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# Scientific and Archaic Arabic Numerals: Origins, Usages and Scribal Traditions of the Two *Abjad* Systems

Johannes Thomann

*University of Zurich*
*Institute of Asian and Oriental Studies*

**Abstract**

There were different methods used for the notation of numbers in the premodern Islamic World. Besides writing down numbers in plain words, the most common way to write numbers in running text, Indian numerals were in common use, and many studies have been devoted to them. Less studied is the system of writing numbers by letters of the Arabic alphabet, the so-called *abjad* numerals—sometimes called *jummal* (or *jumal*) numbers in the sources. There existed two *abjad* systems, generally referred to in scholarly literature as the "eastern" and the "western" systems. The focus of the present study is the origins of these systems. It will be shown that originally they were not regional variants but the result of historical processes over time. The system previously called "western," is, in fact, the older of the two. It will be called the "archaic" system. It is attested in manuscripts of the Qur'ān produced between 703 and 705 CE in Wāsiṭ, in eastern Iraq. The system previously called "eastern" was the result of the project to translate Syriac and Greek astronomical texts into Arabic during the first half of the 9th century CE, in combination with a sound shift of sibilants in the Arabic language. It will be called the "scientific" system. This article provides editions of the earliest technical descriptions of the scientific *abjad* system, and of a late document, which attests the survival of the archaic system in Egypt.

## I Introduction

Grammar books on the Arabic language often show the numerical values of the letters in a table of the Arabic alphabet. In his *Grammaire arabe*, Sylvester de Saçy placed the values of the Arabic letters in a separate column of his table of the Arabic alphabet:

Immediately following the table, he gives a short explanation (Sylvestre de Saçy 1810, 10, English translation by the author):

> The letters of the Arabic alphabet were not always arranged in the order that they are today; the Arabs have preserved for us the memory of an older order, and the value

that they give to their letters when they are used as numerals confirm the existence of this order, which they call *abūjad*, very similar to the way we call the alphabet ABC. In order to remember more easily the letters of the alphabet and their order, they divided them into eight fictional and senseless words, as follows:

ابْجَدِ هَوَزِ حُطِيَ كَلَمُنَ سَعْفَصْ قُرِشَتْ ثَخُذْ ضَظُغْ

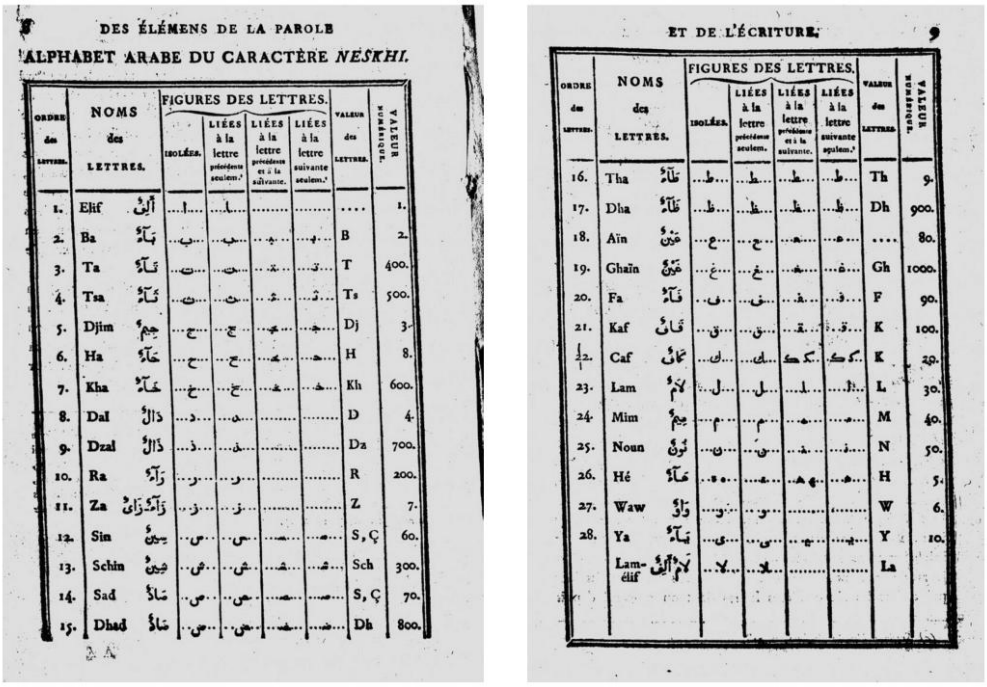



**Figure 1**: The Arabic alphabet table of Sylvestre de Saçy (1810)

In later grammar books, the *abjad* values appear regularly in tables of the Arabic alphabet but generally without further explanation (Caspari 1876; Wright 1896–1898; Fischer 2002).

A modern usage is the pagination of the front matter in printed books. Similar to English books, where page numbers of the front matter are written in Roman numerals, Arabic books may use the Arabic letters as numerals. An early example is an Ottoman printing of *Dekayik-ı ḥarbiye*, published by Muṣṭafā Ḥāmī Paşa (d. 1878 CE), where the first eight pages are numbered in *abjad* letters (Karatay 1956, 272). Otherwise, where numerical notation is needed, Hindu-Arabic numerals are used.

If the 28 Arabic letters are ordered according to their numerical values, a sequence of the letters arises that is quite different from that used in dictionaries and indices. Based on the four first letters, this system is called *abjad*, analogous to the English ABC. The first nine letters have the numerical values of one to nine, the next nine letters have the values of ten to ninety, and the remaining letters, the values of one

hundred to one thousand. Any number from one to one thousand can be written with three letters, or less, by additive composition of the single numerical values. There exist also prescriptions how to write larger numbers, but in practice Hindu-Arabic numerals were used for large numbers.

| | | | | | |
|---|---|---|---|---|---|
| 1 | ا | 10 | ي | 100 | ق |
| 2 | ب | 20 | ك | 200 | ر |
| 3 | ج | 30 | ل | 300 | ش |
| 4 | د | 40 | م | 400 | ت |
| 5 | ه | 50 | ن | 500 | ث |
| 6 | و | 60 | س | 600 | خ |
| 7 | ز | 70 | ع | 700 | ذ |
| 8 | ح | 80 | ف | 800 | ض |
| 9 | ط | 90 | ص | 900 | ظ |
| | | | | 1000 | غ |

Little information is available on the origin of the *abjad* numerals. According to the most comprehensive study on the history of numerical notation it is said to have "originated around 650 AD, at or shortly after the time of the early Islamic conquests of Syria, Egypt, and Mesopotamia" (Chrisomalis 2010, 162–163). The second *abjad* system, called "western," with a different encoding of the numbers 60, 90, 300, 800, 900 and 1000, is said to have been developed "somewhat later than that used elsewhere, probably in the ninth century AD" (Chrisomalis 2010, 162). It will be shown that this is wrong, and that the so-called "western" system is the older of the two.

## II Ḥabash al-Ḥāsib on *Abjad* Numerals

Ḥabash al-Ḥāsib (d. after 869 CE) belonged to the younger generation of astronomers who worked for al-Maʾmūn in Baghdad and Damascus. His astronomical tables are preserved in two manuscripts containing different versions of the text.

### II.1 Edition and Translation

Ḥabash al-Ḥāsib, *al-Zīj al-Dimashqī*, MS Istanbul, Süleymaniye Kütüphanesi, Yeni Cami 784, f. 74r (MS Berlin Staatsbibliothek, Wetzstein I 90 does not contain this section; Debarnot 1987).

وذلك أنّ أهل الحساب جعلو يخرج أجزائهم من ستين وسمّوها دقائق وجعلو كل دقيقـ[ة] ستين ثانية وكذلك إلى السوادس والسوابع وإلى حيث ما انتهت الكسور وعلى هـ[ذا] المثال جعلنا الجدول في كتابنا وجعلناها ما ثبت فيه من الحساب لحروف الجمل وهي حروف ا ب حـ د وهذه صورتها هـ

| ز | و | ه | د | جـ | ب | ا |
|---|---|---|---|---|---|---|
| سبعة | ستّة | خمسة | أربعة | ثلثة | اثنين | واحد |
| ن | م | ل | ك | ي | ط | ح |
| خمسين | أربعين | ثلثين | عشرين | عشرة | تسعة | ثمانية |
| ش | ر | ق | ص | ف | ع | س |
| ثلثمائة | مائتين | مائة | تسعين | ثمانين | سبعين | ستّين |
| غ | ظ | ض | ذ | خ | ث | ت |
| ألف | تسعمائة | ثمان مائة | سبعمائة | ستّمائة | خمسمائة | أربعمائة |

فهذه الحروف المتقدمة تثبّت الأعداد في الجدول واما لجميع الحساب من الجذاول وعند الضرب والقسمة فعلى مثال ما وجدنا الذين عالجوا الحساب قبلنا نستعمل التسعة الأحرف الهندية وهذه صورتها هـ

| منزلة العشة كالواحد | تسعة | ثمانية | سبعة | ستّة | خمسة | أربعة | ثلثة | اثنين | واحد |
|---|---|---|---|---|---|---|---|---|---|
| ١٠ | ٩ | ٨ | ٧ | ٦ | ٥ | ٤ | ٣ | ٢ | ١ |

[...] The calculators undertake to produce their parts out of sixty, and call them minutes. They make each minute as sixty seconds, and similarly up to sixths and sevenths, and up to where the fractions end. Correspondingly we established a table in our book, and show them to make them clear in the calculation with the *jummal*-letters. They are ʾA B J D, and these are their forms:

| | | | | | | |
|---|---|---|---|---|---|---|
| ʾA | B | J | D | H | W | Z |
| One | Two | Three | Four | Five | Six | Seven |
| Ḥ | Ṭ | Y | K | L | M | N |
| Eight | Nine | Ten | Twenty | Thirty | Fourty | Fifty |
| S | ʿ | F | Ṣ | Q | R | Sh |
| Sixty | Seventy | Eighty | Ninety | Hundred | Two hundred | Three hundred |
| T | Th | Kh | Dh | Ḍ | Ẓ | Gh |
| Four hundred | Five hundred | Six hundred | Seven | Eight hundred | Nine hundred | Thousand |

These letters above determine the numbers in the tables. However, for each calculation of the tables and for multiplication and division, we use the nine Indian letters following the example of those who made the calculation before us. These are their forms:

| One | Two | Three | Four | Five | Six | Seven | Eight | Nine | Position of ten like one |
|---|---|---|---|---|---|---|---|---|---|
| ١ | ٢ | ٣ | ٤ | ٥ | ٦ | ٧ | ٨ | ٩ | ١٠ |

## II.2 Commentary

Ḥabash calls these numerals *ḥurūf al-jummal* (or *al-jumal*). Khalīl ibn Aḥmad (d. between 777 and 791 CE) mentions the expression *ḥisāb al-jummal* in his *Kitāb al-ʿAyn*, explaining it as "what is cut according to the letters of the ABC" (*mā quṭiʿa ʿalā l-ḥurūfi Abī Jād*), and next he explains that the word *jummal* means "thick rope" (*al-qals al-ghalīẓ*), but gives no further clarification (Beirut 1988, VI.143). However, Ibn Durayd (d. 321 AH/933 CE) in his *Jamharat al-lugha* says "What concerns *al-jummal* in the domain of calculation, I do not regard it as really Arabic" (*wa-ammā al-jummal mina l-ḥisābi fa-lā ʾaḥsibu-hū ʿarabiyyan ṣaḥīḥan*) (Beirut 1987, I.491). In the *Lisān al-ʿArab* of Ibn Manzūr (d. 1311 CE), we read: "*Ḥisāb al-jummal*, with doubling of the *mīm*: The letters, cut according to the ABC. Ibn Duray said: I do not regard it as Arabic. Someone said: It is *ḥisāb al-jumal*, without doubling.

Ibn Sīda said: I am not sure of it" (Beirut 1955–1956, XI.128a). Obviously *ḥisāb al-jummal* (or *al-jumal* according to others) was considered to be an odd expression. A possible explanation would be that Arabic *jummal* (or *jumal*) was derived from Syriac *jumal*, the name of the third letter, *g*, of the Syriac alphabet (Payne Smith and Payne Smith 1903, 72a; Payne Smith 1879–1927, I.736b). Similarly, the lexicographer Abū ʿAmr al-Shaybānī (d. between 205/820 and 213/828 CE) called his Lexicon, the first ordered by the modern alphabet, *Kitāb al-Jīm* ("Book of the *jīm*"). This might have been an abbreviation for "Book of the ABC" (Haywood 1960, 95). Ḥabash says that in his work he used *abjad* numerals for the numerical tables, and indeed, most of his tables show numbers written with *abjad* numerals. But after the table of the *abjad* numerals Ḥabash immediately goes on to describe the Hindu-Arabic numerals, which he refers to as "the nine Indian letters" (*al-tisʿa al-aḥruf al-hindiyya*). He explains that he used them for multiplication and division and gives examples for these operations. Moreover, Ḥabash preferred Hindu-Arabic numerals for writing calendar years in the tables (Thomann 2013b, 545–546). But generally, *abjad* numerals were also used for calculation with large numbers in sexagesimal notation (Berggren 2016, 46–56).

## III Other Early Sources of the Scientific System of *Abjad* Numerals

Ḥabash was not the first scholar to use *abjad* numerals side-by-side with Hindu-Arabic numerals. Some decades earlier in the *Mumtaḥan zīj* of Yaḥyā ibn Abī Manṣūr (d. 830 CE), the same usage is noted, both in the Escorial manuscript and in a Leipzig manuscript, discovered recently by Benno van Dalen (Dalen 2004, 21, fig. 1; 29, fig. 2; Sezgin 1986; Vernet 1956). Despite the fact that the manuscripts were produced centuries later, it can be said with some confidence that in 9th century Baghdad *abjad* numerals were the dominate system for writing numbers in astronomical works. This is corroborated by a number of 9th century astrolabes. The oldest known astrolabe was kept in the Iraq Museum in Baghdad, but has disappeared (King 2004–2005, II.403–433). The published images are blurred and make reading difficult, but it is clear that the numbering of the degrees was made with *abjad* numerals (Faransīs and al-Naqshbandī 1957, 12–13, fig. 2 and 3). David King suggested that it was made at the end of the 8th or beginning of the 9th centuries. There are other such 9th century astrolabes existing (King 2004–2005, II.458–484). The most recent discovery is an astrolabe in a private collection, signed by Nasṭūlus (see Figure 2; King 2012). *Abjad* numerals were used for numbers of degrees on its scales up to the number 90 (Thomann 2014b, 60 fig. 3). The *abjad* numerals used on these astrolabes agree in their values with the system described by Ḥabash and used by Yaḥyā ibn Abī Manṣūr.

More information on the use of *abjad* numerals in calculation is found in al-Uqlīdisī's *Fuṣūl fī l-ḥisāb al-hindī* ("Chapters on the Indian calculation," written 341

**Figure 2**: An astrolabe signed by Nasṭūlus

AH/952 CE in Damascus). The book is devoted to the use of the Indian numerals and their use in calculation. However, in book II, chapter 20 he explains the use of sexagesimal numbers in degrees, minutes, seconds and thirds, after having explained addition and subtraction with Indian numerals. Then he says (Saidan 1978, 169 (translation); Saidan 1985, 225 (Arabic)),

> If we draw that by *jummal* letters, it would be better and far from ambiguity (*abʿad al-iltibās*). Thus we draw these lines as follows:
>
> حـ نو مد له
> أ يز لح مط
>
> Next we add the lower to the upper, if we want that, or deduct the lower from the upper. We start increasing and decreasing from any side we like.

That is, al-Uqlīdisī explicitly states that *jummal* letters are better suited for writing sexagesimal numbers since they are less ambiguous. Indeed, in the *abjad* system numerals from ten to ninety are represented by Arabic characters that are connected to the following letter. Therefore, all combined numbers from 11 to 99 consist of two connected letters: يا يب يج ... صط.

If a sexagesimal number (degrees, minutes, seconds, thirds, and so on) were to be written with Indian numerals on one line there would be a high risk of mistaken groupings. This seems to be the reason why al-Uqlīdisī writes them in columns, when he gives examples in Indian numerals (Saidan 1978, 265 (translation); Saidan 1985, 336 (Arabic)):

> For example, we want to add one degree, thirteen minutes, 27 seconds, 49 thirds to two degrees, 28 minutes, 45 seconds, 54 thirds. We draw that as follows:

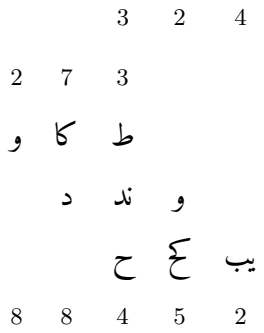
| | |
|---|---|
| 1 | 2 |
| 13 | 28 |
| 27 | 45 |
| 49 | 54 |

This system of notation is convenient for calculations, but in running texts and in tables it would be cumbersome. This corresponds to al-Ḥabash's statement that *abjad* numerals are used in tables and Indian numerals in calculations. According to al-Uqlīdisī, the use of *abjad* numerals was not limited to sexagesimal calculation. In Book IV of his *Fuṣūl* he goes on to explain methods for calculation with ink, pot and paper, not with the dust board (*takht*) as before. In these methods, *abjad* and Indian numerals are used in the same calculation (Saidan 1978, 168 (translation); Saidan 1985, 223 (Arabic)):

> For example we want to multiply 324 by 273; we draw them as

| | | |
|---|---|---|
| 3 | 2 | 4 |
| 2 | 7 | 3 |

> Now we multiply three by two and insert the product under the two by *jummal*, like this:




Similarly, we multiply three by seven and by three and insert that below. It becomes shown. Now we multiply the two by the whole line and insert the products below that; that will be shown. We add, and after addition it becomes 88452.

Obviously, the connected form of *abjad* letters avoids confusion regarding position. However, it remains unclear, whether these methods were common practice at the time of al-Uqlīdisī, and whether they were used later on.

Ghiyāth al-Dīn al-Kāshī (d. 1429 CE) mentioned no such mixed methods in his *Miftāḥ al-ḥisāb* ("Key of calculation"), but included a special chapter to cover calculation with *abjad* numerals. It is significant that he calls it "Calculation method of the astronomers/astrologers" (*ṭarīqat ḥisāb al-munajjimīn*) (Dimirdāsh and al-Ḥifnī 1967, 103–128). This suggests that the real home base of the *abjad* numerals was the sphere of astronomers and astrologers, and that is confirmed by a statement of al-Bīrūnī, to which we now turn.

## IV Al-Bīrūnī on the Two *Abjad* Systems

The eminent scholar Abū Rayḥān al-Bīrūnī (973–1048 CE) wrote an introduction to astronomy and astrology, the *al-Tafhīm li-awā'il ṣinā'at al-tanjīm* (Wright 1934, 140–142, no. 116–117; Mūsā 2003, 46–47). It contains an introduction to calculating with *abjad* numerals in which the two systems are discussed (George 2009, 94).

### IV.1 Text and Translation

هل اختلف فيها

الغرض في هذه الحروف هو اختصار في اثبات الأعداد في الجدول ومن أولى الجداول هم المنجّمون ولم اختلفوا فيها بينهم وإنّما خالفهم من ليس هذه صناعة فجعل مكان سعفص صعفض وحعل الضاد غيرب المعجمة ستين والمعجمة تسعين وجعل مكان قرثت قرست غير معجخة السين وإنما حدا بعضهم على ذلك أبنية للكلام لغويّة وبعضهم تأويلات لأغراض في اعتقاد وهو كالهذيان و لو أطبق المستعملين على هذا الارتيب لجوّزنا خلافهم ولكنّه خروج عن العادة

> Is there disagreement about [*abjad*] numerals?
>
> The intention in these letters is shortening (*ikhtiṣār*) in entering numbers into a table. And those who are concerned with tables (*man awlā l-jadāwil*) are the astronomers (*al-munajjimūn*), and they do not dispute (*lam ikhtalafū*) about [these letters] among themselves. But some whose discipline is not [astronomy] are opposed, and at the place of SʿFṢ they put ṢʿFḌ, and they make the *ḍād* without dot [the sign of] sixty, and [the *ḍād*] with dot [the sign of] ninety. At the place of QRŠT they put QRST [with] a *sīn* without dot. However, some of them insist (*ḥadā*) on that [because of] the linguistic structure of speech (MSS: *abniyya li-l-kalām lughawiyya*/*abniyyat al-kalām lughawiyya*), and some [because of] explanations for the intentions in belief (*ta'wīlāt li-'aghrāḍ li i'tiqād*). But this is like raving (*hadhayān*). If there would [be] an agreement of the users on that order [of the letters], we would accept their opposition, but [in fact] it deviates from [common] usage.

## IV.2 Comments

One can only speculate on the linguistic arguments against the scientific *abjad* order. One reason could be that SʿFṢ was felt to be impossible as an Arabic word, since emphatic *ṣād* and non-emphatic *sīn* do not occur in the same Arabic root, even if in the verbal stem X they can occur in the same word. However, the same is true for *ṣād* and *ḍād*; it could be argued that *ṣād* and *ḍād* are both emphatic and therefore phonetically compatible. The other arguments against the scientific *abjad* order seem to belong to the domain of religion. The term *i'tiqād*, usually translated by the words "croyance," "belief" is not a simple "opinion," but a bond established by contract with a great measure of firmness (Gardet 1974, 279). It is unclear what the word means in this actual context. Early on, the *Tafhīm* was translated from Arabic into Persian. Therefore it is useful to compare the phrase in the Persian version (Humā'ī 1983, 53).

> وماننده این از جهت حدیثهای لغت یا مذهبها کردند
>
> They made the like because of accounts of language, or [because] of the [different] religious creeds.

The term *madhhab* (pl. *madhāhib*, "religious creed") is not exclusively, but predominantly religious. It can mean the theological or legal schools, or the sects in Islam. In the canonical Sunni hadith collections, nothing is found on the topic of *abjad* numerals (Wensinck 1936–1988, 1), but this topic does appear in the Shiite tradition. One account, reported under the authority of the fifth Imam of the Imamites, Muḥammad al-Bāqir, narrates an episode in the life of Jesus (Beirut 1983, 2, 317). When Child Jesus was brought by his mother to the scribes he was first

asked to pronounce the Basmala. After which, he asked about the *abjad* and for each letter he was able to pronounce a religious formula: *ālā' Allāh, bahjat Allāh, jamāl Allāh, dīn Allāh.* In this way he continues with *hawwaz*, *ḥuṭṭī*, *kaliman*, *sa'faṣ*, and *qarshat.* However, since *the series* of letters is identical with that in the scientific system up to 400, it does not help to explain al-Bīrūnī's remark on the use of the archaic system by a religious group.

## V The Use of *Abjad* Numerals for the Encoding of Chronograms

Since ancient times, there existed a tradition in Iranian literatures of including riddles in literary works (Winfuhr 2009, 312–314). From the 10th century CE, it became a custom in Persian poetry to include a chronogram indicating the year in which the work was produced. At first, the year was spelled out in words. In the 11th century CE poets began to encode the year by the sum of the numerical values of the letters in one or two words according to the *abjad* system. The presence of a chronogram was usually indicated by a word like "date" (*tārīkh*) or "year" (*sāl*) (Winfuhr 2009, 327–328). The poet Khāqānī-i Shirwānī (d. July 1199 CE) was a virtuoso in creating complicated encoded chronograms. In a poem in honor of the Shawānshāh Akhsītan I (ruled 1160–1179 CE) he composed the following verse (Sajjādī 1994, 33):

فتح تو به جنگ لشکر روس  تاریخ شد آسمان قران را

Your victory in the battle with the army of the Russians
became the date of the heaven of conjunction

The catchword "date" (*tārīkh*) points to a chronogram, but the following words do not provide a year within the rule of Akhsītan I (555–575 AH). But if the grapheme آسمان (*āsmān* "heaven") is split into اسم آن and read *ism-i ān* ("the name of that/those"), one gets a hint that a second word belongs to the chronogram: namely, *rūs* ("Russians") in the first half of the verse. The letters شد (300 + 4) and روس (200 + 6 + 60) result in the sum 570, corresponding to the year 1174 CE. Khāqānī made a play on words with the two components of the chronogram. Read together (*rūs shud*) they can be understood as "the Russians departed" or "the Russians perished" (Vil'čevskij 1969, 99). The expression "heaven of conjunction" (*āsmān-i qirān*) points to the astrological doctrine of planetary aspects, which were thought to be the indicators of successful events (Barszgar Khāliqī 2008, 216).

There were many more devices in use to make the riddles difficult, and chronograms in Persian poetry remained a constant feature. It was not before the 15th

century CE, however, that similar chronograms were created in Arabic poetry. In the Ottoman empire, encoded chronograms were frequently present both in poetry and inscriptions (Colin 1967; Blair 1998, 221–222).

It is important in our context here to point out that the scientific *abjad* system was also used in chronograms. In the example given above, the value 60 was encoded by the letter *sīn*.

## VI Documentary Evidence for *Abjad* Numerals

In fields other than astronomy *abjad* numerals are rarely found. There seem to be no *abjad* numerals in inscription from the 7th to the 10th centuries CE (Kalus 2017). In that period, year numbers are always written in plain words. *Abjad* numerals in papyri are rare. An early example in which numbers are represented by Arabic letters is a Greek papyrus with one Arabic line. The line "Grain of 4 *faddān* 8 *irdabb*" in Arabic is a translation of the Greek line below "there are for the grain of 4 *arourai* 8 *artabai* of grain" (Morelli 2001, 76–79, Tav. 8 no. 15; Ruska 1917, 40; Karabacek 1894, 154 no. 605). The Greek number signs δ and η are represented with the corresponding Arabic letters د and ح. This is not sufficient proof that a fully developed system of *abjad* numerals existed. It might have been an attempt to express the Greek numerals pedantically in Arabic script. At best it testifies the existence of an alphabetic order in which *ḥā'* was in the eighth position. A correspondence of Greek and Arabic letters is documented in a papyrus document of which Petra Sijpesteijn is preparing an edition (Karabacek 1894, 223 no. 830.). It might have served as a *Schreibvorlage*, and contains the letters of the *abjad* sequence up to *mīm* with the corresponding Greek. These letters correspond to the numerical values from 1 to 40. However, there is no explicit statement that the letters represent numbers. A similar list exists for the correspondence of Hebrew and Arabic letters on a papyrus (Sirat 1985, 93; Karabacek 1894, 223 no. 829). It contains the entire Hebrew alphabet with the Arabic equivalents. The final form of the Hebrew letters are included. This makes it doubtful whether the list was intended as a list of numerals. However, the *abjad* sequence in Arabic is documented up to the letter *shīn*. Actual use of *abjad* numerals seems to be attested on a papyrus document edited by Grohmann (Grohmann 1938–1943, IV.103–104; Grohmann 1952, 146; Ruska 1917, 40; Karabacek 1894, 200–201 no. 761). According to his reading an addition of the numbers appears on one line and the result two lines below. However, Grohmann's readings are doubtful and the proposed numerical values of the letters do not correspond to the known system. Furthermore, the sequence from higher to lower numerals in the notation, is broken. As papyri are concerned, there does not seem to exist a single document that would prove clearly the use of *abjad* numerals for practical purposes. This changes when paper documents are considered. The earliest dated example is an ephemeris for the year 931/932 CE (Thomann 2015b).

The table is filled with *abjad* numerals following the known system. In other astronomical documents of the 10th and early 11th century, the use of *abjad* numerals is standard. Further astronomical documents exist with *abjad* numerals from the years 954/955 CE and 1026/1027 CE (Thomann 2013a; Thomann 2014a).

In purely mathematical works the scientific *abjad* system is used by preference. In the translation of Euclid's optics, which was probably made before 864 CE, the proposition numbers are *abjad* numerals, and proposition sixty is labeled with *sīn* (Kheirandish 1999, I.213). In the Arabic translation of the Arithmetics of Nicomachus of Gerasa made by Thābit ibn Qurra (826–901 ce), *abjad* numerals are used according to the scientific system, both in the text and the tables (Kutsch 1959, 46, 90, 98). However, at one occasion Hindu-Arabic numerals appear as the numerical values below diagrams with circles in a quadratic order (Kutsch 1959, 72). In one and the same table, *abjad* numerals are used up to 256, and Hindu-Arabic Numerals from 324 to 924 (Kutsch 1959, 64). The editions of these texts mentioned are based on later manuscripts, and therefore, the numerical notation might have been changed in the scribal tradition. Some of the oldest known mathematical manuscripts transmitting Euclid's *Elements*, date from the late 10th century: Tehran Malik 3586 and Tehran Danishgah 2120.

In the 10th century CE, *abjad* numerals began to be used in disciplines other than the mathematical sciences. In the alchemical corpus of Pseudo-Jābir, the letters of the *abjad* system are frequently used in his so-called system of balance (Kraus 1942–1943, II.187–316; magic square II.73 n. 1). In works on magic and alchemy, *abjad* numerals are used in magic squares and in these the letter sequence was used to form magic names (Beirut 1957, I.109; El-Bizri 2012, 159, 143 (Arabic); Winkler 1930, 47; Ahrens 1922, 1917). During this same period, the mathematical aspect of magic squares became a topic in treatises by mathematicians, in which *abjad* numerals were used (Sesiano 2017). Later, *abjad* numerals were used more generally. Folios and quires of manuscripts were marked with *abjad* numerals. This is the case in a manuscript written in 969 CE by the mathematician al-Sijzī (MS Paris BNF arabe 2457). However, *abjad* numerals never superseded Hindu-Arabic numerals and remained a notation system used in a few specialized tasks (Gacek 2009, 214).

Turning to the beginnings of the *abjad* system, the question of its origin must be addressed. In most ancient writing systems, special symbols were used for number notation. Roman numerals provide a typical example of how most ancient writing systems encoded numbers (Chrisomalis 2010, 109–116). In that context it was an outstanding innovation when, in the 6th century BCE, the Greeks began to use letters of the alphabet as numerals (Chrisomalis 2010, 134–147). In order to reach the value of 900, three archaic letters were added to the standard alphabet. In the 2nd century BCE, the system was adopted in Hebrew inscriptions (Chrisomalis 2010, 156–160). Much later in the 3rd century CE, alphabetic numerals came to be used in Syriac documents (Chrisomalis 2010, 160–162). This was a long process, in which older number signs were totally superseded by the new system of alphabetic numerals.

It is known from historical sources that during the reign of ʿAbd al-Malik (785–805 CE) Arabic was introduced as the new administrative language of the Islamic empire. For Egypt this process is well documented by administrative documents on papyrus. While the Greek language disappeared towards the end of the eighth century, Greek numerals remained in use up to the 14th century CE in the same form as they were written in earlier Greek and Coptic documents (Grohmann 1954, 101). As for the eastern territories, where Pahlavi was the former administrative language, one can only guess which numerals were used in the early Arabic administration. The few documents from Khurasan and Central Asia do not contain numerals, but numbers are written out in words (Khan 2007). It is difficult to decide if Pahlavi numerals were used in early Arabic administrative documents in the East, or the archaic System of *abjad* numerals. In analogy to Egypt, the use of Pahlavi numerals seems more likely, since they were still used in Abbasid times in Pahlavi works written in Pahlavi book script (Chrisomalis 2010, 89–91).

| | Arabic | | Greek | | Syriac | |
|---|---|---|---|---|---|---|
| 90 | ص | **Ṣād** | Ϙ | Qoppa | ܨ | **Ṣādē** |
| 100 | ق | **Qāf** | Ρ | Rho | ܩ | **Qop** |
| 200 | ر | **Rā**ʾ | Σ | Sigma | ܪ | **Rēsh** |
| 300 | ش | **Shīn** | Τ | Tau | ܫ | **Shīn** |
| 400 | ت | **Tā**ʾ | Υ | Ypsilon | ܬ | **Taw** |
| 500 | ث | **Thā**ʾ | Φ | **Phi** | ܢ | Nūn |
| 600 | خ | **Khā**ʾ | Χ | **Chi** | ܣ | Semkat |
| 700 | ذ | Dhāl | Ψ | Psi | ܥ | ʿĒ |
| 800 | ض | Ḍād | Ω | Omega **Chi** | ܦ | Pē |
| 900 | ظ | **Ẓā**ʾ | Ϡ | **Sampi** | ܨ | **Ṣādē** |

Since Arabic *abjad* numerals appeared in astronomical texts, Greek and Syriac are the most likely candidates as models for the Arabic system. A comparison of the higher numerals reveals that from 90 to 400 Arabic differs considerably in the phonetic values from Greek, but agrees closely with the Syriac. However, from 500 to 900 the Arabic sequence agrees as far as possible with the Greek phonetic values. Therefore, it seems that the Arabic system is a hybrid and was formed in two stages.

The study of Syriac treatises on the astrolabe might have been a first stage. The only known early Syriac text on the astrolabe is that by Severus Sebokht (d. 666 CE) (Nau 1899; Neugebauer 1949). In this work, 18 of the 22 numerals up to 400 are

found. This might have been the basis for the Arabic system of alphabetic numerals found subsequently in astronomical works.

Later, when Greek works with astronomical tables were translated into Arabic, the numbers from 500 to 900 were added to the Arabic *abjad* system, as evidenced in the work of Ḥabash al-Ḥāsib. The phonetic similarity of the pairs *khā'/chi*, and *ẓā'/sampi* and—less closely—*thā'/phi* show that distribution of the remaining letters of the Arabic alphabet was related to the order of the Greek letters rather than to the order of the Syriac letters. This happened at a time when Hindu-Arabic numerals were already in use, since the translation of Indian astronomical text preceded that of Greek texts by half a century.

## VII The Chronological Order of Indian Numerals and Scientific *Abjad* Numerals

It has been claimed that the use of *abjad* numerals in science was earlier than the use of Indian numerals (Sesiano 2017; Ifrah 2000). However, this assumption was made without reference to source material. In fact, there are some indications that point to the contrary position. In the early zījes of Ibn Abī Manṣūr and Ḥabash al-Ḥāsib—still preserved—both systems were use and served different purposes. The *abjad* numerals were primarily used for sexagesimal notation in the tables, and the Indian numerals for calculations with large numbers.

The earlier zīj of al-Khwārizmī is only available in a Latin translation of a revised version, and is therefore an unreliable source. Indian numerals, however, were certainly used throughout in his arithmetical work. In his algebra, numbers in the running text were written out in words, but in the diagrams, numbers are always written with Indo-Arabic numerals (Rashed 2009, 113, 121, 219, 227, 235). Obviously, these were his preferred compact numerical notation. The only direct witness for his use of numerals in astronomy is a treatise on the use of the astrolabe, preserved in an early manuscript, bound together with anonymous treatises on astronomical instruments (Charette and Schmidl 2004). There, numbers are almost exclusively written with Indian numerals. There are only three exception (Charette and Schmidl 2004, 117 § 4b, 122 §§ 26c and 28d). Only the numerals with the same value in both systems occur. They might have been based on the archaic system, or possibly, they were later casual scribal interventions.

Earlier astronomical works of the 8th century CE did not contain numerical tables, since the Sanskrit and Pahlavi originals from which they were translated had no tables either (or at least, such tables were not integral parts of the scribal tradition). The main purpose for the use of *abjad* numerals in later works did not yet exist, while Indian numerals were known in the region at least since the time of Severus Sebokht.

# VIII The Archaic System of *Abjad* Numerals

All examples presented here so far agreed with the scientific system. The archaic system differs from the scientific in the position of six letters.

| | | Scientific | Archaic |
|---|---|---|---|
| س | Sīn | 60 | 300 |
| ص | Ṣād | 90 | 60 |
| ش | Shīn | 300 | 1000 |
| ض | Ḍād | 800 | 90 |
| ظ | Ẓāʾ | 900 | 800 |
| غ | Ghayn | 1000 | 900 |

The position for the number 60 is the most frequently attested. In the scientific system 60 is encoded by the letter *sīn*, in the archaic system by the letter *ṣād*. Modern scholarship has generated little documentation of the archaic system, the so-called "western" system.

The archaic system is, however, documented by early Andalusian astrolabes. A 12th century drawing in a Latin codex was made from a 10th century Andalusian astrolabe (King 2004–2005, II. 382–382 fig. 9.2b; Kunitzsch 1998). On its outer rim, degrees are indicated in *abjad* numerals in steps of five degrees. After *mīm, mīm hāʾ, nūn, nūn hāʾ* follows *ṣād* for the number sixty, and *ṣād hāʾ* for sixty five. At the top, *ḍād* is used for the number ninety. This agrees with the archaic system. This system is also attested in two astrolabes made by Muḥammad ibn al-Ṣaffār (d. 1035 CE) in al-Andalus. The older, Edinburgh NMS T. 1959.62, was made in 417 AH/1026 CE in Cordoba (King 2004–2005, II.895). On the rim of the rete, degrees are indicated in steps of five degrees running from 5° to 360°. *Ṣād* indicates 60, *ḍād* 90 and *sīn* 300 in agreement with the archaic system. The other, Berlin SB Sprenger 2015, with date 420 AH/1029 CE, was also made in Cordoba and shows the same encoding of *ṣād*, *ḍād* and *sīn* for 60, 90 and 300 (King 2004–2005, II.894–895 #116 fig. 15 and 943 fig. 10.4). There are further examples of this encoding on Andalusian astrolabes from the second half of the 11th century CE. An astrolabe by Ibrāhīm ibn al-Saʿīd, Oxford MHS Inv. 55331, has the date Shawwāl 460 AH/August 1068 CE, and was made in Toledo (King 2004–2005, II.388 #118). It has the same encoding as the astrolabes of Ibn al-Ṣaffār. The same applies to the astrolabe made by Muḥammad ibn Saʿīd al-Ṣabbān in Guadalajara, Oxford MHS Inv. 52473, dated 474 AH/1081 (King 2004–2005, II.936–937 #2527).[1]

[1] Later, the archaic system remained in use in *maghribī* manuscripts until the 19th century (Van

The archaic system is well documented by early Andalusian astrolabes, but it was also known in other regions of the Islamic world. Ibn al-Nadīm (d. 995 or 998 CE) reports the following in his account on the origins of Arabic writing (Flügel 1871–1872, I.4, l.20–22; Dodge 1970, I.7 (translation); Sayyid 2009, I.10 l.9–11):

> I have read what was written in the handwriting of Ibn Abī Saʿd in the following form (*ʿalā hādhā l-ṣūra*) and construction (*wa-bi-hādhā l-iʿrāb*): *Abjād, Hāwar, Ḥāṭā, Kalammān* [or *Ḥāṭī, Kalamān* (Sayyid 2009, I.10 l.10)], *Ṣāʿ, Faḍ, Qarasat*. They say that they were foreign peoples who, while camping with ʿAdān ibn Iyād [or Udad (Sayyid 2009, I.10 l.11)] and the like became Arabized and formed the Arabic writing, but it is Allāh who knows.

The seven names of the "foreign peoples" (*al-jibill al-ākhar*) contain the Arabic consonants in the archaic *abjad* sequence up to the letter *tāʾ*, but this is not explicitly mentioned, and no reference to the use as numerals is made. The account is cited by Ibn al-Nadīm after an autograph of ʿAbdallāh ibn ʿAmr ibn ʿAbd al-Raḥmān ibn Bishr ibn Hilāl al-Anṣārī (d. 274 AH/887 CE) (Sayyid 2009, I.10 n. 4 identification). He or his family came from Balkh, but he lived in Baghdad. He was known as a trustworthy transmitter of *akhbār* (Maʿrūf 2001, 11.204–205, no. 5097). According to his entry in the *Fihrist*, he wrote books on grammar, poetry and religious topics (Sayyid 2009, I.ii.334). His book on poets was a source in Abū l-Faraj al-Isfahānī's *Kitāb al-Aghānī* (Sezgin 1975, 95, 542). He was a younger contemporary of Ḥabash al-Ḥāsib, who also lived in Baghdad and wrote the first technical description of the use of the scientific system of *abjad* numerals. But Ibn al-Nadīm knew the scientific system of *abjad* numerals as well. In a chapter on Indian writing he gives a concordance between Indian numerals and the letters of the Arabic aphabet in the order of the scientific *abjad* system (Flügel 1871–1872, I.18–19; Dodge 1970, I.34–35 (translation); Sayyid 2009, I.42–43; Köbert 1978). Both, the archaic and the scientific systems were known contemporarily at the same place, but belonged to different traditions, the archaic system to *akhbār* on ancient Arabia, and the scientific system to works based on Greek astronomy.

A similar account as that in the *Fihrist*, is found in al-Masʿūdī's *Murūj al-dhahab* (ch. 47), but there the forms of names encode the scientific *abjad* system: *Abujad, Hawaz, Ḥuṭī, Kalamun, Saʿfaṣ, Qurishat* (Barbier de Meynard and Pavet de Courteille 1861–1877, III.302; Pellat 1962–1997, II.444; Pellat 1966–1979, II.281; Azamoosh 2008). The names are said to belong to kings, who lived in ancient Arabia. In contrast to Ibn al-Nadīm, al-Masʿūdī explicitly mentioned the fact that the names encode the Arabic alphabet (Barbier de Meynard and Pavet de Courteille 1861–1877, III.302 lines 2–7; Pellat 1962–1997, II.444; Pellat 1966–1979, II.281):

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den Boogert 1989; Mechehed 2007).

> The letters of *jummal* are according to the names of these kings. They are the 24 letters based on that the calculation of *jummal* operates. Other things have been said about these letters, different from whatsoever was mentioned in accordance with what we have provided in this book.[2] This our book is not the place for what people said and carried on a dispute concerning their interpretation and intention.

Similar accounts are found in *al-ʿIqd al-farīd* by Ibn ʿAbd Rabih (d. 940 CE) and in the anonymous Persian *Yawāqīt al-ʿulūm* (al-ʿAryān 1953–1954, IV.212; Dānish-pazhūh 1966, 195; Azamoosh 2008). Later, these were copied in many encyclopedic works and dictionaries. However, in reporting the sequence of letters in accordance with the archaic system of *abjad*, the account of Ibn al-Nadīm is isolated. It would remain of doubtful value for the early history of the Arabic alphabet, if Michael Macdonald would not have pointed out that a similar sequence of letters is documented in pre-Islamic Arabian inscriptions (Macdonald 1986). The South Safaitic alphabet from Khirbet al-Samrāʾ contains, in spiraloid disposition, the letters in Figure 3 (Macdonald 1986, 108 drawing).


**Figure 3**: South Safaitic Alphabet

Despite some irregularities—such as the initial l, which would be expected at the place after *k*, and *r* being placed before ʿ—the sequence is characteristic of that found later in the archaic *abjad*—particularly, the place of *ṣ* after *m* and *n* (Macdonald 1986, 108). This location of *ṣ* corresponds to the placement of *semkath* in the Aramaic order of the alphabet, which represented the sound [s]. It has been argued that *ṣ* "was considered to be, by some at least, to be the nearest sound available to transcribe [s]" (Macdonald 1992, 161). The letter *s* represented a sound closer to [sh] than to [s] (Kogan 2011, 70, Churchyard 1967, 313). In this interpretation, the archaic *abjad* system corresponds to the Aramaic letter-order at a time before the

[2] The French translation of the sentence differs: "Il a été dit beaucoups d'autres choses à propos de ces lettres, comme nous l'avons fait remarquer dans cet ouvrage." The Arabic text reads *wa-qad qīla fī hādhihi l-ḥurūfi ghayru mā dhukira mina l-wujūhi ʿalā ḥasabi mā qaddamnā fī hādhā l-kitābi.*

sound shift in the Arabic language, while the scientific *abjad* system corresponds to the Syriac letter-order after its sound shift, when *s* was pronounced [s] and could be identified with *semkath*.

The Aramaic order of the alphabet goes back to one of the earliest alphabet systems in Ugarit (14th/13th centuries BCE). There, tables with the thirty cuneiform signs representing the consonants were found. One example is shown in Figure 4 (Naveh 1987, 30).

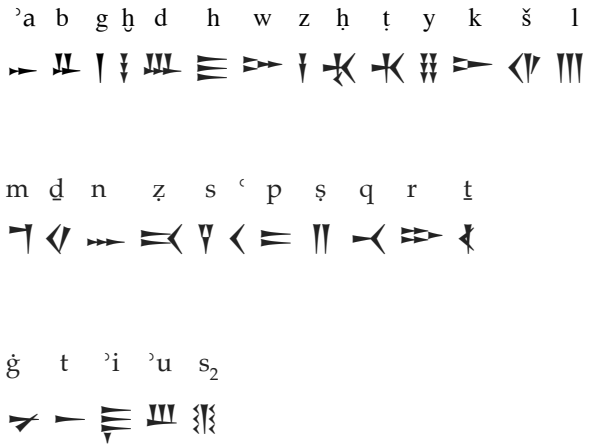


**Figure 4**: Ugaritic Alphabet (Naveh 1987, 30)

The order of the letters is the same as that in the Aramaic alphabet, except for the letters ḫ, š, ḏ, and ẓ, which have no correspondence in Aramaic. However, in Ugaritic, these letters did not have the function of numerals, but special number signs were used instead. This holds also for Aramaic. In Nabatean inscription, signs were used that somehow were derived from Aramaic, and they were also found in pre-Islamic Arabic inscriptions (Chrisomalis 2010, 80).

## IX Numbering of Verses in Early Qurʾān Manuscripts

The logic of the explanation presented in the previous section is compelling, but documentary evidence is far from convincing. It seems more than adventurous to build a bridge from one single Old North Arabian inscription of the 4th century CE (at the latest) to an Andalusian astrolabe of the 10th century CE (at the earliest) (Macdonald 2000, 45; Kunitzsch 1998). However, there is more documentary evidence to support the pillars in this fragile construction.

Alain George has already pointed out, that the so-called "western" system was not restricted to the Muslim west, and therefore its occurrence in a Qurʾān manuscript cannot be a proof of its western origin (George 2009, 92–95). His main argument was the presence of this system in a copy of the Qurʾān which probably predates the Muslim conquest of Spain. More information on this copy, known as the Codex of Fusṭāṭ, will be given in a moment.

Fragments and codices of the Qurʾān in Ḥijāzī script are among the earliest evidence of Arabic book culture. Recent carbon datings show that some of these were produced in the 7th century CE. Even if produced elsewhere, they are likely to preserve elements of a scribal tradition from the Ḥijāz region. A feature of these earliest Qurʾān manuscripts is that the dividers between verses used *abjad* numerals at the beginning of every fifth verse. Even if these numerals were added by later hands, the similarity of form in the different copies suggests a regular practice. A common characteristic among these numberings is the fact that the archaic system of *abjad* numerals is used. The MS Tübingen UB Ma VI 165 has been dated, with a high degree of probability to between 649 and 675 CE (Strotmann 2014; Youssef-Grob and Jocham 2017). On four pages the number 60 is encoded by the letter *ṣ* (Tübingen, UB, Ma VI 165 f. 16v, f. 27v, f. 37r and f. 58r).

**Figure 5**: The so-called Fustat Qurʾān (MS Paris BNF arabe 330 c f.15r)

Another early Qurʾān MS is the so-called Codex Parisino-Petropolitanus. Parts of it are kept in four libraries: Paris BNF arabe 328, St Peterburg BNR Marcel 18, London Khalili Collection KFQ 60, and Vatican BAV Vat. ar. 1605/1 (Déroche 2009, 21). A later hand has added numbers for the verses—eight times the number 60 encoded by the letter ṣ, and seven times the number 90 encoded by the number *ḍ*: P 7v, 14a, 24r, 37v, M 6v, P 48r and 54v with *ṣ* for 60; P 4r, 9r, 16r, 25v, 39r, 42r and 49r with ḍ for 90 (Déroche 2009, 49). In other MSS the numbering by *abjad* numerals was made concurrently with other dividers of the verses (Déroche 2004, 240). The use of *abjad* numerals in the archaic system for the number of verses occurs also in later copies (Altıkulaç 2007, 14 line 14 facsimile part).

While the date of the numbering of verses in the earliest codices remains unclear, it can be determined with some precision in the case of the so-called Fustat Qurʾan (George 2010, 74–89; Déroche 2014; Déroche 2018). Numbering the verses was an innovation at the beginning of the 8th century as part of the so-called Umayyad

canonization project (Hamdan 2006, 149–161; Hamdan 2010). It was initiated by al-Ḥajjāj, the governor of the eastern part of the Islamic empire (d. 714 CE). In the year 702 CE he took Wāsiṭ in eastern Iraq between Kūfa and Baṣra as his residence. In 703 CE he ordered a group of leading Qurʾān scholars from Baṣra to come to Wāsiṭ in order to carry out the task of producing a new edition of the Qurʾān. They were all non-Arabs (*mawālī*), most probably of Iranian descent. The leading scholar was al-Ḥasan al-Baṣrī, a Persian who was enslaved during the conquest of eastern Iraq, and sold to a wealthy woman in Medina. In his youth, al-Ḥasan al-Baṣrī served in the army and went as far east as Kabul, where he could have had some encounters with the still living Buddhist tradition of learning and Indian book culture, a circumstance which will turn out to be of some significance for the Qurʾān project.

The biographies of the other members of the team in Wāsit are less known. But Baṣra, their place of intellectual activities was the maritime door to the east. The Qurʾān canonization project included a reform of othography, tending towards a disctinction in writing between short and long vowels, a standardized style of writing, and a more rigorous structuring of the text. At that time, the division of the suras into sections of five verses (*takhmīs*) or of ten verses (*taʿshīr*) was already a standard practice in teaching. In the so-called Fustat codex we have a copy of the Qurʾān which was produced in Wāsiṭ under the supervision of al-Ḥajjāj and sent to Fustat. Parts of it are preserved in Paris (BNF arabe 330 c) and in St. Petersbourg (Marcel 11, 13, an 15). At the end of a section of ten verses, the number of verses is written with *abjad* numerals according to the archaic system. At the end of verse 60 of sura XIV, a *ṣād* is written in gold (BNF arabe 330 c f. 15r). It cannot be a later addition since the distance between the last word of verse 60 and the first word of verse 61 is four times as wide as the average distance between words and two times as wide as the widest distance otherwise found on the page.

The numbering of verses is a feature which was used in ancient Greek book production in the form of stichometry for poetic texts (Ohly 1928). In prose texts either lines were numbered or small syntactic units (colometry), but this was done occasionally and was not the standard way to display text (Birt 1882, 178–182). The most striking parallel to the numbering of verses are Sanskrit inscriptions. Verses were regularly separated by markers and their numbers in numerals were put at the end of a verse (Plofker 2009, 306; Chhabra and Gai 1981, 203–220 no. 1, 220–224 no. 2. 255–257 no. 11, 257–259 no. 12, 261–266 no.14, 296–305 no. 28, 305–308 no. 29, 312–317 no. 31, 319–321 no. 33, 321–322 no. 34, 322–332 no. 35, 332–334 no. 36, 334–335 no. 37, 335–339 no. 38, 339–341 no. 39, 342–344 no. 40, 352–354 no. 41, 352–354 no. 43; Mirashi 1977, 8–17 no. 4, 17–28 no. 5, 28–35 no. 6; Mirashi 1963, 82–88 no. 19, 103–111 no. 25, 112–119 no. 26, 120–129 no. 27; Salomon 1998, 276–280 no. 8, 281–284 no. 9, 284–296 no. 10, 296–302 no. 11). The final position of the numerals is also common in the Qurʾān manuscripts. Indian inscriptions were found as far west as the coast of the Red Sea in Quṣayr al-Qadīm and prove the presence of

Indian writing culture close to the Arabian Peninsula (Salomon 1991). Furthermore, in Manichaean hymn-cycles in Parthian the cantos are numbered (Boyce 1954, 24). Without more extensive studies of numbering systems in Mediterranean and Asian manuscript traditions it remains doubtful if verse numbering in Qur'ān manuscripts was based on an earlier model. The use of the archaic *abjad* numerals in early Qur'ān MSS proves that in early Islamic times this form of numerical notation was in use in the region of the Ḥijāz, or at places where the Ḥijāzī scribal tradition was alive. There are later examples of Qur'ān MSS with *abjad* numbers and they prove that this system was also active in the western parts of the Islamic world. In a deluxe codex produced in Palermo in the year 372 AH/982 CE, London, Khalili Collection, QUR261, QUR368, the numbers for every tenth verse are included according to the *abjad* system (Déroche 1992, 146–151 no. 81).

## X The Archaic *Abjad* Numerals in P.Vind.inv. A.Ch. 11

A paper document, presumably from the 10th century CE has been acknowledged as one of the earliest documents to contain *abjad* numerals; see Figure 6 (Karabacek 1894, 246 no. 927). However, it was not recognized that the list of *abjad* numerals followed the archaic system. It is a single leaf, and the verso shows a drawing unrelated to the text on the recto. The ink is partly faint and the writing is illegible in parts of the page.

### X.1 Edition and Translation

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | ي | ط | ح | ز | و | ه | د | جـ | ب | ا |
| | | عشرة | تسعة | ثمنية | سبعة | ستة | خمسة | أربعة | ثلاثة | اثنين | واحد |
| ت | س | ر | ق | [ض] | [ف] | ع | ص | ن | م | ل | ك |
| أربعة | ساقط | عشرة | ثمني | أربعة | ستة | ثمنية | عشرة | ساقط | أربعة | ستة | ثمنية |
| | * | عشرة | تسعة | ثمنية | لا | غ | ظ | ش | ذ | خ | ث |
| | | | | سبعة | سبعة | أربعة | ساقط | ثمنية | أربعة | ساقط | ثمنية |

* «أمّا لحرق؟ أحجر ختم بهذه الأحرف»

فتجمع اسمه واسمي « أبويه » وتسقـ[ـط] المجتمع معك اثنا عشر اثنا عشر حتّى يبقا معك اثنا عشر أو ما دونها إلى الواحد فإن بقي واحد فنجمه الحمل وإن بقي اثنين الثور وإن بقي الثلث الجوزا وإن بقي أربعة السرطان وإن بقي خمسة الأسد وإن بقي سدس السنبلة وإن بقي سبعة الميزان وإن بقي ثمنية العقرب وإن بقي تسعة القوس وإن بقي عشرة الجدي وإن بقي اثنا

عشر الحوت كل برج [بيت] لكوكب [...] طالعها الحمل والعقرب بيتا المريخ الثور والميزان بيتا الزهرة الجوزاء والسنبلة بيتا عطارد السرطان بيت القمر الأسد بيت الشمس القوس وال حوب بيتا المشتري الجدي والدلو بيتا زحل الـ[-...-]- بلب؟ القمر الأسد بلب الشمس القوس والحوت [...] والدلو [و]الزحل؟

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| ʾA | B | J | D | H | W | Z | Ḥ | Ṭ | | | |
| One | Two | Three | Four | Five | Six | Seven | Eight | Nine | Ten | | |
| K | L | M | [N] | Ṣ | ʿ | [F] | [Ḍ] | Q | R | S | T |
| Eight | Six | Four | Two | absent | Ten | Eight | Six | Four | Eight | absent | Four |
| Th | Kh | Dh | Sh | Ẓ | Gh | Lʾ | For the engraving (?) of a seal with … | | | | |
| Eight | absent | Four | Eight | absent | Four | Seven | | | | | |

You add his (i.e. the customers) name and the two names of his [parents ?] and you repeatedly subtract twelve from the sum until a value of twelve is reached, or what is less, down to one. If one remains, his sign is the Ram; if two remains, the Bull; if three remains, the Twins; if four remains, the Crab; if five remains, the Lion; if six remains, the Virgin; if seven remains, the Balance; if eight remains, Scorpion; if nine remains, the Sagittarius ; if ten remains, Capricornus; if eleven remains, Aquarius; if twelve remains, the Fish. Each zodiacal sign is a house for a planet [....] their ascendant. The Ram and the Scorpion are the two houses of Mars; The Bull and the Balance are the two houses of Venus; the Twins and the Virgin are the two houses of Mercury; the Crab is the house of the Moon; the Lion is the house of the Sun; Sagittarius and the Fish are the two houses of Jupiter; Capricornus and Aquarius are the two houses of Saturn; the [...]

## X.2 Commentary

The document contains instructions for how to derive from the name of a person and those of their parents a zodiacal sign and the corresponding planet, as if it were at the ascendant of a horoscope. The procedure is to take the sum of the values of the letters in the names. For this purpose it is not necessary to take the full values of the letters determined by the *abjad* system, but it suffices to take their reminder in a division by twelve—that is, mod 12. Therefore in the list of the *abjad* letters at the beginning of the document, the *abjad* values mod 12 are written below the names instead of the full values. When the sum of the values of the letters has been calculated, twelve is repeatedly subtracted until a value of twelve or less remains. This last number identifies one of the zodiacal signs listed in the lower part of the document. Finally, the corresponding planet is found, which is taken as the dominating planet for that person, as if it were at the ascendent of a birth horoscope. The procedure is also described in a treatise which is erroneously attributed to Abū Maʿshar and generally quoted with the title *Kitāb al-mawālīd*, *Kitāb mawālīd al-rijāl wa-l-nisāʾ*, or *Kitāb mawālīd al-rijāl wa-mawālīd al-nisāʾ* (cf. Faddegon 1928, 307; Pingree 1970, 38; Ullmann 1972, 322; Sezgin 1979, 145; MSS: Paris, Bibliothèque Nationale de France, arabe 2583 (1667 CE), f. 1v–2r; arabe 2586 (before 1720 CE),

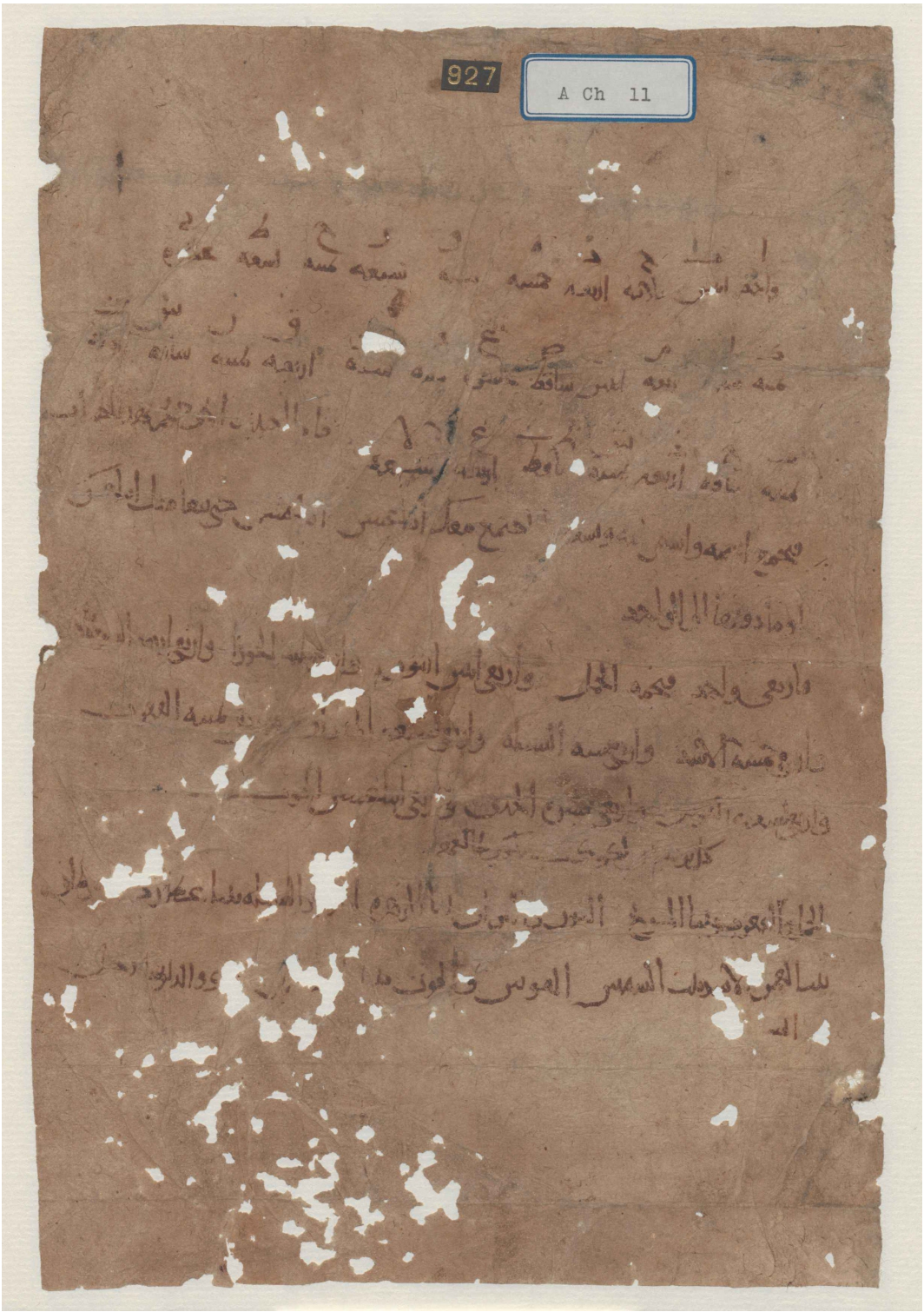


**Figure 6**: *Abjad* numerals in P.Vind.inv. A.Ch. 11

f. 1v; arabe 2587 (1678 CE) f. 3r–3v; printed edition (Cairo 1873, 9–10)). It also became known in Europe and was described by Agrippa of Nettesheim (d. 1535 CE) (Cologne 1533, 144; Compagni 1992, 307). This is one of the surrogate methods for using astrological concepts without involving astronomical calculation and without needing an astronomical ephemeris. Instead, belief in onomatomancy and gematric procedures provided the ideological basis of fortune telling. Methods for deriving astrological information from the numerical value of personal names exited already in late antiquity (Bouché-Leclercq 1899, 537–541). The table of the letters shows the order in the archaic *abjad* system as far as 700 (see the table in the edition and translation, above). Only the last three letters are ordered differently. As these letters are not used on astrolabes, it remains doubtful how the order could otherwise be established in the 10th and 11th centuries. P.Vind.inv. A.Ch. 11 is the earliest full list for archaic *abjad* letters and indirectly, their numerical values. The origin of the document is most probably Egyptian and demonstrates how in the 10th century the "archaic" system was already used in divinatory practice.

## XI Other Examples of the Bifurcation of Scribal Traditions in East and West

There are at least two more examples of the separation of originally mixed scribal traditions into geographically separate traditions. The first case concerns the forms of the Hindu-Arabic numerals. These appear both in the Arabic and the Latin traditions in two distinct shape repertoires, usually called "eastern" and "western" in modern scholarship (Ifrah 2000, 532–538). As has already been said, the history of the Hindu-Arabic numerals and their reception in Latin Europe has been the topic of many studies (Ifrah 2000, 529–541, 577–591; Burnett 2000; Burnett 2002; Kunitzsch 2003; Burnett 2006). It is not necessary to repeat their results here *in extenso*. The only point to make is that the geographical separation of the two form repertoires pertains to a later period in the history of Hindu-Arabic numerals, and must have been the result of regional standardizations. This is indicated by the earliest firm documentary source preserving their actual forms, MS Paris BNF arabe 2457. This manuscript is in the hand of the mathematician al-Sijzī, written between 969 and 972 in Shīrāz. It proves that form variants that later became part of either the "eastern" or the "western" traditions were used at that time side-by-side (Kunitzsch 2003, 5).

Unfortunately, documentary evidence for numerals from Spain and the Maghreb exists only from the 14th century CE onwards (Kunitzsch 2003, 11–15). But Latin works produced earlier in Spain indicate that the so-called "western" numerals were standard there in the 12th century CE (Burnett 2002, 241–243). In manuscripts of Syria, Iraq and Iran the standardization of the forms appears to have been completed in the 11th century CE (Irani 1955, 4).

The second case is the dotting of the Arabic letters. While there are 28 Arabic letters (or 29 if *tā' marbuṭa* is considered as an independent letter), only 15 basic letter forms exist, if they are not at the end of a word. Disambiguation was produced by adding diacritical points above and below the letters. Perhaps, this took place already in pre-Islamic times (Endress 1982, 175; Gruendler 2006, 151). Diacritical dots are found in the two earliest dated Arabic documents, dated 643 CE (Endress 1982, 175). The usage of diacritical points was not standardized until the 9th century CE. In our context, one feature is of particular interest. In the oldest texts, the letter *qāf* is marked with a dot below, in order to distiguish it from *fā'*, which remained without a dot. From the 8th century CE onwards, *qāf* had a dot above and *fā'* a dot below. This feature, sometimes called the Maghribī way, is present in the Qurra Papyri (709–710 CE) and the Inscription of the Jerusalem Dome of the Rock (692 CE) (Kaplony 2008, 95). In Egypt this feature is still found in some documents of the 9th century CE (Grohmann 1935, 439; Froschauer 2007, 106, 134, pl. 3). Later, the modern practice (one dot above *fā'*, two dots above *qāf*) is exclusively found in documents and manuscript produced in Egypt and further east. However the former usage (a dot below *fā'*, a dot above *qāf*) was retained in Spain and the Maghreb until modern times Houdas 1889. As in the case of the *abjad* numerals, an archaic feature of the writing tradition survived in the west, and died out in the other parts of the Islamic world.

## XII Conclusion

The documentary evidence provided in this article is not exhaustive. However, it allows for a broad picture of the early history of the two *abjad* systems. The so-called "eastern" system emerged in a process of scientific innovation when Syriac and Greek astronomical works were translated into Arabic at the end of the 8th or the beginning of the 9th century CE.[3] They were used side-by-side with Indian numerals. *Abjad* numerals were preferred in astronomical tables with sexagesimal numbers, while Indian numerals were used for calculation. With the spread of astronomy and other mathematical disciplines in the Islamic World, *abjad* numerals became known

[3] In an anonymous report on this article it was recommended to refer to George Saliba's "alternative narrative" according to which Greek scientific texts were translated during the reign of ʿAbd al-Malik (65–86 AH/685–705 CE) (Saliba 2007, ch. 1–2). This hypothesis was rightly criticized by Peter Pormann as not well founded and not taking into account relevant scholarship (Pormann 2010, 246, 248)—for example, despite Manfred Ullmann's deconstruction of the legend of Khālid ibn al-Walīd as an alchemist (Ullmann 1978), Saliba refers to it as a historical fact; Saliba does not consider adequately that the first translations of mathematical, astronomical, and astrological texts into Arabic were made from Sanskrit, and to lesser extent from Middle Persian texts (Pingree 1997; Van Bladel 2011, 2015; Thomann 2015a, 2017).

to those educated in these disciplines. The so-called "western" system seems based on an order of the letters established early in the history of Arabic writing in the northern part of the Arabian peninsula. In the 7th century a new scribal culture developed for production of Qurʾān codices. At an early stage, *abjad* numerals were used to number Qurʾānic verses. With the spread of Qurʾān codices and the scribal practice of production in the Islamic world, especially in the West, use of *abjad* numerals became known among persons involved in production and use of Qurʾānic codices. The documents show that both *abjad* systems were known in Egypt in the 10th century. While the "eastern" system was used for mathematical astronomy, the "western" system was used in the context of letter magic. The two contexts probably resulted from two educational systems, one based on Greek science, and the other based on traditions of the Arabs. The first is a child of the scientific Renaissance in early Abbasid times. The second is based on older traditions that were around before the Arabs became acquainted with the mathematical sciences. Questions remain open and it is not clear why the "archaic" system should become the preferred system for scientific purposes in al-Andalus. It is likely that in the future other domains, where *abjad* numerals were used, will be detected. The aim of this article is not merely the provision of positive documentary evidence, but also to indicate unexplored territories in the history of numerical notation in the Islamic world.

## Appendix: Time-Line of Sources

| CE | Spain | Egypt | Syria | Arabia | Iraq | East |
|---|---|---|---|---|---|---|
| 600 | | | | **ṣ D** Inscr. | **# H** Severus | |
| 700 | | **ṣ R** Qur. | **ṣ R** Qur. | | **ṣ R** Qur. | |
| 800 | | **s # D** P.Vind. | | | **# M** Khwāri.<br>**s # M** b.a. Manṣ.<br>**s # M** Ḥabash<br>**ṣ H** b.a. Saʿd | |
| 900 | **ṣ M** Astr. | **s M/D** Eph.<br>**ṣ D** A.Ch. 22 | | | **ṣ/s H** b. Nadīm | |
| 1000 | **ṣ M** Astr.<br>**ṣ M** Astr. | | | | | **ṣ/s M** Bīrūnī |
| 1100 | | | | | | |

### Key to the Time-Line of Sources

| | |
|---|---|
| **s** Scientific system | **D** Documentary source |
| **ṣ** Archaic system | **H** Historical source |
| # Hindu arabic numerals | **M** Mathematical source |
| | **R** Religious source |

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