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Thābit ibn Qurra's Translation of the *Maʾkhūdhāt Mansūba ilā Arshimīdis*

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Abstract

The *Ma'khūdhāt mansūba ilā Arshimīdis* (*Liber Assumptorum*) is a treatise in plane geometry that is attributed to Archimedes. So far, it has been known through an edition due to Naṣīr al-Dīn al-Ṭūsī, but there exists an earlier Arabic translation made by Thābit ibn Qurra. This article provides a critical edition of this Arabic text, along with an English translation.

I Introduction

The *Ma'khūdhāt mansūba ilā Arshimīdis* (also known as *Liber Assumptorum* in modern editions of Archimedes' works) is an Arabic translation of a treatise that consists of fifteen propositions in plane geometry, without any discernible order, attributed to Archimedes. It partly survives in a *risāla* of Abū Saʿīd Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl al-Sijzī (ca. 945–1020 CE), who quotes some of the propositions verbatim; this *risāla* survives in a single manuscript, Arabe 2458 of the Bibliothèque Nationale de France (Paris).[1] There also exists a *taḥrīr* of Naṣīr al-Dīn al-Ṭūsī (597–672 AH/1201–1274 CE), which later became the basis of two Latin translations, one by John Greaves (1659), the other by Ibrāhīm al-Ḥāqilānī (d. 1664 CE, known as "Abrahamus Ecchelensis" in Latin sources) (1661, 377–415),[2] and the modern editions. Al-Ṭūsī's *taḥrīr* is part of his *taḥrīr* of the *Mutawassiṭāt* ("Middle Works"), which survives in numerous manuscripts. Versions of the *Ma'khūdhāt* prior to al-Ṭūsī's *taḥrīr* were thought to be lost;[3] even though al-Sijzī's *risāla* has been known for quite some time.[4] However, it turns out that the *Ma'khūdhāt* is contained in its entirety in the manuscript Fatih 3414 of the Süleymaniye Manuscript Library

[1] Rosenfeld and İhsanoğlu (2003, 111) list another manuscript, Chester Beatty 3652/8 (Dublin), for this *risāla*. Upon examination of this manuscript, however, I have found that it contains a different treatise by al-Sijzī, titled *Jawāb Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl ʿan masā'il handasiyya su'ila ʿanhu ahl Khurāsān*.

[2] On Ibrāhīm al-Ḥāqilānī, see Brock et al. (2011), s.v. "Al-Ḥāqilānī, Ibrāhīm."

[3] See, for example, the remark by Hogendijk (2014, 260, n. 3).

[4] As far as I know, Sédillot (1837) was the first to study this *risāla*, if only briefly. He translated part of its introduction and provided summaries of the propositions.

(İstanbul). This manuscript has already attracted scholarly attention due to the fact that it contains Arabic translations of Archimedes' *On the Measurement of the Circle*, *On the Sphere and the Cylinder*, and Eutocius' (fl. 6th c. CE) commentary to *On the Sphere and the Cylinder*, but the existence of the *Ma'khūdhāt* in it has gone unnoticed.[5] The purpose of this article is to give a critical edition of the *Ma'khūdhāt mansūba ilā Arshimīdis* together with an English translation.

I.1 The Author and the Translator of the *Ma'khūdhāt*

There are two transliterated Greek words appearing in the *Ma'khūdhāt*; it is therefore obvious that at least parts, and possibly the whole, of the treatise were originally composed in Greek. These two words are *arbēlos* (71.11 and 72.11), from Greek ἄρβηλος ("semicircular knife"), and *salinon* (86.7 and 87.6), from Greek σάλινον, a borrowing from Latin *salinum* ("salt-cellar").[6] Despite this, there is nothing in Greek sources that refers directly to any treatise that could be the source of the *Ma'khūdhāt*. As to the immediate source that was used for the Arabic translation, it could be either a Greek or a Syriac version.

The title of the work attributes the *Ma'khūdhāt* to Archimedes. There are, however, problems with this attribution. First, as has already been noted many times, Archimedes' name appears in the treatise twice, which makes it impossible that the treatise we have in its current form is from his pen.[7] A much more serious problem is the fact that some of the propositions of the *Ma'khūdhāt* have connections to results in Pappus' *Collection*, as well as to a small group of short treatises in Arabic, some of which have also been attributed to Archimedes.[8] In light of these connections, it is quite possible that these treatises originated as Greek sources by authors who were

[5] At least in published articles. In private communication (13 January 2016), Abdelkaddous Taha informed me that he was aware of Thābit ibn Qurra's translation.

[6] This is Heath's (1897, xxxiii) suggestion, which I follow.

[7] See, among others, Heath (1897, xxxii).

[8] Among these treatises, we may count the following two. 1. *Kitāb al-mafrūḍāt li-Aqāṭun*, preserved in Aya Sofya 4830/5 (İstanbul). This treatise has been translated into English by Dold-Samplonius (1977). 2. *Kitāb Arshimīdis fī al-uṣūl al-handasiyya*, preserved in Bankipore 2468/29 (Patna), a treatise containing nineteen propositions, all of which are also found in the *Kitāb al-mafrūḍāt li-Aqāṭun*. The fact that the second treatise, attributed to Archimedes, whose propositions are contained entirely in the first, attributed to an otherwise unknown Aqāṭun, is significant. As Dold-Samplonius observes, there are similarities between Proposition 3 of the *Kitāb al-mafrūḍāt li-Aqāṭun* and Proposition 10 of the *Ma'khūdhāt*. We may add that the figure of Proposition 26 of the *Kitāb al-mafrūḍāt li-Aqāṭun* is very similar to that of Proposition 3 of the *Ma'khūdhāt*, which was already known to be related to a proposition in the *Almagest* (See Heath (1897, 303) for details.). A detailed study of the contents of these treatises, along with the connections between them, would be useful.

either anonymous or lesser known, and then mistakenly attributed to Archimedes. Hence, until the connections between these works and their relations to extant Greek mathematical writings (especially of Late Antiquity) are better investigated, the attribution of the *Ma'khūdhāt* to Archimedes, in whole or in part, must also be viewed with considerable skepticism. We also note that the title in Fatih 3414, *Ma'khūdhāt mansūba ilā Arshimīdis*, can be interpreted as expressing some uncertainty about Archimedes' authorship of the work, but even if that were the case, this uncertainty soon disappeared: The titles of both the *risāla* of al-Sijzī and the *taḥrīr* of al-Ṭūsī claim that the work belongs to Archimedes (*li-Arshimīdis*).

The translator's name is given as Thābit ibn Qurra (d. 288 AH/901 CE) in Fatih 3414.

I.2 The *Risāla* of al-Sijzī

The *risāla* of al-Sijzī is a letter written by him to an unnamed recipient; this recipient is saluted throughout with the words *adāma llāhu ʿizzaka* ("may God perpetuate your might"). It is preserved in the unique manuscript Arabe 2458, Bibliothèque Nationale de France (Paris). From the opening of the letter (f. 5r), we understand that this recipient sent al-Sijzī some questions about propositions that he wanted al-Sijzī to explain and interpret (*tas'alunī ḥallahā wa-targhabu ilayya fī tafsīrihā*). Al-Sijzī replies that he found that these propositions were taken exactly as they are from a book of Archimedes known as the *Ma'khūdhāt*, and that they were explained there. Al-Sijzī then says he complied with his correspondent's request, and worked hard so that the demonstrations would be harmonious with the demonstrations given by Archimedes, if not necessarily the same. He says that he did not have anything except the writing itself and the annotations on it. He warns his correspondent that there are some propositions that cannot be proved in an easier manner than that of the "Geometer" and that there remains some difficulty in them.[9] Al-Sijzī then claims to have made the text easier for his correspondent to understand and added many explanations; he proved some propositions in what he regarded as a clearer way than that of Archimedes, that the king may be favorably disposed to the work (*li-yakūna li-naʿmā'i al-maliki adāma llāhu sulṭānahu*). From this last passage, we understand that his correspondent was a ruler.

There are references in Al-Sijzī's *risāla* to other works of his; one of these is of special importance for the dating of the *risāla*. At the beginning of Proposition 8 (f. 6v), Al-Sijzī informs us that this proposition is a *muqaddima* ("preliminary") to his work *Qismat al-zawāyā al-mustaqīmat al-khaṭṭayn bi-thalāthat aqsām mutasāwiya*.[10] As Rashed (2004, 183) already notes, the name of al-Bīrūnī (973–1050 CE) is men-

[9] Although al-Sijzī does not enumerate these propositions explicitly, they are probably Propositions 5, 6, 7, 8, and 12, the propositions he quotes verbatim.

[10] For an Arabic edition and French translation of this text, see Rashed (2004, 333–385).

tioned and his work is referred to in several places in the *Qismat*; Rashed then suggests that the *Qismat* must have been written in the 990s CE. Hence, the *risāla* would have been written even later, probably at the beginning of the 11th century.

I.3 The *Taḥrīr* of Naṣīr al-Dīn al-Ṭūsī

The *taḥrīr* of the *Ma'khūdhāt* by Naṣīr al-Dīn al-Ṭūsī is part of a collection of his *taḥrīr*s of various works, called the *Mutawassiṭāt* ("Middle Works").[11] Judging from the large number of surviving manuscripts, it is clear that these works enjoyed wide circulation in the Middle Ages and beyond. It was no doubt for this reason that al-Ṭūsī's *taḥrīr* of the *Ma'khūdhāt* formed the basis of the European translations by John Greaves (1659) and Ibrāhīm al-Ḥāqilānī (1661, 377–415).

The introduction to al-Ṭūsī's *taḥrīr* of the *Ma'khūdhāt* is simply a quotation from Abū al-Ḥasan ʿAlī ibn Aḥmad al-Nasawī (fl. 1029–1044 CE, referred to throughout the text as *al-ustādh al-mukhtaṣṣ*, or simply *al-ustādh*). Al-Nasawī starts by mentioning the attribution of these "beautiful propositions that are few in number and great in utility" to Archimedes. He goes on to say that the "Moderns" have added them to the collection known as the *Mutawassiṭāt* that should be studied between Euclid and the Almagest. He mentions the references by "Archimedes" to other works of his. Al-Nasawī then goes on to state that Abū Sahl al-Qūhī (fl. ca. 970–1000 CE) composed a treatise named the *Tazyīn kitāb Arshimīdis fī al-Ma'khūdhāt* ("Decoration of the book of Archimedes on the lemmas"), in which he proved Proposition 5 by a more general and elegant method. He finishes by saying that he cited two propositions from al-Qūhī's *Tazyīn* concerning Proposition 5 and left the others out to avoid prolixity.

The body of al-Ṭūsī's *taḥrīr* consists of the fifteen propositions of the *Ma'khūdhāt* in rephrased form. The rephrasing does not touch mathematical substance; in no instance do we find al-Ṭūsī proving the propositions in a different way. The figures are identical to those found in Fatih 3414; the only differences are in the letters used, which are changed so that they are introduced in the *abjad* order, as is common practice in Arabic mathematical texts of the Middle Ages.[12] It is clear that al-Ṭūsī's source was the text of the *Ma'khūdhāt* as found in Fatih 3414; he does not use al-Sijzī's *risāla*.

As an example, we may consider the differences in the demonstrations and figures of Proposition 1 in the *Ma'khūdhāt*, al-Sijzī's *risāla*, and al-Ṭūsī's *taḥrīr*. In the *Ma'khūdhāt*, this proposition is proved by using the properties of parallelograms and isosceles triangles. The strategy of the demonstration is to prove that the angles RKQ and BKQ are equal to two right angles, which implies that RB is a straight line.

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[11] For (nonexhaustive) lists of manuscripts, see Sezgin (1974, 133) and Rosenfeld and İhsanoğlu (2003, 213).

[12] See, however, the notes to Figures 3 and 5 at the end of Section III.

Al-Ṭūsī's *taḥrīr* follows the same outline; moreover, the figures in the *Ma'khūdhāt* and al-Ṭūsī's *taḥrīr* are identical (except for the diagram letters). In contrast, al-Sijzī uses a different approach in his *risāla*; the figure he uses is also different in that it has two sectors of circles less than semicircles that are tangent to each other at a corner, whereas the figure in the *Ma'khūdhāt* and al-Ṭūsī's *taḥrīr* both use two semicircles that are tangent at one point between the two endpoints. Al-Sijzī's demonstration uses the theories of ratios and similarity as found in Euclid's *Elements*, and applies this theory to the ratios of the various lengths in the figure to prove that the triangles BKQ and KRX are similar; this then implies that the angles RKX, XKQ, and QKB (or rather, the angles corresponding to these in his figure) are equal to two right angles.

The demonstrations of the propositions in al-Ṭūsī's *taḥrīr* are sometimes followed by commentaries from al-Nasawī; on occasion al-Ṭūsī adds his own comments. In Proposition 5, al-Nasawī cites two propositions from the *Tazyīn* of Abū Sahl al-Qūhī, as mentioned above. These two propositions of al-Qūhī concern the two cases where the two semicircles in Proposition 5 intersect, as opposed to touch, each other.

I.4 The Latin Translations of the *Ma'khūdhāt* by John Greaves and Ibrāhīm al-Ḥāqilānī

The first appearance of the *Ma'khūdhāt* in Europe, in manuscript or in print, is the Latin translation made by the English mathematician John Greaves (1602–1652 CE) and published in a collection titled *Miscellanea: sive lucubrationes mathematicae* that was edited by John Twysden (1607–1688 CE) and published in London in 1659. This is, in fact, a posthumous collection of seventeen texts, most of them by the English mathematician and astronomer Samuel Foster (d. 1652 CE),[13] on astronomy, geometry, and trigonometry. The texts are in Latin, English, and sometimes both. There is a letter in Latin by Twysden dedicated to Sir Henry Yelverton, 2nd Baronet (1633–1670 CE), and another, in English, to Sir Henry's wife Susanna Longueville, Baroness Grey de Ruthyn. There is also a letter to the reader, in both Latin and English.

The *Lemmata Archimedis* is the eleventh text in this collection. The title page announces that this text had been desired for some time, that it was translated from an old Arabic manuscript by John Greaves, and that this is the first time it was published. Indeed, no version of the *Ma'khūdhāt* is known in Europe prior to 1659 CE, but there was obviously interest in the text. This interest may have been generated by European scholars reading Arabic who became aware of the text before 1659 CE.[14]

[13] For an overview of Samuel Foster's life, his circle, and his astronomical activities, see Frost (2006).

[14] It is not difficult to see how a newly discovered text purportedly by Archimedes would generate

Greaves' Latin translation is made from the *taḥrīr* of al-Ṭūsī. It includes a translation of the introduction, which, as we have noted, is simply a paragraph quoted from al-Nasawī, the propositions, and all the commentaries by al-Nasawī. There is a page of short corrections, also in Latin, by Samuel Foster, and a page of figures at the end.

John Ward (1740, 86), in his biography of Foster, has this to say:

> ... Nor did he only excel in his own faculty, but he was likewise well versed in the ancient languages; as appears by his revising and correcting the *Lemmata* of Archimedes, which had been translated into Latin from an Arabic manuscript, but not published, by Mr. John Greaves.

Hence, we see that his work on the *Lemmata Archimedis* brought Foster enough prestige to justify its mention in his biography.

The next European publication of the *Ma'khūdhāt*, also based on the *taḥrīr* of al-Ṭūsī, is a Latin translation made by Ibrāhīm al-Ḥāqilānī, together with notes by the Italian mathematician Giovanni Alfonso Borelli (1608–1679). This translation is found as the second part of a volume containing Latin translations of Books V, VI, and VII of Apollonius' *Conics*. There is a five-page introduction to the reader by Borelli. Most of this introduction is a discussion of the reasons, for or against, the attribution of the text to Archimedes. At the end of the introduction, Borelli notes that the title indicates the existence of sixteen propositions, but that there are only fifteen in the text.[15] This has consequences, in that two other propositions, not found in any Arabic version of the *Ma'khūdhāt*, were added at the end (see below).

The introduction is followed by the translations of the propositions, al-Nasawī's commentaries, and finally, for some propositions, by notes in Latin written by Borelli. These notes are usually quite extended; they point to connections to other Greek mathematical texts (especially Pappus' *Collection*), to errors found in the text, to possible applications of the propositions to the solutions of problems such as the measurement of the circle and the trisection of the angle, to supplying demonstrations of assumed results.

After his notes to Proposition 15, Borelli adds that he had mentioned that it was not improbable that this booklet of Archimedes was none other than the ancient book of lemmas that Eutocius had said he had seen, in his commentary to Proposition 4 of Book II of *Sphere and the Cylinder*. After that work had been translated

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a great deal of interest in Europe, where Archimedes' treatises were now being translated. This interest may also explain the fact that two translations of the *Ma'khūdhāt* were published within two years of each other. In general, more work on the impact of the *Ma'khūdhāt* on both Islamic and European mathematics remains a *desideratum*.

[15] In fact, all the manuscripts of the *taḥrīr* of al-Ṭūsī that I know of contain fifteen propositions.

by very learned friends, Ibrāhīm al-Ḥāqilānī sent Borelli the translation of a portion of a work of Abū Sahl al-Qūhī, namely a "book of the arrangement of the lemmas of Archimedes" (*librum ordinationis lemmatum Archimedis*). Despite the slight mistranslation, this can be seen to be the *Tazyīn* of Abū Sahl al-Qūhī.[16] Borelli then goes on to cite some words of al-Qūhī's that he says confirms the interpretation that the *Liber Assumptorum* is none other than the lost treatise of Archimedes. Next, he tells us that he is going to add two propositions here, one because Archimedes had promised to prove it, and the other because it must be the one missing from the Arabic manuscript of the *Liber Assumptorum* used by Ibrāhīm al-Ḥāqilānī. (Recall that Borelli is under the impression that the *Liber Assumptorum* ought to contain sixteen propositions.) These propositions have been recently studied in detail, and their texts have been published, by Hogendijk (2014, 264–272); he finds that Proposition 16 in Borelli's edition of the *Liber Assumptorum* has been adapted from a work of al-Qūhī's titled *Filling a Lacuna in the Book by Archimedes on the Sphere and Cylinder* in the edition of Naṣīr al-Dīn al-Ṭūsī.

I.5 Recent Translations

Al-Ḥāqilānī's Latin translation was used by Heiberg (1972, II.509–525) in his edition of Archimedes' works; he reproduced the text of the propositions, and the introduction (in a footnote), but he left out al-Nasawī's commentaries. In modern languages, we have an English translation by Heath (1897, 301–318), a French translation by Ver Eecke (1960, 521–542) and another by Mugler (1970–1972, III.129–164). All these works reproduce the text of the propositions; the commentaries of al-Nasawī are not included.[17]

I.6 The Text of the *Ma'khūdhāt*

The propositions in the *Ma'khūdhāt* follow a slightly different layout from those of Euclid's *Elements*. In general, they start with an *enunciation* (πρότασις) stated, not in general terms, but using lettered objects, with the grammatical structure of an "if-then" statement (*idhā … fa-inna*), so that there is no separate *exposition* (ἔκθεσις) or *specification* (διορισμός), or these are simply folded into the *enunciation*. There are a few irregularities: Proposition 1 starts with an *enunciation* stated in general terms that quickly reverts to the use of lettered objects; in Propositions 13 and 15, the consequence of the opening statement is stated as a *specification*, beginning,

[16] Hogendijk (2014, 268, n. 18) points out that the Latin word *ordinatio* is a mistake for *ornatio*, an exact translation of Arabic *tazyīn*.

[17] There is now a Spanish translation of Archimedes' works by Paloma Ortiz García. The second volume of this translation, published in 2009 according to its amazon.es page, seems to include the *Ma'khūdhāt*, but I regret that I have not been able to consult the book directly.

as is usual in Greek mathematical texts, with ”then I say” (*fa-aqūl*). These are followed by the *construction* (κατασκευή) and the *demonstration* (ἀπόδειξις). None of the propositions contains a *conclusion* (συμπέρασμα).[18] Since the irregularities mentioned above are extremely unlikely to be translation or scribal errors, they must be attributed to the original text from which the *Maʾkhūdhāt* was translated, and they give the strong impression that this original text was compiled from different sources. It should also be noted that the Arabic words *mithāl* (“example”) and *burhān* (“demonstration”) that are used to mark the *exposition* and *demonstration* in some Arabic translations of Greek mathematical works and in many mathematical texts originally composed in Arabic are absent from the *Maʾkhūdhāt*.

There are irregularities in the terminology as well. For example, in Proposition 1, the lines *rāʾ kāf* and *kāf bāʾ* are joined by the expression *wuṣila fīmā bayna nuqṭatay … bi-khaṭṭ* (68.5) instead of *wuṣila khaṭṭ* that will become common in the later propositions (for example, 70.9). The point *bāʾ*, which is the point of tangency of the two circles, is said to be the *mawḍiʿ al-tamāss* (68.7) instead of the expected *nuqṭat al-tamāss*; instead of the expected *khaṭṭ ḍād bāʾ*, we have a *khaṭṭan mustaqīman … ʿalayhi ḍād bāʾ* (68.7). Starting with Proposition 2, this begins to change. Lines are now joined directly, with expressions such as *waṣalnā khaṭṭ* (69.8). The word *qāʿida* (70.2) is sometimes used to denote a cathete, which is again unusual. Perpendiculars are sometimes indicated as *ʿamūd kāf qāf* (70.8, Proposition 3), sometimes as *ʿamūd ʿalayhi ḍād ḥāʾ* (74.7, Proposition 5). The same is true of circles: *dāʾirat rāʾ ḍād* (78.3, Proposition 7) vs. *dāʾirat ʿalayhā rāʾ kāf qāf* (80.12, Proposition 10). Chords, or lines cutting circles, are first described as *qāṭiʿ* (80.12 and 84.2, Propositions 10 and 12 respectively), later as *watar* (87.8, Proposition 15).[19]

The subject matter of the *Maʾkhūdhāt* is plane geometry, which I define as the material covered in Books I–VI of Euclid’s *Elements*. There is no clear arrangement of the propositions, with one notable exception: Propositions 1, 4, 5, and 6 form a group. Propositions 4, 5, and 6 cover properties of the figure known as the ἄρβηλος (“semicircular knife”). I have included Proposition 1 in this group since it is explicitly used in Propositions 5 and 6. The connection of this group to Book IV Pappus’ *Collection* has long been known.[20] It may be possible to include Proposition 14 among these as well, since it concerns the figure known as the σάλινον, from Latin *salinum* (“salt-cellar”). This figure is formed, just like the ἄρβηλος, by semicircles that are tangent to each other.

[18] The divisions of a Euclidean proposition are discussed in some detail by Heath (1926, I.129–131), and with respect to the Arabic texts by Sidoli and Isahaya (2018, 212–218). The presence of Greek terms in parentheses should not be taken to imply that the translator of the *Maʾkhūdhāt* knew this terminology.

[19] These examples are not exhaustive.

[20] See Heath (1897, xxxii–xxxiii, 304–308) and Sefrin-Weis (2010, 207–221).

Finally, we must note the names of some treatises mentioned in the text. These are the "Commentary to the Treatise about Right-Angled Triangles" (Proposition 5), the "Treatise about all Triangles" (Proposition 6), and the "Treatise about Quadrilaterals" (Proposition 12). The mention of two of these treatises, both concerning triangles, in Propositions 5 and 6, strengthens the impression that these two propositions have a common source, as is already implied by their subject matter. None of these treatises is known from Greek sources.

II Description of the Manuscripts

II.1 Fatih 3414 (684 AH/1286 CE)

Physical Description

The manuscript comprises 75 folios, with nineteen lines per page. The folio numbers are written in pencil at the upper left corners of the rectos. The text, the lettering of the figures and the marginal notes are written in very clear and readable *naskhī* from one hand. Pointing is usually, but not always, provided. The figures are well drawn. The marginal notes are not numerous, with the number of the longer notes being less than two dozen; in some of these notes, the writer is explicitly named as Muḥammad ibn ʿUmar ibn Aḥmad ibn Abī Jarāda (7th c. AH/13th c. CE).[21] Some corrections to the main text have been made by covering over the erroneous part with red ink; these are again few in number, and they are corrected right after the erased part, not in the margin. Red ink is also used for the longer marginal comments, and black ink is reserved for the shorter marginal corrections, though there are exceptions. Titles of works, numbers of propositions, figures and the longer marginal notes are in red ink; while the main text is in black ink. Titles of works are fully vowelled and are written in larger letters than usual. Decorated headpieces are also provided for the first two treatises.[22] There is extensive water damage at the bottom parts of the pages; which sometimes impedes reading the bottom lines. There is also minor water damage in various other places.

Contents

The treatises in the manuscript are as follows:[23]

1. *Kitāb Arshamīdas*[24] *fī misāḥat al-dāʾira* (Κύκλου μέτρησις): ff. 2v–6v. On folio

[21] Suter (1900, 158, no. 385) mentions him briefly.

[22] This may explain the fact that only these treatises are listed in the microfilm catalog.

[23] Please note that I am using the folio numbers indicated at the upper left corners of the manuscript, and my numeration may differ slightly from that used by Sezgin (1974).

[24] Even though Archimedes' name is typically vocalized as *Arshimīdis* in Arabic texts, it is vocalized

2r, there is a decorated headpiece for the work. Neither there nor on the next page is the name of the translator given. The colophon gives no information about the copyist or the date of completion; however, the copyist is probably Ibn Abī Jarāda, as he is of all the other treatises, and the hand is the same.

2. *Kitāb Arshamīdas fī shakl al-kura wa-shakl al-usṭuwāna* (Περὶ σφαίρας καὶ κυλίνδρου α' β'): ff. 9v–46v for Part I and 46v–60r for Part II. On folio 9r, there is a decorated headpiece for the work, where the translator's name is given as Qustā ibn Lūqā al-Baʿlabakkī (d. ca. 300 AH/912–913 CE). Part I includes 49 propositions as opposed to 44 in Heiberg's edition (1972), whereas Part II includes nine as in Heiberg. There is a short colophon on folio 46v and a longer one on folio 60r, which latter indicates that the copying was finished on 16 Rabīʿ al-Awwal 676 AH/17 August 1277 CE by Ibn Abī Jarāda.

3. *Min tafsīr kitāb Arshamīdas fī al-kura wa-l-usṭuwāna li-Ūṭūqiyūs*: ff. 61r–67v and 74r–74v. Due to a binding error, folio 74 was bound with the folios of the next treatise, which starts directly after folio 67v (see below). There are seven propositions in this treatise. The colophon on folio 74v indicates that the copying was finished on 6 Dhū al-Qaʿda 684 AH/3 January 1286 CE by Ibn Abī Jarāda. It is interesting to note that this is more than eight years after the previous colophon; this may indicate that the Arabic translation of Eutocius' commentary was found in a different manuscript than the one that contained *On the Sphere and Cylinder*.

4. *Ma'khūdhāt mansūba ilā Arshamīdas*: ff. 68r–73v and 75r–75v.[25] There are fifteen propositions in this treatise. The colophon on folio 75v indicates that the copying was finished on 2 Dhū al-Ḥijja 684 AH/29 January 1286 CE by Ibn Abī Jarāda.

كتبه وشكّله وصحّحه مقابلة وحلّاً الفقير إلى الله تعالى محمّد بن عمر بن أحمد بن أبي جرادة.
وفرغ منه في الثاني من ذي الحجّة سنة أربع وثمانين وستّمائة حامداً لله تعالى ومصلّياً على سيّدنا
محمّد وعلى آله وصحبه ومسلّماً. صحّح مقابلة وحلّاً ولله الحمد كثيراً.

---

as *Arshamīdas* in the titles of some of the treatises contained in Fatih 3414.

[25] Note that Sezgin (1974, 132) erroneously lists this manuscript with another manuscript (Mashhad, Riḍāʾ 5617) as containing the commentary of Abū al-Ḥasan ʿAlī ibn Aḥmad al-Nasawī and then translated into Latin; he then claims that this Latin translation is contained in Heiberg's edition. In fact, the text Heiberg included in his edition is Ibrāhīm al-Ḥāqilānī's translation of al-Ṭūsī's *taḥrīr* of the *Liber Assumptorum,* with the commentaries removed. Sezgin (1974, 132) also lists this treatise as running to folio 73v (72v in his numeration), missing the last page. Krause (1936, 457, no. 21) gives the incipit of the work, which makes it clear that it is not al-Ṭūsī's *taḥrīr*.

Muḥammad ibn ʿUmar ibn Aḥmad ibn Abī Jarāda, the one who needs God the Exalted, wrote, drafted, and verified it by collation. And he finished it on 2 Dhū al-Ḥijja 684, praising God the Exalted, praying for our Master Muhammad, for his family, and for his companions. He verified (it) by collation; much praise is for God.

II.2 Arabe 2458 (539/1145)

Physical Description

The manuscript comprises 32 folios. The folio numbers are written at the upper left corners of the rectos and the upper middle of the versos. The text, the lettering of the figures and the marginal notes are written in *naskhī* in a single hand; but the writing can be difficult to read. Pointing is usually, but not always, provided. The figures are very well drawn, but at the end of the manuscript, four figures are missing. The marginal notes are few in number; the longer notes number less than ten. The marginal notes contain both marginal corrections and mathematical comments. As I have consulted this manuscript from a black-and-white PDF file, I am not able to describe the colors used, but it is clear that the titles were written in a different color than the text itself. The titles are of the same size as the text. There is water damage at the bottom parts of the pages and near the binding, the damage sometimes reaches as far as a third of the page, which sometimes makes it impossible to read the bottom lines of the page.

Contents

The treatises in the manuscript are as follows:

1. *Risāla li-Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl raḥimahu llāh fī ikhrāj al-khuṭūṭ fī al-dawā'ir al-mawḍūʿa min al-nuqaṭ al-muʿṭāh*: ff. 1v–4r. The colophon at the end of folio 4r is difficult to read due to water damage, but the year 539 AH/1144–1145 CE is legible.

2. *Taḥṣīl al-qawānīn al-handasiyya al-maḥdūda li-Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl al-Sijzī raḥimahu llāh*: ff. 4v–5r. The colophon on folio 5r indicates that the copying was finished in 539 AH/1144–1145 CE.

3. *Risālat Aḥmad ibn Muḥammad ibn ʿAbd al-Jalīl fī al-jawāb ʿan al-masā'il allatī su'ila fī ḥall al-ashkāl al-ma'khūdha min kitāb al-ma'khūdhāt li-Arshimīdis*: ff. 5r–8v. The colophon on folio 8v indicates that the copying was finished on 23 Shawwāl 539 AH/18 April 1145 CE. The name of the copyist is not given.

4. *Al-maqāla al-rābiʿat ʿashar min ikhtiṣār al-imām al-Muẓaffar al-Isfazārī li-uṣūl Uqlīdis aḥad ʿashar shaklan*: ff. 9v–10v. The colophon on folio 10v gives no information about when the copying was completed, but it must have been in 539

AH/1144–1145 CE, since both the previous and the next treatises were completed in that year.

5. *Maqāla li-l-Ḥasan ibn al-Ḥasan ibn al-Haytham fī al-maʿlūmāt*: ff. 11v–26r. This treatise has been edited and translated into French by Rashed (2002, 443–583). The colophon on folio 26r indicates that the copying was finished on 9 Dhū al-Ḥijja 539 AH/2 June 1145 CE.

6. *Al-ashkāl allatī yajib an tuḍāf ilā al-ukar ḥattā yufham al-Majisṭī ʿalā al-ḥaqīqa min ghayr taqrīb*: ff. 26v–27v. This is a very short work by Ibn Rushd (520 AH/1126 CE–595 AH/1198 CE) on spherical geometry;[26] including nine figures. There is no title proper, but the text starts with *qāla al-shaykh Abū al-Walīd hādhihi al-askhāl allatī yajib an tuḍāf ilā al-ukar...*, whence the title above. The colophon on folio 27v gives no information about when the copying was completed. If the copying was completed in 539 AH/1144–1145 CE, as were the previous treatises in the manuscript, Ibn Rushd would have been no older than eighteen years of age when he wrote this treatise. We can therefore conclude that this treatise (and the next) were copied well after this date.

7. *Risāla fī al-barāhīn ʿalā masāʾil al-jabr wa-l-muqābala*: ff. 28r–32v. No information about the title or author are given, but the text is a famous treatise by ʿUmar al-Khayyām.[27] There is no colophon.

III Editorial Principles

There exists a well-written manuscript, namely Fatih 3414 (denoted **F**), for *Maʾkhūdhāt mansūba ilā Arshimīdis*. The other witness is part of a letter (*risāla*) by al-Sijzī on the *Maʾkhūdhāt* contained in the manuscript Arabe 2458 (denoted **P**). Al-Sijzī cites several propositions (Props. 5, 6, 7, 8, 12) verbatim, while paraphrasing others; hence his letter can be used to reconstruct the text of the propositions quoted verbatim. Since **P** contains a far greater number of scribal errors than **F**, my approach to establishing the text and the figures has been to rely on **F**. There are a few grammatical mistakes in the text; since they are very minor and do not affect the sense of the text, I have not attempted to correct them. However, I have made some minor changes to the text and two of the figures (see below), if such corrections seemed justified by the variant reading in **P** as well as internal evidence of **F**. I have also occasionally used the (noncritical) Hyderabad edition of al-Ṭūsī's *taḥrīr* of the text (denoted **Hy**) to correct the reading of a few words.

To give one example, in the text of Proposition 6 (76.4) we have the reading وخطّا كع كي مستقيمان in **F**. In that sentence, with one exception at the very beginning,

[26] See Rosenfeld and İhsanoğlu (2003, 190).

[27] See Rosenfeld and İhsanoğlu (2003, 168–169).

every line is indicated by three points on it; moreover, **P** has the reading وخطّا نجع كبي مستقيمان. Therefore, I have accepted the reading of **P** instead of **F**.

Note that in both manuscripts, the figures are placed on the left margin, but their location otherwise does not depend on the text.[28] The lettering of geometric objects is indicated by a series of *connected* letters with an overline. The edition and translation are followed by a list of variant readings in **P** and notes on the critical apparatus. I have given the Arabic originals (in transliteration) of some of the words in English translation; and I have given the Greek loanwords in transliteration, with the Greek original in parentheses. I have supplied the punctuation in the Arabic edition. Finally, the words in square brackets in the English translation indicate that they are not present in the Arabic; they are included to increase readability.

I have reproduced the figures in **F** faithfully, preserving not only the line segments and the arcs appearing in the figures, but also their orientations and the angles made between them. This implies that the figures sometimes have features that are not explicitly stated in the text. The most obvious example is the figure in Proposition 2, where the two tangents to the circle need not be perpendicular to each other. The placing of the letters, however, may differ from that in the manuscripts.

Note on Figure 3: On folio 69v in **F**, there is a line segment connecting *rā'* and *kāf*, which is not constructed in the text.

Note on Figure 5: On folio 70v in **F**, the letter *bā'* appears in the figure, but the text has *thā'*; so I have reproduced the figure with *thā'* instead of *bā'*. Consequently, the figure used in the translation has *V*. The corresponding figure and text in **P**, on folio 6a, both have *bā'*. Also, the figure in **F** has an extra letter *nūn* which, together with *shīn*, forms the diameter of the smaller circle on the right. However, the text constructs only one of the diameters, namely *alif shīn*, the diameter of the smaller circle on the left.

The line segment connecting *rā'* and *kāf* in Proposition 3, the letter *nūn* and the line segment connecting it to *shīn* in Proposition 5 are also missing from the *taḥrīr* of al-Ṭūsī. They were then probably added by Ibn Abī Jarāda, or somebody in the line of transmission to him. Therefore, I have not reproduced them.

[28] This is true of all texts in both manuscripts.

Correspondence of Arabic and Latin Letters

Arabic	Latin	Arabic	Latin	Arabic	Latin
ا	A	ن	N	ت	T
ب	B	ع	O	ض	U
ح	H	ف	P	ث	V
ي	I	ق	Q	خ	W
ك	K	ر	R	ش	X

IV Text and Translation

Conspectus Siglorum

The sigla of the manuscripts used are as follows:

F: Fatih 3414, ff. 68r–73v and 75r–75v. As explained in subsection II.1, folio 74 belongs to another treatise in the manuscript. Folio numbers of this manuscript are indicated on the left side of the Arabic text,
P: Arabe 2458, ff. 5r–8v.

The Hyderabad edition of al-Ṭūsī's *taḥrīr* of the *Ma'khūdhāt* is also referred to as follows:

Hy: al-Ṭūsī (1939, pp. 99–131).

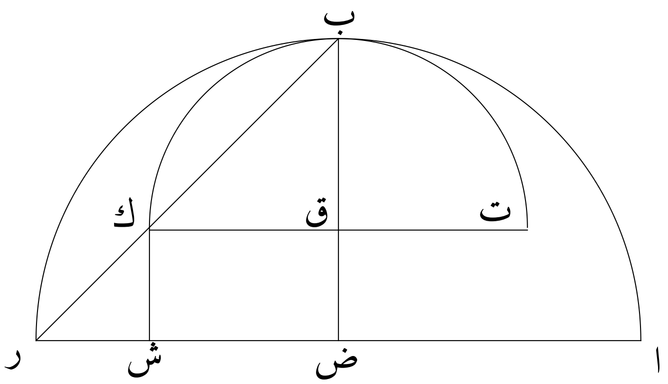
Abbreviations

ante	before
cf.	compare
corr. mg.	corrected in the margin
illeg.	illegible
mg.	margin
om.	omitted
part. illeg.	partially illegible
rep.	repeated (twice)
sup.	above the line
tr.	transposed

مأخوذات منسوبة إلى أَرْشَمِيدَسْ 68r
ترجمة ثابت بن قرّة الحرّانيّ

68v بسم الله الرحمن الرحيم ربّ يسّر

ا إذا كان نصفا دائرتين يماسّ أحدهما الآخر، وكان قطراهما متوازيين كقطري رضا كقت، ووصل فيما بين نقطتي ر ك بخطّ رك وفيما بين نقطتي ك ب بخطّ كب، فإنّ خطّ رب خطّ مستقيم.

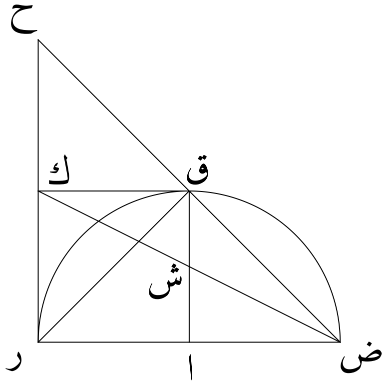


فنجعل مركزي الدائرتين ض ق، ونجيز عليهما خطّاً مستقيماً يصير إلى موضع التماسّ عليه ضب. وليكن خطّ كش موازياً لخطّ قض. فلأنّ خطّ شض مساوٍ لخطّ كق، وخطّ كق مساوٍ لخطّ قب، يكون خطّ شض مساوياً لخطّ قب. ولأنّ جميع خطّ بض مساوٍ لجميع خطّ رض، وخطّ بق من أحدهما مساوٍ لخطّ ضش من الآخر، يبقى خطّ ضق مساوياً لخطّ رش، وهو أيضاً مساوٍ لخطّ كش. فخطّ كش مساوٍ لخطّ شر، وتكون لذلك زاوية شرك مساوية لزاوية ركش. ولأنّ زاوية بقك مساوية لزاوية بضر، وزاوية بضر مساوية لزاوية كشر، تكون زاوية بقك مساوية لزاوية كشر، وتبقى زاويتا

6 خطّ[2]] **F** *mg.*

شرك شكر مساويتين لزاويتي قبك قكب. ولكنّ زاويتي شرك شكر مثلا زاوية شرك، وزاويتا قبك قكب مثلا زاوية قكب، فزاوية قكب مساوية لزاوية شرك. ونجعل زاوية ركق مشتركة، فتكون زاويتا شرك ركق مساويتين لزاويتي ركق بكق. ولكنّ زاويتي شرك ركق مساويتان لزاويتين قائمتين، وذلك أنّهما زاويتان داخلتان فيما بين خطّين متوازيين. فزاويتا ركق بكق مساويتان لزاويتين قائمتين، فخطّ رب خطّ مستقيم. وذلك 69r
ما أردنا أن نبيّن.

ب إذا كان رقض نصف دائرة، وكان خطّا كر كق مماسّين، وكان خطّ قا عموداً، ووصلنا خطّ ضك، فإنّ خطّ قش مساوٍ لخطّ شا.



فنصل خطّ ضق، ونخرجه على استقامة، وليلتق خطّا رح ضقح على نقطة ح. ونصل خطّ رق. فلأنّ زاوية رقض في نصف دائرة، فهي زاوية قائمة، وتبقى زاوية رقح

---

3 مشتركة] **F** *mg.*

5 خطّ] **F** *om.*

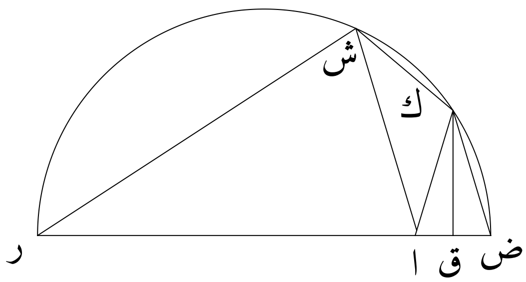
قائمة. وإذا كان مثلّث رحق قائم الزاوية، وقد أخرج فيه عمود† قك فكان مساوياً للخطّ الذي يفصله من القاعدة، وهو كر، وذلك أنّ كلّ واحد منهما مماسّ للدائرة، فإنّ خطّ كر يكون أيضاً مساوياً لخطّ كح، كما بيّنّا في الأشكال التي عملناها في الزوايا القائمة. فلأنّ مثلّث حضر قد أخرج فيه خطّ قا موازياً لقاعدته، وذلك أنّ زاويتي حرا قار الداخلتين مساويتان لزاويتين قائمتين، وقد قسمت القاعدة بنصفين على نقطة ك، وأخرج خطّ كض من نقطة ك فقطع الخطّ الموازي للقاعدة، يكون خطّ قا قد انقسم بنصفين على نقطة ش. وذلك ما أردنا أن نبيّن.

ج إذا كان ركض قطعة من دائرة، وتعلّم عليها نقطة ك، وأخرج منها عمود كق، وكان خطّ قا مساوياً لخطّ ضق، وكانت قوس ضك مساوية لقوس كش، ووصل خطّ رش، فإنّ خطّ رش مساوٍ لخطّ را.

8 وتعلّم] وتُعلّم **F**

10 خطّ] *om.* **F**

1 †عمود] As has been already observed by Foster (Greaves with Foster 1659, 18), Borelli (Ecchelensis and Borelli 1661, 388), Heiberg (1972, II.512, n. 2), and following Borelli, Taha (1998, 267), the line *qāf kāf* need not be perpendicular to the line *rā' ḥā'*, in spite of the figure; nor is this perpendicularity part of the enunciation. I am forced to conclude that the word *'amūd* is corrupt, possibly because a copyist was misled by the figure. At what point in the transmission the corruption occurred, it is impossible to say; therefore, I have not emended it. I have kept the corruption in the English by translating the word as "perpendicular."



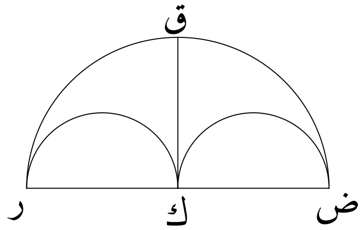
فنصل خطوط ضك كا شا. فلأنّ قوس ضك مثل قوس كش، يكون خطّ ضك مثل خطّ كش. ولانّ خطّ ضق مساوٍ لخطّ قا، وخطّ كق مشترك، وعن جنبتيه زاويتان قائمتان، تكون خطوط ضك كا كش متساوية. والزاوية التي عند نقطة ا كالزاوية التي عند نقطة ض. ولأنّ في الدائرة شكل ذو أربعة أضلاع، وهو رشكض، تكون زاويتا رشك رضك مثل زاويتين قائمتين. وزاوية رضك مساوية لزاوية كاق. فزاويتا رشك كاق مثل زاويتين قائمتين. فزاوية راك مساوية لزاوية رشك. وأيضاً 69v
فإنّ زاوية اشك مساوية لزاوية كاش، لأنّ خطّ كا مساوٍ لخطّ كش. فتبقى زاوية رشا مساوية لزاوية راش، فخطّ رش مساوٍ لخطّ را. وذلك ما أردنا أن نبيّن.

د إذا كان رقض نصف دائرة، وعمل على خطّ رض نصفا دائرتين متماسّتين نصف دائرة رك ونصف دائرة كض، وكان خطّ كق عموداً، فإنّ الشكل الحادث من ذلك الذي يسمّيه أرشميدس أربلوس، وهو السطح الذي تحيط به قوس نصف الدائرة العظمى وقوسا نصفي الدائرتين الصغريين، مساوٍ للدائرة التي يكون قطرها عمود كق.

7 فإنّ] **F** *illeg.*

11 أربلوس] سَارْبَلُوسْ **F**

11 أربلوس] This is the Greek word *arbēlos* (ἄρβηλος, "semicircular knife"), which is found twice in Pappus (Sefrin-Weis 2010, 29, 37).



فلأنّ خطّ كق مناسب لخطّي رك كض فيما بينهما، يكون السطح الكائن من رك في كض مساوياً لمربّع خطّ كق. فنجعل السطح الكائن من رك في كض مع مربّعي خطّي رك كض مشتركاً. فيكون المجتمع من ضرب رك في كض مرّتين مع مربّعي خطّي رك كض مساوياً لمثلي مربّع خطّ كق مع مربّعي خطّي رك كض. ولكنّ المجتمع من ضرب رك في كض مرّتين مع مربّعي خطّي رك كض مساوٍ لمربّع خطّ رض. فمربّع خطّ رض مساوٍ لمثلي مربّع خطّ كق مع مربّعي خطّي رك كض. فالدائرة التي قطرها رض مساوية للدائرتين اللتين قطراهما رك كض مع مثلي الدائرة التي قطرها كق. ولذلك يكون نصف دائرة رقض مثل نصف الدائرتين اللتين قطراهما رك كض مع الدائرة التي قطرها كق. ونسقط المشترك، وهو نصفا دائرتي رك كض، فيبقى الشكل الذي تحيط به قوس رقض وقوسا رك كض، الذي يسمّيه أرشميدس 70r
أربلوس، مساوياً للدائرة التي قطرها كق. وذلك ما أردنا أن نبيّن.

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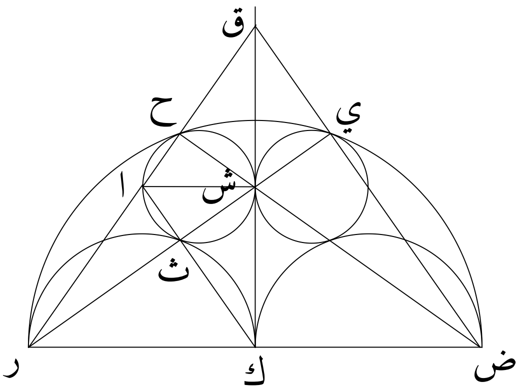
9 ونسقط] منسْقط F *cf.* ونسقط **Hy** (107, l. 20)

11 أربلوس] سارْبلُوسْ F

٥ إذا كان رض نصف دائرة، وتعلّمت على قطره نقطة كيف ما وقعت، وهي ك، وعمل على القطر نصفا دائرتين عليهما رك كض، وكان خطّ كق عموداً على القطر، وعمل عن جنبتيه دائرتان تماسّانه، وتماسّان أنصاف الدوائر، فإنّ الدائرتين متساويتان.



فنخرج قطر إحدى الدائرتين، وهو اش. وليكن موازياً لخطّ رض. ونصل خطّ اح وخطّ ار. فخطّ حر خطّ مستقيم، لأنّا قد بيّنّا في الشكل الأوّل أنّه إذا ماسّ نصف دائرة نصف دائرة أخرى، وكان قطراهما متوازيين، فإنّ الخطّ الذي فيما بين نقطة التماسّ وزاوية نصف الدائرة الصغرى والخطّ الذي فيما بين زاوية نصف الدائرة

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1 إذا] الخامس على ما هو في الكتاب. إذا **P**

1 وتعلّمت] وتُعلّمت **F**

2 نصفا] نصف **P**

2 عموداً] عمود **P**

3 الدوائر] الدوائرة **P**

4 فنخرج] فلنخرج **P**

6 وكان] فكان **P**

الصغرى ونصف الدائرة العظمى متّصلان على استقامة. فليلق خطّ رح خطّ كق على نقطة ق. وأيضاً فإنّا نصل خطّاً فيما بين ح وش، وخطّاً فيما بين ش وض. فيكون أيضاً خطّ حض مستقيماً للّذي تبيّن في الشكل الأوّل. ولأنّه قد وصل خطّ فيما بين ا وث، وخطّ آخر فيما بين ث وك، يكون جميع خطّ اك مستقيماً للّذي بيّنّا في الشكل الأوّل. فإن نحن وصلنا خطّاً فيما بين ش وث، وخطّاً فيما بين ث ور، كان لنا خطّ رش مستقيماً. فلأنّ خطّي قر رض مستقيمان، وقد أخرج من نقطة ق إلى خطّ رض عمود عليه قك، وأخرج من نقطة ض إلى خطّ قر عمود عليه ضح، ووصل خطّ فيما بين ر ش، وأخرج إلى ي، فهو بيّن أنّا إذا أخرجنا إليه عمود ضي، ووصلنا خطّ يق، كان جميع خطّ ضق مستقيماً للّذي بيّنّا في الأشكال التي عملناها في شرح 70v القول في المثلّثات القائمة الزوايا. فلأنّ الزاوية التي عند ث قائمة لأنّها في نصف

---

1 الصغرى ونصف الدائرة] **P** *om.*

4 للّذي] على ما **P**

5 خطّاً] **P** *om.*

5 كان] فإنّ **P**

6 فلأنّ] ولأنّ **P**

7 عليه[1]] **F** *om.*

7 وأخرج من] ومن **P**

7 خطّ[1]] **F** *om.*

7 عمود[2]] *ante* إلى **F**

7–8 ووصل خطّ فيما بين ر ش] ووصل فيما بين ر ش بخطّ مستقيم **P**

9 عملناها] **F** *illeg.*

10–75.1 ث قائمة لأنّها في نصف دائرة، والزاوية التي عند] **P** *om.*

دائرة، والزاوية التي عند ي أيضاً قائمة، تكون المتبادلتان متساويتين. فخطّ اك موازٍ لخطّ قض. ولذلك تكون نسبة خطّ رق إلى خطّ قا كنسبة رض إلى ضك. ولكنّ نسبة رق إلى قا كنسبة رك إلى اش. فالذي يكون من ضرب رك في كض مساوٍ للّذي يكون من ضرب رض في اش. وكذلك أيضاً يتبيّن أنّ الذي يكون من ضرب رك في كض مساوٍ للّذي يكون من ضرب رض في قطر دائرة شي . فالذي يكون من ضرب ضك في كر مساوٍ للّذي يكون من ضرب رض في كلّ واحد من قطري الدائرتين، فقطرا الدائرتين متساويان. ولذلك تكون الدائرتان متساويتين. وذلك ما أردنا أن نبيّن.

و إذا كان رض نصف دائرة، وتعلّمت على قطره نقطة ك فكان رك مرّة ونصفاً مثل كض، وعمل على خطّي رك كض نصفا دائرتين، وعمل دائرة فيما بين نصف الدائرة العظمى ونصفي الدائرتين الصغريين تماسّها، وكان قطر قا موازياً لخطّ رض، وأردنا أن نجد نسبة قطر رض إلى قطر قا.

---

2 تكون] P *om.*

3 رك في كض] رض في اش P

4 رض في اش] رك في كض P

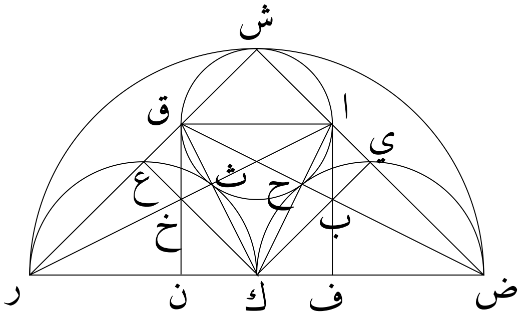
5 شي] شن F

7 فقطرا] فقطر P

9 إذا] الشكل السادس على ما هو في الكتاب. إذا P

10 نصفا] نصف P

11 الصغريين] الصُغراوَين PF



فإنّا نصل خطّين فيما بين ش ونقطتي ق ا، وفيما بين ق، ونقطة ر. فخطّ رش خطّ مستقيم كما بيّنّا في الشكل الأوّل، وبمثل ذلك أيضاً يتبيّن أنّ خطّ شاض مستقيم، وأنّ خطّ قخض مستقيم، وأنّ خطّ اثر مستقيم، وأنّ خطّ قثك مستقيم، وأنّ خطّ احك مستقيم، وخطّا نخع كبي مستقيمان. فلأنّ في مثلّث رقك عمود عليه رث، وعمود آخر عليه كع، يكون الخطّ الذي نصل فيما بين ق خ ونخرج إلى القاعدة، وهو

71r قن، عموداً على القاعدة، لأنّا قد بيّنّا ذلك في التفسير الذي وضعناه في القول في جملة

---

1 فيما بين ش ونقطتي ق ا، وفيما بين ق، ونقطة ر] فيما بين نقطتي ر ش وفيما بين نقطة ق ونقطة ر **P**

2 خطّ[1]] *om.* **F**

2 الشكل] *om.* **P**

2 أيضاً] *om.* **P**

3 قخض مستقيم، وأنّ خطّ اثر مستقيم، وأنّ خطّ] *om.* **P**

3–4 وأنّ خطّ احك] *illeg.* **P**

4 نخع كبي] كي كع **F**

5 يكون] ويكون **P**

5 نصل] نَصِيلُ **F**

5–6 القاعدة، وهو قن، عموداً] *illeg.* **F**

المثلّثات. ولذلك أيضاً يكون خطّ اف عموداً في مثلّث كاض. ولأنّ الزاوية التي عند نقطة ع قائمة، والزاوية التي عند نقطة ش أيضاً قائمة، تكونان متساويتين، ويكون خطّ عك موازياً لخطّ شض. فنسبة رخ إلى خا كنسبة رك إلى كض. ولكنّ نسبة رخ إلى خا كنسبة رن إلى نف، وذلك أنّ قن يوازي اف لأنّهما عمودان. فنسبة رك إلى كض كنسبة رن إلى نف. وخطّ رك مرّة ونصف مثل كض. فخطّ رن أيضاً مرّة ونصف مثل نف. فلأنّ الزاوية التي عند ش قائمة مساوية للزاوية التي عند ي، يكون خطّ كي موازياً لخطّ رش. فنسبة قب إلى بض كنسبة رك إلى كض. ولكنّ نسبة قب إلى بض كنسبة نف إلى فض. فنسبة رك إلى كض كنسبة نف إلى فض. وخطّ رك مرّة ونصف مثل كض. فخطّ نف مرّة ونصف مثل فض. وقد كان تبيّن أنّ رن مرّة ونصف مثل نف. فخطوط رن نف فض الثلاثة متناسبة، وبالمقدار الذي يكون به فض أربعة، يكون به نف ستّة، ويكون به رن تسعة، وجميع رض تسعة عشر. ولأنّ فن مثل قا، وخطّ فن ستّة، يكون قا ستّة. فقد وجدت نسبة رض إلى قا، التي هي كنسبة التسعة عشر إلى الستّة. وكذلك أيضاً تتبيّن هذه النسبة إذا لم يكن

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2 أيضاً] **F** *corr. mg.*

2 خطّ] **P** *sup.*

3 إلى[2]] **P** *sup.*

4 رن] رك **P**

5 وخطّ] وقد كان **P**

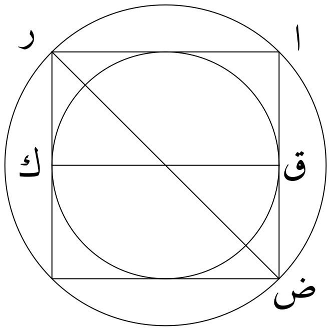
6 فلأنّ] ولأنّ **P**

8 رك] نك **P**

9 مثل[2]] مثل خطّ **P**

رك مرّة ونصفاً مثل كض، لكنّ مثله مرّة وثلثاً أو مرّة وربعاً أو مرّة وخمساً أو نسبة أخرى أيّ نسبة كانت. وذلك ما أردنا أن نبيّن.

ز إذا كان في دائرة رض مربّع عليه أيضاً رض، وعملت في المربّع دائرة كق، فإنّ دائرة رض مثلا دائرة كق.



71v فنخرج قطر المربّع، وهو رض، فهو بيّن أنّ هذا الخطّ هو أيضاً قطر لدائرة رض. ونخرج أيضاً قطر دائرة كق، وهو كق. فلأنّ مربّع خطّ رض مثلا مربّع خطّ را، وخطّ كق مساوٍ لخطّ را، يكون مربّع خطّ رض مثلي مربّع خطّ كق. فدائرة رض مثلا دائرة كق. وذلك ما أردنا أن نبيّن.

---

1 ونصفاً] ونصف P

1 وثلثاً] وثلث P

1 وربعاً] وربع P

1 وخمساً] وخمس P

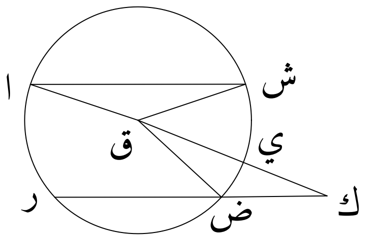
2 وذلك ما أردنا أن نبيّن] *om.* P

3 إذا] الشكل السابع على ما هو في الكتاب. إذا P

3 أيضاً] *om.* P

5 لدائرة] الدائرة P

ح إذا أخرج في دائرة مّا خطّ رض كيف ما خرج غير القطر، وأنفذ على استقامة، وجعل خطّ ضك مساوياً لنصف قطر الدائرة، ووصل خطّ فيما بين ك وبين مركز ق، وأخرج إلى ا، فإنّ قوس را ثلاثة أمثال قوس يض.



فنخرج خطّ اش موازياً لخطّ رض، ونصل خطّي قش قض. فلأنّ خطّي قش قا متساويان، تكون زاوية كقش مثلي زاوية قاش. ولأنّ خطّ ضك مساوٍ لخطّ ضق، تكون زاوية ضكق مساوية لزاوية ضقك. وزاوية كاش أيضاً مساوية لزاوية ضكق، لأنّهما متبادلتان. فزاوية كاش مساوية لزاوية ضقك. فتكون زاوية كقش مثلي زاوية ضقك. فجميع زاوية ضقش ثلاثة أمثال زاوية كقض. ولذلك تكون قوس ضش

---

1 إذا] قال صاحب الكتّاب. إذا **P**

1 غير القطر] *om.* **P**

2 خطّ[2]] *om.* **P**

5 تكون زاوية] *rep.* **P**

5 مثلي] ضعف **P**

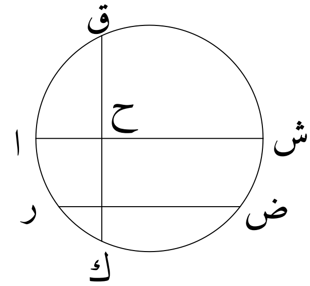
6 أيضاً] *om.* **P**

7 فتكون] وتكون **P**

7 مثلي] ضعف **P**

ثلاثة أمثال قوس ضي. وقوس ضش مساوية لقوس ار، لأنّ خطّ اش موازٍ لخطّ رض. فقوس ار ثلاثة أمثال قوس ضي. وذلك ما أردنا أن نبيّن.

ط إذا تقاطع في دائرة مّا خطّا رض قك على غير المركز، وتقاطعا على زوايا قائمة، فإنّ قوسي رك قض مساويتان لقوسي رق كض.



فنخرج قطر اش موازياً لخطّ رض. فهو بيّن أنّ قطر اش يقطع قك بنصفين، 72r فليقطعه على نقطة ح. وتكون قوس اق مساوية لقوسي ار رك. فلأنّ قوس اكش نصف دائرة، وقوس اق مساوية لقوسي ار رك، تكون قوس قش مع قوسي ار رك مساوية لنصف دائرة اكش. وقوس ار مساوية لقوس شض. فقوس قض مع قوس رك مساوية لنصف دائرة اكش. ولكنّ قوس شض مع قوس رك مساوية لقوس اق. فقوس قض مع قوس رك مساوية لقوس ضك مع قوس رق. وذلك ما أردنا أن نبيّن.

ي إذا كانت دائرة عليها ركق، وكان خطّ رض مماسّاً لها، وخطّ ضك قاطعاً لها، وكان خطّ قض أيضاً مماسّاً لها، وكان خطّ قا موازياً لخطّ ضك، ووصل خطّ ار،

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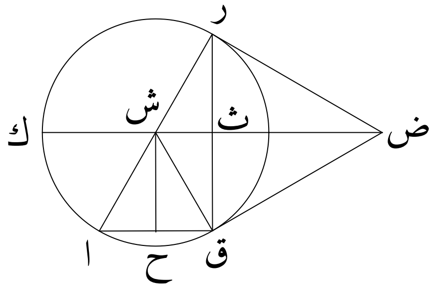
4 لقوسي] **F** *mg.*

وأخرج من نقطة شَ التي يتقاطع عليها خطّا ضك ار عمود على اق عليه شح، فإنّ خطّ اق قد قسم بنصفين على ح.



فنصل خطّ شق. فلأنّ خطّ رض مماسّ، وخطّ رق قاطع للدائرة، تكون زاوية ضرق مساوية لزاوية راق. ولكنّ زاوية راق مساوية لزاوية رشث، فزاوية ضرق مساوية لزاوية رشث. فالذي يكون من ضرب ضش في ضث مساوٍ لمربّع خطّ رض. ومربّع خطّ رض مساوٍ لمربّع خطّ ضق. فالذي يكون من ضرب ضش في ضث مساوٍ لمربّع خطّ ضق. فزاوية ضشق مساوية لزاوية ثقض. ولأنّ زاوية ثقض مساوية لزاوية ضشق، وزاوية ثقض مساوية لزاوية ثرض، وزاوية ضرق مساوية لزاوية رشض، تكون زاوية رشض مساوية لزاوية قشض. وزاوية رشض مساوية لزاوية شاق، وزاوية ضشق مساوية لزاوية اقش. فتكون زاوية شاق مساوية لزاوية اقش. ولكنّ زاوية 72v احش القائمة مساوية لزاوية قحش القائمة. وضلع شح مشترك. فخطّ اح مساوٍ لخطّ حق. وذلك ما أردنا أن نبيّن.

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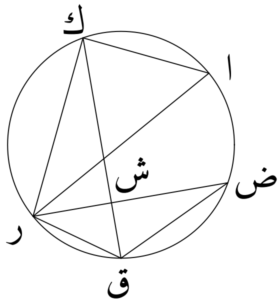
11 القائمة[1]] قائمة **F**

12 وذلك ما أردنا أن نبيّن] **F** *part. illeg.*

يا إذا تقاطع في دائرة خطّا رض كق على زوايا قائمة على غير المركز، فإنّ مربّعات خطوط رش شض كش شق، إذا جمعت، مساوية لمربّع القطر.



فنخرج قطر را، ونصل خطوط رك رق قض كا. فلأنّ زاوية قشر قائمة، تكون مساوية لزاوية ركا. وزاوية رقك مساوية لزاوية كار. وتبقى زاوية ارك مساوية لزاوية ضرق. فقوس ضق مساوية لقوس كا. ويكون لذلك خطّ قض مساوياً لضلع كا. ولأنّ مربّعي خطّي كش رش، إذا جمعا، مساويان لمربّع خطّ كر، ومربّعا خطّي ضش شق مساويان لمربّع خطّ قض، وخطّ قض مساوٍ لخطّ كا، فمربّعات خطوط كش رش ضش شق مساوية لمربّعي رك كا. ومربّعا خطّي كر كا مساويان لمربّع خطّ را الذي هو القطر. فمربّعات خطوط كش رش ضش شق مساوية لمربّع القطر. وذلك ما أردنا أن نبيّن.

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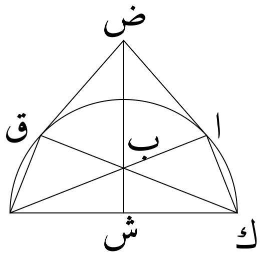
1 دائرة] الدائرة **F** *cf.* دائرة **Hy** (123, l. 3)

8 كا مساويان] **F** *part. illeg.*

يب إذا كان نصف دائرة على قطره رك ، وأخرج من نقطة ض خطّان مماسّان للدائرة عليهما ضق ضا، ووصل خطّا قك ار فتقاطعا على ب، وأخرج خطّ ضبش، فإنّ ضبش عمود على رك .



فنصل خطّي قر اك. فلأنّ زاوية كقر قائمة، تكون زاويتا قرك قكر مساويتين لزاوية قائمة. ولكنّ زاوية كار قائمة. فزاويتا قرك قكر مساويتان لزاوية كار. وزاوية اكق مشتركة. فزوايا قرك قكر اكق مساوية لزاويتي كار اكق. ولكنّ زاوية قبا الخارجة 73r

1 إذا] الشكل الثاني عشر على ما هو في الكتاب. إذا **P**
1 قطره] قطر **P**
1 رك] كب **P**
2 خطّا] خطّ **P**
3 رك] كر **P**
4 فنصل] فلنصل **P**
6 قكر اكق] اكر **P**
6 الخارجة] *om.* **P**

مساوية لزاويتي كار اكق الداخلتين. فزاوية قبا مساوية لزوايا قكر قرك قكا. ولأنّ خطّ قض مماسّ للدائرة، وخطّ قك قاطع للدائرة، تكون زاوية كقض مساوية لزاوية قرك. وبمثل ذلك أيضاً يتبيّن أنّ زاوية ضار مساوية لزاوية اكر. فزاويتا كقض ضار، إذا جمعتا، مساويتان لزاويتي قرك ركا اللتين هما مثل زاوية قبا. وقد تبيّن في قولنا في الأشكال ذوات الأربعة الأضلاع أنّه إذا كان خطّان مستقيمان متساويان قد التقيا، وأخرج فيما بينهما خطّان فتقاطعا، وكانت الزاوية التي يحيط بها الخطّان المتقاطعان مساوية للزاويتين اللتين يحيط بهما هذان الخطّان مع الخطّين الأوّلين، كما أنّ زاوية قبا هاهنا مساوية لزاويتي ضقب ضاب، كان خطّ قض مساوياً لخطّ بض. ولذلك تكون زاوية قبض مساوية لزاوية ضقب. وقد كان تبيّن أنّ زاوية ضقب مساوية لزاوية قرك، فزاوية قبض مساوية لزاوية قرك. فزاوية قرك من شكل رقبش ذي الأربعة الأضلاع مع زاوية قبش منه مثل زاويتين قائمتين. فتبقى زاويتا رشب رقب مساويتين

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1 لزاويتي] *mg.* **P**

1 اكق] اكب **P**

1 قكر قرك] *tr.* **P**

3 أيضاً يتبيّن] *tr.* **P**

4 لزاويتي] *om.* **P**

5 الأربعة] *om.* **P**

7 بهما] بها **F** *om.* **P**

7 هذان] بهذان **P**

8 ولذلك] قد بيّنّا في المقدّمة التي قدّمنا لهذا الشكل. لذلك **P**

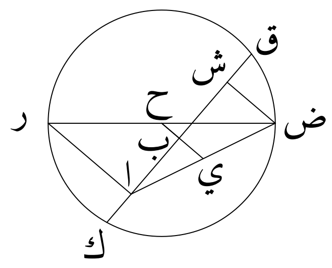
10 قرك، فزاوية] *illeg.* **F**

10 فزاوية قرك] ونجعل قبش مشتركاً. فزاوية قرك **P**

11 زاويتا رشب] *illeg.* **P**

لزاويتين قائمتين. وزاوية رقك قائمة. فتبقى زاوية رشض قائمة. وذلك ما أردنا أن نبيّن.

يج إذا كانت دائرة، وأخرج فيها خطّان على أحدهما رض وعلى الآخر كق، وكان رض قطر الدائرة، وكان كق خطّاً لا يمرّ بالمركز، وأخرج من نقطتي ر ض عمودا ضش را، فأقول إنّهما يفصلان من خطّ كق قطعتين متساويتين، وهما كا شق.



فنصل خطّ اض، ونجعل المركز نقطة ح، ونخرج منه عموداً على كق عليه حب. ولنخرجه إلى نقطة ي من خطّ اض. فلأنّ خطّ حب يمرّ بالمركز، ويقطع كق الذي لا يمرّ بالمركز على زوايا قائمة، فإنّه يقسمه بنصفين. فخطّ كب مساوٍ لخطّ بق. ولأنّ 73v زاوية حبك قائمة، وزاوية راق قائمة، يكون خطّ حي موازياً لخطّ را. ولكنّ خطّ رح مساوٍ لخطّ حض. فيكون اي مثل يض، ويكون خطّ اب مساوياً لخطّ بش. وقد كان تبيّن أنّ جميع كب مساوٍ لجميع بق. فيبقى خطّ كا مساوياً لخطّ شق. وذلك ما أردنا أن نبيّن.

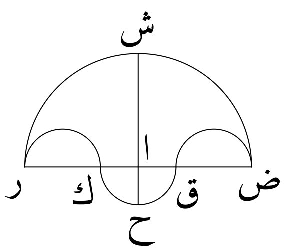
---

1 وزاوية رقك قائمة. فتبقى زاوية] *illeg.* **P**

1 فتبقى] تبقى **F**

7 ولنخرجه] ولنخرج **F**

يد إذا كان رض نصف دائرة، وكان خطّا رك ضق متساويين، وعملت على خطوط رك كق قض أنصاف دوائر، وكان النصفان اللذان على خطّي رك ضق المتساويين معمولين في داخل نصف دائرة رض، والنصف الذي على خطّ كق معمولاً من خارجه، وكان مركز نصف دائرة رض والنصف الذي على خطّ كق معمولاً من خارجه نقطة آ، وكان اش عموداً على رض، وأخرج إلى نقطة ح، فإنّ الدائرة التي يكون قطرها مثل شح مساوية للسطح الذي تحيط به أنصاف الدوائر الصغار ونصف الدائرة العظمى، وهذا الشكل هو الذي يسمّيه أرشميدس سالينون لأنّه يشبه شكل السالن.



فلإنّ خطّ قك قد قسم بنصفين على آ، وزيد عليه كر، يكون مربّعا خطّي قر رك مثلي مربّعي خطّي قا ار. ولكنّ خطّ شح مساوٍ لخطّ قر. فمربّعا خطّي شح رك مثلا مربّعي خطّي قا ار. ولأنّ خطّ رض مثلا خطّ را، وخطّ كق مثلا خطّ قا، يكون مربّعا خطّي رض قك أربعة أمثال مربّعي خطّي را قا. ومثلا هذين المربّعين هما مربّعا

7 سالينون] سْلسُون **F**

7 السالن] *sic* **F**

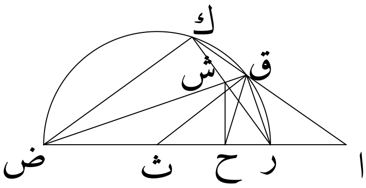
9 مثلي] مثلا **F**

7 سالينون] The word *sālīnūn* (σάλινον) is accompanied in both occurrences in this proposition by the explanation *kadhā wajadtuhu hunā fa-l-yuḥaqqaq*, in red ink, by Ibn Abī Jarāda; presumably because he expected the word to be spelled differently. I take the nonsensical phrase "because it resembles the shape of the *s'ln*" to be an interpolation in the source text that is due to a scribe trying to explain the word *sālīnūn* (σάλινον), no doubt because it seemed strange to him. Incidentally, this strengthens Heath's (1897, xxxiii–iv) argument that this word is a foreign borrowing, in this case from Latin *salīnum* ("salt-cellar").

خطّي شح رك. فمربّعا خطّي رض كق مثلا مربّعي خطّي شح رك.

ولذلك تكون الدائرتان اللتان قطراهما خطّا رض كق مثلي الدائرتين اللتين قطراهما خطّا شح رك. فنصفا الدائرتين اللذان عليهما رض كق مساويان للدائرتين اللتين قطراهما خطّا شح رك. ولكنّ الدائرة التي قطرها رك مساوية لنصفي دائرتي قض رك. فإذا 75r ألقينا هذين نصفي دائرتين المشتركين، كان الشكل الذي تحيط به أربعة أنصاف دوائر رض رك كق قض، الذي يسمّيه أرشميدس سالينون مساوياً للدائرة التي قطرها شح. وذلك ما أردنا أن نبيّن.

يه إذا كان رض نصف دائرة، وكان خطّ رك وتر المخمّس منه، وقسمت قوس رك بنصفين على نقطة ق، ووصل خطّ كق، وأخرج فوقع على نقطة ا، ووصل خطّ قض فقطع خطّ كر على ش، وأخرج من نقطة ش عمود على رض عليه شح، فأقول إنّ خطّ اح مساوٍ لنصف قطر الدائرة.



فنصل خطّ كض، ونجعل المركز ث. ونصل خطوط قث قح قر. فلأنّ زاوية رضك هي زاوية قاعدتها ضلع المخمّس، تكون خمسي قائمة. وتكون كلّ واحدة من زاويتي كضق قضر خمس زاوية قائمة. وزاوية قثر مثلا زاوية قضر، فهي خمسا قائمة. فلأنّ زاوية قضر مساوية لزاوية كضق، وزاوية ركض مساوية لزاوية شحض لأنّهما قائمتان،

5 نصفي] النصفي F

6 سالينون] سْالسُون F

وضلع شض مشترك، يكون ضلع كض مساوياً لضلع ضح. وخطّ ضق مشترك أيضاً. وزاويتا كضق قضر متساويتان. فزاوية ضكق مساوية لزاوية ضحق. وكلّ واحدة منهما مثل ستّة أخماس قائمة. ولكنّ زاوية ضكق مساوية للزاوية الخارجة من شكل ضكقر ذي الأربعة الأضلاع الذي في الدائرة، وهي زاوية قرا. فتبقى زاوية قرض مساوية لزاوية لحر. ويكون لذلك خطّ قر مساوياً لخطّ لحق. ولأنّ زاوية ث خمسا قائمة، وزاوية لحقض ستّة أخماس زاوية قائمة، تبقى زاوية ثقح خمسي قائمة. ولأنّ زاوية رقا هي زاوية

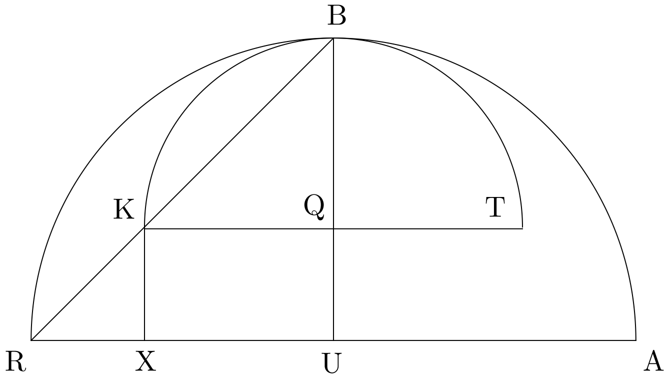
75v خارجة من شكل ذي أربعة أضلاع تحيط به الدائرة، وهي المقابلة لزاوية رضك، تكون خمسي قائمة. فتكون زاوية ثقح مساوية لزاوية اقر. وزاوية ارق مساوية لزاوية ثحق. وضلع قح مساوٍ لضلع قر. فضلع ار مساوٍ لضلع حث. وخطّ رح مشترك. فخطّ اح مساوٍ لخطّ رث الذي هو نصف القطر. وذلك ما أردنا أن نبيّن.

Lemmas Attributed to Archimedes

The Translation of Thābit ibn Qurra al-Ḥarrānī

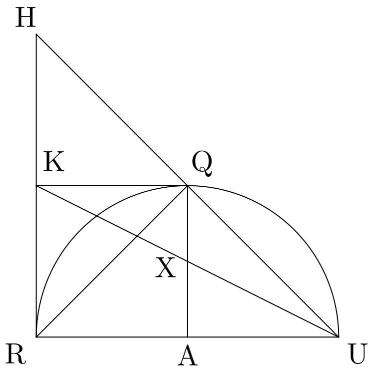
In the name of God, the Most Gracious, the Most Merciful. My Lord, make it easy.

1. If there are two semicircles one of which is tangent to the other, whose diameters are parallel such as the diameters RUA and KQT, between the points R and K the line RK is joined and between the points K and B the line KB [is joined], then the line RB is a straight line.



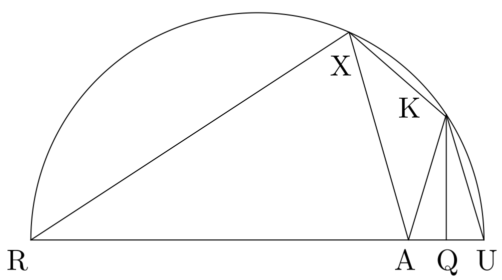
We make the centers of the circles U and Q, and we pass a straight line, on which are UB, through them that reaches the place of tangency. And let the line KX be parallel to the line QU. So since the line XU is equal to the line KQ, and the line KQ is equal to the line QB, the line XU is equal to the line QB. Since the whole line BU is equal to the whole line RU, and the line BQ from one of them is equal to the line UX from the other, the line UQ remains equal to the line RX, and it is also equal to the line KX. So the line KX is equal to the line XR, and therefore the angle XRK is equal to the angle RKX. Since the angle BQK is equal to the angle BUR, and the angle BUR is equal to the angle KXR, the angle BQK is equal to the angle KXR, and the angles XRK and XKR remain equal to the angles QBK and QKB. But the angles XRK and XKR are twice the angle XRK, and the angles QBK and QKB are twice the angle QKB, so the angle QKB is equal to the angle XRK. We make the angle RKQ common, and then the angles XRK and RKQ are equal to the angles RKQ and BKQ. But the angles XRK and RKQ are equal to two right angles—for they are two interior angles between two parallel lines. So the angles RKQ and BKQ are equal to two right angles, and the line RB is a straight line. And that is what we wanted to prove.

2. If RQU is a semicircle, the lines KR and KQ are tangent, the line QA is perpendicular [to RAU], and we join the line UK, then the line QX is equal to the line XA.



We join the line UQ, we extend it straight, and let the lines RKH and UQH meet at the point H. We join the line RQ. Since the angle RQU is in a semicircle, it is a right angle, and the angle RQH remains [equal to a] right [angle]. Since the triangle RQH is right-angled, and the perpendicular† QK was drawn in it so that it is equal to the line that it cuts off from the base, which is KR—for each one of them is tangent to the circle—the line KR is also equal to the line KH, as we proved in the propositions that we constructed about right angles. Since the line QA was drawn parallel to the base in the triangle HUR—for the interior angles HRA and QAR are equal to two right angles—and since the base was bisected at the point K, the line KU was drawn from the point K so that it intersects the line that is parallel to the base, the line QA has been bisected at the point X. And that is what we wanted to prove.

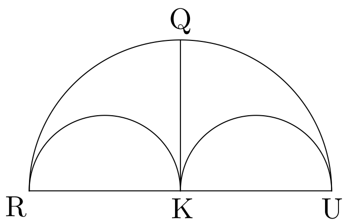
3. If RKU is a segment of a circle, the point K is marked on it, the perpendicular KQ is drawn from it, the line QA is equal to the line UQ, the arc UK is equal to the arc KX, and the line RX is joined, then the line RX is equal to the line RA.



† See note on page 70 above.

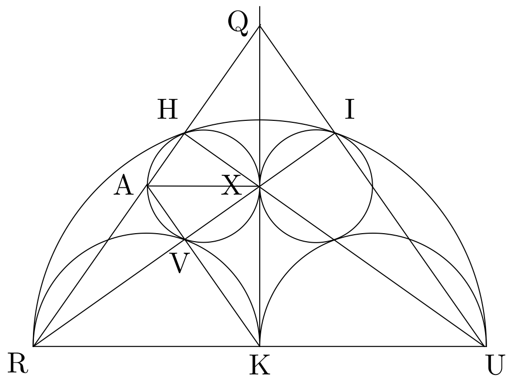
We join the lines UK, KA, and XA. Since the arc UK is equal to the arc KX, the line UK is equal to the line KX. Since the line UQ is equal to the line QA, the line KQ is common, and there are two right angles at its two sides, the lines UK, KA, and KX are equal. And the angle that is at the point A is equal to the angle that is at the point U. And since there is a quadrilateral, which is $RXKU$, in the circle, the angles RXK and RUK are equal to two right angles. And the angle RUK is equal to the angle KAQ. So the angles RXK and KAQ are equal to two right angles. Therefore, the angle RAK is equal to the angle RXK. Also, the angle AXK is equal to the angle KAX, since the line KA is equal to the line KX. So the angle RXA remains equal to the angle RAX; therefore, the line RX is equal to the line RA. And that is what we wanted to prove.

4. If RQU is a semicircle, two mutually tangent semicircles—semicircle RK and semicircle KU—are constructed on the line RU, and the line KQ is perpendicular, then the new figure generated from that—which Archimedes names the *arbilūs* (ἄρβηλος), and which is the surface the arc of the greater semicircle and the arcs of the two smaller semicircles enclose—is equal to the circle whose diameter is the perpendicular KQ.



Since the line KQ is in continuous proportion with the lines RK and KU, the surface that comes into being from RK by KU is equal to the square of the line KQ. So we make the surface that comes into being from RK by KU with the squares of the lines RK and KU common. The result of the multiplication of RK and KU [taken] twice with the squares of the lines RK and KU is equal to twice the square of the line KQ with the squares of the lines RK and KU. But the result of the multiplication of RK and KU [taken] twice with the squares of the lines RK and KU is equal to the square of the line RU. So the square of the line RU is equal to twice the square of the line KQ with the squares of the lines RK and KU. So then the circle whose diameter is RU is equal to the circles whose diameters are RK and KU with twice the circle whose diameter is KQ. Therefore, the semicircle RQU is equal to half of the circles whose diameters are RK and KU with the circle whose diameter is KQ. We subtract the common, which is the semicircles RK and KU, and the figure the arc RQU and the arcs RK and KU enclose, which Archimedes names the *arbilūs* (ἄρβηλος), remains equal to the circle whose diameter is KQ. And that is what we wanted to prove.

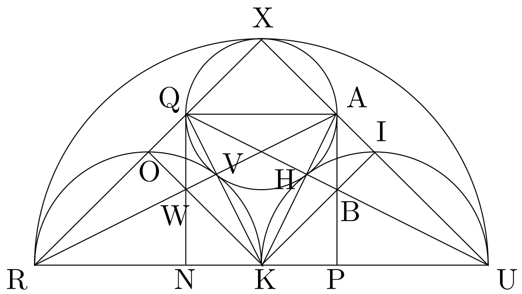
5. If RU is a semicircle, an arbitrary point, which is K, is marked on its diameter, semicircles on which are RK and KU are constructed on the diameter, the line KQ is perpendicular to the diameter, two circles that are tangent to it are constructed on both sides of it, and they are tangent to the semicircles, then the two circles are equal.



We draw the diameter of one of the circles, which is AX. Let it be parallel to the line RU. We join the line AH and the line AR. So the line HR is a straight line, since we proved in the first proposition that if a semicircle is tangent to another semicircle, and their diameters are parallel, then the line between the point of tangency and the angle of the smaller semicircle and the line between the angle of the smaller semicircle and the greater semicircle are adjacent in straight extension. Let the line RH meet the line KQ at the point Q. We also join a line between H and X, and a line between X and U. So the line HU is also straight because of what was proved in the first proposition. And since a line was joined between A and V, and another line between V and K [was joined], the whole line AK is straight because of what we proved in the first proposition. So if we join a line between X and V, and a line between V and R, then we have the straight line RX. Since the lines QR and RU are straight, a perpendicular on which are QK to the line RU was drawn from the point Q, a perpendicular, on which are UH, to the line QR was drawn from the point U, a line was joined between R and X, and it was extended to I, it is evident that if we draw the perpendicular UI to it, and join the line IQ, then the whole line UQ is straight because of what we proved in the propositions we constructed in the commentary [*sharḥ*] to the treatise [*qawl*] about right-angled triangles. Since the angle at V is right—because it is in a semicircle—and the angle at I is right as well, the alternating angles are equal. So the line AK is parallel to the line QU. Therefore, the ratio of the line RQ to the line QA is equal to the ratio of RU to

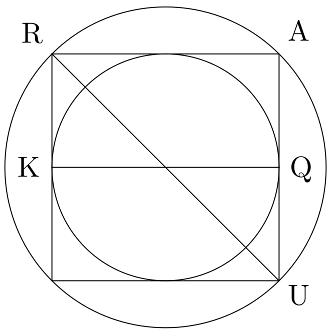
UK. But the ratio of *RQ* to *QA* is equal to the ratio of *RK* to *AX*. So what ensues from the product of *RK* by *KU* is equal to what ensues from the product of *RU* by *AX*. Similarly, it can also be proved that what ensues from the product of *RK* by *KU* is equal to what ensues from the product of *RU* by the diameter of the circle *XI*. So what ensues from the product of *UK* by *KR* is equal to what ensues from the product of *RU* by either one of the diameters of the circles, and the diameters of the two circles are equal. Therefore, the circles are equal. And that is what we wanted to prove.

6. If *RU* is a semicircle, the point *K* is marked on its diameter so that *RK* is one and a half times *KU*, semicircles are constructed on the lines *RK* and *KU*, a circle is constructed between the greater semicircle and the two smaller semicircles that is tangent to them, and the diameter *QA* is parallel to the line *RU*, then we want to find the ratio of the diameter *RU* to the diameter *QA*.



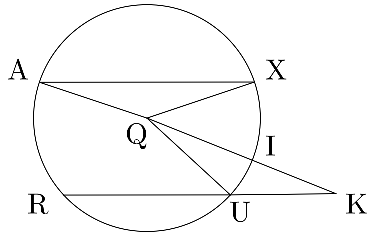
We join lines between *X* and the points *Q* and *A*, and between *Q* and the point *R*. So the line *RX* is a straight line as we proved in the first proposition, and similarly it can be proved that the line *XAU* is straight, the line *QHU* is straight, the line *AVR* is straight, the line *QVK* is straight, the line *AHK* is straight, and the lines *KWO* and *KBI* are straight. Since in the triangle *RQK* there is a perpendicular, on which are *RV*, and another perpendicular, on which are *KO*, the line we join between *Q* and *W* and extend to the base, which is *QN*, is perpendicular to the base, since we proved that in the commentary [*tafsīr*] that we placed in the treatise [*qawl*] about all triangles. Therefore, the line *AP* becomes a perpendicular in the triangle *KAU* as well. Since the angle at the point *O* is right, and the angle at the point *X* is right as well, they are equal, and the line *OK* is parallel to the line *XU*. So the ratio of *RW* to *WA* is equal to the ratio of *RK* to *KU*. But the ratio of *RW* to *WA* is equal to the ratio of *RN* to *NP*—for *QN* is parallel to *AP* since they are perpendiculars. So the ratio of *RK* to *KU* is equal to the ratio of *RN* to *NP*. And the line *RK* is one and a half times *KU*. So the line *RN* is one and a half times *NP* as well. Since the angle at *X* is right and equal to the angle at *I*, the line *KI* is parallel to the line *RX*. So the ratio of *QB* to *BU* is equal to the ratio of *RK* to *KU*. But the ratio of *QB* to BU is equal to the ratio of NP to PU. So the ratio of RK to KU is equal to the ratio of NP to PU. And the line RK is one and a half times KU. So the line NP is one and a half times PU. And it was proved that RN is one and a half times NP. So the three lines RN, NP, and PU are in continuous proportion, and by the amount that makes PU four, NP is six, RN is nine, and the whole of RU is nineteen. Since PN is equal to QA, and the line PN is six, QA is six. So the ratio of RU to QA, which is equal to the ratio of nineteen to six, has been found. And similarly, if RK is not one and a half times KU, but rather one and a third times, one and a fourth times, one and a fifth times or any other ratio, then this ratio can be found as well. And that is what we wanted to prove.

7. If in the circle RU there is also a square on which are RU, and the circle KQ is constructed in the square, then the circle RU is twice the circle KQ.



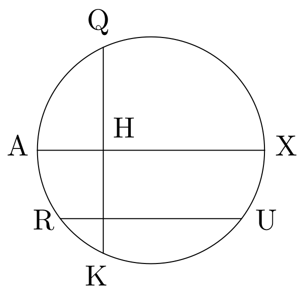
We draw the diagonal of the square, which is RU, and it is evident that this line is also a diameter for the circle RU. We also draw the diameter of the circle KQ, which is KQ. Since the square of the line RU is twice the square of the line RA, and the line KQ is equal to the line RA, the square of the line RU is twice the square of the line KQ. So the circle RU is twice the circle KQ. And that is what we wanted to prove.

8. If in any circle an arbitrary line RU other than the diameter is drawn, it is prolonged straight, the line UK is made equal to half of the diameter of the circle, a line is joined between K and the center Q, and it is extended toward A, then the arc RA is three times the arc IU.



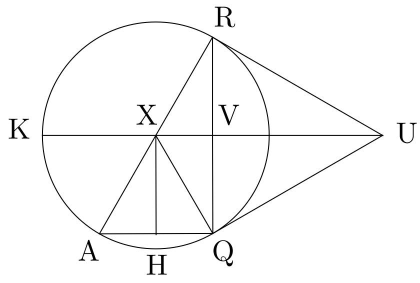
We draw the line AX parallel to the line RU, and we join the lines QX and QU. Since the lines QX and QA are equal, the angle KQX is twice the angle QAX. And since the line UK is equal to the line UQ, the angle UKQ is equal to the angle UQK. And the angle KAX is equal to the angle UKQ as well, since they are alternating. So the angle KAX is equal to the angle UQK. Then, the angle KQX is twice the angle UQK. So the whole angle UQX is three times the angle KQU. Therefore, the arc UX is three times the arc UI. And the arc UX is equal to the arc AR, since the line AX is parallel to the line RU. So the arc AR is three times the arc UI. And that is what we wanted to prove.

9. If in any circle the lines RU and QK intersect anywhere other than the center, and they intersect at right angles, then the arcs RK and QU are equal to the arcs RQ and KU.



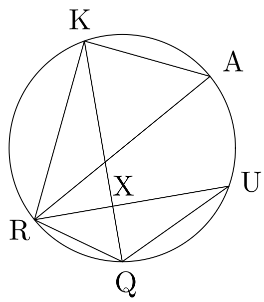
We draw the diameter AX parallel to the line RU. Then it is evident that the diameter AX bisects QK, so let it bisect it at the point H. And the arc AQ is equal to the arcs AR and RK. Since the arc AKX is a semicircle, and the arc AQ is equal to the two arcs AR and RK, the arc QX with the arcs AR and RK is equal to the semicircle AKX. And the arc AR is equal to the arc XU. So the arc QU with the arc RK is equal to the semicircle AKX. But the arc XU with the arc RK is equal to the arc AQ. So the arc QU with the arc RK is equal to the arc UK with the arc RQ. And that is what we wanted to prove.

10. If there is a circle on which are RKQ, the line RU is a tangent of it, the line UK is a secant of it, the line QU is also a tangent of it, the line QA is parallel to the line UK, the line AR is joined, and a perpendicular, on which are XH, to AQ is drawn from the point X, where the lines UK and AR intersect, then the line AQ is bisected at H.



We join the line XQ. Since the line RU is a tangent, and the line RQ is a secant of the circle, the angle URQ is equal to the angle RAQ. But the angle RAQ is equal to the angle RXV, so the angle URQ is equal to the angle RXV. So what ensues from the product of UX by UV is equal to the square of the line RU. And the square of the line RU is equal to the square of the line UQ. So what ensues from the product of UX by UV is equal to the square of the line UQ. And so the angle UXQ is equal to the angle VQU. Since the angle VQU is equal to the angle UXQ, the angle VQU is equal to the angle VRU, and the angle URQ is equal to the angle RXU, the angle RXU is equal to the angle QXU. The angle RXU is equal to the angle XAQ, and the angle UXQ is equal to the angle AQX. So the angle XAQ is equal to the angle AQX. But the right angle AHX is equal to the right angle QHX. And the side XH is common. So the line AH is equal to the line HQ. And that is what we wanted to prove.

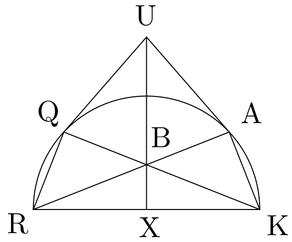
11. If in a circle the lines RU and KQ intersect at right angles and at [a point] other than the center, then the squares of the lines RX, XU, KX and XQ, when added, are equal to the square of the diameter.



We draw the diameter RA, and we join the lines RK, RQ, QU and KA. Since the angle QXR is right, it is equal to the angle RKA. And the angle RQK is equal to the angle KAR. So the angle ARK remains equal to the angle URQ. And so the arc UQ is equal to the arc KA. Therefore, the line QU is equal to the side KA. And since the squares of the lines KX and RX, when added, are equal to the square of the line KR, the squares of the lines UX and XQ are equal to the square of the line QU, and the line QU is equal to the line KA, the squares of the lines KX, RX, UX

and *XQ* are equal to the squares of *RK* and *KA*. And the squares of the lines *KR* and *KA* are equal to the square of the line *RA*, which is the diameter. So the squares of the lines *KX*, *RX*, *UX* and *XQ* are equal to the square of the diameter. And that is what we wanted to prove.

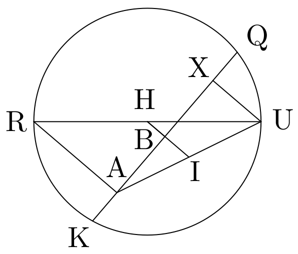
12. If there is a semicircle on its diameter *RK*, tangent lines on which are *UQ* and *UA* are drawn to the circle from the point *U*, the lines *QK* and *AR* are joined so that they intersect at *B*, and the line *UBX* is drawn, then *UBX* is perpendicular to *RK*.



We join the lines *QR* and *AK*. Since the angle *KQR* is right, the angles *QRK* and *QKR* are equal to a right angle. But the angle *KAR* is right. So the angles *QRK* and *QKR* are equal to the angle *KAR*. And the angle *AKQ* is common. So the angles *QRK*, *QKR* and *AKQ* are equal to the angles *KAR* and *AKQ*. But the exterior angle *QBA* is equal to the interior angles *KAR* and *AKQ*. So the angle *QBA* is equal to the angles *QKR*, *QRK* and *QKA*. And since the line *QU* is tangent to the circle, and the line *QK* is a secant of the circle, the angle *KQU* is equal to the angle *QRK*. And similarly it can be shown that the angle *UAR* is equal to the angle *AKR*. So the angles *KQU* and *UAR*, when added, are equal to the angles *QRK* and *RKA*, which are equal to the angle *QBA*. And it was proved in our treatise [*qawl*] about quadrilaterals that if there are two equal straight lines that meet, two lines are drawn between them so that they intersect, and the angle that the two intersecting lines enclose is equal to the two angles that these two lines with the previous lines enclose—as the angle *QBA* here is equal to the angles *UQB* and *UAB*—then the line *QU* is equal to the line *BU*. Therefore, the angle *QBU* is equal to the angle *UQB*. And it was proved that the angle *UQB* is equal to the angle *QRK*, so the angle *QBU* is equal to the angle *QRK*. So the angle *QRK* from the quadrilateral *RQBX* with the angle *QBX* from it is equal to two right angles. So the angles *RXB* and *RQB* remain equal to two right angles. And the angle *RQK* is right. So the angle *RXU* remains [equal] to a right [angle]. And that is what we wanted to prove.

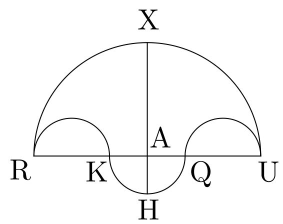
13. If there is a circle, two lines are drawn in it, one of which is *RU* and the other is *KQ*, the line *RU* is the diameter of the circle, *KQ* is a line that does not pass through the center, and perpendiculars *UX* and *RA* are drawn from the points

R and U, then I say that they cut off two equal segments from the line KQ, which are KA and XQ.



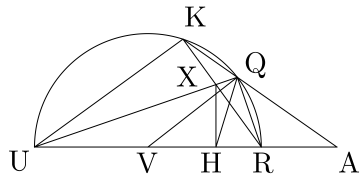
We join the line AU, we make the point H the center, and we draw a perpendicular, on which are HB, from it to KQ. And let us extend it to the point I on the line AU. Since the line HB passes through the center, and intersects KQ which does not pass through the center at right angles, it bisects it. So the line KB is equal to the line BQ. Since the angle HBK is right, and the angle RAQ is right, the line HI is parallel to the line RA. But the line RH is equal to the line HU. So AI is equal to IU, and the line AB is equal to the line BX. And it was proved that the whole of KB is equal to the whole of BQ. So the line KA remains equal to the line XQ. And that is what we wanted to prove.

14. If RU is a semicircle, the lines RK and UQ are equal, semicircles are constructed on the lines RK, KQ and QU, the semicircles on the equal lines RK and UQ are constructed inside the semicircle RU, the semicircle on the line KQ is constructed outside it, the center of the semicircle RU and the semicircle on the line KQ, which is constructed outside it, is the point A, AX is perpendicular to RU, and it is extended to the point H, then the circle whose diameter is equal to XH is equal to the surface the smaller semicircles and the greater semicircle enclose—and this is the figure Archimedes names the *sālīnūn* (σάλινον), because it resembles the shape of the *s'ln* [?].



Since the line QK was bisected at A, and KR was added to it, the squares of the lines QR and RK are equal to twice the squares of the lines QA and AR. But the line XH is equal to the line QR. So the squares of the lines XH and RK are twice the squares of the lines QA and AR. And since the line RU is twice the line RA, and the line KQ is twice the line QA, the squares of the lines RU and QK are four times the squares of the lines RA and QA. And twice these squares are equal to the squares of the lines XH and RK. So the squares of the lines RU and KQ are twice the squares of the lines XH and RK. Therefore, the circles whose diameters are the lines RU and KQ are twice the circles whose diameters are the lines XH and RK. So the semicircles on which are RU and KQ are equal to the circles whose diameters are the lines XH and RK. But the circle whose diameter is RK is equal to the semicircles QU and RK. So if we remove these two common semicircles, the figure the four semicircles RU, RK, KQ and QU enclose—and which Archimedes names the *sālīnūn* (σάλινον)—is equal to the circle whose diameter is XH. And that is what we wanted to prove.

15. If RU is a semicircle, the line RK is a chord of a pentagon inside it, the arc RK is bisected at the point Q, the line KQ is joined, it is extended so that it falls at the point A, the line QU is joined so that it intersects the line KR at X, and the perpendicular, on which are XH, is drawn from the point X to RU, then I say that the line AH is equal to half of the diameter of the circle.



We join the line KU, and we make the center V. And we join the lines QV, QH and QR. Since the angle RUK is an angle whose base is the side of a pentagon, it is two fifths of a right [angle]. And each one of the angles KUQ and QUR are a fifth of a right angle. And the angle QVR is twice the angle QUR, so it is two fifths of a right [angle]. Since the angle QUR is equal to the angle KUQ, and the angle RKU is equal to the angle XHU—since they are both right [angles]—and the side XU is common, the side KU is equal to the side UH. The line UQ is also common. And the angles KUQ and QUR are equal. So the angle UKQ is equal to the angle UHQ. And each one of them is equal to six fifths of a right [angle]. But the angle UKQ is equal to the exterior angle of the quadrilateral $UKQR$ in the circle, which is the angle QRA. So the angle QRU remains equal to the angle QHR. Therefore, the line QR is equal to the line QH. Since the angle V is two fifths of a right [angle], and the angle QHU is six fifths of a right angle, the angle VQH remains equal to two fifths of a right [angle]. Since the angle RQA is an exterior angle of a quadrilateral inscribed in the circle, and it is the opposite of the angle RUK, it is two fifths of a right [angle]. So the angle VQH is equal to the angle AQR. And the angle ARQ is equal to the angle VHQ. And the side QH is equal to the side QR. So the side AR is equal to the side HV. And the line RH is common. So the line AH is equal to the line RV, which is half of the diameter. And that is what we wanted to prove.

V Index

Greek Words:

Personal Name:

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