*. I.10, 40.2–4 ἀλλ’ ἡ ΕΓ ὅλη ἡ ὑπεροχή ἐστιν τῶν ΑΒ καὶ ΑΓ εὐθειῶν· ἡ ἄρα ΖΓ ἡμίσειά ἐστιν τῆς τῶν αὐτῶν ὑπεροχῆς «but ΕΓ as a whole is the difference of straight lines AB and AΓ: therefore ZΓ is half of their difference» *c*) Both statements in the *relatum* could be the target of the scholium; the former entails that EΓ is given as a consequence of *Data* 4 (the difference of given magnitudes is also given), the latter that ZΓ is given as a consequence of *Data* 2 (a magnitude having a given ratio to a given magnitude is also given). Note that, a couple of lines later (40.6–7), Ptolemy expressly states that ZΓ is given because it is half of the difference of AΓ and AB (see sch. **25** and Fig. 4). Theon, *in Alm. I.10*, *iA*, 479.11–17, spells out all deductive steps and formulates them in a canonical way. *d*) Both in **B** and in **C**, sch. **23** is located beside the *relatum*, but the placement does not allow one to decide which of the two statements is the target of the scholium.

24

Text. καὶ τῆς περιφερείας τῆς ἐπὶ τῆς εὐθείας

Transl. And the arc on the chord

Comm. *a*) Vat. gr. 1594, f. 18v interc., Marc. gr. 313, f. 43r marg. sup. *b*) *Ad Alm*. I.10, 40.4–6 ὥστε, ἐπεὶ <u>τῆς ὑπὸ τὴν ΒΓ περιφέρειαν εὐθείας ὑποκειμένης</u> αὐτόθεν δέδοται καὶ ἡ λείπουσα εἰς τὸ ἡμικύκλιον ἡ ΑΒ «so that, since, <u>the chord under arc BΓ being supposed</u>, its complement AB to a semicircle is also immediately given» *c*) The scholium completes the clause by claiming that the arc on the chord should be taken to be «supposed» along with the chord itself (which is among the givens of the problem); according to the scholiast, the *relatum* should as it were be completed as follows: *τῆς ὑπὸ τὴν ΒΓ περιφέρειαν εὐθείας ‹καὶ τῆς περιφερείας τῆς ἐπὶ τῆς εὐθείας› ὑποκειμένης *«the chord under arc BΓ [and the arc on the chord] being supposed». *d*) In **B**, sch. **24** is located beside the *relatum*; in **C**, it is in the upper margin, just above the right corner of the frame reserved to the main text. For Theon’s formulation see sch. **23**. *e*) The chord under arc BΓ is said to be ὑποκειμένη by Ptolemy since it is «supposed» to be given at the beginning of the problem.

25

Text. τουτέστιν ἡ ΑΕ· δοθείσης δὲ καὶ τῆς διαμέτρου τῆς ΑΓ καὶ λοιπὴ ἡ ΕΓ δέδοται· ὥστε καὶ ἡ ἡμίσεια αὐτῆς ἡ ΖΓ

1 τουτέστιν om. Th.

Transl. That is, ΑΕ; and once diameter ΑΓ is also given, ΕΓ is also given as a remainder, so that also the half of it, ΖΓ

Comm. *a*) Vat. gr. 1594, f. 18v interc., Marc. gr. 313, f. 43r marg. ext. *b*) *Ad Alm*. I.10, 40.6–7 δοθήσεται καὶ ἡ ΖΓ ἡμίσεια οὖσα τῆς τῶν ΑΓ καὶ ΑΒ ὑπεροχῆς «ΖΓ, which is half of the difference of ΑΓ and ΑΒ, will also be given» *c*) After identifying ΑΒ with ΑΕ (Fig. 4), the scholium formulates the *relatum* as a chain of principal clauses supplemented by a genitive absolute, mimicking in this way Ptolemy's participial variant to the canonical style. The text coincides with that of the corresponding passage in Theon, *in Alm. I.10*, *iA*, 479.15–17 (the initial τουτέστιν was added by the scholiast). *d*) Both in **B** and in **C**, sch. **25** is located beside the sentence at 40.4–7, taken as a whole to be the *relatum*. *e*) Adjectives such as λοιπός and ὅλος denoting results of operations are always in predicative position, that is, without the article; for this reason, they are translated «as a remainder» and «as a whole», respectively; see also the remark at Toomer (1984, 17).

26

Text. τοῦτο λέγει κατὰ σύνθεσιν

λέγει codd. : expect. λέγεται

Transl. This he calls "by composition"

Comm. *a*) Vat. gr. 1594, f. 18v marg. int., Marc. gr. 313, f. 43v marg. ext. *b*) *Ad Alm*. I.10, 41.4 πάλιν ἔστω κύκλος ὁ ΑΒΓΔ «again, let there be a circle ΑΒΓΔ» ff. *c*) The scholium provides a rigid designator of the third theorem proved by Ptolemy: in a circle with diameter ΑΔ and center Ζ, if two chords ΑΒ, ΒΓ having a common endpoint are given, the chord ΑΓ subtending the sum of the arcs subtended by the given chords is also given (Fig. 5). Sch. **26**–**27** provide two complements to the theorem "by composition." The denomination is also introduced in Theon, *in Alm. I.10*, *iA*, 483.10. *d*) Both in **B** and in **C**, sch. **26** is located beside the beginning of the *relatum*. *e*) The subject of λέγει could be Ptolemy or, more likely, Theon, but one must suspect a scribal corruption for λέγεται, found in all other scholia of similar content (see for instance sch. **19** and **21**). As is customary with him, Ptolemy does not provide this theorem with a general enunciation.

Also in this case, however, he states a general conclusion, preceded by ὥστε «so that» and followed by διὰ τούτου τοῦ θεωρήματος «by this theorem», at 42.3–5.

27

Text. αἱ λείπουσαι εἰς τὸ ἡμικύκλιον

ἡμικύκλιον comp. **B** : τρίγωνον comp. **C**

Transl. The [chords] complement to a semicircle

Comm. *a*) Vat. gr. 1594, f. 18v marg. int., Marc. gr. 313, f. 43v marg. ext. *b*) *Ad Alm*. I.10, 41.15–17 δῆλον δὴ αὐτόθεν ὅτι διὰ μὲν τὴν ΒΓ δοθήσεται καὶ ἡ ΓΕ, διὰ δὲ τὴν ΑΒ δοθήσεται ἥ τε ΒΔ καὶ ἡ ΔΕ «it is immediately clear that [chord] ΓΕ will also be given by means of ΒΓ, and both ΒΔ and ΔΕ will be given by means of ΑΒ». *c*) In the configuration described in sch. **26** (Fig. 5), diameter ΒΖΕ is drawn across, and ΒΔ, ΔΓ, ΓΕ, ΔΕ are joined. The scholium makes explicit the transition to the chord of the complementary arc in the proof of the theorem “by composition.” Note that ΔΕ is the chord of the arc complementary to the arc subtended by ΒΔ, not by ΑΒ; therefore chords ΒΔ and ΔΕ are given in succession. In his proof of the same theorem, Theon, *in Alm. I.10*, *iA*, 484.4–12, makes these derivations explicit. *d*) Both in **B** and in **C**, sch. **27** is located beside the *relatum*. See sch. **22** for the mistake in **C**. *e*) Literally, ἡ λείπουσα εἰς τὸ ἡμικύκλιον is «the [chord] missing to a semicircle» of an assigned chord.

28

Text. οὐχὶ αἱ εὐθεῖαι διπλασιαζόμεναι τρίτον μέρος, ἀλλ’ αἱ περιφέρειαι αἱ κατὰ παραύξησιν τῆς α ∠ μοίρας ἔχουσι μὴ διαιρουμένης τῆς μονάδος, ὧν τὰς εὐθείας τὸν εἰρημένον τρόπον κατείληφεν.

1 αἱ εὐθεῖαι] αὗται Th. **2** ἔχουσι post μέρος habet Th. | ante εὐθείας add. καὶ Th.

Transl. It is not the case that the chords have a third part when doubled, but the arcs ([increased] by increments of $1\frac{1}{2}$ degree since the unit cannot be divided) do have it, whose chords he [*scil*. Ptolemy] took by means of the said method.

Comm. a) Vat. gr. 1594, f. 19r marg. sup., Marc. gr. 313, f. 44r marg. sup., Vat. gr. 184, f. 28r. *b*) *Ad Alm*. I.10, 42.9–10 πάσας ἁπλῶς ἐγγράψομεν, ὅσαι δὶς γινόμεναι τρίτον μέρος ἕξουσιν «we will be able to inscribe [*scil*. in the table] all [chords of arcs] which when multiplied twice have a third part» (= citation in **K**). *c*) The theorems proved by Ptolemy enable him to calculate the numerical values of the chords corresponding to the arc of $1\frac{1}{2}^{\circ}$, and of course of its multiples. The *relatum* contains an abuse of language that also called for an integration in Toomer's translation: what admits of a third part when doubled are not the chords to be entered in the table, but the arcs they subtend. The scholium points out this shortcut. The text of the scholium is an almost verbatim cento of the corresponding passages in Theon, *in Alm. I.10*, *iA*, 486.17–18 (οὐχὶ … ἔχουσι), 486.23–24 (μὴ … μονάδος), 486.21–22 (ὧν … κατείληφεν)—note the different order. Sch. **28**–**30** elaborate on a couple of specific points in Ptolemy's argument about the chords that can be calculated thanks to the theorem "by composition" (42.7–43.5). *d*) In **B**, sch. **28** is preceded by a *signe de renvoi*, also placed beside the *relatum*. In **C**, the conventional sign for the Sun precedes the scholium, but this sign cannot be found in or near the *relatum*. In **K**, sch. **28** immediately follows sch. **22**. In **B**, the scholium is partly shaped as sch. **1**. *e*) Note the initial emphatic οὐχί, whose scope is the whole clause (cf. sch. **22** and **50**). The participle διαιρουμένης at line 2 is meant to imply potentiality; the only other example I know of in a mathematical text (cf. Aristotle, *Top.* Z.4, 142b12) is *El.* VII.def.6 ἄρτιος ἀριθμός ἐστιν ὁ δίχα <u>διαιρούμενος</u> «an even number is that which is <u>divisible</u> into two [equal parts]» (but note that here the participle has a different syntactical function); since Nicomachus (*Ar.* I.7.2) rephrased the definition by making the potential connotation explicit (ἔστι δὲ ἄρτιον μέν, ὃ <u>οἷόν τε</u> εἰς δύο ἴσα διαιρεθῆναι μονάδος μέσον μὴ παρεμπιπτούσης «an even number is what can be divided in two equal [parts], since a unit does not fall in the middle»), the difference triggered a scholium in which the anonymous annotator shows himself unaware of the fact that the potential value of the present indicative can also be assumed by the present participle (see Riedlberger 2013, 130–131 for edition and translation of the scholium, 229–230 for the commentary; in particular, at 230 and n. 448 a preliminary discussion of the linguistic issue is provided; see also sch. **40**). It remains that speaking of an indivisible unit when mentioning $1\frac{1}{2}^{\circ}$ is quite incongruous. The final designation in the *relatum*, taken up in the scholium, means "multiple of $1\frac{1}{2}$."

29

Text. ἐὰν γάρ, ὡς ἐν τῷ γʹ λήμματι ἐμάθομεν, τὴν διχοτομηθεῖσαν περιφέρειαν, ὡς ἐκεῖ τὴν ΒΔΓ, σύνθωμεν μετὰ τοῦ ἡμίσεος αὐτῆς (τουτέστι τῆς ΔΓ), ἵνα γένηται ἡ ὅλη περιφέρεια τριπλῆ τῆς ΔΓ, καὶ ταύτην τὴν ὅλην (τουτέστι τὴν τριπλῆν τῆς ΔΓ) διὰ τοῦ δʹ τοῦ κατὰ σύνθεσιν λήμματος εὑρήσομεν δεδομένην. δῆλον ὅτι πάλιν καὶ τὴν ταύτης ἡμίσειαν διὰ τοῦ γʹ ἐγγράψομεν εἰς τὸν κύκλον, καὶ αὕτη ἡ ἡμίσεια τῆς ὅλης δὶς γενομένη τριπλῆ ἔσται τῆς ΔΓ, ὡς εἴρηται, ὥστε τρίτον μέρος ἕξει.

2 ΒΔΓ] ΒΑΓ **K** | αὐτῆς] αὐτ(ῆς) **BK** : αυτ(ῆς) **C** | ΔΓ] ΑΓ **K** **3** ταύτην] ταύτης **G** | ΔΓ] ΑΓ **K** **4** εὑρήσομεν scripsi : εὕρομεν **BC** : εὕρωμεν **GK** | post πάλιν add. ἐστὶ **G** **5** ἐγγράψομεν] ἐγρ– **K** **6** τριπλῆ] ἡ τριπλῆ **K** | ὡς εἴρηται om. **G**

Transl. For if, as we have learnt in the 3rd lemma, we compose the bisected arc, as there ΒΔΓ, with its own half (namely, ΔΓ), in order that the arc as a whole become triple of ΔΓ, we shall discover that this whole (namely, the triple of ΔΓ) is also given by the 4th lemma “by composition.” Again, it is clear that we shall also inscribe half of this [*scil*. of the triple of ΔΓ] in the circle by means of the 3rd [book of the Elements], and this half of the whole, multiplied twice, is the triple of ΔΓ, as said, so that it will have a third part.

Comm. *a*) Vat. gr. 1594, f. 19r marg. inf., Marc. gr. 313, f. 44r marg. inf., Vat. gr. 184, ff. 28v et 87r marg. inf. *b*) *Ad Alm*. I.10, 42.9–10 πάσας ἁπλῶς ἐγγράψομεν, ὅσαι δὶς γινόμεναι τρίτον μέρος ἕξουσιν «we will be able to enter [*scil*. in the table] all [chords of arcs] which when multiplied twice have a third part». *c*) By calling all preceding propositions «lemmas», the scholiast introduces a different classification of Ptolemy’s results from the one adopted in sch. **11**, where a «lemma» was said to be followed by three «theorems». Given the location of the scholium, it is not easy to identify its *relatum*, that at any rate must be included in the stretch of text at 42.7–43.3 δύναται, since with this word the left column of f. 19r in **B** ends; all in all, the contents and formulation of sch. **29** suggest that one assigns to it the same *relatum* as to sch. **28**. By combining the last two propositions proved by Ptolemy (but the only configuration alluded to is that of the theorem “by bisection,” Fig. 4), the scholium explains in general terms why a chord subtended by an arch which is $^3/_2$ of a given arc is given. It also explains how to inscribe a chord subtended by an arch which is half of a given arc in a circle (the proposition of *El*. here alluded to is III.30), and why twice the half of a chord subtended by an arch which is triple of a given arc has a third part. The second part of the scholium rests on a misunderstanding of Ptolemy’s ἐγγράψομεν «we will be able to inscribe [in the table]», taken to mean «to inscribe [in a circle]». Against this mistake warns Theon at *in Alm. I.10*, *iA*, 486.15–19, taken up in sch. **28**, and also part of the long excerpt at f. 16v of **D** (see n. 17 above). *d*) Both in **B** and in **C**, the scholium is in the lower margin, and it is not preceded by any conventional sign. In **K**, the copyist makes the scholium preceded by an annotation of the most active later scholiast (12th c.) of **B**, taking as citation αὕτη κατά τε τὴν σύνθεσιν καὶ τὴν ὑπεροχήν «this [will enable us to complete] by sum or by difference [all the remaining chords]» (42.14–15). In **B**, the scholium is partly shaped as sch. **1**. *e*) Somehow in opposition to sch. **28**, the abuse of language of identifying a chord and the arc subtending it is present throughout sch. **29**. This fact, the problem with designations, and the misunderstanding pointed out in point *c* above, suggest that sch. **28** and **29** come from different layers. See sch. **3** for the phrase δὶς γενομένη at line 5.

30

Text. ἐπὶ γὰρ τῶν μείζονων περιφερειῶν οὐχ ὁρίζει τὰς πηλικότητας τῶν ὑποτεινουσῶν εὐθειῶν

Transl. For as regards greater arcs he [*scil*. Ptolemy] does not determine the [numerical] values of the chords subtending [them]

Comm. *a*) Vat. gr. 1594, f. 19r marg. ext., Marc. gr. 313, f. 44r marg. ext. *b*) *Ad Alm*. I.10, 43.3–5 κἂν μὴ πρὸς τὸ καθόλου δύνηται τὰς πηλικότητας ὁρίζειν, ἐπί γε τῶν οὕτως ἐλαχίστων τὸ πρὸς τὰς ὡρισμένας ἀπαράλλακτον δύναιτ' ἂν συντηρεῖν «thought it [*scil*. the subsequent lemma] cannot in general exactly determine the [numerical] values, in the case of such very small [values] it can determine them with a negligibly small error». *c*) The *relatum* presents the subsequent «4th short lemma», whose aim is to back up the procedure to be followed (at 43.1, Ptolemy has for this the verbal form μεθοδεύσομεν «set out as a procedure», but the calculation is set out in a demonstrative style—Acerbi 2012, item *b* on 180–181) in order to determine «the [chord] under 1 degree from that under $1\frac{1}{2}$ degree and that under $\frac{1}{2}$ degree» (43.1–2). *d*) Both in **B** and in **C**, sch. **30** is located beside the *relatum*. *e*) The πηλικότης is the «[numerical] value» of any magnitude, in this case of a straight line. As we have seen in the Introduction (n. 67 above), the πηλικότης of a ratio is the main ingredient in the definition of «compounded ratio».

31

Text. λῆμμα

Transl. Lemma

Comm. *a*) Vat. gr. 1594, f. 19r marg. ext., Marc. gr. 313, f. 44r marg. ext. *b*) *Ad Alm*. I.10, 43.6 λέγω γὰρ ὅτι, ἐὰν ἐν κύκλῳ «in fact, I say that, if in a circle» ff. *c*) This theorem is called λημμάτιον by Ptolemy (43.3), a denomination also adopted by Theon, *in Alm. I.10*, *iA*, 490.8. It states that, if in a circle ΑΒΓΔ two unequal chords ΑΒ < ΒΓ are drawn, chord ΓΒ to chord ΒΑ has a lesser ratio than arc ΒΓ to arc ΒΑ (Fig. 6). On this lemma—also proved in Theon, *in Alm. I.10*, *iA*, 490.8–492.5, and as a scholium to Aristarchus' *Magn*., in Fortia d'Urban (1810, 121–122)—and on a similar lemma—attested as *Opt.* 8 **A**; *Opt.* 8 **B**; Theon, *in Alm. I.3*, *iA*, 358.1–11; as sch. 450 to *Sph.* 3.11, in Heiberg (1927, 195.21–196.22) = Czinczenheim (2000, 435.1–20); as a scholium to Pappus, *Coll.* V.4, in Hultsch (1876–1878, 1167.5–23); in *Prol.*, in Acerbi, Vinel, and Vitrac (2010, 121.18–122.5)—see Knorr (1985) and Acerbi, Vinel, and Vitrac (2010, 110–112, 134, nn. 24–30 on 146–148, 177–179). Sch. **31**–**37**, all taken verbatim from Theon's commentary except for sch. **31** and **37**, make some arguments or statements explicit that justify specific deductive steps in the proof of the final theorem of *Alm*. I.10, 43.6–45.8. *d*) Both in **B** and in **C**, sch. **31** is in majuscule and is located beside the beginning of the *relatum*. *e*) Ptolemy provides his theorem with a general enunciation: ἐὰν ἐν κύκλῳ διαχθῶσιν ἄνισοι δύο εὐθεῖαι, ἡ μείζων πρὸς τὴν ἐλάσσονα ἐλάσσονα λόγον ἔχει ἤπερ ἡ ἐπὶ τῆς μείζονος εὐθείας περιφέρεια πρὸς τὴν ἐπὶ τῆς ἐλάσσονος «if in a circle two unequal straight lines are drawn across, the greater to the lesser has a ratio less than the arc on the greater straight line to that on the lesser» (43.6–9).

32

Text. ἐπεὶ καὶ περιφέρεια ἡ ΑΔ τῇ ΔΓ διὰ τὸ καὶ τὰς πρὸς τῷ Β γωνίας ἴσας εἶναι

ΑΔ] ΠΔ **CG** | τὸ καὶ τὰς] τὸ καὶ legi nequit ob truncatum folium **C** : καὶ **G**

Transl. Since arc ΑΔ is also [equal] to ΔΓ, because the angles at Β are also equal

Comm. *a*) Vat. gr. 1594, f. 19r marg. ext., Marc. gr. 313, f. 44v marg. ext., Vat. gr. 184, f. 87r marg. inf. *b*) *Ad Alm*. I.10, 43.21–44.1 ἴση μέν ἐστιν ἡ ΓΔ εὐθεῖα τῇ ΑΔ «straight line ΓΔ is equal to ΑΔ». *c*) In the configuration of sch. **31** and Fig. 6, Δ is the middle point of the arc ΑΓ opposite to Β. The scholium recalls the conditions that are used in *El*. III.29 and III.26 in order to prove the *relatum* from the first of the two conditions (ΑΔ = ΔΓ), and the first condition from the second (angle Β is bisected), respectively. The text of the scholium coincides with Theon, *in Alm. I.10*, *iA*, 491.5–6, where it immediately follows a clause almost identical with the *relatum*. *d*) Both in **B** and in **C**, sch. **32** is located beside the *relatum*.

33

Text. μείζων δὲ ἡ ΓΕ τῆς ΕΑ διὰ τὸ πάλιν ἴσην εἶναι τὴν μὲν ΑΔ τῇ ΔΓ καὶ κοινὴν τὴν ΔΕ, καὶ γωνίαν τὴν ὑπὸ ΒΔΓ τῆς ΒΔΑ μείζονα, ἐπεὶ καὶ περιφέρεια ἡ ΓΒ περιφερείας τῆς ΒΑ μείζων ἐστίν, ὥστε καὶ βάσιν τὴν ΓΕ βάσεως τῆς ΕΑ γίνεσθαι μείζονα, καὶ δῆλον ὅτι ἡ ἀπὸ τοῦ Δ κάθετος ἐπὶ τὴν ΑΓ ἐπὶ τὴν ΕΓ πεσεῖται διὰ τὸ ἴσην τὴν ΔΓ τῇ ΑΔ, τὴν δὲ ΓΕ μείζονα τῆς ΕΑ.

2 ante ΒΔΑ add. ὑπὸ Th. **3** μείζων ἐστίν om. Th. | post βάσιν add. ἄρα Th. | γίνεσθαι **G**Th. : γίν(εται) **BC**
4 ἡ ἀπὸ — ἐπὶ τὴν ΑΓ om. Th. | τὴν^{2}] τῆς Th. | post ἴσην add. εἶναι **G**Th.

Transl. And ΓΕ is greater than ΕΑ because, again, ΑΔ is equal to ΔΓ and ΔΕ in common, and angle ΒΔΓ greater than ΒΔΑ, since arc ΓΒ is also greater than arc ΒΑ, so that base ΓΕ also becomes greater than base ΕΑ, and it is clear that the [straight line] drawn from Δ perpendicular to ΑΓ will fall on ΕΓ because ΔΓ is equal to ΑΔ and ΓΕ greater than ΕΑ.

Comm. *a*) Vat. gr. 1594, f. 19r marg. ext., Marc. gr. 313, f. 44v marg. ext., Vat. gr. 184, f. 87r marg. inf. *b*) *Ad Alm*. I.10, 44.1 μείζων δὲ ἡ ΓΕ τῆς ΕΑ «and ΓΕ is greater than ΕΑ». *c*) In the configuration of sch. **31** and Fig. 6, ΑΒ and ΒΓ are unequal chords with ΑΒ < ΒΓ; ΒΔ is the bisector of the angle at Β, point Δ lying on the circumference; point Ε is the intersection of ΒΔ and chord ΑΓ. The exegesis of the *relatum* is given in form of a short deductive chain with nested explicative clauses (διὰ τὸ …, ἐπεὶ καὶ … «because …, since … also …»), followed by a remark about the position of the foot of perpendicular ΔΖ. The first statement (ΑΔ = ΔΓ), the third statement (arc ΓΒ > arc ΒΑ), and the conclusion (ΓΕ > ΕΑ) in the deductive chain are justified by the statement in sch. **32**, *El.* III.29, and I.24, respectively; the transfer of the inequality from arcs to the angles standing upon them in the same circumference is not a theorem of *El.* The final remark coincides almost exactly with sch. **34**. The text of the short deductive chain coincides with Theon, *in Alm. I.10*, *iA*, 491.6–9; the text of the final remark, once the designation ἡ ἀπὸ τοῦ Δ κάθετος ἐπὶ τὴν ΑΓ ἐπὶ τὴν ΕΓ is omitted, with ibid., 491.10–11. *d*) Both in **B** and in **C**, sch. **33** is located beside the *relatum*. In **G**, sch. **29**, **32**, and **33** are linked so as to produce a seemingly continuous text.

34

Text. καὶ δῆλον ὅτι ἐπὶ τῆς ΕΓ πεσεῖται διὰ τὸ ἴσην εἶναι τὴν ΔΓ τῇ ΑΔ, τὴν δὲ ΓΕ μείζονα τῆς ΕΑ

1 ἴσην] ἴσον **C**

Transl. And it is clear that it [*scil*. perpendicular ΔΖ] will fall on ΕΓ because ΔΓ is equal to ΑΔ, and ΓΕ greater than ΕΑ

Comm. *a*) Vat. gr. 1594, f. 19r interc., Marc. gr. 313, f. 44v marg. sup. *b*) *Ad Alm*. I.10, 44.2–3 ἤχθω δὴ ἀπὸ τοῦ Δ κάθετος ἐπὶ τὴν ΑΕΓ ἡ ΔΖ «let a [straight line] ΔΖ be drawn from Δ perpendicular to ΑΕΓ». *c*) In the configuration of sch. **31** and **33** and Fig. 6, since Δ bisects arc ΑΓ, Ζ is the middle point of chord ΑΓ. The explanation almost coincides with the last statement in sch. **33**. The text of the scholium coincides with Theon, *in Alm. I.10*, *iA*, 491.10–11, where it immediately follows a clause identical with the *relatum*. *d*) In **B**, sch. **34** is located beside the *relatum*; in **C**, it is placed in the upper margin. *e*) The duplication pointed out in point *c* above suggests that sch. **33** and **34** come from different textual layers.

35

Text. τὴν γὰρ μείζονα γωνίαν ὑποτείνει

Transl. For it subtends the greater angle

Comm. *a*) Vat. gr. 1594, f. 19r interc., Marc. gr. 313, f. 44v marg. ext. *b*) *Ad Alm*. I.10, 44.3–4 ἐπεὶ τοίνυν μείζων ἐστὶν ἡ μὲν ΑΔ τῆς ΕΔ, ἡ δὲ ΕΔ τῆς ΔΖ «now, since ΑΔ is greater than ΕΔ, and ΕΔ than ΔΖ». *c*) The scholium gives the condition used in *El*. I.19 in order to prove both statements in the *relatum* (see Fig. 6). The text of the scholium coincides with Theon, *in Alm. I.10*, *iA*, 491.12, where it immediately follows a clause almost identical with the first clause of the *relatum*. *d*) In **B**, sch. **35** is located beside the *relatum*; in **C**, it is misplaced beside the text at 45.1–3 and after sch. **36**.

36

Text. τὸ ΔΕΖ ἄρα τρίγωνον πρὸς τὸν ΔΕΘ τομέα ἐλάττονα λόγον ἔχει ἤπερ τὸ ΔΕΑ τρίγωνον πρὸς τὸν ΔΕΗ τομέα· ἐναλλάξ

Transl. Therefore triangle ΔΕΖ to sector ΔΕΘ has a lesser ratio than triangle ΔΕΑ to sector ΔΕΗ; alternately

Comm. *a*) Vat. gr. 1594, f. 19r marg. ext., Marc. gr. 313, f. 44v marg. ext. *b*) *Ad Alm*. I.10, 44.7–9 καὶ ἐπεὶ ὁ μὲν ΔΕΘ τομεὺς μείζων ἐστὶν τοῦ ΔΕΖ τριγώνου, τὸ δὲ ΔΕΑ τρίγωνον μεῖζον τοῦ ΔΕΗ τομέως «and, since sector ΔΕΘ is greater than triangle ΔΕΖ, and triangle ΔΕΑ than sector ΔΕΗ». *c*) In the configuration of sch. **31**, **33**, and **34**, and of Fig. 6, Η and Θ are the points at which the circle with center Δ and radius ΔΕ meets ΔΑ and ΔΖ produced, respectively. The scholiast supplies a step allowing a smoother transition from the *relatum* to the subsequent assertion. The ἐναλλάξ proposition is *El*. V.16. The text of the scholium coincides with Theon, *in Alm. I.10*, *iA*, 491.17–18, where it immediately follows a clause with a meaning identical with that of the *relatum*. *d*) In **B**, sch. **36** is located beside the *relatum*; in **C**, it is misplaced beside the text at 44.19–21 and before sch. **35**. In **B**, the scholium is shaped as sch. **1**.

37

Text. διὰ τὸ γ′ τοῦ ϛ′ τῶν Εὐκλείδου

Transl. By the 3rd of the 6th of those of Euclid

Comm. *a*) Vat. gr. 1594, f. 19v marg. ext., Marc. gr. 313, f. 44v marg. int. *b*) *Ad Alm*. I.10, 45.3–4 ἀλλ’ ὡς μὲν ἡ ΓΕ εὐθεῖα πρὸς τὴν ΕΑ, οὕτως ἡ ΓΒ εὐθεῖα πρὸς τὴν ΒΑ «but, as straight line ΓΕ to ΕΑ, so is straight line ΓΒ to ΒΑ». *c*) The reference to the proposition of *El.* is correct: ΓΒ and ΒΑ are the sides of triangle ΑΒΓ right-angled at Β, ΒΕ is the bisector of angle Β, ΓΕ and ΕΑ are the segments cut off by the bisector on the hypotenuse (Fig. 6). The text of the scholium is the transformation into a *paragraphê* of the citation in Theon, *in Alm. I.10*, *iA*, 492.1–2, where a compressed version of the enunciation of *El*. VI.3 is quoted. *d*) In **B**, sch. **37** is located four lines above the beginning of the *relatum*, probably in order to save space for the lengthy sch. **40**; in **C**, sch. **37** is located beside the *relatum*.

38

Text. τὸ λῆμμα

ante τὸ expect. διὰ

Transl. [By] the lemma

Comm. *a*) Vat. gr. 1594, f. 19v interc., Marc. gr. 313, f. 45r marg. ext. *b*) *Ad Alm*. I.10, 45.15–18 ἐπεὶ ἡ ΑΓ εὐθεῖα πρὸς τὴν ΒΑ εὐθεῖαν ἐλάσσονα λόγον ἔχει ἤπερ ἡ ΑΓ περιφέρεια πρὸς τὴν ΑΒ «since chord ΑΓ to chord ΒΑ has a lesser ratio than arc ΑΓ to ΑΒ». *c*) Two chords ΑΒ, ΑΓ having a common endpoint Α are drawn in a circle (Fig. 7). The *relatum* is nothing but an instantiated citation of the enunciation of the preceding lemma; see sch. **31** for the denomination λῆμμα. Ptolemy applies the Approximation Lemma twice, and shows that $(\frac{2}{3})ch(1\frac{1}{2}^{\circ}) < ch(1^{\circ}) < (\frac{4}{3})ch(\frac{3}{4}^{\circ})$. Since he also had shown that $ch(1\frac{1}{2}^{\circ})$ = 1;34,15 and $ch(\frac{3}{4}^{\circ})$ = 0;47,8, multiplying by the coefficients and truncating to second sixtieths gives 1;2,50 both as a lower and as an upper bound of $ch(1^{\circ})$; this entails that $ch(1^{\circ})$ = 1;2,50 up to second sixtieths (using the assumed values of $ch(1\frac{1}{2}^{\circ})$ and $ch(\frac{3}{4}^{\circ})$ as if they were exact, the lower bound is exact, the non-truncated upper bound is 1;2,50,40). In the first application, arc ΑΒ is $\frac{3}{4}^{\circ}$, arc ΑΓ is 1°; in the second application, arc ΑΒ is 1°, arc ΑΓ is $1\frac{1}{2}^{\circ}$; see 45.9–46.14. Sch. **38**–**44** complete, mainly by providing more accurate numerical values, Ptolemy's calculation of $ch(1^{\circ})$; in particular, sch. **39**–**42** refer to the deductive chain at 45.20–23: ἀλλὰ ἡ ΑΒ εὐθεῖα ἐδείχθη τοιούτων 𐆊 μζʹ ηʺ, οἵων ἐστὶν ἡ διάμετρος ρκ· ἡ ἄρα ΓΑ εὐθεῖα ἐλάσσων ἐστὶν τῶν αὐτῶν α βʹ νʺ· ταῦτα γὰρ ἐπίτριτά ἐστιν ἔγγιστα τῶν 𐆊 μζʹ ηʺ «but chord ΑΒ was shown [to be] so many 0;47,8 of which the diameter is 120; therefore chord ΓΑ is less than 1;2,50 of the same, for these are very nearly $\frac{4}{3}$ of 0;47,8». *d*) Both in **B** and in **C**, sch. **38** is in majuscule and is located beside the beginning of the *relatum*. It is very likely that an initial διά was lost at some stage of copying. *e*) The correlative τοιούτων ... οἵων … «so many … of which …» in the argument read in point *c* specifies that the unit of measurement is one of the 120 τμήματα «parts» in which the diameter of the reference circle is divided.

39

Text. 𐆊 μζʹ ζʺ λθʹʹʹ

Transl. 0;47,7,39

Comm. *a*) Vat. gr. 1594, f. 19v interc., Marc. gr. 313, f. 45r marg. ext. *b*) *Ad Alm*. I.10, 45.20 ἀλλὰ ἡ ΑΒ εὐθεῖα ἐδείχθη […] 𐆊 μζʹ ηʺ «but chord ΑΒ was shown 0;47,8». *c*) Ptolemy gives at 41.2–3, without calculations, the numerical value 0;47,8 for the chord subtended by ¾°; on that occasion, Theon, *in Alm. I.10*, *iA*, 482.7–483.8, performs the calculations and obtains a more precise numerical value of 0;47,7,39 (see sch. **40**, **41**, and **44**). *d*) Both in **B** and in **C**, sch. **39** is located beside the *relatum*. Both **B** and **C** have bars over the letters representing numerals, except for the second ζ. This shows the extent to which both copyists were faithful to the model.

40

Text. εἰδέναι χρὴ ὅτι, ἐὰν δύο τινῶν μεγεθῶν ἐκκειμένων θελήσωμεν αὐτῶν αὐξῆσαι τὸν λόγον, τὸ μεῖζον αὔξομεν ἢ τὸ ἔλαττον μειοῦμεν. ἐὰν μὲν γὰρ τὸ μεῖζον αὐξήσωμεν, μείζων ‹ὁ› λόγος γενήσεται· τὸ γὰρ μεῖζον πρὸς τὸ αὐτὸ μείζονα λόγον ἤπερ τὸ ἔλαττον. κἂν τὸ ἔλαττον μειώσωμεν, μείζων ὁ λόγος πάλιν γενήσεται· τὸ γὰρ αὐτὸ πρὸς τὸ ἔλαττον μείζονα λόγον ἤπερ πρὸς τὸ μεῖζον. ἐὰν δὲ θελήσωμεν αὐτῶν μειῶσαι τὸν λόγον, τὸ ἀνάπαλιν ποιήσομεν· ἢ τὸ μεῖζον μειοῦμεν ἢ τὸ ἔλαττον αὔξομεν. ἐὰν γὰρ τὸ μεῖζον μειώσωμεν, ἐλάττων ὁ λόγος γενήσεται· τὸ γὰρ ἔλαττον πρὸς τὸ αὐτὸ ἐλάττονα λόγον ἔχει ἤπερ τὸ μεῖζον. ἐὰν δὲ τὸ ἔλαττον αὐξήσωμεν, ἐλάττων ὁ λόγος πάλιν γενήσεται· τὸ γὰρ αὐτὸ πρὸς τὸ μεῖζον ἐλάττονα λόγον ἔχει ἤπερ πρὸς τὸ ἔλαττον. ἐπεὶ οὖν ἡ ΑΒ περιφέρεια μοίρας ∠ δ′, ἡ δὲ ΑΓ μοίρας α, ἡ ΑΓ ἄρα περιφέρεια τῆς ΑΒ περιφερείας ἐπίτριτός ἐστιν· ἔστι δὲ καὶ τὰ α β′ ν″ τῶν ō μζ′ η″ ἐπίτριτα ἐγγύς (ἔθος γὰρ αὐτῷ τῶν τρίτων ἑξηκοστῶν καταφρονεῖν, ὅταν ὃ πλεῖον ὑπερέχει μὴ πολυπλασιάζῃ τὸ παραληφθέν)· ἔστιν ἄρα ὡς ἡ ΑΓ περιφέρεια πρὸς τὴν ΑΒ περιφέρειαν οὕτως τὰ α β′ ν″ πρὸς τὰ ō μζ′ η″· ἐδείχθη δὲ ἡ ΑΓ | περιφέρεια πρὸς τὴν ΑΒ περιφέρειαν μείζονα λόγον ἔχουσα ἤπερ ἡ ΑΓ εὐθεῖα πρὸς τὴν ΑΒ εὐθεῖαν· ἡ ΑΓ ἄρα εὐθεῖα πρὸς τὴν ΑΒ εὐθεῖαν ἐλάττονα λόγον ἔχει ἤπερ τὰ α β′ ν″ πρὸς τὰ ō μζ′ η″· δεῖ ἄρα μειῶσαι τὸν τῶν α β′ ν″ πρὸς τὰ ō μζ′ η″ λόγον ἵνα ὁ αὐτὸς γένηται τῷ τῆς ΑΓ πρὸς ΑΒ. καὶ ἐπειδή, ὡς εἴρηται, μειοῦται λόγος ἢ τοῦ μείζονος μειουμένου ἢ τοῦ ἐλάττονος αὐξομένου καὶ οὐ δυνατὸν αὐξῆσαι τὸν ἐλάσσονα (εἴληπται γὰρ ἐκ τῶν γραμμῶν τῷ τῆς διχοτομίας θεωρήματι), τὸν ἄρα μείζονα μειώσομεν· ἡ ΑΓ ἄρα εὐθεῖα πρὸς τὴν ΑΒ εὐθεῖαν λόγον ἔχει ὃν ἐλάττων τῶν α β′ ν″ πρὸς τὰ ō μζ′ η″. πάλιν ἡ ΑΓ περιφέρεια μοίρας ἐστὶ α ∠, ἡ δὲ ΑΒ μοίρας α· ἡ ‹ἄρα› ΑΓ περιφέρεια τῆς ΑΒ περιφερείας ἡμιολία ἐστίν· ἔστι δὲ καὶ τὰ α λδ′ ιε″ τῶν α β′ ν″ ἡμιόλια· ἔστιν ἄρα ὡς ἡ ΑΓ περιφέρεια πρὸς τὴν ΑΒ περιφέρειαν οὕτως τὰ α λδ′ ιε″ πρὸς τὰ α β′ ν″· ἐδείχθη δὲ ἡ ΑΓ περιφέρεια πρὸς τὴν ΑΒ περιφέρειαν μείζονα λόγον ἔχουσα ἤπερ ἡ ΑΓ εὐθεῖα πρὸς τὴν ΑΒ εὐθεῖαν· ἡ ΑΓ ἄρα εὐθεῖα πρὸς τὴν ΑΒ εὐθεῖαν ἐλάττονα λόγον ἔχει ἤπερ α λδ′ ιε″ πρὸς τὰ α β′ ν″· δεῖ ἄρα μειῶσαι τὸν τῶν α λδ′ ιε″ πρὸς α β′ ν″ λόγον ἵνα ὁ αὐτὸς γένηται τῷ τῆς ΑΓ πρὸς ΑΒ. ἐπεὶ οὖν οὐ δύναται ὁ μείζων μειωθῆναι (εἴληπται γὰρ ἐκ τῶν γραμμῶν τῷ τῆς διχοτομίας θεωρήματι), τὸν ἄρα ἐλάττονα αὐξήσομεν· ἡ ἄρα ΑΓ εὐθεῖα πρὸς τὴν ΑΒ εὐθεῖαν λόγον ἔχει ὃν α λβ′ ιε″ πρὸς μείζονα τῶν α β′ ν″· ἐδείχθη δὲ ἡ ΑΓ πρὸς τὴν ΑΒ εὐθεῖαν λόγον ἔχουσα ὃν τὰ ἐλάττονα τῶν α β′ ν″ πρὸς τὰ ō μζ′ η″· ἡ ἄρα τὴν μοῖραν α ὑποτείνουσα καὶ μείζων ἐστὶ τῶν α β′ ν″ καὶ ἐλάττων, καὶ ἐχρῆν ἐπαγαγεῖν· ὅπερ ἄτοπον. ἀλλ᾽ ἐπειδὴ ἐκ τῶν ἀκριβεστέρων ἀριθμῶν ἡ ὑποτείνουσα μοῖραν α ἐλάττων μέν ἐστι α β′ ν″ ιβ‴ μείζων δὲ α β′ ν″ καὶ οὐδὲν ἄτοπον συνήγετο, οὐκ εἶπε τὸ ὅπερ ἄτοπον ἀλλ’ ἐπήγαγεν· ὥστε, ἐπεὶ τῶν αὐτῶν ἐδείχθη καὶ μείζων καὶ ἐλάττων ἡ τῆν μοῖραν α ὑποτείνουσα εὐθεῖα, καὶ ταύτην δηλονότι ἕξομεν τοιούτων α β′ ν″.

3 ὁ addidi : om. codd. **5** δὲ om. **C** | αὐτῶν] αυ$^{\tau}$ **BC** : αὐτὸν **K** **6** ποιήσομεν] –σωμεν codd. **10** δὲ ex δ′ fecit m. 2 **K** : δ′ **BC** | μοίρας] μοίραν **K** | ΑΓ] ΓΑ **C** **11** τὰ] τὸ **C** | τῶν] τὴν **K** | αὐτῷ om. **BK** **12** ὃ πλεῖον ὑπερέχει] ο πλει^{o} (χρόνος) **BC** : ὁ πλείων χρόνος **K** **13** τὰ ex ἡ fecit m. 2 **K** : ἡ **B** : om. **C** **16** τὰ^{1} supra lineam **K** : om. **BC** | τῶν] τὴν **K** **17** τῷ] τῶν **K** **19** τὸν ἐλάσσονα] τὸ ἔλαττον **K** | τῷ — θεωρήματι] τῷ — θεω$^{\rho}$ **BC** : τῶν — θεωρίων **K** **20** ἐλάττων] ἔλαττον **C** **22** ἄρα addidi : om. codd. | α^{2}] λ **C** **25** πρὸς τὴν ΑΒ^{1} supra lineam m. 2 **K** **26** ἐλάττονα] ἔλαττον **K** | ante α^{1} add. ἡ m. 2 **K** | τὸν] τὴν m. 2 **K** | α λδ′ ιε″^{1} — **27** οὖν marg. m. 2 **K** | τῷ] τῶν m. 2 **K** | δύναται ob truncatum folium des. **C** **28** τῷ — θεωρήματι] τῶν — θεω$^{\rho}$ **B** : τῶν — θεωρημάτων **K** | τὸν — **29** ἐλάττονα] τὸ ἄρα ἔλαττον **K** **30** τῶν] τὸν **BK** | εὐθεῖαν] εὐ$^{\theta}$ **B** : εὐθεῖα **K** **31** μείζων comp. **B** : μείζονα **K** **33** μοῖραν] μοίρα **K**

Transl. One must know that, any two magnitudes being set out, if we want to increase their ratio, we may increase the greater or lower the lesser. For if we increase the greater, the ratio will become greater, for the greater has to the same a greater ratio than the lesser. Even if we lower the lesser, the ratio will again become greater, for the same has to the lesser a greater ratio than to the greater. If, on the contrary, we want to lower their ratio, we shall do the opposite: either we lower the greater or we increase the lesser. For if we lower the greater, the ratio will become lesser, for the lesser has to the same a lesser ratio than the greater. And if we increase the lesser, the ratio will again become lesser, for the same has to the greater a lesser ratio than to the lesser. Then since arc AB is of ½ ¼ degree, ΑΓ of 1 degree, therefore arc ΑΓ is $^{4}\!/_{3}$ of arc AB; and 1;2,50 also is approximately $^{4}\!/_{3}$ of 0;47,8 (for [Ptolemy] used to disregard third sixtieths whenever what is in excess does not happen to multiply what has been taken); therefore as arc ΑΓ is to arc AB, so 1;2,50 is to 0;47,8; and it was shown that arc ΑΓ has to arc AB a greater ratio than chord ΑΓ to chord AB; therefore chord ΑΓ has to chord AB a lesser ratio than 1;2,50 to 0;47,8; therefore one has to lower the ratio of 1;2,50 to 0;47,8 in order that it become the same as that of ΑΓ to AB. And since, as said, a ratio is lowered either if the greater is lowered or if the lesser is increased, and it is not possible to increase the lesser (for it was among the chords obtained by means of the theorem “by bisection”), therefore we shall lower the greater; therefore chord ΑΓ has to chord AB the ratio that [a number] less than 1;2,50 [has] to 0;47,8. Again, since arc ΑΓ is of 1½ degree, AB of 1 degree, [therefore] arc ΑΓ is $^{3}\!/_{2}$ of arc AB; and 1;34;15 also is $^{3}\!/_{2}$ of 1;2,50; therefore as arc ΑΓ is to arc AB, so 1;34;15 is to 1;2,50; and it was shown that arc ΑΓ has to arc AB a greater ratio than chord ΑΓ to chord AB; therefore chord ΑΓ has to chord AB a lesser ratio than 1;34;15 to 1;2,50; therefore one has to lower the ratio of 1;34;15 to 1;2,50 in order that it become the same as that of ΑΓ to AB. Then since the greater cannot be lowered (for it was among the chords obtained by means of the theorem “by bisection”), therefore we shall increase the lesser; therefore chord ΑΓ has to chord AB the ratio that 1;34;15 [has] to [a number] greater than 1;2,50; and it was shown that ΑΓ has to chord AB the ratio that less than 1;2,50 [has] to 0;47,8; therefore the [chord] subtending 1 degree is both greater and less than 1;2,50, and one should have concluded: which is absurd. Yet, since, according to more accurate numbers, the [chord] subtending 1 degree is less than 1;2,50,12 and greater than 1;2,50, and nothing absurd could ensue, he did not utter "which is absurd" but concluded: "so that, since the [chord] subtending 1 degree was shown to be both greater and less than the same amount, we shall clearly also establish this as 1;2,50."

Comm. *a*) Vat. gr. 1594, f. 19v marg. ext. et inf., Marc. gr. 313, f. 45r marg. inf., Vat. gr. 184, ff. 28v–29r. *b*) *Ad Alm*. I.10, 45.21–22 ἡ ἄρα ΓΑ εὐθεῖα ἐλάσσων ἐστὶν τῶν αὐτῶν α β′ ν″ «therefore chord ΓΑ is less than 1;2,50 of the same [parts in which diameter is 120]» (= citation in **K**). *c*) The scholium repeatedly applies and quotes *El*. V.8 in an attempt to provide some omitted deductive steps in—and to outline the rationale behind—Ptolemy's use of inequalities and of lower/upper bounds in the calculation of *ch*(1°)—as usual, the ratios are taken to be greater-to-lesser. The scholiast implicitly charges Ptolemy with a mistake in his dealing with such inequalities (see the commentary on sch. **38** for Ptolemy's argument), and holds that the mistake has to be corrected by adopting more accurate numerical values of one of the chords at issue; in this way one would get 1;2,50 < *ch*(1°) < 1;2,50,12, and «nothing absurd could ensue». The second part of the scholiast's argument is invalid because he forgot that chord AB (Fig. 7) refers to different objects in the two inequalities he compares. Nevertheless, the mathematical argument is redacted in a most formal style, and one may surmise that it was adapted from some commentary (maybe Pappus'; it was not extracted from Theon's, even if the assertion about uttering the clause "which is absurd" is found at 494.7). The argument is first expounded in general terms, then instantiated both in the configuration of the preceding theorem (see again sch. **38**) and by means of the numbers actually set out by Ptolemy. The final quote from *Alm*. is at 46.9–11, the scholiast having astutely omitted the ἔγγιστα «very nearly» that qualifies number 1;2,50. A similar discussion of the upper bound of Prolemy's inequality we find, developed at length but without additional mathematical lapses, in Theon, *in Alm. I.10*, *iA*, 494.6–495.11, who uses as better upper bounds first 1;2,50,40 (⁴⁄₃ of 0;47,8; see sch. **42**) and then 1;2,50,12 (⁴⁄₃ of 0;47,7,39). The latter value is introduced to forestall the objection that he had adopted an upper bound larger than one allowed by the subsequent rounding off (see also sch. **39**, **41**, and **44**)—in fact, a truncation. *d*) In **B**, the scholium is preceded by a *signe de renvoi*, also placed in the intercolumnar space, beside the *relatum*. In **C**, the scholium is in the lower margin, no *signe de renvoi* being added. The citation in **K** exactly corresponds to one single line in **B**. Both **B** and **C** have bars over the letters representing numerals. Note the corrupted clause at line 12, almost surely induced by the similarity of the abbreviations for the stems υπερεχ– and χρον–. *e*) Note the "citational article" τὸ in front of ὅπερ ἄτοπον at line 34; the metadiscursive connotation is here stronger than for the clauses introduced by forms of ἐπάγειν at lines 32 and 34–36. At line 2, note the connotation of potentiality of the present indicatives αὔξομεν and μειοῦμεν (cf. sch. **28**); the different structure of the parallel sentence at line 6 suggests that here we should not retain such a connotation in translation.

41

Text. α β′ ν″ ιβ‴

Transl. 1;2,50,12

Comm. *a*) Vat. gr. 1594, f. 19v interc., Marc. gr. 313, f. 45r marg. ext. *b*) *Ad Alm*. I.10, 45.22 α β′ ν″ «1;2,50». *c*). This numerical value comes from taking $\frac{4}{3}$ of the value 0;47,7,39, given by Theon, for the chord subtended by $\frac{3}{4}^{\circ}$ (see sch. **39**, **40**, and **44**). *d*) Both in **B** and in **C**, sch. **41** is located beside the *relatum*. Both **B** and **C** have bars over the letters representing numerals.

42

Text. ἔστι διὰ τὸ ἀκριβῶς ἐπίτριτα α β′ ν″ μ‴

τὸ om. **C** | ἀκριβῶς codd. : expect. ἀκριβὲς

Transl. For the sake of exactness, $\frac{4}{3}$ [of 0;47,8] is 1;2,50,40

Comm. *a*) Vat. gr. 1594, f. 19v marg. int., Marc. gr. 313, f. 45r marg. ext. *b*) *Ad Alm*. I.10, 45.22–23 ταῦτα γὰρ ἐπίτριτά ἐστιν ἔγγιστα τῶν ο̄ μζ′ η″ «for these [*scil*. 1;2,50] are very nearly $\frac{4}{3}$ of 0;47,8». *c*). This numerical value comes from taking $\frac{4}{3}$ of the value 0;47,8, given by Ptolemy, for the chord subtended by $\frac{3}{4}^{\circ}$. The text of the scholium corresponds to Theon, *in Alm. I.10*, *iA*, 494.15 (ταῦτα γὰρ ἀκριβῶς ἐπίτριτά ἐστιν τῶν ο̄ μζ′ η″). The scholiast adapted Theon's text but produced the incongruous expression διὰ τὸ ἀκριβῶς (note also the variant of **C**). *d*) Both in **B** and in **C**, sch. **42** is located beside the *relatum*; in **C** it is placed just under sch. **41**, in **B** exactly on the opposite side of the column. Both **B** and **C** have bars over the letters representing numerals.

43

Text. τὰ γὰρ α λδ′ ιε″ ἡμιόλιά ἐστι ἀκριβῶς τῶν α β′ ν″

Transl. For 1;34,15 is exactly $\frac{3}{2}$ of 1;2,50

Comm. *a*) Vat. gr. 1594, f. 19v interc., Marc. gr. 313, f. 45r marg. ext. *b*). *Ad Alm*. I.10, 46.5–7 ἀλλὰ τὴν ΑΓ ἀπεδείξαμεν […] οὖσαν α λδ′ ιε″ «but we showed that ΑΓ […] is 1;34,15». *c*) The scholiast's statement is correct; no rounding off or truncation is necessary. The numerical value 1;34,15 of the chord ΑΓ subtended by $1\frac{1}{2}^{\circ}$ was stated, without calculations, by Ptolemy at 40.21–41.2; on that occasion, Theon, *in Alm. I.10*, *iA*, 481.4–482.6, had performed the calculations and obtained the same value. At 46.7–8, Ptolemy makes the same statement as the scholium: ἡ ἄρα AB εὐθεῖα μείζων ἐστὶν τῶν αὐτῶν α β′ ν″· τούτων γὰρ ἡμιόλιά ἐστιν τὰ προκείμενα α λδ′ ιε″ «therefore chord AB is greater than the 1;2,50 themselves, for the proposed 1;34,15 are $\frac{3}{2}$ of these». The text of the scholium corresponds both to this sentence and to Theon, *in Alm. I.10*, *iA*, 494.21 ([…] α β′ ν″· τούτων γὰρ ἡμιόλιά ἐστι τὰ α λδ′ ιε″). *d*) In **B**, sch. **43** is located beside the *relatum*; in **C** it lies in the extreme outer margin, beside the last line of the main text and the intervening diagram. Both **B** and **C** have bars over the letters representing numerals.

44

Text. ἐπεὶ τῶν μὲν α β′ ν″ ιβ‴ ἐλάττων ἐδείχθη, τῶν δὲ α β′ ν″ μείζων ὡς ἔγγιστα α β′ ν″

Transl. Since it [*scil*. the chord subtending 1º] was shown [to be] less than 1;2,50,12, and on the other hand 1;2,50 is as nearly as possible greater than 1;2,50

Comm. a) Vat. gr. 1594, f. 19v marg. int., Marc. gr. 313, f. 45r marg. ext. *b*) *Ad Alm*. I.10, 46.7–9 τῶν αὐτῶν α β′ ν″· τούτων γὰρ ἡμιόλιά ἐστιν τὰ προκείμενα α λδ′ ιε″· ὥστε, ἐπεὶ τῶν αὐτῶν ἐδείχθη καὶ μείζων καὶ ἐλάσσων «than the same 1;2,50, for the proposed 1;34,15 are $\frac{3}{2}$ of these; so that since it [*scil*. the chord subtending 1º] was proved greater and less than» ff. *c*) The scholiast explains the gist of Ptolemy's argument, providing his own favorite numbers for insertion in the final statement of the *relatum*: the chord subtended by 1º was shown on the one hand to be less than $\frac{4}{3}$ of the chord subtended by $\frac{3}{4}$º (= $\frac{4}{3}$×0;47,7,39 = 1;2,50,12), on the other to be greater than $\frac{2}{3}$ of the chord subtended by $1\frac{1}{2}$º (= $\frac{2}{3}$×1;34,15 = 1;2,50); as a consequence, 1;2,50 ≤ *ch*(1º) < 1;2,50,12. The second clause in the scholium formulates the first inequality. As said, Ptolemy states 1;2,50 both as a lower and as an upper bound of *ch*(1º). The numerical value 1;2,50,12 is the one given in sch. **41** (see also sch. **30** and **40**) and derives from Theon, who in his turn performs again the entire calculation both using the value 1;2,50,40 = $\frac{4}{3}$×0;47,8 (cf. sch. **42**)—the latter being the value of the chord subtended by $\frac{3}{4}$º given by Ptolemy—and the more accurate value 1;2,50,12 that he himself had calculated (see again sch. **41**). *d*) Both in **B** and in **C**, sch. **44** is located beside the *relatum*. Both **B** and **C** have bars over the letters representing numerals.

45

Text. τοῦ ἡμιμοιρίου

Transl. Of a half-degree

Comm. *a*) Vat. gr. 1594, f. 20r marg. int., Marc. gr. 313, f. 45v marg. ext. *b*) *Ad Alm*. I.10, 46.18–19 ἐκ δὲ τῆς ὑπεροχῆς τῆς πρὸς τὰς γ μοίρας καὶ τῆς ὑπὸ τὰς β ∠ διδομένης «and the [chord subtending] $2\frac{1}{2}$ degrees being also given by difference with respect to 3 degrees». *c*) The scholium restores the determination τοῦ ἡμιμοιρίου «of $\frac{1}{2}$ degree» of the ὑπεροχή «difference» allowing one to calculate the chord subtending $2\frac{1}{2}^{\circ}$ from that subtending 3°. In the immediately preceding clause, which exemplifies the use of the theorem "by composition" and whose structure is exactly parallel to the *relatum*, Ptolemy had in fact written ἐκ μὲν τῆς πρὸς τὴν μίαν ἥμισυ μοῖραν […] συνθέσεως τοῦ ἡμιμοιρίου δεικνυμένης τῆς ὑπὸ τὰς β μοίρας «the [chord subtending] 2 degrees being shown by composition of a half-degree […] with respect to $1\frac{1}{2}$ degree» (46.15–18). Sch. **45**–**46** clarify two deductive steps in Ptolemy's description of the procedure for completing the Table of Chords (46.14–20). *d*) Both in **B** and in **C**, sch. **45** is in majuscule and is located beside the *relatum*.

46

Text. εὑρὼν ἐν τεταρτημορίῳ ἔχει τὴν λείπουσαν εἰς τὸ ἡμικύκλιον

Transl. After operating in a quadrant, he gets the complement to a semicircle

Comm. *a*) Vat. gr. 1594, f. 20r marg. int., Marc. gr. 313, f. 45v marg. ext. *b*) *Ad Alm*. I.10, 46.19–20 ὡσαύτως δὲ καὶ ἐπὶ τῶν λοιπῶν «and similarly also for the remaining [chords]». *c*) The scholium explains that it is enough to carry out the procedure described by Ptolemy (namely, adding or subtacting $\frac{1}{2}^{\circ}$ from given arcs) only for the first quadrant. For the other quadrants one operates by calculating chords of supplementary arcs. A remark to the same effect is developed in Theon, *in Alm. I.10*, *iA*, 503.12–19. *d*) Both in **B** and in **C**, sch. **46** is located beside the *relatum*. In the manuscripts, the term τεταρτημόριον is always written δʹ M^{o}. See sch. **11** and **27** for the expression ἡ λείπουσα εἰς τὸ ἡμικύκλιον «the [chord] complement to a semicircle».

47

Text. τῶν μὲν μεταξὺ τοῦ ἡμιμοιρίου μερῶν λαμβάνομεν τὰς ἐπιβαλλούσας πηλικότητας τῶν εὐθειῶν (οἷον τῶν μοιρῶν ι λεπτῶν ιε) πεντεκαιδεκάκι ποιοῦντες τὰ παρακείμενα ταῖς ι μοίραις τῆς περιφερείας ἐν τῷ τρίτῳ σελιδίῳ καὶ τὰ γινόμενα προστιθέντες τῇ ὑποτεινούσῃ τὰς ι μοίρας ἐν τῷ δευτέρῳ σελιδίῳ. ἀνάπαλιν δὲ δοθείσης τινὸς εὐθείας μεταξὺ τῶν ἐν τῷ κανόνι ἐκτεθειμένων πιπτούσης, ἡ ἐπ' αὐτῆς περιφέρεια δίδοται λαμβανόντων ἡμῶν τὴν ὑπεροχὴν τῆς εὐθείας πρὸς τὴν ἔγγιστα ἐλάττονα παρακειμένην καὶ ἔτι τὰ παρακείμενα ὡς ἐν τῷ τρίτῳ σελιδίῳ ἑξηκοστὰ τῇ ἔγγιστα ἐλάττονι, καὶ παρὰ ταῦτα μερίζοντες τὴν ὑπεροχὴν τῶν β εὐθειῶν καὶ τὰ γινόμενα ἐκ τοῦ μερισμοῦ προστιθέντες τῇ ἐκκειμένῃ ἐλάττονι περιφερείᾳ. ἢ διὰ τοῦ ἐξ ἀναλόγου, ὡς ἐν τῷ προχείρῳ κανόνι ἐμάθομεν, ἑκάστην λαμβάνομεν.

1 μερῶν codd. : expect. μοιρῶν **2** λεπτῶν comp. **BC** : λεπτὰ **K** | πεντεκαιδεκάκι] –κάδι **K** **3–4** τῇ ὑποτεινούσῃ] τῆς ὑποτείνουσης **K** **7** ὡς **K** : comp. ἐστι **BC** | ἑξηκοστὰ comp. **BC** : ἑξικοστῶν **K** **8** ὑπεροχὴν comp. **BC** : ἐλαττόνων **K** **9** ἐλάττονι περιφερείᾳ comp. **BC** : ἐλαττόνου περιφερείας **K** **10** λαμβάνομεν **K** : λαμβανομένην **BC**

Transl. We may take the successive [numerical] values of the chords [corresponding to] the parts between the half-degrees (for instance, 10 degrees 15 minutes) by making 15 times what in the third column corresponds to the 10 degrees of the arc and adding the result to the [chord] in the second column subtending the 10 degrees. Conversely, some given chord falling between those set out in the table, the arc on it is given if we take the difference of the chord with respect to the nearest lesser tabulated [chord], and again [we take] the sixtieths corresponding in the third column to the nearest lesser [chord], and dividing by these the difference of the 2 chords, and adding the result of the division to the lesser arc set out. Alternatively, we may take each of them by linear interpolation, as we have learnt in the handy table.

Comm. *a*) Vat. gr. 1594, f. 20r in spatio vacuo in textu, Marc. gr. 313, f. 45v marg. inf., Vat. gr. 184, f. 29r. *b*) *Ad Alm*. I.10, 46.22–47.2 ἵνα δέ, ὡς ἔφην, ἐφ’ ἑκάστης τῶν χρειῶν ἐξ ἑτοίμου τὰς πηλικότητας ἔχωμεν τῶν εὐθειῶν ἐκκειμένας «but, as I said, in order that we may have the actual [numerical] values of the chords readily available for every occasion» (= citation in **K**). *c*) The scholium explains how to interpolate between the tabulated numerical values of arcs/chords. This procedure shortcuts some computational steps in the basic procedure of linear interpolation (see sch. **48**) by using the numbers tabulated in the third column of the Table of Chords. These are the coefficients of linear interpolation within each half-degree: in order to find the sought chord (arc), it is enough to multiply (divide) by the suitable coefficient the difference of the arcs (chords) in the table nearest to the assigned arc (chord) and add it to the chord (arc) corresponding to the lesser of the arcs (chords) in the table nearest to the assigned arc (chord). The coefficient is calculated by dividing the difference between consecutive chords in the table by 30 minutes (= $\frac{1}{2}^{\circ}$ = the difference of consecutive arcs in the table); this division simply amounts to taking the double of such a difference. Actually, Ptolemy enters the 60^{th} part as the tabulated value (the heading of the column is «sextieths»), in order to directly get τὴν τοῦ ἑνὸς ἑξηκοστοῦ μέσην ἐπιβολὴν «the average increment corresponding to one minute» (cf. *Alm*. I.10, 47.7–11, and sch. **50**). The text of the scholium amounts to a rewriting, with elimination of the numerical counterparts of most of the designations, of the «procedure» in Theon, *in Alm. I.10*, *iA*, 510.1–13 (Theon uses the verb ἐφοδεῦσαι). As a consequence of this elimination, the scholium has an even more decidedly procedural turn than the original text has (cf. Acerbi 2012, 183–189, on the "language of procedures"). It is quite surprising that the scholium mentions the *Handy Tables* (viz., a commentary on them; Ptolemy’s *Handy Tables* does not include the Table of Chords; a worked-out example of linear interpolation can be found at Theon, *PC*, 221), for the basic procedure of linear interpolation is explained in full details by Theon, *in Alm. I.10*, *iA*, 507.18–509.18, just before the passage the scholiast is summarizing. One possibility is that the annotator resorted in his turn to excerpts. Sch. **47–53** complete Ptolemy’s description of the Table of Chords (46.21–47.21) and expound some of its features. *d*) In **B**, the scholium lies above sch. **50**, within the outer column of the main text. The text of *Alm*. I.10 ends in fact before the end of the column and leaves, after ornamentation, 15 blank lines; a *signe de renvoi* is added, both beside the scholium and, within the intercolumnar space, beside the line containing the first word of the *relatum*. In **C**, sch. **47** is located, without a *signe de renvoi*, in the lower margin. Both **B** and **C** make this text accompanied by a calculation in tabular form, here edited as sch. **48**. In **K**, sch. **47** immediately precedes sch. **50**. Both **B** and **C** have bars over the letters representing numerals. *e*) The scholium introduces some items of the technical lexicon of table-building. The verb ἐπιβάλλειν refers to the terms of a table insofar as they «follow each other» (the construct with a double genitive in the first sentence is rather wild). The παρακείμενα are the terms in a table «corresponding to» (lit. «lying by the side of») terms taken as reference; the relation is symmetric, as it turns out from the very text of the scholium; the occurrence παρακειμένην at line 6 assumes a less strict signification. The very compressed qualifier ἔγγιστος ἐλάττων «nearest lesser» is canonical (many occurrences are found in *Prol.*). Given a number and a list of numerical values organized as an increased sequence, the ἔγγιστος ἐλάττων in the list is the greatest of the values that are less than the given number. The prepositional phrase ἐξ ἀναλόγου «linear interpolation» at line 9 means literally «by proportion». It is absent in *Alm*. but one can find it with this signification in Pappus, *in Alm.*, *iA*, 47.7, 81.5, 194.11 (*bis*), 195.13 (*bis*), 196.6 (*bis*), and Theon, *in Alm.*, *iA*, 500.3, 501.8, 596.21, 650.8, 910.5. One also finds 8 occurrences in *Prol*. The singular ἐν τῷ προχείρῳ κανόνι «in the handy table» at lines 9–10 is the canonical way to refer to Ptolemy’s treatise. At line 2 we read the only occurrence of λεπτά «minutes» in our scholia.

48

Text.

M^{o}	ι		ι	κζ	λβ	
M^{o}	ι	ιε	ι	μγ	ι	
M^{o}	ι	λ	ι	νη	μθ	
ΥΠΕΡΟ^{X}						
ō	λ		ō	λα	ιζ	
ō	ιε		ō	ιε	λη	λ′

1 – 3 M^{o} om. Th. **4** ΥΠΕΡΟ^{X} om. Th. **5** ō^{2} Th. : β codd. post **6** ō ō υξε σνε scripsit Th.

Transl.

U^{n}	10		10	27	32	
U^{n}	10	15	10	43	10	
U^{n}	10	30	10	58	49	
DIFFE^{R}						
0	30		0	31	17	
0	15		0	15	38	30′

Comm. *a*) Vat. gr. 1594, f. 20r marg. ext., Marc. gr. 313, f. 45v marg. inf. *b*) Without a specific *relatum* in *Alm*.; it is a complement to the last statement of sch. **47**. *c*) This is the tabular arrangement of a worked-out example of the basic procedure of linear interpolation (see also sch. **102**); one has to find the chord associated to arc 10;15. This much is announced in the last statement of sch. **47**. One takes the chords 10;27,32 and 10;58,49 corresponding to the nearest neighbours of 10;15 among the tabulated arcs, namely, 10 and 10;30, and arranges all these data as in the prospect; a blank space must be preliminarily left between the numerical values of the chords, to be occupied by the final result 10;43,10. To calculate the amount to be added to 10;27,32 [= *ch*(10)] in order to find *ch*(10;15), one proceeds by linear interpolation: take the quotient of the differences *ch*(10;30) – *ch*(10) and 10;30 – 10 between the known chords and arcs, respectively, and use it as a coefficient to multiply the difference 10;15 – 10 between the assigned arc and its lesser nearest neighbour (the said quotient is what we find in the third column of the Table of Chords). These numbers are on the one side 0;31,17 and 0;30, on the other 0;15. The operations just described amount to taking the appropriate fourth proportional of these numbers, namely [(0;31,17)×(0;15)]/(0;30). This is done by putting the numbers on three of the four corners of a fictitious rectangle (the arcs on one side, the chords on the other), the fourth corner being then occupied by the result, here 0;15,38,30. The diagonally opposed numbers (namely, those to be multiplied to each other) are normally joined by mutually intersecting line segments, here absent. The result of the cross-multiplication of the known numerical values can be marked below this X-shaped array; this result is here also absent. Adding 0;15,38,30 to 10;27,32 [= *ch*(10)] and neglecting third sixtieths one finally obtains 10;43,10 = *ch*(10;15). The example coincides with that worked out by Theon, *in Alm. I.10*, *iA*, 508.5–509.6. Theon also organized his data in a table, which he describes and to which he makes explicit reference (508.6–10). Theon's table also includes the result of the multiplication (0;31,17)×(0;15) = 0;0,465,255, which is transcribed below the rectangular array of proportionality; some manuscripts put the last three lines beside the first three; the indications M^{o} and ΥΠΕΡΟ^{X} are always omitted (see 509.16–18 *app.*). Since sch. **47** ends by referring to the *Handy Tables*, we may safely surmise that sch. **48** was redacted in order to complete sch. **47**, and hence after it. *d*) Both **B** and **C** place sch. **48** beside sch. **47**. Neither **B** nor **C** have bars or apices over the letters representing numerals; only λ′ is assigned an apex.

49

Text. διὰ τὸ ἑξῆς ἐν τοῖς τῶν ἀνωμαλιῶν κανόσι φανησόμενον εὔχρηστον

Transl. Because it will prove useful in the subsequent tables of anomalies

Comm. *a*) Vat. gr. 1594, f. 20r marg. int., Marc. gr. 313, f. 45v marg. ext. *b*) *Ad Alm*. I.10, 46.22–47.3 ἵνα δέ, ὡς ἔφην, ἐφ' ἑκάστης τῶν χρειῶν ἐξ ἑτοίμου τὰς πηλικότητας ἔχωμεν τῶν εὐθειῶν ἐκκειμένας, κανόνια ὑποτάξομεν ἀνὰ στίχους με διὰ τὸ σύμμετρον «but, as I said, in order that we may have the actual [numerical] values of the chords readily available for every occasion, we shall set out tables below, arranged on 45 lines to achieve a symmetrical appearance». *c*) The scholium states that the usefulness of the arrangement on 45 lines of the Table of Chords will become apparent later, when the tables of anomalies will be introduced (*Alm*. III.6: Sun's anomaly; V.8: Moon): these are in fact set out on 45 lines; from 0º to 90º and from 270º to 360º the increment is of 6º, from 90º to 180º and from 180º to 270º it is of 3º. If one wants to set out a single table whose tabulation value runs from 0º to 180º and increases by a constant amount over half of the range, while increasing twice that amount for the remaining half, the only solution is to have the table on 45 lines. Of course, it might well be that the peculiar format of the tables of anomalies derives from the requirement of having all of them set out on 45 lines, and not the opposite, as the scholiast suggests. The text coincides with Theon, *in Alm. I.10*, *iA*, 500.12–501.1, where it follows a clause having the same meaning as the *relatum*. *d*) Both in **B** and in **C**, sch. **49** is located beside the *relatum*. *e*) The term "symmetrical" in Ptolemy's statement can also be taken to refer to the fact that 45 is σύμμετρος «commensurable» with 360, that is the number of lines in the whole Table of Chords. The only other allusion by Ptolemy to the *mise en table* of his data is at 209.14–15 διὰ τὸ φανησόμενον σύμμετρον τῆς κανονογραφίας «because it [*scil.* taking 18 years as the basis of the long-term cycle of the mean motion of the Sun] will produce symmetry in the layout of the tables», when presenting in III.1 the Table of the Mean Motion of the Sun; the clause has quite obviously served as a model for the one we read in the scholium. As Rome (*iA*, 500–501, n. 4) and Toomer (1984, 56, n. 67) argue, the choice between 30, 45, and 60 as the most suitable submultiple of 360 in view of a convenient *mise en table* of *Alm*.'s data was very likely dictated by some standard height of papyrus roll; on the issue see also Acerbi (2013, 125–129). The syntax of the nominal clause making up the scholium has been changed in translation.

50

Text. οὐχ ὡς τοῦ λʹ μέρους τῆς παραυξήσεως τὸ λʹ τοῦ ἡμιμοιρίου ὑποτείνονται (οὐδὲ γὰρ αἱ ὑπεροχαὶ τῶν εὐθειῶν ἄνισοι οὖσαι τὰ ἡμιμοίρια ἴσα ὄντα ὑποτείνουσιν), ἀλλ’ ὡς ἐξ ἀναλόγου τῇ παραυξήσει τῆς περιφερείας καὶ τῆς εὐθείας παραυξανομένης.

1 τῆς παραυξήσεως] αὐτῆς Th. | ὑποτείνονται (οὐδὲ] ὑποτείνοντος οὐ Th.

Transl. It is not the case that [Ptolemy took $^1\!/_{30}$ of the difference of the chords] because the 30th part of the increment [of a chord] subtends the 30th part of a half-degree (for it could not be case, either, that the differences of the chords, which are unequal, subtend the [successive] half-degrees, which are equal), but because the chord is also increased by linear interpolation along with the increment of the arc.

Comm. *a*) Vat. gr. 1594, f. 20r in spatio vacuo in textu, Marc. gr. 313, f. 45v marg. ext., Vat. gr. 184, f. 29r. *b*) *Ad Alm*. I.10, 47.7–9 τὰ δὲ τρίτα τὸ λʹ μέρος τῆς καθ’ ἕκαστον ἡμιμοίριον τῶν εὐθειῶν παραυξήσεως «and the third [portion of the table will contain] the 30th part of the increment in the chord for each [interval of] $^1\!/_2$ degree». In **K**, it is not preceded by a citation. *c*) Ptolemy’s statement was regarded as potentially ambiguous; the scholium points out that the (variable) 30th part of the increment tabulated in the third column must be used as the (variable) coefficient of a linear interpolation between adjacent numerical values of the chords, and not as the (constant) value of the chord subtending the 30th part of an arc of $^1\!/_2$°. The text of the scholium coincides, with two minor variants, with Theon, *in Alm. I.10*, *iA*, 501.6–9 (but changing ὑποτείνοντος to ὑποτείνονται partly destroys the syntax of the sentence, originally made of two conjoined clauses ὡς + participle; the scholiast neglected to put τοῦ λʹ μέρους in the plural nominative); the initial clause added in the translation comes from Theon’s text. *d*) In **C**, sch. **50** is located beside the *relatum*. In **B**, it lies under sch. **47**, still within the outer column of the main text; this entails that sch. **50** is shifted four lines below the first word of the *relatum*. In **K**, sch. **50** immediately follows sch. **47**. In **B**, the scholium is shaped as sch. **1**. *e*) Note the initial emphatic οὐχ, whose scope is the whole clause (cf. sch. **22** and **28**). The terms παραύξησις «increment» and ὑπεροχή «difference» are synonyms, the amount designated by them being as it were seen from opposite perspectives. For the expression ἐξ ἀναλόγου, see sch. **47**.

51

Text. τοσαύτης οὔσης ὅση ἐστὶν ἡ ὑποτεινομένη περιφέρεια ὑπὸ τῆς ζητουμένης εὐθείας

Transl. Being that much as it is the arc subtended by the sought chord

Comm. *a*) Vat. gr. 1594, f. 20r marg. ext., Marc. gr. 313, f. 45v marg. ext. *b*) *Ad Alm*. I.10, 47.18–19 ἢ ‹ἐκ› τῆς πρὸς ἄλλας τινὰς τῶν δεδομένων ὑπεροχῆς «or [from] the difference with respect to some other given [chord]». *c*) The scholium identifies the numerical value of some unspecified arc with that subtended by the sought chord; this identification does not fit any statement in *Alm*., nor is it excerpted from Theon's commentary. The *relatum* suggested by the position of the scholium is the second item of a disjunctive list of ways of recalculating the numerical value of a chord in the table, in order to test its correctness and possibly to amend it. These ways are: by bisection of a given chord, by difference of given chords, or by transition to the complement to a semicircle of a given chord. One possibility is that the annotator did not have in his text of *Alm*. the disjunctive particle introducing the *relatum*, and that he tried to amend the text by creating a genitive absolute: *ἤτοι ἀπὸ τῆς ὑπὸ τὴν διπλασίονα τῆς ἐπιζητουμένης ~~ἢ~~ τῆς πρὸς ἄλλας τινὰς τῶν δεδομένων ὑπεροχῆς τοσαύτης οὔσης ὅση ἐστὶν ἡ ὑποτεινομένη περιφέρεια ὑπὸ τῆς ζητουμένης εὐθείας ἢ […] *«either from the [chord] under double the [arc of that] of the sought [chord], ~~or~~ the difference with respect to some other given [chord] being that much as it is the arc subtended by the sought chord, or […]». The resulting statement is true but trivially so: the difference between some number and its double is equal to the number itself. *d*) Both in **B** and in **C**, sch. **51** is located beside the *relatum*. In **B**, the scholium is shaped as sch. **1**.

52

Text. αἱ τῶν ἐλαττόνων εὐθειῶν πηλικότητες ἐν μείζοσι διαφοραῖς παρηύξηνται κατὰ τὸ ἑξῆς, τῶν παραυξήσεων τῶν περιφερειῶν ἴσων οὐσῶν. αἱ ὑπὸ τὰς ἐλάττους τῶν ξ μοιρῶν ὑποτείνουσαι εὐθεῖαι μείζους εἰσὶ τῷ ἀριθμῷ τῶν κατ' αὐτὰς περιφερειῶν, αἱ δὲ ὑπὸ τὰς μείζους ἐλάττονες.

2 ἐλάττους] ἐλάττονας Th. | ξ Th. : τξ codd. **3** μείζους] πλείους fere cuncti codd. Theonis **4** μείζους] πλευρὰς codd. fortasse ex π^{λ}(είους)

Transl. The [numerical] values of the lesser chords successively increase in greater differences, while the increments of the arcs being equal. The chords subtending the [arcs] less than 60 degrees are larger in numerical [value] than the corresponding arcs, those under the [arcs] greater [than 60 degrees] are smaller.

Comm. *a*) Vat. gr. 1594, f. 20v marg. inf., Marc. gr. 313, f. 46r marg. inf., Vat. gr. 184, f. 29v. *b*) *Ad Alm.* I.11, Table of Chords. *c*) The scholium points out two features of the Table of Chords: *i*) the increments are monotonically decreasing (in our language: it is a sine table; the derivative of a sine is monotonically decreasing from 0º to 90º); *ii*) the numerical value of a chord is greater than that of the arc it subtends if the arc is less than 60º, otherwise it is smaller. This is a consequence of the different units of measurement that Ptolemy adopted for the diameter and the circumference: there are 120 parts in the diameter and 360 parts in the circumference; adopting the same unit for the diameter and the circumference (for instance, if angles are measured in radians), the numerical value of a chord is obviously smaller than that of the arc it subtends. The scholium is a reformulation of the two exegetic goals stated by Theon at *in Alm. I.10*, *iA*, 504.1–5, and that he then sets out to prove. The first clause is identical with that at 504.1–3. The second clause is repeated twice, at 505.15–18 (in the form of a title, almost certainly a scholium; the text is here almost identical with that of the scholium, see *app*.) and at 505.20–506.2 (with some variants: αἱ ἐλάσσονες τῶν ὑπὸ τὰς ξ μοίρας ὑποτείνουσαι εὐθεῖαι μείζονές εἰσιν τῷ ἀριθμῷ τῶν κατ’ αὐτὰς περιφερειῶν, αἱ δὲ ὑπὲρ τὰς ξ ἐλάττονες). Quite unusually, the original scholium contained two obvious mistakes. Sch. **52–53** refer to *Alm*. I.11. *d*) Both in **B** and in **C**, sch. **52** is placed under the first of the 16 sub-tables making up the entire Table of Chords (arcs from ½º to 22½º). In **B**, the scholium is shaped as sch. **1**; the two clauses are separated by a *dicolon* and a blank space. *e*) Note the construct ὑποτείνειν ὑπὸ «to subtend» + accusative, a variant of the more common ὑποτείνειν + accusative. The variant is sanctioned by *El*. I.18–19.

53

Text. λʹ μέρος τῶν ὑπεροχῶν τῶν ὑποτεινουσῶν εὐθειῶν τὰς καθ’ ἕκαστον ἡμιμοίριον παραυξήσεις

1 ante λʹ add. τὸ Th. | τῶν ὑπεροχῶν] τῆς ὑπεροχῆς Th. | εὐθειῶν om. Th. | ἕκαστον om. Th.

Transl. 30th part of the differences of the chords subtending the increments at each half-degree

Comm. *a*) Vat. gr. 1594, f. 20v marg. inf., Marc. gr. 313, f. 46r marg. inf. *b*) *Ad Alm.* I.11, Table of Chords. *c*) The scholium slightly reformulates the noun phrase designating what is contained in the third column of the Table of Chords, namely, τὸ λʹ μέρος τῆς καθ’ ἕκαστον ἡμιμοίριον τῶν εὐθειῶν παραυξήσεως «the 30th part of the increment in the chord for each [interval of] half of a degree» (*Alm*. I.10, 47.8–9). The text of the scholium almost coincides also with Theon, *in Alm. I.10*, *iA*, 501.4–6. The formulation is closer to Theon's than to Ptolemy's. *d*) Both in **B** and in **C**, sch. **53** is placed under the second of the 16 sub-tables making up the entire Table of Chords (arcs from 23º to 45º).

54

Text. καὶ τῶν διὰ μέσων τῶν ζῳδίων κύκλου τμημάτων

post τῶν[1] expect. τοῦ

Transl. And of segments of [the] ecliptic

Comm. *a*) Vat. gr. 1594, f. 23v marg. int., Marc. gr. 313, f. 49v marg. ext. *b*) *Ad Alm*. I.13, 68.15–19 ἀκολούθου δ’ ὄντος ἀποδεῖξαι καὶ τὰς κατὰ μέρος γινομένας πηλικότητας τῶν ἀπολαμβανομένων περιφερειῶν μεταξὺ τοῦ τε ἰσημερινοῦ καὶ τοῦ διὰ μέσων τῶν ζῳδίων κύκλου τῶν γραφομένων μεγίστων κύκλων διὰ τῶν τοῦ ἰσημερινοῦ πόλων «our next task is to demonstrate the [numerical] values of the individual arcs cut off between the equator and the ecliptic along a great circle traced through the poles of the equator» (= citation in **K**). *c*) This is a complement to the initial sentence of the chapter. The import of the sentence is not clear; maybe it refers to the arcs of the ecliptic set out as tabulation values of the Table of Declination. Sch. **54–91** refer to *Alm*. I.13. *d*) Both in **B** and in **C**, sch. **54** is located beside the *relatum*.

55

Text. ταῦτα τὰ λήμματα καθολικὰ παραδίδωσιν ὁ Πτολεμαῖος, καί εἰσιν ἐν τῷ πρώτῳ τῷ κατὰ σύνθεσιν θεωρήματι πτώσεις μέν (ἤτοι λήψεις Λ) ις, ἀποδείξεις δὲ ξδ, οἷόν ἐστιν ὁ τῆς ΓΑ πρὸς ΓΕ καὶ ὁ τῆς ΕΑ πρὸς ΑΓ καὶ ὁ τῆς ΕΓ πρὸς ΓΑ. ἰδού· ἐπὶ μιᾶς εὐθείας δ Λ· ὡσαύτως καὶ ἐπὶ ἑκάστης τῶν λοιπῶν γ τῶν ΑΒ ΒΕ ΓΔ δ εἰσὶ Λ. ἀποδείξεις δὲ πασῶν ξδ· ἢ γὰρ διὰ τοῦ Γ ἢ διὰ τοῦ Α ἄγεται ἡ παράλληλος ἢ διὰ τοῦ Ε, ποτὲ μὲν τῇ ΑΒ ποτὲ δὲ τῇ ΕΒ ποτὲ δὲ τῇ ΔΓ, ὡς νῦν. καθ’ ἑκάστην πτῶσιν δ ἀποδείξεις εἰσὶν οὕτως· ἐάν τε γὰρ ἀπὸ τοῦ κατὰ τὴν ἀρχὴν πέρατος τοῦ συντιθεμένου λόγου (ὡς τοῦ τῆς ΓΑ πρὸς ΑΕ), οἷον τοῦ Γ, ἐάν τε διὰ τοῦ κατὰ τὸ τέλος (οἷον τοῦ Ε) παράλληλον ἀγάγωμεν, ἡ ἀπόδειξις προχωρήσει. ἄγεται δ’ ἐξ ἑκατέρου τοῦ κατὰ τὴν ἀρχὴν καὶ πέρας ‹τοῦ› συντιθεμένου λόγου σημείου ἡ παράλληλος διχῶς· ἀπὸ γὰρ τοῦ Γ ἄγεται παράλληλος ἢ τῇ ΕΒ ἢ τῇ ΑΒ, οὐκέτι δὲ καὶ ταῖς λοιπαῖς ταῖς ΑΓ ΓΔ (ἐφ’ ἑκατέρας γὰρ αὐτῶν ἐστι τὸ Γ σημεῖον), ἀπὸ δὲ τοῦ Ε ὁμοίως β μόναι ἄγονται παράλληλοι τῇ τε ΓΔ καὶ τῇ ΑΒ, οὐκέτι δὲ καὶ ταῖς λοιπαῖς ταῖς ΕΒ ΑΓ (ἐφ’ ἑκατέρας γὰρ αὐτῶν ἐστι τὸ Ε). ὁμοίως καὶ ἐπὶ ἑκάστου λόγου τῶν Λ τῶν λοιπῶν, δ ἀγομένων εὐθειῶν παραλλήλων, β μὲν ἀπὸ τοῦ κατὰ τὴν ἀρχὴν πέρατος τοῦ συντιθεμένου λόγου, β δὲ ἀπὸ τοῦ πέρατος, γίνεται ἀποδείξεις ξδ. ἐπὶ δὲ τοῦ κατὰ διαίρεσιν πτώσεις μέν εἰσιν η, ἀποδείξεις δὲ λβ· ἐπὶ γὰρ ἑκάστης εὐθείας δύο εἰσὶ λόγοι, οἷον ὁ τῆς ΓΕ πρὸς ΕΑ καὶ ὁ τῆς ΑΕ πρὸς ΕΓ. πάλιν δὲ ἕκαστος τούτων τετραχῶς δείκνυται, τῶν παραλλήλων ὡς εἴρηται ἀγομένων. ἐὰν οὖν τὸν κανόνα τοῦ Θέωνος ἐπὶ πασῶν τῶν πτώσεων φυλάξωμεν, ἔχομεν αὐτὰς ἐξ ἑτοίμου εὑρημένας. ἔστι δὲ ὁ κανὼν τοιοῦτος· δεῖ τὸν πρῶτον λόγον τῶν συντιθέντων ἄρχεσθαι μὲν ὅθεν καὶ ὁ συντιθέμενος, τὸν δὲ δεύτερον τῶν συντιθέντων ἄρχεσθαι μὲν ὅθεν ὁ πρῶτος τῶν συντιθέντων ἔληξε, λήγειν δὲ ὅπου καὶ ὁ συντιθέμενος. ὁ δὲ μέγας φιλόσοφος προσδιορισμὸν προσέθηκε λέγων ὅτι δεῖ ἐπὶ ἑκάστου λόγου [εὐθείας] | τὸν πρόλογον καὶ τὸν ὑπόλογον ἐν μιᾷ εὐθείᾳ εἶναι· ἐὰν γὰρ εἴπω ὅτι ὁ τῆς ΓΑ πρὸς ΑΕ λόγος σύγκειται ἐκ τοῦ τῆς ΓΔ πρὸς ΔΒ καὶ τοῦ τῆς ΒΖ πρὸς ΖΕ, ἰδού· οἱ κανόνες τοῦ Θέωνος ἐφυλάχθησαν καὶ ὅμως οὐκ ἀληθεύουσι· διὰ τοῦτο δὲ οὐκ ἠλήθευσεν ὅτι οὐκ ἔστιν ἐν μιᾷ εὐθείᾳ ὁ τῆς ΓΔ πρὸς ΔΒ.

1 – 26 uix legitur **B** **1** παραδίδωσιν] –οται **G** | τῷ[2]] τῶν **K** **2** θεωρήματι **G** : comp. **BC** : θεωριῶν **K** | ἐστιν] καὶ comp. **G** sed καὶ ὁ fecit ὁ m. 2 **3** ΑΓ] ΕΓ **G** **4** Λ] λοι(πὸν) **G** **5** ἢ[2]] καὶ comp. **G** **7** συντιθεμένου] συντεθειμένου **G** | λόγου] Λ **BK** **9** προχωρήσει] προχωρήσῃ **C** | δ’] δʹ **BC** : ex δʹ fecit m. 2 **GK** | πέρας codd. : expect. τέλος sed cfr. u. 15 | τοῦ addidi : om. codd. | συντιθεμένου λόγου] συντιθέμενον λ[o] codd. **10** παράλληλος[2] comp. **BC** : διάμετρος **G** **11** δὲ ex corr. **G** : δʹ alii codd. | ταῖς λοιπαῖς ταῖς] τῆς λοιπῆς τῆς **G** **12** δὲ[2] ex corr. **G** : δʹ alii codd. **13** λόγου] Λ **B** **14** τῶν Λ om. **G** | ἀγομένων εὐθειῶν] ἀγομ[ε] εὐθεία **CG** **15** συντιθεμένου] συντεθειμένου **G** | πέρατος] πέρατα **K** | λόγου, β δὲ ex corr. **G** : Λ β δʹ **BC** : λόγου βδ **K** **16** δὲ **G** : δʹ alii codd. | εἰσὶ om. **G** **17** λόγοι] παραλλήλων comp. **G** : Λ alii codd. | ΕΑ] ΕΔ **K** | ΕΓ] ΘΓ **K** **19** ἔχομεν om. **CG** | εὑρημένας] εὕρομεν **G** | δὲ om. **K** **20** λόγον] Λ **K** | συντιθέντων] –τεθέντων **BCG** | μὲν fort. delendum | συντιθέμενος] –τεθείμενος **G** **21** δεύτερον] δύο **G** | συντιθέντων[1]] –τεθέντων **CG** | συντιθέντων[2]] –τεθέντων **G** **22** ὅπου] ὅπως **C** | συντιθέμενος] –τεθείμενος **G** | προσδιορισμὸν] προσδιωρισμένων **G** **23** λόγου] Λ **B** | πρόλογον] πρῶτον λόγον **G** **24** ΓΑ] ΓΔ **B** : ex ΓΔ fecit **K** | λόγος] Λ **B** **26** ἠλήθευσεν] ἀλήθευσεν **CG**

Transl. Ptolemy hands down these general lemmas; in the first theorem “by composition” there are 16 cases (or choices, [henceforth] C) and 64 proofs, such as is the [ratio] of ΓΑ to ΓΕ, or that of ΕΑ to ΑΓ, or that of ΕΓ to ΓΑ. Look: 4 C for one single straight line; similarly there also are 4 C for each of the remaining 3 [straight lines], ΑΒ, ΒΕ, ΓΔ. There are in all 64 proofs, for the parallel is drawn either through Γ or through Α or through Ε, [and it is parallel] either to ΑΒ, or to ΕΒ, or to ΔΓ (as now). There are 4 proofs in each case, as follows: whether we draw a parallel from the endpoint (namely, Γ) at the beginning of the compounded ratio (as that of ΓΑ to ΑΕ), or we draw it through the [endpoint] at the end (namely, Ε), the proof will apply. Now, the parallel can be drawn in two ways from each of the points at the beginning and end[point] of the compounded ratio: for a [straight line] can be drawn from Γ parallel either to ΕΒ or to ΑΒ, but no longer to the remaining [straight lines] ΑΓ, ΓΔ (for point Γ is on each of them), and similarly, only two [straight lines] can be drawn from Ε parallel either to ΓΔ or to ΑΒ, but no longer to the remaining [straight lines] ΕΒ, ΑΓ (for Ε is on each of them). Similarly, also for each ratio of the remaining C, once 4 [straight lines] are drawn parallel—2 from the endpoint at the beginning of the compounded ratio, 2 from the endpoint [at the end]—, the result is 64 proofs. In the [theorem] “by separation,” instead, there are 8 cases and 32 proofs, for there are two ratios for each straight line, namely, the [ratio] of ΓΕ to ΕΑ and that of ΑΕ to ΕΓ. Again, each of these [cases] is proved in four ways, once the parallels are drawn as said. Then if we adhere to Theon’s rule for all cases, we may have them immediately discovered. The rule is as follows: the first of the compounding ratios must begin from where the compounded [ratio] also [begins], the second of the compounding [ratios] must begin from where the first of the compounding [ratios] ends, and end where the compounded [ratio] also [ends]. Yet, the great scholar added an additional determination, claiming that, for each of the ratios, the antecedent and the consequent must be in one single straight line, for, if I say that ratio ΓA to AE is compounded of that of ΓΔ to ΔB and of that of BZ to ZE, look: Theon's rules are adhered to, still they do not hold, and the reason for their not holding is that the [ratio] of ΓΔ to ΔB is not in one single straight line.

Comm. *a*) Vat. gr. 1594, f. 23v marg. ext. et inf., Marc. gr. 313, ff. 49v marg. ext. et inf. – f. 50r marg. sup., Vat. gr. 184, ff. 30r et 90v marg. ext. et inf. *b*) *Ad Alm*. I.13, 68.19–22 προεκθησόμεθα λημμάτια βραχέα καὶ εὔχρηστα, δι' ὧν τὰς πλείστας σχεδὸν δείξεις τῶν σφαιρικῶς θεωρουμένων, ὡς ἔνι μάλιστα, ἁπλούστερον καὶ μεθοδικώτερον ποιησόμεθα «we shall preliminarily set out some short and useful lemmas which will enable us to carry out most demonstrations involving spherical theorems in the simplest and most methodical way possible» (= citation in **K**). *c*) The scholium provides an *a priori* reckoning of the different configurations of the rectilinear lemmas pertaining to the Sector Theorem, and of the ways they can be proved. The "base" configuration shared by the two rectilinear lemmas is as follows. From endpoints B, Γ of two mutually intersecting straight lines AB, AΓ, two lines BE, ΓΔ are drawn across, meeting at Z and intersecting lines AΓ, AB at E, Δ, respectively. It is required to show that ΓA:AE = (ΓΔ:ΔZ)∘(ZB:BE) (lemma "by composition") and that ΓE:EA = (ΓZ:ZΔ)∘(ΔB:BA) (lemma "by separation"). The proof (see Figs. 8–9) draws a suitable parallel to one of the straight lines and readily argues by similar triangles and substitutions in compounded ratios written in "normal form", thus: ΓA:AE::ΓΔ:HE = (ΓΔ:ZΔ)∘(ZΔ:HE). The parameters taken into account by the scholiast are the following. First, the number of ratios associated to each punctuated straight line in the "base" configuration of each lemma: four ratios including trivial inversions in the case "by composition," two ratios in the case "by separation;" since four punctuated straight lines are involved, one gets by multiplication sixteen and eight configurations, respectively. Second, the number of parallels to straight lines of the "base" configuration that can be drawn in the auxiliary construction of each lemma; there are four of these parallels in any instance, as the scholiast shows: two points are available for drawing paralles, two parallels can be drawn at each of these point. A complete classification of the different cases of the Sector Theorem (not of the rectilinear lemmas) was worked out by Thābit ibn Qurra (Lorch 2001); most of the valid cases can be deduced by simple manipulations of ratios, without any geometric argument. Theon's rule is first formulated in general and in instantiated form at *in Alm. I.13*, *iA*, 539.17–25, and repeated in general form at 543.14–17 and 564.1–5 (Sector Theorem). The scholiast slightly changes Theon's wording. There is no passage parallel to the whole scholium in Theon's commentary, even if of course his «rule» is taken from it; most notably, Theon does not formulate the "additional determination:" we have no elements to identify the μέγας φιλόσοφος who added it; maybe it is Ammonius, maybe Syrianus (see n. 41 above). *d*) Both in **B** and in **C**, sch. **55** starts in the outer margin and covers most of the lower margin of the page at which *Alm*. I.13 begins; no original *signe de renvoi* is perceptible; a later hand in **B** adds a sign just beside the beginning of the setting-out of the subsequent theorem (69.3). Of particular interest is the use of the abbreviation Λ for λῆψις: maybe it is a trait of the original scholium, maybe the syntagm ἤτοι λήψεις is a gloss on the scholium, aimed at explaining a sign used more than once. The manuscripts have it with a subscript *omicron*, as if it were the sign for λοιπός; it is not said that this is an original feature of the abbreviation here employed. The standard sign for παράλληλος (two parallel horizontal strokes, surmounted by the compendium of a suitable termination) is employed throughout the scholium; at line 10, it induced the copyist of **G** into error. See sch. **83** for the similar sign for εὐθεῖα, whose form may explain the useless εὐθείας at line 23: it may be that this was in origin a stroke "—" marking a break in the scholium like the change of page in **C**, that occurs exactly after that word but marked by an asterisk ※. Since the stretch of text at f. 50r in **C** would occupy no more than two lines in the format of the preceding part of text at f. 49v, and since there still is a wide blank space in the lower margin of f. 49v, I surmise that **C** preserves the original layout of the scholium. *e*) The imperative/interjection ἰδού «look» (lines 3 and 25) and the personal form εἴπω «I say» (line 24) are exclusive to this scholium. At line 19, note the periphrastic construct ἔχομεν + perfect participle. Note also the translation «scholar» for φιλόσοφος at line 22: thinking that there were as many "philosophers" in Greek Antiquity as there are individuals to whom the epithet φιλόσοφος was attached is the same kind of lexical anachronism that has made, and still makes, the Aristotelian Coriscos someone «musical» and not, as it should be, a «literate» (he is called μουσικός at *SE* 17). Other occurrences of the noun προσδιορισμός «additional determination» (line 22) in the Greek mathematical corpus are Diophantus, *Ar*. I.14 and V.10, *DOO* I, 36.6 and 340.9–10; Eutocius, *in Sph. cyl. II.4*, *AOO* III, 150.15; Proclus, *in Eucl.*, 240.27 and 349.21. The substantives πρόλογος and ὑπόλογος at line 23 designate the «antecedent» and the «consequent» of a ratio, respectively; see again sch. **94** and **98**. The plural οἱ κανόνες «the rules» at line 25 is an idiomatic trait of Greek language: as the scholiast himself asserted at line 18, there is only one such rule (see sch. **11**, **54** and **106**).

56

Text. α′ λῆμμα εὐθύγραμμον κατὰ σύνθεσιν

Transl. 1st rectilinear lemma, by composition

Comm. *a*) Vat. gr. 1594, f. 23v marg. int., Marc. gr. 313, f. 49v marg. ext. *b*) *Ad Alm*. I.13, 68.23–69.1 εἰς δύο δὴ εὐθείας τὰς ΑΒ καὶ ΑΓ διαχθεῖσαι δύο εὐθεῖαι ἥ τε ΒΕ καὶ ἡ ΓΔ «now, two straight lines BE and ΓΔ, which are drawn to meet two straight lines AB and AΓ» ff. *c*) The scholium provides a rigid designator of the first theorem proved by Ptolemy. The denomination is not employed by Theon, who however calls the second rectilinear configuration "by separation" (see sch. **59**) and expressly resorts to the two designations for the two spherical configurations, at *in Alm. I.13*, *iA*, 558.2 and 562.12. Ptolemy only refers to the second rectilinear configuration as obtained “by separation” (69.21–22; see again sch. **59**); he does not assign denominations to the two spherical configurations. Note that Ptolemy himself calls λημμάτια βραχέα καὶ εὔχρηστα «short and useful lemmas» (68.19–20; cf. the *relatum* of sch. **55**) the six subsequent theorems: the two rectilinear lemmas and the four cyclic lemmas. Sch. **56–58** refer to the first rectilinear lemma. *d*) Both in **B** and in **C**, sch. **56** is located beside the beginning of the *relatum*. It is in majuscule in **B**. *e*) As is customary with him, Ptolemy does not provide any of the theorems in *Alm*. I.13–14 with a general enunciation; we read these enunciations in Theon’s commentary, apparently a qualifying point of his exegesis.

57

Text. ἰσογώνιον γάρ ἐστι τὸ ΑΓΔ τρίγωνον τῷ ΑΕΗ τριγώνῳ

Transl. For triangle ΑΓΔ is equiangular to triangle ΑΕΗ

Comm. *a*) Vat. gr. 1594, f. 23v marg. int., Marc. gr. 313, f. 49v marg. ext., Vat. gr. 184, f. 30v. *b*) *Ad Alm*. I.13, 69.6–8 ἐπεὶ παράλληλοί εἰσιν αἱ ΓΔ καὶ ΕΗ, ὁ τῆς ΓΑ πρὸς ΕΑ λόγος ὁ αὐτός ἐστιν τῷ τῆς ΓΔ πρὸς ΕΗ «since ΓΔ and ΕΗ are parallel, the ratio of ΓΑ to ΕΑ is the same as that of ΓΔ to ΕΗ» (underlined the citation in **K**). *c*) In the configuration “by composition” (sch. **55** and Fig. 8), straight line ΗΕ is drawn from Ε parallel to ΔΓ; it meets ΑΒ in Η. The scholium gives the condition used in *El*. VI.4 to prove the statement in the apodosis of the *relatum*; such a condition is in its turn a consequence of the protasis of the *relatum*. The text coincides with that of the corresponding passage in Theon, *in Alm. I.13*, *iA*, 539.9. *d*) Both in **B** and in **C**, sch. **57** is located beside the *relatum*.

58

Text. καὶ ἰσογώνιον πάλιν γίνεσθαι τὸ ΒΖΔ τρίγωνον τῷ ΒΕΗ τριγώνῳ

τριγώνῳ om. Th.

Transl. And [because] triangle ΒΖΔ is again equiangular to triangle ΒΕΗ

Comm. *a*) Vat. gr. 1594, f. 23v marg. int., Marc. gr. 313, f. 49v marg. int. *b*) *Ad Alm*. I.13, 69.16–18 ἔστιν δὲ καὶ ὁ τῆς ΔΖ πρὸς ΗΕ λόγος ὁ αὐτὸς τῷ τῆς ΖΒ πρὸς ΒΕ διὰ τὸ παραλλήλους πάλιν εἶναι τὰς ΕΗ καὶ ΖΔ «and the ratio of ΔΖ to ΗΕ is also the same as that of ΖΒ to ΒΕ because ΕΗ and ΖΔ are again parallel». *c*) The scholium gives the condition used in *El*. VI.4 to prove the statement in the principal clause of the *relatum*; in its turn, such a condition is a consequence of the causal subordinate clause in the *relatum* (see Fig. 8). The text coincides with that of the corresponding passage in Theon, *in Alm. I.13*, *iA*, 539.13–14 (the scholiast added the final τριγώνῳ). *d*) Both in **B** and in **C**, sch. **58** is located beside the *relatum*.

59

Text. β′ λῆμμα κατὰ διαίρεσιν

Transl. 2nd rectilinear lemma, by separation

Comm. *a*) Vat. gr. 1594, f. 24r marg. int., Marc. gr. 313, f. 50r marg. int., Vat. gr. 184, f. 90v marg. int. *b*) *Ad Alm*. I.13, 69.21–22 κατὰ τὰ αὐτὰ δὲ δειχθήσεται ὅτι καὶ κατὰ διαίρεσιν «and in the same way we shall also show that, by separation» ff. *c*) The scholium provides a rigid designator of the second theorem proved by Ptolemy. Theon, *in Alm. I.13*, *iA*, 542.13–14, also uses this denomination when he introduces the theorem (cf. sch. **56**). Sch. **59**–**62** refer to the second rectilinear lemma. *d*) Both in **B** and in **C**, sch. **59** is located beside the beginning of the *relatum*. It is in majuscule in **B**.

60

Text. καὶ ἰσογώνιον τὸ ΓΕΖ τρίγωνον τῷ ΓΑΗ τριγώνῳ

Transl. And triangle ΓΕΖ is equiangular to triangle ΓΑΗ

Comm. *a*) Vat. gr. 1594, f. 24r marg. int., Marc. gr. 313, f. 50r marg. ext., Vat. gr. 184, f. 90v marg. int. *b*) *Ad Alm*. I.13, 69.25–70.2 ἐπεὶ γὰρ πάλιν παράλληλός ἐστιν ἡ ΑΗ τῇ ΕΖ, ἔστιν ὡς ἡ ΓΕ πρὸς ΕΑ, ἡ ΓΖ πρὸς ΖΗ «in fact, since AH is again parallel to EZ, as ΓE is to EA, ΓZ is to ZH». *c*) In the configuration "by separation" (sch. **55** and Fig. 9), straight line AH is drawn from A parallel to EB; it meets ΓΔ produced in H. The scholium gives the condition used in *El*. VI.4 to prove the statement in the apodosis of the *relatum*; in its turn, such a condition is a consequence of the protasis of the *relatum*. There is no corresponding passage in Theon. *d*) Both in **B** and in **C**, sch. **60** is located beside the *relatum*.

61

Text. καὶ ἰσογώνια ἐστὶ τὰ ΑΔΗ ΒΔΖ τρίγωνα

Transl. And triangles ΑΔΗ, ΒΔΖ are equiangular

Comm. *a*) Vat. gr. 1594, f. 24r marg. int., Marc. gr. 313, f. 50r marg. ext. *b*) *Ad Alm*. I.13, 69.25–70.2 ἐπεὶ γὰρ πάλιν παράλληλός ἐστιν ἡ ΑΗ τῇ ΕΖ, ἔστιν ὡς ἡ ΓΕ πρὸς ΕΑ, ἡ ΓΖ πρὸς ΖΗ «in fact, since ΑΗ is again parallel to ΕΖ, as ΓΕ is to ΕΑ, ΓΖ is to ΖΗ». *c*) The scholium gives the condition used in *El*. VI.4 to prove the statement in the apodosis of the *relatum*; in its turn, such a condition is a consequence of the protasis of the *relatum* (see Fig. 8). The text adapts the corresponding passage in Theon, *in Alm. I.13*, *iA*, 539.9–10 (διὰ τὸ ἰσογώνια εἶναι κτλ.). *d*) Both in **B** and in **C**, sch. **61** is located beside the *relatum*.

62

Text. ἐν τοῖς τοιούτοις εὐθεῖαι μέν εἰσι δ, σημεῖα δὲ ς, παράλληλοι δὲ ιβ

Transl. In such [theorems] there are 4 straight lines, 6 points, 12 parallels

Comm. *a*) Vat. gr. 1594, f. 24r marg. int. et inf., Marc. gr. 313, f. 50r marg. ext., Vat. gr. 184, f. 90v marg. inf. *b*) The *relata* are both rectilinear lemmas, namely, that “by composition” and that “by separation.” *c*) The scholium points out the obvious fact that there are 4 straight lines and 6 points (= their intersections) in the configuration of these theorems; the 12 parallels come from the fact that, from any of the 6 points, parallels to only 2 of the 4 straight lines can be drawn; cf. sch. **55**. *d*) Both in **B** and in **C**, sch. **62** is located beside the diagram of the lemma “by separation.” In **G**, sch. **63** and **62** (in this order) are linked so as to produce a seemingly continuous text.

63

Text. γ′ λῆμμα κυκλικόν

Transl. 3rd lemma, cyclic

Comm. *a*) Vat. gr. 1594, f. 24r marg. ext., Marc. gr. 313, f. 50r marg. ext., Vat. gr. 184, f. 90v marg. inf. *b*) *Ad Alm*. I.13, 70.17 πάλιν ἔστω κύκλος ὁ ΑΒΓ «again, let there be a circle ΑΒΓ» ff. *c*) The scholium provides a rigid designator of the third theorem proved by Ptolemy. Theon uses κυκλικός to designate any or all of the cyclic lemmas (*in Alm. I.13*, *iA*, 545.13, 548.9, 551.1, 555.1, 557.27). Ptolemy does not provide denominations of his four cyclic lemmas. Sch. **63–65** refer to the first cyclic lemma. *d*) Both in **B** and in **C**, sch. **63** is located beside the beginning of the *relatum*. It is in majuscule in **B**.

64

Text. ἰσογώνιον ἄρα ἐστὶ τὸ ΑΕΖ τρίγωνον τῷ ΓΕΗ τριγώνῳ

ἰσογώνιον] ἰσογώνια codd.

Transl. Therefore triangle AEZ is equiangular to triangle ΓΕΗ

Comm. *a*) Vat. gr. 1594, f. 24r marg. ext., Marc. gr. 313, f. 50r marg. ext. *b*) *Ad Alm*. I.13, 71.5–8 ἐπεὶ παράλληλός ἐστιν ἡ ΑΖ τῇ ΓΗ, καὶ διῆκται εἰς αὐτὰς εὐθεῖα ἡ ΑΕΓ, ἔστιν, ὡς ἡ ΑΖ πρὸς τὴν ΓΗ, οὕτως ἡ ΑΕ πρὸς ΕΓ «since AZ is parallel to ΓΗ, and straight line ΑΕΓ has been drawn across them, as AZ is to ΓΗ, so AE is to ΕΓ». *c*) The configuration of the first cyclic lemma is as follows (Fig. 10). In a circle ΑΒΓ of center Δ, mark two consecutive arcs AB, ΒΓ, any of which is less than a semicircle, join ΔΒ and ΑΕΓ intersecting at E, draw from A, Γ perpendiculars AZ, ΓΗ to radius ΔΒ, respectively. It is required to show that *ch*(2AB):*ch*(2ΒΓ)::AE:ΕΓ. The scholium gives the condition used in *El*. VI.4 to prove the statement in the apodosis of the *relatum*; in its turn, such a condition is a consequence of the protasis of the *relatum*. The text coincides with the corresponding passage in Theon, *in Alm. I.13*, *iA*, 547.3. *d*) Both in **B** and in **C**, sch. **64** is located beside the *relatum*.

65

Text. αἱ ΑΒ ΒΓ περιφέρειαι· ἢ ἐλάττονές εἰσι τεταρτημορίου ἑκατέρα, ἢ ἴσαι ἢ ἄνισοι· ἢ ἡ μὲν τεταρτημορίου ἡ δὲ ἐλάττων [τῶν] τεταρτημορίου· ἢ ἑκατέρα αὐτῶν τεταρτημορίου· ἢ ἡ μὲν ἐλάττων τεταρτημορίου ἡ δὲ μείζων τεταρτημορίου· ἢ ἡ μὲν τεταρτημορίου ἡ δὲ μείζων μὲν τεταρτημορίου ἥττων δὲ ἡμικυκλίου· ἢ ἑκατέρα μείζων μὲν τεταρτημορίου ἥττων δὲ ἡμικυκλίου, ἢ ἴσαι ἢ ἄνισοι

1 ἢ ἴσαι ἢ ἄνισοι om. **CG** **2** τῶν codd. : delendum **4** δὲ[1] om. **G** | μὲν[1]] V **G** | μείζων μὲν] μὲν μείζων **G** | τεταρτημορίου] δ′ om. **G** **5** ἥττων] ἧττον **C**

Transl. Arcs AB, ΒΓ: either each of them is less than a quadrant, either equal or unequal; or the one is of a quadrant, the other is less than a quadrant; or each of them is of a quadrant; or the one is less than a quadrant, the other greater than a quadrant; or the one is of a quadrant, the other greater than a quadrant but less than a semicircle; or each [of them] is greater than a quadrant but less than a semicircle, either equal or unequal.

Comm. *a*) Vat. gr. 1594, f. 24r marg. inf., Marc. gr. 313, f. 50r marg. inf., Vat. gr. 184, f. 91r marg. inf. *b*) The *relatum* is the entire third lemma (the first cyclic lemma). *c*) The scholium reorganizes the material expounded in Theon, *in Alm. I.13*, *iA*, 547.17–548.8. The scholium lists all possible combinations by successively increasing the size of either arc, the threshold value being a quadrant. All items in the disjunction except for the second and the last are on the same level and are connected by line segments with their common root (= the first clause); the second and the last dichotomy (equal/unequal) are subordinated to the items preceding them; the branching is again represented by diverging line segments. Theon, instead, groups the cases that have the same proof and identifies them as follows: unequal and both less than 90º; equal, without further qualification; the one equal to, the other less than 90º; both greater than 90º. *d*) This is a schematic scholium; both **B** and **C** have it in the lower margin, just after the end of the *relatum*. In **B**, the *relatum* is followed, in the outer column, by the associated diagram and by the initial line of the subsequent lemma; in **C**, the *relatum* exactly closes both the text of the third lemma and the page (the final ἔδει δεῖξαι of Ptolemy's proof is centered in the last line), the diagram being on the next page. It is an easy guess that this was the *mise en page* of the common model of **BC**. The ordering of the items is left-right. *e*) This is the sole occurrence of the comparative ἥττων in our corpus of scholia.

66

Text. οὐ μόνον τῆς ΑΓ περιφερείας δοθείσης δίδοται καὶ ἑκατέρα τῶν ΑΒ ΒΓ, ἀλλὰ καὶ ὁποτέρας δοθείσης δοθήσονται καὶ αἱ λοιπαὶ δύο περιφέρειαι.

Transl. Not only: once arc ΑΓ is given, each of ΑΒ, ΒΓ is also given, but also: once any of the latter is given, the remaining two arcs will also be given.

Comm. *a*) Vat. gr. 1594, f. 24v marg. ext., Marc. gr. 313, f. 50v marg. ext., Vat. gr. 184, ff. 30v et 91r marg. sup. *b*) *Ad Alm*. I.13, 71.14–20 παρακολουθεῖ δ' αὐτόθεν ὅτι, κἂν δοθῶσιν ἥ τε ΑΓ ὅλη περιφέρεια καὶ ὁ λόγος ὁ τῆς ὑπὸ τὴν διπλῆν τῆς ΑΒ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΒΓ, δοθήσεται καὶ ἑκατέρα τῶν ΑΒ καὶ ΒΓ περιφερειῶν «and it immediately follows that, even if both the whole of arc ΑΓ and the ratio of the [straight line] under the double of [arc] ΑΒ to that under the double of ΒΓ be given, each of arcs ΑΒ and ΒΓ will also be given» ff. (no citation in **K**). *c*) In the configuration of the first cyclic lemma (sch. **64** and Fig. 10), neglect perpendiculars ΑΖ, ΓΗ and from center Δ draw ΔΖ perpendicular to ΑΕΓ. To show that, once arc ΑΓ and ratio *ch*(2ΑΒ):*ch*(2ΒΓ) are given, each of ΑΒ, ΒΓ is also given (Fig. 11). The scholium points out that the said ratio and any of the lesser arcs will determine the other two arcs. The proof is immediate: if, for instance, arc ΑΒ is given, then also *ch*(2ΑΒ) is given by the Table of Chords; but also *ch*(2ΑΒ):*ch*(2ΒΓ) is given; therefore *ch*(2ΒΓ) is also given (*Data* 2); by the Table of Chords, arc ΒΓ will also be given; therefore arc ΑΓ is also given as a whole (*Data* 3). See sch. **76**. Sch. **66**–**73** refer to the second cyclic lemma. *d*) Both in **B** and in **C**, sch. **66** is located beside the beginning of the *relatum*. In **K**, it follows sch. **69**. *e*) Note the correlative emphatic οὐ μόνον … ἀλλὰ καὶ … «not only: … but also: …»; the scope of each correlate is the entire subsequent clause (cf. sch. **22**, **28** and **50**).

67

Text. ὡς μείζονος δηλαδὴ οὔσης τῆς ΑΒ περιφερείας τῆς ΒΓ· εἰ γὰρ ἦσαν ἴσαι, ἡ κάθετος ἐπὶ τὸ Ε ἔπιπτεν, εἰ δὲ ἐλάττων ἡ ΑΒ τῆς ΒΓ, ἐπὶ τῆς ΕΓ ἔπιπτεν.

1 ὡς — περιφερείας om. **G** **2** ἐλάττων] ἔλαβεν **G**

Transl. Since, clearly, arc AB is greater than ΒΓ; for if they were equal, the perpendicular would fall on E; if instead AB were less than ΒΓ, it would fall on ΕΓ.

Comm. *a*) Vat. gr. 1594, f. 24v marg. ext., Marc. gr. 313, f. 50v marg. ext., Vat. gr. 184, ff. 30v et 91r marg. sup. *b*) *Ad Alm*. I.13, 71.22–23 καὶ ἤχθω ἀπὸ τοῦ Δ κάθετος ἐπὶ τὴν ΑΕΓ ἡ ΔΖ «and let from Δ a [straight line] ΔΖ be drawn perpendicular to ΑΕΓ» (no citation in **K**). *c*) The scholium justifies the position of perpendicular ΔΖ within the configuration and on the corresponding diagram (Fig. 11), for Ptolemy's argument assumes without proof that perpendicular ΔΖ falls inside triangle ΑΔΕ. The text coincides with the corresponding passage in Theon, *in Alm. I.13*, *iA*, 549.10–12. *d*) Both in **B** and in **C**, sch. **67** is located beside the *relatum*. In **K**, it follows sch. **66**; in **G**, it is linked with it so as to produce a seemingly continuous text (note the initial omission). In **B**, the scholium is shaped as sch. **1**. *e*) This is the sole occurrence of adverb δηλαδή in our corpus of scholia.

68

Text. ἐὰν γὰρ προσεκβληθῇ ἡ ΔΖ, ἐπὶ τῆς διχοτομίας τῆς ΑΒΓ περιφερείας πεσεῖται

Transl. For, if ΔΖ be produced, it will fall on the middle point of arc ΑΒΓ

Comm. *a*) Vat. gr. 1594, f. 24v interc., Marc. gr. 313, f. 50v marg. int. *b*) *Ad Alm*. I.13, ρπ. 71.24–72.2 τῆς ΑΓ περιφερείας δοθείσης ἥ τε ὑπὸ ΑΔΖ γωνία τὴν ἡμίσειαν αὐτῆς ὑποτείνουσα δεδομένη ἔσται [...] δῆλον «[that], arc ΑΓ being given, angle ΑΔΖ, which subtends half of it, will be given [...] it is clear». *c*) The scholium justifies the first of two statements regarded as «clear» by Ptolemy (see sch. **69** and Fig. 11): the angle at the center subtending an arc that is half a given arc is also given. See *El*. III.3 and III.30; the angle is given since its sides ΑΔ and ΔΖ are given in position and meet at a given point. The text coincides with the corresponding passage in Theon, *in Alm. I.13*, *iA*, 549.13–14. *d*) Both in **B** and in **C**, sch. **68** is located beside the *relatum*.

69

Text. δέδοται δὲ καὶ ἡ πρὸς τῷ Ζ ὀρθὴ γωνία· καὶ λοιπὴ ἄρα ἡ πρὸς τῷ Α γωνία δοθήσεται· δέδοται δὲ καὶ ἡ ΔΑ εὐθεῖα ἐκ τοῦ κέντρου (παντὸς γὰρ κύκλου ἡ ἐκ τοῦ κέντρου ξ ἐστι οἵων ἡ διάμετρος ρκ), καὶ ἔτι ἡ ΑΖ ἡμίσεια οὖσα τῆς ΑΓ δεδομένης ἐκ τῶν ἐν κύκλῳ εὐθειῶν, ἐπεὶ καὶ ἡ ΑΒΓ περιφέρεια δέδοται· καὶ ἔστι τὸ ἀπὸ τῆς ΑΔ ἴσον τοῖς ἀπὸ τῶν ΑΖ ΖΔ· ὥστε καὶ λοιπὴ τοῦ ΑΔΖ τριγώνου πλευρὰ ἡ ΔΖ ἔσται δεδομένη.

1 καὶ ἡ] ἡ καὶ **CG** | ὀρθὴ γωνία comp. **BC** : ὀρθογώνιον **G** : ex ὀρθογώνιον corr. m. 2 **K** | λοιπὴ] λοιπῇ **C** | τῷ Α γωνία ex τῶν Α γωνιῶν corr. m. 2 **K** | Α] ΙΑ **CG** **2** παντὸς — **3** ρκ om. Th. | κύκλου comp. **BC** : κέντρου **G** | οἵων] οἷον **G** | ἐκ Th. : εἰς codd. **4** τῶν — εὐθειῶν] τὴν — εὐθεῖαν **G** | ΑΒΓ om. **G** | καὶ^2 om. **G** | τὸ fecit ex τῷ **G** **5** τοῖς] τῶν **K** | ante λοιπὴ add. ἡ Th. | λοιπὴ] λοιπὸν **G** | ἔσται comp. **B** : περιφέρεια **K**

Transl. And the right angle at Z is also given; therefore the angle at A will also be given as a remainder; and stright line ΔA is also given as a radius (for the radius of every circle is so many 60 of which the diameter is 120), and again AZ [is given], which is half of AΓ—which is given from the chords in a circle, since arc ABΓ is also given—; and the [square] on AΔ is equal to those on AZ, ZΔ; so that side ΔZ of triangle AΔZ will also be given as a remainder.

Comm. *a*) Vat. gr. 1594, f. 24v marg. sup., Marc. gr. 313, f. 50v marg. ext. et inf., Vat. gr. 184, ff. 30v et 91r marg. ext. *b*) *Ad Alm*. I.13, 72.2 καὶ ὅλον τὸ ΑΔΖ τρίγωνον, δῆλον «and [that] triangle AΔZ [is given] as a whole, it is clear». *c*) The scholium justifies the second of two statements regarded as «clear» by Ptolemy (see sch. **68** and Fig. 11): a right-angled triangle having one of the acute angles and two sides given is also given. The deductive steps are justified by *Data* def. 1, prop. 4, 2, *El*. I.47, respectively; the final conclusion requires a combination of theorems of *Data*: 52 (formation of squares), 4 (subtraction of squares), 55 (square root of the remainder); the radius ΔA and straight line AΓ are given since their numerical values can be provided (the former by stipulation, the latter by simply computing with the Table of Chords—but note that in this case one might also use *Data* 87 to show that the radius and AΓ are given). As a consequence, triangle AΔZ is given since its sides are given (*Data* 39 and 52). The text almost coincides with the corresponding passage in Theon, *in Alm. I.13*, *iA*, 549.15–19, in which it is followed by καὶ ὅλον δηλονότι τὸ ΑΔΖ τρίγωνον «and, clearly, triangle AΔZ as a whole», which is a restatement of the *relatum*; the scholiast added the explanation of the fact that the radius of a circle of given diameter is also given. *d*) Both in **B** and in **C**, sch. **69** is located in the upper margin. In **B**, a *signe de renvoi* is added by a later hand and placed just above ἡμίσειαν at 72.1 (see the *relatum* of sch. **68**); no first-hand *signe de renvoi* can be found in the manuscripts. In **K**, sch. **69** precedes sch. **66**. The mistake of **G** at line 2 was induced by the similarity of the signs for κέντρον and κύκλος (both in the form of a K, the former having an additional short stroke at the middle of the letter = majuscule ligature *kappa*-*epsilon*). See sch. **25** for λοιπή here and in sch. **72**.

70

Text. ἐπειδὴ καὶ ἡ ΑΓ περιφέρεια δέδοται

Transl. Since arc ΑΓ is also given

Comm. *a*) Vat. gr. 1594, f. 24v marg. ext., Marc. gr. 313, f. 50v marg. ext. *b*) *Ad Alm*. I.13, 72.2–3 ἐπεὶ δὲ τῆς ΑΓ εὐθείας ὅλης δεδομένης «and since, chord ΑΓ being given as a whole». *c*) The scholium justifies the statement in the *relatum* (see Fig. 11): arc ΑΓ is among the givens of the theorem, and the chord ΑΓ associated to it is given since it is provided by the Table of Chords (= it is a result of a calculation that in principle can be formulated in the language of the givens—but note that one might also use *Data* 87). *d*) In **B**, sch. **70** is located beside the *relatum*; in **C**, it is shifted four lines below it.

71

Text. δοθέντι

Transl. Which is given

Comm. *a*) Vat. gr. 1594, f. 24v marg. ext., Marc. gr. 313, f. 50v marg. ext. *b*) *Ad Alm*. I.13, 72.3–5 ὑπόκειται καὶ ὁ τῆς ΑΕ πρὸς ΕΓ λόγος ὁ αὐτὸς ὢν τῷ τῆς ὑπὸ τὴν διπλῆν τῆς ΑΒ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΒΓ «and ratio ΑΕ to ΕΓ was supposed to be the same as that of the [straight line] under the double of [arc] ΑΒ to that under the double of ΒΓ». *c*) The participle δοθέντι is in the dative: therefore it qualifies the underlined syntagm; the mentioned ratio is in fact among the givens of the theorem (see Fig. 11). *d*) In **B**, sch. **71** is located beside the *relatum*; in **C**, it is shifted 5 lines below it.

72

Text. δέδεικται ἐν τοῖς δεδομένοις ὅτι ἐὰν δεδομένον μέγεθος εἰς δεδομένον λόγον διαιρεθῇ, ἑκάτερον τῶν τμημάτων ἔσται δεδομένον. ἢ καὶ οὕτως· ἐπεὶ δέδοται ὁ τῆς ΑΕ πρὸς ΕΓ λόγος, καὶ συνθέντι δέδοται ὁ τῆς ΑΓ πρὸς ΓΕ λόγος· καὶ δέδοται ἡ ΑΓ· δέδοται ἄρα καὶ ἡ ΓΕ· δέδοται δὲ καὶ ἡ ΖΓ ἡμίσεια οὖσα τῆς ΑΓ· καὶ λοιπὴ ἄρα ἡ ΖΕ ἔσται δεδομένη· δέδοται δὲ καὶ ἡ ΖΔ· καὶ ὀρθὴ ἡ ὑπὸ ΔΖΕ· δέδοται ἄρα καὶ ἡ ΔΕ.

1 post δέδεικται add. γὰρ Th. | δεδομένον^{2}] δέδο(ται) **G** **2** ἐπεὶ] ἐπὶ **C** **3** καὶ συνθέντι — λόγος om. **G** | συνθέντι] συντεθέντων **C** **4** ἡ^{1} om. **CG** **5** δὲ om. **G** | ΖΔ] ΖΘ **G** | ἡ^{3} om. **CG**

Transl. It is shown in the Data that, if a given magnitude be divided in a given ratio, each of the segments will be given. Or also as follows. Since the ratio of ΑΕ to ΕΓ is given, by composition the ratio of ΑΓ to ΓΕ is also given; and ΑΓ is given; therefore ΓΕ is also given; and ZΓ is also given, since it is half of AΓ; therefore ZE will also be given as a remainder; and ZΔ is also given; and also right angle ΔZE; therefore ΔE is also given.

Comm. *a*) Vat. gr. 1594, f. 24v marg. ext., Marc. gr. 313, f. 50v marg. sup., Vat. gr. 184, ff. 31r et 91r marg. sup. *b*) *Ad Alm*. I.13, 72.5–8 ἥ τε AE ἔσται δοθεῖσα καὶ λοιπὴ ἡ ZE. καὶ διὰ τοῦτο καὶ τῆς ΔZ δεδομένης δοθήσεται καὶ ἥ τε ὑπὸ EΔZ γωνία τοῦ EΔZ ὀρθογωνίου «both AE will be given and ZE, as a remainder. And because of this and of the fact that ΔZ is given, angle EΔZ of the right-angled [triangle] EΔZ will also be given» (underlined the citation in **K**). *c*) The scholium justifies some of the statements in the *relatum* (the relevant magnitudes are underlined in what follows). The quotation of the enunciation of *Data* 7 is applied to show that both AE and EΓ are given once AΓ (sch. **70**) and ratio AE:EΓ::*ch*(2AB):*ch*(2BΓ) are (see Fig. 11). The “alternative” deduction, which does not mention AE, first applies *Data* 6, 2 (twice), and 4 to show that ZE is given; then, after claiming that ZΔ (by *Data* 30, 25, 26) and right angle ΔZE (*Data* def. 1) are given, deduces that ΔE is also given (*Data* 52, 4, 55; see sch. **69**), and hence that angle EΔZ is also given since triangle EΔZ is given in form (*Data* 39). The scholium is a cento of two passages in Theon, *in Alm. I.13*, *iA*, 549.24–26 (quotation of the enunciation of *Data* 7) and 549.27–550.1; the second passage is the beginning of an alternative proof of the fact that, in the second cyclic lemma, both AE and EΓ are given (Rome wrongly puts a paragraph at 549.27, but what follows is not an alternative proof of the entire lemma); see sch. **79**. *d*) In **B**, sch. **72** is *figuratum* (an altar) and its beginning is located just beside the *relatum*; in **C**, it is in the upper margin, without a *signe de renvoi*.

73

Text. διὰ τὸ καὶ τὴν AZ δεδόσθαι ἡμίσειαν οὖσαν τῆς AΓ

Transl. Because AZ is also given, since it is half of AΓ

Comm. *a*) Vat. gr. 1594, f. 24v interc., Marc. gr. 313, f. 50v marg. int. *b*) *Ad Alm*. I.13, 72.5–6 ἥ τε AE ἔσται δοθεῖσα καὶ λοιπὴ ἡ ZE «both AE will be given and ZE, as a remainder». *c*) The scholium provides and justifies the coassumption necessary to find ZE, as Ptolemy suggests, by subtracting AZ (= $\frac{1}{2}$AΓ: *Data* 2) from AE (*Data* 4; for AE see sch. **72**). *Data* 2 and 4 are here applied (see Fig. 11). *d*) Both in **B** and in **C**, sch. **73** is located beside the *relatum*. In **B**, the scholium is shaped as sch. **1**. It is further enriched by an ornamental motif, placed just below the last two signs.

74

Text. καὶ ὅμοιον ποιεῖν τῷ ΓΕΗ τριγώνῳ τὸ ΒΕΖ τρίγωνον

ποιεῖν] ποιοῦσαι Th. | τῷ ΓΕΗ τριγώνῳ τὸ ΒΕΖ τρίγωνον] τὸ ΓΕΗ τρίγωνον τῷ ΒΕΖ τριγώνῳ Th.

Transl. And make triangle BEZ similar to triangle ΓΕΗ

Comm. *a*) Vat. gr. 1594, f. 24v marg. int., Marc. gr. 313, f. 51r marg. int. *b*) *Ad Alm*. I.13, 73.6–7 ἔσται διὰ τὸ παραλλήλους αὐτὰς εἶναι, ὡς ἡ ΓΗ πρὸς τὴν ΒΖ, οὕτως ἡ ΓΕ πρὸς τὴν ΕΒ «because they are parallel, as ΓΗ is to BZ, so ΓΕ will be to EB». *c*) The configuration of the third cyclic lemma is as follows (see Fig. 12). In a circle ΑΒΓ of center Δ, mark two consecutive arcs AB, ΒΓ, any of which is less than a semicircle, join ΔA and ΓB meeting at E once produced, draw from B, Γ perpendiculars BZ, ΓΗ to radius ΔA, possibly produced. It is required to show that *ch*(2ΓA):*ch*(2AB)::ΓE:BE. The scholium completes the explicative clause of the *relatum* by giving the condition used in *El*. VI.4 in order to prove the statement in the principal clause. It corresponds to Theon, *in Alm. I.13*, *iA*, 552.2–3; the syntax is adapted to fit Ptolemy's sentence. Sch. **74**–**75** refer to the third cyclic lemma. *d*) Both in **B** and in **C**, sch. **74** is located beside the *relatum*.

75

Text. διπλασία γὰρ ἑκατέρα ἑκατέρας τῶν ΓΗ ΒΖ

Transl. For each of them is the double of each of ΓΗ, BZ, respectively

Comm. *a*) Vat. gr. 1594, f. 24v marg. int., Marc. gr. 313, f. 51r marg. int. *b*) *Ad Alm*. I.13, 73.8–9 ὥστε καί, ὡς ἡ ὑπὸ τὴν διπλῆν τῆς ΓΑ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΑΒ «so that, as the [straight line] under the double of [arc] ΓΑ to that under the double of AB». *c*) The scholium explains how the ratio mentioned in the *relatum* can be obtained from the ratio of ΓΗ to BZ (see sch. **74** and Fig. 12); this is simply because the two chords involved in the former ratio are double of ΓΗ and BZ, respectively. The scholium modifies Theon, *in Alm. I.13*, *iA*, 553.5–6, who halves the chords instead of doubling ΓΗ and BZ. *d*) Both in **B** and in **C**, sch. **75** is located beside the *relatum*.

76

Text. καὶ ἐνταῦθα οὐ μόνον τῆς ΒΓ δοθείσης δίδονται καὶ αἱ λοιπαί, ἀλλὰ καὶ ὁποτέρας τῶν ΓΑ ΑΒ δοθείσης δίδονται καὶ αἱ λοιπαὶ δύο περιφέρειαι.

Transl. Here too, not only: once [arc] ΒΓ is given, the remaining ones are also given, but also: once any of ΓΑ, ΑΒ is given, the remaining two arcs are also given.

Comm. *a*) Vat. gr. 1594, f. 24v marg. inf., Marc. gr. 313, f. 51r marg. ext. *b*) *Ad Alm*. I.13, 73.11–14 καὶ ἐνταῦθα δὲ αὐτόθεν παρακολουθεῖ, διότι, κἂν ἡ ΓΒ περιφέρεια μόνη δοθῇ, καὶ ὁ λόγος ὁ τῆς ὑπὸ τὴν διπλῆν τῆς ΓΑ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΑΒ δοθῇ, καὶ ἡ ΑΒ περιφέρεια δοθήσεται «and here too, it immediately follows that, even if both the single arc ΓΒ is given and the ratio of the [straight line] under the double of [arc] ΓΑ to that under the double of ΒΓ is given, arc ΑΒ will also be given» ff. *c*) In the configuration of the third cyclic lemma (sch. **74** and Fig. 12), neglect perpendiculars ΒΖ, ΓΗ and from center Δ join ΒΔ and draw ΔΖ perpendicular to ΕΒΓ. It is required to show that, once arc ΓΒ and ratio *ch*(2ΓΑ):*ch*(2ΑΒ) are given, arc ΑΒ is also given (Fig. 13). The scholium points out that the ratio and any of the arcs will determine the other two arcs: see sch. **66** for the same statement in the case of the second cyclic lemma. Theon has a remark partly to the same effect at *in Alm. I.13*, *iA*, 557.24–26 (but the statement is only witnessed by Laur. Plut. 28.18): ὁμοίως δὲ κἂν ἡ ΑΒ περιφέρεια μόνη δοθῇ καὶ ὁ λόγος τῆς ὑπὸ τὴν διπλῆν τῆς ΑΓ πρὸς τὴν ‹ὑπὸ τὴν διπλῆν τῆς› ΓΒ, δοθήσεται καὶ ἡ ΒΓ περιφέρεια, τῆς συμπτώσεως ἐπὶ τὰ πρὸς τοῖς Γ γινομένης «similarly, even if the single arc ΑΒ is given and the ratio of the [straight line] under the double of [arc] ΑΓ to that [under the double of] ΓΒ, arc ΒΓ will also be given, the intersection occurring on the side of Γ». The proof of the remaining case is immediate: if arc ΑΓ is given, then also *ch*(2ΑΓ) is given by the Table of Chords; but *ch*(2ΑΓ):*ch*(2ΒΓ) is given; therefore *ch*(2ΒΓ) is also given (*Data* 2); by the Table of Chords, arc ΒΓ will also be given; therefore arc ΑΒ is also given as a remainder (*Data* 4). The same argument applies if the ratio is *ch*(2ΑΓ):*ch*(2ΑΒ). Sch. **76**–**81** refer to the fourth cyclic lemma. *d*) In **B**, sch. **76** is located under the column containing the beginning of the *relatum*. In **C**, it is beside the last four lines of the proof. In **B**, the scholium is partly shaped as sch. **1**. *e*) The identical καὶ ἐνταῦθα «here, too» at the beginning of both the *relatum* and the scholium show to what extent the commentators tend to mimic the language of the original.

77

Text. πολλάκις ἐν ταῖς κατὰ σύνθεσιν πτώσεσι παράλληλος γίνεται ἡ ΒΓ τῇ ΔΑ· διὸ τότε αὐτόθεν δίδοται ἡ ΒΑ περιφέρεια, διὰ τὸ δοθείσης τῆς ὑπὸ ΖΔΒ δίδοσθαι καὶ τὴν λείπουσαν εἰς τὴν μίαν ὀρθήν, τουτέστι τὴν ὑπὸ ΒΔΑ· δίδοται ἄρα καὶ ἥ τε ΒΑ καὶ ὅλη ἡ ΓΒΑ.

1 ΔΑ] ΒΑ codd. **2** ΒΑ] ΓΑ **C** | ΖΔΒ] ΖΑΒ **K**

Transl. In the cases by composition, ΒΓ often becomes parallel to ΔΑ; this is the reason why in that case arc ΒΑ is immediately given, because, once [angle] ΖΔΒ is given, the complement to one right [angle] is also given, that is, ΒΔΑ; therefore both [arc] ΒΑ and ΓΒΑ as a whole are also given.

Comm. *a*) Vat. gr. 1594, f. 25r marg. sup., Marc. gr. 313, f. 51r marg. sup., Vat. gr. 184, f. 31v. *b*) *Ad Alm*. I.13, 73.11–14 καὶ ἐνταῦθα δὲ αὐτόθεν παρακολουθεῖ, διότι, κἂν ἡ ΓΒ περιφέρεια μόνη δοθῇ καὶ ὁ λόγος ὁ τῆς ὑπὸ τὴν διπλῆν τῆς ΓΑ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΑΒ δοθῇ, καὶ ἡ ΑΒ περιφέρεια δοθήσεται «here too, it immediately follows that, even if both the single arc ΓΒ is given and the ratio of the [straight line] under the double of [arc] ΓΑ to that under the double of ΒΓ is given, arc ΑΒ will also be given» ff. (no citation in **K**). *c*) This is a scholium to the fourth cyclic lemma, showing that the result is also valid when ΑΔ and ΒΓ are parallel (Fig. 13). That angle ΖΔΒ is given is stated by Ptolemy, 73.16–74.2, and then one applies *Data* 4, the fact that the arcs on a circumference and the angles at the center subtending them are in one-to-one correspondence (use *Data* 89, *El*. III.20, *Data* 2), and *Data* 3. In this case one also immediately gets that, since ΒΖ = ΓΗ in the configuration of the third cyclic lemma, the ratio mentioned in the *relatum* is that of equality, a fact that will prove crucial in the proof of the "parallel" configuration of the Sector Theorem outlined in sch. **87**. Theon, *in Alm. I.13*, *iA*, 554.11 and 554.16, asserts instead that in this case ἀσύστατον ἔσται τὸ θεώρημα «the theorem [*scil*. the fourth cyclic lemma] will be unsolvable» (lit. «non-constructible»), and points out that Ptolemy οὐ προσχρῆται ταῖς οὕτως ἀσύστατον ποιούσαις τὸ πρόβλημα «does not use those [straight lines] that make the problem in this way unsolvable». *d*) In **B**, the scholium is above the column in which the fourth cyclic lemma ends. In **C**, it is in the upper margin of the page containing the same lemma. In either case, no *signe de renvoi* is added. In **K**, it precedes sch. **78**. *e*) The scholiast's πολλάκις «often» at line 1 is something of a cheat: as a matter of fact, the "parallel" configuration of the Sector Theorem is never required in *Alm*.; see Theon's statement read above and Rome's remarks at *iA*, 554–556, n. 1, and (1933, 45, n. 1). The αὐτόθεν «immediately» at line 2 is both imitative of the *relatum* (cf. sch. **76**) and a typical metadiscursive modifier, of which Ptolemy is specially fond; one finds 69 occurrences in *Alm*.; 5 in Pappus, *in Alm*. V–VI; 26 in Theon, *in Alm*. I–IV; 3 in our corpus of scholia (sch. **77**, **84**, and **98**). The operator ἡ λείπουσα εἰς «the complement to» at lines 2–3 is in this scholium applied to an angle; otherwise the expression ἡ λείπουσα εἰς τὸ ἡμικύκλιον means «the [chord] complement to a semicircle»: see sch. **11**, **27**, and **46**; see also sch. **107**.

78

Text. δέδοται δὲ καὶ ἡ ΒΓ εὐθεῖα ἐκ τῶν ἐν κύκλῳ εὐθειῶν καὶ δίχα αὐτὴν τέμνει ἡ ΔΖ· καὶ ἡ ἡμίσεια ἄρα αὐτῆς ἡ ΒΖ δέδοται· δέδοται δὲ καὶ ἡ ἐκ τοῦ κέντρου ἡ ΔΒ· καὶ ἔστι τὸ ἀπὸ τῆς ΔΒ ἴσον τοῖς ἀπὸ τῶν ΖΒ ΖΔ· δέδοται ἄρα καὶ ἡ ΖΔ.

1 δὲ om. **G** | ΒΓ] ΑΓ **G** | ΔΖ] ΑΖ **K** **3** τῶν] τῆς **BK** : τὴν **CG** | ἄρα] ἔτι **G**

Transl. Straight line ΒΓ is also given from the chords in a circle, and ΔΖ bisects it; therefore half of it, ΒΖ, is also given; and radius ΔΒ is also given; and the [square] on ΔΒ is equal to those on ΖΒ, ΖΔ; therefore ΖΔ is also given.

Comm. *a*) Vat. gr. 1594, f. 25r marg. sup., Marc. gr. 313, f. 51r marg. ext., Vat. gr. 184, ff. 31v et 91r marg. ext. *b*) *Ad Alm*. I.13, 73.16–74.2 ἡ μὲν ὑπὸ ΒΔΖ γωνία τὴν ἡμίσειαν ὑποτείνουσα τῆς ΒΓ περιφερείας ἔσται δεδομένη· καὶ ὅλον ἄρα τὸ ΒΔΖ ὀρθογώνιον «angle ΒΔΖ, subtending half of arc ΒΓ, will be given; therefore the right-angled [triangle] ΒΔΖ is also [given] as a whole» (no citation in **K**). *c*) The scholium supplies the steps needed to prove the conclusion from the stated premisses (Fig. 13). The propositions applied are *Data* 87 (here replaced by a look at the Table of Chords), *El*. III.3, *Data* 2, the fact that the radius is given since its numerical value is provided by stipulation as 60 parts (see sch. **69**), *El*. I.47. The last step is justified by *Data* 52, 4, 55: see again sch. **69**, and, further, sch. **80**. As a consequence, triangle ΒΔΖ is given since its sides are given (*Data* 39 and 52). The text of the scholium coincides with Theon, *in Alm. I.13*, *iA*, 557.5–8. *d*) In **B**, the scholium is above the intercolumnar space on the right of the column in which the fourth cyclic lemma ends; it happens to be quite close to the *relatum*. In **C**, it is beside the space left, between the third and the fourth lemma, for the diagram of the third lemma. In neither case, any *signe de renvoi* is added. In **K**, it follows sch. **77**.

79

Text. καὶ διελόντι

Transl. And by separation

Comm. *a*) Vat. gr. 1594, f. 25r marg. int., Marc. gr. 313, f. 51r marg. ext. *b*) *Ad Alm*. I.13, 74.3–5 ἐπεὶ δὲ καὶ ὅ τε τῆς ΓΕ πρὸς τὴν ΕΒ λόγος δέδοται καὶ ἔτι ἡ ΓΒ εὐθεῖα, δοθήσεται καὶ ἥ τε ΕΒ «since both the ratio of ΓΕ to ΕΒ is given and again straight line ΓΒ, [straight line] EB will also be given». *c*) The scholium supplies a missing step in Ptolemy's one-sentence deduction (Fig. 13): before applying *Data* 2 to get EB, one has to perform separation of ratios on the ratio of ΓE to EB in order to have the ratio of ΓB to EB given (but there is no theorem to that effect in *Data*); ΓB is among the givens of the theorem. The διελόντι deductive step is made explicit in Theon, *in Alm. I.13*, *iA*, 557.15, within an alternative proof of a deductive step of the fourth cyclic lemma very much in the style of the alternative proof of the corresponding step in the second cyclic lemma (see sch. **72**—also in this case, Rome wrongly puts a paragraph at 557.15). *d*) In **B**, sch. **79** is located beside the *relatum*. In **C**, it is misplaced beside the second line of the proof, seven lines above the *relatum*.

80

Text. ἐπεὶ καὶ ἡ BZ δέδοται ἡμίσεια οὖσα τῆς BΓ δεδομένης καὶ ἔστι τὰ ἀπὸ τῶν EZ ZΔ ἴσα τῷ ἀπὸ τῆς EΔ, δέδοται ἄρα καὶ ἡ EΔ· ὥστε καὶ τὸ EΔZ τρίγωνον ὀρθογώνιον.

1 οὖσα comp. **BC** : om. **G**

Transl. Since BZ is also given because it is half of BΓ, which is given, and the [squares] on EZ, ZΔ are equal to that on EΔ, therefore EΔ is also given; so that the right-angled triangle EΔZ is also [given]

Comm. *a*) Vat. gr. 1594, f. 25r marg. int., Marc. gr. 313, f. 51r marg. ext., Vat. gr. 184, f. 91r marg. ext. *b*) *Ad Alm*. I.13, 74.4–6 δοθήσεται καὶ ἥ τε EB καὶ ἔτι ὅλη ἡ EBZ· ὥστε καί, ἐπεὶ ἡ ΔZ δέδοται, δοθήσεται καὶ ἥ τε ὑπὸ EΔZ γωνία «[straight line] EB will also be given, and again EBZ as a whole; so that, since ΔZ is given, angle EΔZ will also be given». *c*) The scholium supplies some steps missing in Ptolemy's two-sentence deduction (Fig. 13): they almost coincide with those in sch. **78**. The text of the scholium is a rewriting, more faithful towards the end, of Theon, *in Alm. I.13*, *iA*, 557.17–19, within an alternative proof of a deductive step of the fourth cyclic lemma very much in the style of the alternative proof of the corresponding step in the second cyclic lemma (see sch. **72** and **79**). *d*) In **B**, sch. **80** is located beside the *relatum*. In **C**, it starts beside the fourth line of the proof, six lines above the *relatum*.

81

Text. ἐπειδὴ ἡ ὑπὸ BΔZ γωνία δίδοται

1 ἐπειδὴ] ἐπεὶ **B**

Transl. Since angle BΔZ is given

Comm. *a*) Vat. gr. 1594, f. 25r interc., Marc. gr. 313, f. 51r marg. ext., Vat. gr. 184, f. 91r marg. ext. *b*) *Ad Alm*. I.13, 74.6–7 δοθήσεται καὶ ἥ τε ὑπὸ ΕΔΖ γωνία τοῦ αὐτοῦ ὀρθογωνίου καὶ λοιπὴ ἡ ὑπὸ ΕΔΒ «angle ΕΔΖ of the same right-angled [triangle] [*scil.* ΕΔΖ] will also be given, and ΕΔΒ as a remainder». *c*) The scholium supplies the coassumption necessary to find angle ΕΔΒ by subtracting ΒΔΖ from ΕΔΖ (Fig. 13). That angle ΒΔΖ is given is stated by Ptolemy, 73.16–74.2, and then one applies *Data* 4. *d*) In **B**, sch. **81** is located beside the *relatum*; in **C**, it is placed four lines above it. In **G**, sch. **78**, **80**, and **81** are linked so as to produce a seemingly continuous text. *e*) The oscillations in our corpus of scholia between the non-canonical present tense δίδοται (5 occurrences, never in extracts from Theon) and the canonical perfect tense δέδοται (19 occurrences) reflect Late-Antiquity stylistic choices: one finds 0/146 occurrences of δίδοται/δέδοται in *Data*; 15/11 in *Alm*.; 16/14 in Pappus, *in Alm*. V–VI, but 0/10 in *Coll*.; 23/73 in Theon, *in Alm*. I–IV. On the issue of the participial forms of διδόναι, see Acerbi (2011b, 127–128). I have retained in the main text the variant reading of the **C**-branch on no other grounds than a sense of stylistic appropriateness.

82

Text. θεώρημα κατὰ διαίρεσιν

Transl. Theorem by separation

Comm. *a*) Vat. gr. 1594, f. 25r marg. int., Marc. gr. 313, f. 51r marg. int. *b*) *Ad Alm*. I.13, 74.9–10 τούτων προληφθέντων γεγράφθωσαν ἐπὶ σφαιρικῆς ἐπιφανείας μεγίστων κύκλων περιφέρειαι «this being preliminarily established, let two arcs of great circles be drawn on a spherical surface» ff. *c*) The scholium provides a rigid designator of the fifth theorem proved by Ptolemy: this is the first configuration, "by separation," of the Sector Theorem. Theon, *in Alm. I.13*, *iA*, 558.1–2 and repeatedly since then, calls the Theorem σφαιρικὸν θεώρημα «spherical theorem» and its configurations κατὰ διαίρεσιν «by separation» and κατὰ σύνθεσιν «by composition». Sch. **82–91** refer to the Sector Theorem, configuration "by separation." *d*) Both in **B** and in **C**, sch. **82** is located beside the beginning of the *relatum*. It is in majuscule in **B**.

83

Text. ἐν τοῖς σφαιρικοῖς τούτοις λημματίοις αἱ μὲν κατὰ διαίρεσιν πτώσεις πᾶσαι σώζουσιν τὴν εὐθύγραμμον καταγραφήν, αἱ δὲ κατὰ σύνθεσιν οὔτε σώζουσιν ἀεὶ τὴν εὐθύγραμμον καταγραφὴν οὔτε δείκνυνται πᾶσαι· δυνατὸν δέ ἐστι δεικνῦναι πάσας τὰς κατὰ σύνθεσιν ἐκ τῶν κατὰ διαίρεσιν δεδειγμένων· ἡ γὰρ ὑποτείνουσα τὴν διπλῆν τῆς ΓΕ ὑποτείνει καὶ τὴν διπλῆν τῆς συνεχοῦς τῇ ΓΕ ὡς ἐπὶ τὰ πρὸς τὸ Ε μέρη καὶ ἀναπληρούσης τὸ ἡμικύκλιον· ἡ γὰρ αὐτὴ εὐθεῖα ὑποτείνει τό τε μεῖζον τμῆμα τοῦ κύκλου καὶ τὸ ἔλαττον. τὰ δ' αὐτὰ δεῖ νοεῖν καὶ ἐπὶ τῶν ΓΖ ΖΔ περιφερειῶν.

5 ὑποτείνει] ὑπο– comp. **B** : ἀπο– **CKG** | μέρη] μέρους **G** | ἀναπληρούσης] –πλήρωσιν codd. **6** τὸ ἡμικύκλιον] τοῦ κύκλου codd. (κύκλου symbolon **B**) | αὐτὴ] αὕτη **K** | εὐθεῖα] α supra lineolam scripserunt **BCG** : idem sed δ scripsit **K** **7** περιφερειῶν om. **G**

Transl. In these short spherical lemmas, all cases by separation preserve the rectilinear diagram, whereas the [cases] by composition neither preserve in every instance the rectilinear diagram, nor can they all be demonstrated; but it is possible to demonstrate all [cases] by composition from those demonstrated by separation, for the [straight line] subtending the double of [arc] ΓΕ also subtends the double of the [arc] adjacent to ΓΕ on the side of Ε and completing a semicircle, for the same straight line subtends both the greater and the lesser segment of the circle. The same must also be conceived of for arcs ΓΖ, ΖΔ.

Comm. *a*) Vat. gr. 1594, f. 25r marg. inf., Marc. gr. 313, f. 51r marg. inf., Vat. gr. 184, ff. 32r et 91v marg. ext. *b*) *Ad Alm*. I.13, 74.9–10 τούτων προληφθέντων γεγράφθωσαν ἐπὶ σφαιρικῆς ἐπιφανείας μεγίστων κύκλων περιφέρειαι «this being preliminarily established, let two arcs of great circles be drawn on a spherical surface» ff. (= citation in **K**). *c*) The configuration of the Sector Theorem is as follows (Fig. 14). From the end-points Β, Γ of two mutually intersecting arcs ΑΒ, ΑΓ of great circles on the surface of a sphere, two arcs ΒΕ, ΓΔ are drawn across, meeting at Ζ and intersecting arcs ΑΓ, ΑΒ at Ε, Δ, respectively; all these arcs must be less than a semicircle. Then

$$ch(2\Gamma E):ch(2EA) = [ch(2\Gamma Z):ch(2Z\Delta)]\circ[ch(2\Delta B):ch(2BA)] \text{ ("by separation")},$$

$$ch(2\Gamma A):ch(2AE) = [ch(2\Gamma\Delta):ch(2\Delta Z)]\circ[ch(2ZB):ch(2BE)] \text{ ("by composition")}.$$

The first relation is proved by Ptolemy (74.9–76.2); the second relation is only enunciated (76.3–9). The proof brings into existence the configuration of a suitable rectilinear lemma: from the center Η of the sphere, radii ΗΒ, ΗΖ, ΗΕ are joined; ΗΒ is produced to meet ΑΔ produced at Θ; ΓΔ, ΓΑ are joined and they meet ΗΖ, ΗΕ at Κ, Λ, respectively; one shows that points Θ, Κ, Λ are on one and the same straight line. The resulting rectilinear configuration is that in which from the endpoints Θ, Γ of two mutually intersecting straight lines ΑΘ, ΑΓ two lines ΘΛ, ΓΔ are drawn across, meeting at Κ and intersecting straight lines ΑΓ, ΑΘ at Λ, Δ, respectively. The preceding lemmas allow "lifting" to the spherical case the relations obtaining in the underlying rectilinear configuration. The scholium points out the main differences between the configurations "by separation" and those "by composition" of the Sector Theorem. "To preserve the linear diagram" means that the straight lines and the arcs involved in the ratios that feature in corresponding rectilinear and spherical configurations have the same position in the associated diagrams. This does not happen, for instance, in the case “by composition” proved by Theon at *in Alm. I.13*, *iA*, 564.9–565.20. It is not said that the scholiast had a general proof of the affirmative side of his statement in mind; he could simply have guessed by induction on the cases proved in Theon’s commentary. The case that “cannot be demonstrated” is the “parallel” configuration of the Sector Theorem (see sch. **77** and **87**). A proof by paradigmatic example that every case “by composition” can be deduced from an appropriate case “by separation” can be found at Theon, *in Alm. I.13*, *iA*, 567.1–570.12, exactly along the lines indicated by the scholiast. The denomination λημμάτια «short lemmas» for the several cases of the Sector Theorem does not coincide with those of Ptolemy or of Theon (see sch. **56** and **82**). *d*) Both in **B** and in **C**, sch. **83** is located in the lower margin of the page; no *signe de renvoi* is identifiable in the manuscripts. In **K**, it follows sch. **86**. Note, at line 6, the sign for εὐθεῖα, that baffled the copyist of **K**. *e*) Here and in sch. **84**, the annotator introduces a material connotation by the fact of referring to a καταγραφή and not to a σχῆμα. The difference between the two terms is well-established in the technical (meta-)terminology: σχῆμα is a geometric figure, καταγραφή its graphic representation. At line 6, as elsewhere in our corpus of scholia, note that the noun ἡμικύκλιον is preceded by an article, even if modern Western languages idiomatically resort to an indeterminate expression.

84

Text. ἐπιζευχθεῖσα ἡ κοινὴ τομὴ ἡ ΘΚΛ, μετὰ τῶν τοῦ τριγώνου πλευρῶν ποιεῖ τὴν εὐθύγραμμον καταγραφήν, καὶ οὕτως ἐπὶ παντός. δεῖ δὲ τρίγωνον λαμβάνειν τὸ περιεχόμενον ὑπὸ τῶν εὐθειῶν τῶν ἐπὶ τὰ πέρατα τῶν λόγων ἐπιζευγνυμένων, τὴν δὲ κοινὴν τομὴν τὴν ἐπὶ τὰ σημεῖα ἐπιζευγνυμένην καθ’ ἃ συμβάλλουσιν αἱ ἀπὸ τοῦ κέντρου ἐπὶ τὰ μέσα σημεῖα τῶν λόγων ἐπιζευγνύμεναι ταῖς ἐπὶ τὰ ἄκρα σημεῖα τῶν λόγων ἐπιζευγνυμέναις, εἴτε αὐτόθεν ὡς ἡ ΕΗ τῇ ΓΑ εἴτε καὶ ἐκβαλλόμεναι ὡς ἡ ΗΒ τῇ ΑΔ· τὰ γὰρ τρία σημεῖα καθ’ ἃ συμβάλλουσιν αἱ ἀπὸ τοῦ κέντρου τῆς σφαίρας ἐπιζευγνύμεναι ἐπὶ τὰ μέσα σημεῖα τῶν λόγων ταῖς ἐπὶ τὰ ἄκρα σημεῖα ἐπιζευγνυμέναις ἐπ’ εὐθείας γίνονται ἀλλήλοις. | καὶ ἐπειδὴ ἐπὶ τῶν εὐθυγράμμων τὸν συντιθέμενον λόγον ἐκλαμβάνομεν ἢ κατὰ διαίρεσιν ἢ κατὰ σύνθεσιν, οὕτως καὶ ἐπὶ τῶν σφαιρικῶν ὁμοίως ποιήσομεν, μιγνύντες τὴν εὐθύγραμμον θεωρίαν τῇ κυκλικῇ, ἐξ ὧν ἡ σφαιρικὴ ἀνίσταται θεωρία νῦν.

1 ΘΚΛ] Θ codd. **2** καταγραφήν] γραφήν **K** | παντός] παν$^{\tau}$ **BC** : πάντων **G** | τρίγωνον symbolon **BC** : ποιεῖν καὶ **G** **3** ἐπιζευγνυμένων] –γνύμενα **K** **4** τὴν — ἐπιζευγνυμένην] τῶν — ἐπιζευγνυμ$^{\varepsilon}$ **C** : τῶν (ex corr. **K**) — ἐπιζευγνυμένων **GK** **6** ΗΒ] ΝΒ **CG** **8** ἐπιζευγνύμεναι] –μέναις **K** **9** τὸν συντιθέμενον] τὴν συντιθεμένην **C** : τὴν συντεθειμένην **G** | λόγον] λόγων **G** **10** ἐκλαμβάνομεν] ἐλαμ– **K** | σφαιρικῶν] –κὴ **C** **11** θεωρίαν] γωνίαν **G** | post ὧν habet νῦν **G**

Transl. Once the common section ΘΚΛ has been joined, it makes a rectilinear configuration with the sides of the triangle, and similarly in each case. One must take as a triangle the one contained by the straight lines joining the endpoints [*scil*. Γ, Δ, A] of the ratios, and as a common section the [straight line] joining the points at which the [straight lines] joining the center [of the sphere] and the middle points [*scil*. E, Z, B] of the ratios meet the [straight lines] joining the extreme points of the ratios, either immediately as EH [meets] ΓA or even if produced as HB [meets] AΔ, for the three points, at which the [straight lines] joining the center of the sphere and the middle points of the ratios meet the [straight lines] joining the extreme points, come to be on a straight line one to another. And since, in the case of rectilinear [configurations], we indeed work out the compounded ratio either by separation or by composition, so we shall also do in a similar way—namely, by mixing rectilinear and cyclic theory—in the case of spherical [configurations], starting from which the discipline of spherics is actually set up.

Comm. *a*) Vat. gr. 1594, f. 25r marg. sup. et ext., Marc. gr. 313, f. 51v marg. sup. et ext., Vat. gr. 184, ff. 31v–32r et 91v marg. ext. *b*) *Ad Alm*. I.13, 74.9–10 τούτων προληφθέντων γεγράφθωσαν ἐπὶ σφαιρικῆς ἐπιφανείας μεγίστων κύκλων περιφέρειαι «this being preliminarily established, let two arcs of great circles be drawn on a spherical surface» ff. (no citation in **K**). *c*) The scholium describes in the most general terms the construction of the rectilinear configuration associated to a generic case of the Sector Theorem (Fig. 14); see also point *c* of sch. **83**. There is no parallel passage in Theon's commentary. *d*) Both in **B** and in **C**, sch. **84** is located in the upper margin of the page containing the main body of the proof of the Sector Theorem; no *signe de renvoi* is identifiable in the manuscripts. In **K**, it follows sch. **78**. In **G**, sch. **83** and **84** are linked so as to produce a seemingly continuous text. *e*) On καταγραφή at line 2, see sch. **83**. Here and in sch. **86**, the noun phrase αἱ ἀπὸ τοῦ κέντρου (lines 4–5 and 7) is not a denomination of «radius» alternative to ἡ ἐκ τοῦ κέντρου, but a part of the formula identifying a straight line joined <u>from</u> a point <u>to</u> another (ἀπὸ ** ἐπὶ ##), here and in what follows simplified in translation as «joining ** and ##»; see also sch. **5**, **96**, and **100**. The αὐτόθεν «immediately» at line 6 has a different connotation from the one, eminently metadiscursive, expounded when commenting on sch. **77**: it means «without the straight lines being produced».

85

Text. τῆς EA λόγος συνῆπται ἔκ τε τοῦ τῆς ὑπὸ τὴν διπλῆν

EA] EΔ codd.

Transl. The ratio […] of [arc] EA is compounded of that of the [straight line] under the double

Comm. *a*) Vat. gr. 1594, f. 25r marg. int., Marc. gr. 313, f. 51v marg. ext. *b*) *Ad Alm*. I.13, 74.16–17 πρὸς τὴν ὑπὸ τὴν διπλῆν <u>τῆς EA λόγος συνῆπται ἔκ τε τοῦ τῆς ὑπὸ τὴν διπλῆν</u>

τῆς ΓΖ «the ratio […] to the [straight line] under the double of [arc] ΕΑ is compounded of that of the [straight line] under the double of ΓΖ». *c*) This is a pericope witnessed by **D** but that was omitted by *saut du même au même* by the copyist of the common model of **ABC**, and there added in the margin by some reviser. Heiberg wrongly ascribes the integration to a later hand in **C**. *d*) In **B**, sch. **85** is located beside the *relatum*; in **C**, it is shifted nine lines under it. No *signe de renvoi* is added. In **B**, the 12th-century annotator added a sign and the indication κείμενον «standing» [in the text]; in **C**, a later hand added the same clause as sch. **85** *supra lineam* in the main text. *e*) The verbal form συνῆπται «is compounded» is a synonym, widely used by Ptolomy, of σύγκειται.

86

Text. καθ' ἕκαστον λόγον τριῶν ὄντων τῶν σημείων τῶν μὲν ἄκρων τοῦ δὲ μέσου, αἱ μὲν τῶν εὐθειῶν ἐπὶ τὰ ἄκρα τῶν λόγων σημεῖα ἐπιζευγνύουσιν, αἱ δὲ ἀπὸ τοῦ κέντρου τῆς σφαίρας ἐπὶ τὸ μέσον τοῦ τε συντιθεμένου λόγου καὶ ἑκατέρου τῶν συντιθέντων.

1 ante τριῶν posuit τῶν **G** **3** συντιθεμένου] συντεθειμένου **G** | συντιθέντων] συντεθέντων **CG**

Transl. Since there are three points for each ratio, namely, the extremes and the middle [term], some of the straight lines join the extreme points of the ratios, other [join] the center of the sphere and the middle [term] both of the compounded ratio and of each of the compounding [ratios].

Comm. *a*) Vat. gr. 1594, f. 25r marg. inf., Marc. gr. 313, f. 51v marg. ext., Vat. gr. 184, ff. 32r et 91v marg. sup. *b*) *Ad Alm*. I.13, 74.20–75.13 εἰλήφθω γὰρ τὸ κέντρον τῆς σφαίρας […] κατὰ τὸ Κ σημεῖον «in fact, let the center of the sphere be taken […] at point K» ff. (no citation in **K**). *c*) This is a scholium to the construction of the Sector Theorem (Fig. 13). It points out which straight lines must be joined in order to produce the rectilinear configuration associated to the assigned spherical configuration; see sch. **84**. There is no parallel passage in Theon's commentary. *d*) In **B**, sch. **86** is located in the lower margin, just below the first two lines of the *relatum*; in **C**, it is beside the last lines of the *relatum*. No *signe de renvoi* is added. In **K**, it follows sch. **87**. *e*) Here and in the subsequent scholia, the subtleties connected with expressing the components of compounded ratios (see n. 67 above) frequently carry to error the copyists, in particular as far as participial forms of συντιθέναι are concerned.

87

Text. ὅταν μὲν ἡ ἀπὸ τοῦ Η ἐπὶ τὸ Α ἐπιζευγνυμένη ποιῇ τὰς ὑπὸ ΔΑΗ ΑΗΒ γωνίας δύο ὀρθῶν ἐλάττονας, τότε ἡ ΑΔ συμπεσεῖται τῇ ΗΒ κατὰ τὸ Θ, ὡς νῦν· ὅταν δὲ δύο ὀρθῶν μείζονας, τότε ἡ ΔΑ τῇ ΒΗ ἐπὶ θατέρῳ μέρει συμπεσεῖται, προσαναπληρωθέντων τῶν

ΒΔΑ ΒΖΕ ἡμικυκλίων καὶ τῆς ΒΗ διαμέτρου, καὶ ἡ δεῖξις προβαίνει· ὅταν δὲ παράλληλος ᾖ ἡ ΑΔ τῇ ΒΗ, τότε καὶ τῇ ΚΛ παράλληλος γίνεται ἐξ ἀνάγκης, καὶ ὁ τῆς ΓΛ πρὸς ΛΑ λόγος ἀναφθήσεται ἐκ τοῦ τῆς ΓΚ πρὸς ΚΔ· ὁ γὰρ τῆς ὑπὸ τὴν διπλῆν τῆς ΑΒ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΒΔ τότε ἰσότητός ἐστι λόγος, τουτέστι τοῦ αὐτοῦ πρὸς τὸ αὐτό· ὥστε καὶ οὕτως ἡ δεῖξις προβήσεται.

1 ὑπὸ] ἀπὸ **C** **2** ὀρθῶν[1]] ὀρ$^{\theta}$ **BC** : ὀρθὸν **K** **3** μείζονας] comp. **B** : γωνίας comp. **C** : μιᾶς **K** | ΔΑ] ΑΔ **K** | θατέρῳ μέρει] **C** : θάτερα μὲν οὐ **BK** | προσαναπληρωθέντων] (προσ)ανα– **BC** : πρὸς ἀνα– **K** **5** ᾖ] ῇ̣ **K** **6** ΓΚ] Γ (καὶ) **K** | ΚΔ scripsi : ΚΓ codd. | ὁ γὰρ τῆς scripsi : ὃ γί/$^{\tau}$ **K** : ὃ γίνεται **K** **8** οὕτως] (ου)$^{\tau}$ **B** : οὖ **K**

Transl. When the [straight line] joining H and A makes angles ΔAH, AHB less than two right angles, then AΔ will meet HB at Θ, as now; when [it makes angles ΔAH, AHB] greater than two right angles, then ΔA will meet HB on the other side, once semicircles BΔA, BZE and diameter BH have been completed, and the proof can proceed; when AΔ is parallel to BH, then it necessarily becomes parallel to KΛ too, and the ratio of ΓΛ to ΛA will be compounded of that of ΓK to KΔ, for the [ratio] of the [straight line] under the double of [arc] AB to the [straight line] under the double of [arc] BΔ is in that case the ratio of equality, that is, of the same to the same, so that also in this way the proof will proceed.

Comm. *a*) Vat. gr. 1594, f. 25r marg. ext., Marc. gr. 313, f. 51v marg. ext. et inf., Vat. gr. 184, f. 32r. *b*) *Ad Alm*. I.13, 74.20–76.2 εἰλήφθω γὰρ τὸ κέντρον τῆς σφαίρας […] καὶ <u>τοῦ τῆς ὑπὸ τὴν διπλῆν τῆς ΔΒ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΒΑ</u> «in fact, let the center of the sphere be taken […] and <u>of the [ratio] of the [straight line] under the double of [arc] ΔB to that under the double of BA</u>» (no citation in **K**). *c*) This is a scholium to the construction and proof of the Sector Theorem, describing the “parallel” configuration as a limiting case of the configuration actually assumed by Ptolemy (the straight line joining H and A does not feature in Ptolemy’s construction; it just serves to formulate a criterion of intersection/parallelism of straight lines AΔ and HB). The scholiast also summarizes in few but careful words the gist of the proof in that case, not treated by Ptolemy and declared ἀσύστατον «unsolvable» by Theon (see sch. **77**). Take the relation stemming from the rectilinear configuration assumed by Ptolemy: ΓΛ:ΛA = (ΓK:KΔ)(ΔΘ:ΘA). Now, as seen in sch. **77**, in the “parallel” configuration of the third cyclic lemma the ratio between chords mentioned in the *relatum* (underlined above) is that of equality. On the other hand, as the scholiast points out (Fig. 14), when AΔ becomes parallel to BH and hence to KΛ, by *El*. VI.2 the ratios ΓΛ:ΛA and ΓK:KΔ become “equal,” and this entails that the case “by separation” (sch. **83**) still applies, with *ch*(2ΓE):*ch*(2EA)::*ch*(2ΓZ):*ch*(2ZΔ); the actual proof of this statement, that we read, in the quite complex Arabo-Latin tradition of Menelaos’ *Sphaerica*, as a further case after the case “by separation,” is just a formalization of the argument outlined by the scholiast. Sch. **77** and **87** constitute the first direct evidence that a proof of the “parallel” configuration of the Sector Theorem was elaborated in Greek (see Acerbi 2015). *d*) In **B**, sch. **87** is located in the outer margin, beside the construction of the Sector Theorem; in **C**, its beginning is placed beside the last five lines of the proof; the remaining portion of the scholium continues beside the sketch of proof of the configuration "by composition" (76.3–9). No *signe de renvoi* is added. In **K**, sch. **87** follows sch. **84**. *e*) At line 3, the form of προσαναπληροῦν with double preverb is slightly more canonical, in case of parts of circles, than the form of ἀναπληροῦν we have read at sch. **83**, line 5: after the isolated, seminal occurrences of the former at *El*. III.25 (what is completed is a circle) and of the latter at *El*. XII.2 (what is completed is a parallelogram), a mathematical Atticist such as Pappus only resorts to the former when completing circles (11 occurrences in *Coll*.). Note the non-canonical ἀναφθήσεται at line 6: the point is that there is only one compounding ratio, namely, ΓΚ:ΚΔ, that «makes up completely» ratio ΓΛ:ΛΑ.

88

Text. διὰ τὸ β′ λῆμμα

Transl. By the 2nd lemma

Comm. *a*) Vat. gr. 1594, f. 25r marg. ext., Marc. gr. 313, f. 51v marg. int. *b*) *Ad Alm*. I.13, 75.13–15 ὁ ἄρα τῆς ΓΛ πρὸς ΛΑ λόγος συνῆπται ἔκ τε τοῦ τῆς ΓΚ πρὸς ΚΔ καὶ τοῦ τῆς ΔΘ πρὸς ΘΑ «therefore ratio ΓΛ to ΛΑ is compounded of that of ΓΚ to ΚΔ and of that of ΔΘ to ΘΑ». *c*) The scholium provides a reference to the relevant lemma: it is the rectilinear lemma "by separation." *d*) Both in **B** and in **C**, sch. **88** is located beside the *relatum*. It is in majuscule in **B**.

89

Text. διὰ τὸ γ′ λῆμμα

Transl. By the 3rd lemma

Comm. *a*) Vat. gr. 1594, f. 25r marg. ext.; the annotation is missing in Marc. gr. 313. *b*) *Ad Alm*. I.13, 75.15–17 ἀλλ’ ὡς μὲν ἡ ΓΛ πρὸς ΛΑ, οὕτως ἡ ὑπὸ τὴν διπλῆν τῆς ΓΕ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΕΑ περιφερείας «but, as ΓΛ is to ΛΑ, so the [straight line] under the double of [arc] ΓΕ is to that under the double of arc ΒΓ». *c*) The scholium provides a reference to the relevant lemma: it is the first cyclic lemma. *d*) In **B**, sch. **89** is in majuscule and is located beside the *relatum*.

90

Text. διὰ τὸ γʹ λῆμμα

Transl. By the 3rd lemma

Comm. *a*) Vat. gr. 1594, f. 25r marg. ext.; the annotation is missing in Marc. gr. 313. *b*) *Ad Alm*. I.13, 75.17–19 ὡς δὲ ἡ ΓΚ πρὸς ΚΔ, οὕτως ἡ ὑπὸ τὴν διπλῆν τῆς ΓΖ περιφερείας πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΖΔ «and, as ΓΚ is to ΚΔ, so the [straight line] under the double of [arc] ΓΖ is to that under the double of ΖΔ». *c*) The scholium provides a reference to the relevant lemma: it is again the first cyclic lemma. *d*) In **B**, sch. **90** is in majuscule and is located beside the *relatum*.

91

Text. διὰ τὸ ἀνάπαλιν τοῦ εʹ λήμματος

Transl. By the inverse of the 5th lemma

Comm. *a*) Vat. gr. 1594, f. 25r marg. ext.; the annotation is missing in Marc. gr. 313. *b*) *Ad Alm*. I.13, 75.19–21 ὡς δὲ ἡ ΘΔ πρὸς ΘΑ, οὕτως ἡ ὑπὸ τὴν διπλῆν τῆς ΔΒ περιφερείας πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΒΑ «and, as ΘΔ is to ΘΑ, so the [straight line] under the double of [arc] ΔΒ is to that under the double of ΒΑ». *c*) The scholium provides a reference to the relevant lemma: it is the fifth cyclic lemma. The adverb ἀνάπαλιν only means that the inverse of the proportion in the lemma is used, not the inverse of the lemma itself. *d*) In **B**, sch. **91** is in majuscule and is located beside the *relatum*.

92

Text. τῶν κατὰ μέρος λοξώσεων

Transl. Of the individual declinations

Comm. *a*) Vat. gr. 1594, f. 25v marg. ext., Marc. gr. 313, f. 52r marg. int. *b*) *Ad Alm*. I.14, 76.12–14 τούτου δὴ τοῦ θεωρήματος προεκτεθειμένου ποιησόμεθα πρώτην τὴν τῶν προκειμένων περιφερειῶν ἀπόδειξιν οὕτως «this theorem being preliminarily set out, we shall first of all demonstrate the [numerical values] of the arcs we set ourselves to determine, as follows». *c*) The expression specifies that the arcs alluded to by Ptolemy are in fact the individual arcs of the declination of points of the ecliptic with respect to the equator. The text coincides with Theon, *in Alm. I.13*, *iA*, 571.14. Sch. **92–104** refer to *Alm*. I.14. *d*) Both in **B** and in **C**, sch. **92** is located beside the *relatum*. In **B**, it is placed exactly beside the underlined pericope of the *relatum* (amounting to one line in the column).

93

Text. τεταρτημόριον γάρ ἐστι ἑκατέρα τῶν ΒΕ ΕΔ

Transl. For each of [arcs] BE, EΔ is a quadrant

Comm. *a*) Vat. gr. 1594, f. 25v marg. int., Marc. gr. 313, f. 52r marg. int. *b*) *Ad Alm*. I.14, 76.23–24 ὥστε τὸ μὲν Β χειμερινὸν τροπικὸν εἶναι, τὸ δὲ Δ θερινόν «so that B is the winter solstice, Δ the summer [solstice]». *c*) The geometric configuration assumed in *Alm*. I.14 is as follows (Fig. 15). In a representation of the celestial sphere whose circular outline ABZΓΔ is the circle through pole Z of the equator and the pole of the ecliptic, arc AEΓ within the circle is the equator, arc BEΔ the ecliptic (and hence B and Δ are the winter and summer solstices, respectively, and E is the spring equinox). Arc ZHΘ is drawn across from Z, meeting the ecliptic at H and the equator at Θ. It is required to find HΘ (the declination of point H of the ecliptic with respect to the equator) given EH (the distance of point H along the ecliptic from the spring equinox). The scholium specifies that arcs BE, EΔ are of a quadrant, for, as noted by Ptolemy in the immediately preceding clause, point E is the spring equinox. *d*) Both in **B** and in **C**, sch. **93** is located beside the *relatum*. It is in majuscule in **B**.

94

Text. ὅταν ἐπιταττώμεθα λόγον ἀφελεῖν ἐκ λόγου, δῆλον ὅτι οὐδὲν ἄλλο τί ἐστιν ἢ διαλῦσαι τὸν λόγον ἀφ’ οὗ ἡ ἀφαίρεσις γίνεται εἴς τε τὸν ἀφαιρούμενον καὶ τὸν μετὰ τοῦτο καταλειπόμενον· καὶ γὰρ σύγκειται ἐκεῖνο ἀφ’ οὗ ἡ ἀφαίρεσις γίνεται ἔκ τε τοῦ ἀφαιρουμένου καὶ τοῦ μετ’ ἐκεῖνο λοιποῦ (τὸ γὰρ ἔλαττον ἀπὸ τοῦ μείζονος ἀφαιρεῖται). πῶς οὖν ἡ διάλυσις γενήσεται; ἢ δῆλον ὅτι ἀνάπαλιν τῇ συνθέσει; λόγος δὲ ἐκ λόγων συγκεῖσθαι λέγεται ὅταν αἱ τῶν λόγων πηλικότητες ἐφ’ ἑαυτὰς πολλαπλασιασθεῖσαι ποιῶσί τινα· δύο γὰρ ὅρων ἄλλου μεταξὺ τιθεμένου, ὁ τοῦ πρώτου πρὸς τὸν δεύτερον λόγος ἐπὶ τὸν τοῦ δευτέρου πρὸς τὸν τρίτον πολλαπλασιασθεὶς ποιεῖ τὸν τῶν ἄκρων λόγον, ὁποῖός ποτ’ ἂν ὁ μέσος εἴη. οἷον τοῦ β καὶ τοῦ η ἔστω μέσος ὁ δ. ἐπεὶ οὖν ὁ μὲν β πρὸς τὸν δ τὸν τοῦ ἡμίσεος ἔχει λόγον καὶ ὁ δ πρὸς τὸν η ὁμοίως, πολλαπλασιάζω τὸ ἥμισυ ἐπὶ τὸ ἥμισυ καὶ ποιῶ δʹ καὶ εὑρίσκω τὰ β τῶν η δʹ μέρος. ἀλλὰ κἂν ἀντὶ τοῦ δ τὸν μ θῶ μέσον ὅρον, πάλιν ὁ τῶν β πρὸς τὸν η σύγκειται ἔκ τε τοῦ τῶν β πρὸς τὸν μ καὶ τοῦ μ πρὸς η· τὰ γὰρ β τῶν μ κʹ ἐστι μέρος, τὰ δὲ μ τῶν η πενταπλάσια, τὸ δὲ κʹ ἐπὶ τὰ ε ποιεῖ δʹ. ἐπειδὴ οὖν λόγος ἐκ λόγων συγκεῖσθαι λέγεται ὅταν αἱ τῶν λόγων πηλικότητες ἐφ’ ἑαυτὰς πολλαπλασιασθεῖσαι ποιῶσί τινα, δῆλον ὅτι ἐὰν ἀπὸ δεδομένου λόγου ἀφαιρεθῇ λόγος δεδομένος, ὁ λοιπὸς δεδομένος ἔσται· ἔχοντες γὰρ τήν τε τοῦ λόγου ἀφ’ οὗ ἡ ἀφαίρεσις γίνεται πηλικότητα καὶ τὴν τοῦ ἀφαιρουμένου, ἕξομεν καὶ τὴν λοιπὴν πηλικότητα, ἥτις ἐπὶ τὴν τοῦ ἀφαιρεθέντος γενομένη πηλικότητα ποιεῖ τὴν τοῦ συνθέτου, δ′. εἰ τοίνυν τὴν τοῦ ἐλάττονος τῶν διδομένων λόγων πηλικότητα ἀπὸ τῆς τοῦ μείζονος ἀφαιρεῖται, τόδ’ ἔσται διχῶς, ἢ τοῦ ἐλάττονος λόγου μετατιθεμένου εἰς τὸν μείζονα καὶ ἐναρμοζομένου αὐτῷ, τότε τοῦ μείζονος μεταφερομένου εἰς τὸν ἐλάττονα | καὶ περιέχοντος αὐτόν· καὶ τούτων ἑκάτερον διχῶς ἐπιτελεσθήσεται· καθ’ ἑκάτερον γὰρ τῶν λόγων ὄντων προλόγου καὶ ὑπολόγου, εἴ τε εἰς τὸν μείζονα λόγον ὁ ἐλάχιστος ἐναρμόσει, ἡ ἀφαίρεσίς ποτε μὲν πρὸς τῷ προλόγῳ (ὡς ἐπὶ τοῦ ιβ δ η) ποτὲ δὲ πρὸς τῷ ὑπολόγῳ τοῦ μείζονος λόγου γενήσεται (ὡς ἐπὶ τοῦ ιβ δ ∠ γ), ἀφ’ οὗ ἡ ἀφαίρεσις γίνεται, εἴ τε ὁ μείζων εἰς τὸν ἐλάττονα μετενεχθῇ, πάλιν ἢ πρὸς τῷ προλόγῳ τοῦ ἐλάττονος γενήσεται ἡ ἀφαίρεσις ἢ πρὸς τῷ ὑπολόγῳ. οὕτω μὲν οὖν καθόλου ποιητέον. εἰ μέντοι εὕρομεν ἐν τῷ μείζονι καὶ ἐλάττονι λόγῳ τὸν αὐτὸν ὅρον ἢ ἐν προλόγῳ ἢ ἐν ὑπολόγῳ, εὐμαρεστέρα ἡμῖν ἔσται ἡ λῆψις· οἱ γὰρ παρὰ τοὺς αὐτοὺς ὅρους, οὗτοι τὸν λοιπὸν λόγον περιέξουσιν. οἷον δὲ ἔστω ἀπὸ τοῦ λόγου τῶν δώδεκα πρὸς γ τὸν τῶν ιβ πρὸς δ ἀφελεῖν· ἐναρμοζομένου τοῦ ιβ τῷ ιβ καὶ μέσου τιθεμένου τοῦ δ, ἀφῄρηται ὁ τῶν ιβ πρὸς δ λόγος, περιλείπεται δὲ ὁ τῶν δ πρὸς γ, ἐπίτριτος· ἐπίτριτος γὰρ ἐπὶ τριπλάσιον τετραπλάσιον ποιεῖ. κἀνταῦθα δ’ ὁμοίως δύναται ἀπό τε τοῦ μείζονος εἰς τὸν ἐλάττονα καὶ ἀπὸ τοῦ ἐλάττονος εἰς τὸν μείζονα ποιεῖσθαι τὴν μεταγωγήν ποτε μὲν πρὸς τῷ προλόγῳ ποτὲ δὲ τῷ ὑπολόγῳ γινομένης τῆς ἀφαιρέσεως.

1 ὅταν] ὅτ’ἂν et sic semper **K** | δῆλον ὅτι] δηλονοτι **C** | ἄλλο τί] αλλοιουτο **C** : ἄλλο τοῦτο **D** **2** διαλῦσαι] διδόναι **G** **4** ἐκεῖνο] –ον **D** | τὸ ἔλαττον] τὸν — χ **B** (sed vix legitur) : τον — ελαττον dein truncatum folium **C** : τὸν — ἐλ/^{ά} **G** : τὸν — ἔλαττον **K** | ἀφαιρεῖται] –εῖ **BK** : αφαιρὴν **C** : ἀφ^{αῖ} **D** : –εῖν **G** **5** διάλυσις] δηλου **G** | δῆλον ὅτι] δηλονότι **DG** : δηλ dein truncatum folium **C** | τῇ συνθέσει] τῆς συνθέσεως **G** | λόγων] λο$^{\gamma}$ **BC** : λόγου **GK** **7** ποιῶσί] –σιν **BC** : ποιοῦσι **G** | ἄλλου] –ος **D** : ἀν ἁ **G** | τιθεμένου] –ος **D** | πρώτου] α^{ου} **D** | πρὸς] καὶ **C** **8** λόγος] λόγον **CG** | τὸν om. **G** | δευτέρου] β^{ου} **D** | τὸν] τὸ **K** **9** ποτ’ ἂν] ποτὲ **G** | οἷον] οἱ **K** corr. m. 2 | καὶ τοῦ] πρὸς τὸν **D** | μέσος] μ^{εσ} **K** fecit μέσον m. 2 | οὖν] γοῦν **G** | post ὁ add. δὲ m. 2 **K** **10** πολλαπλασιάζω] –σθὲν **DG** : –σθεὶς **B** comp. **C** **K** corr. m. 2 **11** καὶ om. **D** : deleuit m. 2 **K** | ποιῶ] ποιεῖ **K** corr. m. 2 | τῶν om. **G** | δ′] δ″ **D** | τὸν] τὸ **G** **12** ante μέσον scripsit μὲν **K** | μέσον] μὲν **G** | τὸν] τὸ **K** corr. m. 2 | τῶν om. **D** | β^{2} — καὶ τοῦ] β ε πρὸς **G** | τὸν] τὸ **K** corr. m. 2 **13** κ′] κ″ **D** : κ″ ex κ′ **K** | ἐστι om. **G** | τὸ] τὰ **G** : ex τὸ fecit τὰ m. 2 **K** | κ′] κ **BCK** | ε] ε′ **DG** **14** δ′] δ **BD** : supra lineam add. ον m. 2 **K** | λόγων^{1}] λο$^{\gamma}$ **BG** : λόγος **C** : λόγου **K** **15** πολλαπλασιασθεῖσαι] πολλαπλ^{α} **G** : –ασθεῖσαι supra lineam suppl. m. 2 **K** | ποιῶσί] –σιν **BC** | δῆλον ὅτι] δηλονοτι **CG** | ἐὰν supra lineam **D** | δεδομένου] –ος **C** | ἀφαιρεθῇ] αφαιρ/ **C** : ἀφαιρεῖται **G** **16** ἔχοντες] ἔχων τε **G** **17** πηλικότητα] πηλίκη **K** corr. m. 2 | λοιπὴν] λοιπ **C** : λοιπὸν **D** **18** γενομένη] γίνεται **GK** | πηλικότητα] –τες **K** corr. m. 2 et postea add. καὶ supra lineam | τοῦ συνθέτου] τούτου σύνθεσιν **K** **19** δ′] δ **BCD** | τῆς om. **G** **20** ἀφαιρεῖται] fortasse –οῦμεν **D** : –εθῇ **G** : –ήσομεν ex corr. m. 2 **K** | τόδ’] τοῦτο **D** : τὸ δ **K** | διχῶς] δι$^{\chi}$ **BCG** : δίχα **K** | μετατιθεμένου] μετατίθεται **BCGK** | τὸν] τὸ **K** corr. m. 2 | μείζονα] μεῖζον **K** corr. m. 2 **21** ἐναρμοζομένου] –νην **DG** comp. **C** fortasse **B** et ex corr. m. 2 **K** | αὐτῷ] αὐτὴν **DG** comp. **CK** : legi nequit **B** | τοῦ μείζονος] suppleui | μεταφερομένου] –φερομεν **BCDGK** | τὸν ἐλάττονα] τὸ ἔλαττον **K** corr. m. 2 **22** περιέχοντος] –έχον^{τ} **B** : –εχον^{τ} **C** : περιέχο^{ντ} **D** : –έχοντα **G** : ex –έχον^{τ} fecit –έχοντα m. 2 **K** | διχῶς] δι$^{\chi}$ **C** | ἑκάτερον] ἕκαστον fecit m. 2 **K** **23** προλόγου καὶ ὑπολόγου] προλόγων καὶ ὑπολόγων **D** | ἐλάχιστος] ἐλάττων **GK** | ἐναρμόσει] –οσθῇ **G** **24** ἀφαίρεσίς ex corr. **G** | δ subscr. **BC** : om. **G** | πρὸς τῷ ὑπολόγῳ τοῦ] καὶ τὸ ὑπόλογον **G** **24–25** τοῦ μείζονος λόγου **D** : τὸν μ$^{\zeta}$ λο$^{\gamma}$ **B** : τον μει$^{\zeta}$ λο$^{\gamma}$ **C** : τὸν μείζονα λόγον **GK** | γενήσεται (ὡς] γεν/^{τ} ὡς **B** : γένηται καὶ **G** : γένηται ὡς **K** **26** εἰς τὸν ἐλάττονα] εἴ τε ὁ ἐλάττων **BCGK** | πρὸς] ἐν **G** | τοῦ ἐλάττονος] τῷ ἐλάττονι **G** et ex τοῦ ἐλάττονος fecit m. 2 **K** **27** ἀφαίρεσις des. **G** | ὑπολόγῳ] ζέτει ἐπάνω σημεῖον τοῦτο εἰς τὰ ἐπίλοιπα adnotauit **C** | οὕτω] τοῦ **K** **27–28** τῷ μείζονι καὶ ἐλάττονι λόγῳ] τοῖς μείζοσι καὶ ἐλάττοσι λόγοις **K** | τῷ om. **C** | μείζονι] μείζοσι **B** | εὐμαρεστέρα] ευμαρεστω **C** **29** οὗτοι τὸν λοιπὸν] ου^{τ} τοῦ λοιποῦ **BC** : ὄντες τὸν λοιπὸν **D** : αὐτὸν τοῦ λοιποῦ **K** | περιέξουσιν] περιέξει **K** **30** ἔστω] ἐστιν comp. **C** **K** corr. m. 2 : legi nequit **B** | δώδεκα] ιβ **DK** | τὸν] τὸ **BCK** | τῶν] τοῦ **D** | πρὸς δ om. **K** **31** ἐναρμοζομένου τοῦ] ἐναρμόζομεν τὸν **BK** | τῷ ιβ om. **K** | μέσου τιθεμένου] μ^{ε} τιθεμεν **C** : μεσον τι dein legi nequit **B** : μετατιθεμένου **D** : μέσον τίθεμεν **K** | τοῦ] τὸν **BCK** **32** τῶν] τοῦ **D** | ἐπίτριτος^{1}] ἐπεὶ γ (γ cum circumflexu) **BC** **K** corr. m. 2 | τριπλάσιον τετραπλάσιον] –ον bis ex corr. m. 2 **K** **33** τε om. **D** | τὸν ἐλάττονα] τὸ ἔλαττον **K** | τὸν] τοὺς **C** **34** εἰς τὸν] εισοι **C** | τὸν μείζονα] τὸ μεῖζον **K** | τῷ] το **C**

Transl. When we undertake to remove a ratio from a ratio, it is clear that this is nothing other than to resolve the ratio from which the removal is going to occur into the removed [ratio] and the remaining [ratio] after this. And in fact that from which the removal is going to occur is compounded of the removed [ratio] and the remaining [ratio] after that (for the lesser [ratio] is removed from the greater). Now, how will the resolution come to pass? Or is it clear that [it will] conversely to composition? A ratio is said to be compounded of ratios when the [numerical] values of the ratios multiplied by each other make some [numerical value of a ratio]. If, in fact, another term is set between two terms, the ratio of the first to the second, multiplied by that of the second to the third, makes the ratio of the extremes, whatever the middle [term] might be. For instance, let 4 be a middle of 2 and 8. Then since 2 has to 4 the ratio of $\frac{1}{2}$, and similarly 4 to 8, I multiply $\frac{1}{2}$ by $\frac{1}{2}$ and I make $\frac{1}{4}$, and I find that 2 is a 4th part of 8. Yet even if I set 40 instead of 4 as a middle term, the [ratio] of 2 to 8 is, again, compounded both of that of 2 to 40 and of that of 40 to 8, for 2 is a 20th part of 40, 40 is quintuple of 8, and $\frac{1}{20}$ times 5 makes $\frac{1}{4}$. Then since a ratio is said to be compounded of ratios when the [numerical] values of the ratios multiplied by each other make some [numerical value of a ratio], it is clear that if a given ratio be removed from a given ratio, the remaining [ratio] will be given. For if we have both the [numerical] value of the ratio from which the removal is going to occur and the [numerical value] of the removed [ratio], we will also have the remaining [numerical] value, which makes the one of the compound [ratio], $\frac{1}{4}$, when it is multiplied by the [numerical] value of the removed [ratio]. Now, if one removes the [numerical] value of the lesser of the given ratios from the one of the greater, this will come about in two ways, either by the lesser ratio being transposed to the greater and fitted to it, or by the greater being transformed into the lesser and made to contain it; and each of these cases will be accomplished in two ways. For, each of the ratios having an antecedent and a consequent, if the least is going to be fitted into the greater ratio, the removal will occur either with respect to the antecedent (as for 12 4 8) or with respect to the consequent of the greater ratio (as for 12 $4\frac{1}{2}$ 3), from which the removal is going to occur; if the greater [ratio] has been transformed into the lesser, again, the removal will occur either with respect to the antecedent or with respect to the consequent of the lesser. Now, this is the way it should be done in general. But if we should happen to find in the greater and the lesser ratio the same term in either the antecedent or the consequent, finding the value will be easier for us, for these terms other than the identical terms will contain the remaining ratio. For instance, from the ratio of twelve to 3, let it be required to remove that of 12 to 4; 12 being fitted to 12 and 4 being set as middle [term], the ratio of 12 to 4 turns out to be removed, and that of 4 to 3 remains, which is $\frac{4}{3}$; and $\frac{4}{3}$ multiplied by triple makes quadruple. Here, too, it is similarly possible to carry out the transition from the greater to the lesser or from the lesser to the greater, and the removal can occur with respect to either the antecedent or the consequent.

Comm. *a*) Vat. gr. 1594, f. 25v marg. ext. et inf., Marc. gr. 313, f. 52r marg. ext, inf. et sup., Vat. gr. 184, ff. 32r–v et 91v–92r, Vat. gr. 180, f. 23v marg. sup., ext. et inf. *b*) *Ad Alm*. I.14, 78.3–5 ἐὰν ἄρα ἀπὸ τοῦ τῶν ρκ πρὸς τὰ μη λα′ νε″ λόγου ἀφέλωμεν τὸν τῶν ξ πρὸς τὰ ρκ «therefore if from the ratio of 120 to 48;31,55 we remove that of 60 to 120». The citation in **K** coincides with the title of *Alm*. I.14 περὶ τῶν μεταξὺ τοῦ ἰσημερινοῦ καὶ τοῦ λοξοῦ κύκλου περιφερειῶν «on the arcs between the equator and the ecliptic». *c*) The scholium first expounds basics about compounded ratios, and then provides general rules to perform the operation of removal of ratios; see the introduction for some details on the procedure, in particular on the crucial operation of ἐναρμόζειν «fitting» a ratio to another. The scholium is the Ur-text of the treatise on removal of ratios ascribed to Domninus of Larissa (Riedlberger 2013, whose translation I have adapted); the relationships between these two texts are studied in detail in Acerbi and Riedlberger (2014). Ancient expositions on removal of ratios include sch. **98–99** below, Pappus, *Coll.* VII.240; Theon, *in Alm.*, *iA*, 532.1–535.9 and 759.8–762.2; the final section of *Prol.* (edited in a preliminary form in Knorr 1989, 190–201—but, misguided by a calculation error in the Greek text, he misunderstood a part of the procedure: 188–189); Eutocius, *in Con.*, *AGE* II, 218.6–220.25, and *in Sph. cyl. II.4*, *AOO* III, 120.3–126.20. Add a few splinters in scholia 2–11 to Book VI of *El.* (*EOO* V, 320.5–331.4—but scholium 4 coincides with the first of Theon's passages just mentioned), where we read a definition of compounded ratio (*El.* VI.def.5). Of some interest is also a ὑπόμνημα σχόλιον εἰς τὰς τῶν λόγων σύνθεσίν τε καὶ ἀφαίρεσιν ascribed to a Leon (ibid., 341.9–345.7). This is transcribed by the main copyist, together with other exegetic material and after the end of *El*. VI, at ff. 120–121 of Bodl. Dorv. 301; for a detailed discussion of these and other Greek and Byzantine texts on removal of ratios see Acerbi (2018a). *d*) The scholium is long and, both in **B** and in **C**, it has been located in the most spacious margins about the first application of the Sector Theorem. Since another long scholium (sch. **98**) had to be transcribed after sch. **94** and before the end of the chapter (sch. **94** outlines a general approach to the operation of removal of ratios, sch. **98** works out a specific example), it happened that sch. **94** and its *relatum* lie on different pages. In **K**, sch. **94** follows sch. **83**.

95

Text. μεγίστου

Transl. Of a great

Comm. *a*) Vat. gr. 1594, f. 25v marg. int., Marc. gr. 313, f. 52r marg. int. *b*) *Ad Alm*. I.14, 77.11 ἡ τοῦ μεγίστου κύκλου περιφέρεια «the circumference of a great circle». *c*) This is a word witnessed by **A** (**D** has a different text) but that was omitted by the copyist of the common model of **BC**, and there added in the margin by some reviser or by the copyist himself. *d*) Both in **B** and in **C**, sch. **95** is located beside the *relatum*.

96

Text. ἡ γὰρ ΖΑ ἐκ πόλου οὖσα τοῦ ἰσημερινοῦ τεταρτημόριόν ἐστι

ἡ γὰρ ΖΑ ἐκ τοῦ πόλου οὖσα ἐπὶ τὸν ἰσημερινὸν τῶν τοῦ τεταρτημορίου μοιρῶν ἐστιν ϙ Th.

Transl. For [arc] ZA is a quadrant since it is [an arc] from a pole of the equator

Comm. *a*) Vat. gr. 1594, f. 26r marg. int., Marc. gr. 313, f. 52r marg. int. *b*) *Ad Alm*. I.14, 77.21–22 ἀλλ' ἡ μὲν τῆς ΖΑ περιφερείας διπλῆ μοιρῶν ἐστιν ρπ «but the double of arc ZA is of 180 degrees». *c*) The scholium explains why the double of arc ZA is 180º (Fig. 15): ZA is a quadrant because Ptolemy took A to be on the equator and Z to be a pole of the equator (76.18–20 and 76.24–77.1). The text corresponds to Theon, *in Alm. I.14*, *iA*, 573.10–11. *d*) Both in **B** and in **C**, sch. **96** is located beside the *relatum*. *e*) Note the noun phrase ‹ἡ› ἐκ τοῦ πόλου «[arc] from a pole», formed on the model of ἡ ἐκ τοῦ κέντρου «radius»; ἐκ πόλου here lacks the article ἡ because it is the noun of the predicate. It is not clear whether the absence of the article τοῦ is a slip of the scholiast or a subtle correction of his, induced by the fact that every circle on the surface of a sphere has two poles (Theon, more faithful to the model, has the article); cf. sch. **5**, **84**, **86**, **100**. The first occurrences of the expression ἡ ἐκ τοῦ πόλου with the meaning of «polar radius» are in Theodosius, *Sph.* I.16, 17, 19–21, II.5, 14, 17, 22; one also finds this expression in Heron, *Metr.* I.39 (Acerbi and Vitrac 2014, 244.6–7), and in Pappus and Theon.

97

Text. ἡ γὰρ ΑΒ μεγίστη ἐστὶ λόξωσις μοιρῶν κγ να′ κ″

Transl. For AB is the greatest declination, namely, of 23;51,20 degrees

Comm. *a*) Vat. gr. 1594, f. 26r interc., Marc. gr. 313, f. 52r marg. int. *b*) *Ad Alm*. I.14, 77.23–26 ἡ δὲ τῆς ΑΒ διπλῆ κατὰ τὸν συμπεφωνημένον ἡμῖν τῶν πγ πρὸς τὰ ια λόγον μοιρῶν μζ μβ′ μ″ «and, according to the ratio of 83 to 11, with which we agreed, the double of [arc] AB is of 47;42,40 degrees». *c*) In the configuration described in sch. **93** (Fig. 15), arc AB is cut off, on the great circle through pole Z of the equator and the winter solstice B, by the equator itself and the ecliptic; therefore, it is the greatest declination of the ecliptic on the equator. Of course, this is half of the value given in the *relatum*; Ptolemy deals with this matter in *Alm*. I.12. *d*) Both in **B** and in **C**, sch. **97** is located beside the beginning of the *relatum*.

98

Text. ἐὰν ἀπὸ δεδομένου λόγου βουληθῶμεν δεδομένον λόγον ἀφελόντες τὸν καταλειπόμενον εὑρεῖν, εἰ μὲν εὑρεθῇ ὁ πρόλογος τοῦ ἀφαιρουμένου ὁ αὐτὸς τῷ προλόγῳ τοῦ ἀφ’ οὗ ἀφαιρεῖται, ἢ ὁ ὑπόλογος τῷ ὑπολόγῳ, αὐτόθεν οἱ διάφοροι ἐν τοῖς λόγοις ὅροι περιέχουσι τὸν καταλειπόμενον λόγον (ὡς ἐπὶ ιβ ς δ ἢ ιβ δ γ)· ἔσται γὰρ ὁ λοιπὸς ὅρος τοῦ ἀφαιρουμένου λόγου μέσος γινόμενος τῶν περιεχόντων τὸν ἀφ’ οὗ ἀφαιρεῖται. ἐὰν δὲ μὴ οὕτως ἔχωσι, δεῖ ποιεῖν ἢ ὡς τὸν ἡγούμενον τοῦ ἀφαιρουμένου πρὸς τὸν ἑπόμενον οὕτως τὸν ἡγούμενον τοῦ ἀφ’ οὗ ἡ ἀφαίρεσις γίνεται πρὸς ἄλλον τινά (ὅταν ὁ ἀφαιρούμενος λόγος πρῶτος ᾖ τῶν συντιθέντων), ἢ ὡς τὸν ἑπόμενον πρὸς τὸν ἡγούμενον οὕτως τὸν ἑπόμενον τοῦ ἀφ’ οὗ ἡ ἀφαίρεσις πρὸς ἄλλον τινά (ὅταν ὁ ἀφαιρούμενος λόγος δεύτερος ᾖ τῶν συντιθέντων). ὁ γὰρ τέταρτον ἀνάλογον, μετὰ τοῦ λοιποῦ τοῦ περιέχοντος τὸν ἀφ’ οὗ ἡ ἀφαίρεσις γίνεται, ποιήσει τὸν καταλειπόμενον λόγον.

ις η$^{δ'}$ δ ις η$^{δ'}$ δ

ς ι ς γ

ὅπως δὲ ταῦτα χρὴ ποιεῖν καὶ ὡς δεῖ τοὺς ἀριθμοὺς τάττειν, εἰσόμεθα οὕτως. προτάττομεν τοὺς τοῦ συντιθεμένου λόγου, καὶ πρῶτον μὲν τὸν πρῶτον λαμβάνομεν ἐν τῷ λόγῳ καὶ δεύτερον τὸν δεύτερον (οἷον πρῶτον τὸν ρκ καὶ δεύτερον τὸν μη λα′ νε″). εἶτα τῶν συντιθέντων β λόγων, ἐπειδὴ γ ἀριθμοὺς ἔχομεν δεδομένους καὶ λόγον ἕνα τῶν συντιθέντων δεδομένον, ὑποτάττομεν τὸν δεδομένον λόγον. καὶ εἰ μὲν τελευταῖός ἐστιν ὁ δεδομένος λόγος, ὡς ἐπὶ τοῦ προκειμένου θεωρήματος, τάττομεν τὸν τελευταῖον ἀριθμὸν τοῦ δεδομένου λόγου ὑπὸ τὸν τελευταῖον ἀριθμὸν τοῦ συντιθεμένου λόγου (οἷον τὸν ρκ ὑπὸ τὸν μη λα′ νε″), τὸν δὲ πρῶτον τοῦ δεδομένου λόγου τάττομεν μεταξὺ καὶ ὥσπερ ἐν τῇ μέσῃ χώρᾳ (οἷον τὸν ξ), καὶ πολλαπλασιάσαντες τὸν ξ ἐπὶ τὸν μη λα′ νε″ καὶ μερίσαντες παρὰ τὸν ρκ εὑρίσκομεν τὸν κδ ιε′ νζ″ καὶ τάττομεν αὐτὸν ἐπάνω τοῦ ξ, καὶ δηλονότι ἀφείλομεν ἀπὸ τοῦ λόγου τοῦ τῶν ρκ πρὸς τὰ μη λα′ νε″ τὸν δεδομένον λόγον τὸν τῶν κδ ιε′ νζ″ πρὸς τὰ μη λα′ νε″ (ὁ αὐτὸς γάρ ἐστι τῷ τῶν ξ πρὸς ρκ), καὶ λοιπὸν τὸν τῶν ρκ πρὸς κδ ιε′ νζ″ λόγον ἔχομεν δεδομένον, ὃν καὶ ἐπιζητοῦμεν. ἐπεὶ οὖν εὕρομεν τὸν ζητούμενον λόγον τὸν τῶν ρκ πρὸς κδ ιε′ νζ″, ἦν δὲ ὁ ζητούμενος λόγος ὁ τῆς ὑπὸ τὴν διπλῆν τῆς ΖΘ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΘΗ, ἔχομεν ἄρα τὸν τῆς ὑπὸ τὴν διπλῆν τῆς ΖΘ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΘΗ λόγον, τὸν τῶν ρκ πρὸς κδ ιε′ νζ″· καὶ ἔστιν ἡ ὑπὸ τὴν διπλῆν τῆς ΖΘ ρκ, ὧν καὶ ἡ ὑπὸ τὴν διπλῆν τῆς ΖΑ· καὶ ἡ ὑπὸ τὴν διπλῆν ἄρα τῆς ΘΗ ἐστι κδ ιε′ νζ″. καὶ οὕτως εὑρίσκομεν πόσου ἐστὶν ἡ ὑπὸ τὴν διπλῆν τῆς ΘΗ. εἰ δὲ μὴ ἦν ἡ ὑπὸ τὴν διπλῆν τῆς ΖΘ ρκ, ἀλλὰ καθ’ ὑπόθεσιν μ, ἐποιοῦμεν ὡς ρκ πρὸς κδ ιε′ νζ″, οὕτως μ πρὸς ἄλλον τινά, καὶ οὕτως εὑρίσκομεν πόσου ἐστὶν ἡ ὑπὸ τὴν διπλῆν τῆς ΘΗ. ὁμοίως δὲ ποιήσομεν κἂν ὁ ζητούμενος λόγος ὑπάρχῃ τελευταῖος.

1 post ἐὰν add. γὰρ **K** | ἀπὸ suprascr. m. 2 **K** | δεδομένον λόγον] δεδομένου λόγου **G** | ἀφελόντες] ἀφέλ– **K** corr. m. 2 | τὸν] τὸ **K** corr. m. 2 **2** καταλειπόμενον] διελόντα μὲν **G** | ὁ^{2} om. **G** **3** τοῦ] τῷ **CK** : om. **G** **4** ὅροι] ὅρος **K** corr. m. 2 | περιέχουσι usque hac legi nequit ob truncatum folium **D** | ὡς ἐπὶ] ὡς — ε^{π} **B** : — ε^{π} **K** | ιβ ς δ ἢ ιβ δ γ **D** : ιβ ς δ ἢ ιβ δ^{λ} γ **B** : ιβ ς δη ἢ ιβ λ^{δ} γ **C** : ιβ ς λη β λ γ **G** : ιβ ς δ ἢ ιβ λ^{δ} γ **K** | ὡς ἐπὶ

— δ γ post ἀφαιρεῖται omissis ὡς ἐπὶ ac ἢ locauit **G** **5** τῶν περιεχόντων] τῷ περιέχοντι **DG** et ex τῶν περιεχόντων fecit m. 2 **K** | τὸν om. **CDG** : τῶν **B** **K** corr. m. 2 **6** μὴ οὕτως] μειοῦται **C** **G** corr. m. 2 | δεῖ om. **G** | post ἡγούμενον add. μὲν **G** et supra lineam m. 2 **K** **7** οὕτως] οὖ **K** corr. m. 2 | τὸν] τοῦ **CG** : τὸ **K** corr. m. 2 | ἡγούμενον] –μένου **G** | τοῦ om. **G** | ἄλλον τινά] ἄλλόν τινα et sic semper **K** | ὅτ’ ἂν] ὅτ’ἂν cum duobus accentibus ἂν et sic semper **K** **9** οὕτως] οὖ **K** corr. m. 2 | οὕτως τὸν ἑπόμενον om. **G** **10** τέταρτον] δ′δ′ (ad pr. supra lineam add. ον m. 2) **G** : primum add. m. 2 **K** | ἀνάλογον] –ος m. 2 **K** **11** ποιήσει] ποιῇ **G** | καταλειπόμενον] –ον add. m. 2 **K** **15** δὲ] δεῖν **G** | εἰσόμεθα] ἰσ– **C** **K** corr. m. 2 **16** τοὺς] τὸν fecit m. 2 **K** | λόγου] –ον fecit m. 2 **K** | πρῶτον^{2}] πρόλογον **G** et fecit m. 2 **K** | λαμβάνομεν] λαμβ$^{\alpha}$ **CD** : om. **G** **17** τῷ] τι **G** | λόγῳ] λς codd. | τὸν] τοῦ **C** **K** corr. m. 2 **18** β] δύο **G** | γ] τοὺς **G** et τούτους fecit m. 2 **K** | δεδομένους καὶ] –νους add. et καὶ deleuit m. 2 **K** | δεδομένους καὶ λόγον] δεδομένον πρόλογον **G** **19** δεδομένον^{1}] –νον add. m. 2 **K** | τὸν δεδομένον λόγον] τοῦ δεδομένου λόγου codd. | τελευταῖός ἐστιν ex corr. m. 2 **K** **20** ἀριθμὸν ex corr. **B** : suprascr. et καὶ deleuit m. 2 **K** **21** λόγου] λόγον **B** **22** τὸν^{2}] τῶν **G** **23** τῇ μέσῃ χώρᾳ] τι μὴ χ$^{\omega}$ **G** | τὸν^{1}] τοῦ **G** | πολλαπλασιάσαντες] πολλαπλ$^{\alpha}$ **BD** : πολλ/$^{\alpha}$ **C** : πο$^{\lambda\lambda\backslash}$ **G** : –άσαντες add. m. 2 **K** | τὸν (sec. ac ter.)] τῶν **G** **24** μερίσαντες] –ες add. m. 2 **K** | τὸν^{2}] τὸ **K** corr. m. 2 | αὐτὸν supra lineam **G** **25** δηλονότι] δηλονοτι **C** : δῆλον ὅτι **B** **K** corr. m. 2 | ἀφείλομεν legi nequit **D** : ex corr. m. 2 **K** | πρὸς] καὶ **G** | πρὸς τὰ] καὶ τοῦ **D** **26** τὰ] τὸν **CDG** **29** διπλῆν] διπλὴν et sic semper **C** | τῆς **D** : legi nequit **C** : τοῦ **BGK** | ὑπὸ τὴν bis **G** | τῆς] τοῦ **G** | τὸν] τὸ **GK** **30** ΖΘ πρὸς — διπλῆν τῆς add. supra lineam m. 2 **K** | τὸν] τὸ **G** **K** corr. m. 2 **31** τῆς] τῶν **BC** **K** corr. m. 2 | ὧν] ὧν **DGK** **32** οὕτως] οὖ **K** corr. m. 2 **33** ἡ om. **BG** : add. supra lineam m. 2 **K** | μ om. **G** **34** οὕτως] οὖ **K** corr. m. 2 | μ] μὲν **G** | οὕτως om. **G** : οὖ **K** corr. m. 2 | ἐστὶν] εἰσὶν fecit m. 2 **K** **35** κἂν] καὶ **K** et add. ἐὰν supra lineam m. 2

Transl. If, removing a given ratio from a given ratio, we want to find the remaining [ratio], if the antecedent of the removed [ratio] is found to be the same as the antecedent of the ratio from which one removes, or the consequent the same as the consequent, the terms in the ratios that are different immediately contain the remaining ratio (as for 12 6 4 or 12 4 3), for the remaining term of the removed ratio will come to be a middle [term] of the [terms] containing the [ratio] from which one removes. If, instead, there is no such relationship among the [terms], one must make either (namely, when the removed ratio is first among the compounding [ratios]), as the antecedent of the removed [ratio] to its consequent, so the antecedent of the ratio from which the removal is going to occur to some other [number], or (namely, when the removed ratio is second among the compounding [ratios]), as the consequent to the antecedent [of the removed ratio], so the consequent of the ratio from which the removal [is going to occur] to some other [number]—for the fourth proportional, together with the remaining [term] that contains the ratio from which the removal is going to occur, will make the remaining ratio.

16 8^{4th} 4 16 8^{4th} 4

6 10 6 3

In which way must these things be carried out, and in which way must we arrange the numbers, we shall learn as follows. We first arrange those of the compounded ratio, and we take first the first in the ratio, and second the second (namely, first 120 and second 48;31,55). Since we have 3 numbers given and one single ratio given among the compounding [ratios], we arrange next underneath the ratio which is given among the 2 compounding ratios. And, if the given ratio comes last in order (as in the theorem at issue), we arrange the last number in order of the given ratio under the last number in order of the compounded ratio (namely, 120 under 48;31,55), and we arrange the first [number] of the given ratio (namely, 60) in between, and so to speak in the intermediate space, and multiplying 60 by 48;31,55 and dividing by 120 we find 24;15,57, and we arrange it above 60, and, clearly, we have removed from the ratio of 120 to 48;31,55 the given ratio of 24;15,57 to 48;31,55 (for it is the same as that of 60 to 120), and we have the ratio of 120 to 24;15,57—which we also seek—given as a remainder. Then since we have found the sought ratio as that of 120 to 24;15,57, and the sought ratio was that of the [chord] under the double of [arc] ZΘ to that under the double of ΘH, therefore we have the ratio of that under the double of [arc] ZΘ to that under the double of ΘH, namely, that of 120 to 24;15,57; and that under the double of ZΘ is 120 (which is also that under the double of ZA); therefore that under the double of ΘH also is 24;15,57. And in this way we may find how much is it that under the double of ΘH. If instead that under the double of ZΘ had not been 120, but by hypothesis 40, we would have made, as 120 is to 24;15,57, so 40 to some other [number], and in this way we may find how much is it that under the double of ΘH. We shall operate in a similar way even if the sought ratio happens to be the last in order.

Comm. a) Vat. gr. 1594, f. 26r marg. sup. et ext., Marc. gr. 313, f. 52v marg. sup. et ext., Vat. gr. 184, ff. 32v et 92r–v, Vat. gr. 180, f. 24r marg. sup., int. et inf. *b*) *Ad Alm.* I.14, 78.3–5 ἐὰν ἄρα ἀπὸ τοῦ τῶν ρκ πρὸς τὰ μη λα′ νε″ λόγου ἀφέλωμεν τὸν τῶν ξ πρὸς τὰ ρκ «therefore if from the ratio of 120 to 48;31,55 we remove that of 60 to 120». No citation in **K**. *c*) The scholium works out the removal of ratios alluded to in the *relatum*. It does so by outlining first a general prescription, and then describing a tabular set-up allowing to perform the removal in an orderly way; the tabular set-up so described coincides with sch. **99**. In the general prescription of sch. **98**, the case in which the antecedent (consequent) of the compounded ratio coincides with the antecedent (consequent) of the removed ratio is treated first: the terms different from those that coincide will form the ratio resulting from the removal. This case is exemplified by setting out two triples of numbers, 12 6 4 and 12 4 3, joined by suitable arcs, so as to make clear that the compounded ratio coincides with 12:4 or 12:3, respectively, the removed ratio with 12:6 or 4:3, respectively (that is: no arc joins 6 and 4 in the sequence 12 6 4, no arc joins 12 and 4 in the sequence 12 4 3). The triple 12 4 3 is identical with the one set out to the same effect in sch. **94**, but there the removed ratio is 12:4. The general case is instead dealt with by taking a suitable fourth proportional; the numerical examples at lines 13–14, set out in a standard X-shaped scheme of calculation (see sch. **48**), are heavily corrupt and cannot be reconstructed without introducing arbitrary corrections. The scholium next treats the tabular set-up associated with the removal alluded to in the *relatum* and here edited as sch. **99**. This amounts to a modification of the standard X-shaped scheme of calculation of a fourth proportional; the gist of the description resides in the general indications about the places to be assigned, in the tabular set-up, to the relevant terms of the given ratios, namely, the compounded ratio and the removed ratio. After performing the calculation and identifying its result with the relevant chord, the scholiast points out that the case at issue is particular since ZΘ = ZA; in case ZΘ ≠ ZA, one simply has to take again a fourth proportional between *ch*(2ZA), *ch*(2ZΘ), and the value just found of *ch*(2ΘH). The scholiast omits the case in which the removed ratio comes first in order: as he himself claims, one has to «operate in a similar way», the only difference being that one has to put the first number in order of the removed ratio under the first number in order of the compounded ratio. All in all, this is the simplest and most concise exposition (see sch. **94**) of the operation of removal of ratio. The exposition has no exact parallel in Theon's commentary, who, however, performs a calculation to the same effect. *d*) The scholium is long and, both in **B** and in **C**, it occupies most of the available marginal space in the second page containing the first applications the Sector Theorem. Since another long scholium with the same *relatum* (sch. **94**) had to be transcribed before sch. **98** (sch. **94** outlines a general approach to the operation of removal of ratios, sch. **98** works out a specific example), only one page remained (namely, the last before the Table of Declination *Alm*. I.15) for the transcription of sch. **98**; it so happened that sch. **98** and its *relatum* lie on the same page. The part of the scholium lying in the outer margin is very nicely *figuratum* in **B**: it is a Latin cross above an amphora. In **K**, sch. **98** follows sch. **94**.

99

Text.

ὁ τῆς ὑπὸ τὴν διπλῆν τῆς ZA πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς AB λόγος

δʹ

ρκ κδ ιεʹ νζʺ μη λαʹ νεʺ

ξ ρκ

ὁ τῆς ὑπὸ τὴν διπλῆν τῆς ZΘ πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς ΘH λόγος

ὁ τῆς ὑπὸ τὴν διπλῆν τῆς HE πρὸς τὴν ὑπὸ τὴν διπλῆν τῆς EB λόγος

apices non adhibet **C** **2** δʹ] τεταρτημόριον comp. **B** **5–6** ὁ τῆς ὑπὸ τὴν διπλῆν τῆς HE — EB λόγος bis scripsit uarias lectiones ΘIE ac HE adhibens **G** | HE] ΘIE sed dein correxit m. 1 **C**

Transl.

The ratio of that under the double of ZA to that under the double of AB

4th

120 24 15 57 48 31 55

60 120

The ratio of that under the double of ZΘ to that under the double of ΘH

The ratio of that under the double of HE to that under the double of EB

Comm. *a*) Vat. gr. 1594, f. 26r marg. inf., Marc. gr. 313, f. 52v in spatio figurae, Vat. gr. 184, f. 92r. *b*) The *relatum* is sch. **98**. *c*) This tabular scholium is described in sch. **98**; the denominations of the three ratios involved also feature in sch. **99**; the ratios themselves are made visually prominent by arcs. *d*) In **B**, sch. **99** is in the lower margin, alongside a similar schema (not edited here) for the second removal of ratios performed in *Alm*. I.14. In **C**, it is beside the last ten lines of the *relatum*, whithin a blank space left by the diagram. In **G**, the denominations of the three ratios involved are separated from the tabular set-up, that in its turn is disintegrated by the copyist.

100

Text. ἐπειδὴ ἡ ΕΒ τεταρτημορίου ἐστὶ διὰ τὸ β μεγίστους κύκλους τόν τε ΑΕΓ ἰσημερινὸν καὶ τὸν ΒΕΔ ζῳδιακὸν τέμνειν ἀλλήλους εἰς ἡμικύκλιον, καὶ τὸν ΑΒΓΔ διὰ τῶν πόλων αὐτῶν ὄντα τέμνειν τὰ ἀπειλημμένα αὐτῶν ἡμικύκλια δίχα.

1 ἐπειδὴ] ἐπειδήπερ Th. | ἐστὶ διὰ τὸ] ἐστὶ τὸν **B** : διά ἐστι τὸ **C** | μεγίστους κύκλους] μεγίστου κύκλου comp. codd. | post κύκλου add. τοῦ **C** **2** ἀλλήλους] ἄλληλα **B** : comp. **C**

Transl. Since [arc] ΕΒ is of a quadrant because 2 great circles—the equator ΑΕΓ and the zodiac ΒΕΔ—cut each other in a semicircle, and because [circle] ΑΒΓΔ, being through their poles, bisects the semicircles cut off from them.

Comm. *a*) Vat. gr. 1594, f. 26r interc., Marc. gr. 313, f. 52v marg. int. *b*) *Ad Alm*. I.14, 78.2–3 ἡ δὲ τῆς ΕΒ διπλῆ μοιρῶν ρπ καὶ ἡ ὑπ' αὐτὴν εὐθεῖα τμημάτων ρκ «and the double of ΕΒ is of 180 degrees and the straight line under it is of 120 parts». *c*) The scholium explains why the double of arc ΕΒ is of 180º (Fig. 15): it is because ΕΒ is the arc between the intersection of two great circles (the ecliptic and the equator) and the intersection of one of them with the great circle passing through the poles of both: therefore, ΕΒ is of a quadrant (Theodosius, *Sph*. I.11 and II.9). The text almost coincides with Theon, *in Alm. I.13*, *iA*, 573.18–21, a text that has exactly the same function. The transcription from the common model of **B** and **C** must have been careless, as the mistakes recorded in the apparatus suggest. *d*) Both in **B** and in **C**, sch. **100** is located beside the *relatum*. *e*) For the noun phrase διὰ τῶν πόλων see sch. **5**, **84**, **86** and **96**. On the difference between the units of measurement μοῖρα «degree» and τμῆμα «part», mentioned in the *relatum*, see sch. **12**.

101

Text. ἥντινα εἰσαγαγόντες εἰς τὸν κανόνα τῶν ἐν κύκλῳ εὐθειῶν εὑρήσομεν τὴν ἐπ' αὐτῆς περιφέρειαν

Transl. By entering at it into the table of the chords in a circle, we shall find the arc on it.

Comm. *a*) Vat. gr. 1594, f. 26r interc., Marc. gr. 313, f. 52v marg. int. *b*) *Ad Alm*. I.14, 78.9–10 καὶ ἡ ὑπὸ τὴν διπλῆν ἄρα τῆς ΘΗ τῶν αὐτῶν ἐστιν κδ ιε′ νζ″ «therefore the [chord] under the double of [arc] ΘΗ also is 24;15,57 of the same [parts]». *c*) The scholium asserts that one must enter into the Table of Chords at the value provided in the *relatum* in order to get the double of arc ΘΗ; actually, a linear interpolation is required: it is carried out in sch. **102**. The text coincides with Theon, *in Alm. I.13*, *iA*, 574.8–9, where it has exactly the same function. *d*) In **C**, sch. **101** is located beside the *relatum*. In **B**, it is misplaced, beside the first occurrence of the value 24;15,57 (78.7).

102

Text.

κγ κγ νε′ κζ″

κ′ λ″

κγ κδ ιε′ νζ″

λ′ μς″

κγ ∠ κδ κς′ ιγ″

ΥΠΕΡΟ^{X}

λ′ λ′ μς″

χιε″ **‴

ιθ′ νθ″ κ′ λ″

χιε″

χιδ″ μθ‴ ιδ⁗

apices non adhibent **CG** **2** κ λ′ om. **CG** **4** λ μς′ om. **CG** **5** ∠ om. **C** : add. m. 2 **G** **6** ΥΠΕΡΟ^{X} om. **G** **8** χιε″ **‴ uix legitur **B** : om. **CG** **11** χιδ μθ′ ιδ″ om. **CG**

Transl.

23 23 55 27

20 30

23 24 15 57

30 46

23 ½ 24 26 13

DIFFE^{R}

30 30 46

615 **

19 59 20 30

615

614 49 14

Comm. *a*) Vat. gr. 1594, f. 26r marg. inf., Marc. gr. 313, f. 52v marg. ext., Vat. gr. 184, f. 92v. *b*) *Ad Alm*. I.14, 78.10–11 ὥστε καὶ ἡ μὲν διπλῆ τῆς ΘΗ περιφερείας μοιρῶν ἐστιν κγ ιθʹ νθʺ «so that the double of arc ΘΗ also is of 23;19,59 degrees». *c*) This is a tabular arrangement of the procedure of linear interpolation (see also sch. **48**); the calculation provides the number mentioned in the next-to-last sentence of the first application of the Sector Theorem. This number is the double of arc ΘΗ, whose chord was stated by Ptolemy to be 24;15,57 (the calculation of this value, by removal of ratios, is performed in sch. **98–99**). By searching in the Table of Chords, one takes the nearest neighbours 23;55,27 and 24;26,13 of the assigned chords, as well as the corresponding arcs, namely, 23 and 23;30. These five data are arranged as in the table; the provisional value 23 for the degrees of the arc associated to chord 24;15,57 is also placed in front of the latter; a blank space is left on the right of value 23, to be filled by first and second sixtieths. To calculate them, one proceeds by linear interpolation: take the quotient of the corresponding differences *arch*(24;26,13) – *arch*(23;55,27) and 24;26,13 – 23;55,27; use it as a coefficient to multiply 24;15,57 – 23;55,27. The values of these differencies are on the one side 0;30 and 0;30,46, on the other 0;20,30; the latter two numbers are also transcribed in the table, on the right of the values of the corresponding pairs of chords and enclosed by arcs indicating the terms of the subtraction of which they are the result. The operation described above amounts to taking the appropriate fourth proportional of these numbers, namely, [(0;20,30)×(0;30)]/(0;30,46). This is done by putting the numbers on three of the four corners of a fictitious rectangle, the fourth corner being then occupied by the result, here 0;19,59. The diagonally opposed numbers are joined by mutually intersecting line segments. The result of the multiplication (0;20,30)×(0;30) is marked below this X-shaped array: this is 0;0,615. The division by 0;30,46 is not carried out; as a check, the result 0;0,614,49,14 of the multiplication of 0;19,59 by 0;30,46 is marked below 0;0,615. Finally, adding 0;19,59 to 23 = *arch*(23;55,27) one gets 23;19,59 = *arch*(24;15,57). I am unable to justify the presence, on the right of the X-shaped array in **B**, of number 0;0,615,**. There is no parallel calculation in Theon's commentary; he only adds the clause excerpted as sch. **101**. *d*) Both in **B** and in **C**, sch. **102** is misplaced with respect to the *relatum*; it is quite likely that in their common model the calculation was performed in a space left blank within the abundant scholiastic apparatus of the page before the Table of Declination. In **B**, it is just below sch. **104**, in the "shadow" of the right column; the *relatum* occupies three of the last four lines of the left column. In **C**, sch. **102** is in the outer margin, beside the first half of the second application of the Sector Theorem.

103

Text. τῶν ΖΑ ΑΒ

Transl. Of ZA, AB

Comm. *a*) Vat. gr. 1594, f. 26r insterc., Marc. gr. 313, f. 52v marg. int. *b*) *Ad Alm*. I.14, 78.14 τῶν ἄλλων μενόντων τῶν αὐτῶν «the other remaining the same». *c*) This is a scholium to the data that are assigned in the second application of the Sector Theorem. In the *relatum*, Ptolemy asserts that all but one of the values assumed in the first application of the Theorem are unchanged. The scholiast specifies that such unchanged values are those of arcs ZA and AB. Actually, he should have added arcs ZΘ and BE, both of a quadrant. *d*) Both in **B** and in **C**, sch. **103** is located beside the *relatum*.

104

Text. ἐπεὶ καὶ ἐπὶ τῶν λοιπῶν γ τεταρτημορίων τὰ αὐτὰ μεγέθη εὑρίσκονται· τὰ γὰρ ἴσον ἀπέχοντα ἀφ' ἑκατέρου τῶν ἰσημερινῶν τοῦ διὰ μέσων τῶν ζῳδίων τμήματα τὴν ἴσην λόξωσιν λοξοῦται.

2 μέσων **G** : μέσον **BC**

Transl. Since the same magnitudes can also be found for the remaining 3 quadrants, for segments of the ecliptic equally distant from each side of the equator are inclined at an equal inclination.

Comm. *a*) Vat. gr. 1594, f. 26r marg. inf., Marc. gr. 313, f. 52v marg. ext., Vat. gr. 184, f. 92v. *b*) *Ad Alm*. I.14, 79.2–5 ἐκθησόμεθα κανόνιον τῶν τοῦ τεταρτημορίου μοιρῶν ϙ παρακειμένας ἔχον τὰς πηλικότητας τῶν ὁμοίων ταῖς ἀποδεδειγμέναις περιφερειῶν «we shall set out a table giving for the 90 degrees of a quadrant the [numerical] values corresponding to the arcs computed in a similar way as those [above]». *c*) The scholium initially coincides with Theon, *in Alm. I.13*, *iA*, 578.8–10 (ἐπεὶ καὶ ἐπὶ τῶν λοιπῶν γ τὰ αὐτὰ μεγέθη συνάγεται, <u>ὡς ἑξῆς δείξομεν</u>, διὰ τὸ καὶ μίαν τινὰ καὶ τὴν αὐτὴν εἶναι ἔγκλισιν τοῦ ζῳδιακοῦ πρὸς τὸν ἰσημερινόν «since the same magnitudes can also be derived, <u>as we shall show in the sequel</u>, for the remaining 3 [quadrants], because the inclination of the zodiac on the equator is both one and the same»), but then replaces Theon's quite vague explanation with one that is in fact a gloss on Theon's gloss: the values of the declination are symmetric with respect to the points of intersection of ecliptic and equator. In other words, if the arcs of the ecliptic are reckoned from one of the equinoxes or of the solstices, the values of the declination associated to arcs on the ecliptic of x° and $360^{\circ} - x^{\circ}$ coincide. See sch. **107** for the details promised by Theon (underlined clause), who only highlights here, as the scholium does, this property of symmetry. As a matter of fact, the property as formulated by the scholiast gives rise to the rules enunciated later by Theon only after a little elaboration. *d*) In **B**, sch. **104** is exactly in the "shadow" of the column where *Alm*. I.14 ends. In **C**, it is beside the *relatum*. *e*) At line 3, note the etymological figure of speech in λόξωσιν λοξοῦται.

105

Text. ἐνταῦθα ἀπὸ τῆς ἐλαχίστης λοξώσεως ἄρχεται

Transl. Here it starts from the smallest declination

Comm. *a*) Vat. gr. 1594, f. 26v marg. sup., Marc. gr. 313, f. 53r marg. sup., Vat. gr. 184, f. 93r marg. sup. *b*) *Ad Alm.* I.15, Table of Declination. *c*) The ἐνταῦθα «here» refers to the different convention adopted in the *Handy Tables*, where the reckoning of arcs of the ecliptic in the Table of Declination starts from the largest declination, namely, from the beginning of Cancer (summer solstice). See also the remarks in Theon, "Little Commentary" 13, *PC*, 237.10–238.3, and "Great Commentary" I.20, *GC* I, 155.12–18, a portion of which can be read as a scholium in the manuscript Leiden, B.P.G. 78, of the *Handy Tables*. In *Alm*., the Table of Declination is set out as two columnar sub-tables that occupy an entire page: see Par. gr. 2389, f. 24v, Vat. gr. 1594, f. 26v, Marc. gr. 313, f. 53r, Vat. gr. 180, f. 24v, Vat. gr. 184, f. 93r. The name of the table is κανόνιον λοξώσεως «Table of Declination»; the title of each of the two sub-tables is περιφέρειαι «arcs»; in the column of the values of arcs of the ecliptic (the tabulation value), τοῦ διὰ μέσων «of the ecliptic» is marked; in the column of the values of declination, μεσημβρινοῦ «of a meridian». The tabulation value of the leftmost sub-table ranges from 1º to 45º, that of the rightmost sub-table from 46º to 90º. The step of the tabulation value is 1º, its origin at one of the equinoctial points. The tabulated declination starts thus with its minimum value (since by definition the ecliptic and the equator intersect at the equinoxes), and this is the fact the scholium draws attention to. Owing to the properties of symmetry alluded to in sch. **104** and more diffusely explained in sch. **107**, the range of the tabulation value goes in the table from 1º to 90º. The declination is tabulated up to second sixtieths. Sch. **105–107** refer to *Alm*. I.15. *d*) Sch. **105–107** are placed in the same way in **B** and **C**; this must have been the position in their common model. Sch. **105** (resp. **106**) occupies the left (resp. right) outer corner of the page; sch. **107** is transcribed over the whole length of the lower margin, below the two columnar sub-tables of declination.

106

Text. ὅτι ἀεὶ αἱ πρὸς τοῖς ἰσημερινοῖς λοξώσεις ἐν μείζοσιν ὑπεροχαῖς παρηύξηνται τῶν ἀπώτερον

1 ὅτι om. Th. | ἀεὶ] αἰεὶ Th. **2** ἀπώτερον] ἀπότερον codd. : ἀπώ– fecit m. 2 **K**

Transl. That the declinations closer to the equator always increase by greater diffences than those farther from it

Comm. *a*) Vat. gr. 1594, f. 26v marg. sup., Marc. gr. 313, f. 53r marg. sup., Vat. gr. 184, ff. 32v et 93r marg. sup. *b*) *Ad Alm.* I.15, Table of Declination. No citation in **K**. *c*) This is an obvious remark about the values in the table: the ecliptic is "almost flat" about the solstices. A correlated astronomical phenomenon is also conspicuous: near the solstices the length of the daylight increases or decreases very slowly. Apart from the initial ὅτι, the text coincides with Theon, *in Alm. I.15*, *iA*, 584.10–11, who also provides a proof, based on Theodosius, *Sph.* III.5, of this statement (*iA*, 584.13–585.14). *d*) See sch. **105**. In **K**, sch. **106** follows sch. **98**. *e*) The plural τοῖς ἰσημερινοῖς at line 1 is an idiomatic trait of ancient Greek language (see sch. **11** and **55**).

107

Text. ἐπειδὴ ἐν τῷ ἐκτεθειμένῳ τῆς λοξώσεως κανονίῳ ἀπὸ τοῦ ἰσημερινοῦ τυγχάνοντι κατὰ | τὸ πρῶτον σελίδιον ἑνὸς μόνου τεταρτημορίου τοῦ διὰ μέσων ἔκκεινται μοῖραι ϙ, ἀναγκαῖον δηλῶσαι ὃν τρόπον εἰσαγαγεῖν ὀφείλομεν εἰς τὸ κανόνιον πλειόνων τῶν ϙ μοιρῶν διδομένων. ὅτι μὲν οὖν ἐὰν λόγου ἕνεκεν ἀπὸ ἀρχῆς Κριοῦ δοθῶσιν μέχρι μοιρῶν ϙ, αὐτὰς εἰσαγαγόντες κατὰ τὸ πρῶτον σελίδιον τοῦ κανόνος τὰς παρακειμένας αὐταῖς ἐν τῷ δευτέρῳ σελιδίῳ ἐροῦμεν λελοξῶσθαι τὸ δεδομένον τοῦ ζῳδιακοῦ τμῆμα φανερόν· ἐὰν δὲ ὑπὲρ τὰς ϙ ὦσιν αἱ διδόμεναι τοῦ ζῳδιακοῦ μοῖραι ἕως ρπ, τὰς λειπούσας εἰς τὰς ρπ μοίρας εἰσαγαγόντες ὁμοίως ληψόμεθα τὴν ἐπιζητουμένην τῆς λοξώσεως πηλικότητα· ἐὰν δὲ ὑπὲρ τὰς ρπ ὦσιν ἕως σο, τὰς λοιπὰς μετὰ ἀφαίρεσιν τῶν ρπ εἰσάγομεν· ἐὰν δὲ ὑπὲρ τὰς σο, τὰς λειπούσας εἰς τὰς τξ.

1 ἐπειδὴ] ἐπεὶ Th. | κανονίῳ] κανόνι **K** **3** εἰσαγαγεῖν — πλειόνων] εἰσάγειν ὀφείλομεν ἐν τῷ κανόνι πλεόνων Th. | εἰς τὸ — **4** διδομένων om. **K** | Κριοῦ symbolon **BGK** **5** εἰσαγαγόντες] εἰσάγοντες **DG** **7** ϙ] ενενι(ους) **CG** sed corr m. 2 **G** | μοῖραι] μοίρας **K** | ἕως ρπ om. Th. **8** ante ὁμοίως add. et dein erasit ἐὰν δὲ m. 1 **K** | ληψόμεθα] λημψόμεθα Th. **9** ἀφαίρεσιν om. Th. | εἰσάγομεν] εἰσαγάγομεν **BCK** : –γωμεν **G**

Transl. Since, in the first column of the Table of Declination (which happens to have been set out [starting] from the equator), 90 degrees of one single quadrant of the ecliptic are set out, it is necessary to clarify in which way shall we enter into the table if more than 90 degrees are given. Well now, if, for the sake of argument, [such values] are given [starting] from the beginning of Aries as far as 90 degrees, by entering at them into the first column of the table we shall find that the given visible segment of the zodiac turns out to be inclined of the [amount] corresponding to them in the second column; if the given degrees of the zodiac exceed 90 [and go] as far as 180, by entering at the complement to 180 degrees we shall similarly get the sought [numerical] value of the declination; if they exceed 180 as far as 270, we enter at the remainder after subtraction of 180; if they exceed 270, [we enter] at the complement to 360.

Comm. *a*) Vat. gr. 1594, f. 26v marg. inf., Marc. gr. 313, f. 53r marg. inf., Vat. gr. 184, ff. 32v–33r et 93r marg. ext. *b*) *Ad Alm.* I.15, Table of Declination. No citation in **K**. *c*) As the Table of Declination only sets out arcs of the ecliptic in the first quadrant starting from an equinox, a rule must be given as to how to handle arcs greater than 90°. To this end, the scholiast transcribes an extensive excerpt from Theon's commentary, namely, the passage at *in Alm. I.15*, *iA*, 582.2–13 (see sch. **103**). Since the ecliptic and the equator are great circles on a sphere, their mutual inclination has the following properties of symmetry: if the arcs of the ecliptic are reckoned from one of the equinoxes or of the solstices, the values of the declination associated to arcs on the ecliptic of x^{o} and $180^{\mathrm{o}} - x^{\mathrm{o}}$, with $x^{\mathrm{o}} < 180^{\mathrm{o}}$, coincide, as do values of the declination associated to arcs on the ecliptic of x^{o} and $360^{\mathrm{o}} - x^{\mathrm{o}}$. By combining these two properties one gets that the same property holds for values of the declination associated to arcs on the ecliptic of x^{o} and $180^{\mathrm{o}} + x^{\mathrm{o}}$. The rules enunciated by Theon are an immediate consequence of these three properties. *d*) See sch. **105**. In **K**, sch. **107** follows sch. **106**. *e*) At line 8, the scholiast eliminates the parasite *my* in the form λημψόμεθα, a characteristic trait of Theon's language (*iA*, LXXXVI and CXVII–CXIX). The operator ἡ λείπουσα εἰς «the complement to» at lines 7 and 10 is in this scholium applied to a numerical value; otherwise, the expression ἡ λείπουσα εἰς τὸ ἡμικύκλιον means «the [chord] complement to a semicircle»: see sch. **11**, **27**, and **46**; see also sch. **77** for the same expression applied to taking the complement with respect to a right angle. The verb εἰσάγειν here and in sch. **101** is the standard one to denote the operation of «entering» into table at a value to find the value παρακείμενον «corresponding» to it (cf. sch. **47**).

Appendix. The Diagrams of *Alm*. I.10, 13, 14

General Remarks

The diagrams of *Alm*. in the oldest manuscripts normally are accurate and very well executed, even to modern standards. A wide space in the page is usually allotted to the figures; in Par. gr. 2389 and Vat. gr. 1594, written on two columns, the indentations reserved for the diagrams extend over the whole width of a column. In Marc. gr. 313, the indentations initially extend in width over the whole page; starting from f. 50v, they occupy only a part of the main frame; the diagrams are very often partly drawn in the margins. In Vat. gr. 180, the size of the indentations is of 7–9 lines × ⅓ of the main frame. In Vat. gr. 184, the diagrams are all in the margins; the same diagram was frequently traced more than once, and by different hands. Since the hands are difficult to assign and the figures often are very inaccurate, Vat. gr. 184 will not be taken into account in this appendix.

As for Vat. gr. 1594, the diagrams are missing in *Prol*. (except for f. 4v) and starting from f. 31v (*Alm*. II.5); the spaces reserved for them (usually of 7–9 lines at full column) are present but empty. The first figure drawn by the hand of the main copyist of *Alm*. is at f. 17r; diagrams related to the main text but placed in the lower margin are at ff. 18v, 19v, 24r, 26r, 29v. One or several later hands have drawn the missing figures at ff. 31v–120v (*Alm*. II.5–V.19; but the diagrams are absent at ff. 99v, 100r, 108v), 133v, 136v, 186v–192r (*Alm*. IX.6–9), 229v–232v (*Alm*. XII.1–2; but the diagrams are absent at ff. 230v, 232r); after this, the correctors gave up. Among the oldest manuscripts of *Alm*., the phenomenon of missing diagrams is unique to Vat. gr. 1594; the hyparchetype of its textual family had all its diagrams, since they are regularly present in Marc. gr. 313. In its turn, this manuscript has a feature that singles it out among all mathematical *codices vetustissimi*: the diagrams were traced before the text surrounding them; this is proved by the fact that the script is sometimes nicely adapted to the contour of the figure; see for instance ff. 284r–v and 353r–365r.

As was customary in the oldest mathematical manuscripts, the diagrams are located after the text of the proposition whose geometric configuration they represent; the indentations not extending over a whole column or page usually are on the right side of the main frame (but Marc. gr. 313 displays many exceptions to this rule, as the diagrams are frequently located on the left side).

One must bear in mind that the diagrams in Greek manuscripts usually are more symmetric than the geometric configurations they are intended to represent; this phenomenon is called “overspecification:” see Saito (2006) and, for a more general assessment, Saito and Sidoli (2012). Overspecification may be the outcome of at least three covariant factors:

- The diagrams were overspecified in the original treatises, maybe as a way to make, by contrasting an overspecified figure with the real configuration it is intended to represent, the general character of a mathematical proposition more manifest.[81] I must say that I now find this possibility quite implausible: I do not see any reason why Archimedes would like to have his diagrams overspecified. On the contrary, in Greek mathematical *texts* we find the opposite phenomenon of "underspecification:" geometric objects may happen to be designated by expression that make them more generic than they are; a case in point is Archimedes' denomination παραλληλόγραμμον (ὀρθογώνιον *Stom.*) for a square in *Stom.* and *Meth.* (see *AOO* II, 418.5, 426.11.24, 428.2–3).
- As any teacher of mathematics knows, less-skilled students asked to draw a figure usually make it oversymmetrized. In a sense, symmetric diagrams are (felt as) easier to draw and, for this reason, more reassuring: if a parallelogram is to be drawn, drawing an (approximate) rectangle is easier and "safer" than drawing an (approximate yet generic) parallelogram. Since diagrams were normally copied by mathematically unskilled copyists, one might surmise that the same phenomenon also applies to the reproduction of the figures of Greek mathematical treatises. The problem of this proposal is that it presupposes that some or all copyists redrew afresh the diagrams, maybe by taking a look at the construction-part of the proposition. This is implausible.
- I would favour a third explanation. The goal of medieval copyists surely was to reproduce as faithfully as possible the diagrams they found in their models. Diagrams in different manuscripts often are so stunningly identical as to suggest that some copyists contrived conformal reproductions of the figures by employing suitable devices or simple tricks (such as superposing the sheets of parchment before a source of light). Now, the successive steps of the process of copy of a figure can quite obviously be modelled as the evolution of a dynamical system: what evolves, by discrete time-steps, is the form of the diagram, that any act of copying modifies in a more or less appreciable way. It so happens that, if external constraints are not imposed, the forms of a diagram enjoying additional symmetries work as points of stable equilibrium in such an evolution.[82] An external constraint may be, for instance, a mismatch between the diagram and the indentation reserved to it, making it necessary to deform the diagram (don't forget that figures are usually added after the text was written). What makes oversymmetrized diagrams work as points of stable equilibrium is the obvious perceptional and psychological mechanism that makes a limiting case to be

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[81] This proposal is suggested in Acerbi (2007, 296). See also Saito and Sidoli (2012, 143), who, however, do not venture to propose an explanation, but simply contend that «[f]or us, an irregular triangle is somehow a more satisfying representation of 'any' triangle, whereas for the ancient and medieval mathematical scholars an arbitrary triangle might be just as well, if not better, depicted by a regular triangle».
[82] That this is the case when geometric patterns are to be reproduced by memory was proved experimentally within the framework of Gestaltpsychologie: see for instance Perkins (1932), and references therein. Even if, contrary to what happens in these experiments, the period during which the copyist is exposed to the original pattern (a text or a diagram) can be arbitrarily long, one must not forget that the act of copying is first and foremost a process involving memory.

perceptally (and hence graphically) more significant than a less limiting one: an isosceles triangle *versus* a scalene triangle, a diameter *versus* a generic chord, the middle point of a segment *versus* a generic point on it. Add to this that, from a graphical point of view, the notion "isosceles triangle" is not as sharply defined as the mathematical notion is: quasi-isosceles yet scalene triangles are simply perceived as isosceles. After all, *we ourselves* are victim of the same psychological mechanism when we deem the diagrams in medieval manuscripts overspecified—since of course a diagram cannot display *exact* properties of symmetry. Now, copyists *do not* take a look at the construction-part of a proposition in order to draw their diagrams: they simply reproduce pre-existing figures, and, if they have to correctly do their job without resorting to conformal copying by mechanical tricks, they must ask themselves some questions about the structure of the drawing they have before their eyes: basic—such as which points must be assigned the denotative letters, which lines intersect, etc.—and less basic issues must be addressed just by looking at the model diagram. The less basic issues include deciding whether a chord drawn near to the center of a circle really is a diameter or not, whether two nearly equal segments are equal or not, whether an angle is right or not, or, apparently a very difficult task, which angle of a right-angled triangle is right. As no one will transform a triangle perceived as isosceles into a decidedly scalene triangle, convergence towards limiting cases is the only alternative to stationary evolution. In this way, elements of a diagram intended to be drawn in a generic position tend to drift, during the process of copying, towards a limiting position, that thereby works as an "attractor" in the sense of the theory of dynamical systems: a chord near the center of a circle will converge towards a diameter. If mistakes or external factors do not intervene, such limiting positions, being perceived as such, will keep stable under the subsequent acts of copying. I am fairly confident that just a few steps in the process of copying are enough to make a diagram converge towards an overspecified form.

Specific Diagrams

The reproductions below, that I owe to the kindness of Ramon Masià, conform to the prescriptions set out in Ptolemy's constructions; when possible, they also conform to the diagrams contained in all or in some of the manuscripts. Graphic variants of some relevance not to be regarded as mere mistakes are described in a short commentary after the indication of the diagrams' position in the manuscripts; special attention is paid to features tending to symmetrize the diagram. The variants show that **ABC** on one side and **D** on the other belong to different traditions as far as the diagrams are concerned.

Fig. 1. Calculation of specific chords (sch. **4–8**)
Par. gr. 2389, f. 12v; Vat. gr. 1594, f. 17r; Marc. gr. 313, f. 41v; Vat. gr. 180, f. 14v.

In the manuscripts, triangle ZBE is nearly isosceles on base ZE. The line segments ΑΓ, BZ, BΔ, BE, ZΔ, and ZE are redrawn in **D** by a later hand, that also adds appropriate numerical values to them.

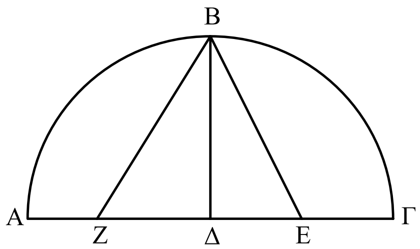


Fig. 2. Ptolemy's Theorem (sch. **17–18**)
Par. gr. 2389, f. 13v; Vat. gr. 1594, f. 18r; Marc. gr. 313, f. 42v; Vat. gr. 180, f. 15v.

The diagrams of **ABC** are virtually identical, the quadrilateral being nearly equilateral. In **D**, the inscribed quadrilateral is nearly an isosceles trapezium on diameter AΔ.

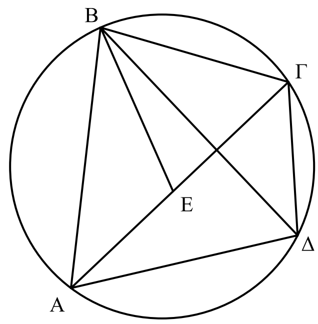


Fig. 3. Theorem "by difference" (sch. **19–20**)
Par. gr. 2389, f. 14r; Vat. gr. 1594, f. 18r; Marc. gr. 313, f. 43r; Vat. gr. 180, f. 16r.
The diagrams of **A** and **C** are nearly identical, with arc AB ≫ arc BΓ. In **B**, instead, arc AB is nearly equal to arc BΓ. In **D**, point Γ is in the upper right quadrant. Two later hands in **A** add numerical values to the line segments AB, BΓ, AΓ, BΔ, ΓΔ, and AΔ, also indicating the values of the squares on some of them. A later hand in **B** adds numerical values to the line segments AB, BΓ, and AΓ.

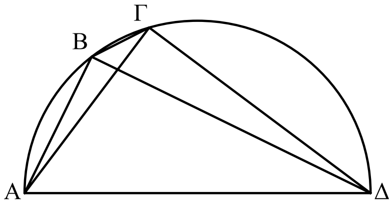


Fig. 4. Theorem “by bisection” (sch. **22**, **23**, **25**, **29**)
Par. gr. 2389, f. 14r; Vat. gr. 1594, f. 18v; Marc. gr. 313, f. 43r; Vat. gr. 180, f. 16r.

The diagrams of the manuscripts are nearly identical. In **AD**, point B almost coincides with the uppermost point of the semicircle.

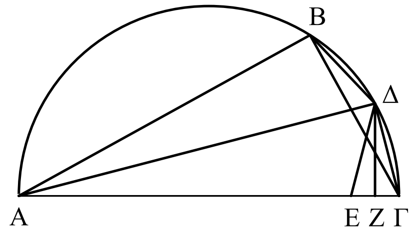


Fig. 5. Theorem “by composition” (sch. **26**–**27**)
Par. gr. 2389, f. 14v; Vat. gr. 1594, f. 18v marg. inf.; Marc. gr. 313, f. 43v; Vat. gr. 180, f. 16v.

In **ABC**, the diagram is almost exactly symmetric with respect to line ZΓ. In **C**, neither AΔ nor BE pass through the center Z of the circle. In **D**, arc AB = arc BΓ and both lie in the upper left quadrant (point Γ almost coincides with the uppermost point of the semicircle).

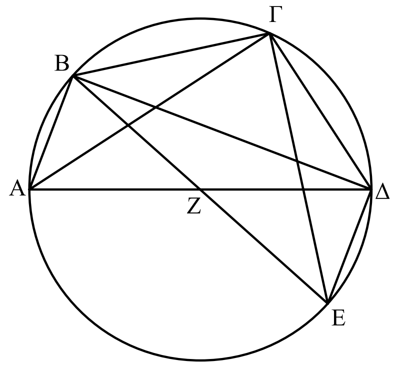


Fig. 6. Approximation Lemma (sch. **31**–**37**)
Par. gr. 2389, f. 15v; Vat. gr. 1594, f. 19v; Marc. gr. 313, f. 44v; Vat. gr. 180, f. 17r.

Manuscripts **ABC** have the diagram here reproduced, with the differences that chords AB and BΓ are not so markedly different and ΔZ is definitely not perpendicular to AEΓ. In **C**, a further circle is drawn in the lower margin, beside the diagram. Heiberg prints a diagram that is identical with that of **D**, namely, reversed left-right, with chord AΓ horizontal, and arc BΓ ≫ arc AB.

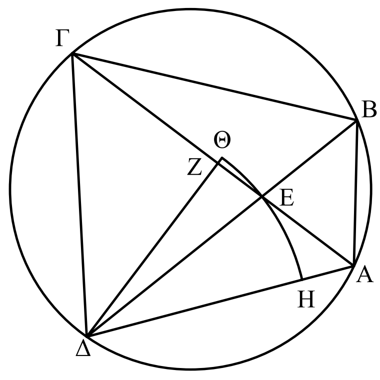


Fig. 7. Application of the Approximation Lemma (sch. **38**, **40**)
Par. gr. 2389, f. 16r; Vat. gr. 1594, f. 19v marg. inf.; Marc. gr. 313, f. 45r; Vat. gr. 180, f. 17v.

In **B**, chords AB, AΓ are drawn both in the upper and in the lower semicircle, symmetric with respect to those in the diagram here reproduced. In **D**, chord AΓ is drawn horizontal, is located in the lower semicircle and almost coincides with a diameter; point B almost coincides with the uppermost point of the circle.

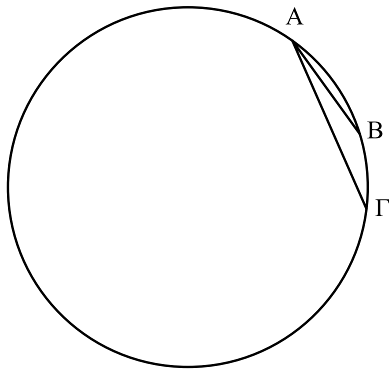


Fig. 8. First rectilinear lemma, "by composition" (sch. **57**, **58**, **61**)
Par. gr. 2389, f. 20v; Vat. gr. 1594, f. 24r; Marc. gr. 313, f. 49v marg. inf.; Vat. gr. 180, f. 21v.

The diagrams of the manuscripts are nearly identical; the "base" configuration is almost symmetric with respect to line AZ.

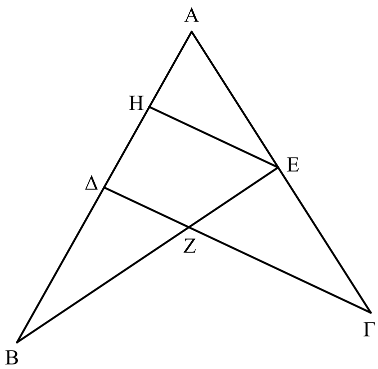


Fig. 9. Second rectilinear lemma, "by separation" (sch. **60**)
Par. gr. 2389, f. 21r; Vat. gr. 1594, f. 24r marg. inf.; Marc. gr. 313, f. 50r; Vat. gr. 180, f. 22r.

The diagrams of the manuscripts are nearly identical; the one in **C** is slightly rotated clockwise. The "base" configuration is almost symmetric with respect to line AZ.

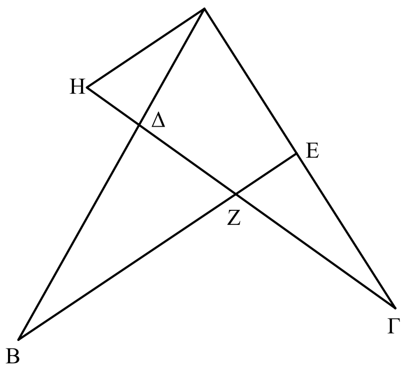


Fig. 10. First cyclic lemma (sch. **64**, **66**)
Par. gr. 2389, f. 21v; Vat. gr. 1594, f. 24r; Marc. gr. 313, f. 50v; Vat. gr. 180, f. 22r.

In **AC**, the line segments AZ and HΓ are definitely not perpendicular to diameter BΔ. In **D**, point A is located in the lower semicircle, the center Δ of the circle lying between E and Z.

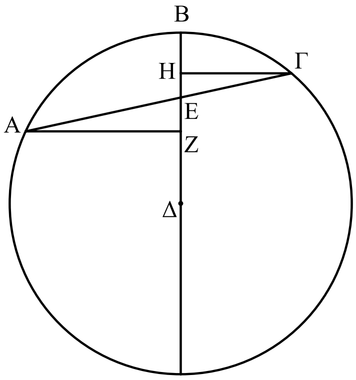


Fig. 11. Second cyclic lemma (sch. **66**–**73**)
Par. gr. 2389, f. 21v; Vat. gr. 1594, f. 24v; Marc. gr. 313, f. 50v; Vat. gr. 180, f. 22v.

I reproduce, as Heiberg did, a diagram that is identical with that of **D**. In **AC** (angle AZΔ is obtuse) and in **B** (angle AZΔ is acute), ΔZ is not perpendicular to AEΓ. In **B**, the radii AΔ and ΔB are not perpendicular to each other (as in fact they are not required to be). The diagrams of **AC** are identical.

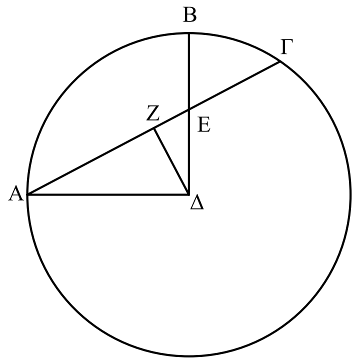


Fig. 12. Third cyclic lemma (sch. **74**–**76**)
Par. gr. 2389, f. 22r; Vat. gr. 1594, f. 24v; Marc. gr. 313, f. 51r; Vat. gr. 180, f. 22v.
The diagrams of the manuscripts are nearly identical; the one in **C** is slightly rotated counterclockwise. In **D**, the line segment EΔH is produced as far as the circumference, that meets at a point called Θ.

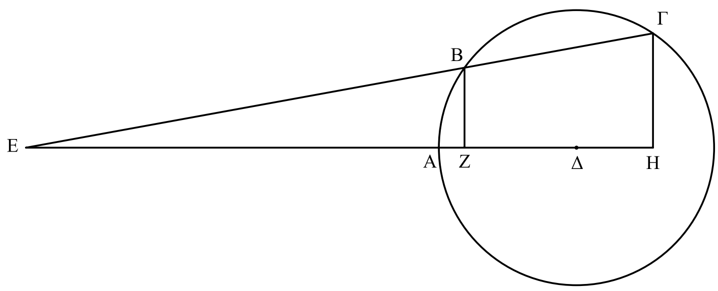


Fig. 13. Fourth cyclic lemma (sch. **76–81**, **86**)

Par. gr. 2389, f. 22r; Vat. gr. 1594, f. 25r; Marc. gr. 313, f. 51r; Vat. gr. 180, f. 23r.

In **AC**, the line segment ΔZ is perpendicular to EAΔ and not to chord BΓ. The diagram in **C** is rotated counterclockwise. In **D**, the line segment ΔZ is produced as far as the circumference.

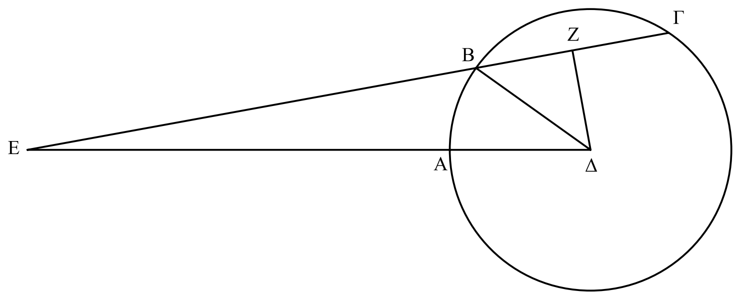


Fig. 14. Sector Theorem (sch. **83**, **84**, **87**)

Par. gr. 2389, f. 22v; Vat. gr. 1594, f. 25v; Marc. gr. 313, f. 51v marg. inf.; Vat. gr. 180, f. 23v.

In **ABC**, arcs BΔA and ΓEA are slightly produced after point A. In **B**, the whole diagram is inscribed in a sphere of center H, point Θ being on its circular outline. In **D**, the diagram is reversed left-right.

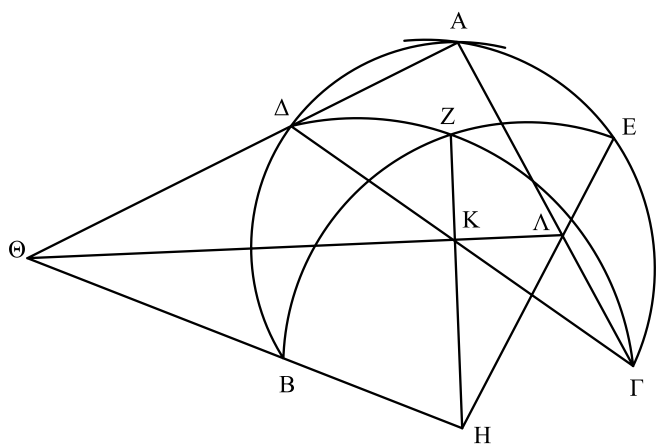
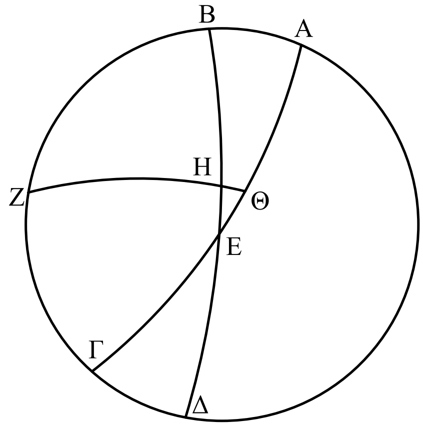


Fig. 15. First application of the Sector Theorem (sch. **93**, **96**, **97**, **100**)
Par. gr. 2389, f. 23v; Vat. gr. 1594, f. 26r marg. inf.; Marc. gr. 313, f. 52v; Vat. gr. 180, f. 24r.



In **B**, the diagram has points A/B, Γ/Δ, and Θ/H interchanged; moreover, the entire circles to which semicircles AΓ, BΔ, and quadrant ZHΘ belong are completed (in the usual mandala-shaped form) and the solstitial and equinoctial points are marked by their canonical signs. In **D**, arcs AΓ and BΔ have the concavity on the opposite side.

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